CHAPTER 2

Wideband Probe-Fed Microstrip Patch Antennas and Modelling Techniques

2.1 INTRODUCTORY REMARKS

During recent years, much effort has gone into bandwidth enhancement techniques for microstrip antennas in general. As such, there is a great amount of information in the open literature and it covers a very broad range of solutions that have been proposed thus far [1]. The entire spectrum of approaches that have been suggested is too comprehensive to discuss here and therefore the discussion in this chapter will be restricted to techniques that have been applied to enhance the bandwidth of probe-fed microstrip patch antennas in particular. In this chapter, a broad overview will be given in terms of the various techniques that are currently available to enhance the bandwidth of probe-fed microstrip patch antennas. The performance, advantages and disadvantages of the most practical approaches will also be discussed. With these in mind, the new antenna element that forms the basis of this study, will be presented.

Another field in which there has been a tremendous amount of activity during recent years, is that of computational electromagnetics [15, 16]. These tools are essential for an accurate analysis and design of complex antenna elements and arrays. Although probe-fed microstrip patch antennas are structurally quite simple, an accurate analysis of their various characteristics proves to be rather intricate. This is partly due to the singular and rapidly-varying nature of the current in the vicinity of the probe-to-patch junction and also due to the presence of a multilayered substrate. This chapter will give a brief overview of the methods that are currently available, together with their associated strengths and shortcomings. Once again, with these in mind, an overview of the formulation that was implemented for the purposes of this study, will also be presented.

In this chapter, Section 2.2 gives an overview of wideband probe-fed microstrip antennas, while Section 2.3 presents the new wideband microstrip antenna elements, employing capacitive feed probes. Section 2.4 gives an overview of the modelling techniques that are currently available, while Section 2.5 presents the basic aspects of the theoretical formulation that was implemented for the purposes of this study.

2.2 OVERVIEW OF WIDEBAND PROBE-FED MICROSTRIP PATCH ANTENNAS

The impedance bandwidth of microstrip patch antennas is usually much smaller than the pattern bandwidth [17]. This discussion on bandwidth-enhancement techniques will therefore focus on input impedance rather than radiation patterns. There are a number of ways in which the impedance bandwidth of probe-fed microstrip patch antennas can be enhanced. According to Pozar [18], the various bandwidth-enhancement techniques can be categorised into three broad approaches: impedance matching; the use of multiple resonances; and the use of lossy materials. For the purpose of this overview, it has been decided to rather categorise the different approaches in terms of the antenna structures that are normally used. These include: wideband impedance-matching networks; edge-coupled patches; stacked elements; shaped probes; and finally capacitive coupling and slotted patches. In terms of Pozar's categories, all these approaches can be identified as making use of either impedance matching or multiple resonances. In practice, lossy materials are not frequently used as it limits the radiation efficiency of the antenna. It will therefore not be considered here.

2.2.1 Wideband Impedance-Matching Networks

One of the most direct ways to improve the impedance bandwidth of probe-fed microstrip antennas, without altering the antenna element itself, is to use a reactive matching network that compensates for the rapid frequency variations of the input impedance. As shown in Figure 2.1, this can typically be implemented in microstrip form below the ground plane of the antenna element. The method is not restricted to antenna elements on either thin or a thick substrates, but the thick substrate will of course add some extra bandwidth.

Pues and Van de Capelle [19] implemented the method by modelling the antenna as a simple resonant circuit. A procedure, similar to the design of a bandpass filter, is then used to synthesise the matching network. With this approach, they have managed to increase the bandwidth from 4.2% to 12% for a voltage standing-wave ratio (VSWR) of 2:1. Subsequently to that, An *et al.* [20] used the simplified real frequency technique in order to design the matching network for a probe-fed microstrip patch antenna. They have managed to increase the bandwidth of one antenna element from 5.7% to 11.1% for a VSWR of 1.5:1, and that of another from 9.4% to 16.8% for a VSWR of 2:1. Recently, De Haaij *et al.* [21] have shown how a parallel resonant circuit can increase the bandwidth from 3.2% to 6.9% for a VSWR of 1.5:1.

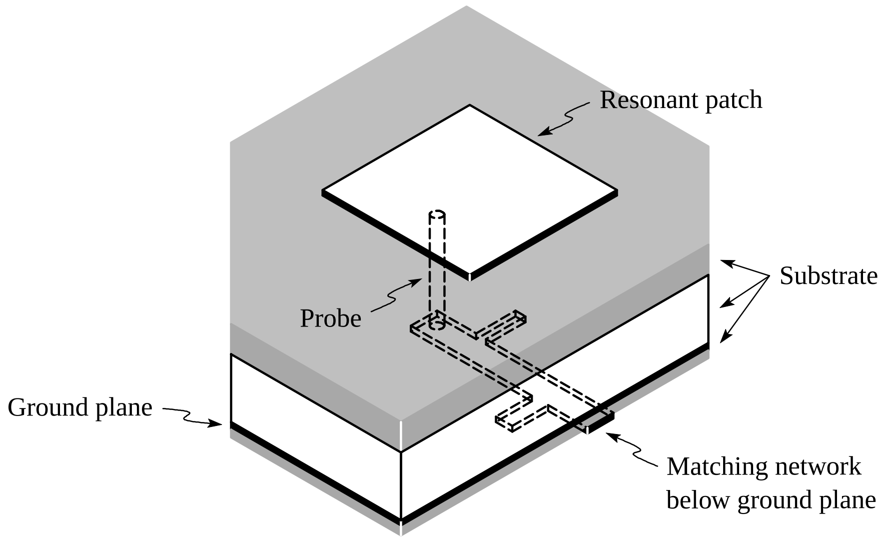

Figure 2.1 Geometry of a probe-fed microstrip patch antenna with a wideband impedance-matching network.

The advantages of using impedance-matching networks are that the antenna elements do not get altered and that the matching network can be placed behind the antenna's ground plane. As such, the radiation characteristics of the antenna element stay unchanged, while radiation from the matching network is also minimised. The drawback of this method is that the matching network can potentially take up space that is very limited when microstrip feed networks are used to excite the individual elements in an antenna array. Another drawback is that, for single-element antennas, more than one substrate layer is required to support the antenna element and the matching network.

2.2.2 Edge-Coupled Patches

The basic idea behind edge-coupled patches, is to increase the impedance bandwidth of a microstrip patch through the introduction of additional resonant patches. By doing so, a few closely-spaced resonances can be created. Only one of the elements is driven directly. The other patches are coupled through proximity effects. An example of such an arrangement is shown in Figure 2.2.

This approach has been investigated by Wood [22] as well as Kumar and Gupta [23–25]. The parasitic patches can be coupled to either the radiating edges, the non-radiating edges or to both pairs of edges. The approach in [25] uses short transmission lines to couple the parasitic patches directly to the driven patch. With the edge-coupled approach, impedance bandwidths of up to 25.8% have been obtained for a VSWR of 2:1. This was achieved with four parasitic patches coupled to the driven patch.

