A MODEL PREDICTIVE CONTROL STRATEGY FOR LOAD SHIFTING IN A WATER PUMPING SCHEME WITH MAXIMUM DEMAND CHARGES

by

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The aim of this research is to affirm the application of closed-loop optimal control for load shifting in plants with electricity tariffs that include time-of-use (TOU) and maximum demand (MD) charges. The water pumping scheme of the Rietvlei water purification plant in the Tshwane municipality (South Africa) is selected for the case study. The objective is to define and simulate a closed-loop load shifting (scheduling) strategy for the Rietvlei plant that yields the maximum potential cost saving under both TOU and MD charges.

The control problem is firstly formulated as a discrete time linear open loop optimal control model. Thereafter, the open loop optimal control model is converted into a closed-loop optimal control model using a model predictive control technique. Both the open and closed-loop optimal control models are then simulated and compared with the current (simulated) level based control model. The optimal control models are solved with integer programming optimization. The open loop optimal control model is also solved with linear programming optimization and the result is used as an optimal benchmark for comparisons.

Various scenarios with different simulation timeouts, switching intervals, control horizons, model uncertainty and model disturbances are simulated and compared. The effect of MD charges is also evaluated by interchangeably excluding the TOU and MD charges.

The results show a saving of 5.8% to 9% for the overall plant, depending on the simulated scenarios. The portion of this saving that is due to a reduction in MD varies between 69% and 92%. The results also shows that the closed-loop optimal control model matches the saving of the open loop optimal control model, and that the closed-loop optimal control model compensates for model uncertainty and model disturbances whilst the open loop optimal control model does not.

**KEYWORDS**

Model predictive control, load shifting, maximum demand, optimal control.
OPSOMMING

Die doel van hierdie navorsing is om die applikasie van geslote-lus optimale beheer vir las verskuiwing in aanlegte met elektrisiteit tariewe wat tyd-van-gebruik (TVG) en maksimum aanvraag (MA) kostes insluit te bevestig. Die water pomp skema van die Rietvlei water reiniging aanleg in die Tshwane munisipaliteit (Suid-Afrika) is gekies vir die gevalle studie. Die objekief is om 'n geslote-lus las verskuiwing (skedulering) strategie vir die Rietvlei aanleg te definieer en te simuler wat die maksimum potensiaal vir koste besparing onder beide TVG en MA kostes lever.

Die beheer probleem is eerstens gevormuleer as 'n diskreet tyd lineêre ope-lus optimale beheer model. Daarna is die ope-lus optimale beheer model aangepas na 'n geslote-lus optimale beheer model met behulp van 'n model voorspellende beheer tegniek. Beide die ope- en geslote-lus optimale beheer modele is dan gesimuleer en vergelyk met die huidige (gesimuleerde) vlak gebaseerde beheer model. Die optimisering van optimale beheer modelle is opgelos met geheeltallige programmering. Die optimisering van die ope-lus optimale beheer model is ook opgelos met lineêre programmering en die resultaat is gebruik as 'n optimale doelwit vir vergelykings.

Verskeie scenarios met verskillende simulasie stop tye, skakel intervalle, beheer horisonne, model onsekerheid en model versteurings is gesimuleer en vergelyk. Die effek van MA kostes is ook gegreeleer deur inter uitruiling van die TVG en MA kostes.

Die resultate toon 'n besparing van 5. 8% tot 9% vir die algehele aanleg, afhangend van die gesimuleerde scenarios. Die deel van die besparing wat veroorsaak is deur 'n vermindering in MA wissel tussen 69% en 92%. Die resultate toon ook dat die geslote-lus optimale beheer model se besparing dieselfde is as die besparing van die ope-lus optimale beheer model, en dat die geslote-lus optimale beheer model kompenseer vir model onsekerheid en model versteurings, terwyl die ope-lus optimale beheer model nie kompenseer nie.

SLEUTELWOORDE

Model voorspellende beheer, las verskuiwing, maksimum aanvraag, optimale beheer.
LIST OF ABBREVIATIONS

BIP  Binary integer programming
DME  Department of Minerals and Energy
DSM  Demand side management
IP   Integer programming
LP   Linear programming
MD   Maximum demand
MILP Mixed integer linear programming
MINLP Mixed integer non-linear programming
ML   Mega litre
MPC  Model predictive control
NERSA National Energy Regulator of South Africa
NMD  Notified maximum demand
TOU  Time-of-use
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1. INTRODUCTION

This dissertation defines and simulates a closed-loop optimal control strategy for load shifting in a specific water pumping scheme with an electricity tariff that includes both time-of-use (TOU) and maximum demand (MD) charges.

1.1. Outline of the dissertation

This chapter introduces the background to the research problem and briefly describes the research approach. Chapter 2 covers the literature survey, which includes a detail description of the research problem, the research approach and the contributions of this research. In Chapter 1 a generic optimal control model is defined. In Chapter 4 this general control model is applied to a specific case study. Chapter 5 reports on the simulated results for the case study, and Chapter 6 concludes and makes recommendations for further research.

1.2. Background

The demand for electricity is increasing throughout the world, which results in higher cost and additional greenhouse gas emissions. This is of particular concern in South Africa where large scale load shedding was recently required to reduce the electricity demand [1], [2]. To assist with this problem demand side management (DSM) initiatives are pursued. DSM initiatives are less expensive, cleaner, faster and have lower risks than building new power plants [3].

DSM can be divided into two categories: energy efficiency and load management. Energy efficiency aims to reduce the net amount of energy consumed, whilst load management aims to reduce the load in peak demand periods. Load shifting is an aspect of load management where the load is moved from peak demand periods to lower demand periods, i.e., the load is simply moved, but the net amount of consumed energy may not change.

To encourage load shifting utilities have structured electricity tariffs with time-of-use
(TOU) and/or maximum demand (MD) charges [4]-[7]. TOU charges are based on higher kWh rates during high demand periods, whilst MD charges are based on fixed fees per maximum kVA or kW for a month, typically in high demand periods [5]. MD is measured as the highest average demand in kVA or kW during any integrating period –the integrating period is generally 30 minutes, and it coincides with the TOU periods [5].

Optimal control [8], [9] is new to the field of load shifting; specifically where both TOU and MD charges are applicable. Therefore, the goal of this research is to affirm that an optimal control model can be used to solve industrial load shifting problems for customers whose tariffs include TOU and MD charges.

1.3. Approach

The research objective is to define and simulate a closed-loop load shifting (scheduling) strategy that yields the maximum potential cost saving under both TOU and MD charges for a specific application.

A generic open loop optimal control model is defined for a water pumping scheme. This generic open loop optimal control model is then converted into a closed-loop optimal control model using a model predictive control (MPC) approach. The generic open and closed-loop optimal control models are then applied to a case study. The water pumping scheme at the Rietvlei water purification plant in the Tshwane municipality (South Africa) is selected for the case study.

Thereafter, the applied control models are simulated and compared with the current (simulated) level based control model. Integer programming (IP) is used to solve the optimization problem for the open and closed-loop optimal control models. The open loop optimal control model is also solved with linear programming (LP) to determine an optimal benchmark for comparisons.

A number of scenarios with varying simulation timeout, switching intervals and control horizons are also simulated to evaluate the effect of compromises in practical applications.
The effect of MD charges is also evaluated by interchangeably excluding the TOU and MD charges. And finally, the affect of model uncertainty and model disturbances on the open and closed-loop optimal control models is evaluated.
2. LITERATURE SURVEY

This chapter covers the literature survey. It includes a detail description of the research problem, the research approach and the contributions of this research.

2.1. Demand side management (DSM) in South Africa

Approximately 95% of South Africa’s electricity is generated and distributed by the state owned utility Eskom [2]. Eskom has an installed electrical generation capacity of approximately 43,037 MW with an operating capacity of approximately 38,744 MW [2]. The difference between the installed and operating capacity is referred to as the net reserve margin. Due to fast economic growth and slow capital expansion the net reserve margin in 2008 has decreased to 8%, compared to an internationally accepted margin of 15% [2]. As a result unplanned outages resulted in large scale load shedding [1], [2].

In order to correct the situation Eskom has embarked on capital expansion and DSM programs [2]. The aim of the capital expansion program is to refurbish old mothballed power stations and to build new peak and base load power stations. The aim of the DSM programme is to affect the timing or amount of electricity used by customers [2], [10]. The DSM program is very important, because DSM initiatives are less expensive, cleaner, faster and have lower risks than building new power stations [3].

The current Eskom DSM program is an extension of an exiting DSM programme which is in line with the energy efficiency strategy for South Africa from the Department of Minerals and Energy (DME) [10], [11]. The aim of this overall program is to reduce 3,000 MW from April 2007 to March 2012, and a further 5,000 MW by March 2025. The Eskom DSM program is approved and monitored by the National Energy Regulator of South Africa (NERSA). With these aggressive targets and the reality of load shedding it is clear that DSM is an important initiative in South Africa.

DSM can be divided into two categories: energy efficiency and load management [12], [13]. Note that in some literature energy efficiency and DSM are referred to as two
2.2. The energy efficiency component of DSM

Energy efficiency aims to reduce the net amount of energy consumed. Various techniques are used to reduce energy consumption, for example, energy efficient lighting [15], variable speed drives [16], solar water heating systems [17], energy efficient motors [18], etc. These techniques reduce the net energy consumed, help in lowering the demand during peak periods and also reduce greenhouse gas emissions.

2.3. The load management component of DSM

Load management aims to schedule load out of peak demand periods to lower demand periods. As opposed to energy efficiency, the load is simply moved, but the net amount of consumed energy is not changed. Various techniques are used for load management, e.g., direct load control [19], real-time pricing [20], TOU pricing [21], MD pricing [22], etc. The focus of this dissertation is on the TOU and MD pricing techniques. With these techniques utilities do not actively manage the load, but customers are encouraged to re-schedule loads to reduce electricity costs. This technique is commonly referred to as load shifting.

2.4. Electricity tariffs applicable to load shifting

As mentioned, to encourage load shifting utilities have structured electricity tariffs with TOU and/or MD charges [4]-[7]. MD charges are also used by utilities to represent infrastructure costs due to high peak demands. In other words, utilities charge for the MD to encourage more uniform load profiles. This reduces the cost of reticulation equipment, e.g., power transformers. MD charges are also typically applicable in high demand periods, which encourage load shifting to lower demand periods [4]-[7].

TOU charges are based on higher kWh rates during high demand periods, whilst MD charges are based on fixed fees per maximum kVA or kW for a month [5]. MD is measured as the highest averaged demand in kVA or kW during any integrating period.
The integrating period is generally 30 minutes, and it coincides with the TOU periods [5].

The MD is different from the notified maximum demand (NMD) [4], [5]. The NMD is the agreed limit on the monthly MD. In other words, a customer agrees that his MD will not exceed his NMD. A customer that exceeds the NMD is typically charged with a penalty fee. This dissertation focuses on the MD only, because by optimizing the MD the NMD is automatically adhered to.

2.5. Load shifting with optimal control models

Various techniques are used to solve load shifting problems in different applications. Fuzzy logic is used in [23] for load shifting of a domestic hot water cylinder. A neural network is used in [24] for load shifting in a petrochemical plant. In [25]-[32] load shifting problems are modelled as optimization problems, and in [33]-[36] load shifting problems are modelled as optimal control problems [8], [9].

A load shifting problem can be considered as an optimal control problem, because the objective is to control a process, within a set of constraints, whilst minimising a cost function. The cost function in this case is the cost of electricity. It is assumed that the applicable tariff is structured to encourage load shifting.

The optimization techniques used in [25]-[32] do not consider external disturbances or inaccurate system models. In other words, no feedback and subsequent re-optimization is included to compensate for these deficiencies. From a control theory point of view [37], these types of applications can be referred to as open loop control models, because no feedback is used to determine if the controller’s input has achieved the desired output. This means that the system does not observe the output of the processes that it is controlling.

The optimization techniques (open loop optimal control models) used in [25]-[32] are valuable starting points to quantify the potential for load shifting. However, they cannot actively control a load shifting process with disturbances. To actively control a load
shifting process a controller with feedback and subsequent re-optimization is required. This type of controller is referred to as a closed-loop optimal controller [37].

Table 2-1 shows that the load shifting techniques in [25]-[36] can be further categorized as follows: [25]-[27] considers optimization (open loop optimal control) with TOU charges, [28]-[32] considers optimization (open loop optimal control) with TOU and MD charges, [33] and [34] considers closed-loop optimal control models with TOU charges, and only [35] and [36] considers closed-loop optimal control models with TOU and MD charges. Furthermore, the closed-loop optimal control models in [35] and [36] are re-optimized daily, but should ideally be re-optimized more frequently to react to disturbances closer to real-time. Table 2-1 also shows that only [30]-[32] and [35]-[36] covers load shifting in water supply systems.

Table 2-1: Summary of the optimal control based references.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Open loop control</th>
<th>Closed-loop control</th>
<th>TOU charges</th>
<th>MD charges</th>
<th>Water supply system</th>
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<tbody>
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</table>

*Includes daily feedback only

The remainder of this section covers more details about the references [25]-[36] in Table
2.5.1. Open loop optimal control models with TOU charges only

In [25] an open loop optimal control model is applied to a flour mill in India that is charged on both TOU and MD. However, the MD charge is not included in the objective function of the optimal control model. It is argued that the MD charges can be added later, because it is a constant charge throughout a 24 hour period. The problem in [25] is formulated as a discrete time linear problem and solved using IP. The total connected load of the plant is 235 kW and the reduction in electricity cost after load shifting is 1.5%. With a different working shift strategy and additional storage capacity the cost saving increases to 29%.

