THE RELATIONSHIP BETWEEN TEACHERS' INSTRUCTIONAL PRACTICES AND LEARNERS' LEVELS OF GEOMETRY THINKING

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ABSTRACT

The relationship between teachers' instructional practices and learners' levels of geometry thinking

The aim of this study was to investigate the relationship between teachers' instructional practices in terms of specific areas of focus pertaining to the teaching and learning of geometry described in literature and, their learners' levels of geometry thinking as elaborated in the Van Hiele theory. A review of literature on the development of geometry understanding was conducted to frame what is meant by 'teachers' instructional practices' as they pertain to the teaching and learning of geometry in this study. These instructional practices are understood to include the appropriate allocation of time for the facilitation of geometry concept development, the use of concrete apparatus, the use of relevant and level appropriate language as well as the use of level appropriate geometry activities. The structure of the curriculum in terms of its content and opportunity for conceptual progression was also considered.

Literature reveals continuing discourse regarding the levels of thinking described in the Van Hiele theory, and even though there is no consensus regarding the nature of the levels and that assessing learners' levels of thinking remains difficult and inconclusive, it is generally accepted that the Van Hiele test is a reliable measure in assessing learners' levels of geometry thinking.

An exploratory case study design was chosen for this study. The phenomenon being explored is the teaching and learning of geometry in the Foundation and Intermediate Phases of a particular private school. In order to do this, the teachers' timetables and Work Schedules were analysed to determine how much time was allocated to the instruction of Mathematics in general and for the instruction of geometry in particular. These documents also yielded data regarding the type of geometry experiences included in the implemented curriculum. The learners' level of geometry understanding according to the Van Hiele theory was assessed using an instrument designed by Usiskin (1982). This assessment was facilitated by the researcher in the learners' home class and happened in June after six months of instruction in a particular grade level. Data regarding the teachers' perception of geometry and the best method to facilitate the learning of geometry was gathered through a teacher's questionnaire. The teachers were requested to facilitate geometry lessons, which were digitally recorded by the researcher. Each grade level (0-5) was regarded as a sub-unit and analysed as the case for that grade level. The data was then assimilated to present the case of geometry teaching and learning in the Foundation and Intermediate Phases in the school.

The findings report that when juxtaposed alongside research, geometry instructional practices in this school, compare favourably with regards to the teachers' professed and observed practice of using concrete aids and tasks that engage the learners actively in developing geometry insight. There is also evidence that these instructional practices support progression through the levels however the shortfall of time allocated to facilitating this progression and the lack of conclusive data regarding the language used and the types of experiences may justify further research into whether this progression is satisfactory.

KEY WORDS

- Geometry instructional practices
- Van Hiele levels
- Geometry teaching
- Teaching and learning geometry
- Learners' geometry understanding
- Primary school geometry

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LIST OF ABBREVIATIONS

NCS	National Curriculum Statement
DoE	Department of Education
TIMSS	Trends in International Mathematics and Science Study
NCTM	National Council of Teachers of Mathematics
CDASSG	Cognitive Development and Achievement in Secondary School Geometry
PDP	Parallel Distributed Processing Network Model
GRA	Grey Relational Analysis
CBMS	Conference Board of the Mathematical Sciences
WMGT	Wu – Ma Geometry Test
SAAMSTE	Southern African Association for Research in Mathematics, Science and Technology Education

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CHAPTER 1
INTRODUCTION AND OVERVIEW

1.1 ORIENTATION AND BACKGROUND

Much has been written on Mathematical teaching and ways of thinking, and much of the writing re-iterates that Mathematical learning is underpinned by a progression from concrete experiences to abstract generalizations. This journey reveals a tremendous scope for dialogue and research. The central role of the teacher in guiding the learner to take ownership of the geometry idea magnifies the importance of the type of instruction that takes place in the classroom. De Villiers (1996) states:

> "*The main reason for the failure of the traditional geometry curriculum was attributed by the van Hieles to the fact that the curriculum was presented at a higher level than those of the pupils.*"

This *Mismatch* of levels of thinking means that instruction is happening outside of the learners' knowledge structure and thinking skills and hence is ineffective in scaffolding the learners' growth in understanding (Bransford, Brown & Cocking, 2000). An American study on cognitively guided instruction shows that learners whose teachers knew more about their thinking achieved higher levels of problem-solving than learners whose teachers knew less about their learners' thinking (Carpenter, Fennema, Franke, Levi & Empson, 2000). Without insight into the learners' cognitive processes and an ability to establish where the learners' level of understanding lie, the teacher and learner are often left with no other alternative than to learn by rote (Atebe & Schäfer, 2008; Lithner, 2008). These memorized facts are unconnected bits of information, which take a great deal of effort to maintain, making learning Mathematics an onerous and less enjoyable task. Sans the development of relevant schema, this information is inaccessible to the learners (Bransford, Brown & Cocking, 2000; Van de Walle, 2007). It is no wonder that many learners feel unmotivated to learn geometry (Bleeker & Goosen, 2008; Kyriacou & Goulding 2006; Mogari, 2003).