The advantages of the edge-coupled approach include the fact that the structure is coplanar in nature and that it can be fabricated on a single-layer substrate. However, this approach also has

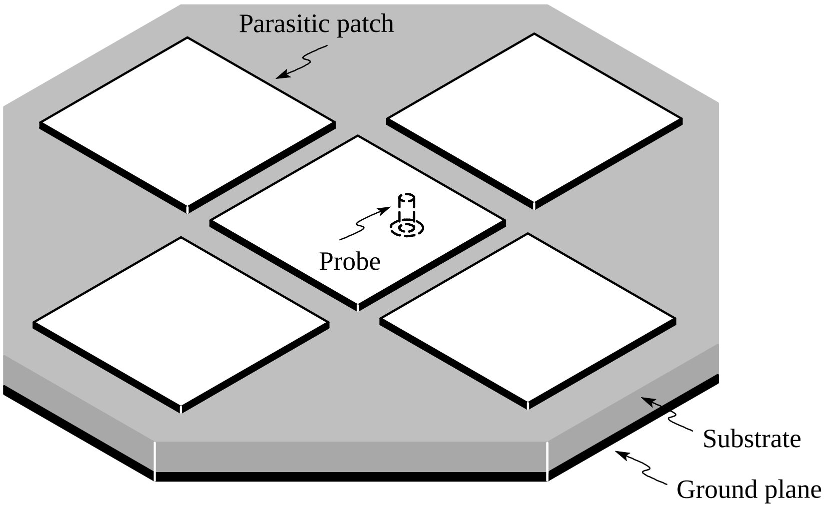

Figure 2.2 Geometry of a probe-fed microstrip patch element that is edge-coupled to the parasitic patches.

a few drawbacks. Due to the fact that the different patches radiate with different amplitudes and phases at different frequencies, the radiation patterns change significantly over the operating frequencies. The enlarged size of the structure can also be a potential handicap in many applications. For example, in phased-array applications, the large separation distances between elements can introduce grating lobes.

2.2.3 Stacked Patches

A very popular technique, which is often used to increase the impedance bandwidth of microstrip patch antennas, is to stack two or more resonant patches on top of each other [2]. As with the edge-coupled resonators, this technique also relies on closely-spaced multiple resonances. However, in this case, the elements take up less surface area due to the fact that they are not arranged in a coplanar configuration. Figure 2.3 shows the geometry of such an antenna element where the bottom patch is driven by a probe and the top patch, which is located on a different substrate layer, is proximity-coupled to the bottom one.

In practice, the patches are usually very close in size so that the generation of two distinct resonances can be avoided. Different shapes of patches can be used. These commonly include rectangular patches [26, 27], circular patches [26, 28, 29] and annular-ring patches [26, 30]. It has also been shown how a combination of shapes can be used [31, 32]. As far as impedance bandwidth goes, Waterhouse [26] reported a 26% 10 dB return-loss bandwidth for rectangular patches, Mitchell *et al.* [29] reported a 33% 10 dB return-loss bandwidth for circular patches, while Kokotoff *et al.* [30] reported a 22% 10 dB return-loss bandwidth for annular-ring patches. These bandwidths were all obtained for two patches stacked on top of each other. It is possible to stack more patches, but the performance may not be much better than with only two patches [2,18].

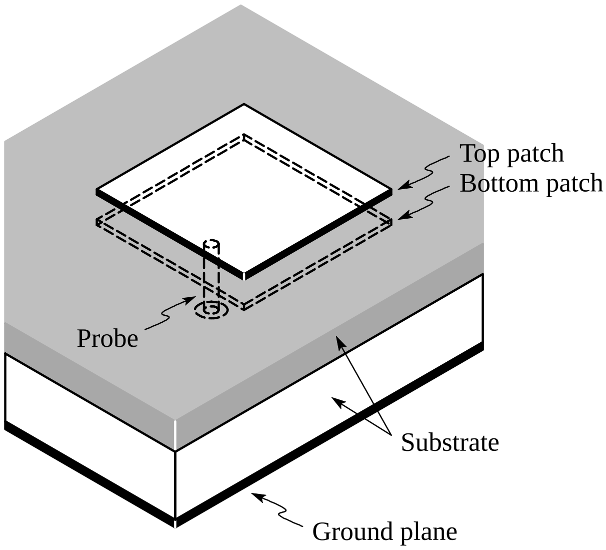

Figure 2.3 Geometry of a probe-fed stacked microstrip patch antenna.

Instead of aligning the patches vertically, some researchers have also used a horizontal offset between the patches [33]. However, due to the structural asymmetry, these configurations exhibit beam dispersion.

The stacked-patch configuration has a number of advantages over the edge-coupled configuration. Since it does not increase the surface area of the element, it can be used in array configurations without the danger of creating grating lobes. Its radiation patterns and phase centre also remains relatively constant over the operating frequency band. It has a large number of parameters that can be used for optimisation. However, due to this, the design and optimisation process can also be very complex. Another drawback of stacked patches, is that it requires more than one substrate layer to support the patches.

2.2.4 Shaped Probes

As was shown in Chapter 1, a thick substrate can be used to enhance the impedance bandwidth of microstrip patch antennas. However, the input impedance of probe-fed microstrip patch antennas become more inductive as the substrate thickness is increased. In order to offset this inductance, some capacitance is needed in the antenna's feeding structure. One way to implement such a capacitive feed is to alter the shape of the probe. There are basically two approaches. In one approach, the probe is connected directly to the patch [34, 35], while in the other approach, the probe is not connected to the patch at all [36–39].

The direct feed can be implemented as shown in Figure 2.4(a), where the feeding structure consists of a stepped probe. The horizontal part of the probe couples capacitively to the patch. Chen and Chia [34] reported an impedance bandwidth of 19.5% for a VSWR of 2:1. Another option is to

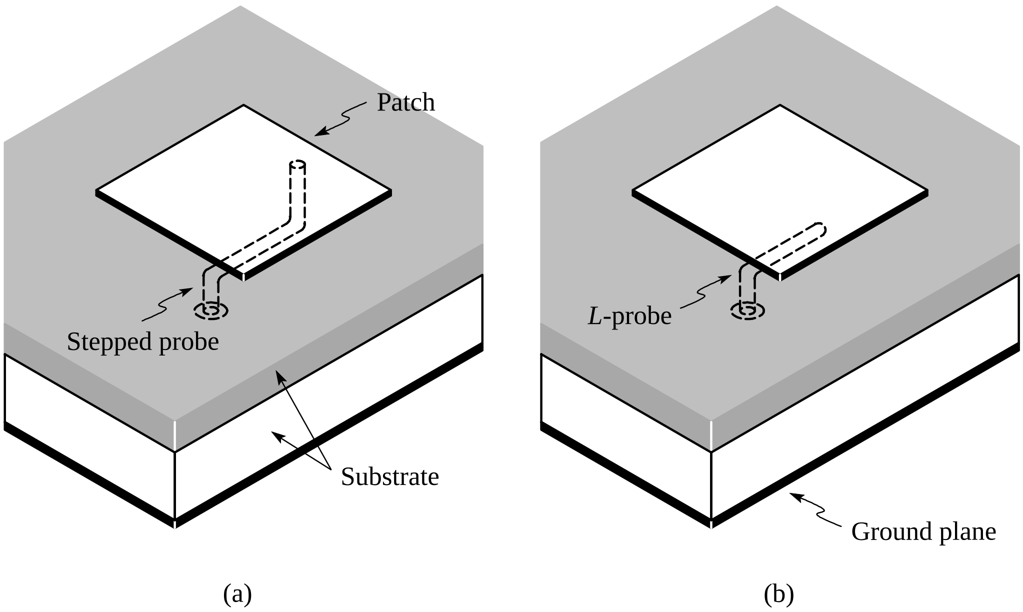

Figure 2.4 Geometries of microstrip patch antennas with shaped probes. (a) Stepped probe. (b) *L*-shaped probe.

add a stub to one of the radiating edges of the patch and to feed the stub directly with a probe. For such an approach, Chen and Chia [35] reported an impedance bandwidth of 25%, once again for a VSWR of 2:1.

