AN INITIAL SOLUTION HEURISTIC FOR THE VEHICLE ROUTING AND SCHEDULING PROBLEM

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Abstract

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South Africa provides a fascinating interface between the developed and the developing world and poses a multitude of opportunities for enhancing the sustainable development of local cities. The concept of City Logistics is concerned with the mobility of cities, and entails the process of optimizing urban logistics activities by considering the social, environmental, economic, financial, and energy impacts of urban freight movement. Vehicle routing and scheduling has the potential to address a number of these key focus areas. Applying optimization to vehicle routing and scheduling results in a reduced number of trips, better fleet utilization, and lower maintenance costs; thereby improving the financial situation of the fleet owner. Improved fleet utilization could have a positive environmental impact, while also improving the mobility of the city as a whole. Energy utilization is improved while customer satisfaction could also increase through on-time deliveries and reliability.

The Vehicle Routing Problem (VRP) is a well-researched problem in Operations Research literature. The main objective of this type of problem is to minimize an objective function, typically distribution cost for individual carriers. The area of application is wide, and specific variants of the VRP transform the basic problem to conform to application specific requirements. It is the view of this dissertation that the various VRP variants have been researched in isolation, with little effort to integrate various problem variants
into an instance that is more appropriate to the South African particularity with regards to logistics and vehicle routing.

Finding a feasible, and integrated initial solution to a hard problem is the first step in addressing the scheduling issue. This dissertation attempts to integrate three specific variants: multiple time windows, a heterogeneous fleet, and double scheduling. As the problem is burdened with the added constraints, the computational effort required to find a solution increases. The dissertation therefor also contributes to reducing the computational burden by proposing a concept referred to as time window compatibility to intelligently evaluate the insertion of customers on positions within routes.

The initial solution algorithm presented proved feasible for the integration of the variants, while the time window compatibility decreased the computational burden by 25%, and as much as 80% for specific customer configurations, when using benchmark data sets from literature. The dissertation also improved the quality of the initial solution, for example total distance travelled, by 13%. Finding an initial solution is the first step in solving vehicle routing problems. The second step is to improve the initial solution iteratively through an improvement heuristic in an attempt to find a global optimum. Although the improvement heuristic falls outside the scope of this dissertation, improvement of the initial solution has a significant impact on the quality of improvement heuristics, and is therefore a valuable contribution.
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Chapter 1

Research problem

South Africa’s level of urbanization closely follows international trends in developed countries, with the highest level of economic activity focused in a few metropolitan areas; attracting both people and investments. The good functioning of these metropolitan areas is of strategic importance to the country, as these areas are the main focus for economic and social development. The level of service of transport provided impacts directly on the efficiency and the quality of the development in the metropolitan areas. South African metropolitan areas are experiencing rapid growth, and are having difficulties in controlling the physical urban expansion. Both public and freight transport costs are negatively impacted by these phenomena. As demand for transport increases faster than the supply of these services, commuting and freight transportation costs increase at a rate higher than inflation. The community at large also experiences the higher expenses required to support demands for more extensive infrastructure and services.

1.1 Motivation for the project

South Africa provides a fascinating interface between the developed and the developing world. In a critical review, Leinbach and Stansfield [28] have emphasized that Industrial Engineers should re-adopt a systematic view. They argue that the perception of Industrial Engineers has been negatively impacted by their ability to model the obvious, and in the over-simplification of their models, to the extent that the reality is not represented comprehensively. Industrial engineers should therefore appreciate the complex and intertwined relationships between social, political, and economic factors influencing urban freight transport systems.

Freight carriers are sharing the road network with various modes of public transport. The use of private vehicles have rapidly increased. The increase can be attributed to both an increase in the number of trips under-
taken, and increased journey lengths [5, 48]. Road network performance is negatively impacted by the higher usage of private vehicles and results in higher levels of congestion, and a significant reduction in operating speeds. Public transport performance is impacted negatively when operating speeds decrease, resulting in increased operating costs for the carriers, and thus impacting negatively on its attractiveness. As a result, the economically able part of the population turn to their private vehicles for a reliable source of transport, and unknowingly contributes to the hyper-congestion phenomenon.

The concept of City logistics has emerged to address a new area in transport planning. The objective of the concept is to support the sustainable development of cities and to address challenging problems such as high levels of traffic congestion, negative environmental impacts, high energy consumption and a shortage of trained labour. Taniguchi et al. [57] states that City logistics is the process of totally optimizing urban logistics activities by considering the social, environmental, economic, financial and energy impacts of urban freight movement. In an earlier article Taniguchi et al. [55] emphasizes that the optimization drive is focussed on private shippers and carriers in a free market economy, although consideration is given to the costs and benefits for both the private and public stakeholders. Usually one or more of the following initiatives are included:

- **Advanced information systems** utilize the ever-increasing computational ability to analyze and rationalize existing logistics operations.

- **Co-operative freight transport systems** allow for a reduced number of vehicles in the system by means of load consolidation.

- **Public logistics terminals** are implemented with success in the northern hemisphere. These terminals are typically located on the outskirts of cities, and are helpful promoters of the co-operative transport systems mentioned previously.

- **Load factor controls** are also new initiatives where certificate systems are introduced for carriers. The concept has been successfully implemented for freight vehicles in parts of Europe. Service vehicles, such as telecommunication service providers, refuse removal, plumbing and electrical contractors, are dealt with ineffectively. Service vehicles transport service providers, as opposed to freight. The objective of service vehicles is to provide a reliable service, either on schedule, or when requested. Load factor controls, as a performance measure, are therefore ineffective. The load factor initiative aims to reduce the number of vehicles in congested central business districts by promoting load consolidation. This results in higher fleet utilization, and thus reducing the number of large freight vehicles with small loads.
• *Underground freight transport systems* are innovative, yet costly solutions for freight transport problems. Ooishi and Taniguchi [40] evaluate a proposed initiative in Tokyo, and conclude that the overall effects upon the society and the economy is positive.

It is important to quantify the consequences of such *City Logistics* initiatives as this will enhance the evaluation of the significance and benefits thereof, especially when designing new, or improving existing, urban infrastructure and freight transport activities. Models, representing the various stakeholders and their particular objectives, should be used to quantify the changes in logistics costs, traffic congestion, fleet utilization, hazardous gas emissions, accident occurrences, etc. of proposed initiatives.

### 1.2 Overview of the subject

The existing urban structures in South Africa have been brought about by apartheid policies over many generations. Spence [48] argues that the extent of the separation between people – black and white, rich and poor, advantaged and disadvantaged – and the resources and opportunities which they require, has produced urban conditions that are morally, socially, politically, and economically, unsustainable. The separation resulted in inequitable spatial development and economic structures that favored growth in existing well-resourced areas. More specifically, the urban land use disposition produced low-density residential development and urban sprawl with opportunities concentrated in the vicinities of the more affluent and privileged areas. Conversely, the majority of the population are settled in remote areas with few opportunities or social amenities. Lipman and Monaghan [30] elaborate on the spatial planning, indicating that the legacy from past spatial planning policies has resulted in long travel distances and insufficient residential densities for effective transport services. With the national transport policies under revision, a deliberate focus on transport is essential [34]. Transport in itself is a key factor in creating sustainable economic growth.

#### 1.2.1 Stakeholders

Various key stakeholders participate in the economy and often have competing and egocentric objectives.

**Residents**

The community, or *residents*, are the people that live, work, shop, and entertain in the metropolitan areas. Their objectives include minimizing traffic congestion, noise, air pollution due to traffic, and traffic accidents. Residents do not welcome large freight carrying vehicles in residential areas [42]. Nevertheless, these carriers are required as residents have an expectation to
Table 1.1: Distances and average time spent in commuting in the main cities

<table>
<thead>
<tr>
<th>Distance between black township(s) &amp; CBD (in kms)</th>
<th>Average time spent in transport (minutes per journey)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Johannesburg (bus \ train)</td>
<td>20.0</td>
</tr>
<tr>
<td>Johannesburg (taxi \ cars)</td>
<td>20.0</td>
</tr>
<tr>
<td>Pretoria</td>
<td>52.0</td>
</tr>
<tr>
<td>Durban</td>
<td>20.0</td>
</tr>
<tr>
<td>Bloemfontein</td>
<td>58.4</td>
</tr>
<tr>
<td>Port Elizabeth</td>
<td>16.1</td>
</tr>
<tr>
<td>East London</td>
<td>21.4</td>
</tr>
<tr>
<td>Cape Town</td>
<td>18.8</td>
</tr>
<tr>
<td>Average</td>
<td>28.3</td>
</tr>
</tbody>
</table>

receive their commodities at convenience stores scattered all over residential areas. The South African transport system, from the residents’ frame of reference, is a dual system. For the more affluent portion of the population the system imitates the American way of life with houses, malls, services and offices distributed abundantly, necessitating automobile use, as they are geographically dispersed. The malls are small, and do not offer the shopper a comprehensive range of products. This is in contrast with European models where cities have a central business district, offering the visitor a full range of products and services in a small area, accessible on foot, and in close proximity of a well-established public transport system. On the other hand, given the distorted spatial structure, a large portion of the urban population simply aspires to minimize their commuting expense and ensure their safety while commuting. Here the less affluent move through various modes, from on-foot transportation or bikes, to busses, minibus taxis, commuter trains, and to a lesser extent cars and trucks. Saint-Laurent [18] presents the distances and average time spent commuting by commuters in South-Africa in table 1.1.

Carriers

Carriers represent both public and private stakeholders executing the logistic and distribution functions. The cargo is not limited to freight, but also encompasses passengers in the form of public transport. Freight carriers are continuously expected to provide higher levels of service at lower rates, and therefore try to minimize their logistic costs, and maximize their profits.

Modes of public transport in South African urban areas are limited to commuter trains (9%), busses (16%), minibus taxis (24%), and cars (50%).
The values in brackets indicate the market share of each mode with respect to urban trips, based on the number of passengers multiplied by the distance travelled [18]. These different modes of public transport are often in competition for passengers. This is in contrast to the ideal of being cooperative and integrated. An ideal configuration would see trains forming the backbone of the network and supplying the major (primary) routes; busses on secondary routes, and taxis on local routes; feeding into primary and secondary routes, as is indicated in figure 1.1.

**Shippers**

*Shippers* are the customers of carriers, and often receive (or send) goods from (to) other shippers, or residents. Examples of shippers include manufacturing plants, wholesale and retail outlets, and mail centers. The objective of the shippers is to maximize their level of service, which can be a function of cost, reliability, and/or traceability. Shippers place requirements on carriers for specific collection and delivery times. These requirements are referred to as time windows.

**Administrators**

*Administrators* represent local, provincial, and national government whose objective is to resolve conflict between stakeholders involved in urban freight transport, while facilitating sustainable development of urban areas.

Transport authorities are responsible for planning, coordinating, implementing, monitoring, funding and applying law-enforcement of land transport in provincial and local government spheres. Traffic densities and road construction costs are much higher in urban areas. Municipal resources for transport is also severely restricted due to issues such as housing, safety, and education that enjoy preference in the urbanized environment. Public transport has a better cost and space effectiveness for mass transportation.
Public-transport should therefor be considered a serious alternative in urban areas. Not only do administrators appreciate the alternative, but they also directed the focus of the National Land Transport Transition Bill towards public passenger transport [37]. Although the freight transport industry is largely deregulated in South Africa, attention has been given to freight operations in urban areas, as it interacts with other modes and traffic streams. Nothnagel [34] outlines the history of transport legislation as it progressed since South Africa’s independence in 1994.

Regulation of urban freight transport is a powerful tool for improving the efficient use of transport networks and infrastructure. Numerous regulations have been implemented internationally [55], and include:

- Weight limits
- Load factor control
- Designated routes
- Zoning
- Time windows

A recent, and controversial, event has seen the mayor of London introduce a congestion charge for all vehicles entering the central district [7]. Base- man [6] illustrates that congestion is neither a new, nor a unique problem to our century, and mentions that Julius Caesar placed limits, and raised taxes, on the number of vehicles entering Rome in A.D. 125.

1.2.2 The City Logistics approach

It is appropriate to distinguish between the various impacts that the transport network and infrastructure can have on the stakeholders identified in section 1.2.1.

- Financial impacts relate to, but is not limited to, commuting costs for residents; the payback period of investments that shippers consider making in establishing facilities in new locations; fuel and fleet costs for shippers; and internal rate of return (IRR) for administrators investing in public transport capacity such as train wagons and busses.

- Social impacts relate to equity for various user and non-user groups of transport, and could include the impact of accidents, accessibility to transport, or even business competition in the case of taxi operators servicing the same area.
• Economic impacts are more comprehensive than financial impacts. Cost benefit analysis does not simply focus on the immediate financial implications, but rather the viability of a transport scheme over the period of the scheme’s entire life. Costs could include the upfront acquisition of capital equipment and maintenance costs throughout the operational life, while benefits could represent the reduced travel times experienced by commuters or reduced fleet operating costs experienced by carriers on items such as consumables and maintenance.

• Energy conservation is becoming more important since there is a limited amount of natural resources. Petroleum and automotive diesel oil are the two main sources of energy for the transport industry. City Logistics initiatives, such as route optimization, could potentially reduce the total amount of fuel consumed by freight and public transport vehicles, if the objective of the exercise is to reduce route lengths.

• Environmental effects pose a direct risk on human health. Greenhouse gasses produced by vehicles constitute of the following pollutants:

  - Carbon monoxide (CO)
  - Oxides of nitrogen (NOx)
  - Suspended particulate material (SPM)
  - Hydrocarbons (HC)

Noise pollution is also an area of concern, especially in urban areas.

To be able to limit the scope of an investigation or optimization exercise, Taniguchi et al [57] introduce a third spatial boundary to the transport system, as figure 1.2 indicates.

A terminal refers to a single location, node, or venue, in the transport system, for example a distribution center or a bus stop. The infrastructure along which carriers move between two terminals are referred to as a link. Examples could include a bus route between two stops, or a segment of rail track between two stations. Multiple links make up an area, for example Centurion in Gauteng, South Africa. A corridor refers to a number of directly-connected areas. The Mabopane-Centurion corridor would serve as an example. It is significant to distinguish between the various spatial elements, as each element holds unique improvement opportunities.

To address problems, or opportunities for improvement, models could be created that is used to represent the transport system, and predict the effects of proposed solutions and improvements. Taniguchi et al. [55] elaborate on three distinct types of models:

• Supply models are created when the performance of the road system is represented or predicted through, for example, simulation models.
Figure 1.2: System boundaries
These models include the various cost factors associated with routes, vehicles, and terminals. Supply models are created with the suppliers' objective in mind, such as minimizing outbound logistic costs, distribution strategies for a given network, etc.

- **Demand models** focus on commodity flows and vehicle trips. Models include mode choice (e.g. rail, road, etc.) that is based on commodity (freight) generation; distribution of freight; and vehicle routing and scheduling.

- **Impact models** are used to estimate and predict various environmental, economic, and social impacts that a transportation system might have on stakeholders.

In establishing the scope of a *City Logistics* project, it is therefore necessary to identify the stakeholders that will form the object of the study. The next step would be to determine the need of the stakeholders, and thus identify the relevant impacts under consideration. Once these foci have been set, an appropriate model can be developed.

### 1.2.3 Vehicle routing and scheduling

Vehicle routing and scheduling problems are well-researched in the field of *Operations Research*. The main objective of these types of problems are to minimize the distribution costs for individual carriers. Given the complexity of the type of problem, extensive research has been conducted to develop exact and heuristic solution techniques for urban distribution problems. The Vehicle Routing Problem (VRP) can be described as the problem of assigning optimal delivery or collection routes from a depot to a number of geographically distributed customers, subject to side constraints. The most basic version of the VRP can be defined with $G = (V, E)$ being a directed graph where $V = \{v_0, v_1, \ldots, v_n\}$ is a set of vertices representing customers, and with $v_0$ representing the depot where $m$ identical vehicles, each with capacity $Q$, are located [61]. $E = \{(v_i, v_j)|v_i, v_j \in V, i \neq j\}$ is the edge set connecting the vertices. Each vertex, except for the depot $(V \setminus \{v_0\})$, has a non-negative demand $q_i$ and a non-negative service time $s_i$. A distance matrix $C = \{c_{ij}\}$ is defined on $E$. In some contexts, $c_{ij}$ can be interpreted as travel cost or travel distance. Hence, the terms distance, travel cost, and travel time are used interchangeably. The VRP consists of designing a set of $m$ vehicle routes having a minimum total length such that

- each route starts and ends at the depot,
- each remaining vertex $(V \setminus \{v_0\})$ is visited exactly once by one vehicle,
- the total demand of a route does not exceed $Q$, and
• the total duration (including service and travel time) of a route does not exceed a preset limit $L$

The VRP is a hard combinatorial ($np$-hard) optimization problem for which Laporte [27] has indicated several exact and approximate solution algorithms. An $np$-hard problem implies that the solution space will increase at an exponential or factorial rate (non-polynomial) as the number of customers/vertices increases. Early researchers such as Clarke and Wright [14] realized that exact algorithms can only solve relatively small problems, but a number of heuristic (near-optimal) algorithms have proved very satisfactory.

The basic VRP makes a number of assumptions, including utilizing a homogeneous fleet, a single depot, one route per vehicle, etcetera. These assumptions can be eliminated by introducing additional constraints to the problem. This implies increasing the complexity of the problem, and, by restriction, classifies the extended problem as an $np$-hard problem. The various side constraints, and their application, are discussed in more detail in chapter 2. It should be noted that most of these additional side constraints are implemented in isolation, without integration, due to the increased complexity of solving such problems.

1.3 Research questions

The concept of City Logistics is as important in South Africa as it is in the rest of the world. To contribute to the school of subject knowledge, the following general research question could be asked.