The work of the Van Hieles is summarized succinctly by Pegg and Davey (1998, p. 110):

> *In summary, the* Van Hiele *theory is directed at improving teaching by organizing instruction to take into account students' thinking, which is described by a hierarchical series of levels. According to the theory, if students' levels of thinking*

are addressed in the teaching process, students have ownership of the encountered material and the development of insight (the ability to act adequately with intention in a new situation) is enhanced. For the Van Hieles, the main purpose of instruction was the development of this insight.

1.2 PROBLEM STATEMENT

Literature published in recent years has described the low level of Mathematics achievement of South African learners (Howie, 2000; Makgato, 2007). In 1995, South Africa took part in the Third International Mathematics and Science Study (TIMSS) and again in 1998 (TIMSS-R). Of the 41 countries that completed the assessment in 1995, South Africa ranked last and showed no significant improvement in the subsequent repeat in 1998 ranking last out of 38 countries. South Africa’s scores on the TIMSS and TIMSS-R are discouraging and sparked research into the factors that play a part in the learning of Mathematics as it relates to learners (Chard, Baker, Clarke, Jungjohann, Davis & Smolkowski, 2008; Da Ponte & Chapman, 2006; Dumay & Dupriez, 2007; Howie, 2000; Makgato, 2007). Although this research provided valuable insight, the complexity of our educational environment should not be underestimated. In other words, this is by no means a straightforward problem. Furthermore, the disquiet regarding Mathematics achievement is not a new problem, nor is it unique to South Africa.

1.3 RATIONALE FOR THE STUDY

Many studies on mathematics education deals with the concept of number and fewer studies deal with the area of geometry and spatial reasoning. This provides a small opening in which to place this study as it deals with geometry levels of thought. As a teacher who has taught for twenty years and has had the privilege of teaching from Grade 0 to Grade 11 during the course of this time, I have had the opportunity to experience a wide range of age groups. I have also taught in a number of different schools including a rural missionary school, a long-standing public school, and a well-resourced private school. In my experience, learners generally say that they do not like Mathematics; they perceive it to be difficult and they seem think that only a few people can do it, and even fewer, do it well. It has also been my experience that geometry is often the most neglected section in the primary school, as teachers do not fully understand the thought processes involved in constructing a meaningful schema for geometry shapes and concepts. Geometry is often dealt with within an art lesson, as many cannot see what there is to teach about a shape beyond the fact that a triangle has three sides and a square four.

The level of learners' Mathematical ability triggered research into the preparedness of educators in terms of their competency to facilitate effective learning. Research regarding the pedagogical content knowledge of Mathematics teachers (Dumay & Dupriez, 2007; Parker, 2004) was one approach in the search for a solution to the poor TIMSS ranking; another approach was to investigate the content knowledge of teachers (Knight, 2006; Leikin & LevavWayberg, 2007; Mayberry, 1983; Van der Sandt & Nieuwoudt, 2005; Van Putten, 2008; Wong, 1970). In short, there are multiple factors that play a role in the learners' Mathematical outcomes. The fact that one starts with different types of learners with varying levels of competency, from different socio-economic backgrounds in schools with varied environments and resources, and then introduces teachers with varying competencies, results in a very irregular playing field (Balfanz & Byrnes, 2006; Zvoch & Stevens, 2006). In addition, although the aim is to 'finish' with the same goal that being competent, creative and confident Mathematical problem-solvers, one can hardly expect a 'one size fits all' solution, especially considering these varied and diverse starting points. Working from a constructivist paradigm it is essential that the prior knowledge and prior experience of the learner is recognized and used as a foundation upon which to build new knowledge, skills and attitudes. Furthermore, it is assumed that effectively to build on what an institution is already achieving, the existing practices, policies and procedures need to be recognized.