The proximity-coupled probe is implemented as shown in Figure 2.4(b), where the probe is bent into a *L*-shape. The horizontal part of the probe runs underneath the patch and also couples capacitively to it. This solution has been implemented for a variety of patch shapes. Mak *et al.* reported an impedance bandwidth of 36% for a rectangular patch in [36] and 42% for a triangular patch in [39], while Guo *et al.* reported an impedance bandwidth of 27% for an annular-ring patch in [38]. These bandwidth figures were all quoted for a VSWR of 2:1. Instead of a *L*-shaped probe, Mak *et al.* [40] also used a *T*-shaped probe and managed to achieve an impedance bandwidth of 40% for a VSWR of 2:1.

A microstrip patch antenna with a shaped probe, be it directly driven or not, can usually be supported on a single substrate layer. This makes it extremely suitable for antenna arrays where cost has to be minimised. Most of these elements have radiation patterns with a slight squint in the *E*-plane and slightly high cross-polarisation levels in the *H*-plane. These are characteristics of probe-fed microstrip patch antennas on thick substrates. The stepped probe, though, exhibits somewhat lower cross-polarisation levels. The patches that are directly driven should be more robust that those with the proximity coupled probes. For the latter ones, care has to be taken with respect to the proper alignment of the pathes and probes. Another advantage of both approaches is that, since they do not increase the surface area of the element, they can be used in array config-

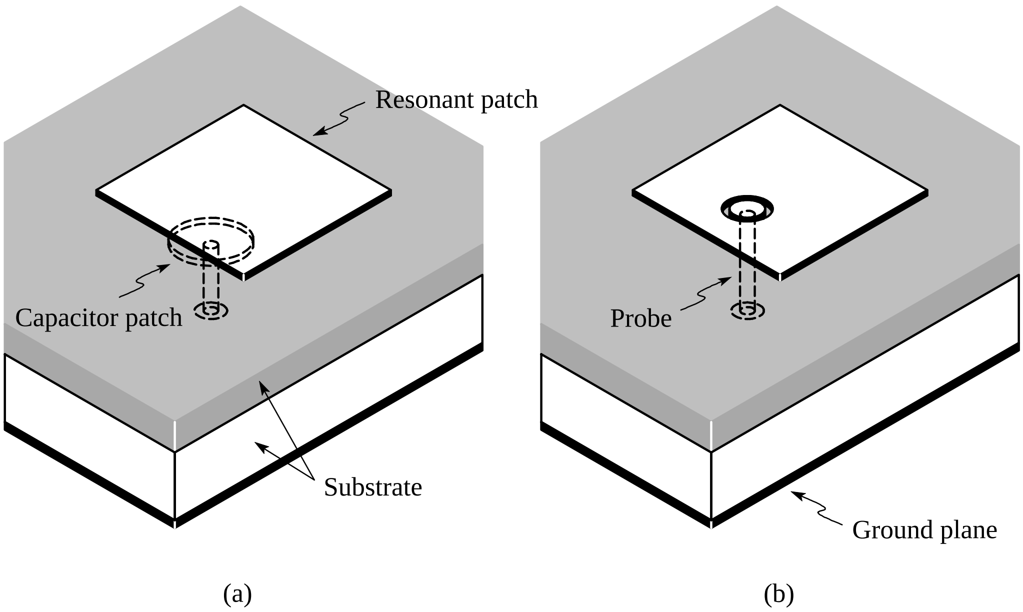

Figure 2.5 Geometries of probe-fed microstrip patch antennas where capacitive coupling and slots are used. (a) Capacitive coupling. (b) Slot in the surface of the patch.

urations without the danger of creating grating lobes.

2.2.5 Capacitive Coupling and Slotted Patches

There are two alternative approaches that can also be used to overcome the inductive nature of the input impedance associated with a probe-fed patch on a thick substrate. These are capacitive coupling or the use of slots within the surface of the patch element. Examples of such approaches are shown in Figures 2.5(a) and (b) respectively. It can be argued that these two approaches are structurally quite similar. The approach in Figure 2.5(a) has a small probe-fed capacitor patch, which is situated below the resonant patch [41–43]. The gap between them acts as a series capacitor. Similarly, the annular slot in Figure 2.5(b) separates the patch into a small probe-fed capacitor patch and a resonant patch [44, 45]. In this case, the slot also acts as a series capacitor. In principle, both of these approaches employ some sort of capacitive coupling and are functionally also, to some degree, equivalent to the *L*-probe and *T*-probe as described in Section 2.2.4.

Liu *et al.* [46] combined the capacitively-coupled feed probe with stacked patches and reported a impedance bandwidth of 25.7% for a VSWR of 2:1. To achieve this, they used two stacked patches with a small probe-fed patch below the bottom resonant patch. In another approach, González *et al.* [47] placed a resonant patch, together with the small probe-fed capacitor patch just below it, into a metallic cavity. With this configuration, they managed to obtain a impedance bandwidth of 35.3% for a VSWR of 2:1.

As far as the slotted patches go, Hall [44] showed that, for a circular resonant patch with a annular slot around a small circular probe-fed capacitor patch in the surface of the resonant patch, a 10 dB return-loss bandwidth of 13.2% could be obtained for the TM_{11} mode and 15.8% for the TM_{21} mode. Anguera *et al.* [45] used a rectangular resonant patch, also with a small circular probe-fed capacitor patch in the surface of the resonant patch, and managed to achieve an impedance bandwidth of 21.7% for a VSWR of 1.5:1. Chen and Chia [48] used a small rectangular probe-fed capacitor patch, located within a notch that was cut into the surface of the resonant patch. They managed to obtain an impedance bandwidth of 36% for a VSWR of 2:1. Some authors also used a rectangular resonant patch with a *U*-slot in its surface. The metallic area inside the slot is then driven directly with a probe. Here, Tong *et al.* [49] reported a impedance bandwidth of 27% for a VSWR of 2:1, while Weigand *et al.* [50] reported an impedance bandwidth of 39%, also for a VSWR of 2:1. In yet another approach, Nie *et al.* [51] placed a circular probe-fed patch within a annular-ring patch, with the circular patch exciting higher-order modes on the annular-ring patch. They managed to obtain a 8 dB return-loss bandwidth of 20%. Kokotoff *et al.* [52] placed a small shorted circular probe-fed patch within a annular-ring patch, but with the circular patch exciting the dominant TM_{11} mode on the annular-ring patch. They reported a 10 dB return-loss of 6.6%.