*How can we "Glocalize" the concept of City Logistics in South African urban environments?*

Urban environments are complex societies with various stakeholders. Although the perceptions and motivation of stakeholders vary, their objective towards freight movement is uniform: civilize the truck. One way of achieving the objective is to reduce the number of freight vehicle trips. Residents will appreciate the alleviated congestion; shippers welcome reduced shipping and receiving transactions; and carriers are happy to reduce their fleet operating costs as a result of the reduced number of trips. Trip optimization is a key focus area of City Logistics, and vehicle routing and scheduling procedures provide core techniques in the modelling of City Logistics. Most practical distribution problems are not limited to occurrences of isolated VRP variants. No literature, however, could be found that integrate an appropriate number of these variants. To make the proposed demand model for carriers more applicable to the South African environment, the aim of this dissertation is to answer the following research question:

1To have a problem, or opportunity, with global relevance, yet with local application.
Is it possible to solve a vehicle routing problem with multiple integrated constraints?

The dissertation aims to investigate the feasibility of integrating a number of these side constraints into a single problem instance, referred to as the Vehicle Routing Problem with Multiple Constraints (VRPMC).

1.4 Research design and methodology

Mouton [33] notes that theory-building studies aim to develop new models to explain particular phenomena. The phenomena in this particular dissertation would be the routing of vehicles to service a number of geographically dispersed customers. The VRP and its variants are well-researched problems in academic literature but, as stated, lack integrated models. One of the limitations of model-building studies are their ineffectiveness if the models make implausible claims on reality. To ensure that the most common trap – over-abstract formulation – is avoided, figure 1.3 illustrates the proposed Operations Research process, as adapted from Rardin [43].

The objective of the dissertation is to address a particular problem; both empirical data and actual problem instances exist that could be used to understand the actual problem and its underlying assumptions. Given the problem and the available literature, a representative model could be formulated. The model will consist of:

- decision variables that will indicate the decisions (solution space) open to decision makers,
- a set of constraints that limit the decision choices, and
- an objective function that will indicate preferred decisions.

The modelling task includes the process of evaluating the assumptions made in literature, and criticizing the assumed reality the model is based on. Figure 1.4 from Taha [49] illustrates the modelling task appropriately.

It is believed that, as is the case for most VRP variants, the modelling of the problem is relatively simple. In the case of the VRP variants, the model is only the first step of the Operations Research process. Solving the model involves finding an initial solution, and then optimizing the solution until a local, or optimistically, a global, optimum is found. The objective of this dissertation is to document the initial solution algorithm, and to code the algorithm in an appropriate language, such as MATLAB, for testing purposes using empirical data. To make conclusions from the results, it is necessary to evaluate the quality of the results generated. Solomon [46] developed sets of test problems representing various environments. These test sets have become a popular benchmark for result comparisons. Although
Figure 1.3: *Operations Research* process
Figure 1.4: Levels of abstraction in model development
the sets may not adhere to the perceived model reality of the dissertation’s proposed model, it could be used as point of departure when developing test sets for the model. Only once the model solutions are verified and validated, could they be used to support decisions regarding the problems experienced in reality.

1.5 The project outline

A comprehensive two-part literature review is undertaken in Chapter 2. The aim of the review is to firstly introduce the concept of City Logistics with specific reference to the South African particularity. The latter part of the literature review elaborates on vehicle routing and scheduling, focussing on the VRP variants, and finding an appropriate point of departure for the proposed model.

Due to the complex nature of the proposed model, and as a result of additional constraints introduced to the model, a new concept of Time Window Compatibility is introduced in Chapter 3. The concept serves as a means to reduce the computational complexity of the solution algorithm. In Chapter 4 the proposed model is defined, and the initial solution algorithm introduced.

Results are presented in Chapter 5, followed by interpretation and related discussions. The dissertation is concluded with a research agenda for City Logistics studies and applications.
Chapter 2

Literature Review

The aim of the review is to establish the broader environment within which the optimization of logistic operations occur. The process of City Logistics is introduced. The multi-actor environment is emphasized, where a number of non-transport influences affect the transport activity within cities. The need exists to describe and predict influencing factors in the urban network. For this purpose a new modelling approach is introduced that takes the various perceptions of stakeholders into account. Vehicle routing and scheduling procedures are formalized as core techniques to model the logistic system in urban areas. Realistic variants of the vehicle routing problem are introduced, and their impact on the modelling task is highlighted.

The complex nature of vehicle routing problems requires the introduction of heuristic solution algorithms. The second half of the chapter reviews the process of finding a good initial solution – a critical parameter in the solution quality of the optimization process. Finally, three improvement heuristics are introduced that proved successful in vehicle routing problems.

2.1 Fundamental concepts

Transportation, as the business of conveying passengers and/or goods [2], has evolved tremendously in South Africa over the past century [1]. Transportation occurs even in the absence of infrastructure: nature transports seeds and natural materials through forces, such as wind and water; people in remote areas travel by foot in the absence of accessible road surfaces, and get from point A to point B, even across inhospitable areas. It is, however, as a result of the frequent use of transport routes that the users of the routes collectively demand improvements. The improvements apply to both travel conditions, such as shorter journeys and smoother rides, and travel reliability, for example the accessibility of roads due to weather conditions.
It is also not uncommon for users to pay for improvements. The Chair of Transportation Engineering elaborates on the evolution of transportation engineering as a discipline [38]. The demand for improvements gave rise to the development of transport infrastructure such as hardened road surfaces, bridges, ports, etc., and established disciplines such as structural, pavement, railway, and traffic engineering.

The discipline of traffic engineering is based on developing a mathematical understanding of the operation of vehicles on sections of the road network. This ability to model vehicular movement is extended when the need emerges to forecast what transport facilities would be required in the future. The discipline of transport planning, that applies to single and multi-modal problems, emerged. It can also be used to understand the inter-relationship of land-use and the demographic aspects that generate the need for transportation.

2.2 Modelling City Logistics

It is appropriate to elaborate on a few basic transport concepts to emphasize the complex relationships between various stakeholders in society.

2.2.1 Transport Concepts

The function of transport is to move passengers and goods from where they are to where they want to be or where their relative value is greater. The demand for transport arises from the fact that not all places are equally endowed with resources, and surpluses are then moved to areas experiencing shortages [39]. Transport is a service and does not occur for its own purpose, and is therefore referred to as a derived demand. In the development of industries, transport plays a vital part in linking the sources of raw material, manufacturing or processing centers, and the markets. Raw material are moved to, and between processing centers, while finished goods are moved via wholesalers and retailers to the point of consumption or utilization. Transport is essential to enable people to travel between their homes and places of employment. This is even more true in South Africa where the average commuting time for residents is in excess of one hour (Table 1.1).

Transport and development are closely linked, and effective transport is a prerequisite for the development of a country. Investment alone, however, does not guarantee prosperity. Whilst transport is focussed on the physical movement of objects, be it freight or passengers, logistics, in the context of this dissertation, is concerned with the activity of transport within a larger environment. The environment of the logistics system is illustrated in
The City Logistics process has numerous interfaces with various built environment, development, engineering, and geographical disciplines. It is concerned with the mobility, sustainability, and liveability of cities [56]. A few non-transport aspects of logistics are explained:

**Economic aspects of transport**

Transport cost consists mainly of a fixed portion, $C_f$ at a point $A$, and a variable portion based on the distance travelled from point $A$ to a point $B$, as indicated in figure 2.1. The cost function need not be linear as indicated,

![Cost diagram](image)

**Figure 2.1: Transport cost components**

and the slope of the cost function could be calculated as $\frac{\delta C}{\delta D}$. The transport cost between points $A$ and $B$, $C_b$, can be calculated as $C_b = C_f + D_{ab} \times \frac{\delta C}{\delta D}$. The transport cost function is influenced by improvements in transport facilities and infrastructure, e.g. improved transshipment methods, more efficient vehicles, and optimized vehicle routes.

Competing carriers and shippers react differently to these advances, and could gain a *competitive advantage* by effectively implementing the advances. Consider the example illustrated in figure 2.2 where two carriers, $A$ and $B$, compete on the basis of cost. At present, carriers $A$ and $B$ supply their services at a cost of $C_{a1}$ and $C_{b1}$ respectively. A technological advance becomes available to both carriers, and carrier $A$ increases the fixed portion of its transport cost by investing in the advance – to a greater extent than carrier $B$. This results in carrier $A$ being able to compete in the market at a cost of $C_{a2}$, lower than carrier $B$’s cost of $C_{b2}$.
Economics of scale influences the cost per unit transported, and is illustrated in figure 2.3. This can be achieved should a carrier is able to consolidate loads on a vehicle, or acquire new business for that vehicle.

Although transport services is generally initiated by the private sector, the concept of public-private-partnerships have gained prominence, and could benefit both sectors. It is important to preserve transport through joint initiatives. The term preserve is used to emphasize the perishable nature of the service. Once a vehicle, such as a delivery van, or a bus, have departed with a load factor less than 1, the opportunity to sell the empty
seats for that trip is immediately, and permanently, destroyed. The South African government has introduced commercialization and privatization policies to share the control over transport with the private sector. Examples include the creation of the *Airports Company of South Africa* and the concession of rail services to private companies such as *MetroRail* [36, 37]. Government subsidizes various aspects of transport activities in both the public and private sector. Bus and rail subsidies amount to R3,737 million in 2002/03, which include maintenance, as well as the creation of additional infrastructure [15]. Government is implementing a *Taxi Re-capitalization Programme* in the private sector whereby economic potential is unlocked in a taxi industry that provides 65% of the 2.5 billion annual passenger trips in urban areas [8, 12, 15].

**Political aspects of transport**

The relationship between government and transport is bidirectional. As a user of transport, the government communicates its decisions to all areas. This happens indirectly when provincial and local government officials travel between parliament and their representative offices. Government also uses transport to move its defence force to defend the country's borders. Transport is used more strategically to achieve developmental goals in the country, and to achieve political goals in the form of incentives and investments to attract industrial parties to settle in specific areas. As a transport influencer, government provides funds for infrastructure, and supports innovations through pilot and demonstration projects.

**Social aspects of transport**

The main social impact of improved transport is the reduced friction that results from distance: people are given the opportunity to participate, socially and economically, in a larger geographical area. This results in various cultures interacting and communicating different ideas and frames of reference. Mutual understanding is fostered that should result in the elimination of suspicion between races and/or cultures. The improvement of transport, specifically air travel, has widened the scope of opportunities between nations to the extent that the world is often referred to as a *global village*.

Developments in the automotive industry, and the accessibility to cars in South Africa, have contributed to increased use of this mode of transport. In the beginning of 2002, there were some 6.9 million registered vehicles in South Africa, more than 3.98 million of which were cars [15]. Although the private car provides spatial freedom to travel easily, and give status in terms of prosperity and adulthood, it does produce some significant social
problems:

- Residential development occur at a lower density, and further away from the working place.

- Specialized urban activities, such as shopping and entertainment, are concentrated in a way that is supporting the use of private cars, as opposed to infrastructure supporting pedestrian access.

- A reduction in the number of public transport trips. This results in inefficient public transport use, decreased service levels, and eventually more people reverting to private car use.

The higher number of vehicles also results in a demand for more roads: infrastructure that, in itself, create pedestrian barriers, force inhabitants to relocate, change land use patterns, and reduce residential (social) quality. The effects of freeways are even more profound. De Boer [16] introduces transport sociology and states that traffic, and the desires of planners to provide additional infrastructure, should be tamed. The emphasis should be shifted from the expansion of infrastructure, to the management of the available transport infrastructure. When a transport system is in place, its utilization and physical condition should be improved. Additional infrastructure creates potential competition, resulting in suboptimal utilization of all infrastructure.

Environmental aspects of transport

Transportation is a major source of air pollution, emitting tons of carbon monoxide (CO), hydrocarbons (HO), and nitrogen oxides (NOx) into the air [35]. The environmental effects of a transport system is not limited to a negative change in the chemical composition of the atmosphere, but also impacts the social environment. Noise pollution created by traffic results in sleep and speech interference, annoyance, and impairment of hearing after exposure over long periods of time [39].

2.2.2 A new approach to City Logistics Modelling

It is clear from the previous section that stakeholders do not participate as individual entities, but rather as complex networks. De Bruijn and tenHeuvelhof [17] identifies four important characteristics of stakeholder networks:

- Interdependence. Actors cannot achieve their goals without cooperation, as they are dependent on the resources of others actors, such as funding, information, and statutory powers.
- **Pluriformity.** Corporate actors do not behave as individuals, but as coalitions, since their constituents may have diverging and competing interests.

- **Self-containment.** Corporate actors are inclined to close themselves off from their environment, developing their own frame of reference and norms, making it harder to induce the cooperation.

- **Instability.** Positions and relations in policy networks are continually undergoing changes.

Villa [63] confirms autonomous decision making in networks. These characteristics impede the ability of stakeholders to make decisions that is optimal for the sustainability of the system as a whole. It does open a window of opportunity to address the dynamics of the relationships according to each stakeholder’s perception. The Thomas-theorem states that if man define situations to be real, they are real in their consequences [60]. It implies that the operations research practitioner should not pursue a comprehensive model of a specific situation, but rather a set of models reflecting the diversity in actor perceptions. The soundness of the set of Thomas-models is not determined by the degree of correspondence to reality, but by the acuteness with which it mirrors the assumptions that actors make about their reality. It may appear as if the Thomas-models are in conflict with figure 1.4 and the discussion in paragraph 1.4 that argues that the model assumptions should be challenged to ensure that a real representation of the problem is modelled. It is the opinion of the author that the Thomas-model approach emphasizes the importance of the operations research practitioners’ skillful definition of the target audience (object) before engaging in the modelling task.

Van Duin et al [62] address the perspectives of individual actors and their strategic behavior through *perception based modelling*. There are various analysis and modelling techniques to capture decision-making processes. Taniguchi et al [57] discuss a computerized support tool, called *Dynamic Actor Network Analysis* (DANA), and proceed to introduce a new approach to city logistics modelling.

**Dynamic actor network analysis**

The first step towards modelling the stakeholder network is to capture the perceptions of multiple actors. The DANA tool is set up as an open database into which several perceptions of actors can be submitted. Perceptions are modelled in terms of assumptions. *Factual assumptions* represent how an actor perceives the current state of his environment. *Causal assumptions* represent changes that will occur in the perception of the actor, and
uses *if-then* statements. *Teleological assumptions* represent the actor’s view on his desirability, or purposefulness in the network. An important feature of the tool is a query generator. Queries include questions like: *which actors have conflicting goals on a specific factor, or which actors have different definitions for a factor?* These conflicts are brought to the modeler’s attention during the design of a logistics model.

**Performance measures**

Performance measures, or performance indicators, are the normative values of the perception-based factors that represent how an actor perceives the current state of his environment. It is used to compare actual performance against a pre-defined norm. An actor creates the norm by attaching a quantified value, based on his perceptions, to a factor influencing his environment.

**Logistics modelling**

The last part of the approach is directed towards the calculation of the value part of the performance indicators. Dedicated models are developed to measure the impact of logistics concepts. The models are based on the important factors identified during the dynamic actor network analysis, as interpreted from the operations research practitioner’s perspective. The challenge is to develop a model at such a level that it is both comprehensive and easy to understand for the actors involved, yet sufficiently detailed to be validated in practice.

2.3 **Vehicle routing**

There are significant features of truck operations in urban areas affecting mobility in cities:

- Pickup/delivery trucks visit a number of customers on a single trip. Optimized route schedules decreases operational expenses
- Several pickup/delivery trucks are usually operated as a group by a shipper/courier. It is possible to reduce the number of vehicles used, as improved load factors increase fleet efficiency.
- Each customer specifies a time window to be visited by the pickup/delivery vehicle, complicating route schedules for carriers

Vehicle routing and scheduling procedures address the mobility in cities [57]. *Vehicle travel time/distance, total fleet cost, and customer satisfaction* are factors that are valued by both carriers and shippers. Vehicle routing and scheduling problems involve an optimization process of assigning customers
to trucks and determining the visiting order of customers on truck routes.

According to Van Breedam [61] and Laporte [27], the general Vehicle Routing Problem (VRP) can be defined as the problem of finding a set of routes for a fleet of vehicles, which have to serve a number of customers (also referred to as stops or nodes) by offloading their goods. The vehicles depart from, and return to, a single depot. Vehicles must complete their individual routes within a maximum total route time. Although the basic VRP has been described mathematically in section 1.2.3, it will now formally be defined.

Let:

- \( N \) be the total number of customers
- \( q_i \) be the known demand for node \( i \), where \( i = \{1, 2, \ldots, N\} \)
- \( s_i \) be the service time at node \( i \), where \( i = \{1, 2, \ldots, N\} \)
- \( d_{ij} \) be the distance between nodes \( i \) and \( j \), where \( i, j = \{1, 2, \ldots, N\} \)
- \( c_{ij} \) be the cost incurred on the arc between nodes \( i \) and \( j \), where \( i, j = \{1, 2, \ldots, N\} \)
- \( t_{ij} \) be the travel time between nodes \( i \) and \( j \), where \( i, j = \{1, 2, \ldots, N\} \)
- \( K \) be the total number of vehicles available
- \( p \) be the capacity of each vehicle in the homogeneous fleet

The principle decision variable, \( x_{ijk} \), is defined as

\[
x_{ijk} = \begin{cases} 
1 & \text{if vehicle } k \text{ travels from node } i \text{ to node } j, \text{ where } \\
& i, j = \{1, 2, \ldots, N\}|i \neq j}, k = \{1, 2, \ldots, K\} \\
0 & \text{otherwise}
\end{cases}
\]

\[
\min z = \sum_{i=0}^{N} \sum_{j=0, j \neq i}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} \tag{2.1}
\]
subject to

\[
\sum_{j=1}^{N} x_{0jk} = \sum_{j=1}^{N} x_{j0k} = 1 \quad \forall k \in \{1, 2, \ldots, K\} \quad (2.2)
\]

\[
\sum_{j=1}^{N} \sum_{k=1}^{K} x_{0jk} \leq K \quad (2.3)
\]

\[
\sum_{i=1}^{N} \sum_{j \neq i} x_{ijk} = 1 \quad \forall j \in \{1, 2, \ldots, N\} \quad (2.4)
\]

\[
\sum_{i=1}^{N} \sum_{j \neq i} x_{ijk} = 1 \quad \forall i \in \{1, 2, \ldots, N\} \quad (2.5)
\]

\[
\sum_{i=1}^{N} q_i \sum_{j \neq i} x_{ijk} \leq p \quad \forall k \in \{1, 2, \ldots, K\} \quad (2.6)
\]

\[
x_{ijk} \in \{0, 1\} \quad (2.7)
\]

The objective of the problem is to minimize the total travel cost incurred during the process of servicing the \( N \) customers. The element \( c_{ij} \) in (2.1) can be replaced with either \( t_{ij} \) to minimize travel time, or with \( d_{ij} \) to minimize travel distance. Constraint (2.2) ensures that all routes start and end at the depot, node 0, while (2.3) ensures that the maximum number of vehicles/routes are not exceeded. Constraints (2.4) and (2.5) limit the number of visits to each node to 1, while (2.6) ensures that the cumulative demand on any route does not exceed the vehicle capacity.