This case study explored the educational terrain of an independent school with particular reference to geometry and aimed to describe and investigate the relationship between the levels of geometry thought of Foundation and Intermediate Phase learners and the instructional practices of their teachers. It is hoped that the data collected and the subsequent analysis provides a rich and accurate description of the current educational landscape of the school to inform school visionaries, local policy-makers, subject heads and heads of departments, to assist them in planning and plotting a successful Mathematical journey for all concerned.

1.4 CONTEXT OF THE INQUIRY

The purpose of this exploratory case study was to plot the educational landscape of an independent school in terms of the geometry that is being taught and learnt in the Foundation and Intermediate Phases. The school in which this research was conducted has two classes per grade and the number of learners per class is limited to twenty-five. The relatively small class size is pedagogically advantageous as it allows the learners to enjoy more individual attention. The opportunity that allows the teacher and the learner to interact on a one-to-one basis to facilitate understanding is later described as 'cognitive dialogue'. In addition, the classes in the Foundation Phase have a teacher's assistant assigned to them whose purpose is to assist the teacher with administrative duties to allow the teacher to be able to engage more with the pupils. The school also has a library of Mathematics manipulatives of which the teachers may make use.

1.5 OBJECTIVES OF THE INQUIRY

This is an exploratory case study and therefore it did not assume a research problem (Cohen, Manion & Morrison, 2000). The objective of this study was to investigate the nature of the current educational terrain in terms of geometry teaching and learning. The findings from this study provide reliable data from which to suggest possible strategies if and where needed to develop the teaching and learning of geometry in the school.

In this investigation mapping was dependent on the following:

- using literature to describe instructional practices which support the learning of geometry;
- describing the current instructional practices in this specific school; and
- comparing the current intended and actual instructional practices in the school with the practices suggested by the literature.

These aims are encompassed in the research question:

What is the nature of the teaching and learning of geometry in this private independent primary school?

The following sub-questions put in operation the main research question:

1. What does the literature tell us about effective teaching and learning of geometry?
2. What are the instructional strategies currently employed by these teachers?
3. To what extent may the learning and teaching of geometry in this school be described as in line with what the literature tells us?

This study provides evidence supported by literature to sketch a meaningful landscape upon which relevant and effective educational decisions for this school can be made (Carpenter *et al.*, 2000; Clements, Swaminathan, Hannibal & Sarama, 1999). This study may form the front-end analysis of a larger action research study or may reveal ways in which to design an instrument or programme that enables a quick, cost-effective strengths and weaknesses analysis of a school's geometry curriculum. Using an exploratory study as a means to determine the educational terrain, the local policy-makers, those being the subject-heads and members of the executive committee, in an institution could more effectively use the strengths and assets already present in the school to inform decisions (Merriam, 1998). An asset-based approach to decision-making where the focus is shifted away from what is lacking to the resources and strengths already present, is a more positive approach to bringing change (Rose, 2006). Tailor-made innovation would also seem more efficient. Arguably, it is more effective. To revisit the landscape analogy: any sustainable construction must sincerely consider the terrain on which it is founded.

1.6 RESEARCH DESIGN

A descriptive exploratory case study design was chosen for this study (Cohen *et al.*, 2000; Creswell, 2005; Bergman, 2009; Herman, 2009). The phenomenon being explored is the teaching and learning of geometry in the Foundation and Intermediate Phases of a particular private independent school. In order to do this, the teachers' timetables and Work Schedules were analysed to determine how much time was allocated to the instruction of Mathematics in general and for the instruction of geometry in particular. These documents also yielded data regarding the type of geometry experiences included in the implemented curriculum. The learners' level of geometry understanding according to the Van Hiele theory was assessed using an instrument designed by Usiskin (1982). This assessment was facilitated by the researcher in the learners' home classroom and happened in June after six months of instruction in a particular grade level. Data regarding the teachers' perception of geometry and the best method to facilitate the learning of geometry was gathered through a teacher's questionnaire (Maree & Pietersen, 2007), which replaced the originally intended teacher interviews. The teachers were requested to facilitate geometry lessons, which were digitally recorded by the researcher. Information from these classroom observations was used to validate the integrity of the other data collected and to determine what instructional practices are currently employed at the school including the use of level specific language, appropriate activities and the use of manipulatives.