The advantage of the approach where the capacitor patch is located below the resonant patch, is that the cross-polarisation levels in the *H*-plane are lower than what can be achieved with the approach where the capacitor patch is located within the surface of the resonant patch. However, in order to support the capacitor patch below the resonant patch, an additional substrate layer might be required. In contrast, only one substrate layer is required to support the configuration where the capacitor patch is located inside the surface of the resonant patch. Furthermore, the capacitor patch below the resonant patch is prone to alignment errors and can complicate the fabrication process. On the other hand, when using a capacitor patch within the surface area of a resonant patch, there can potentially be many design parameters that can complicate the design of such antenna elements. Here also, an advantage of both approaches is that, since they do not increase the surface area of the element, they can be used in array configurations without the danger of creating grating lobes.

2.3 NEW WIDEBAND MICROSTRIP PATCH ANTENNAS WITH CAPACITIVE FEED PROBES

As was stated earlier on, the basis of this study is a new wideband microstrip patch antenna element with a capacitive feed probe. Figure 2.6 shows the general geometry of the new antenna structure. As can be seen, it consists of a rectangular resonant patch with a small probe-fed capacitor patch right next to it. Both patches reside on the same substrate layer. For this study, both circular and rectangular capacitor patches, as shown in Figures 2.6(a) and (b) respectively, were used.

For wideband applications, the two patches can be manufactured on a thin substrate with a thick low-loss substrate, such as air, right below it. The antenna element is functionally very similar to most other capacitively-coupled elements. The gap between the resonant patch and the capacitor

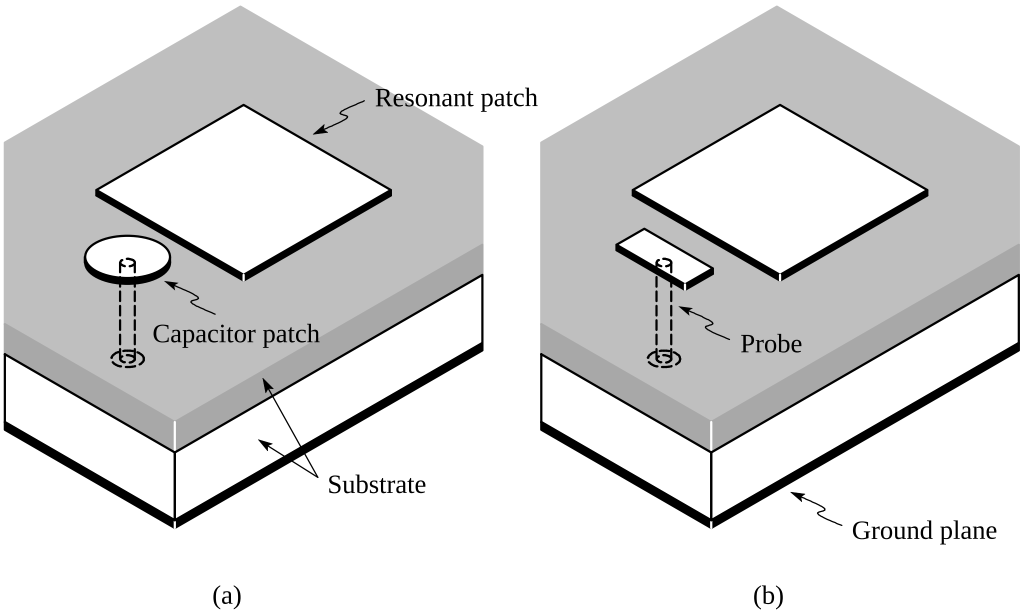

Figure 2.6 Geometries of the new wideband microstrip patch antennas employing capacitive feed probes. (a) Circular capacitor patch. (b) Rectangular capacitor patch.

patch acts as a series capacitor, thereby offsetting the inductance of the long probe. Once the size of the resonant patch and the thickness of the substrate have been fixed for a certain operating frequency and impedance bandwidth, there are basically two parameters that can be used to control the input impedance of the antenna element. These are the size of the capacitor patch and the size of the gap between the two patches. In Chapter 4, it will be shown how these parameters affect the input impedance.

Given that most of the approaches for wideband probe-fed microstrip patch antennas have been described in Section 2.2, the structural properties of the new antenna element can be viewed in context. First of all, the new antenna element can be manufactured on a single substrate layer due to both the resonant patch and the capacitor patch residing on the same layer. This is very important for large antenna arrays where laminates can be very expensive. The fact that the capacitor patch is driven directly by a probe, gives the structure some rigidity. The structure is also less prone to alignment errors, which can be a factor for antenna elements where the probe does not make physical contact with any of the patches or where the capacitor patch is located on a different layer than the resonant patch. The surface area of the element is not much larger than that of a resonant patch and therefore it is very suitable for use within antenna arrays. An advantage that might not be very obvious at first, is that the antenna element, as opposed to slotted antenna elements, consists of parts that are canonical in shape. As will be shown in Section 2.5, it has huge benefits for the analysis of such antennas, especially for large antenna arrays. Finally, the design of such an antenna element, as well as tuning of the input impedance, is very straightforward due to the few parameters that have to be adjusted. In terms of performance, it is expected that the new antenna element should be comparable to other probe-fed patch antennas on thick substrates. This will be investigated thoroughly in Chapter 4.

2.4 OVERVIEW OF MODELLING TECHNIQUES

There are a number of methods that can be used for the analysis of probe-fed microstrip patch antennas. Most of these methods fall into one of two broad categories: approximate methods and full-wave methods [1]. The approximate methods are based on simplifying assumptions and therefore they have a number of limitations and are usually less accurate. They are almost always used to analyse single antenna elements as it is very difficult to model coupling between elements with these methods. However, where applicable, they normally do provide good physical insight and the solution times are usually very small. The full-wave methods include all relevant wave mechanisms and rely heavily upon the use of efficient numerical algorithms. When applied properly, the full-wave methods are very accurate and can be used to model a wide variety of antenna configurations, including antenna arrays. These methods tend to be much more complex than the approximate methods and also provide less physical insight. Very often they also require much computational resources and extensive solution times.

In the remainder of this section, an overview of both approximate and full-wave methods will be given. Since the full-wave moment method, also known as the method of moments, is arguably the most popular method for the analysis of microstrip antennas, it will be discussed separately. The moment method can be implemented in either the spatial domain or the spectral domain. Both of these will be considered.

2.4.1 Approximate Methods

Some of the popular approximate models include the *transmission-line model*, the *cavity model* and the *segmentation model*. These models usually treat the microstrip patch as a transmission line or as a cavity resonator.