2.3.1 The vehicle routing problem and its variants

The basic VRP assumes that nodes can be visited anytime during the route, that all vehicles are homogeneous in terms of cost and capacity, and that each vehicle can only service one route during a scheduling period. There exist numerous extensions to the VRP. These arise when additional side constraints are added to adapt the basic VRP to many real-life business scenarios. The three variants considered in this dissertation are described in the following paragraphs.

Time windows

A time window can be described as a window of opportunity for deliveries. It is an extension of the VRP that has been researched extensively. Examples include the work of Ibaraki et al. [25], Taillard [50], Taillard et al. [51], and Tan et al. [54]. A time window is the period of time during which deliveries can be made to a specific customer \( i \), and has three main characteristics:
• the earliest allowed arrival time, \( e_i \) (also referred to as the opening time),

• the latest allowed arrival time, \( l_i \) (also referred to as the closing time),

• whether the time window is considered soft or hard.

Consider the example, illustrated in figure 2.4, where customer \( i \) requests delivery between 07:30 and 17:00.

\[
\begin{align*}
\text{Figure 2.4: Double sided hard time window}
\end{align*}
\]

To distinguish between the actual and the specified times of arrival, the variable \( a_i \) denotes the actual time of arrival at node \( i \). Should the actual arrival time at node \( i \), \( a_i \), be earlier than the earliest allowed arrival at the node, \( e_i \), then the vehicle will incur a waiting time, \( w_i \), which can be calculated as \( w_i = \max\{0, e_i - a_i\} \). The introduction of time windows to the basic VRP sees the introduction of three new constraints.

\[
\begin{align*}
0 & = w_0 = s_0 = 0 \quad (2.8) \\
\sum_{k=1}^{K} \sum_{i=0}^{N} x_{ij}(a_i + w_i + s_i + t_{ij}) & \leq a_j \quad \forall j = \{1, 2, \ldots, N\} \quad (2.9) \\
e_i & \leq (a_i + w_i) \leq l_i \quad \forall i = \{1, 2, \ldots, N\} \quad (2.10)
\end{align*}
\]

Constraint (2.8) assumes that vehicles are ready and loaded by the time the depot opens, which is indicated as time 0 (zero). Constraint (2.9) calculates the actual arrival time, while (2.10) ensures that each customer \( i \) is serviced within its time window.

When both an earliest and latest allowed arrival is stipulated, the time window is referred to as double sided. If no arrivals are allowed outside of the given parameters, the time window is said to be hard, as is the case in figure 2.4. When delivery is allowed outside the specified time window, the time window is said to be soft, and the lateness is penalized at a cost of \( \alpha_i \). Customer \( i \) may specify a maximum lateness, \( L^\text{max}_i \). The example illustrated in figure 2.5 sees customer \( i \) specifying a time window between 07:30 and 15:30. The customer will, however, allow late deliveries until 17:00. A hard time window is therefor a special type of soft time window where \( L^\text{max}_i = 0 \).
Should a vehicle arrive after the latest allowed arrival time, $l_i$, but prior to the maximum lateness, $L_i^{max}$, the lateness at node $i$, $L_i$, can be calculated as $L_i = \max\{0, a_i - l_i\}$ if $a_i \leq L_i^{max}$. The lateness is penalized by introducing a penalty term to the VRP objective function (2.1).

$$\min \sum_{i=0}^{N} \sum_{j=0, j \neq i}^{N} \sum_{k=1}^{K} c_{ij}x_{ijk} + \sum_{i=1}^{N} \alpha_i \times \max\{0, L_i\}$$  \hspace{1cm} (2.11)

The time window for the depot, node 0, can be specified. The case illustrated in figure 2.6 sees the depot specifying operating hours (time window) from 06:00 to 18:00, while the first customer on the route, customer 1, specifies a time window between 07:00 and 09:00, and the last customer, customer $n$, requests delivery between 15:00 and 17:00.

Should a customer specify multiple time windows, an indexing symbol, $a_i$, is introduced as superscript to the earliest and latest allowed arrival times, respectively, where $a_i \in \{1, 2, \ldots, A\}$ in which $A$ indicates the maximum number of time windows allowed for each customer. Consider the example where customer $n$ requests delivery either between 06:30 and 09:00, or between 16:00 and 17:30. The case is illustrated in figure 2.7. This example

is typical of residents requesting home shopping deliveries outside business
hours.

**Heterogeneous fleet**

Gendreau et al. [20] propose a solution methodology for cases where the fleet is heterogeneous, that is, where the fleet is composed of vehicles with different capacities and costs. Their objective is to determine what the optimal fleet composition should be, and is referred to as either a Heterogeneous Fleet Vehicle Routing Problem (HVRP), or a Fleet Size and Mix Vehicle Routing Problem (FSMVRP). Taillard [50] formulates the Vehicle Routing Problem with a Heterogeneous fleet of vehicles (VRPHE) where the number of vehicles of type $t$ in the fleet is limited; the objective being to optimize the utilization of the given fleet. Salhi and Rand [45] incorporates vehicle routing into the vehicle composition problem, and refer to it as the Vehicle Fleet Mix problem (VFM).

The implication of a heterogeneous fleet on the standard VRP is that $T$ type of vehicles are introduced, with $t \in \{1, 2, \ldots, T\}$. The vehicle capacity parameter $p$ is changed. The new parameter, $p_t$, represents the capacity of vehicles of type $t$, resulting in each vehicle $k$ having a unique capacity, $p_k$. The use of one vehicle of type $t$ implies a fixed cost $f_t$. A unique fixed cost, $f_k$, is introduced to each vehicle $k$, based on its vehicle type. The objective function changes to

$$\min \sum_{i=0}^{N} \sum_{j=0, j \neq i}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} + \sum_{k=1}^{K} \sum_{j=1}^{N} f_k x_{0jk}$$

while (2.6) changes to indicate the new capacity parameter

$$\sum_{i=1}^{N} q_i \sum_{j=0, j \neq i}^{N} x_{ijk} \leq p_k \quad \forall k = \{1, 2, \ldots, K\}$$

Taillard [50] introduces a variable $c_{ijt}$ to represent the cost of travelling between nodes $i$ and $j$, using a vehicle of type $t$. It is possible to introduce the variable portion of the vehicle cost into the objective function (2.12). The introduction will lead to

$$\min \sum_{i=0}^{N} \sum_{j=0, j \neq i}^{N} \sum_{k=1}^{K} \sum_{t=1}^{T} c_{ijt} x_{ijk} \xi + \sum_{k=1}^{K} \sum_{j=1}^{N} f_k x_{0jk}$$

where

$$\xi = \begin{cases} 
1 & \text{if vehicle } k \text{ is of type } t, \text{ where } k = \{1, 2, \ldots, K\}, \\
t = \{1, 2, \ldots, T\} \\
0 & \text{otherwise}
\end{cases}$$
Double scheduling

It is often not viable to assume that each vehicle will only complete a single route. Double scheduling is concerned with the case where a vehicle could complete deliveries on a scheduled route, return to the depot where its capacity is renewed, after which a second, or consecutive trip is executed with the renewed capacity. Taillard et al. [53] refer to this type of problem as the Vehicle Routing Problem with Multiple use of vehicles (VRPM). Butt and Ryan [11] consider the Multiple Tour Maximum Collection Problem (MTMCP) and assumes that the routes are constrained in such a way that all of the customers cannot be visited. Their approach aims to maximize the number of customers serviced. Brandão and Mercer [9] introduce the Multi-Trip Vehicle Routing Problem (MTVRP), and address the combination of multiple trips with time windows.

This dissertation considers a vehicle that starts and ends its tour at the depot. A tour consists of one or more routes, each starting and ending at the depot. The same vehicle can only be used for two or more routes if the routes do not overlap. As opposed to (2.8), multiple routes require a service time to be specified for the depot. Consider the example illustrated in figure 2.8. The depot has a time window from 06:00 to 18:00. A vehicle fills its

![Figure 2.8: Double scheduling](image)

capacity at the depot for a time period of $s_0 = 0.5$ hours. It leaves the depot at 06:30, services the first route, and returns to the depot at 11:00, where its capacity is renewed. A second route, of five hours, is serviced before the vehicle returns to the depot.

The mathematical implication of double scheduling on the basic VRP could not be established from literature. Taillard et al. [53] confirm that this type of problem has received very little attention in literature. This dissertation proposes a way to deal with multiple routes. The proposed solution involves a time verification process. If a vehicle arrives back at the depot at time $a_m$, and the service time is specified as $s_0$, then the vehicle is considered for an additional route on its current tour if, after the capacity has been renewed, the depot’s time window is still open. The case is represented
in (2.15).

\[ a_m + s_0 \leq l_0 \]  \hspace{1cm} (2.15)

2.3.2 Computational complexity of the VRP

Problems that are considered hard to solve are those problems for which there are not polynomial solution algorithms, and are referred to as nonde-
terministic (NP) class problems. The problems that require an inordinate
amount of computer processing time in the NP class are identified as NP-
hard problems. Lenstra and Rinnooy, and Laporte, have classified vehicle
routing and scheduling problems as NP-hard problems [27, 29].

When confronted with difficult problems to be solved (NP-hard), opera-
tions research (OR) practitioners have used approximation techniques to
arrive at a good solution. The approximate rule-of-thumb techniques where
optimality cannot be assured are classified as heuristic techniques in OR
practice [65].

2.3.3 Solving the vehicle routing problem

Heuristics typically uses a greedy approach to obtain a good initial
solution in efficient time and then incrementally improve the solution by
neighborhood exchanges or local searches. As a result, heuristics tend to
get trapped in a local optimal solution and fail to find a global optimum.
Heuristics have evolved into global optimization heuristics. These are gen-
eral master strategies to solve problems, and are based on intelligent search
techniques. Where heuristic searches are limited to steps that will improve
the objective function, global optimization heuristics allow steps that will
temporarily decrease the objective function value, in an attempt to escape
the local optimum and look for the global optimum, or at least a better local
optimum. These global optimization heuristics are often called metaheuris-
tics because the procedure used to generate new solution out of the current
one, is embedded in a heuristic which determines the search strategy. The
main drawback of metaheuristics is that they do not have definitive stop-
ning criteria; the longer the computation time, the higher the probability of
finding the global optimum [61].

Initial solution algorithms

Solomon divides VRP tour-building algorithms into either sequential or
parallel methods [46]. Sequential procedures construct one route at a time
until all customers are scheduled. Parallel procedures are characterized by
the simultaneous construction of routes, while the number of parallel routes
can either be limited to a predetermined number, or formed freely. Solomon
concludes that, from the five initial solution heuristics evaluated, the sequential insertion heuristic (SIH) proved to be very successful, both in terms of the quality of the solution, as well as the computational time required to find the solution.

Initialization criteria refers to the process of finding the first customer to insert into a route. The most commonly used initialization criteria is the farthest unrouted customer, and the customer with the earliest deadline, or the earliest latest allowed arrival. The first customer inserted on a route is referred to as the seed customer. Once the seed customer has been identified and inserted, the SIH algorithm considers, for the unrouted nodes, the insertion place that minimizes a weighted average of the additional distance and time needed to include a customer in the current partially constructed route. This second step is referred to as the insertion criteria. The third step, the selection criteria, tries to maximize the benefit derived from inserting a customer in the current partial route rather than on a new direct route. Note that the terms nodes and customers are used interchangeably. The insertion and selection criteria can be simplified using the example illustrated in figure 2.9. The partially constructed route in the example consists

![Figure 2.9: Sequential insertion of customers](image)

of the depot and three routed nodes, namely B, C, and E. The route can be expressed as Depot − B − C − E − Depot. Nodes A and D are currently unrouted.

The insertion criteria, \( c_1(i, u, j) \), calculates the best position and associated cost, between two adjacent nodes \( i \) and \( j \) on the partial route, to insert a customer \( u \), and is calculated for each of the unrouted nodes. Consider node A in the example, there are currently four edges where the node can be inserted, namely Depot − B, B − C, C − E, or E − Depot, as illustrated in
Figure 2.10: Selection criteria

Figure 2.10. Dullaert et al. [19] extends Solomon’s heuristic and determines $c_1(i, A, j)$ for the unrouted node $A$ as

$$c_1(i, A, j) = \min_p [c_1(i_{p-1}, A, i_p)], p = \{1, 2, \ldots, m\}$$

(2.16)

in which $m$ represents the routed nodes in the partially constructed route. If the expressions are generalized for all unrouted nodes $u$, the insertion criteria is calculated as

$$c_1(i, u, j) = \alpha_1 c_{11}(i, u, j) + \alpha_2 c_{12}(i, u, j) + \alpha_3 c_{13}(i, u, j)$$

(2.17)

with

$$c_{11}(i, u, j) = d_{iu} + d_{uj} - \mu d_{ij}, \mu \geq 0$$

(2.18)

$$c_{12}(i, u, j) = a_j^{new} - a_j$$

(2.19)

$$c_{13}(i, u, j) = ACS, AOOS, or AROS$$

(2.20)

With the extension to Solomon’s heuristic, the weighting factors $\alpha_i$ need not add up to 1. $c_{11}(i, u, j)$ denotes the additional distance, and $c_{12}(i, u, j)$ the additional time needed to serve customer $u$ after customer $i$, but before customer $j$. The new actual arrival time at node $j$ is denoted by $b_j^{new}$ in (2.19). The vehicle savings criteria, denoted by $c_{13}(i, u, j)$, considers any one of three parallel approaches to vehicle cost, where Dullaert et al. [19] adapts the savings concepts first introduced by Golden et al. [22]. To elaborate on the concepts, let
\( F(z) \) be the fixed cost of the smallest vehicle that can service a cumulative route demand of \( z \)

\( F'(z) \) be the fixed cost of the largest vehicle whose capacity is less than or equal to \( z \)

\( P(z) \) be the capacity of the smallest vehicle that can service a demand of \( z \)

\( Q \) be the load of the vehicle currently servicing the route

\( \bar{Q} \) be the maximum capacity of the vehicle currently servicing the route

\( Q^{\text{new}} \) be the new load of the vehicle after the customer has been inserted into the route

\( \bar{Q}^{\text{new}} \) be the (new) capacity of the vehicle after the customer has been inserted into the route

The Adapted Combined Savings (ACS) is defined as the difference between the fixed costs of the vehicles capable of transporting the load of the route after, and before, inserting customer \( u \), and is calculated as

\[
ACS = F(Q^{\text{new}}) - F(Q)
\] (2.21)

The Adapted Optimistic Opportunity Savings (AOOS) extends the ACS by subtracting the fixed cost of the vehicle that can service the unused capacity, and is calculated as

\[
AOOS = [F(Q^{\text{new}}) - F(Q)] - F(\bar{Q}^{\text{new}} - Q^{\text{new}})
\] (2.22)

The Adapted Realistic Opportunity Savings (AROS) takes the fixed cost of the largest vehicle smaller than or equal to the unused capacity, \( F'(\bar{Q}^{\text{new}} - Q^{\text{new}}) \), into account as an opportunity saving. It only does so if a larger vehicle is required to service the current route after a new customer has been inserted. AROS is calculated as

\[
AROS = [F(Q^{\text{new}}) - F(Q)] - \delta(\omega)F'(\bar{Q}^{\text{new}} - Q^{\text{new}})
\] (2.23)

where

\[
\delta(\omega) = \begin{cases} 
1 & \text{if } Q + q_u > \bar{Q} \\
0 & \text{otherwise}
\end{cases}
\]

Any one of these savings criteria can be used as all three outperformed previous best published results for the initial solution [19]. Once the best position for each unrouted node has been determined, as illustrated in figure 2.11, the customer that is best according to the selection criteria, is selected. The procedure can be expressed mathematically as

\[
c_2(i, u^*, j) = \min_u [c_2(i, u, j)], u \text{ unrouted and feasible}
\] (2.24)

\[
c_2(i, u, j) = \lambda (d_{ou} + t_{ou}) + s_u + F(q_u) - c_1(i, u, j), \lambda \geq 0
\] (2.25)
The best customer, $u^*$, is then inserted into the partially created route between its specific nodes $i$ and $j$. From Figure 2.11, consider node $D$ to be the best node. After inserting $D$ into the current route, node $A$ remains the only unrouted node, and the new route is illustrated in Figure 2.12, and can be expressed as $Depot - B - D - C - E - Depot$. The insertion process is repeated until no remaining unrouted nodes have a feasible insertion place. A new route is then initialized and identified as the current route.

Van Breedam [61] introduces an initial solution parameter in his evaluation of improvement algorithms, and finds that, in most cases, a good initial solution gives significantly better final results.
Solution improvement algorithms

Initial solutions serve as input to improvement heuristics, and although these improvement heuristics are not the main focus of this dissertation, the three main metaheuristics is described briefly.

The evaluation of any heuristic is subject to the comparison of a number of criteria that relate to different aspects of the algorithm’s performance. Such criteria include computational time, quality of solution, ease of implementation, and flexibility. Bräysy and Gendreau [10] state that flexibility is important criteria for algorithms that are designated to to be used in real-world problems, as an algorithm should be able to handle changes to the constraints and objective function. There is generally a trade-off between computational time and solution quality. This characteristic is a key feature of metaheuristics.

Tan et al. [54] compare the three popular meta heuristic methods with one another and concludes that there is no single heuristic that is generic enough to solve problems for all situations. Instead, they are inevitably problem specific. Each of the three popular metaheuristics has its advantages and disadvantages.