In [26] an open loop optimal control model is applied to a steel plant (arch furnace) in India that is charged on both TOU and MD. However, the MD charge component is not included in the objective function of the optimal control model. The same argument is used as in [25]. The problem is formulated as a discrete time linear problem and is solved using IP. The contracted demand for the plant is 70 MVA. The reduction in energy cost is not given against the current usage, but a comparison is done between the cost for an optimal schedule under a flat tariff and an optimal schedule under a TOU tariff. The saving in this comparison is 5.7%.

In [27] an open loop optimal control model is developed and simulated for a domestic hot water cylinder. Only TOU charges have been included in the objective function of the control model. A commercial TOU based tariff is used for illustration purposes, because TOU charges are not applicable to residential customers in South Africa. The problem is formulated as a continuous time switched optimal control model –based on a technique in [38]. The total cost saving is 35%, which is based on a simulated cost comparison between the traditional control model and the new optimal control model. Both control models are simulated with the selected TOU tariff.

2.5.2. Open loop optimal control models with TOU and MD charges
In [28] an open loop optimal control model is developed and simulated for a fertilizer plant in India that is charged on both TOU and MD. The total connected load of the plant is 24.7 MW and the reduction in electricity cost after load shifting is approximately 3%. This includes the saving on MD and TOU charges.

In [29] an open loop optimal control model is developed and simulated for an electrolytic process (chlorine) plant in India that is charged on both TOU and MD. The problem is formulated as a discrete time linear problem and is solved using mixed integer non-linear programming (MINLP). The switching intervals are selected as 30 minute intervals, which coincides with the MD integrating periods. The MD charge component is included in the objective function of the optimal control model. The connected load of the plant is 40 MW and the contracted demand is 25 MVA. The reduction in electricity cost after load shifting is 3.9%. Although the reduction in MD is 19%, the affect on the total cost saving is small, i.e., the portion of saving due to MD reduction can be calculated from the paper as 0.078% of the total 3.9%. One of the reasons is that the MD charges are small compared to the TOU charges in the applicable tariff. However, in South African tariffs the MD charges are higher in relation to the TOU charges [5], and it is expected that the MD charges will play a more significant role in South Africa. For example, the MD charges in [29] would account for approximately 7% of the total electricity cost for a constant load over a month, whilst the MD charges in South Africa for the same scenario would account for approximately 15% (high demand season) [5].

In [30] an open loop optimal control model is developed and simulated for a water supply network in England. The water supply network consists of a number of pumping stations that are charged on both TOU and MD. The control horizon is divided into control intervals that coincide with the TOU periods. The required operating time in each control interval is then solved with linear programming. The advantage of this approach is that the variables are not binary integer constrained and can therefore be solved faster. The disadvantage is that a discrete pump schedule still needs to be determined from the results. The MD cost is optimized by repeated linear optimization for different pump combinations. The total load of the plant is not given. The cost saving for TOU optimization only is 7.7%, and for both TOU and MD optimization it is 15.6%. This
proves that in certain applications the optimization of MD costs is important. An indication of computation time is also given which shows reasonable computation times of 2 to 42 seconds, depending on the size of the problem.

In [31] an open loop optimal control model is developed and simulated for a hypothetical water supply system. The main idea in [31] is the consideration of future water demand in the optimization of MD. The daily water demand is modelled as a Markov process, which is then used in a dynamic programming algorithm to determine daily MD limits for a month. This calculation is done occasionally, e.g., once per month. The daily MD limits are then used as constraints in the daily optimization scheduler. A specific daily optimization scheduler is not described.

In [32] an open loop optimal control model is developed and simulated for a hypothetical water distribution system. The hypothetical system is charged on both TOU and MD. The problem is formulated as a discrete time non-linear model and solved with a generalized reduced gradient optimization algorithm. The control variables are continuous and they represent the required flow from a pump station. This means that the discrete pump schedule at each pump station still needs to be determined from the results. The control horizon is 24 hours and it is divided into 24 hourly control intervals. The TOU electricity saving is 6.3 %, and the total saving (including MD charges) is 8.3%. The computation time for the problem is approximately 5 minutes; it is assumed that it would be faster on a more modern computer.

2.5.3. Closed-loop optimal control models with TOU charges only

In [33] a closed-loop optimal control model is developed and simulated for a colliery conveyor system in South Africa. The plant is charged on TOU only. The problem is formulated as a discrete time linear model and solved with binary integer programming (BIP). The closed-loop model is re-optimized on the arrival of a new train. The control horizon is the time period between the arrival of coal trains, and the control intervals are selected as one hour. The TOU cost saving on a specific conveyor is 49%.
In [34] a closed-loop optimal control model is developed and simulated for a heating cogeneration system of a sample building. Although, this is not a typical load shifting application, the principles are the same. A simple tariff with a day-night TOU charge is used. The problem is formulated as a discrete time linear problem, and solved with linear programming. The control variables are continuous because the output of the cogeneration engine is variable, i.e., not binary controlled. The control horizon is 24 hours, the switching intervals are 15 minutes, and the simulation is executed over a 1 year period. The results are compared with a conventional simulated system without any cogeneration. The results show a 29% saving in overall energy cost.

2.5.4. Closed-loop optimal control models with TOU and MD charges

In [35] a closed-loop optimal control model is developed and simulated for a water supply system in the USA that is charged on both TOU and MD. The MD costs are first optimized with a long term model over a month to determine an optimal MD. This information is then used in a daily optimization schedule. Continuous variables are used in the long term model and binary integer variables are used in the short term model. The long and short term control horizons are divided into switching intervals of one hour. The long and short term models are formulated as non-linear problems and solved with a dynamic programming algorithm. The total cost saving is 20%.

In [36] a closed-loop optimal control model is developed and simulated for a water pumping scheme in the USA that is charged on both TOU and MD. The problem is formulated as a continuous time linear problem and solved using mixed integer linear programming (MILP). The control horizon is divided into varying control intervals that coincide with the TOU periods. The control variables are continuous and they represent the pump duration required for each pump in a control interval. This means that the discrete pump schedules still need to be determined from the results. The MD charge component is included in the objective function of the optimal control model as a continuous variable. The total demand in each MD interval is then constrained to this variable. At the beginning of each month a monthly optimal schedule is determined based on forecasted water demands. The schedule is then followed until demand levels deviate from the predicted
demand to a point that the reservoir levels are not acceptable. To compensate for this deviation a short term (daily) schedule is determined using the same optimization model. The short term schedule then restores reservoir levels to the predicted levels of the monthly model. Thereafter, the monthly model is followed again. The total connected load of the plant is not given, but it is estimated as 5 MW based on the ratings of the individual motors. The reduction in electricity cost after load shifting varies between 3% and 40% per month over a 7 month period. The actual cost is compared to the cost of an optimized model.

2.6. Rationale for this study

As discussed, only [33]-[36] uses closed-loop optimal control models that includes TOU charges, and only [35]-[36] uses closed-loop optimal control models that includes both TOU and MD charges. As mentioned, the closed-loop optimal control models in [35]-[36] are re-optimized daily. Ideally they should be re-optimized more frequently to react to disturbances closer to real-time.

Therefore, little evidence could be found to prove the applicability of closed-loop optimal control for load shifting in different applications, and specifically where both TOU and MD charges are included in the objective functions of the control models.

Although the problem and the optimization techniques are similar between an open and closed-loop optimal control models, it is important to prove the applicability of the closed-loop control model, because there are certain issues that are not addressed by an open loop control model. For example, the optimization time of a closed-loop optimal control model has to be short in order to actively control a process. This might require a compromise on the optimality of the solution, like the selection of shorter control horizons to improve the optimization time.

Furthermore, there are differences in application of the optimal control models in [25]-[36], which means that the techniques and assumptions are not universally applicable. For example, only [30]-[32] and [35]-[36] covers load shifting in water supply systems,
and these solutions are also not universally applicable to all water supply problems. Therefore, an additional case study adds to the available body of knowledge.

Note that the switching intervals in [28]-[32] and [35]-[36] are greater than or equal to the MD integrating period. This means that optimization within the MD integrating interval is not considered. This dissertation shows that smaller switching intervals can reduce MD costs in certain applications.

A two step optimization approach is used in [35]-[36] to optimize the TOU and MD costs. In both cases the first optimization step uses continuous variables, which are then scheduled into binary statuses in the second optimization step. This dissertation considers a single optimization step that is based on the technique used in [36]. This is covered in more detail in section 2.7 and Chapter 1.

2.7. Approach of this study

2.7.1. Hypothesis

The hypothesis is that a closed-loop optimal control model can be used for load shifting problems in industrial applications; including customers that are charged on TOU and MD.

2.7.2. Motivation for the model predictive control (MPC) approach

The control problem of a specific case study is formulated as a discrete time linear closed-loop optimal control problem. The closed-loop optimal control model is implemented with an MPC approach [39]-[43].

MPC is an optimal control approach that uses an explicit model of the plant to predict future responses (outputs). With an MPC approach an open loop optimal control problem is solved repeatedly over a finite control horizon at each switching interval, and only the first control step (input) is implemented after each iteration. At the next sampling interval the state of the plant is re-sampled (measured) and the process of optimization is repeated.
Chapter 2  Literature survey

[40], [43]. The basic structure of the MPC approach is shown in Figure 2-1. The MPC approach is explained in more detail in section 3.2.

![Diagram of Basic Structure of an MPC Model](adapted from [40]).

An MPC approach is selected for the following reasons [43]:

1. The periodic re-optimization characteristic of an MPC model provides stability during external disturbances.
2. The re-optimization also compensates for inaccurate or simplified system models i.e. model uncertainty.
3. An MPC model converges to the solution of the open loop optimal control model.
4. An MPC controller can be started and restarted at any time, whilst an open loop controller must be started at the correct simulated start time.
5. An MPC approach is a generic closed-loop optimal control solution that is not only applicable to a specific control problem.

2.7.3. Research process and modelling

The research comprises of the following sequential activities:

1. A generic open loop optimal control model is defined with an objective function that includes TOU and MD charges. The control model is formulated as a discrete time linear problem so that standard optimization algorithms can be applied to solve the problem [44],[45],[46]. One of these standard optimization algorithms is
IP, which is selected for the optimization in this dissertation.

2. The generic open loop control model is then converted to a closed-loop optimal control model using the MPC approach with IP optimization.

3. The generic open and closed-loop optimal control models are then applied to a specific case study.

4. The current level based control model for the case study is then defined and simulated as the baseline for comparisons.

5. The applied open loop optimal control model is then simulated with an LP optimization algorithm. The LP simulation is included to calculate a benchmark to evaluate the effectiveness of the IP solutions. The LP model solves very quickly (less than 10 seconds), and the result is considered optimal, because the variables are not constrained to integer values. Note that this is not a practical solution, because the pumps are binary controlled.

6. The applied open loop optimal control model is then simulated with IP optimization for a specific scenario with different simulation timeout values. The simulated scenario uses two switching intervals per MD period, a control horizon of 24 hours is used and it is simulated over a one day period. Based on the results a practical timeout value is selected for subsequent simulations. Note that with 2 switching intervals per MD period the switching intervals are 15 minutes long, because the MD integrating period is 30 minutes long.

7. The open loop optimal control model with IP optimization is then simulated over 30 days to get a monthly average. The results are also used for comparison with the closed-loop optimal control model in step 11. This scenario uses two switching intervals per MD period, and a control horizon of 24 hours.

8. The open loop optimal control model with IP optimization is then simulated with different switching intervals to evaluate the effect on the TOU and MD costs. For these scenarios a control horizon of 24 hours is selected and the control model is simulated over 30 days. It is assumed that the results from the closed-loop model will match the results from the open loop model for all the scenarios with different switching intervals if the results for one of the scenarios between the open and closed-loop control models match.
9. The open loop IP control model is then interchangeably simulated with TOU and MD charges. The purpose of this simulation is to evaluate the effect of the TOU and MD optimization separately. This scenario uses 11 switching intervals per MD period (i.e., 2.73 minutes), a control horizon of 24 hours, and is simulated over 30 days. Initially two switching intervals per MD period were used for this simulation. However, the optimal schedule coincidently resulted in a low MD cost, and therefore 11 switching intervals is selected which results in a better example of the problem.

10. The closed-loop control model is then simulated with different control horizons to evaluate the effect on TOU and MD costs. For these scenarios 2 switching intervals per MD period is used and the control model is simulated over 30 days. The purpose of this simulation is to evaluate the effect of possible compromises in practical applications where shorter control horizons are required to improve the simulation time.

11. The open and closed-loop control models are then compared over 30 days for a specific scenario, i.e., two switching intervals per MD period and a control horizon of 24 hours.

12. The open and closed-loop control models are then simulated with an inaccurate system model and with model disturbances to show how the closed-loop control model compensates for the model uncertainty and the model disturbances, whilst the open loop control model does not.