The **transmission-line model** represents the antenna by radiating slots that are separated by a length of low-impedance transmission line [4,53–57]. A good implementation of this model is the one by Pues and Van de Capelle [57]. Each radiating slot is represented by a parallel equivalent admittance. The analysis then basically boils down to normal circuit theory. This method is the simplest method for the analysis and design of microstrip patch antennas, but often yields the least accurate results and also lacks versatility. It can be used to calculate the resonant frequency and input resistance of an antenna element. With this method, it is difficult to model the coupling between antenna elements, although it has been done successfully [55,58]. The method only works reasonably well for antennas with thin substrates and low dielectric constants, while it becomes increasingly less accurate as either the substrate thickness or dielectric constant is increased [1]. The transmission-line model has mostly been applied to directly-driven rectangular patches.

Some of the drawbacks associated with the transmission-line model can be overcome with the **cavity model**. The cavity model is a modal-expansion analysis technique whereby the patch is viewed as a thin cavity with electric conductors above and below it, and with magnetic walls along its perimeter [56, 59, 60]. With this method, the electric field between the patch and the ground plane is expanded in terms of a series of cavity resonant modes or eigenfunctions, along with its eigenvalues or resonant frequencies associated with each mode. Due to the cavity being thin, only transverse magnetic (TM) field configurations, with respect to the height of the cavity, are considered. The field variation along the height of the cavity is also assumed to be constant. Furthermore, due to the fact that a lossless cavity cannot radiate and exhibits a purely reactive input impedance, the radiation effect is modelled by introducing an artificially-increased loss tangent for the substrate. The cavity model can model the resonant frequency and input impedance more accurately than the transmission-line model, but it is also limited to patch shapes for which the two-dimensional Helmholtz equation admits an analytical solution [61]. As with the transmission-line model, the cavity model also becomes less accurate as the substrate thickness or dielectric constant is increased [1]. It does not take into account the effect of guided waves in the substrate. It is difficult to model mutual coupling with the cavity model, although it has been done successfully [62, 63]. Gómez-Tagle and Christodoulou [64] even used it successfully for the modelling of stacked patches in a multilayered substrate. Microstrip patch antennas are often represented by an equivalent network model [43, 45, 65, 66]. In such a model, the values of the lumped elements can be determined by using the cavity model [67].

The **segmentation method** is more versatile than both the transmission-line model and the cavity model, especially in terms of its ability to treat patches with arbitrary shapes. It is an extension of the cavity model, but instead of treating the patch as a single cavity, the patch is segmented into sections of regular shapes. The cavity model is then applied to each section, after which the multiport-connection method is used to connect the individual sections. This method has been used, for example, by Palanisamy and Garg [68] for the modelling of a square-ring patch, while Kumar and Gupta [23–25] used it for the modelling of edge-coupled patches. As with the other approximate methods that have been described, this method also works best for thin, low dielectric-constant substrates.

2.4.2 Full-Wave Methods

Three very popular full-wave methods that can be used to model probe-fed microstrip patch antennas, are the *moment method* (MM), the *finite-element method* (FEM) and the *finite-difference time-domain* (FDTD) *method*. These are the three major paradigms of full-wave electromagnetic modelling techniques [69]. Unlike the approximate methods, these methods include all the relevant wave mechanisms and are potentially very accurate. They all incorporate the idea of discretising some unknown electromagnetic property. For the MM, it is the current density, while for the FEM and FDTD, it is normally the electric field (also the magnetic field for the FDTD method). The discretisation process results in the electromagnetic property of interest being approximated by a set of smaller elements, but of which the complex amplitudes are initially unknown. The amplitudes are determined by applying the full-wave method of choice to the agglomeration of elements. Usually, the approximation becomes more accurate as the number of elements is increased. Although these methods all share the idea of discretisation, their implementations are very different and therefore each of the three methods will now be considered in some more detail.

The **MM** is undoubtedly the most widely-used full-wave method for the analysis of microstrip antennas [69]. It is synonymous with the method of weighted residuals and was popularised by Harrington [70–72] who first demonstrated its power and flexibility for the solution of electromagnetic problems in the 1960s. This method is mostly applied in the frequency domain, where only a single frequency is considered at any one time.

When using the MM, the current density on the antenna is usually the working variable from which all the other antenna parameters are derived. The method is implemented by replacing the antenna with an equivalent surface current density. The surface current density is then discretised into a set of appropriate current-density elements, also known as basis functions (or expansion functions), with variable amplitudes. For example, these elements can take the form of wire segments and surface patches. Now, the Green's function for the problem is used to express the electric and/or magnetic fields everywhere in terms of the current-density elements on the surface of the antenna. Boundary conditions for the electric and/or magnetic fields are then enforced on the surface of the antenna by using testing functions (also known as weighting functions). This process is often called the testing procedure and results in a system of linear integral equations (IEs). This system of equations can be expressed in matrix form, where the interaction between each basis (expansion) function and each testing (weighting) function is taken into account. For most MM implementations, this matrix, also known as the interaction matrix, is usually a dense matrix. If the basis functions and testing functions are chosen to be the same set of functions, the method is known as the Galerkin method. Finally, the system of linear equations is solved to yield the amplitudes of the current-density elements. After the surface current density on the surface of the antenna has been solved for, the other antenna parameters, such as the input impedance, radiation patterns and gain, can easily be derived. For more detailed descriptions of the MM, texts such as those by Balanis [4, 73], Stutzman and Thiele [74], as well as Sarkar *et al.* [75], can be consulted.

A major advantage of the MM, when compared to other full-wave methods, is its efficient treatment of highly conducting surfaces. With the MM, only the surface current density is discretised and not the fields in the surrounding medium. Furthermore, it inherently includes the far-field radiation condition, while antennas that are embedded in multilayered media, can be simulated efficiently when using the appropriate Green's function for such a configuration. On the down side, the MM is not very well suited for the efficient analysis of problems that include electromagnetically penetrable materials. Also, while the formulations for multilayered media are very powerful, they are very intricate and complex to implement. For an efficient multilayered-media formulation, an infinite ground plane and laterally infinite layers have to be assumed. The implementation for finite ground planes and layers is much more inefficient as the substrate and ground plane also have to be discretised.

Despite some of its drawbacks, the MM is still the preferred method for frequency-domain problems that involve highly conducting surfaces [15]. For the analysis of microstrip antennas, the method can either be implemented in the so-called spatial domain or the so-called spectral domain. Due to the fact that these two implementations have been used so extensively for the analysis of microstrip antennas, they will be considered separately in Sections 2.4.3 and 2.4.4 respectively.

The **FEM** is widely used in structural mechanics and thermodynamics. It was introduced to the electromagnetic community towards the end of the 1960s [15, 69]. Since then, great progress has been made in terms of its application to electromagnetic problems. As is the case with the MM, the FEM is also mostly applied in the frequency domain. What makes the FEM very attractive, is its inherent ability to handle inhomogeneous media.