The Simulated Annealing (SA) methodology is similar to the annealing process of solids. When a metal is heated to high temperatures it is structurally weak and unstable. If the metal is allowed to cool slowly, it orders itself into a stable, structurally strong configuration. This process is called annealing. Kirkpatrick et al. [26] first proposed the use of SA for optimization, while Osman [41] applied it to vehicle routing and scheduling problems. The states of the solids correspond to the feasible solution, and the temperature at each state corresponds to the improvement in objective function, with the minimum temperature being the optimal solution. SA involves a process in which the temperature is gradually reduced during the simulation. Often, the system is first heated and then cooled. The system is given the opportunity to surmount energy barriers in a search for conformations with temperatures lower than the local-minimum temperature found by energy minimization. At each step of the simulation algorithm, a new state of the system is constructed from the current state by giving a random displacement to a randomly selected particle. If the energy associated with this new state was lower than the energy of the current state, the displacement was accepted, that is, the new state becomes the current state. This basic step, called a Metropolis step, can be repeated indefinitely. The whole procedure is called a Metropolis loop. Some of the choices that need to be made with a SA strategy are

- the expression used to calculate the initial temperature,
• the cooling function used for the reduction of temperature,

• the conditions for thermal equilibrium – where the temperature can be lowered, and

• the stopping criteria – usually a number of consecutive degrading values.

SA produces good solutions much faster than other metaheuristics, in terms of computational time, although the solution quality of the final solution is inferior.

Reeves [44] describes the Genetic Algorithm (GA) as the intelligent exploitation of a random search that was first presented by Holland [23]. The name originates from the analogy between the representation of a complex structure by means of a vector of components, and the idea of the genetic structure of a chromosome. GA is described as follows:

* A population of solutions are maintained and a reproductive process allows parent solutions to be selected from the population. Offspring solutions are produced which exhibit some of the characteristics of each parent. The fitness of each solution can be related to the objective function value, in this case the total distance travelled [4].

Analogous to biological processes, offspring with good fitness levels are more likely to survive and reproduce. Selection of the fittest ensures that fitness levels throughout the population will improve with evolution. The result is a population of chromosomes, in this case vehicle routes, with high performance characteristics. In most applications the component vector, or chromosome, is represented as a string of bits (0 and 1). Tan et al. [54] argue the case of replacing the binary digits with integer digits. A chromosome can now easier represent the order in which customers are visited. Genetic operators are used to improve the quality of the population. The most common genetic operators used to manipulate theses chromosomes are crossover and mutation. Crossover is the exchange of sections of the parents’ chromosomes, while mutation is the random modification of the chromosome.

Researchers have explored the adaptations of GA to form hybrid-GA [54, 59]. The highest potential for hybrid-GA seems to be when the basic principles of GA are combined with adaptive memory features introduced by Taillard et al. [52]. This flexibility of the hybrid-GA makes it a very popular heuristic and holds promising prospects for the application of GA. Thangiah [58] recognizes that GA does not perform well for problems in which customers are geographically clustered together.
The Tabu Search metaheuristic (TS), a memory based search strategy, deals with the problem of being trapped at a local optimum by temporarily forbidding moves that would return to a solution recently visited (cycling). The result is that the tabu search heuristic prevents short-term cycling, although solutions can be repeated over a longer period of time. This heuristic was proposed in its present form by Glover [21] and has been applied to many optimization problems besides vehicle routing. A tabu list is used to record these forbidden moves, which means that each iteration chooses a non-tabu move. After each step, a collection of moves that includes any immediate return to the previous point is added to the tabu list [43]. These recently added moves are not allowed for a fixed number of iterations, but are eventually removed from the tabu list.

As with other metaheuristics, any particular iteration can either improve or degrade the objective function value. It is therefore important to continuously update the best feasible solution found so far, referred to as the incumbent solution. When the tabu search is complete, usually after a finite number of iterations have been completed, the incumbent solution is accepted as the final solution: an approximation of the optimum solution. It is important to design the criteria of the TS carefully. The decision of which moves to add or remove from the tabu list is imperative as it has a direct effect on both the accuracy and computational time of the search. If too few moves are disallowed, it may lead to cycling; too many disallowed moves restrict the search from finding superior solutions quickly.

The tabu list contains record of three elements: the list position, the original route and the string of stops moved. The list is implemented as a queue that operated on a first-in, first-out principle. The memory of the tabu list can be recency or frequency based. The recency based list, called short-term memory, contains the last \( x \) number of moves the algorithm has encountered and sets them as tabu, assuming that the tabu list size is \( x \). The frequency based list, called long-term memory, complements the short-term memory by providing the additional information of how many times each tabu move have been attempted. Tan et al. [54] propose the use of a multi-functional list structure that serves both purposes of a recency and frequency-based list, and state that frequency-based memory provides better incentive as to the choice of the next move. Van Breddam [61] concludes that the use of long-term memory gives significantly worse solutions for the majority of problems. The impact of these contradictory opinions is that, should TS be used, the easier implementable option of short-term memory is preferred. A way of still retaining long-term memory is to increase the length of the tabu list.

Van Breddam [61] notes that unlike the SA heuristic, the performance of
the TS is highly dependent on the quality of the initial solution. The use of TS requires careful consideration of the mentioned characteristics of TS, as it should be chosen to best represent the problem environment of the project.

TS solutions are generally closest to optimal, but the computational time is about two to three times that of GA, and almost twenty times that of SA. The current computational power of modern computers makes the slower performance of TS less of a problem. Tan et al. [54] report that TS was able to solve 56 problem instances, each containing 100 customers, in an average of 1,500 seconds. Carriers furthermore require an operational scheduling system on a daily basis, with real-time scheduling as a future functionality.

2.4 Conclusion

The term City Logistics refer to the process of optimizing urban freight movement in a multi-actor environment. Changes occur in the environment, and stakeholders have conflicting perceptions with regards to how the changes will impact them, and their co-stakeholders. It is not contested that fleet optimization, in the form of vehicle routing and scheduling, will have advantages for all stakeholders. Literature indicates that, to address the specific needs and perceptions of logistic stakeholders, more realistic models are required to manage and predict factors influencing urban networks. Realistic models require the introduction of additional constraints to the solution space and results in problems being harder to solve. The quality of the final solution is impacted by the quality of the initial solution generated through heuristic methods.

Chapter 3 introduces a new approach, time window compatibility, to improve the quality of the initial solution. The aim is to have a mechanism that will assist in identifying seed customers in an innovative way during the route building process, and also ease the computational burden.
Chapter 3

Time window compatibility

The purpose of this chapter is to introduce a new concept, Time Window Compatibility (TWC). A matrix, referred to as the Time Window Compatibility Matrix (TWCM), is used as the mechanism to calculate the compatibility between all nodes in the network. A brief motivation for the concept is given. Various scenarios of the concept exist, and are illustrated individually. An attempt is made to define the concept in a generalized form.

An example is given to illustrate the use of the concept. Other than reducing the computational burden of existing heuristics, the example also illustrates the opportunity to improve the quality of the initial solution. A modified copy of the concepts discussed in this chapter have been communicated the the European Journal of Operational Research.

3.1 Motivation for a new approach

A shortcoming of the Sequential Insertion Heuristic (SIH) of Solomon [46] is that it considers all unrouted nodes when calculating the insertion and selection criteria for each iteration. The fact that all unrouted nodes are considered makes it computationally expensive. The VRP variant considered in this dissertation has multiple additional constraints. The occurrence of obvious infeasible nodes in a partially constructed route therefore becomes significant. The introduction of the time window compatibility concept assists in identifying, and eliminating, the obvious infeasible nodes. This results in a more effective and robust route construction heuristic.

3.2 Time window compatibility defined

The purpose of TWC is to determine the time overlap of all edges, or node combinations, \((i, j)\), where \(i, j \in \{0, 1, 2, \ldots, N\}\), and \(N\) the total number of nodes in the network. During the route construction phase, time win-
dow compatibility can be checked, and obvious infeasible nodes can be eliminated from the set of considered nodes. The TWCM is a non-symmetrical matrix as the sequence of two consecutive nodes, \( i \) and \( j \), is critical.

Let:

- \( N \) be the total number of nodes
- \( e_i \) be the earliest allowed arrival time at customer \( i \), where \( i = \{0, 1, \ldots, N\} \)
- \( l_i \) be the latest allowed arrival time at customer \( i \), where \( i = \{0, 1, \ldots, N\} \)
- \( s_i \) be the service time at node \( i \), where \( i = \{0, 1, \ldots, N\} \)
- \( t_{ij} \) be the travel time from node \( i \) to node \( j \), where \( i, j = \{0, 1, \ldots, N\} \)
- \( a_{ij}^{e_i} \) be the actual arrival time at node \( j \), given that node \( j \) is visited directly after node \( i \), and that the actual arrival time at node \( i \) was \( e_i \), where \( i, j = \{0, 1, \ldots, N\} \)
- \( a_{ij}^{l_i} \) be the actual arrival time at node \( j \), given that node \( j \) is visited directly after node \( i \), and that the actual arrival time at node \( i \) was \( l_i \), where \( i, j = \{0, 1, \ldots, N\} \)
- \( TWC_{ij} \) be the time window compatibility when node \( i \) is directly followed by node \( j \)

\( TWC_{ij} \) indicates the entry in row \( i \), column \( j \) of the TWCM. Consider the following five scenarios illustrating the calculation of the time window compatibility. Each scenario assume customer \( j \) to be serviced directly after customer \( i \), a service time of one hour, and a travel time of two hours from node \( i \) to node \( j \).

**Scenario 1:** if \( a_{ij}^{e_i} > e_j \) and \( a_{ij}^{l_i} < l_j \), illustrated in figure 3.1. Customer \( i \)

![Time window compatibility scenario 1](image)

Figure 3.1: Time window compatibility scenario 1

specifies a time window \([e_i, l_i] = [08:00, 12:00]\), while customer \( j \) requires service during the time window \([e_j, l_j] = [09:00, 16:00]\). If service
at customer $i$ starts at the earliest allowed time, $e_i$, then the actual arrival time at customer $j$ would be calculated as

$$a_j^{ei} = e_i + s_i + t_{ij}$$  \hspace{1cm} (3.1)

In this scenario $a_j^{ei} = 11:00$. Similarly, $a_j^{li}$ would be the actual arrival time at customer $j$, given that the actual arrival time at customer $i$ was $l_i$, and is calculated as

$$a_j^{li} = l_i + s_i + t_{ij}$$  \hspace{1cm} (3.2)

The difference between $a_j^{ei}$ and $a_j^{li}$ indicates the time window overlap between the two nodes. The time window compatibility is calculated as

$$TW C_{ij} = a_j^{li} - a_j^{ei}$$  \hspace{1cm} (3.3)

For this example, the time window compatibility is four hours (04:00).

**Scenario 2:** if $a_j^{ei} > e_j$ and $a_j^{li} > l_j$, illustrated in figure 3.2. Customer $i$

![Figure 3.2: Time window compatibility scenario 2](image)

specifies a time window $[e_i, l_i] = [08:00, 12:00]$, while customer $j$ requires service during the time window $[e_j, l_j] = [09:00, 13:00]$. The calculations for $a_j^{ei}$ and $a_j^{li}$ are similar to (3.1) and (3.2), respectively. The time windows of customer $i$ and customer $j$ only partly overlap, and the time window compatibility is calculated as

$$TW C_{ij} = l_j - a_j^{ei}$$  \hspace{1cm} (3.4)

For this example, the time window compatibility is two hours (02:00).

**Scenario 3:** if $a_j^{ei} < e_j$ and $a_j^{li} < l_j$, illustrated in figure 3.3. Customer $i$ specifies a time window $[e_i, l_i] = [08:00, 12:00]$, while customer $j$ requires service during the time window $[e_j, l_j] = [12:00, 16:00]$. The
calculations for \( a_j^{e_i} \) and \( a_j^{l_i} \) are similar to (3.1) and (3.2), respectively. The time windows of customer \( i \) and customer \( j \) only partly overlap, and the time window compatibility is calculated as

\[
TW C_{ij} = a_j^{l_i} - e_j
\]  

(3.5)

For this example, the time window compatibility is three hours (03:00).

**Scenario 4:** if \( a_j^{e_i}, a_j^{l_i} < e_j \), illustrated in figure 3.4. Customer \( i \) specifies a time window \([e_i, l_i] = [08:00, 12:00]\), while customer \( j \) requires service during the time window \([e_j, l_j] = [17:00, 18:00]\). The calculations for \( a_j^{e_i} \) and \( a_j^{l_i} \) are similar to (3.1) and (3.2), respectively. The time windows of customer \( i \) and customer \( j \) do not overlap. Even if customer \( i \) is serviced as late as possible, \( l_i \), a waiting time is incurred at customer \( j \). The time window compatibility is calculated as

\[
TW C_{ij} = a_j^{l_i} - e_j
\]  

(3.6)

For this example, the time window compatibility is negative two hours (-02:00). The significance of the negative time is that it is possible, in
In this case, to service customer \( j \) after customer \( i \), although the waiting time is penalized.

**Scenario 5:** If \( a_j^{c_i}, a_j^{l_i} > l_j \), illustrated in figure 3.5. Customer \( i \) specifies

![Diagram showing a time window with nodes and time intervals for Scenario 5](image)

**Figure 3.5:** Time window compatibility scenario 5

a time window \([e_i, l_i] = [08:00, 12:00]\), while customer \( j \) requires service during the time window \([e_j, l_j] = [07:00, 11:00]\). The calculations for \( a_j^{c_i} \) and \( a_j^{l_i} \) are similar to (3.1) and (3.2), respectively. Although the time windows of customer \( i \) and customer \( j \) partly overlap, it is impossible to service customer \( j \), even if customer \( i \) is serviced as early as possible, \( e_i \). Therefore, no time window compatibility exist.

A generalized equation is proposed that will address all five scenarios illustrated, and is given as

\[
TW_{C_{ij}} = \begin{cases} 
\min\{a_j^{l_i}, l_j\} - \max\{a_j^{c_i}, e_j\}, & \text{if } l_j - a_j^{c_i} > 0 \\
-\infty, & \text{otherwise}
\end{cases} \tag{3.7}
\]

The higher the value, the better the compatibility of the two time windows considered. Therefore an incompatible time window is defined to have a compatibility of negative infinity.

**Example.** Consider the following example with five nodes geographical distributed around a depot in figure 3.6. In the example, node \( c \) has indicated two possible time windows. As discussed in section 2.3.1, the customer is artificially split and treated as two separate nodes, \( c^1 \) and \( c^2 \), respectively. The time windows for each customer, including the depot, as well as the service time at each node, are given in table 3.1.

The distance matrix, \( \overline{D} \), is calculated using the rectangular distance
Figure 3.6: Geographical distribution of nodes around a depot

Table 3.1: Time windows and service times

<table>
<thead>
<tr>
<th>Node</th>
<th>Time window</th>
<th>Service time (in hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>((e_i; l_i))</td>
<td>(s_i)</td>
</tr>
<tr>
<td>Depot</td>
<td>07:00 - 18:00</td>
<td>0.00</td>
</tr>
<tr>
<td>a</td>
<td>08:00 - 12:00</td>
<td>0.50</td>
</tr>
<tr>
<td>b</td>
<td>11:00 - 13:00</td>
<td>0.25</td>
</tr>
<tr>
<td>c^1</td>
<td>08:00 - 09:00</td>
<td>0.25</td>
</tr>
<tr>
<td>c^2</td>
<td>15:00 - 17:00</td>
<td>0.25</td>
</tr>
<tr>
<td>d</td>
<td>08:00 - 12:00</td>
<td>0.50</td>
</tr>
<tr>
<td>e</td>
<td>10:00 - 15:00</td>
<td>0.25</td>
</tr>
</tbody>
</table>
between nodes, as taken from figure 3.6.

\[ \mathbf{D} = \begin{bmatrix}
0 & 60 & 60 & 50 & 50 & 70 & 60 \\
60 & 0 & 20 & 70 & 70 & 110 & 120 \\
60 & 20 & 0 & 50 & 50 & 110 & 120 \\
50 & 70 & 50 & 0 & 0 & 80 & 90 \\
50 & 70 & 50 & 0 & 0 & 80 & 90 \\
70 & 110 & 110 & 80 & 80 & 0 & 70 \\
60 & 120 & 120 & 90 & 90 & 70 & 0
\end{bmatrix} \]

If the average speed is known, the time matrix, \( \mathbf{T} \), can be calculated. In this example, \( \mathbf{T} \) is given. Values are given in hours.

\[ \mathbf{T} = \begin{bmatrix}
0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0.5 & 1 & 1 & 2 & 2 \\
1 & 0.5 & 0 & 1 & 1 & 2 & 2 \\
1 & 1 & 1 & 0 & 0 & 1.5 & 1.5 \\
1 & 1 & 1 & 0 & 0 & 1.5 & 1.5 \\
1 & 2 & 2 & 1.5 & 1.5 & 0 & 1 \\
1 & 2 & 2 & 1.5 & 1.5 & 1 & 0
\end{bmatrix} \]

With the information at hand, the time window compatibility matrix can be calculated. For the given example,

\[ \mathbf{TWCM} = \begin{bmatrix}
11 & 4 & 2 & 1 & 2 & 4 & 5 \\
4 & 3.5 & 2 & -\infty & -1.5 & 1.5 & 4 \\
2 & 0.25 & 1.75 & -\infty & -0.75 & -\infty & 1.75 \\
1 & 1 & -0.75 & 0.75 & -5.75 & 1 & 0.75 \\
1.75 & -\infty & -\infty & -\infty & 1.75 & -\infty & -\infty \\
4 & 1.5 & 2 & -\infty & -1 & 3.5 & 3.5 \\
5 & -\infty & 0.75 & -\infty & 1.75 & 0.75 & 4.75
\end{bmatrix} \]

### 3.3 Using the time window compatibility matrix

The time window compatibility matrix, \( \mathbf{TWCM} \), is calculated before the route building heuristic is evoked.