2.7.4. Selected case study

The water pumping scheme at the Rietvlei water purification plant in the Tshwane municipality (South Africa) is selected for the case study. A tariff that includes TOU and MD charges is selected for the simulations [5]. A tariff is selected because the plant is not currently charged on a TOU based tariff by the Tshwane municipality [5]. This is covered in more detail in section 4.2. A specific part of the plant is selected for this research, i.e., the part of the plant that has potential for load shifting without large infrastructure expenditure. The detail of the plant is described in Chapter 4.
2.7.5. How this approach addresses current deficiencies

The selected approach addresses the deficiencies described in section 2.6 as follows:

1. The selected case study affirms the application of closed-loop optimal control for load shifting in plants with electricity tariffs that include TOU and MD charges.
2. The selected case study also specifically affirms the use of a closed-loop optimal control model in water supply systems.
3. The MPC approach optimizes regularly (at every control step) as oppose to just daily, which enables a faster reaction to disturbances.
4. The MPC approach is a generic approach that is not specific to the optimization problem. The only difference is the modelling of the optimization problem.
5. This approach evaluates the effect of various control parameters that can be considered in practical applications of closed-loop optimal control models, i.e., simulations timeouts, switching intervals and prediction horizons.
6. The selected approach optimizes within the MD period, e.g., at 15 minute intervals.
7. The selected approach determines the optimal binary pumping schedule for both TOU and MD charges in a single step.

2.7.6. Challenges and limitations of the selected approach

The challenges and limitations of this approach are:

1. The research is limited to a specific case study only. The aim is simply to affirm the hypothesis by adding to the existing body of knowledge.
2. The interpretation and modelling of the specific plant might not be 100% accurate. A real implementation is required as part of a further research initiative to affirm the results.
3. The control model is simplified for simulation purposes and cannot be implemented as is. For example, the design does not factor in disturbances like motor failures and maintenance outages. The design also does not consider the maturity of process
control technology, i.e., existing process controllers might not be able to compute complicated optimization algorithms.

4. Modelling and optimization under both TOU and MD charges is challenging. The control model and the simulation results provide information to guide future applications.

2.8. Contributions of this study

The contributions of this study can be summarized as follows:

1. This study affirms that a closed-loop optimal control model can be used to solve industrial load shifting problems for customers whose tariffs include TOU and MD charges. In other words, it adds value by adding to the available case studies [35], [36].

2. This study affirms that an MPC approach can be used to design the closed-loop controller for load shifting problems [33].

3. This study considers scheduling within the MD integrating period, where other work does not [28]-[32], [35], [36].

4. This study validates the convergence of the electricity cost of the closed-loop MPC solution to the open loop controller [43].

5. This study validates the robustness of the MPC solution [43].

6. This study evaluates the effect of various control parameters that can be considered in practical compromises. This kind of comparative information is not available from exiting references [25]-[36].

7. This study validates the importance of including MD charges in the optimization where applicable, i.e., it adds to the available case studies [28]-[32], [35], [36].

8. This study proves that an optimal binary pump schedule that optimizes both TOU and MD charges can be obtained in a single optimization step. This is done as a two step approach in existing literature [28]-[32], [35], [36].
3. GENERIC CONTROL MODEL FOR A WATER PUMPING SCHEME

This chapter defines a generic open and closed-loop optimal control model for load shifting in a water pumping scheme.

As described in section 2.7.2, with an MPC approach an open loop optimal control problem is solved repeatedly over a finite control horizon at each switching interval, and only the first control step is implemented after each iteration. Therefore, the first step required to define a generic closed-loop optimal model is to define a generic open loop optimal control model. The second step is to convert the open loop model to a closed-loop model.

3.1. Generic open loop optimal control model

The state model of the open loop optimal control model is defined as

\[ L_{(t+1)r} = L_{tr} + \sum_n A_{rn} \cdot u_{tn} + B_r, \]  

where:

- \( n \): The \( n \)-th pump, and \( n = 1, \ldots, N \).
- \( N \): The total number of pumps.
- \( t \): The \( t \)-th discrete switching interval, and \( t = 1, \ldots, T \).
- \( T \): The total number of discrete switching intervals.
- \( u_{tn} \): The binary switching status of the \( n \)-th pump at the \( t \)-th switching interval; \( u_{tn} = 0 \) when the pump is off and \( u_{tn} = 1 \) when the pump is on.
- \( r \): The \( r \)-th reservoir, and \( r = 1, \ldots, R_0 \).
- \( R_0 \): The total number of reservoirs.
- \( L_{tr} \): The level of the \( r \)-th reservoir at the \( t \)-th switching interval.
The flow rate of the \( n \)-th pump at the \( r \)-th reservoir.

The constant inflow or outflow rate of the \( r \)-th reservoir, e.g., gravitational flow.

The aim is to optimize the switching of a number of pumps (\( N \)) to reduce the cost of both TOU and MD charges over a control horizon (\( H \)), for example 24 hours. The objective function to minimize is defined as

\[
\min_{u_m, z_n} \sum_{n=1}^{N} \left( \lambda_1 \sum_{t=1}^{T} u_m \cdot p_n \cdot c_t + \lambda_2 \frac{p_n}{S} \cdot z_n \cdot C \right),
\]

where:

\( s \)  The \( s \)-th switching interval in any MD integrating period, and \( s = 1, \ldots, S \).

\( S \)  The total number of switching intervals in an MD integrating period.

\( p_n \)  The power consumption of the \( n \)-th pump.

\( c_t \)  The TOU energy cost in the \( t \)-th switching interval.

\( z_n \)  The \( n \)-th MD integer variable for the \( n \)-th pump, and \( 0 \leq z_n \leq S \).

\( C \)  The MD charge in R/kW or R/kVA, and ‘R’ represents the South African currency called Rand (US $1 \approx R8.00).

\( \lambda_1 \)  The weight assigned to the TOU energy cost.

\( \lambda_2 \)  The weight assigned to the MD cost.

The variables \( u_m \) and \( z_n \) needs to be solved by the optimization algorithm over the control horizon (\( H \)).

The relationship between the variables can be explained as follows: a control horizon is divided into \( T \) switching intervals, and each MD interval consists of one or more switching intervals (\( S \)). For example, if a control horizon (\( H \)) of 24 hours is divided into 15 minute switching intervals, then the total number of switching intervals is 96 (\( T=96 \)). This means
that each MD interval is divided into two switching intervals \((S=2)\), if the MD integrating intervals are 30 minutes long.

Note that the technique to represent the MD costs with a variable in the objective function is based on [36]. In this dissertation the variables \(u_n\) and \(z_n\) are modeled as binary integer and pure integer variables respectively, whilst in [36] all the variable are modeled as continuous variables. The benefit of the integer variables is that it avoids a second scheduling step within the MD period.

The reservoir level constraints are defined as

\[
D_r \leq L_n \leq E_r
\]  

(3-3)

where \(D_r\) represents the minimum level constraint of the \(r\)-th reservoir and \(E_r\) represents the maximum level constraint of the \(r\)-th reservoir.

The following constraints are required to force \(z_n\) to represent the highest MD across all MD integrating periods:

\[
\sum_{n=1}^{N} \left( \sum_{t=1+kS}^{kS+S} u_n \cdot p_p - p_p \cdot z_n \right) \leq 0 \text{ for } k = 0, \ldots, \left( \frac{T}{S} - 1 \right),
\]  

(3-4)

where \(k\) represents the number of MD periods in the control horizon. In other words, the purpose of (3-4) is to constrain each individual MD period to the maximum value of the MD that is represented by \(z_n\) in the objective function in (3-2).

### 3.2. Generic closed-loop optimal control model

The closed-loop optimal control model is defined with the same state model as the open loop model in (3-1), and the objective function is defined as
where \( m = 1, \ldots, M \), and \( M \) represents the last switching interval of the controller. The last switching interval could be considered as infinite unless the controller is stopped.

In (3-5) the open loop optimal control problem is solved repeatedly over a finite control horizon \((H)\) at each switching interval \( t \), and only the first control step \( u_m \) is implemented after each iteration. At the next sampling interval \((t+1)\) the state of the plant \((L)\) is re-sampled and the process of optimization is repeated over the new control horizon \([m, m+T]\).

The same constraints for the open loop model in (3-3) and (3-4) apply to the closed-loop model. The only difference is that the constraints need to be updated after each switching interval is implemented.

The MPC control strategy can be explained further with Figure 3-1, which shows the result of a hypothetical controller that controls the level of one reservoir. The reservoir has a constant inflow rate and the outflow is controlled with only one pump.

The control model in Figure 3-1 uses 15 minute switching intervals \((S=2)\), and a control horizon \((H)\) of 8 hours. Figure 3-1 shows the level of the reservoir \( L_t \) (output), the statuses of the pump \( u_t \) (inputs) and the TOU energy charges \( c_t \) over 14 hours.

Figure 3-1 shows that the current time is 6h00, which means that the inputs and output prior to 6h00 are historical and the inputs and output after 6h00 are the future predicted values. Note that the MPC sampling intervals are chosen to coincide with the switching intervals of the pumps.
The process of the MPC controller in Figure 3-1 can be described as follows: At the current time (6h00) the controller samples the current reservoir level, applies all the constraints, and predicts the future statuses of the pump that will optimize cost over the next 8 hours. The calculated statuses of the pump are referred to as the predicted inputs \(u_{t1}\). The results in Figure 3-1 show that the pump needs to be switched on for the next 15 minutes (6h00 to 6h15), and that pump needs to be switched on for a few more 15 minute intervals between 10h00 and 14h00. Figure 3-1 also shows how the level of the reservoir is predicted over the next 8 hours from 6h00 to 14h00.

However, once the predicted inputs are calculated only the first predicted input is implemented and the rest of the predicted inputs are discarded. After the first predicted input is implemented the entire optimization process is repeated. This means that the pump is switched on for 15 minutes, and when the 15 minute interval lapses the level of the reservoir is sampled again, the constraints are re-applied and the future statuses of the
pump over the next 8 hours are predicted again. When the new statuses of the pump are
determined only the first status (predicted input) is implemented again. Therefore, the
optimization process repeats indefinitely at each switching interval \((t)\).

3.3. Impact of tariff and system parameters on the optimal control model

The generic optimal control model is defined for a water pumping scheme with an
electricity tariff that includes TOU and/or MD charges. It is expected that this generic
optimal control model is applicable to most water pumping schemes with an electricity
tariff that includes TOU and/or MD charges. For example, the generic optimal control
model is still applicable if the structure of the plant is similar, i.e., only the system
parameters such as the number of pumps, number of reservoirs, flow rates, and reservoir
capacities change. Therefore, it is expected that the prices in the electricity tariff and the
system parameters will only affect the optimal scheduling (computation results) of each
plant.

It is expected that the optimal control model will change if the structure of the plant is
different or if the tariff structure is different, e.g., when the plant is not a water pumping
scheme or when a third charge component is added to the tariff. However, it is expected
that the modelling idea is applicable to load shifting applications in general, even if the
plant structure or tariff structure is different.

3.4. Validation of the generic optimal control model

The generic open and closed-loop optimal control models are validated in Chapter 4 where
the generic control models are applied to a specific case study. In Chapter 5, both the
applied open and closed-loop optimal control models are then simulated and compared
with the current level based control model. The optimal control models are solved with
integer programming optimization. The results show that the applied control model reduces
electricity costs within the constraints of the plant.
4. APPLICATION OF THE GENERIC CONTROL MODEL TO A CASE STUDY

This chapter describes the selected case study, and applies the generic open and closed-loop optimal control models from Chapter 3 to the case study.

4.1. Plant overview

A water purification plant in the Tshwane municipality in South Africa is selected for the case study. The plant can be divided into the purification plant itself and the pumping scheme of the purified water (see Figure 4-1). This dissertation focuses on the water pumping scheme, because it consumes the bulk of the electricity and it has potential for load shifting.

Figure 4-1: Pumping scheme of the Rietvlei water purification plant.
Water flows from the dam through the purification plant into a reservoir (R1) at 40 ML/day (mega liter per day). R1 is also supplied with water from a fountain at 5 ML/day. R1 has a capacity of 1.4 ML.

The water from R1 is pumped to two reservoirs: R2 and R3, with a capacity of 120 ML and 60 ML respectively. The water to R2 is pumped by motors K1, K2 and K3; each rated at 300 kW with the ability to pump 22 ML/day per motor. The water to R3 is pumped by motors G1, G2 and G3; each rated at 275 kW with the ability to pump 10 ML/day per motor.

The primary source of water to R2 and R3 is a water utility in the province called Randwater, and R3 is also supplied by boreholes at a rate of 10 ML/day.

The remainder of this section focuses on the water pumping scheme at the purification plant, which includes reservoir R1 and motors K1, K2, K3, G1, G2 and G3.

The constraints of R1 and the relevant pumps are:

1. At least one of the pumps to both R2 and R3 must run continuously, otherwise the water in the pipes flows back into R1.
2. As much water as possible must be pumped to R3, because the reservoir is small and therefore the risk of running out of water is high.
3. Running three pumps to either R2 or R3 is not desirable, because the mechanical losses are too high.
4. A motor should not be started more than three times per hour.
5. There is no limit on the amount of water that can be pumped to R2 and R3.
6. Randwater supplies most of the water to R2 and R3 at a cost of R 2.98/kL (Rand per kilolitre). The boreholes and the purification plant are used as alternative water supplies with significant lower costs, i.e., R 0.30/kL and R 1.03/kL respectively. This means that the maximum amount of water from R1 must be pumped to R2 and
The pumps K1, K2, K3, G1, G2 and G3 are currently controlled with a level based control system. This means that each pump switches on and off when R1 reaches a specific level. The current switching levels are shown in Table 4-1. Based on the on/off switching levels in Table 4-1, G1, G2 and K1 are running most of the time, whilst K2 switches on and off to control the level. The exception is during outages when the level of R1 may drop into the switching ranges of the other pumps. Note that K3 and G3 are used as back-up pumps. As a result this configuration pumps approximately 25 ML/day to R2 and 20 ML/day to R3, as shown in Figure 4-1.