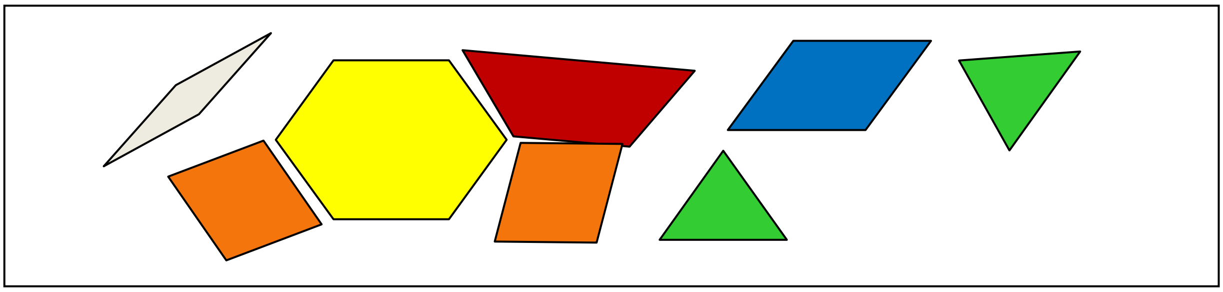
Each grade level (0-5) was regarded as a sub-unit and analysed as the case for that grade level. The data was then assimilated to present the case of geometry teaching and learning in the Foundation and Intermediate Phases in the school.

1.7 CLARIFICATION OF TERMS

Instructional practices: In this study, only those teaching practices directly concerned with the teaching of geometry concepts will be considered. In particular, the use of level specific and appropriate language and the use of geometry models and manipulatives will be observed. In addition, the time allocated for geometry lessons will be analysed as well as the type and progression of activities. It is important that the reader be aware that the researcher recognizes and celebrates the multi-faceted and diverse role of the teacher and that this study is an exploration of a single facet. The findings of this study therefore cannot and should be not used to evaluate the effectiveness of a teacher in terms of their contribution to the school as a whole.

Manipulatives: This term is used extensively by the participants of the study and describes any physical or concrete teaching resources used in a lesson, either by the teacher in a demonstration or by the learners in an activity to facilitate understanding of a Mathematical concept. Manipulatives may include models such as regular polyhedral (cones, cubes, and prisms), objects brought from the learners' home environment such as boxes, measuring equipment such as scales and tape measures, and may also include such things as string and play-dough.

Cuisenaire pattern blocks: These are a specific type of Mathematical manipulative of French origin. They may be described as colourful plastic tiles in the shape of regular



polygons.

FIGURE1: Cuisenaire Pattern blocks

The tiles or pattern blocks form part of a learning programme designed to help children develop problem-solving skills and spatial understanding through hands-on activities.

Language of instruction: This term implies the use of age appropriate vocabulary. For example, the learners in Grades 0 and 1 would refer to the vertex of a pyramid as the point and the angles of a square as the corners. Although this is true in the Van Hiele theory and so too in the context of this study, the language of instruction not only includes age appropriate vocabulary but also refers to a verbal presentation by the teacher that presumes a level of understanding. Each Van Hiele level has its own linguistic symbols and has its own systems of relations connecting these symbols. Thus, a relation that is 'correct' at one level may be modified at another level. The concept of class inclusion is such an example. The language of instruction in the teaching and learning of geometry is described by Atebe and Schäfer (2008, 50) who cite Van Hiele as:

> *People reasoning at different levels speak different languages and the same term is interpreted differently. The mismatch between instruction and students' cognitive levels in geometry is caused largely by teachers' failure to deliver instruction to the pupils in a language that is appropriate to students' thinking level.*

Cognitive dialogue: The notion of cognitive dialogue may be described as both the teacher and the learner together establishing where the learners' level of understanding lie, and serves to inform the teacher how to best scaffold the learners' construction of new ideas. The notion of cognitive dialogue draws from the work of other researchers (Bransford *et al.*, 2000; Carpenter *et al.*, 2000; Cobb, Wood, Yackle, Nicholls, Wheatly, Trigatti & Perlwitz, 1991; Gerace, 1992; Webb, Franke, Tondra, Chan, Freund, Shein & Melkonian, 2009). The significance of cognitive dialogue rests in the role played by the teachers' use of language in the teaching and learning of geometry (Eloff, Maree & Ebersöhn, 2006; Roux, 2005; Rudd, Lambert, Satterwhite & Zaier, 2008).