#### 3.3.1 Reducing computational complexity

Solomon [46] calculates the insertion and selection criteria by means of (2.17) and (2.25), respectively. In each iteration, these criteria are calculated for each edge on the partially constructed route, irrespective of the compatibility of the time window of the node considered for insertion with the time windows of the two nodes forming the edge.
This chapter presents an improved case. Consider the example where node \( u \) is considered for insertion between nodes \( i \) and \( j \). As the TWCM is already calculated, it is possible to check the compatibility of node \( u \) with the routed nodes \( i \) and \( j \). If either \( TWCM_{iu} \) or \( TWCM_{uj} \) is negative infinity \((-\infty)\), indicating an incompatible time window, the insertion heuristic moves on and considers the next edge, without wasting computational effort on calculating the insertion and selection criteria.

In the earlier example, eleven instances of infeasible time windows occur. If these instances are identified and eliminated, a computational saving in excess of 22% is achieved.

### 3.3.2 Identifying the seed customer

When looking at the TWCM for the example, it is clear that the incompatibility is distinct for specific nodes. It is therefore possible to identify *incompatible* nodes. As opposed to the two most common initialization criteria, namely *customer with earliest deadline*, and *furthest customer*, as suggested by Dullaert [19], this dissertation proposes the use of the TWCM to identify seed nodes based on their time window compatibility.

Table 3.2 indicates the number of instances where a node has an infeasible time window with another node, either as origin, or as destination. Both

<table>
<thead>
<tr>
<th>Node</th>
<th>Number of infeasible time windows, as origin</th>
<th>Number of infeasible time windows, as destination</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depot</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( a )</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( b )</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( c^1 )</td>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>( c^2 )</td>
<td>5</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( d )</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( e )</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

nodes \( c^1 \) and \( c^2 \) have five infeasible instances. The two artificial nodes are representing the same customer \( c \). It can be concluded that customer \( c \) is the most incompatible node, and is therefore identified as the seed customer. Ties are broken arbitrarily. Should two nodes have the same number of infeasible time window instances, any of the two customers could be selected as seed customer.
It may be possible to not have any infeasible time window instances. In such a scenario, a total compatibility value can be determined for each node \( a \), and is calculated as

\[
\sum_{i=1,i \neq a}^{M} TW C_{ia} + \sum_{j=1,j \neq a}^{M} TW C_{aj} + TW C_{aa}
\]  

(3.8)

or

\[
\sum_{i=1}^{M} TW C_{ia} + \sum_{j=1}^{M} TW C_{aj} - TW C_{aa}
\]  

(3.9)

where \( M \) refers to all the unrouted nodes, including all instances of those nodes that are split artificially. The customer with the lowest total compatibility is selected as seed customer.

### 3.4 Conclusion

A new concept, Time Window Compatibility (TWC) is introduced in this chapter. A matrix, referred to as the Time Window Compatibility Matrix (TWCM), is used as the mechanism to calculate the compatibility between all nodes in the network. A numerical example illustrates the use of the concept to reduce the computational burden of existing heuristics, and proposes an improved criteria for seed customer selection.

The time window compatibility is used in the next chapter when the formal model and initial solution algorithm is defined.
Chapter 4

Model definition

The purpose of the chapter is to depart on the first leg of the journey towards obtaining an initial solution to the extended vehicle routing problem, currently referred to as the Vehicle Routing Problem with Multiple Constraints (VRPMC). The model development process used to develop the model is taken from Taniguchi et al. [57] and is presented in figure 4.1.

Problem definition – The conceptual problem is defined in chapter 1.

Objective – As the model is concerned with determining an initial solution to a routing and scheduling problem, the result produced by the algorithm becomes the objective of the model. The choice of solution candidates are influenced by the mathematical objective function of the problem model.

Criteria – To elaborate on the criteria, a comprehensive mathematical model of the VRPMC is presented in section 4.1. The criteria define the solution space through multiple mathematical constraints.

System analysis – The analysis process involves identifying the essential components and interaction within the solution algorithm. Section 4.2.1 describe the interaction of the algorithm’s logical processes at a high level.

System synthesis – Although Taniguchi et al. [57] specify that this involves expressing the model in mathematical terms, it was already formulated in chapter 2, and presented in its entirety in section 4.1. Synthesis, in this dissertation, is the process of constructing and documenting a robust algorithm that will serve as direct input to the coding stage.

Software development – A computer based procedure will be developed in MATLAB. This will allow the mathematical and logical procedures, developed during the synthesis stage, to be used to produce actual
quantitative results. The software development is further discussed in chapter 5.

Verification – Procedures are tested and checked for correct logical structure. This iterative process makes use of manually simulated and calculated instances, and compares the algorithm’s output with its anticipated behavior.

Validation – At this stage the algorithm’s output is compared with published results. The objective of validation is to determine if the initial solution created by the algorithm is comparable with those generated by accepted algorithms, as the result should only be marginally better, or worse, than previously published results.

Application – The algorithm will be tested in parallel with current scheduling applications. Given the nature of the algorithm, and the fact that
the output is only an initial solution, and will act as an input to an optimization algorithm, the quality of the algorithm’s output can not be compared with that of the final algorithm.

4.1 The mathematical model definition

All variables and concepts are defined in chapter 2, and the mathematical model is therefore presented without any declaration of variable or explanation of constraint.

\[
\begin{align*}
\min z &= \sum_{i=0}^{N} \sum_{j=0,j\neq i}^{N} \sum_{k=1}^{K} c_{ij} x_{ijk} + \sum_{k=1}^{K} \sum_{j=1}^{N} f_k x_{0jk} \\
&\quad + \sum_{i=1}^{N} \alpha_i \times \max\{0, L_i\} \\
\text{subject to} \\
\sum_{j=1}^{N} x_{0jk} &= \sum_{j=1}^{N} x_{j0k} = 1 \quad \forall k = \{1, 2, \ldots, K\} \quad (4.2) \\
\sum_{j=1}^{N} \sum_{k=1}^{K} x_{0jk} &\leq K \quad (4.3) \\
\sum_{i=1,i\neq j}^{N} \sum_{k=1}^{K} x_{ijk} &= 1 \quad \forall j = \{1, 2, \ldots, N\} \quad (4.4) \\
\sum_{j=1,j\neq i}^{N} \sum_{k=1}^{K} x_{ijk} &= 1 \quad \forall i = \{1, 2, \ldots, N\} \quad (4.5) \\
\sum_{i=1}^{N} q_i \sum_{j=0,j\neq i}^{N} x_{ijk} &\leq p_k \quad \forall k = \{1, 2, \ldots, K\} \quad (4.6) \\
a_0 = w_0 = s_0 = 0 \\
\sum_{k=1}^{K} \sum_{i=0;i\neq j}^{N} x_{ijk}(a_i + w_i + s_i + t_{ij}) &\leq a_j \quad \forall j = \{1, 2, \ldots, N\} \quad (4.8) \\
e_i &\leq (a_i + w_i) \leq l_i \quad \forall i = \{1, 2, \ldots, N\} \quad (4.9) \\
x_{ijk} &\in \{0, 1\} \quad (4.10)
\end{align*}
\]
4.2 System analysis

It is the objective of this dissertation to promote the use of a systematic approach to model development, as opposed to the rapid-prototyping approach often experienced in practise. To ensure that the algorithm acts in a coherent and logical manner, the algorithm is modelled at various levels prior to being coded.

4.2.1 Overview

A graphical overview of the algorithm is presented in figure 4.2. The

![Flowchart Diagram]

Figure 4.2: Overview of initial solution algorithm

number in the lower right-hand corner of a procedure, or decision in the
flowchart, refers to the sequence of discussions in the following subsection with regards to specific algorithm detail that are highlighted.

4.2.2 Algorithm detail

Sections of the overview model is represented using Structured English – a language and syntax, based on the relative strengths of structured programming, and natural English [64]. Structured English is not pseudocode, as it does not concern itself with the declaration and initialization of variables, linking, and other technical issues. The Structured English sections aims to communicate unambiguous logic about the algorithm which is easy to understand, yet not open to misinterpretation [3]. Readability takes preference over programming preferences. It is a strict and logical form of English, and the following constructs reflect structured programming:

- **Sequencing** shows the order of processing a group of instructions – simple, declarative sentences, following one another – without repetition and branching. Compound sentences are avoided, as they create ambiguity. Strong action verbs, such as GET, FIND, CALCULATE, UPDATE, SORT, etc. are used.

- **Selection or decision structure** facilitates the choice of actions under well-specified conditions. Variations of sequencing include:
  - the IF-THEN-ELSE construct specifies the actions that must be taken if a specific condition, or set of conditions, are all true.
  - The CASE construct is an elegant substitute for multiple IF-THEN-ELSE statements. The CASE construct is used where there are more than two sets of actions, based on well-specified conditions, to choose from.

- **Iteration or repetition** facilitates the same action, or set of actions, to be carried out a number of times. Two variations are
  - The REPEAT-UNTIL construct indicates that certain actions are repeated one or more times, based on the value of a stated condition.
  - The DO-WHILE construct indicates that certain actions are to be repeated zero, one, or more times, based on the value of a stated condition. Note that this construct need not be executed, as opposed to the REPEAT-UNTIL construct that will execute the set of actions at least once.

Blocking and indentation are used to indicate the beginning and end of constructs, as opposed to terms such as ENDIF, ENDCASE, ENDDO and ENDREPEAT, as these give the algorithm too much of a programming look
and feel. Uppercase terms in the algorithm with italicized bold typeface indicate a variable set that is used in the coding of the algorithm. These sets are treated in square brackets in the document text, for example [VEHICLE] indicates the set of vehicles. The row numbers on the left indicate the line number in the complete algorithm. The algorithm presented in the dissertation is aggregated to eliminate unnecessary technical information helpful during the programming of the algorithm, hence the irregular numbering.

Figure 4.3 describes the capturing of input information. A list of all the technical field names appear in Appendix A.

1. Capture input information
2. Capture vehicle information in \textbf{VEHICLES}
3. Set average speed as 55 km/h
4. Sort available vehicles
5. Clear and set \textbf{VEHAVAIL} as an available vehicle matrix
6. for all available vehicles in \textbf{VEHICLES}
7. Add vehicle to \textbf{VEHAVAIL}
8. Sort \textbf{VEHAVAIL} in ascending order on \textit{volumetric capacity}
9. Capture general \textbf{CUSTOMER} information
10. Capture customer information in \textbf{CUSTOMER}
11. for each entry, \( i \), in \textbf{CUSTOMER}
12. if \textbf{CUSTOMER} has multiple time windows
13. Split customer into \textit{customer}(\( i \),\textit{tv} artificial customers
14. Add \textit{artificial customer} to \textbf{ARTIF}
15. Capture the time window information for each \textbf{ARTIF}icial customer
16. else
17. Add the \textbf{CUSTOMER} as a single \textbf{ARTIF}icial customer
18. Capture the time window information for the single \textbf{ARTIF}icial customer
19. Calculate the \textbf{DIST}ance matrix between all the \textbf{ARTIF}icial nodes

Figure 4.3: Capture input information

The depot is captured as the first customer. If a customer specifies more than one time window, the customer is artificially split into \( n \) customers, each with a single time window, where \( n \) indicate the number of time windows specified. Once the customers are split artificially, reference will only be made to nodes – with each node indicating an artificial customer in the \textbf{ARTIF} set. Figure 4.4 describes the initialization process.

29. \textbf{Initialise algorithm}
30. Set the \textbf{ROUTED} matrix as empty
31. for all the \textbf{ARTIF}icial nodes, except the depot (node 1)
32. Add the \textbf{ARTIF}icial node to the \textbf{UNROUTED} matrix

Figure 4.4: Initialize algorithm

52
If there are vehicles available, a new tour is created. A tour can be made up of one or more routes. The initialization of a tour involves assigning the smallest available vehicle to the tour, and matching the tour capacity to that of the vehicle. This is indicated in figure 4.5.

44 Initialise TOUR
45 Set the TOUR index (i) to 1
46 Establish the starting time for the TOUR
47 Starting time for the current TOUR is $t_d + s_d$
48 (It is assumed that vehicles are not loaded at the beginning of the depot’s time window)
49 Assign vehicle to TOUR
50 Set the first vehicle in VEHAVAIL as the current vehicle for the TOUR
51 Update vehicle availability
52 Locate the current vehicle in VEHICLE
53 Set vehicle(k).availability = 0
54 Recalculate VEHAVAIL

Figure 4.5: Initialize new tour

Once a tour has been created, one or more routes are established to make up the route. The iterative route creation process starts with the initialization of a new route. This entails assigning the route to the current tour, adding the depot as first and last node on the route, and identifying and inserting the seed customer: the first customer, other that the depot, to be added onto the route. The theory behind determining the seed customer has been elaborated upon in section 3.3. The algorithmic procedure for route initialization is indicated in figure 4.6. Nodes in the [UNROUTED] set are evaluated for insertion on the partially constructed route. The iterative route-building procedure is indicated in figure 4.7.

The concept of scheduling a vehicle to complete multiple routes (referred to as double scheduling), is difficult to implement in solution algorithms. The procedure followed in this dissertation to determine multi-route feasibility in a tour, is indicated in figure 4.8. When a vehicle returns to the depot at the end of a route, the multi-route feasibility check procedure determines if the depot’s time window is still open after the vehicle’s capacity has been replenished/renewed. It might be realistic to add some time to the potential route to allow the vehicle to at least service one node. The additional time added, conveniently referred to as minimum route time parameter, is different for each environment, and has been set to one hour in this dissertation. The effect will be that an empty route may be assigned to a number of tours when the initial solution is presented. To overcome the effect of empty routes, the final reporting procedure have been adapted to check for empty routes prior to reporting the initial solution. The procedure in indicated in figure 4.9.
Initialise `ROUTE` with seed customer
Set `ROUTE` index (`r`) to 1
Assign current `ROUTE` to current `TOUR`
Establish the starting time for the `TOUR`
Set `ROUTE` load to zero

Assign the depot as starting and ending node for the current `ROUTE`
Select a seed customer from the `UNROUTED` nodes
Calculate the time window compatibility matrix (`TWCM`) for all `UNROUTED` nodes
for each node combination `(a,b)` where node `b` is serviced after node `a`
Calculate the earliest possible arrival at `b` as `arrival_earliest`
Calculate the latest possible arrival at `b` as `arrival_latest`
if the earliest possible arrival at `b` is before the latest allowed arrival at `b`
Calculate time window compatibility (`TWC`)
\[ TWC_{ab} = \min (arrival_{latest}, t_b) - \max (arrival_{earliest}, a_b) \]
else
`TWC` is negative infinity

Calculate the number of infeasible time windows for each `UNROUTED` node
for each `UNROUTED` node (`i`)
Determine how many times in row `i` of `TWCH` is `TWC` negative infinity
Determine how many times in column `i` of `TWCH` is `TWC` negative infinity
Calculate the total number of infeasibilities by adding row and column count
if there are infeasible time windows for any `UNROUTED` node
The seed customer is the node with the most number of infeasible time windows
else
Calculate the `COMPATIBILITY` vector
for each `UNROUTED` node (`a`) in the `TWCM`
row = `TWCM`(a,:)
column = `TWCM`(:,a)
\[ \text{compatibility}(a) = \text{sum}(\text{row}) + \text{sum}(\text{column}) - \text{TWCM}(a,a) \]
The seed customer is the node with the lowest `COMPATIBILITY`

Insert seed customer
Insert seed customer on current `ROUTE`
Update `UNROUTED` customers
Remove seed customer from `UNROUTED`
Remove any other artificial nodes related to seed customer from `UNROUTED`
Update `ROUTE` load

Figure 4.6: Initialize new route
Expand partial ROUTE

while UNROUTED is not empty and there are customers that fit into the current ROUTE

Clear the node selection matrix C2

for each UNROUTED node (u)

Clear the node insertion matrix C1

Select the best position to insert node u on the current ROUTE

for each edge (i,j) on the current ROUTE

Determine feasibility to add node u

Infeasible if either TWC_i or TWC_j is unfeasible

Infeasible if TOUR capacity is exceeded by u

if it is feasible to evaluate node u between i and j

Update the C1 vector for the insertion positions

Calculate c_{i,j}(u,v)

Add the c_{i,j}(u,v) value to C1(i).value

else

Check next edge on current ROUTE

Select the best edge (i',j') based on the lowest C1 matrix value

Update the C2 matrix for the insertion position

Calculate c_{i',j'}(u,v)

Add the c_{i',j'}(u,v) value to the C2 matrix

Sort C2 in ascending order

Find first time-feasible node (u*), starting at the beginning of C2

While no u* has been found, and end of C2 has not been reached

Check for time feasibility

if feasible

Identify applicable node as u*

else

Check next element of C2

if a unique u* node has been identified

Insert node u*

Update UNROUTED customers

Remove u* from UNROUTED

Remove any other artificial nodes related to u* from UNROUTED

Update ROUTE

Update ROUTE load

if new vehicle has been indicated

if Q^new > Q

Find the smallest available vehicle to service Q^new

Update VEHAVAIL

Change the availability status of the current vehicle to available

Change the availability status of the new vehicle to unavailable

Assign new vehicle to current TOUR

Recalculate VEHAVAIL

Recalculate ROUTE schedule for nodes

Actual start-time at origin (a_x) is the start-time indicated for the current route

for each node (y) on the current ROUTE, except the depot at both ends

a_y = max [a_x, a_{x+1} + s_{x+1} + t_{x+1,y}]

w_y = max [0, t_{x,y} - (a_x + s_x + t_{x,y})]

Calculate actual arrival at the depot (a_y) at the end of the current ROUTE

a_y = a_{y-1} + s_{y-1} + t_{y-1,y}

else

Initialize new ROUTE

Figure 4.7: Expand partially constructed route
Expand TOUR

Determine multi-route feasibility

Check the actual arrival time at the depot of the previous ROUTE of the current TOUR \( a_{k} \)

if \( a_{k} + s_{k} + 1 \) hour < \( l_{k}^{\text{new}} \)

then feasible

else

infeasible

if feasible

Initialize new ROUTE

else

if the last ROUTE of the TOUR has no nodes other than the depot

Eliminate ROUTE from TOUR

else

Initialize new TOUR

Figure 4.8: Checking for multi-route feasibility

Define ORPHANs

if UNROUTED is not empty

Assign all elements in UNROUTED to ORPHANs

Clear UNROUTED

Report initial solution

Calculate the Objective function value for the initial solution

Report initial solution

for each TOUR

Report all TOUR and ROUTE information

Figure 4.9: Report initial solution
4.3 Conclusion

The chapter introduced the model development process. The objectives and criteria are stipulated in the mathematical definition of the problem. This chapter elaborates on the system analysis and synthesis. The proposed initial solution algorithm is presented at a high level, with selective detail given in *Structured English*. The complete algorithm is presented in Appendix B. Chapter 5 discusses the implementation, and the results, of the proposed algorithm as coded in *MATLAB*. 
Chapter 5

Results

This chapter reports on the results of the initial solution algorithm, as programmed in MATLAB. The chapter begins with a discussion on how comparative data sets were established for computational purposes, along with a motivation for modifying existing data sets. The results are presented and discussed, focusing on the contribution of the time window compatibility on both the computational burden, and the quality of the initial solution.