Therefore, the most viable load shifting option for the pumping scheme at the purification plant, within the listed constraints, and without large infrastructure expenditure, is the switching times of K2. The problem with the current switching times of K2 is that K2 operates approximately six times per day, for more than 30 minutes at a time. This means that the maximum demand per month equals the maximum capacity of the motors. With the current control model K2 could also operate during peak demand periods, because the motor simply starts when the reservoir level is too high. Therefore, it would be more desirable if K2 can run more frequently, for shorter periods, and preferable not during peak demand periods.

<table>
<thead>
<tr>
<th>Pump</th>
<th>Current “on” level</th>
<th>Current “off” level</th>
<th>Revised “off” level</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>0.6</td>
<td>0.3</td>
<td>0.05</td>
</tr>
<tr>
<td>K2</td>
<td>1.3</td>
<td>0.9</td>
<td>0.2</td>
</tr>
<tr>
<td>K3</td>
<td>back-up</td>
<td></td>
<td></td>
</tr>
<tr>
<td>G1</td>
<td>0.8</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>G2</td>
<td>1.0</td>
<td>0.7</td>
<td>0.15</td>
</tr>
<tr>
<td>G3</td>
<td>back-up</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To enable more effective load shifting of K2 the “off” switching levels of all the motors
are revised to allow K2 with a wider operating band. The revised switching levels are shown in Table 4-1. Based on the revised switching levels G1, G2 and K1 will still run most of the time, whilst K2 switches on and off to control the level.

4.2. Electricity tariff

The purification plant is supplied with electricity from the Tshwane municipality on the standard 11 kV bulk supply tariff. This tariff includes a flat energy charge and an MD charge [5]. However, for this case study the municipality’s 11 kV TOU tariff is used instead. The 11 kV TOU tariff includes a TOU and an MD charge [5]. Note that the electricity cost is more or less the same on the flat and TOU tariffs if the load is not shifted. The variable fees of the TOU tariff for September 2008 are summarized in Table 4-2.

<table>
<thead>
<tr>
<th>Period</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Off-peak (0h00 to 6h00 and 22h00 to 24h00)</td>
<td></td>
</tr>
<tr>
<td>High demand (winter)</td>
<td>0.1187 R/kWh</td>
</tr>
<tr>
<td>Low demand (summer)</td>
<td>0.1049 R/kWh</td>
</tr>
<tr>
<td>Standard (6h00 to 7h00 and 10h00 to 18h00)</td>
<td></td>
</tr>
<tr>
<td>High demand (winter)</td>
<td>0.1411 R/kWh</td>
</tr>
<tr>
<td>Low demand (summer)</td>
<td>0.1383 R/kWh</td>
</tr>
<tr>
<td>Peak (7h00 to 10h00 and 18h00 to 22h00)</td>
<td></td>
</tr>
<tr>
<td>High demand (winter)</td>
<td>0.8205 R/kWh</td>
</tr>
<tr>
<td>Low demand (summer)</td>
<td>0.2628 R/kWh</td>
</tr>
<tr>
<td>Maximum demand charge (applicable in peak and standard times)</td>
<td>R 66.50/kVA</td>
</tr>
</tbody>
</table>

4.3. Current electricity costs

The calculated electricity cost of the overall pumping scheme over 30 days is shown in Table 4-3. The overall cost calculation is required to determine the overall saving for the plant.
Chapter 4  Application of the generic control model to a case study

The simulated energy cost for K2 from section 5.1 is used in Table 4-3, whilst the energy cost for K1, G1 and G2 is calculated with the high demand (winter) TOU tariff. A uniform load throughout the month is assumed in the calculations.

Table 4-3: Calculated electricity costs of the overall pumping scheme over 30 days.

<table>
<thead>
<tr>
<th>Pump</th>
<th>kWh</th>
<th>MD (kVA)</th>
<th>Energy cost</th>
<th>MD cost</th>
<th>Total costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1 (calculated)</td>
<td>216,000</td>
<td>300</td>
<td>R 59,438</td>
<td>R 19,950</td>
<td>R 79,388</td>
</tr>
<tr>
<td>K2 (simulated)</td>
<td>30,348</td>
<td>300</td>
<td>R 8,351</td>
<td>R 19,950</td>
<td>R 28,301</td>
</tr>
<tr>
<td>G1 (calculated)</td>
<td>198,000</td>
<td>275</td>
<td>R 54,485</td>
<td>R 18,288</td>
<td>R 72,772</td>
</tr>
<tr>
<td>G2 (calculated)</td>
<td>198,000</td>
<td>275</td>
<td>R 54,485</td>
<td>R 18,288</td>
<td>R 72,772</td>
</tr>
<tr>
<td>Total</td>
<td>642,348</td>
<td>1,150</td>
<td>R 176,758</td>
<td>R 76,475</td>
<td>R 253,233</td>
</tr>
</tbody>
</table>

The calculated results in Table 4-3 correlates with the actual consumption for the plant during June 2008 where an MD of 1,152 kVA was registered with a total energy consumption of 640,416 kWh. As mentioned, the actual cost is not used, because the plant is currently charged on a flat energy rate.

4.4. Assumptions for the optimal control model

The following assumptions are made for the optimal control models:

1. The revised switching levels from Table 4-1 are used.
2. Only K2 is considered in the optimal control model. Pumps K1, K3, G1, G2 and G3 are assumed to be controlled with the existing level based control model, which (as explained) results in K1, G1 and G2 to always run.
3. The high demand season (winter) tariffs are used.
4. The off-peak, standard and peak times for all days are considered the same as a week day. This is to simplify simulation over a 30 day period.
5. A motor power factor of one is used for the simulations. This means the kVA and kW consumption is equal.
6. A utilization factor of one is used for the simulations. This means that the motors
run at full load.

7. The pumping scheme consumes more than 90% of the electricity of the total plant, and therefore the overall cost saving is evaluated against the consumption of the pumping scheme only.

4.5. Formulation of the current level based control model

The current level based control model is also defined as a discrete time model, which is based on the state model in (3-1). Since there is only one reservoir (R1) and one pump (K2) to consider, the level of reservoir R1 at the t-th switching interval is defined as

\[ L_{t+1} = L_0 + \sum_{r=1}^{t} \text{FLOWIN}_r - \text{FLOWOUT}_r \cdot u_t, \]  \hspace{1cm} (4-1)

where \( L_0 \) is the initial level of reservoir R1, and \( t=1,\ldots,T \). The upper level limit is used as the initial level, i.e., \( L_0 = 1.3 \) ML. \( \text{FLOWIN}_r \) is the relative inflow to R1 over the t-th switching interval, which is a constant flow from the fountain and the purification plant, minus the outflows from the level controlled motors, i.e., \( \text{FLOWIN}_r = \text{purification plant} + \text{fountain} - \text{G1} - \text{G2} - \text{K1} = 3 \) ML/day. \( \text{FLOWOUT}_r \) is the outflow of K2 for the t-th switching interval, which is a constant value of 22 ML/day.

The control status of the pump (K2) is based on the revised switching levels, and it is defined as

\[ u_t = \begin{cases} 0, & \text{when } L_t \leq 0.2 \text{ ML} \\ 1, & \text{when } L_t \geq 1.3 \text{ ML} \end{cases} \]  \hspace{1cm} (4-2)

4.6. Open loop optimal control model with IP optimization

Since these is only one reservoir (R1) and pump (K2) to consider, the state model in (4-1) also applies to the open loop optimal control model, and the generic objective function in
(3-2) is simplified as

\[
\min_{u_t, z} \left( \lambda_1 \sum_{i=1}^{T} u_i \cdot p \cdot c_i + \lambda_2 \frac{P}{S} \cdot z \cdot C \right),
\]  

(4-3)

where \( C = R \ 66.50 \) for the maximum kVA/kW over any 30 minute integrating period, \( p = 300 \ kW, \ S = 2, \ T = 2HS, \) and \( \lambda_1 = \lambda_2 = 1. \) Setting \( \lambda_1 = \lambda_2 = 1 \) means that the total cost in (4-3) represents actual costs with no preference between a TOU or MD reduction. The objective function in (4-3) is subject to the constraints in (4-5)-(4-7).

Based on Table 4-2, the cost of energy in Rands for the high demand season is defined as

\[
c_t = \begin{cases} 
0.1187 \ / \ 2S, & t \in [0.12S] \cup [44S, 48S] \\
0.1411 \ / \ 2S, & t \in [12S, 14S] \cup [20S, 36S] \\
0.8205 \ / \ 2S, & t \in [14S, 20S] \cup [36S, 44S] 
\end{cases}.
\]  

(4-4)

The generic level constraints in (3-3) are defined as the upper and lower level constraints as

\[ L_t \leq 1.3 \text{ ML for } t = 1, \ldots, T, \]

(4-5)

and

\[ L_t \geq 0.2 \text{ ML for } t = 1, \ldots, T. \]

(4-6)

The generic MD constraints in (3-4) (applicable in peak and standard times) are simplified and defined as
Chapter 4 Application of the generic control model to a case study

Note that the constraint of the number of allowable starts per hour for K2 is automatically adhered to as part of the optimization, i.e., K2 will be switched on at most once per MD period, which results in a worst case of two starts per hour.

The optimization problem is solved with IP, because the variables $u_t$ and $z$ are defined as binary integer and pure integer variables.

4.7. Open loop optimal control model with LP optimization

If the integer variables ($u_t$ and $z$) in the open loop optimal control model (4-3)-(4-7) are treated as real variables, then the optimization problem becomes a linear programming problem. This strategy is not practical for the selected case study, because the pumps are binary controlled. However, the LP optimization strategy is included as a benchmark to evaluate the effectiveness of the IP solutions.

The LP solution is considered as a benchmark, because the variables are not constrained to integers, which results in one optimal result that satisfies all the constraints. The LP optimization also solves very quickly (less than 10 seconds).

Therefore, the result from the LP optimization is better or equal to the result form the IP optimizations, which provides a possible least bound for the objective function value evaluated at integer feasible solutions.

The formulation of the open loop optimal control model with LP optimization is the same as the formulation of the open loop optimal control model with IP optimization in section 4.6. The only difference is that $u_t$ and $z$ are not constrained to integer values – $u_t$ is only constrained as $0 \leq u_t \leq 1$. 

\[
\sum_{t=1}^{S} u_t \cdot p - p \cdot z_s \leq 0 \text{ for } k = 0, ..., \left(\frac{T}{S} - 1\right). \quad (4-7)
\]
4.8. Closed-loop optimal control model with IP optimization

The closed-loop optimal control model is defined with the same state model as the current control model and the open loop optimal control model in (4-1).

Since there is only one pump (K2) and one reservoir (R1) to consider, the generic closed loop objective function in (3-5) is simplified as

\[
\min_{u_t, z} \left( \lambda_1 \sum_{t=1}^{T_m} u_t \cdot p \cdot c_t + \lambda_2 \frac{p}{S} \cdot z \cdot C \right). \tag{4-8}
\]

The objective function in (4-8) is minimized subject to the constraints in (4-5), (4-6), and (4-7) over the prediction horizon \([m, m+T]\), as described in section 3.2. This means that the closed-loop model is not a simple optimization problem, but a series of optimization solutions with iterative implementations of obtained solutions.

Note that the TOU cost function in (4-4) applies to the objective function in (4-8) as well.

4.9. Choice of simulation timeout

Solving an IP problem could take very long, depending on the number of variables, the optimization algorithm, the level of LP relaxation, etc. [50]. This is often referred to as the curse of dimensionality. However, optimization software provides parameters that can be set to improve the optimization time. For example, maximum number of iterations, maximum optimization time, gap between bounds, etc. In many cases this results in near-optimal solutions as opposed to the absolute optimal solutions.

If a timeout parameter is used, the best available result is presented when the optimization terminates. This is important in a closed-loop control model where the optimization can not run indefinitely, i.e., a less optimal solution that satisfies the constraints in a practical timeframe is preferred. Therefore, a timeout parameter is selected for this case study. The
timeout value for this case study is selected as 10 seconds. This selection is based on an evaluation that is covered in section 5.3.1. The 10 second value is also a practical selection that gives near optimal results for this case study.

4.10. Choice of switching intervals

The switching intervals are chosen to coincide with the TOU and MD periods, i.e., at least one switching interval per 30 minute integrating period ($S=1$). To reduce the MD charge in this application $S$ needs to be bigger than one ($S>1$), which divides the MD integrating period into smaller intervals.

However, the size of $S$ is a trade-off between computational time, equipment constraints and cost saving. For example, with $S=6$ the switching intervals are only five minutes long. This enables fine optimization, especially for the MD charges, because a motor can be started every 30 minutes for only five minutes. However, with $S=6$ the number of variables over a 24 hour period is very high, which has a significantly affect on computation time and it could make the solution impractical.

Furthermore, there might also be a limitation on the number of times a motor can be started in an hour and/or in the expected lifetime of the motor [47],[48]. One of the assumed limitations in section 4.1 is that a motor may not be started more than three times per hour. As mentioned in section 4.6, this limitation is automatically adhered to as part of the optimization

The effect of the switching intervals ($S$) on the TOU and MD cost for $S=1$ to $S=13$ is covered in section 5.3.3.

4.11. Choice of control horizon

Like the switching interval, the control horizon ($H$) is a trade-off between computational time and cost saving. The control horizon is selected as 24 hours for most of the simulations in Chapter 5. The only exception is the scenarios in section 5.4.1 where the
effect of different control horizons is simulated.