When using the FEM for electromagnetic problems, the electric field is the unknown variable that has to be solved for. The method is implemented by discretising the entire volume over which the electric field exists, together with its bounding surface, into small elements. Triangular elements are typically used on surfaces, while tetrahedrons can be used for the volumetric elements. Simple linear or higher-order functions on the nodes, along the edges or on the faces of the elements, are used to model the electric field. For antenna problems, the volume over which the electric field exists, will have one boundary on the antenna and another boundary some distance away from the antenna. The latter boundary is an absorbing boundary, which is needed to truncate the volume. One viewpoint from which the FEM can be derived, is that of variational analysis [15]. This method starts with the partial differential equation (PDE) form of Maxwell's equations and finds a variational functional for which the minimum (or extremal point) corresponds with the solution of the PDE, subject to the boundary conditions. An example of such a functional is the energy functional, which is an expression describing all the energy associated with the configuration being analysed, in terms of the electric field [76]. After the boundary conditions have been enforced, a matrix equation is obtained. This equation can then be solved to yield the amplitudes that are associated with the functions on the elements used to model the electric field. The matrix associated with the FEM, is a sparse matrix due to the fact that every element only interacts with the elements in its own neighbourhood. Other parameters, such as the magnetic field, induced currents and power loss, can be obtained from the electric field. For more detailed descriptions of the FEM, texts such as those by Jin [77], Silvester and Ferrari [78], Volakis *et al.* [79], as well as Peterson *et al.* [80], can be consulted.

The major advantage of the FEM is that the electrical and geometrical properties of each element can be defined independently [76]. Therefore, very complicated geometries and inhomogeneous materials can be treated with relative ease. This implies that the analysis of microstrip antennas with finite ground planes and layers is also possible. However, the FEM has a few weak points when compared to methods such as the MM. The fact that the entire volume between the antenna surface and the absorbing boundary has to discretised, makes the FEM very inefficient for the analysis of highly conducting radiators. Also, for large three-dimensional structures, the generation of the mesh, into which the problem is discretised, can become very complex and time-consuming.

The FEM is usually not the preferred method for the analysis of most antenna problems, but is frequently used for the simulation of microwave devices and eigenvalue problems. An interesting approach is where the FEM is hybridised with the MM. These methods are very useful for the analysis of microstrip antennas inside cavities [47, 81]. Like most other full-wave modelling techniques, the FEM has been implemented in a few commercial codes. A typical example is HFSS from Ansoft.

The **FDTD method**, which was introduced by Yee [82] in 1966, is also very well suited for the analysis of problems that contain inhomogeneous media. However, unlike the MM and the FEM, the FDTD method is a time-domain method and is not restricted to a single frequency at any one time. As compared to the MM and the FEM, the FDTD method is much easier to implement as it makes limited demands on higher mathematics [15].

The FDTD method is also a PDE-based method. However, unlike the FEM, it does not make use of variational analysis, but directly approximates the space- and time-differential operators in Maxwell's time-dependant curl equations with central-difference schemes. This is facilitated by modelling the region of interest with two spatially interleaved grids of discrete points [76]. One grid contains the points at which the electric field is evaluated, while the other grid contains the points at which the magnetic field is evaluated. A time-stepping procedure is used where the electric and magnetic fields are calculated alternatively. The field values at the next time step are calculated by using those at the current and previous time steps. In such a way, the fields are then effectively propagated throughout the grid. The time stepping is continued until a steady-state solution is obtained. The source that drives the problem is of course also some time-dependant function. Frequency-domain results can be obtained by applying a discrete Fourier transform to the time-domain results. Unlike the MM and the FEM, no system of linear equations has to be solved and therefore no matrix has to be stored. As with the FEM, the grid has to be terminated with an absorbing boundary. For more detailed descriptions of the FDTD method, texts such as those by Taflove [83, 84] and Elsherbeni *et al.* [85] can be consulted.

The FDTD method has a number of attractive features, which include its relatively simple implementation, its straightforward treatment of inhomogeneous materials, its ability to generate wideband data from a single run and the fact that no system of linear equations need to be solved.

The analysis of microstrip antennas with finite ground planes and layers is of course also possible. On the down side, however, the regular orthogonal grid that is normally used, is not very flexible. This implies that curved surfaces have to be approximated with a staircase approach. For configurations with sharp edges, such an approach may require a very fine grid and therefore many field points. As with the FEM, the fact that the volume around the antenna is discretised, makes the FDTD inefficient for the analysis of highly conductive radiators.

Like the FEM, the FDTD method is usually not the preferred method for the analysis of most antenna problems, but it is very useful for wideband systems and in situations where time-domain solutions are required. Despite of this, the FDTD method has been used for the analysis of probe-fed microstrip patch antennas [49, 85, 86] and can indeed yield very accurate results. The FDTD method has been implemented in a few commercial codes. Typical examples of these are Fidelity from Zeland Software and XFDTD from Remcom.

As has already been mentioned, the MM is by far the most widely-used method for the analysis of microstrip antennas. There are two broad implementations of this method: the spatial-domain implementation and the spectral-domain implementation. Each of these implementations has its own advantages and will now be discussed in Sections 2.4.3 and 2.4.4 respectively.

2.4.3 Spatial-Domain Moment Method

As the name indicates, the spatial-domain MM is characterised by the fact that all the entries in the interaction matrix are expressed in terms of spatial variables. As mentioned before, each entry in the interaction matrix represents the interaction between a basis (expansion) function and a testing (weighting) function. This is accomplished by using the Green's function for the problem at hand. Of course, in this case, it would be the Green's function for a grounded planarly multilayered medium. The Green's function for planarly multilayered media is available in closed form, but only in the spectral domain. Its spatial-domain counterparts are then commonly obtained by applying an inverse Fourier-Bessel transform (also known as a Sommerfeld integral) to the spectral-domain Green's function. Apart from this, each entry in the interaction matrix also contains two surface integrals. One integral is associated with the convolution between the basis function and the Green's function, in order to find the electric field due to the basis function, while the other integral is associated with the testing procedure, in order to apply the relevant boundary conditions over the support (footprint) of the testing function.

In the spatial domain, the so-called mixed-potential form of the electric-field integral-equation (EFIE), or in short, the MPIE, is usually preferred for the analysis of microstrip antennas embedded in planarly multilayered media [42, 87–95]. With the MPIE formulation, the electric field is expressed in terms of the induced current density and induced charge density through the vector and scalar potentials respectively. The MPIE is preferred due to the fact that the potential forms of the Green's function are less singular than the field forms [96]. This becomes more apparent when the interaction between two very closely-spaced basis and testing functions is calculated. The evaluation of the Sommerfeld integrals, which are required to find the spatial-domain Green's function, is usually a very laborious process due to the integrand being highly oscillatory and slowly decaying. As such, much effort has been invested into finding methods to speed up the evaluation of this type of integral. One popular approach is to integrate along the Sommerfeld integration path [97] and to use techniques such as the partition-extrapolation method to speed up the integration [98]. An approach that has become very popular recently, is to derive closed-form expressions for the Green's function [99–103]. The basic idea behind this approach is to approximate the Green's function by a sum of complex exponentials to which the Sommerfeld identity can be applied, resulting in closed-form expressions for the Green's function. However, the process is not entirely trivial as it involves the extraction of surface-wave contributions that are associated with the poles of the Green's function. Another technique that can be used with any of the aforementioned approaches, is to precompute the integrals, whereafter interpolation techniques can be used on the precomputed values.