5.1 The basic Solomon sets

Solomon [46] discusses the generation of data sets for the Vehicle routing and scheduling problems with time window constraints (VRPSTW), and indicates that the design of these data sets highlight several factors that affects the behavior of his routing and scheduling heuristics. The corresponding six data sets, referred to as R1, R2, C1, C2, RC1, and RC2, are often used and referred to in literature.

5.1.1 Geographical distribution

The data used for the customer coordinates and demands are based on the work of Christofides et al. [13], and are classified into one of three categories:

- Randomly distributed customers (denoted by an R prefix)
- Clustered customers (denoted by a C prefix)
- Semi-clustered customers (denoted by an RC prefix)

By semi-clustered is implied a random mix of both randomly distributed and clustered customers.
5.1.2 Scheduling horizon

The length of the route-time is regarded as a capacity constraint and, along with the vehicle capacities, limit the number of customers serviced by a specific vehicle. The short scheduling horizon problems are denoted by a “1” as a suffix. The problems denoted by a “2” suffix, on the other hand, have a large scheduling horizon, and along with the vehicle capacities, allow a larger number of customers to be serviced by a single vehicle.

5.2 Test data

Solomon’s data sets have become a benchmark for vehicle routing problem variants, although the sets are often used with some modification due to the specific variant’s particularity. The published data sets \[47\] assume a homogeneous fleet, indicate a given vehicle capacity, and assume infinite availability of vehicles. Furthermore, the sets only indicate a single time window for each geographically distributed customer.

The aim of the algorithm developed in this dissertation is to test the feasibility of integrating multiple time windows, a heterogeneous fleet, and double scheduling into an initial solution heuristic. As none of these three specific variants are addressed by the Solomon data sets, it was necessary to generate a unique data set to accommodate, and integrate, the problematic variants, yet still have resemblance to the familiar benchmarks in literature. The problem of developing an appropriate data set meant addressing both multiple time windows, and a heterogeneous fleet. Double scheduling is a function of the algorithm, and does not require any manipulation of the data sets.

5.2.1 Incorporating multiple time windows

It originally seemed appropriate to use two of the Solomon sets for a specific class of problem, each with 100 customers and a single, unique time window, to create a new set of 100 customers with multiple time windows. It turned out to be futile, as the time windows for different data sets of the same classification, as presented by Solomon \[47\], had very similar, if not exactly the same, time windows. The extended data sets presented by Homberger \[24\] are developed in the same manner as Solomon’s sets, but have sets with 200, 400, 600, 800, and 1000 customers.

For each of the six problem classes, an extended set with 200 customers from Homberger’s sets is used to create a new set with 100 customers, but with two time windows. Table 5.1 illustrates an excerpt from the data used to create a new set for the $C1$ class. In the Homberger data sets, the cus-
customer number, x-coordinate, y-coordinate, demand, service time, and the start- and end times of a single time window, are given. Table 5.1 indicates

<table>
<thead>
<tr>
<th>Customer</th>
<th>x</th>
<th>y</th>
<th>Time window 1</th>
<th>Time window 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$e_i^1$</td>
<td>$l_i^1$</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
<td>...</td>
<td>616</td>
<td>661</td>
</tr>
<tr>
<td>5</td>
<td>26</td>
<td>128</td>
<td>179</td>
<td>478</td>
</tr>
<tr>
<td>6</td>
<td>86</td>
<td>616</td>
<td>531</td>
<td>583</td>
</tr>
<tr>
<td>7</td>
<td>109</td>
<td>680</td>
<td>351</td>
<td>1011</td>
</tr>
<tr>
<td>8</td>
<td>32</td>
<td>79</td>
<td>655</td>
<td>721</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
the case where the single time windows of customers 101 through 200 have been used as second time windows for customers 1 through 100. For example, the time window for customer 4 is given as (616, 661), measured in minutes from 0:00 am. Customer 104's time window is given as (128, 195) in the original data set. Customer 104's time window now becomes the second time window for customer 4, while all other data about customers 101 through 200 is disregarded. Observe, however, that the two time windows specified for customer 5 overlap, and do not yield two unique time windows as is the case for customer 4.

Procedure 1 indicates the procedure used to manipulate the time windows from multiple sets into a single data set. Where time windows overlap,

**Procedure 1 Creating a data set with multiple time windows**

if either $e_i^1 > l_i^2$ or $e_i^2 > l_i^1$ then

Number of time windows is 2

$E_i^1 = \min\{e_i^1, e_i^2\}$

$L_i^1 = \min\{l_i^1, l_i^2\}$

$E_i^2 = \max\{e_i^1, e_i^2\}$

$L_i^2 = \max\{l_i^1, l_i^2\}$

else

Number of time windows is 1

$E_i^1 = \min\{e_i^1, e_i^2\}$

$L_i^1 = \max\{l_i^1, l_i^2\}$

end if

the new, single time window, is defined to start at the opening of the earlier
time window, and end at the closing of the later time window. After the manipulation of the data, the start of the first time window for customer \( i \) is denoted by \( E_1^i \), and the end of the time window by \( L_1^i \). Where only one time window exist, no second time window is specified. Alternatively, the start of the second time window for customer \( i \) is denoted by \( E_2^i \), and the end of the time window by \( L_2^i \).

5.2.2 Incorporating a heterogeneous fleet

Liu and Shen [31, 32] propose a specific fleet structure with the introduction of their insertion-based savings heuristic for a heterogeneous fleet. The proposed cost structure sees the cost of a vehicle more than doubles when its capacity doubles. Although Dullaert et al. [19] challenges the cost structure presented by Liu and Shen, they did not propose a new cost structure. It was therefore considered appropriate to use the given fleet composition as indicated in Table 5.2. It should be noted that Liu and Shen assumed an infinite number of each of the vehicles types. To accommodate the infinite number

<table>
<thead>
<tr>
<th>Problem class ( R1 )</th>
<th>Problem class ( R2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vehicle</strong></td>
<td><strong>capacity</strong></td>
</tr>
<tr>
<td>A</td>
<td>30</td>
</tr>
<tr>
<td>B</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>80</td>
</tr>
<tr>
<td>D</td>
<td>120</td>
</tr>
<tr>
<td>E</td>
<td>200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem class ( C1 )</th>
<th>Problem class ( C2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vehicle</strong></td>
<td><strong>capacity</strong></td>
</tr>
<tr>
<td>A</td>
<td>100</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
</tr>
<tr>
<td>C</td>
<td>300</td>
</tr>
<tr>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Problem class ( RC1 )</th>
<th>Problem class ( RC2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>vehicle</strong></td>
<td><strong>capacity</strong></td>
</tr>
<tr>
<td>A</td>
<td>40</td>
</tr>
<tr>
<td>B</td>
<td>80</td>
</tr>
<tr>
<td>C</td>
<td>150</td>
</tr>
<tr>
<td>D</td>
<td>200</td>
</tr>
<tr>
<td>E</td>
<td>500</td>
</tr>
<tr>
<td>F</td>
<td>1000</td>
</tr>
</tbody>
</table>

Table 5.2: Heterogeneous fleet data
of vehicles into the data sets generated for the algorithm, the number of vehicles for each type is said to be the number of vehicles needed to service the demand of all customers, if only that type of vehicle was available.

5.3 Results

Six data sets, one for each class of problem, were generated. The algorithm was coded in MATLAB 6.5 release 13. The algorithm was executed on three similar Pentium IV 1.6GHz computers, each with 256Mb RAM, with each class being executed at least once on each of the computers. For each class of problem, five runs of the algorithm were executed. The results presented in this chapter is in each case the average of the five runs. Appendix C contains the solution details with respect to the specific customers assigned to the various routes, the orphaned customers (if any), as well as the total scheduling distance.

A summary of the initial solutions generated by the proposed algorithm is presented in Table 5.3. The influence of the problem characteristics (depicted by the problem class) can be appreciated when comparing the average number of customers per route for the type 1 and type 2 classes. The type 1 classes, with a narrow scheduling horizon, has an average of 2.13 customers per route, while the type 2 classes with longer scheduling horizons have, on average, 9.38 customers per route.

It should be noted, however, that the average CPU time is extremely high. The reason for the computationally expensive CPU results is twofold:

- The algorithm was coded in MATLAB with the specific intent of the candidate to learn the software package. The code structure may therefore be inefficient, with numerous opportunity for code optimization.

<table>
<thead>
<tr>
<th>Problem class</th>
<th>Average CPU time (seconds)</th>
<th>Number of Tours</th>
<th>Number of Routes</th>
<th>Total scheduling distance (kilometers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>3180</td>
<td>18</td>
<td>71</td>
<td>11260</td>
</tr>
<tr>
<td>R2</td>
<td>31730</td>
<td>5</td>
<td>7</td>
<td>8722</td>
</tr>
<tr>
<td>C1</td>
<td>2170</td>
<td>11</td>
<td>22</td>
<td>7330</td>
</tr>
<tr>
<td>C2</td>
<td>65650</td>
<td>4</td>
<td>6</td>
<td>7240</td>
</tr>
<tr>
<td>RC1</td>
<td>1120</td>
<td>26</td>
<td>48</td>
<td>8706</td>
</tr>
<tr>
<td>RC2</td>
<td>6960</td>
<td>12</td>
<td>19</td>
<td>7748</td>
</tr>
</tbody>
</table>
The file format in which MATLAB was executed is a non-compiled 
*.M file, which is, from a computational point of view, significantly 
slower than a compiled *.dll file. The decision to not compile was 
influenced by compiling software availability at the time of testing the 
algorithm.

Although there is ample opportunity to improve the algorithms technical 
performance, the results, in terms of number of tours, number of routes per 
tour, as well as the actual scheduling distance, proves to have significant 
contribution to the field of vehicle routing problem algorithms. Figures 5.1 
through 5.6 indicate the cumulative CPU time for each class of problem.

5.4 Evaluating the contribution of TWC

It is important to evaluate the contribution that the proposed time win-
dow compatibility has on the results of the algorithm. For this purpose, a 
comparative control algorithm is created. The control algorithm differs only 
in two respects from the proposed algorithm:

- It does not evaluate nodes for time window compatibility when cal-
culating the insertion criteria, and therefore considers every node for 
insertion on every edge of a partially constructed route.

- As no time window compatibility is calculated for any node, the initial-
ization criteria is changed to identify the seed customer as the unrouted 
customer with the earliest deadline.

The control algorithm is executed for two extreme problem classes, namely 
clustered customers with a short scheduling horizon (C1), and uniformly dis-
tributed customers with a long scheduling horizon (R2). Five runs for each 
class were executed in a similar fashion to the proposed algorithm, with 
regards to the computers used, and the distribution of runs on the computers.

Figure 5.7 illustrates the significant improvement that the time window 
compatibility has on the computational burden for the C1 class of problems. 
The average CPU time is down from 10950 seconds to 2170, an improvement 
in excess of 80%. Not only did the time window compatibility reduce the 
computational burden, but it also improved the quality of the initial solution 
by almost 13%. By the quality of the solution is implied the total scheduling 
distance. The comparative results summary is given in Table 5.4.

Figure 5.8 indicates a computational saving of more than 24% for the 
R2 class problems, while the quality of the initial solution itself is improved 
by almost 13%. The R2 results correspond with the expectation that the
Figure 5.1: Cumulative progress for the $R_1$ class problem

Figure 5.2: Cumulative progress for the $R_2$ class problem
Figure 5.3: Cumulative progress for the $C1$ class problem

Figure 5.4: Cumulative progress for the $C2$ class problem
Figure 5.5: Cumulative progress for the RC1 class problem

Figure 5.6: Cumulative progress for the RC2 class problem
Figure 5.7: The effect of time window compatibility on the C1 class

Figure 5.8: The effect of time window compatibility on the R2 class
Table 5.4: Summary of comparative results

<table>
<thead>
<tr>
<th>Problem class</th>
<th>Average CPU time (seconds)</th>
<th>Number of Tours</th>
<th>Number of Routes</th>
<th>Total scheduling distance (kilometers)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 with TWC</td>
<td>2170</td>
<td>11</td>
<td>22</td>
<td>7330</td>
</tr>
<tr>
<td>C1 without TWC</td>
<td>10950</td>
<td>12</td>
<td>39</td>
<td>8876</td>
</tr>
<tr>
<td>R2 with TWC</td>
<td>31730</td>
<td>4</td>
<td>7</td>
<td>8722</td>
</tr>
<tr>
<td>R2 without TWC</td>
<td>41920</td>
<td>2</td>
<td>13</td>
<td>9990</td>
</tr>
</tbody>
</table>

*Time window compatibility* will have less of an impact if the scheduling horizon is relatively large, as is the case for the type 2 problems. Still, the computational saving is significant.

5.5 Research agenda

Chapter 1 states the research question. The objective is to attempt to create an initial solution that caters for multiple time windows, a heterogeneous fleet, as well as double scheduling. Although the results generated during this dissertation is computationally expensive, the aim has been achieved, and with unexpectedly high returns. The concept of *time window compatibility* has proved to have a staggering impact, especially for clustered customers, and problem areas where tight time windows apply.

The issue of the computational burden of the algorithm, and technical code optimization, will be addressed in future research to ensure that the value that is obtained from this research is realized in implementable applications. As part of the code optimization, one option would be, after evaluating for time window compatibility, to first evaluate for time feasibility before calculating the insertion and selection criteria. In the proposed algorithm, time feasibility is only evaluated after the selection criteria have been calculated.

Improving the initial solution into a final solution through meta heuristics is also also required before the algorithms can be implemented. In the development of the improvement heuristic, the concept of *time window compatibility* should again be introduced as a potential performance improvement tool.

A number of minor parameters in the calculation of the insertion and selection criteria have been taken directly from literature, based on their
relative performance. These parameters could be challenged to ensure that they contribute to the particularity of the proposed algorithm.

Initial solutions are only the first step in an optimization process such as the vehicle routing problems. The next step would be to create a number of unique (different) initial solutions. The value of having varying initial solutions is that the Tabu Search improvement meta heuristic, which is often used for vehicle routing problems, requires a set of unique initial solutions before being invoked. It is envisaged that a single customer will be removed from the original customer data list before invoking the initial solution algorithm. A simple insertion heuristic will then be used to insert the omitted customer into the generated solution after it is created. It is anticipated that if the process is repeated for different customers, different solutions will be generated.

5.6 Conclusion

City Logistics is concerned with the mobility of cities, and often aims to establish best practices and initiatives to improve the state of transport planning. The algorithm proposed in this dissertation contributes to the process of optimizing urban logistics as it proves that initiatives such as time windows and load factors can be planned for by shippers and carriers. The results prove that multiple variants of the vehicle routing problem can be integrated into the initial solution algorithm. The increased complexity is addressed by the time saving concept of time window compatibility, which proved to have a significant impact on both the computational burden, and the quality of the initial solution.
## Appendix A

### Technical fields

<table>
<thead>
<tr>
<th>Description</th>
<th>Field name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle's description</td>
<td>vehicle(k).description</td>
</tr>
<tr>
<td>Vehicle’s volumetric capacity</td>
<td>vehicle(k).cv</td>
</tr>
<tr>
<td>Vehicle’s weight capacity</td>
<td>vehicle(k).cw</td>
</tr>
<tr>
<td>Vehicle’s fixed cost</td>
<td>vehicle(k).f</td>
</tr>
<tr>
<td>Vehicle’s availability</td>
<td>vehicle(k).avail</td>
</tr>
<tr>
<td>Customer’s name</td>
<td>customer(i).name</td>
</tr>
<tr>
<td>Customer’s geographical position</td>
<td>customer(i).x</td>
</tr>
<tr>
<td>Customer’s geographical position</td>
<td>customer(i).y</td>
</tr>
<tr>
<td>Customer’s volumetric demand</td>
<td>customer(i).dv</td>
</tr>
<tr>
<td>Customer’s weight demand</td>
<td>customer(i).dw</td>
</tr>
<tr>
<td>Service time at customer</td>
<td>customer(i).st</td>
</tr>
<tr>
<td>Maximum lateness allowed at customer</td>
<td>customer(i).Lmax</td>
</tr>
<tr>
<td>Lateness penalty factor for customer</td>
<td>customer(i).alpha</td>
</tr>
<tr>
<td>Number of time windows for customer</td>
<td>customer(i).tw</td>
</tr>
<tr>
<td>Artificial customer index</td>
<td>customer(i).index</td>
</tr>
<tr>
<td>Artificial customer reference</td>
<td>customer(i).ref</td>
</tr>
<tr>
<td>Earliest allowed time of arrival at artificial customer</td>
<td>customer(i).earliest</td>
</tr>
<tr>
<td>Latest allowed time of arrival at artificial customer</td>
<td>customer(i).latest</td>
</tr>
<tr>
<td>Actual time of arrival at artificial customer</td>
<td>customer(i).actual</td>
</tr>
</tbody>
</table>
Appendix B

Complete algorithm

1. **Capture input information**
   - Capture vehicle information in `VEHICLES`
     - for each vehicle $k$
       - Capture vehicle registration number as `vehicle(k).description`
       - Capture vehicle's volumetric capacity as `vehicle(k).cv`
       - Capture vehicle's weight capacity as `vehicle(k).cw`
       - Capture vehicle's fixed cost as `vehicle(k).f`
       - Capture vehicle availability as `vehicle(k).avail`