4.12. Solving the problem with Matlab and LPSOLVE

The IP optimization problem is solved with the Matlab [49] and LPSOLVE [50]. LPSOLVE is an open source library that is callable from Matlab and it solves mixed integer linear programming (MILP) and IP problems. LPSOLVE is required, because Matlab only provides a binary integer programming function (called \textit{bintprog}) with the Matlab optimization toolbox.

However, in an initial exercise, the problem was formulated as a pure binary integer programming problem and the Matlab \textit{bintprog} function was used. In this initial exercise $z$ was formulated as a number of binary integer variables (one for each switching interval). However, the simulation times were impractical, especially when $S>2$. For example, a scenario with $S=2$ that provides good results within 10 seconds with LPSOLVE takes approximately 20 minutes with the Matlab \textit{bintprog} function, for the same results. The same problem with \textit{bintprog} was encountered in [51]. Therefore, LPSOLVE is used for this case study.

LPSOLVE uses a branch and bound strategy with LP relaxation to solve the IP problem [50],[52]. This means that the solution is not necessarily the true optimal solution, but a good approximation of the true optimal solution. Although the Matlab \textit{bintprog} function uses the same strategy the LPSOLVE algorithm is more effective, especially in scenarios with many variables.

There are many settings that can be used to increase the optimization performance of LPSOLVE. Some of the settings are a trade-off between performance and the true optimal solution. The following LPSOLVE settings are used in the IP simulations in this case study:

1. The LPSOLVE bound on the objective function is set to 15,000 for all the open and
closed-loop optimal control model simulations. The only exception is the scenario in section 5.3.3 where \( S=1 \) where the bound was set to 30,000. This setting improves performance, because the solver can ignore unwanted solutions.

2. The tolerance on the integers is selected as 0.01 for all the open and closed-loop optimal control model simulations.

3. As mentioned, the timeout on the optimization is selected as 10 seconds for all the open and closed-loop optimal control model simulations. The justification for this value is covered in section 5.3.1.

LPSOLVE does not provide the capability to suggest a solution to the optimization algorithm. This capability is available in the Matlab `bintprog` function though. The advantage of this capability in a closed-loop model is that the optimization results from the previous step can be suggested to the next optimization step. However, this limitation is not critical, because LPSOLVE (as mentioned) is effective and provides the desired result in a very short time.

The LP optimization problem is solved with the Matlab LP function (`linprog`). As mentioned, the Matlab LP function is included to calculate a benchmark to evaluate the effectiveness of the integer solutions. The LP model solves very quickly (less than 10 seconds), and the result is considered optimal, because the variables are not constrained to integer values.

The LPSOLVE and `linprog` functions are defined as

\[
\min_x f^T \cdot x \quad \text{such that} \quad A \cdot x \leq b \quad \text{and} \quad Aeq = beq, \quad (4-9)
\]

where \( f, b, \) and \( beq \) are vectors; \( A \) and \( Aeq \) are matrices; and the solution \( x \) is a integer vector for `bintprog` and a decimal value for `linprog`. For LPSOLVE the values of \( x \) that represent \( u_t \) are constrained to binary integers, whilst the value of \( x \) that represents \( z \) is constrained to an integer only.
The Matlab simulation environment is summarized in Table 4-4.

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer</td>
<td>Dell PowerEdge, 1955 blade</td>
</tr>
<tr>
<td>Processor</td>
<td>Intel Xeon, 5355, Quad-Core, 2.66GHz</td>
</tr>
<tr>
<td>Random access memory</td>
<td>8 GB</td>
</tr>
<tr>
<td>Operating system</td>
<td>Debian Linux</td>
</tr>
<tr>
<td>Matlab version</td>
<td>Version 7.7.0.471 (R2008b)</td>
</tr>
<tr>
<td>LPSOLVE version</td>
<td>5.5.0.10 for 64 bit operating systems</td>
</tr>
</tbody>
</table>

Note that LPSOLVE is a single threaded application which means that only one of the four processors is utilized for the optimization.
5. SIMULATION RESULTS OF THE APPLIED CONTROL MODEL

This chapter simulates and compares the control models that are defined in Chapter 4. This comparison includes:

1. The current level based control model.
2. The open loop optimal control model with LP optimization.
3. The open loop optimal control model with IP optimization.
4. The closed-loop optimal control model with IP optimization.

The effect on TOU and MD costs with each control model is evaluated in section 5.1 to 5.4. Significant results from this evaluation is summarized and compared in section 5.5. The robustness of the closed-loop optimal control model is evaluated in section 5.6, and the practicality of the optimal control model is discussed in section 5.7.

The results in this chapter are listed in Table 5-1 and Table 5-2. The saving on the overall plant is covered in Table 5-1, and the saving on K2 in isolation is covered in Table 5-2.

Table 5-1: Overall saving for the pumping scheme for significant scenarios.

<table>
<thead>
<tr>
<th>S</th>
<th>Energy cost</th>
<th>% From baseline (energy cost)</th>
<th>MD Cost</th>
<th>% From baseline (MD cost)</th>
<th>Total costs</th>
<th>% From baseline (total)</th>
<th>MD portion of saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>176,758</td>
<td>0.00%</td>
<td>76,475</td>
<td>0.00%</td>
<td>253,233</td>
<td>0.00%</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Current level based control model – the baseline

Open loop optimal control model with IP optimization – different switching intervals (S) and H=24.

<table>
<thead>
<tr>
<th>S</th>
<th>Energy cost</th>
<th>% From baseline (energy cost)</th>
<th>MD Cost</th>
<th>% From baseline (MD cost)</th>
<th>Total costs</th>
<th>% From baseline (total)</th>
<th>MD portion of saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>172,101</td>
<td>-2.63%</td>
<td>58,306</td>
<td>-23.76%</td>
<td>230,407</td>
<td>-9.01%</td>
<td>79.6%</td>
</tr>
</tbody>
</table>

Open loop optimal control model with LP optimization – the optimal benchmark

<table>
<thead>
<tr>
<th>S</th>
<th>Energy cost</th>
<th>% From baseline (energy cost)</th>
<th>MD Cost</th>
<th>% From baseline (MD cost)</th>
<th>Total costs</th>
<th>% From baseline (total)</th>
<th>MD portion of saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>172,158</td>
<td>-2.60%</td>
<td>66,500</td>
<td>-13.04%</td>
<td>238,658</td>
<td>-5.76%</td>
<td>68.4%</td>
</tr>
<tr>
<td>11</td>
<td>172,105</td>
<td>-2.63%</td>
<td>58,339</td>
<td>-23.71%</td>
<td>230,444</td>
<td>-9.00%</td>
<td>79.6%</td>
</tr>
</tbody>
</table>

Closed-loop optimal control model with IP optimization (H=24)

<table>
<thead>
<tr>
<th>S</th>
<th>Energy cost</th>
<th>% From baseline (energy cost)</th>
<th>MD Cost</th>
<th>% From baseline (MD cost)</th>
<th>Total costs</th>
<th>% From baseline (total)</th>
<th>MD portion of saving</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>172,175</td>
<td>-2.59%</td>
<td>66,500</td>
<td>-13.04%</td>
<td>238,675</td>
<td>-5.75%</td>
<td>68.5%</td>
</tr>
</tbody>
</table>
### Chapter 5  
Simulation results of the applied control model

#### Table 5-2: Summary of results for K2 (timeout=10 seconds, simulated over 30 days).

<table>
<thead>
<tr>
<th>$H$</th>
<th>$S$</th>
<th>Energy cost</th>
<th>MD Cost</th>
<th>Total costs</th>
<th>MD portion of saving</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(energy cost)</td>
<td>(MD cost)</td>
<td>(total)</td>
<td>saving</td>
</tr>
<tr>
<td>N/A</td>
<td>2</td>
<td>8,351</td>
<td>19,950</td>
<td>28,301</td>
<td>0.0%</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>3,694</td>
<td>1,781</td>
<td>5,475</td>
<td>-80.7%</td>
</tr>
<tr>
<td>24</td>
<td>1</td>
<td>3,800</td>
<td>19,950</td>
<td>23,750</td>
<td>-16.1%</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>3,751</td>
<td>9,975</td>
<td>13,726</td>
<td>-51.5%</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>3,737</td>
<td>6,650</td>
<td>10,387</td>
<td>-63.3%</td>
</tr>
<tr>
<td>24</td>
<td>4</td>
<td>3,728</td>
<td>4,988</td>
<td>8,716</td>
<td>-69.2%</td>
</tr>
<tr>
<td>24</td>
<td>5</td>
<td>3,717</td>
<td>3,990</td>
<td>7,707</td>
<td>-72.8%</td>
</tr>
<tr>
<td>24</td>
<td>6</td>
<td>3,715</td>
<td>3,325</td>
<td>7,040</td>
<td>-75.1%</td>
</tr>
<tr>
<td>24</td>
<td>7</td>
<td>3,709</td>
<td>2,850</td>
<td>6,559</td>
<td>-76.8%</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>3,706</td>
<td>2,494</td>
<td>6,200</td>
<td>-78.1%</td>
</tr>
<tr>
<td>24</td>
<td>9</td>
<td>3,706</td>
<td>2,217</td>
<td>5,923</td>
<td>-79.1%</td>
</tr>
<tr>
<td>24</td>
<td>10</td>
<td>3,704</td>
<td>1,995</td>
<td>5,699</td>
<td>-79.9%</td>
</tr>
<tr>
<td>24</td>
<td>11</td>
<td>3,698</td>
<td>1,814</td>
<td>5,512</td>
<td>-80.5%</td>
</tr>
<tr>
<td>24</td>
<td>12</td>
<td>3,704</td>
<td>3,325</td>
<td>7,029</td>
<td>-76.1%</td>
</tr>
<tr>
<td>24</td>
<td>13</td>
<td>3,704</td>
<td>3,070</td>
<td>6,774</td>
<td>-76.1%</td>
</tr>
<tr>
<td>0.5</td>
<td>2</td>
<td>7,448</td>
<td>9,975</td>
<td>17,423</td>
<td>-38.4%</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>6,728</td>
<td>9,975</td>
<td>16,703</td>
<td>-41.0%</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>5,122</td>
<td>9,975</td>
<td>15,097</td>
<td>-46.7%</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4,139</td>
<td>9,975</td>
<td>14,114</td>
<td>-50.1%</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>3,858</td>
<td>9,975</td>
<td>13,833</td>
<td>-51.1%</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3,838</td>
<td>9,975</td>
<td>13,813</td>
<td>-51.2%</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>3,813</td>
<td>9,975</td>
<td>13,788</td>
<td>-51.3%</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>3,791</td>
<td>9,975</td>
<td>13,766</td>
<td>-51.4%</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>3,772</td>
<td>9,975</td>
<td>13,747</td>
<td>-51.4%</td>
</tr>
<tr>
<td>12</td>
<td>2</td>
<td>3,763</td>
<td>9,975</td>
<td>13,738</td>
<td>-51.5%</td>
</tr>
<tr>
<td>24</td>
<td>2</td>
<td>3,768</td>
<td>9,975</td>
<td>13,743</td>
<td>-51.4%</td>
</tr>
</tbody>
</table>

Current level based control model –the baseline

Open loop optimal control model with LP optimization –the optimal benchmark

Open loop optimal control model with IP optimization –different switching intervals ($S$)

Closed-loop optimal control model with IP optimization –different control horizons ($H$)
The overall calculated cost from section 4.3 is used to calculate the overall saving in Table 5-1. Note that Table 5-1 lists the simulated scenarios with significant results only, whilst Table 5-2 lists all the simulated scenarios.

### 5.1. Current control model – the baseline

The current control model is simulated over a 30 day period. The current model is simulated with the revised switching band for K2 as defined in Table 4-1. The total energy and MD costs are shown in Table 5-2. Figure 5-1 shows the results of the current control model on the first day. From the figure it is clear that the current model results in an undesirably high MD, and K2 runs for 1½ hours in standard time.

![Current control model for K2 with no optimization on the first day.](image)

Figure 5-1: Current control model for K2 with no optimization on the first day.

Figure 5-2 shows the results of the current control model on the 30th day. The 30th day is selected, because it is a good example where K2 switches on in peak times, i.e., in Figure
Chapter 5  Simulation results of the applied control model

5-2 K2 runs for 2½ hours in peak time and a ½ hour in standard time.

Note that the resulting MD is registered in the applicable 30 minute MD integrating period with the highest average usage. As mentioned, MD charges are applicable in standard and peak TOU times, i.e., between 6h00 and 22h00. If more than one period results in the same MD, the first MD period represents the maximum. In Figure 5-1 K2 runs for the full 30 minutes in an applicable MD period, which result in the highest possible MD for K2.

Note that switching intervals are not really applicable in the current control model, because the controller constantly monitors the reservoir level and start the pump when required. However, for simulation purposes a switching interval of 15 minutes is used, i.e., $S=2$.

![Figure 5-2: Current control model for K2 with no optimization on the 30th day.](image)

### 5.1.1. Validation of the current control model
The simulation results in Figure 5-1 and Figure 5-2 shows that the definition of the current control model corresponds with the description of the actual control model in section 4.1. Furthermore, as mentioned in section 4.3, the actual electricity cost for the plant closely matches the simulated electricity costs.

5.2. Open loop optimal control model with LP optimization – the benchmark

As mentioned, the open loop optimal control model with LP optimization is simulated to get a benchmark result that is considered optimal. The control model is simulated over a 30 day period. The reservoir level at the end of each day is used as the initial reservoir level for the next day. Figure 5-3 shows the simulated results on the first day. The resulting energy cost is the same on each of the simulated days, and the monthly energy and MD costs are shown in Table 5-2.