1.8 OUTLINE AND ORGANIZATION OF THE DISSERTATION

Chapter 1 of this dissertation presents the researcher's approach to, and rationale for the inquiry. Chapter 2 positions the study in relation to existing literature in terms of instructional practices generally used in the teaching of Mathematics and more specifically the teaching of geometry. In addition to expounding on the Van Hiele model, which forms the theoretical framework for the study, practical examples of Van Hiele-based experiences are given for two reasons:

- to elaborate on the model by means of examples emphasizing the S*equential* property of the Van Hiele model and
- to give the reader a clearer idea of what the researcher was looking for during observations.

Research methods and methodological considerations are continued in Chapter 3. Chapter 4 presents the findings of the inquiry followed by a brief discussion of these findings. The final section, Chapter 5, draws conclusions from the findings and offers recommendations relative to the objectives of this study.

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CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

The purpose of this chapter is to present the most pertinent contributions from a large body of literature on the development of geometry thinking. Two reasons for this chapter are to firstly, position the study in relation to existing research; and secondly, to provide this investigation with a scholarly framework. From a constructivist paradigm, how one defines, geometry is dependent on one's experience and depth of understanding. For some, geometry may mean identifying and naming shapes, for others it may be largely about measurement. The definition below is consistent with the views expressed by the teachers in this study (Chapter 4) and therefore useful as a point of reference.

2.2 WHAT IS GEOMETRY

In defining what is understood when geometry at school level is referred to, Clements and Battista describe it as follows:

> *When the term "school geometry" is used, it almost universally refers to Euclidean geometry, even though there are numerous approaches to the study of the topic (for example, synthetic, analytical, transformational and vector). The traditional, secondary school version of geometry is axiomatic in nature; elementary school geometry traditionally has emphasized measurement and informal development of those basic concepts needed in high school* (Clements & Battista, 1992, p. 420).

The emphasis on axiomatic systems in geometry curricula tends to downplay the foundational role played by spatial reasoning in the development of geometry concepts. Battista (2007), states that spatial reasoning provides critical cognitive tools as well as the 'input' for formal geometry reasoning and analyses. Since the scope of this study is limited to Grades 0-5, the study of geometry is seen to include spatial reasoning and extend to Euclidean geometry.

2.3 THE DEVELOPMENT OF GEOMETRY THINKING

Research suggests that Mathematical knowledge is constructed relationally (Arzarello, Robutti & Bazzini, 2005; Bransford *et al.*, 2000; Rudd *et al.*, 2008; Van de Walle, 2007) and that new knowledge is learnt more effectively when it draws on real and relevant contexts – re-enforcing the idea of building on learners' prior knowledge (Barnes, 2004; Freudenthal, 1971). If a learner cannot relate new information to any existing concepts, or accommodate it into their knowledge schema, it remains largely inaccessible and unbeneficial (Kilpatrick, Swafford & Findell, 2001; Van de Walle, 2007). At most, memorizing unrelated facts may cause unnecessary cognitive strain and, where the links are at best weak, they may cause the learner to make incorrect associations impeding further learning (Clements & Battista, 1992).

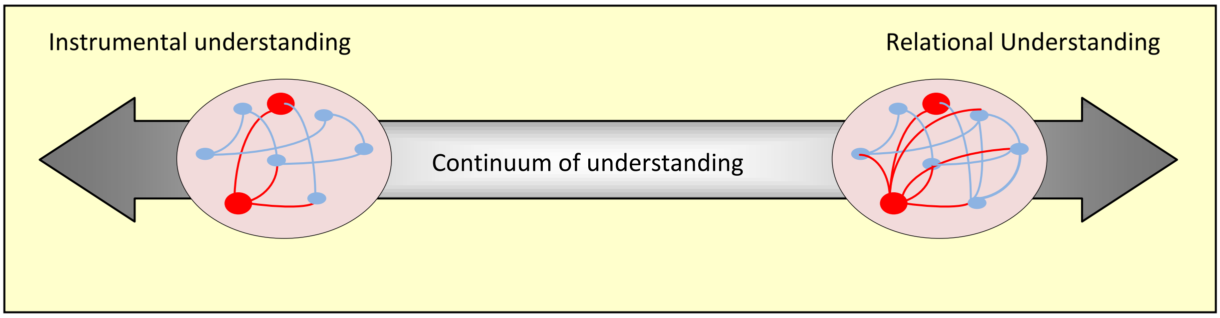


FIGURE 2: **Understanding is a measure of the quality of connections that a new idea has with existing ideas. The greater the number of connections, the better the understanding** (Van de Walle, 2007, p. 25)

In terms of geometry learning, these relationally weak concepts manifest in the learner as imitative reasoning, where the learners may imitate or recite the educator's use of symbols and language and perform definitions and procedures by rote (Atebe & Schäfer, 2008; Lithner, 2008). If this is the case, it is likely that learners may be able to use the imitated reasoning and language in situations that very closely mirror classroom experiences, but it is quite improbable that they would be able to use the information in new and different contexts (Kilpatrick *et al.*, 2001). However, successful learning is visible when learners are able to transfer or extend what they have learned to different situations. Moreover, successful transfer is influenced by the degree of mastery or level of understanding of the concept taught (Bransford *et al.*, 2000; Jones, Langrall, Thorton & Nisbet, 2002; Lithner, 2008).