The spatial-domain MM is well suited for the use of subdomain basis functions (i.e. basis functions that exist only over a small part of the total surface area to be modelled). This is due to the fact that the two surface integrals, which are associated with each entry in the interaction matrix, can be evaluated much more easily for a simple function over a small support than for a complex function over a much larger support. Two types of subdomain basis functions that are often used on surfaces, are rooftop functions with rectangular support [87] and Rao, Wilton and Glisson (RWG) basis functions with triangular support [104]. The RWG basis functions are very useful for the modelling of geometries with arbitrary shapes [97, 105]. Special basis functions, known as attachment modes, are required to model the current behaviour at junctions between wires and surface patches. A number of attachment modes have been developed for use within the spatial-domain MM. For some of them, the attachment point has to coincide with the corners of the surface basis functions [87, 90], while for others, the attachment point can be anywhere on the surface basis functions [89, 106–108].

Due to the use of subdomain basis functions in the spatial-domain MM, the number of basis functions that are required to model the current on a structure, increases very rapidly with the size of the structure. This implies that the interaction matrix can very easily become quite large, resulting in potentially very large computer-memory requirements for storage of the interaction matrix. For large problems, the bottleneck in the spatial-domain approach is the solution time associated with the matrix equation. It usually exceeds the time that is required to set up the interaction matrix. In this area too, many efforts have led to more efficient methods of solving the matrix equation [109]. Some of the approaches include: the conjugate-gradient (CG) method combined with the fast Fourier transform (FFT) [110–113]; the fast-multipole method (FMM) [114]; and the impedance-matrix localisation (IML) method [115].

The spatial-domain MM is the method of choice for most commercial MM codes. Typical examples of these are IE3D from Zeland Software, Ensemble from Ansoft and FEKO from EM Software and Systems. For the modelling of surfaces, IE3D uses basis functions with both rectangular and triangular support, while Ensemble and FEKO only use basis functions with triangular support.

2.4.4 Spectral-Domain Moment Method

The spectral-domain MM is characterised by the fact that the entries in the interaction matrix are expressed in terms of spectral variables rather than spatial variables [116]. This is achieved by applying a two-dimensional Fourier transform to the basis functions, the testing functions and the Green's function. In effect, the two transverse spatial variables are transformed to their spectral counterparts. These spectral variables are actually two wavenumbers that are associated with the same directions as the spatial variables. A general entry in the interaction matrix would now contain two infinite integrals over the two spectral variables. As has already been mentioned, the spectral-domain Green's function can be found in closed form [117]. However, the spectral-domain formulation do place certain requirements on the basis functions and testing functions that can be used. One of these is that the two-dimensional Fourier transforms of the basis functions should be available in closed form. Another is that the two-dimensional Fourier transforms of the basis functions should decay faster than what the Green's function grows asymptotically [118, 119].

In the spectral domain, the Green's function does not become singular for small separation distances between the basis functions. Therefore, the electric field is usually expressed directly in terms of the induced current density through the EFIE [120]. Furthermore, the spectral-domain MM is well suited for the use of entire-domain basis functions (i.e. basis functions that exist over the entire surface area to be modelled). This is due to the fact that a function with a larger support in the spatial domain, transforms to a function that decays faster in the spectral domain. An example of such an entire-domain basis function, is the set of resonant modes that would be excited on a microstrip patch [31, 121–125]. These basis functions may not be as versatile as the subdomain basis functions, but the solution can be very efficient if the antenna structure mainly consists of canonical shapes. As with the spatial-domain approach, various kinds of attachment modes have also been developed to model the current behaviour at junctions between wires and surface patches [30, 126–135] or between wires and microstrip lines [136]. In the spectral-domain, however, it might be harder to find an attachment mode that models the current density at the junction accurately and of which the two-dimensional Fourier transform can be found in closed form. Subdomain basis functions can also be used with the spectral-domain formulation [137–142]. Some codes use volumetric currents for vertical connections [140–143]. The use of such basis functions, however, place a restriction on probe lengths that can be modelled accurately.

The most computationally-intensive part of the spectral-domain MM is the evaluation of the two infinite integrals over the spectral variables for each entry in the interaction matrix. This is mainly due to the integrand becoming more oscillatory as the separation distance between the basis and testing function is increased. The integrand also exhibits poles, branch points and branch cuts that further complicate the evaluation of the integrals. The efficient evaluation of these integrals have been the topic of many research papers and, as such, various methods have been developed to deal with these integrals. One approach is to transform the spectral variables to a polar coordinate system [123,144,145]. By doing so, the two infinite integrals are transformed to one finite integral and one semi-infinite integral. Numerical integration algorithms, which take advantage of the cancellation effect that is associated with a fast oscillating integrand, can then be used to speed up the evaluation of the semi-infinite integral [146, 147]. It is also possible to use asymptotic extraction techniques [61, 125, 140–142, 148–159]. In both cases, however, it is important to realise that, for the semi-infinite integral, the contributions from the poles and branch points have to be extracted or the integration path has to be deformed so as to avoid these singularities [123, 160]. Another approach is to leave the spectral variables in a rectangular coordinate system. The separation of variables can then be used to speed up the evaluation of the integrals [161–163]. However, this method places some restrictions on the basis functions and testing functions that can be used. Another method is to use complex integration paths that dampens the oscillatory nature of the integrand [164–168]. The FFT can also be used to speed up numerical integration, but it implies that the basis functions should all be of equal size and should be spaced on a underlying rectangular grid [150, 169, 170]. Although the spectral-domain formulation has been used for the analysis of finite arrays on a uniform grid [125, 171–173], it is often used to model antenna arrays of infinite size by only analysing one unit cell of the array [129, 131, 174, 175].

When entire-domain basis functions are used with the spectral-domain MM, the interaction matrix is usually much smaller than with subdomain basis functions. In contrast with the spatial-domain MM, the bottleneck is not the solution time associated with the matrix equation, but rather the processing time that is required to set up the interaction matrix. This is due to the spectral integrations that have to be evaluated numerically.

The spectral-domain MM has been implemented in some commercial codes. Typical examples of these are Sonnet from Sonnet Software and EMSight from Applied Wave Research. These codes use subdomain basis functions on an underlying rectangular grid.

2.5 PROPOSED FORMULATION

From the previous discussion on analysis techniques, it is clear that the approximate methods are inappropriate for the modelling of the new antenna elements and also for antenna arrays that are based on these elements. This is partly due to the thick multilayered substrate as well as the fact that accurate coupling calculations between the various patches are crucial. As for the full-wave methods, the FEM and FDTD method are usually not one of the first choices when it comes to the modelling of microstrip antennas. This is of course due to the huge number of elements that is needed to discretise the region around the antennas. The MM is far more efficient for such analyses as it only discretises the surface of the antenna. As was mentioned in the previous section, the spectral-domain MM formulation is very well suited for the analysis of problems that involve canonical shapes. Canonical shapes permit the usage of entire-domain basis functions that brings about significant savings in computer-memory requirements. The new antenna elements consist entirely of canonically-shaped patches and therefore naturally lend themselves to the use of the spectral-domain MM. A brief overview of the spectral-domain MM formulation that was implemented, will now be given. The details of the implementation can be found in Chapter 3.