Figure 5-3: Open loop optimal control model for K2 with LP optimization (\(H=24\), \(S=2\), timeout=N/A).
The load in Figure 5-3 is moved out of the peak energy charge periods, and the load in the standard energy charge periods is reduced. The MD is also reduced by spreading the load evenly over the applicable 30 minute MD periods. Note that the LP optimization completes quickly (less than 10 seconds), and the timeout value is therefore not applicable. Furthermore, the value of $S$ does not affect the TOU or MD costs.

Also note that the open loop LP model is not considered as a control solution for the specific case study, because the pumps are binary controlled, therefore this solution is only a benchmark for comparisons.

5.3. **Open loop optimal control model with IP optimization**

5.3.1. **Effect of optimization time**

The open loop control model with $S=2$ is simulated with a number of different timeout values over a 24 hour period. The effect of the different timeout values is shown in Table 5-3. Figure 5-4 shows a graphical representation of the results in Table 5-3.

A one day (24 hours) simulation period is selected instead of a 30 day period, because the longest timeout value of 24 hours is impractical over 30 days, i.e., the simulation will have to run for 30 days. This is considered an acceptable compromise, because the aim of this simulation is to analyse the relative affect of the timeout values.

Table 5-3 shows that the MD cost is the same across all timeout values. However, the energy cost changes slightly, i.e., the energy cost with a 24 hours timeout is 1.3% lower than the energy cost with a one second timeout. Note that the results only improve from two hours onwards, but it does not change between two and 24 hours.

Therefore, a timeout value of 10 seconds is selected for all subsequent simulations. The results for the 10 seconds simulation is considered near-optimal and practical for the closed-loop control models that are covered in section 5.4. Furthermore, the results from
section 5.3.3 for the open loop IP model with $S=11$ (timeout=10s) shows the results closely match the benchmark LP control model.

Table 5-3: Effect of simulation timeout on the open loop optimal control model with IP optimization ($H=24h$, $S=2$, simulated over a one day period).

<table>
<thead>
<tr>
<th>Timeout</th>
<th>Daily energy cost</th>
<th>Monthly MD cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1s</td>
<td>133.0</td>
<td>9,975</td>
</tr>
<tr>
<td>10s</td>
<td>133.0</td>
<td>9,975</td>
</tr>
<tr>
<td>1 min</td>
<td>133.0</td>
<td>9,975</td>
</tr>
<tr>
<td>1h</td>
<td>133.0</td>
<td>9,975</td>
</tr>
<tr>
<td>2h</td>
<td>131.4</td>
<td>9,975</td>
</tr>
<tr>
<td>4h</td>
<td>131.4</td>
<td>9,975</td>
</tr>
<tr>
<td>8h</td>
<td>131.4</td>
<td>9,975</td>
</tr>
<tr>
<td>12h</td>
<td>131.4</td>
<td>9,975</td>
</tr>
<tr>
<td>24h</td>
<td>131.4</td>
<td>9,975</td>
</tr>
</tbody>
</table>

Figure 5-4: Effect of simulation timeout on the open loop optimal control model with IP optimization ($H=24h$, $S=2$, simulated over a one day period).

Figure 5-5 shows the results of the open loop control model with IP optimization for the 10 second timeout scenario. The load is moved out of the peak periods, and the load in the standard periods is also reduced. The MD is reduced by 50%, because K2 runs more
frequently, but for shorter times, i.e., only 15 minutes per 30 minute MD period. Note that between 17h00 and 18h00 K2 runs for two consecutive switching intervals. However, these two switching intervals fall within two separate MD charge periods and therefore the resulting MD is still 50% lower.

Figure 5-5: Open loop optimal control model for K2 with IP optimization ($H=24h$, $S=2$, timeout=10s).

Figure 5-6 is the same scenario as in Figure 5-5, but with a timeout period of two hours. The main difference between the two figures is that K2 runs for five switching intervals in standard time in Figure 5-5, and for only four switching intervals in standard time in Figure 5-6. Note that the final reservoir level in Figure 5-6 and Figure 5-5 is the same, i.e., 1.092 ML.

A disadvantage with the pump schedule in Figure 5-5 and Figure 5-6 is that K2 is switched on and off unnecessarily in off-peak times, e.g., between 0h00 and 6h00. Since the MD
charge is not applicable it would be ideal to keep K2 running for less longer intervals as opposed to many short intervals. It is recommended that this is considered in a future research initiative. It might be possible to modify the objective function and/or constraints to cater for this. Note that this problem is applicable to all the IP based optimal control models in this chapter.

5.3.2. Open loop optimal control model over 30 days

Figure 5-7 shows the daily energy cost over 30 days for the open loop IP model. The final reservoir level of each day is used as the initial reservoir level for the next day Figure 5-7 shows that the daily energy cost is not the same for all days in the 30 day period; the energy cost is R 133.0 on days 1, 12 and 23, but only R 124.1 on the rest of the days. The cumulative average daily energy cost is also shown in Figure 5-7, which results in a daily average of R 125.0 after 30 days. The total monthly energy cost is therefore R 3,751 (i.e.,
The energy cost is higher on days 1, 12 and 23 because on these days K2 runs for nine switching intervals in off-peak time and five switching intervals in standard time. On the rest of the days K2 runs only eight switching intervals in off-peak time and five intervals in standard time. The additional running interval is required on days 1, 12 and 23 because the initial reservoir level on these days is too high to run for only eight off-peak and five standard intervals. As mentioned the final reservoir level of each day is used as the initial reservoir level for the next day.

Figure 5-7: Daily energy cost over 30 days for the open loop optimal control model with IP optimization ($H=24\text{h}$, $S=2$, timeout=10s).

5.3.3. Effect of the switching interval ($S$)
Table 5-2 shows the effect of the switching interval on the TOU and MD cost over a 30 day simulation period. Figure 5-8 shows a graphical representation of the results in Table 5-2.

The energy cost in Figure 5-8 (Table 5-2) does not change significantly, i.e., only a 6.2% difference between the lowest ($S=1$) and highest ($S=11$) values. However, the MD cost changes significantly, i.e., a 90.9% difference between the lowest ($S=1$) and highest ($S=11$) values. This is caused by the shorter switching intervals which allow K2 to run more frequently but for shorter intervals, which in turn reduces the MD cost. The results for $S=11$ also closely matches the optimal benchmark. This means that $S=11$ can be considered an optimal switching interval for this case study. As mentioned, with $S=1$ the switching intervals are 30 minutes long, and with $S=11$ the switching intervals are 2.73 minutes long.

![Figure 5-8: Cost vs. switching interval for the open loop optimal control model with IP optimization (H=24h, timeout=10s).](image)

Note that the energy and MD cost started increasing again from $S=12$, because the switching intervals for $S=12$ are too short, i.e., only 2.5 minutes. This results in K2 running two consecutive switching intervals in certain MD periods to control the reservoir level. As $S$ increases beyond $S=12$ the costs decreases again, and it is assumed that it will again
reach an optimal value. However, this is not practical, because the number of variables will be unnecessarily high.

Figure 5-9 shows the results of the open loop IP control model with \( S=1 \). The load is moved out of the peak periods, and the load in the standard periods is also reduced. The MD is not reduced, because K2 runs the full MD integrating period in MD applicable times.

![Figure 5-9: Open loop optimal control model for K2 with IP optimization (\( H=24 \text{h}, S=1 \), timeout=10s).](image)

Figure 5-10 shows the results of the open loop IP control model with \( S=11 \). The load is also moved out of the peak periods, and the load in the standard periods is also reduced. However, the MD is reduced significantly, because K2 runs more frequently, but for shorter times, i.e., only 2.73 minutes per 30 minute MD period.

![Figure 5-10: Open loop optimal control model for K2 with IP optimization (\( H=24 \text{h}, S=11 \), timeout=10s).](image)
5.3.4. Effect of the maximum demand (MD) optimization

Figure 5-11 shows the results of the open loop optimal control model with TOU optimization only, i.e., the MD constraints in (4-7) is removed, $\lambda_2=1$ and $\lambda_2=0$ in (4-3). Note that the simulation in Figure 5-11 is done with $S=11$, because with $S=2$ it coincidently results in the lowest MD. Therefore, $S=11$ is selected, because it results in a better example to prove the effect of MD optimization.

The load in Figure 5-11 is moved out of the peak periods and the load in the standard periods is reduced, however the MD is not significantly reduced to the desired level because K2 runs for four consecutive switching intervals in the applicable MD period, i.e., 109 kW as opposed to 27 (see Figure 5-10). The TOU cost is the same as the model that optimizes for both TOU and MD, i.e., it matches the TOU results for $S=11$ in Table 5-2 and Figure 5-10.
Figure 5-11: Open loop optimal control model for K2 with IP optimization on TOU charges only $(H=24h, S=11, \text{timeout}=10s)$.

Figure 5-12 shows the results of the open loop control model with MD optimization only, i.e., $\lambda_1=0$ and $\lambda_2=1$ in (4-3). The MD cost is reduced by running K2 more frequently, but for shorter intervals. However, the TOU cost is reduced by only 21%, i.e., R 6,564 from R 8,351 per month (simulated over a 30 day period). The TOU cost is reduced because the net usage in the standard and peak periods is reduced by the MD optimization.

### 5.4. Closed-loop optimal control model with IP optimization

#### 5.4.1. Effect of the control horizon $(H)$

Table 5-2 shows the effect of the cost vs. the control horizon $(H)$ for the closed-loop control model with IP optimization, simulated over a 30 day period. The switching interval is selected as $S=2$ for all the scenarios where the effect of the control horizon $(H)$ is evaluated.
The results in Table 5-2 shows that the MD cost is not affected by the control horizon. The energy cost, on the other hand, is affected by $H$. The affect is more apparent in Figure 5-13, which is a graphical representation of Table 5-2. Figure 5-13 shows that the energy cost does not significantly decrease for $H \geq 4$, i.e., only a 2.3% reduction from $H = 4$ to $H = 24$. This is due to the fact that the longest peak TOU period is only three hours, which enables the optimal control model to shift the load out of the peak periods.

This result is important, because it allows the selection of shorter control horizons, which is easier to optimize, because of less integer variables. For example, in the scenarios with the shorter control horizons ($H \leq 6$) the optimization completes before the timeout value of 10 seconds.

Note that the energy cost for $H = 12$ is slightly lower than the energy cost for $H = 24$. This
might be as a result of better optimization due to fewer variables or just an integer tolerance error.

![Graph showing energy cost vs. prediction horizon](image)

**Figure 5-13:** Costs vs. the control horizon ($H$) for the K2 closed-loop optimal control model with IP optimization ($S=2$, timeout=10s).

Figure 5-14 shows the results of the closed-loop IP model with $H=24$ for the first day. The load is moved out of the peak periods, and the load in the standard periods is also reduced. The MD is reduced by 50%, because K2 runs more frequently, but for shorter times, i.e., only 15 minutes per 30 minute MD period.

Note that the final reservoir level on the first day in Figure 5-14 is lower than the open loop control model for the same scenario in Figure 5-5, i.e., 0.6 ML for the closed-loop control model as oppose to 1 ML for the open loop control model. This is caused by the closed-loop control model which predicts past the 24th hour, and is already compensating for the next 24 hours. As a result the energy cost on the first day is higher with the closed-loop control model than with the open loop control model. This is covered in more detail in section 5.4.2.

Figure 5-15 shows the results of the closed-loop IP model with $H=4$ for the first day. The load is moved out of the peak periods, and the load in the standard periods is also reduced. The MD is reduced by 50%, because K2 runs more frequently, but for shorter times, i.e.,
only 15 minutes per 30 minute MD period.

Figure 5-14: Closed-loop optimal control model for K2 with IP optimization ($H=24h$, $S=2$, timeout=10s).

Note that the final reservoir level on the first day is the same for $H=24$ and $H=4$. The main difference between the control models ($H=24$ and $H=4$) is that K2 runs for five switching intervals in standard TOU times with $H=24$, whilst K2 runs for six switching intervals in standard TOU times with $H=4$. This results in the mentioned 2.3% difference.

5.4.2. Closed-loop optimal control model over 30 days

Figure 5-16 shows the daily energy costs over 30 days for the closed-loop IP model. Figure 5-16 shows that the daily energy cost is not the same for all days in the 30 day period; the energy cost is R 150.8 on day 1, R 133.0 on days 11 and 22, but only R 124.1 on the rest of the days. The cumulative average daily energy cost for the closed-loop model is also
Chapter 5  Simulation results of the applied control model

shown in Figure 5-16, which results in a daily average of R 125.6 after 30 days. The total monthly energy cost is therefore R 3,768 (i.e., R 125.616 × 30 days).

Figure 5-15: Closed-loop optimal control model for K2 with IP optimization ($H=4h$, $S=2$, timeout=10s).

The energy cost is higher on days 1, 11 and 22, because on day 1 K2 runs for eleven switching intervals in off-peak time and five switching intervals in standard time, and on days 11 and 22 K2 runs for nine switching intervals in off-peak time and five switching intervals in standard time. On the rest of the days K2 runs only eight switching intervals in off-peak time and five intervals in standard time.

The additional running interval is required on days 1, 11 and 22 because the initial reservoir level on these days is too high to run for only eight off-peak and five standard intervals. As mentioned, the final reservoir level of each day is used as the initial reservoir level for the next day.
The main difference between the open and closed-loop models is the energy cost on the first day. The energy cost of the closed-loop model on the first day is R 150.8 whilst the energy cost of the open loop model on the first day is R 133.0. As mentioned, this is caused by the closed-loop model which predicts past the 24th hour, and is already compensating for the next 24 hours.