The theories of the Van Hieles, Piaget and other cognitive psychologists have provided substance and momentum to the development of understanding the teaching and learning of geometry.

2.3.1 EXISTING UNDERSTANDING AS A STARTING POINT

The idea that new knowledge is hooked onto an existing network of understanding is not current and although it precedes the work of Piaget, it is most often associated with him since he is a well-known constructivist. The work of Vygotsky (Case, 1998) elaborates a constructivist view by emphasizing the role of the learners' environment/context and social interaction in the learning process. The socio-historic theories of Vygotsky and other researchers brought an awareness of the learning context and the use of language as tools to facilitate construction of conceptual schema. The means of assisting learners to gain a greater understanding using appropriate contexts and language is often referred to as *scaffolding* conceptual construction. Vygotsky's idea of the *'zone of proximal development'* describes instruction and hence learning as most effective when new concepts are presented to the learner in a theoretical space that links the learners' known ideas and possible developments in the learners' understanding. From Vygotsky we glean that effective teaching and learning begin with what the learner already understands and is facilitated by relevant communication in an appropriate context.

2.3.2 STAGES OF DEVELOPMENT

2.3.2.1 Stages as related to maturity

Topological Primacy, developed by Piaget and Inhelder (1967), presented a theory proposing that the spatial representation of a child developed in stages and corresponded to Mathematical structures. According to Piaget, the stages the child progressed through, in developing a more accurate representation of their spatial environment corresponded to the child's age. The sequential nature of gaining understanding proposed by Piaget is reflected in the *fixed sequence* characteristic of the Van Hiele model. In contrast to Piaget's theory where development is age-related, development according to the Van Hiele theory is experience-related. At the core of Topological Primacy is the "claim that the development of more sophisticated spatial concepts involves increasingly systematic and coordinated action" (Clements & Battista, 1992). The varying levels of young children's' fine-motor/ drawing abilities produced anomalies in Piaget and Inhelder's results and proved to be a stumbling block in comfortably establishing the acceptability of their theory. Freudenthal (1979) argued that the assumption that Mathematical learning is hierarchical is flawed and supported some of Vygotsky work by re-emphasizing the need for the learner to be able to relate to the Mathematical material with which they are presented (Jones *et al.*, 2002; Kilpatrick *et al.*, 2001; Van de Walle, 2007). Other criticisms of Piaget and Inhelder's theory include the inaccurate use of terms and the classification of figures as topological or Euclidean. Although Topological Primacy is not supported, it has not been totally disproven. Piaget and Inhelder's second major theme: "Children's representation of space is not a perceptual 'reading off' of their spatial environment, but is constructed from prior active manipulation of that environment" (Clements & Battista, 1992, p. 426), is supported implicitly by their results, and substantiates a constructivist epistemology and a perceptuo-motor approach to learning geometry (Arzarello *et al*., 2005; Owens & Outhred, 2006; Sullivan, Tobias & McDonough, 2006).

2.3.2.2 Stages as related to levels of thinking

Mirroring a Piagetian hierarchical view of knowledge, Pierre and Dina van Hiele developed a geometry model of understanding using five sequential and discrete levels of thought. Each level is characterized not only by qualitatively different levels of thinking but also by different internal knowledge and processing (Battista, 2007). Most significantly, the Van Hieles asserted that these levels of thinking are not as closely related to the learners' physical development or maturity as to the types of geometry activities the learners have experienced. In general, learners are assigned to a Van Hiele level when their overall cognitive organization and processing allows them to reason at that particular level. The nature of these levels and the assigning of learners to a specific level have been two of the three most troublesome characteristics of the theory. The third, being the application of the model, will be discussed briefly a little later.