2. Set average speed as 55 km/h
3. Sort available vehicles
   - Clear and set `VEHAvAIL` as an available vehicle matrix
     - for all available vehicles in `VEHICLES`
     - Add vehicle to `VEHAvAIL`
     - Sort `VEHAvAIL` in ascending order on <volumetric capacity>

4. Capture general `CUSTOMER` information
   - Capture customer information in `CUSTOMER`
     - for each customer $i$
       - Capture customer's name as `customer(i).name`
       - Capture customer's position as `customer(i).x` and `customer(i).y`
       - Capture customer's volumetric demand as `customer(i).dv`
       - Capture customer's weight demand as `customer(i).dw`
       - Capture service time at customer as `customer(i).st`
       - Capture maximum lateness at customer as `customer(i).Lmax`
       - Capture lateness penalty factor for customer as `customer(i).alpha`
       - Capture number of time windows for customer as `customer(i).tw`
for each entry, $i$, in CUSTOMER

if CUSTOMER has multiple time windows

Split customer into customer(i),tw artificial customers

Add artificial customer to ARTIF

Capture the time window information for each ARTIFcial customer

else

Add the CUSTOMER as a single ARTIFcial customer

Capture the time window information for the single ARTIFcial customer

Calculate the DISTance matrix between all the ARTIFcial nodes

Initialise algorithm

Set the ROUTED matrix as empty

for all the ARTIFcial nodes, except the depot (node 1)

Add the ARTIFcial node to the UNROUTED matrix

Initialise TOUR

Set the TOUR index ($t$) to 1

Establish the starting time for the TOUR

Starting time for the current TOUR is $e_d + s_d$

(It is assumed that vehicles are not loaded at the beginning of the depot's time window)

Assign vehicle to TOUR

Set the first vehicle in VEHAVAIL as the current vehicle for the TOUR

Update vehicle availability

Locate the current vehicle in VEHICLE

Set vehicle(k).availability = 0

Recalculate VEHAVAIL

Build TOUR

While UNROUTED is not empty

Initialise ROUTE with seed customer

Set ROUTE index ($r$) to 1

Assign current ROUTE to current TOUR

Establish the starting time for the TOUR

Set ROUTE load to zero

Assign the depot as starting and ending node for the current ROUTE

Select a seed customer from the UNROUTED nodes

Calculate the time window compatibility matrix (TWCM) for all UNROUTED nodes

for each node combination ($a, b$) where node $b$ is serviced after node $a$

Calculate the earliest possible arrival at $b$ as $arrival_{earliest}$

$e_a$ - the earliest allowed arrival at node $u$

$s_a$ - the service time at node $a$

$t_{ab}$ - the travel time between node $a$ and node $b$

arrival_{earliest} = e_a + s_a + t_{ab}$

Calculate the latest possible arrival at $b$ as $arrival_{latest}$

$I_a$ - the latest allowed arrival at node $a$
$s_a$ - the service time at node $a$
$t_{ab}$ - the travel time between node $a$ and node $b$
arrival$_{latest} = l_a + s_a + t_{ab}$
if the earliest possible arrival at $b$ is before the latest allowed arrival at $b$
Calculate time window compatibility (TWC)
$TWC_{ab} = \min \{arrival\_latest, l_b\} - \max \{arrival\_earliest, e_a\}$
else
TWC is negative infinity

Calculate the number of infeasible time windows for each UNROUTED node
for each UNROUTED node ($i$)
Determine how many times in row $i$ of TWC is TWC negative infinity
Determine how many times in column $i$ of TWC is TWC negative infinity
Calculate the total number of infeasibilities by adding row and column count

if there are infeasible time windows for any UNROUTED node
The seed customer is the node with the most number of infeasible time windows
else
Calculate the compatibility vector
for each UNROUTED node ($a$) in the TWC
row = TWC($a;i$)
column = TWC($i;a$)
compatibility($a$) = sum(row) + sum(column) - TWC($a,a$)
The seed customer is the node with the lowest compatibility

Insert seed customer
Insert seed customer on current ROUTE
Update UNROUTED customers
Remove seed customer from UNROUTED
Remove any other artificial nodes related to seed customer from UNROUTED
Update ROUTE load

Expand partial ROUTE
while UNROUTED is not empty and there are customers that fit into the current ROUTE
Clear the node selection matrix $C_2$
for each UNROUTED node ($u$)
Clear the node insertion matrix $C_1$
Select the best position to insert node $u$ on the current ROUTE
for each edge $(i,j)$ on the current ROUTE
Determine feasibility to add node $u$
Infeasible if either $TWC_{ib}$ or $TWC_{if}$ is unfeasible
Infeasible if TOUR capacity is exceeded by $u$
if it is feasible to evaluate node $u$ between $i$ and $j$
Update the $C_1$ vector for the insertion positions
Calculate $c_{11}(i,u,j)$
\( d_{ij} \) - the distance between nodes \( i \) and \( j \)

\[ c_{11}(i,u) = d_{ui} + \mu d_{ij} - \mu d_{ij}, \mu \geq 0 \text{ (in Dullaert, } \mu = 1) \]

Calculate \( c_{12}(l,u) \)

\( e_{l} \) - earliest allowed arrival at node \( l \)

\( w_{l} \) - the waiting time before node \( l \)

\( a_{l} \) - the actual start time before node node \( l \)

\( s_{l} \) - the service time at node \( l \)

\( t_{ij} \) - the travel time between nodes \( i \) and \( j \)

\( b_{l} \) - the original start of service at node \( l \)

\( b_{l}^{\text{new}} \) - the start of service at node \( l \) after node \( u \) has been inserted

\[ b_{u} = \max \{ e_{u}, a_{l} + s_{l} + t_{ul} \} \]

\[ b_{l} = a_{l} \]

\[ b_{l}^{\text{new}} = \max \{ e_{l}, b_{u} + s_{u} + t_{ul} \} \]

Calculate \( c_{12}(l,u) = b_{l}^{\text{new}} - b_{l} \)

Calculate \( c_{ij}(l,u) \)

Select to use either ACS, AOOS, or AROS

ACS

\( Q \) - the load of the current vehicle before node \( u \)

\( Q^{\text{new}} \) - the max capacity of the current vehicle before node \( u \)

\( Q^{\text{new}} \) - the new load of the vehicle after node \( u \)

\( Q^{\text{new}} \) - the new max capacity of the vehicle after node \( u \)

\( F(C) \) - the fixed cost of the smallest available vehicle that can service a demand of size \( C \) for a subtour

\[ ACS = F(Q^{\text{new}}) - F(Q) \]

if new vehicle is indicated

flag new vehicle number

AOOS

\[ AOOS = ACS - F(Q^{\text{new}} - Q^{\text{new}}) \]

if new vehicle is indicated

flag new vehicle number

AROS

\( F(C) \) - the fixed cost of the largest available vehicle whose capacity is less than or equal to \( C \)

\( P(x) \) - the capacity of the smallest available vehicle that can service a demand of \( x \)

\[ L = \begin{cases} 
  P(z_1+z_2) - P(\max(z_1,z_2)) & \text{if } \omega > 0 \text{, otherwise } 0 \end{cases} \]

\( \delta(\omega) = 1 \text{ if } Q^{\text{new}} > Q, \text{ otherwise } 0 \)

\[ AROS = ACS - \delta(\omega)F(Q^{\text{new}} - Q^{\text{new}}) \]

Calculate \( c_{13}(l,u) \) = any of ACS, AOOS, or AROS

if new vehicle is indicated

flag new vehicle number
\(\sigma_j\) - weight factors. The weight need not add up to 1
\[ c_{1j}(i,u,j) = \sigma_1 c_{11}(i,u,j) + \sigma_2 c_{12}(i,u,j) + \sigma_3 c_{13}(i,u,j) \]

Add the \(c_{1j}(i,u,j)\) value to \(C1(m)\).

else

Check next edge on current ROUTE.
Select the best edge \((i^*,j^*)\) based on the lowest \(C1\) matrix value

Update the \(C2\) matrix for the insertion position

Calculate \(c_{2}(i^*,j^*,u)\)
\[
d_{aw} - the distance between the depot (node o) and node u  
\]
\[
t_{aw} - the travel time between the depot (node o) and node u  
\]
\[
s_{u} - the service time at node u  
\]
\[
\lambda - in Solomon combinations of 1 and 2 are used  
\]
\[
F(q_{u}) - the fixed cost of the smallest available vehicle that can service the demand of node u  
\]
\[
c_{2}(i^*,j^*,u) = \lambda(d_{aw} + t_{aw}) + s_{u} + F(q_{u}) - c_{1j}^{best}(i,u,j)  
\]

Add the \(c_{2}(i^*,j^*,u)\) value to the \(C2\) matrix

Sort \(C2\) in ascending order.
Find first time-feasible node \((u^*)\), starting at the beginning of \(C2\)
While no \(u^*\) has been found, and end of \(C2\) has not been reached

Check for time feasibility

Check node \(u\)'s time feasibility
\[
a_u = \max \{ a_u, a_i + s_i + t_{ui} \}  
\]
if \(a_u \leq 1_u + L_l^{max}\)

Check node \(j\)'s time feasibility
\[
a_{j^*}^{new} = \max \{ a_j, a_{j^*} + s_{j^*} + t_{uj^*} \}  
\]
if \(a_{j^*}^{new} \leq 1_{j^*} + L_l^{max}\)

Check rest of ROUTE's \(r\) nodes for time feasibility
While feasible
\[
a_{r}^{new} = \max \{ a_r, a_{r+1} + s_{r+1} + t_{ur+1} \}  
\]
if \(a_{r}^{new} \leq 1_r + L_l^{max}\)
then feasible
else
infeasible
else
infeasible
else
infeasible

if feasible
Identify applicable node as \(u^*\)
else
  Check next element of C2

if a unique $u^*$ node has been identified
  Insert node $u^*$

  Update UNROUTED customers
  Remove $u^*$ from UNROUTED

  Remove any other artificial nodes related to $u^*$ from UNROUTED

Update ROUTE
  Update ROUTE load
  if new vehicle has been indicated
    if $Q^{\text{new}} > Q$
      Find the smallest available vehicle to service $Q^{\text{new}}$

      Update VEHAVAIL
      Change the availability status of the current vehicle to available
      Change the availability status of the new vehicle to unavailable
      Assign new vehicle to current TOUR
      Recalculate VEHAVAIL

Recalculate ROUTE schedule for nodes
  Actual start-time at origin ($a_o$) is the start-time indicated for the current route
  for each node ($i$) on the current ROUTE, except the depot at both ends
  $a_i = \max \{a_i, a_{i-1} + s_{i-1} + t_{i-1,i}\}$
  $w_i = \max \{0, a_{i-1} - (a_i + s_i + t_{i,i+1})\}$

  Calculate actual arrival at the depot ($n$th node) at the end of the current ROUTE
  $a_n = a_{n-1} + s_{n-1} + t_{n-1,n}$

else
  Initialize new ROUTE

Expand TOU:
  Determine multi-route feasibility
  Check the actual arrival time at the depot of the previous ROUTE of the current TOUR ($a_n$)
  if $a_n + s_n + 1$ hour $< l_o^{\text{max}}$
    then feasible
  else
    infeasible
  if feasible
    Initialize new ROUTE
  else
    if the last ROUTE of the TOUR has no nodes other than the depot
      Eliminate ROUTE from TOUR
    Initialize new TOUR
Define ORPHANs

if UNROUTED is not empty
  Assign all elements in UNROUTED to ORPHANs
  Clear UNROUTED

Report initial solution
  Calculate the OBJECTive function value for the initial solution
  Report initial solution
    for each TOUR
      Report all TOUR and ROUTE information
Appendix C

Output

The output presented in this appendix are representations of the actual text files (*.txt) generated by MATLAB 6.5. Consider the following excerpt from the output for the R1 problem class.

```
:   :
Tour: 2
Vehicle: v2
Route: 1
Customer   Actual time
    c11        0.00
    c881       13.00
    c511       86.00
    c272      347.00
    c11      358.31
Route: 2
Customer   Actual time
    c11      358.31
    c412     412.00
:   :
```

The last digit (either 1 or 2) represents the customer’s specific time window during which it will be serviced. For example, the second tour is assigned vehicle 2. The first and the last nodes on every route is c11. The c1 denotes the depot, while the last digit, 1, denotes the first time window (and for the depot, the only time window).

The third customer (the fourth node) on the first route of the second tour is c272. The c27 denotes customer 27, while the last digit, 2, indicates that customer 27 is serviced during its second specified time window, and to be specific, at 347 minutes after the vehicle left the depot.
At the end of each output file, any orphans are indicated. Orphans are defined to be customers that could not be inserted into any routes. The total scheduling distances is also indicated.

C.1 Problem class R1

The following output was generated by MATLAB for the problem class R1: randomly distributed customers with a short scheduling horizon.

Tour: 1
Vehicle: v1
Route: 1
Customer  Actual time
  c11  0.00
  c561 17.00
  c681  62.00
  c671 161.00
  c11  172.89
Route: 2
Customer  Actual time
  c11  172.89
  c401  179.00
  c231  267.00
  c482  411.00
  c11  422.44
Route: 3
Customer  Actual time
  c11  422.44
  c912  492.00
  c992  512.00
  c11  523.65
Route: 4
Customer  Actual time
  c11  523.65
  c112  559.00
  c11  570.33

Tour: 2
Vehicle: v2
Route: 1
Customer  Actual time
  c11  0.00
  c881  13.00
  c511  86.00
c272 347.00
C11 358.31
Route: 2
Customer Actual time
C11 358.31
c412 412.00
c432 422.67
c712 448.00
c11 459.20
Route: 3
Customer Actual time
C11 459.20
c842 459.95
c782 503.00
c11 514.84
Route: 4
Customer Actual time
C11 514.84
c282 516.67
c982 531.00
c11 541.40
Route: 5
Customer Actual time
C11 541.40
c92 560.00
c11 571.91
Tour: 3
Vehicle: v3
Route: 1
Customer Actual time
C11 0.00
c451 60.00
c121 345.00
c11 356.91
Route: 2
Customer Actual time
C11 356.91
c802 420.00
c921 430.67
c11 441.49
Route: 3
Customer Actual time
C11 441.49
<table>
<thead>
<tr>
<th>Customer</th>
<th>Actual time</th>
</tr>
</thead>
<tbody>
<tr>
<td>c32</td>
<td>443.45</td>
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<tr>
<td>c752</td>
<td>456.00</td>
</tr>
<tr>
<td>c11</td>
<td>467.00</td>
</tr>
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Route: 4

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<td>c622</td>
<td>535.00</td>
</tr>
<tr>
<td>c11</td>
<td>546.80</td>
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</tbody>
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Tour: 4

Vehicle: v4

Route: 1

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<tr>
<td>c861</td>
<td>68.00</td>
</tr>
<tr>
<td>c651</td>
<td>216.00</td>
</tr>
<tr>
<td>c261</td>
<td>240.00</td>
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<tr>
<td>c11</td>
<td>251.49</td>
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Route: 2

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<tr>
<th>Customer</th>
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<td>c11</td>
<td>251.49</td>
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<tr>
<td>c571</td>
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<tr>
<td>c812</td>
<td>309.00</td>
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<tr>
<td>c11</td>
<td>320.27</td>
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Route: 3

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<th>Customer</th>
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<tr>
<td>c11</td>
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<td>c182</td>
<td>346.00</td>
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<tr>
<td>c11</td>
<td>357.87</td>
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Route: 4

<table>
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<tr>
<td>c792</td>
<td>413.00</td>
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<tr>
<td>c11</td>
<td>424.87</td>
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Route: 5

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<tr>
<td>c502</td>
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<tr>
<td>c202</td>
<td>490.00</td>
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<tr>
<td>c11</td>
<td>501.49</td>
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Route: 6

<table>
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<tr>
<td>c602</td>
<td>502.02</td>
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<tr>
<td>Customer</td>
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<tr>
<td>----------</td>
<td>-------------</td>
</tr>
<tr>
<td>c11</td>
<td>512.55</td>
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<tr>
<td></td>
<td></td>
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<tr>
<td>Tour:</td>
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</tr>
<tr>
<td>Vehicle:</td>
<td>v5</td>
</tr>
<tr>
<td>Route:</td>
<td>1</td>
</tr>
<tr>
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<td>Actual time</td>
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<td>10.00</td>
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<td>c211</td>
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<td>c851</td>
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<td>411.95</td>
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Tour: 6  
Vehicle: v6  
Route: 1  
Customer  Actual time  
c11  0.00  
c641  42.00  
c191  63.00  
c11  73.91  
Route: 2  
Customer  Actual time  
c11  73.91  
c311  75.15  
c541  105.00  
c11  116.49  
Route: 3  
Customer  Actual time  
c11  116.49  
c162  117.64  
c341  188.00  
c11  199.62  
Route: 4  
Customer  Actual time  
c11  199.62  
c901  206.00  
c11  216.91  
Route: 5  
Customer  Actual time  
c11  216.91  
c592  334.00  
c11  345.71  
Route: 6  
Customer  Actual time  
c11  345.71  
c932  378.00  
c11  389.49  
Route: 7  
Customer  Actual time  
c11  389.49  
c492  426.00  
c11  437.47  
Route: 8  
Customer  Actual time  
c11  437.47
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Tour: 7
Vehicle: v7
Route: 1

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Route: 3

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Route: 4

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<tr>
<td>c301</td>
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Tour: 8
Vehicle: v8
Route: 1

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Route: 2

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Route: 3

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Route: 4
Customer Actual time
  c11  297.91
  c702 313.00
  c11  323.67

Route: 5
Customer Actual time
  c11  323.67
  c422 397.00
  c11  407.82

Tour: 9
Vehicle: v9
Route: 1
Customer Actual time
  c11  0.00
  c171 150.00
  c11  162.27

Route: 2
Customer Actual time
  c11  162.27
  c52  244.00
  c11  254.82

Route: 3
Customer Actual time
  c11  254.82
  c962 271.00
  c72  285.00
  c11  297.29