![Graph showing daily energy cost over 30 days for the closed-loop optimal control model with IP optimization (H=24h, S=2, timeout=10s).]

The actual days with higher energy cost is also different between the open and closed-loop models, i.e., days 1, 12 and 23 for the open loop model, and days 1, 11 and 22 for the closed-loop model –the closed-loop model is one day earlier. This is caused by the closed-loop model which always predicts further than the open loop model, and as a result the closed-loop model is already compensating for the following day. As explained in section 5.3.2, the additional running interval is required on days 1, 11 and 22, because the initial reservoir level on these days is too high to run only eight off-peak and five standard
Figure 5-16 shows that the cumulative average daily energy cost of the closed-loop model converges towards the cumulative average daily energy cost of the open loop control model, i.e., after 30 days there is only a 0.5% difference in the cumulative average daily energy cost which equates to 0.5% difference in the monthly energy cost.

5.5. Summarized comparison of the control models

This section gives a brief summary of significant results from section 5.1 to 5.4. The scenarios from Table 5-2 with the most significant results are shown in Figure 5-17.

Figure 5-17: Summary of significant scenarios (simulated over a 30 days, timeout=10 seconds).

Figure 5-17 (Table 5-2) shows that the open and closed-loop optimal control models with $H=24$ and $S=2$ results in approximately the same total saving (MD and TOU) for K2 over 30 days, i.e., 51.5% and 51.4% respectively. This results in a total saving for the overall plant off 5.8% (see Table 5-1). Note that with $S=2$ the switching intervals are 15 minutes long.

Figure 5-17 (Table 5-2) shows that the open loop IP optimal control model ($H=24$, $S=11$) and the open loop LP optimal control model results in approximately the same total saving.
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(MD and TOU) for K2 over 30 days, i.e., 80.5% and 80.7% respectively. This results in a total saving for the overall plant of 9% (see Table 5-1). Note that with $S=11$ the switching intervals are 2.73 minutes long.

Figure 5-17 (Table 5-2) shows that the open loop IP optimal control model with $H=24$ and $S=2$ to $S=11$ saves approximately 55% on energy (TOU) cost for K2, which matches the result of the optimal benchmark. The overall plant saving on energy cost is 2.6%. This means that the open and closed-loop control models with IP can be considered optimal as far as TOU charges are concerned. This also means that the choice of the switching interval has an insignificant affect on the optimization for TOU charges.

Figure 5-17 (Table 5-2) shows that the MD saving for K2 is significantly affected by the switching interval. With $S=2$ the MD saving is 50%, and with $S=11$ the MD saving is 90.9% (overall plant 23.7%), which is close to the 91.1% saved by the benchmark LP model.

Note that the closed-loop control model is not simulated with different switching intervals, because it is assumed that the closed-loop control model would converge to the open loop control model for all the values of $S$ if the control models converge for $S=2$.

Figure 5-17 (Table 5-2) shows that the MD portion of the saving is higher than the energy saving for all the scenarios, except the scenario in Table 5-2 where $S=1$. This confirms that MD charges needs to be included in the objective function of the optimal control model where applicable.

Table 5-2 also shows that the control horizon ($H$) in the closed-loop simulation did not have an affect on the MD cost, but it did affect the TOU cost. The best results are with $H \geq 4$.

Note that the total amount of energy consumed is the same for the current control model and all the scenarios of the optimal control models. The electricity cost is only optimized
by improving the timing of the energy consumption in the optimal control models.

5.6. Robustness of the optimal control models

This section evaluates the robustness of the open and closed-loop optimal control models (IP optimization) against disturbances and an inaccurate system model.

5.6.1. Effect of disturbances on the optimal control models

Figure 5-18: Open loop optimal control model with inflow disturbances for K2 with IP optimization ($H=24\text{h}$, $S=2$, timeout=10s).

Figure 5-18 and Figure 5-19 show the results of the open and closed-loop optimal control models (IP optimization) with a positive random inflow disturbance, i.e.,

$$FLOWIN_i = FLOWIN_i + 0.2 \cdot FLOWIN_i \cdot r(m).$$  \hspace{1cm} (5-1)
where \( r(m) \) is a random number between 0 and 1. This means that the assumed constant inflow rate is altered with a random disturbance.

Figure 5-18 shows that the level of \( R1 \) exceeds the maximum level constraint (1.3 ML) in the open loop control model, whilst Figure 5-19 shows that the closed-loop control model compensates for the disturbances and keeps the level of \( R1 \) within the maximum level constraint. This result is important, because the inflow and outflow rates in practical applications could be affected by external factors such as temperature, rain, equipment age, etc.

The control models in Figure 5-18 and Figure 5-19 are simulated over a 24 hour period only. The aim is simply to show the relative effect of the disturbances on the control models.
The daily energy cost of the closed-loop control model increases from R 150.80 to R 170.30 due to the additional pumping time required to control the reservoir level. The MD cost remains the same though, i.e., R 9,975/month.

**5.6.2. Effect of an inaccurate system model on the optimal control models**

Figure 5-20 and Figure 5-21 show the results of the open and closed-loop optimal control models (IP optimization) with an inaccurate outflow model for K2.

For demonstration purposes is assumed that the actual outflow is only 90% of the assumed outflow, i.e.,

\[
FLOWOUT_t = 0.9 \cdot 22 \cdot ML.
\]  

(5-2)
Figure 5-20 shows that the level of $R1$ exceeds the maximum level constraint (1.3 ML) in the open loop control model, whilst Figure 5-21 shows that the closed-loop control model compensates for the inaccurate system model and keeps the level of $R1$ within the maximum level constraint. This result is important for practical applications, because it is likely that the defined plant model will contain some error. The plant model could also be incorrect due to a simplification of the model for practical reasons.

The control models in Figure 5-20 and Figure 5-21 are simulated over a 24 hour period only. The aim is simply to show the relative effect of the disturbances on the control models.

The daily energy cost of the closed-loop control model increases from R 150.80 to R 161.40 due to the additional pumping time required to control the reservoir level. The
MD cost remains the same though, i.e., R 9,975/month.

5.7. Practicality of the optimal control model

The main problem with the optimal control model from a practical point of view is the number of times that K2 needs to operate per day, which is related to the number of switching intervals per MD period ($S$). As mentioned in section 4.10, the number of switching intervals in an MD period ($S$) is a trade-off between computational time, equipment constraints and cost saving.

The current implemented control model K2 operates approximately six times per day, for more than 30 minutes at a time (see section 4.1). This matches the number of times that K2 will operate with the optimal control model when $S=1$ (see Figure 5-9). This means that the control model with $S=1$ can be implemented without additional strain on the motor, motor soft starter, or associated switchgear. However, as shown in this chapter, the control model with $S=1$ results in a saving on TOU cost only.

The control model with $S=2$ in Figure 5-6 and Figure 5-14 can also be implemented without additional strain on the equipment, if the unnecessary switching in off-peak times is eliminated. This unnecessary switching problem is discussed in section 5.3.1, and it is recommended as a consideration for a future research initiative. Note that with $S=2$, both the TOU and MD costs are reduced.

Another approach that can also be considered to reduce unnecessary switching is by running back-to-back intervals. This allows K2 to run for longer periods, but split across two MD intervals. The effect of this technique is shown in Figure 5-5. Note that the “back-to-back” result in Figure 5-5 is a coincidence, however, it may be possible to modify the control model to derive the required back-to-back switching. This is also recommended as a consideration for a future research initiative.

If the unnecessary switching in off-peak period is eliminated and back-to-back switching is
considered then K2 will operate fewer times per day. For example, with $S=4$ K2 will operate only 6 times per day, instead of 14 times per day, and with $S=11$ K2 will operate only 13 times per day, instead of 30 times day. This means that a control model with $S \leq 4$ can be implemented without additional strain on the equipment.

However, K2 will operate more times per day than in the current implemented control model if the unnecessary switching is not eliminated, or if $S>4$. In these cases the additional saving needs to be compared against the additional strain on the equipment, which in turn will have an impact on maintenance cost. Note that all the motors in the Rietvlei pumping scheme have soft starters installed, which reduce the stress on the equipment during starting [47]. Therefore more frequent operation can be considered for the Rietvlei plant.
6. CONCLUSIONS AND RECOMMENDATIONS

This dissertation defined and evaluated the efficiency of a closed-loop optimal control strategy for load shifting in a specific plant with TOU and MD charges. The closed-loop optimal control model was implemented with an MPC approach, and the water pumping scheme at the Rietvlei water purification plant in the Tshwane municipality was selected for the case study.

The current control model of the plant was compared against optimal control models with LP and IP optimization. The LP optimization was included as a benchmark which is considered optimal. The LP optimization was used in an open loop control model only, whilst the IP optimization was used in open and closed-loop control models.

The simulations focussed on the saving for a specific motor (K2), and it was assumed that the remaining part of the plant operates as is. The results showed that the optimal control models reduce both TOU and MD costs. The TOU cost is reduced by shifting load out of peak TOU periods, and the MD cost is reduced by pumping more frequently, but for shorter intervals.

The total cost saving for both TOU and MD charges varied between 52% and 81% for K2, with a control horizon \( H \) of 24 hours and varying switching intervals. This resulted in a total saving of 5.8% to 9% for the overall plant.

The TOU cost saving alone was approximately 55% for K2 for all the optimal control models across most of the simulated scenarios. The exception was the simulations with short control horizons (i.e., \( H < 4 \) hours). This resulted in a TOU cost saving of 2.6% for the overall plant.

The MD cost saving alone varied between 50% and 91% for K2 for the open and closed-loop IP control models, depending on the switching intervals. This resulted in an MD cost saving of 13% to 23% for the overall plant. The 91% saving for K2 matched the saving of
the benchmark optimal LP control model.

The open and closed-loop control model for a specific scenario was compared, i.e., $H=24$, $S=2$, and simulated over 30 days. The results showed that the MD cost is the same throughout the 30 days, but the energy cost varies between days. That is, the energy cost for the closed-loop model was higher on the first day, but it was the same for the rest of the simulated days. As a result, the saving of closed-loop model converged to the saving of the open loop control model with only a 0.5% difference over 30 days.

The effect of MD charges on the optimization was evaluated by interchangeably excluding the TOU and MD charges from the objective function of the open loop optimal control model. This resulted in an undesirably high MD cost when the MD charges were excluded, and an undesirably high TOU cost when the TOU charges were excluded.

Across all the control models the largest portion of the savings was due to a reduction in the MD costs. The MD portion of the saving varied between 69% and 92% across all the simulated scenarios, except where $S=1$. In this case there was no MD saving. This confirms that MD charges needs to be included in the objective functions of optimal control models, where MD charges are part of the applicable tariff.

The effect of different control parameters was also simulated to evaluate the effect of practical compromises. The control parameters that were considered are: the simulation time, the number of switching intervals ($S$), and the control horizon ($H$) of the closed-loop control model.

The simulation time was evaluated to help select a practical time limit for subsequent simulations. The results showed that there is not a significant difference between a 1 second and a 24 hour timeout. A 10 second timeout was then selected, and the results from subsequent simulations corresponded closely with the results from the benchmark LP optimal control model.
The number of switching intervals ($S$) in the MD period had a significant affect on the MD cost, and a less significant affect on the TOU cost. For example, with one switching interval per MD period ($S=1$) the MD cost for K2 is not reduced, with two switching intervals ($S=2$) the MD cost for K2 is reduced by 50% and with 11 switching intervals ($S=11$) the MD cost for K2 is reduced by 91%, which matches the benchmark LP optimal control model. The TOU costs, for K2 varied by only 6.3% between the lowest and highest switching interval (i.e., $S=1$ and $S=11$). Note that with $S=11$ K2 runs for many short pump cycles which is not necessarily practical, depending on physical equipment limitations. Even without equipment limitations, an interesting discovery is that with $S=12$ the MD cost increased slightly, and then started to decrease again from $S=13$. It is assumed that with $S>13$ the cost will again converge to the optimal benchmark. For clarity, with $S=1$ the switching intervals are 30 minutes long, with $S=2$ the switching intervals are 15 minutes long, and with $S=11$ the switching intervals are 2.73 minutes long.

The control horizon ($H$) in the closed-loop control model did not have an affect on the MD cost, but a longer control horizon did reduce the energy cost. However, the energy cost for K2 did not significantly decrease for $H \geq 4$, i.e., only a 2.3% reduction from $H=4$ to $H=24$. This result is important, because it allows the selection of shorter control horizons, which is easier to optimize, because of fewer integer variables.

The affect of disturbances and an inaccurate system model was simulated for the open and closed-loop IP control models. The results showed that an open loop control model does not compensate for the disturbances or an inaccurate system model, whilst the closed-loop control model does.

The contributions of this research in the context of existing work were also discussed. In summary, the contributions are: the application of a closed-loop (MPC based) control strategy for load shifting in plants that are charged on TOU and MD is affirmed; scheduling within the MD integrating period is considered; convergence of the open and closed-loop controller is shown; the robustness of the MPC controller is demonstrated; the effect of various control parameters is evaluated, which can be considered in practical
compromises; the importance of including MD charges in the optimization is affirmed; and it is shown that an optimal binary pump schedule that optimizes both TOU and MD charges can be obtained in a single optimization step.

The practicality of the optimal control model was discussed, and it is shown that the control model with $S=1$ can be implemented as is. However, for $S>1$ the additional strain on the equipment due to the additional switching needs to be considered. Otherwise, the control model needs to be enhanced to reduce unnecessary switching, e.g., in off-peak times.