2.3.3 MODELS OF COGNITIVE DEVELOPMENT

Battista argues that spatial reasoning is the foundation of geometry reasoning and, in citing other researchers, he defines spatial reasoning as follows: "Spatial reasoning includes generating images, inspecting images to answer questions about them, transforming and operating on images, and maintaining images in the service of other mental operations" (Battista, 2007, p. 844). Our understanding regarding the nature and conceptualisation of the images that Battista refers to is influenced by the work of cognitive psychologists. Anderson's Model of Cognition (Clements & Battista, 1992) postulates two types of knowledge: *declarative and procedural.* Very simplistically, learning according to this model could be described as receiving content or facts then applying this knowledge in some tasks whereby the learner intuitively develops an understanding of the types of tasks/problems the procedure being used is applicable to and hence reinforcing the declarative and procedural knowledge. This type of deductive learning is also evident in Greeno's Model of Geometry Problem Solving. A Parallel Distributed Processing (PDP) Network Model has at its core the pattern of interconnections among the units as the systems knowledge structure. In explaining this theoretical conjecture Clements and Battista (1992, p. 435) write:

> *With appropriate instruction, property recognition units begin to form; that is, visual features become sentient in isolation and are linked to a verbal label. The student becomes capable of reflecting on the visual features and, thus recognizing the shapes' properties, eventually leading to (Van Hiele) Level 2 thought.*

This premise validates the relationships between shapes, their properties and relationships among properties postulated in the Van Hiele model and supports other research, which highlights the role of metacognition in the learning process (Bransford *et al.*, 2000; Ritter, Anderson, Koedinger & Corbett, 2007). Clements and Battista (1992, p. 436) add, "research has also substantiated the PDP-postulated existence of multiple schemas. That is, students may possess several different visual schemas for figures (for example, a vertically- and horizontally-oriented rectangle) without accepting the 'average' case (for example, an obliquely-oriented rectangle)". Although this view is not directly addressed in the van Hiele model, it is consistent with other studies on the Van Hiele theory (Burger & Shaughnessy, 1986; Fuys, Geddes & Tischler, 1988) and may provide valuable insight into why it appears that some learners develop at different levels of thinking simultaneously. Pegg and Davey (1998) integrated the Van Hiele theory with the SOLO theory of Biggs and Collis (Battista, 2007) to address the lack of detail in the Van Hiele model in explaining the learners' cognitive development as they progress through the levels. Although the SOLO taxonomy proposes five levels similar to the Piagetian stages, Pegg and Davey only discuss three major modes of thinking. Battista (2007, p. 849) explains these as follows:

> *In the ikonic mode, students form and operate on mental images of objects with which they have had contact. In the concrete-symbolic mode, students link concepts and operations to written symbols, as long as the context falls within their personal experience. In the formal mode, students are no longer restricted to concrete*

referents and can systematically consider principles, theories, and ranges of possibilities and constraints; they become capable of formal proof.

Battista (2007, p. 858) noted that to accurately to describe the cognitive processes of learners is very difficult as these are internal and cannot be observed, but research indicates that: "perceptual wholes are built up from parts-based representations, but accessing these parts-based representations is initially impossible and only occurs 'with some work'.

2.3.4 COGNITIVE PROCESSES

Battista draws on the work of a number of researchers (Hoyles & Healy, 1997; Kosslyn, 1994; Steffe & Cobb, 1988; Von Glaserfeld, 1982) to explain the complexities of the process of abstraction. Battista also states that this process is at the root of the construction of entities into family groups or in other words from single facts or observations into ideas and concepts. Battista explains that for learners to move from seeing physical images to understanding relationships in geometry figures, learners must first be able to 'see' the parts of the shape so that they can build a spatial relationship between these parts (Bingolbali & Monaghan, 2008; Presmeg, 2006). In other words, once learners have perceived the general shape of a triangle, they need to identify that a triangle comprises of three sides and three angles or corners. Secondly, learners must mentally build explicit spatial relationships between the parts. For example, does a shape with three corners always have three sides? Battista refers to the learner as having "consciously to attempt to conceptualise these relationships"; this he says requires "intentional reflection". Battista's description here highlights the learners' role during the third phase of learning, described in the Van Hiele theory, that of *Explication* where learners verbalize their understanding and hence begin to make the systems of relations at the level more explicit. By all accounts, the learners' actions (physical and mental) are central to their developing even more sophisticated mental models and thus growing their understanding of geometry objects, relationships and processes (Jones *et al.*, 2002; Hazzan & Zazkis, 2005; Kilpatrick *et al.*, 2001).