Route: 4
Customer Actual time
  c11  297.29
  c392 325.00
  c11  336.13

Route: 5
Customer Actual time
  c11  336.13
  c242 347.00
  c11  358.56

Tour: 10
Vehicle: v10
Route: 1
Customer Actual time
c11  0.00
c1002  225.00
c11  235.84
Route:  2
Customer  Actual time
  c11  235.84
c772  237.65
c11  249.47
Route:  3
Customer  Actual time
  c11  249.47
c632  268.00
c11  279.31
Tour:  11
Vehicle: v11
Route:  1
Customer  Actual time
  c11  0.00
c942  182.00
c11  193.36
Route:  2
Customer  Actual time
  c11  193.36
c522  219.00
c11  230.29
Tour:  12
Vehicle: v12
Route:  1
Customer  Actual time
  c11  0.00
c472  229.00
c11  239.69
Tour:  13
Vehicle: v13
Route:  1
Customer  Actual time
  c11  0.00
c142  189.00
c11  200.31
Route:  2
Customer  Actual time
c11  200.31
cl32  209.00
c11  220.31

Tour:  14
Vehicle: v14

Tour:  15
Vehicle: v15

Tour:  16
Vehicle: v16

Tour:  17
Vehicle: v17

Tour:  18
Vehicle: v18

Tour:  19
Vehicle: v19

Tour:  20
Vehicle: v20

Tour:  21
Vehicle: v21

Tour:  22
Vehicle: v22

Tour:  23
Vehicle: v23

Tour:  24
Vehicle: v24

Tour:  25
Vehicle: v25

Tour:  26
Vehicle: v26

Tour:  27
Vehicle: v27

Tour: 28
Vehicle: v28

Tour: 29
Vehicle: v29

Tour: 30
Vehicle: v30

Tour: 31
Vehicle: v31

Tour: 32
Vehicle: v32

Tour: 33
Vehicle: v33

Tour: 34
Vehicle: v34

Tour: 35
Vehicle: v35

Tour: 36
Vehicle: v36

Tour: 37
Vehicle: v37

Tour: 38
Vehicle: v38

Tour: 39
Vehicle: v39

Tour: 40
Vehicle: v40

Tour: 41
Vehicle: v41
Tour: 42
Vehicle: v42

Tour: 43
Vehicle: v43

Tour: 44
Vehicle: v44

Tour: 45
Vehicle: v45

Tour: 46
Vehicle: v46

Tour: 47
Vehicle: v47

Tour: 48
Vehicle: v48

Tour: 49
Vehicle: v49

Tour: 50
Vehicle: v50

Tour: 51
Vehicle: v51

Tour: 52
Vehicle: v52

Tour: 53
Vehicle: v53

Tour: 54
Vehicle: v54

Tour: 55
Vehicle: v55

Tour: 56
Vehicle: v56
Tour: 57
Vehicle: v57

Tour: 58
Vehicle: v58

Tour: 59
Vehicle: v59

Tour: 60
Vehicle: v60

Tour: 61
Vehicle: v61

Tour: 62
Vehicle: v62

Tour: 63
Vehicle: v63

Tour: 64
Vehicle: v64

Tour: 65
Vehicle: v65

Tour: 66
Vehicle: v66

Tour: 67
Vehicle: v67

Tour: 68
Vehicle: v68

Tour: 69
Vehicle: v69

Tour: 70
Vehicle: v70

Tour: 71
Vehicle: v71
Route: 1
Customer  Actual time
  c11      0.00
  c1012    411.00
  c11      422.60
Route: 2
Customer  Actual time
  c11      422.60
  c532     495.00
  c11      505.67

Tour:  72
Vehicle: v72
Route: 1
Customer  Actual time
  c11      0.00
  c872     410.00
  c11      422.36
Route: 2
Customer  Actual time
  c11      422.36
  c292     474.00
  c11      485.38
Route: 3
Customer  Actual time
  c11     485.38
  c62      495.00
  c11      506.71

Tour:  73
Vehicle: v73
Route: 1
Customer  Actual time
  c11      0.00
  c822     309.00
  c11      319.36
Route: 2
Customer  Actual time
  c11     319.36
  c552     384.00
  c11      395.02

Tour:  74
Vehicle: v74
Route:  1
Customer Actual time
  c11  0.00
  c742 104.00
  c11  114.44
Route:  2
Customer Actual time
  c11  114.44
  c692 154.00
  c11  165.47
Route:  3
Customer Actual time
  c11  165.47
  c382 346.00
  c11  356.65
Route:  4
Customer Actual time
  c11  356.65
  c102 385.00
  c11  395.40

Tour:  75
Vehicle: v75
Route:  1
Customer Actual time
  c11  0.00
  c582  76.00
  c11  87.11
Route:  2
Customer Actual time
  c11  87.11
  c222 263.00
  c11  274.38
Route:  3
Customer Actual time
  c11  274.38
  c42  336.00
  c11  347.78
Route:  4
Customer Actual time
  c11  347.78
  c22  366.00
  c11  376.80
Orphans: none

Total distance: 11260

C.2 Problem class R2

The following output was generated my MATLAB for the problem class R2: randomly distributed customers with a long scheduling horizon.

Tour: 1
Vehicle: v1
Route: 1

<table>
<thead>
<tr>
<th>Customer</th>
<th>Actual time</th>
</tr>
</thead>
<tbody>
<tr>
<td>c11</td>
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</tr>
<tr>
<td>c821</td>
<td>25.00</td>
</tr>
<tr>
<td>c651</td>
<td>37.89</td>
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<tr>
<td>c571</td>
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</tr>
<tr>
<td>c321</td>
<td>91.02</td>
</tr>
<tr>
<td>c611</td>
<td>102.25</td>
</tr>
<tr>
<td>c691</td>
<td>113.27</td>
</tr>
<tr>
<td>c681</td>
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<td>c521</td>
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<tr>
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<td>891.00</td>
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<tr>
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<tr>
<td>c501</td>
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<tr>
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<tr>
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Tour: 2
Vehicle: v2
Route: 1

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<tr>
<td>c51</td>
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</table>
Orphans: none

Total distance: 11260

C.2 Problem class $R2$

The following output was generated by MATLAB for the problem class $R2$: randomly distributed customers with a long scheduling horizon.

Tour: 1
Vehicle: v1
Route: 1

<table>
<thead>
<tr>
<th>Customer</th>
<th>Actual time</th>
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<tbody>
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Tour: 2
Vehicle: v2
Route: 1

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Tour: 3  
Vehicle: v3  
Route: 1

Tour: 4  
Vehicle: v4
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c111  1636.85
c551  1691.00
c702  1797.00
c541  1807.91
c912  1818.78
c1002  1938.00
c22  1973.00
c872  1985.36
c11  1995.89

Tour:  5
Vehicle:  v5
Route:  1

Customer  Actual time

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<tr>
<th>Customer</th>
<th>Actual time</th>
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<tbody>
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Orphans: none

Total distance: 8722

C.3 Problem class C1

The following output was generated my MATLAB for the problem class C1: clustered customers with a short scheduling horizon.

Tour:  1
Vehicle:  v1
Route:  1

Customer  Actual time

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<tbody>
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</tr>
<tr>
<td>c11</td>
<td>561.02</td>
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Route:  2
Customer  Actual time
   c11    561.02
   c801   595.00
   c1011  686.33
   c681   777.25
   c181   867.35
   c402   957.42
   c82    1049.42
   c11    1141.38

Tour:  2
Vehicle: v2
Route:  1
Customer  Actual time
   c11    0.00
   c221   72.00
   c241   162.16
   c511   252.67
   c531   343.07
   c132   435.24
   c11    526.33

Route:  2
Customer  Actual time
   c11    526.33
   c291   527.31
   c152   617.53
   c751   723.00
   c881   813.76
   c162   904.31
   c542   998.04
   c992   1087.42
   c11    1179.67

Tour:  3
Vehicle: v3
Route:  1
Customer  Actual time
   c11    0.00
   c631   23.00
   c451   113.11
   c201   203.44
   c811   294.58
   c322   384.62

97
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Tour: 6
Vehicle: v6

Route: 1

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Route: 3

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| Tour: | 8 |
| Vehicle: | v8 |
| Route: | 1 |

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| Route: | 2 |

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<td>c602</td>
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| Tour: | 9 |
| Vehicle: | v9 |
| Route: | 1 |

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<td>985.13</td>
</tr>
<tr>
<td>c11</td>
<td>1076.35</td>
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</tbody>
</table>
c11  0.00
503  84.00
c271  201.00
c761  291.89
c701  508.00
c622  599.44
c11  689.87

Route:  2
Customer  Actual time
  c11  689.87
  c62  754.00
  c92  950.00
  c11  1042.35

Tour:  10
Vehicle: v10
Route:  1
Customer  Actual time
  c11  0.00
  c491  70.00
  c281  178.00
  c951  517.00
  c11  609.44

Tour:  11
Vehicle: v11
Route:  1
Customer  Actual time
  c11  0.00
  c561  79.00
  c41  169.87
  c11  262.18

Orphans: none

Total distance: 7330

C.4 Problem class C2

The following output was generated by MATLAB for the problem class C2: clustered customers with a long scheduling horizon.

Tour:  1
Vehicle: v1
Route: 1

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Route: 2

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Tour: 2
Vehicle: v2
Route: 1

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Route: 1
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Vehicle: v4
Route: 1

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c121 2910.00  
c11  3000.58

Orphans: none

Total distance: 7240

C.5 Problem class RC1

The following output was generated my MATLAB for the problem class RC1: both randomly distributed, and clustered customers with a short scheduling horizon.

Tour: 1  
Vehicle: v1  
Route: 1  
Customer Actual time  
c11  0.00  
c441  79.00  
c492  524.00  
c11  534.87  
Route: 2  
Customer Actual time  
c11  534.87  
c762  549.00  
c11  560.84

Tour: 2  
Vehicle: v2  
Route: 1  
Customer Actual time  
c11  0.00  
c251  52.00  
c961  63.04  
c372  550.00  
c11  561.07

Tour: 3  
Vehicle: v3  
Route: 1  
Customer Actual time  
c11  0.00  
c701  53.00

105
c591  172.00
C982  446.00
c11  457.00
Route:  2
Customer Actual time
   c11  457.00
   c402  473.00
   c682  501.00
   c11  511.29

Tour:  4
Vehicle: v4
Route:  1
Customer Actual time
   c11  0.00
   c431  10.00
   c471  138.00
   c212  438.00
   c11  449.53
Route:  2
Customer Actual time
   c11  449.53
   c652  482.00
   c842  498.00
   c11  509.84

Tour:  5
Vehicle: v5
Route:  1
Customer Actual time
   c11  0.00
   c711  101.00
   c201  114.20
   c312  485.00
   c11  495.67

Tour:  6
Vehicle: v6
Route:  1
Customer Actual time
   c11  0.00
   c571  41.00
   c862  420.00
   c11  431.69
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Vehicle: v9
Route:  1
Customer | Actual time
---|---
c11  | 0.00
C851 | 51.00
c121 | 156.00
c882 | 232.00
c11  | 243.13

Route:  2
Customer | Actual time
---|---
c11  | 243.13
C382 | 261.00
c222 | 287.00
c342 | 334.00
c11  | 345.49

Route:  3
Customer | Actual time
---|---
c11  | 345.49
c742 | 347.11
c362 | 357.75
c972 | 372.00
c11  | 382.91

Route:  4
Customer | Actual time
---|---
c11  | 382.91
c22  | 411.00
c612 | 421.05
c11  | 432.49

Tour:  10
Vehicle: v10
Route:  1
Customer | Actual time
---|---
c11  | 0.00
c781 | 72.00
c91  | 97.00
c142 | 337.00
c11  | 348.56

Route:  2
Customer | Actual time
---|---
c11  | 348.56
c182 | 349.58
c672 | 372.00
c11  | 382.78
Route: 3
Customer  Actual time
  c11   382.78
  c352  383.71
  c32   396.00
  c11   407.13

Tour: 11
Vehicle: v11
Route: 1
Customer  Actual time
  c11   0.00
  c641  26.00
  c261  117.00
  c941  376.00
  c11   387.60

Tour: 12
Vehicle: v12
Route: 1
Customer  Actual time
  c11   387.60
  c772  391.00
  c11   401.95

Tour: 13
Vehicle: v13
Route: 1
Customer  Actual time
  c11   0.00
  c81   57.00
  c811  90.00
  c922  263.00
  c11   274.87

Route: 2
Customer  Actual time
  c11   274.87
  c802  281.00
c822     303.00
c792     314.80
  c11     325.35
Tour:  14
Vehicle: v14
Route:  1
Customer   Actual time
     c11       0.00
     c531     42.00
     c321     52.09
     c752     318.00
     c11     329.98

Tour:  15
Vehicle: v15
Route:  1
Customer   Actual time
     c11       0.00
     c871   161.00
     c562   283.00
     c11   294.25
Route:  2
Customer   Actual time
     c11   294.25
     c272   299.00
     c11   310.56

Tour:  16
Vehicle: v16
Route:  1
Customer   Actual time
     c11       0.00
     c41    83.00
     c162   310.00
     c11   321.24

Tour:  17
Vehicle: v17
Route:  1
Customer   Actual time
     c11       0.00
     c291    15.00
     c151   68.00

110
c72  202.00  
c11  212.45

Tour:  18  
Vehicle: v18  
Route:  1  
Customer  Actual time  
c11  0.00  
c51  9.00  
c501  51.00  
c11  61.49  

Route:  2  
Customer  Actual time  
c11  61.49  
c412  70.00  
c232  192.00  
c11  203.04  

Tour:  19  
Vehicle: v19  
Route:  1  
Customer  Actual time  
c11  0.00  
c541  15.00  
c391  110.00  
c11  121.78  

Route:  2  
Customer  Actual time  
c11  121.78  
c61  123.33  
c301  135.93  
c112  190.00  
c11  201.98  

Tour:  20  
Vehicle: v20  
Route:  1  
Customer  Actual time  
c11  0.00  
c621  101.00  
c992  176.00  
c11  186.82  

Tour:  21
Vehicle: v21
Route: 1
Customer   Actual time     
c11         0.00           
c1011       32.00          
c902        164.00         
c11         174.76         

Tour: 22
Vehicle: v22
Route: 1
Customer   Actual time     
c11         0.00           
c481        30.00          
c631        40.95          
c11         52.51          

Route: 2
Customer   Actual time     
c11         52.51          
c601        72.00          
c1002       104.00         
c11         116.09         

Route: 3
Customer   Actual time     
c11         116.09         
c892        117.40         
c11         128.71         

Route: 4
Customer   Actual time     
c11         128.71         
c282        144.00         
c11         155.47         

Tour: 23
Vehicle: v23
Route: 1
Customer   Actual time     
c11         0.00           
c131        58.00          
c332        121.00         
c11         132.80         

Route: 2
Customer   Actual time     
c11         132.80         

112
c732 135.00
c11 145.76
Route: 3
Customer Actual time
  c11 145.76
c102 153.00
c11 163.93

Tour: 24
Vehicle: v24
Route: 1
Customer Actual time
  c11 0.00
c521 127.00
c11 138.71

Tour: 25
Vehicle: v25
Route: 1
Customer Actual time
  c11 0.00
c691 70.00
c11 81.33
c11 81.33

Tour: 26
Vehicle: v26
Route: 1
Customer Actual time
  c11 0.00
c191 82.00
c11 93.29
c11 93.29

Orphans: none

Total distance: 8706

C.6 Problem class RC2

The following output was generated my MATLAB for the problem class RC1: both randomly distributed, and clustered customers with a long
scheduling horizon.

Tour:  1
Vehicle: v1
Route:  1

Customer: Actual time
  c11  0.00
  c411  26.00
  c431  38.05
  c161  136.00
  c961  146.64
  c251  209.00
  c372  2255.00
  c11  2266.07

Tour:  2
Vehicle: v2
Route:  1

Customer: Actual time
  c11  0.00
  c51  9.00
  c851  144.00
  c321  156.04
  c701  210.00
  c91  387.00
  c141  588.00
  c762  2194.00
  c11  2205.84

Tour:  3
Vehicle: v3
Route:  1

Customer: Actual time
  c11  0.00
  c481  61.00
  c631  71.95
  c291  83.15
  c801  180.00
  c441  314.00
  c492  2100.00
  c11  2110.87

Tour:  4
Vehicle: v4
Route: 1

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<tr>
<td>c661</td>
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<td>1783.00</td>
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<td>1794.00</td>
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Route: 2

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Tour: 5

Vehicle: v5

Route: 1

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Tour: 6

Vehicle: v6

Route: 1

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Tour: 7
Vehicle: v7
Route: 1
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Route: 2
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Tour: 8
Vehicle: v8
Route: 1
Customer Actual time
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  c541 15.00
  c31  197.00
  c641 207.98
  c61  324.00
  c772 1564.00
  c11  1574.95

Tour: 9
Vehicle: v9
Route: 1
Customer Actual time
  c11  0.00
  c731 211.00
  c501 221.89
  c811 358.00
  c521 507.00
  c922 1054.00
  c11  1065.87

Route: 2
Customer Actual time
  c11  1065.87
  c562 1132.00
  c792 1204.00
  c222 1215.47
  c172 1226.49
  c822 1237.11
  c11  1248.36

Tour: 10
Vehicle: v10
Route: 1
Customer Actual time
  c11  0.00
  c781 230.00
  c71  241.51
  c191 329.00
  c601 340.60
  c261 470.00
  c272 1196.00
  c11  1207.56
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| Tour: | 12 |
| Vehicle: v12 |
| Route: | 1 |
| Customer | Actual time |
| c11 | 0.00 |
| c151 | 270.00 |
| c901 | 417.00 |
| c691 | 428.65 |
| c301 | 513.00 |
| c871 | 646.00 |
| c11 | 657.56 |
| Route: | 2 |
| Customer | Actual time |
| c11 | 657.56 |
| c112 | 760.00 |
| c11 | 771.98 |

Orphans: none

Total distance: 7748
Bibliography