It is recommended that further research is conducted where an MPC strategy is implemented to control a real plant. This will identify the practical issues that will guide future implementations, for example, physical equipment limitations, the maturity of MPC technology in industrial controllers, plant availability, etc. It is also recommended that the unnecessary switching in off-peak periods is investigated, which might be resolved with a modification to the control model formulation.
7. REFERENCES


ADDENDUM A –MATLAB PROGRAMS (SOFTWARE ARCHITECTURE)

Baseline.m
- Solves the current control model (i.e., the baseline)

RietvleiSolve.m
- Solves all the optimal control models

lp_solve.m
- Interface to the LPSOLVE application

mdlimits.m
- Calculates the applicable MD constraints

tariff.m
- Calculates the applicable TOU and MD charges
function [ output_args ] = Untitled1( input_args )

p = 48; % Prediction horizon in 30 minute MD intervals.
s = 1; % Number of switching intervals in an MD interval.

uplimit = 1.3;
lowlimit = 0.2;
initialLevel = 1.3;

% Flow per switching interval
flowin = 3/24/60*(30/s); % per 5 minute
flowout = 22/24/60*(30/s); % per 5 minute

% Motor rated power in kW
power = 300;

% Initialize the current level
currentLevel = initialLevel;

% Declare variables that accumulates.
ffinal = ones(0);
zfinal = ones(0);
level = currentLevel*ones(1);
totalEnergyCost = zeros(1);
x = 0;
days = 1000;
for j = 1:days
    xfinal = ones(0);
    for i = 1:p*s
        if currentLevel >= uplimit
            x = 1;
        elseif currentLevel <= lowlimit
            x = 0;
        end
        currentLevel = currentLevel+flowin-x*flowout;
        level = [level; currentLevel];
        xfinal = [xfinal; x];
    end
    [mdcost, energycost] = tariff(1, p*s, s);
    a = totalEnergyCost;
    b = power*energycost'*xfinal;
    totalEnergyCost = a(1)+b(1);
end

totalEnergyCost = totalEnergyCost/30

totalMDCost = 0;
figure;
hold on;
axisx = [0:1/2/s:p/2];
stairs([axisx],[xfinal; 0],'g');
plot([axisx],level((days-1)*p*s+1:(days-1)*p*s+p*s+1),'b');
%plot([axisx],level,'b');

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University of Pretoria
stairs([axisx],[energycost(1:p*s)*60/30/30*s; 0], 'r');

legend('K2 Status (on/off)', 'R1 Level (ML)', 'TOU Charge (cents)');
legend('Location', 'NorthWest');
xlabel('Hours');
function [ argout ] = RietvleiSolve(p, s, startTime, endTime, step, stepSave)

% Add the linux path to LPSOLVE for when we are on the cluster
addpath( '/afs/ee.up.ac.za/user/n/jvstaden/lp_solve_5.5/extra/MATLAB/lpsolve' );

% Add the windows path to LPSOLVE for when we are on the laptop
addpath( '..\Tools\lpsolve\matlab' );

% Example of variables
p = 48;  % Prediction horizon in 30 minute MD intervals.
s = 2;   % Number of switching intervals in an MD interval.
startTime = 1;
endTime = p*s;
step = p*s;
stepSave = 0;

% LPSOLVE Optimization parameters
timeout = 10;
obj_bound = 15000;

% Levels
uplimit = 1.3;
lowlimit = 0.2;
initialLevel = 1.3;

% Flow per switching interval
flowin = 3/24/60*(30/s);
flowout = 22/24/60*(30/s);
randomFlow = 0; % 0.2*flowin;

% Motor rated power in kW
power = 300;

% Initialize the current level
currentLevel = initialLevel;

% Declare variables that accumulates.
xfinal = ones(0);
ffinal = ones(0);
zfinal = ones(0);
level = currentLevel*ones(1);

% Initialize currentZ
z=1;  % MD variables. Set z=s if binary variables are required.
currentZ = 0;

% Initialize x
x = zeros((p*s+z),1);

% Variable to keep all the objective functions
objs = ones(0);
Addendum C –Matlab programs (ReitvlciSolve.m)

% Flag to load saved file on initial iteration
loadFile = 1;

startIter = startTime;

% Load the saved file to solve the LPSOLVE hang problem
if stepSave == 1 & loadFile == 1
    try
        load('temp.mat');
    catch
        disp('No file to load');
    end
    loadFile = 0;
end

for currenttime = startIter:step:endTime

    % Display the current time
    currenttime

    % Get the applicable tariff
    [mdcost, energycost] = tariff(currenttime, currenttime+p*s-1,s);

    % MD Constraints
    mdLimits = mdlimits(p, s, power, currentZ, currenttime-startTime+1, mdcost);
    [r_mdLimits, c_mdLimits] = size(mdLimits);
    Aflow = ones(0,0);
    bflow = ones(0,0);

    % Upper limit constraints
    for i = 1:p*s
        outFlow = [ flowout*ones(1,i) zeros(1,p*s-i) zeros(1,z)];
        Aflow = [Aflow; -outFlow];
        bflow = [ bflow; uplimit-flowin*i-currentLevel];
    end;

    % Lower limit constraints
    for i = 1:p*s
        outFlow = [ flowout*ones(1,i) zeros(1,p*s-i) zeros(1,z)];
        Aflow = [Aflow; outFlow];
        bflow = [ bflow; flowin*i+currentLevel-lowlimit];
    end;

    % Combine constraints
    A = [ Aflow; mdLimits ];
    b = [ bflow; zeros(1,r_mdLimits) ];

    % Function to minimize
    f = [power*energycost' power*mdcost*ones(1,z)];
    vlb = zeros(p*s+z,1);
    %vub = ones(p*s+z,1); Use instead of next for binary variables.
    vub = [ones(p*s,1); inf*ones(z,1)];
% Optimize
[obj,x,duals,stat] = lp_solve(f,A,b,-
1,vlb,vub,vub,[],[],timeout,obj_bound);
objs = [objs; obj];
obj

% Round x values to force integers. Comment out when linprog is used.
x = round(x);

% If linprog is used
%x = linprog(f,A,b,[],[],vlb,vub);

% Add new x values to xfinal so that we can plot at the end
if length(xfinal) == 0
    xfinal = x(1:step);
else
    xfinal = [xfinal; x(1:step)];
end

% Set new current level
for i = 1:step
    newLevel = currentLevel+flowin-flowout*x(i)+
    randomFlow*abs(randn(1));
    currentLevel = newLevel;
    level = [level;currentLevel];
end;

% Update zfinal with the highest maximum demand
ztemp = x( (length(x)-z+1):length(x) );
if length(zfinal) == 0
    zfinal = ztemp;
elseif sum(ztemp) > sum(zfinal)
    zfinal = ztemp;
end

% Reset current Z when a new MD period is entered
if floor(currenttime/s) == currenttime/s
    currentZ = 0;
else currentZ = currentZ + x(1);
end

% Save each step to overcome the LPSOLVE hang problem
if stepSave == 1
    startIter = startTime + currenttime;
    save('temp');
end
end

% Get the cost function for the entire interval
[mdcost, energycost] = tariff(startTime, endTime, s);
f = [power*energycost' power*mdcost*ones(1,z)];

% Calculate costs
totalEnergyCost = f(1:(length(f)-z))*xfinal;
totalMDCost = f((length(f)-z+1):length(f))*zfinal;
totalCost = [totalEnergyCost totalMDCost totalEnergyCost+totalMDCost];
disp('Energy    MD    Total');
disp(totalCost);

% Calculate the daily costs
dailyEnergyCosts = []; aveDailyEnergyCosts = [];
for i = 1:48*s:endTime
    dailyEnergyCost = f(i:i+48*s-1)*xfinal(i:i+48*s-1);
    dailyEnergyCosts = [dailyEnergyCosts; dailyEnergyCost];
    aveDailyEnergyCosts = [aveDailyEnergyCosts; mean(dailyEnergyCosts)];
end

% Save the results
save(strcat('Results2/p',num2str(p),'_s',num2str(s),'_start',num2str(startTime),'_end',num2str(endTime),'_step',num2str(step),'_to',num2str(timeout),'_bound',num2str(obj_bound),'_R',num2str(randomFlow),'_',date));

% Print some results
zfinal

% Plot the results
figure;
hold on;
axisx = [0:1/2/s:(endTime-startTime+1)/2/s];
stairs([axisx],[xfinal(1:(endTime-startTime+1))]; 0), 'g';
plot([axisx],level,'b');
stairs([axisx],[energycost*60/30/30*s; 0 ],'r');
legend('K1 Status (on/off)','R1 Level (ML)','TOU Charge (cents)');
xlabel('Hours');
hold off;

% Plot the daily costs
figure;
hold on;
plot(dailyEnergyCosts);
plot(aveDailyEnergyCosts);
function [ mdCost, energyCost ] = tariff( startInterval, endInterval, s )

day = 48*s;

% Calculate the minutes in a switching interval
k = 30/s;

cost = ones(day,1);

% Winter tariffs
peak = 0.8205;
standard = 0.1411;
off_peak = 0.1187;

for i = 1:day
    if i <= 360/k
        cost(i) = off_peak;
    elseif i <= 420/k
        cost(i) = standard;
    elseif i <= 600/k
        cost(i) = peak;
    elseif i <= 1080/k
        cost(i) = standard;
    elseif i <= 1200/k
        cost(i) = peak;
    elseif i <= 1320/k
        cost(i) = standard;
    elseif i <= 1440/k
        cost(i) = off_peak;
    end
end

tariff =
[ cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;
cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;
cost;cost;cost;cost;cost;cost;cost;cost;cost;cost;cost]

cost = tariff(startInterval:endInterval);

% Multiply energy cost with 30 to get the equivalent monthly cost. This is % required so that the MD and Energy cost carries the same weight.
energyCost = 30*k*cost/60;

% MD cost
mdCost = 66.5/s;
ADDENDUM E –MATLAB PROGRAMS (MDLIMITS.M)

function [ mdLimits ] = mdlimits( p, s, power, currentZ, currenttime, mdcost )

% Shift data to the first day.
currenttime = currenttime - round(currenttime/48/s)*48*s;

% MD charge interval
mdstart = 360/30+1;
mdend = 1320/30;

mdPeriodStart = floor((currenttime-1)/s)+1;
maskPosition = currenttime - floor((currenttime-1)/s)*s;

% extend p to an extra period so that we can apply the mask
p = p+1;

z = 1; % Makes z=s if binary z values are required.

% MD Constraints
mdLimits = [];

for i = 1:p
    if mdstart <= mdPeriodStart+i-1 && mdPeriodStart+i-1 <= mdend
        mdLimit = [ zeros(1,i*s-s) power*mdcost*ones(1,s) zeros(1,p*s-i*s)];
        mdLimits = [mdLimits; mdLimit];
    end;
end;

% Apply the mask
[r,c] = size(mdLimits);
if r >= 1
    if maskPosition == 1  % No mask required, just remove the extra p
        mdLimits = [ mdLimits(1:r,1:(p-1)*s)];
        % Add the Z variables
        Z = -power*mdcost*ones(r,z);
        %Z = -power*ones(r,z);
        mdLimits = [ mdLimits Z];
    else  % mask required
        % Add the mdcost occured up to now for the MD interval
        mdLimits(1,maskPosition:s) = mdLimits(1,maskPosition:s)+power*mdcost*currentZ;
        % mdLimits(1,maskPosition:s) = mdLimits(1,maskPosition:s)+power*currentZ;
        % Apply the mask in the x variables
        mdLimits = [ mdLimits(1:r,maskPosition:s) mdLimits(1:r,s+1:(p-1)*s+maskPosition-1)];
        % Add the Z variables according to the mask
        Z = -power*mdcost*ones(r,z);
        mdLimits = [ mdLimits Z];
    end;
end
function [obj, x, duals, stat] = lp_solve(f, a, b, e, vlb, vub, xint, scalemode, keep, timeout, obj_bound)

[m, n] = size(a);
lp = mxlpsolve('make_lp', m, n);
mxlpsolve('set_verbose', lp, 4);
mxlpsolve('set_mat', lp, a);
mxlpsolve('set_rh_vec', lp, b);
mxlpsolve('set_obj_fn', lp, f);
mxlpsolve('set_int', lp, [vub]);
mxlpsolve('set_timeout', lp, timeout);
mxlpsolve('set_epsint', lp, 0.01);
mxlpsolve('set_print_sol', lp, 1);
for i = 1:length(e)
    if e(i) < 0
        con_type = 1;
    elseif e(i) == 0
        con_type = 3;
    else
        con_type = 2;
    end
    mxlpsolve('set_constr_type', lp, i, con_type);
end
if nargin > 4
    for i = 1:length(vlb)
        mxlpsolve('set_lowbo', lp, i, vlb(i));
    end
end
if nargin > 5
    for i = 1:length(vub)
        mxlpsolve('set_upbo', lp, i, vub(i));
    end
end
if nargin > 6
    mxlpsolve('set_obj_bound', lp, obj_bound);
    for i = 1:length(xint)
        mxlpsolve('set_binary', lp, vub(i), 1);
    end
end
if nargin > 7
    if scalemode ~= 0
        mxlpsolve('set_scaling', lp, scalemode);
    end
end
result = mxlpsolve('solve', lp);
if result == 0 | result == 1 | result == 11 | result == 12
    [obj, x, duals] = mxlpsolve('get_solution', lp);
    stat = result;
else
    obj = [];
    x = [];
    duals = [];
    stat = result;
end
if nargin < 9
    mexlpsolve('delete_lp', lp);
end
mexlpsolve('is_integerscaling', lp);