As was explained in Section 2.3, the bandwidth of the new antenna element is obtained by introducing a low-loss substrate, such as air, between the patch laminate and the ground plane. The analysis of such a structure calls for the use of the planarly multilayered Green's function. In the spectral domain, the EFIE, which relates the electric field directly to the current density on the antenna, is normally used. For the purposes of this study, the Green's function was calculated as reported by Chen [6] and Lee *et al.* [176]. As the currents on the antenna structure can flow both in a horizontal and vertical direction, all nine components of the dyadic Green's function are required. A point to note here is that the currents flow either in a completely horizontal direction or a completely vertical direction (i.e. there are no oblique current flow through the layers of the substrate). The analysis of such a problem is often termed a two-and-one-half-dimensional analysis.

An antenna array basically consists of antenna elements and a feed network. With probe-fed antennas, the feed network is usually situated below the ground plane and is therefore isolated from the antenna elements. If an infinite ground plane is assumed, as is the case with the planarly multilayered Greens's functions, there will be no coupling between the feed network and the antenna elements, neither will there be any radiation from the feed network. Due to this, the formulation that was implemented, only models the antenna elements and not the feed network. The feed network can take on many forms and can usually be modelled quite effectively with most full-wave electromagnetic simulators. The feed network will of course excite the antenna elements in a certain way and there will also be certain impedance-matching issues between the feed network and the antenna elements. This can, however, be handled very easily if the scattering parameters of both the antenna elements and the feed network are connected by means of network-analysis techniques.[1]

In terms of basis functions, the proposed formulation draws on the benefits of entire-domain functions, while subdomain functions are only used on small features of the antenna elements. It is assumed that the patches are infinitely thin, but that the probes have finite radii that can be specified. The electric current density on each rectangular resonant patch is modelled by a set of entire-domain sinusoidal functions. These functions originate from a cavity-model analysis of

[1] Commercial codes, such as IE3D and Sonnet, have specific modules that are capable of network analyses.

rectangular patch antennas [125]. They account for most of the memory savings that are associated with this formulation. It has been found that the current density on each circular capacitor patch, as well as the current density at the junction where it connects to the probe, can be modelled with an extension to the circular attachment mode of Pinhas and Shtrikman [127]. The same attachment mode, together with subdomain rooftop basis functions, are used to model the current density on each rectangular capacitor patch. In this study, a rectangular attachment mode [6, 7, 129, 132] was also investigated, but as will be shown in Chapter 4, it is not as versatile as the circular one. Finally, due to the probes being fairly long, the current density on each probe is modelled by piecewise-sinusoidal functions along the length of the probe. The basis functions on the probes take the probe radii into account, but it is assumed that the current density on the probes has no axial variation. The piecewise-sinusoidal functions along the length of each probe have been used instead of piecewise-linear functions that are normally used [6,7,125]. This choice ensures that the analytical integration associated with terms in the interaction matrix that contain vertical currents, are much simpler.

Due to the many different basis and testing functions that are used to model the current density on different parts of the antenna elements, the MM interaction matrix contains entries for many different types of interactions. The spectral integrations that are associated with the different types of interactions do not all behave in a similar manner and therefore different integration strategies had to be implemented. The integrations associated with entries that describe the interaction between basis and testing functions on the probes and/or circular attachment modes, can be reduced to a single integral. The integration path for this integral is slightly deformed in order to avoid any singularities. When the basis functions are laterally separated, the method of averages [146, 177] is used to speed up the numerical integration process. The integration strategy for the remaining entries depends on whether the basis functions overlap or not. For basis functions that overlap, the integrand is smooth and can usually be evaluated quite easily. In this formulation, these integrals are evaluated by transforming to a polar coordinate system. For basis functions that are widely separated, the integrand is highly oscillatory and not easy to evaluate. These integrals are evaluated in a rectangular coordinate system, where the integration path is deformed in an appropriate way so that the oscillations in the integrand are damped. This is an extension to the work of Sereno-Garino *et al.* [164–168]. Unlike most other spectral-domain formulations, the basis functions in this formulation can be arbitrarily orientated and positioned. This implies that methods such as the FFT, cannot easily be used to speed up the numerical integrations. These methods require basis functions that are aligned on a rectangular grid. Given that the basis functions are not necessarily aligned and that there are many different types of interactions, it is impractical to use asymptotic extraction techniques to speed up the integrations. In previous work, the analytical parts associated with these techniques, have only been derived for the interaction between specific types of basis and testing functions, and for the special case where they are all aligned.

Another method that was used to speed up the process of setting up the interaction matrix, is to eliminate the recalculation of any duplicate entries. As a first step in this direction, the Galerkin method was used, ensuring that the interaction matrix is symmetric. For the remaining entries, special algorithms were developed to identify all duplicates before the interaction matrix is actually generated. Due to the different types of basis functions and their arbitrary orientations, this is not a trivial process. However, especially for the analysis of antenna arrays, this process saves a huge amount of computational time.

Due to the choice of basis functions, the interaction matrix is usually relatively small and therefore the matrix equation can be solved by direct inversion of the interaction matrix. In this case, Gaussian elimination was used. The excitation for each antenna element is modelled with the delta-gap model at the base of each probe. With this excitation model, the Y-parameters associated with the antenna can be calculated directly once the basis-function coefficients have been solved for. The radiation patterns are calculated by using the stationary-phase method [6, 141].

Many of the ideas that have been used in this formulation, have been implemented separately to some extent before. However, this formulation combines these ideas, together with some new ones, in a unique way, thereby resulting in an efficient analysis of the new antenna elements and associated antenna arrays.

2.6 CONCLUDING REMARKS

This chapter presented a broad overview of several approaches that can be used to enhance the impedance bandwidth of probe-fed microstrip patch antennas, as well as the different techniques that can be used for the modelling of these antennas. The new antenna element, which forms the basis of this study, has also been introduced, together with a qualitative description of the theoretical formulation that was implemented for the modelling thereof. The new antenna element makes use of capacitive coupling in its feed structure, but unlike other approaches, the capacitor patch is positioned next to the resonant patch. This has several advantages: all the patches can be manufactured on a single laminate and is therefore relatively cheap and easy to manufacture; the input impedance can be controlled by adjusting only two parameters; and the fact that all the patches have canonical shapes, lends itself to more efficient modelling techniques. The theoretical formulation that was implemented for the analysis of these new antenna elements and arrays, is based on the spectral-domain MM that implements the planarly multilayered Green's function as well as a mixture of entire-domain and subdomain basis functions. The details of this implementation will be addressed in Chapter 3.