The work of Bransford *et al.* (2000) expounding on various aspects of how people learn supports Battista's (2007) commentary on the progressive levels of abstraction that individuals go through in order to move away from concrete stimuli to mental models. The first level of abstraction is that of *perceptual abstraction* which isolates an item from experience and recognizes it as an object. At this level, the item is entered into the working memory but cannot be visualized. The next level is *internalisation* and this is when objects can be visualized after sufficient perceptual abstraction. At this level, learners can only re-present an object not reflect on it or analyse it. *Interiorisation*, the ultimate level of abstraction, consists of two sub-levels; the first happens when an object is sufficiently abstracted to be 'dis-embedded' from its original perceptual context and can be operated on in the mind and applied to new or novel contexts. The final stage, being the second sub-level of *Interiorisation*, is when the item can be operated upon without having to re-present it and symbols can replace the originally abstracted material. What Battista's commentary makes apparent is that learners must be active in the construction of their geometry concepts and that sufficient and various opportunities should be provided for successful abstraction to happen.

As mentioned, the work of Bransford *et al.* (2000) on the differences between novices and experts supports these notions of abstraction. They propose that experts unlike novices are able to transfer knowledge and concepts to new situations more successfully because, among other factors, their knowledge is organized around big ideas and not dependent on the context which originally was used to present the knowledge (Gerace, 1992; Ritter *et al.*, 2007). Furthermore, with sufficient experience the experts' knowledge is 'conditionalised' which clarifies conditions of applicability of that knowledge. Bransford *et al.* (2000) also state that it is an active and dynamic process whereby learners are effectively able to apply their learning to novel contexts.

2.3.5 SUMMARY

These "levels" of understanding in geometry have proved difficult to grasp with clarity (Gutiérrez, Jaime & Fortuny, 1991). The "levels" were first related to the learners' maturity and then to their experiences. Both of these approaches proved problematic. Although an hierarchical view of learning provides impetus for a developmentally structured curriculum and supports a constructivist epistemology, continued developments in the teaching and learning of geometry have shifted to a more flexible, connected view which emphasizes that learning geometry is an active construction of relationships among concepts. So in terms of the effective teaching of Mathematical concepts, literature posits that new ideas should be presented to learners within their level of understanding and that these ideas are more likely to be accessible to learners if they are linked to an existing network of ideas that learners already have. In addition to these instructional guidelines, literature also suggests that learners learn best by doing Mathematics using appropriate concrete apparatus. Using manipulatives to do

Mathematics, described and advocated by Arzarello *et al.*, as a perceptuo-motor approach, dovetails well with the socio-constructivist approach to Mathematical learning (Bransford *et al.*, 2000; Cobb *et al.*, 1991).

2.4 THE VAN HIELE MODEL

According to the Van Hiele theory, learners progress through five sequential and hierarchical levels of thought. Each level characterized not only by qualitatively different levels of thinking but also different internal knowledge and processing (Battista, 2007). In general, learners are assigned to a Van Hiele level when their overall cognitive organization and processing allows them to reason at that particular level.

The nature of these levels and the assigning of learners to a specific level have been two of the three most troublesome characteristics of the theory (Gutiérrez *et al.*, 1991; Gutiérrez & Jaime, 1998; Usiskin & Senk, 1990; Wilson, 1990). The third, being the application of the model, will be discussed briefly a little later. Although these levels were considered by the Van Hieles to be discrete, other researchers have found these levels to be dynamic and continuous. Therefore assigning a learner to a particular level, particularly those in transition from one level to the next, becomes extremely problematic (Battista, 2007; Burger & Shaughnessy, 1986; Crowley, 1987; Senk, 1989; Usiskin, 1982).

Gutiérrez *et al.* (1991) argue that learners develop several Van Hiele levels simultaneously and that acquisition of a specific level can take months and even years. Considering this argument, for Mathematics educators to be able to meet their learners at their current level of understanding or within their conceptual schema, the educator would have to have an intimate knowledge of the Van Hiele theory and of how geometry understanding is formed in each of their learners. This would be an insanely difficult task in schools where the teacher-pupil ratio is high.

However, bearing in mind that the classroom may be filled with individuals at different levels of geometry thinking for any particular shape, instructional practices should span the range of levels and be relevant to the varying degrees of acquisition of those learners being taught. In this regard, a robust assessment of learners' Van Hiele levels would suffice for this study. A detailed account of each learner's geometry understanding would not only be inhibitive and time consuming but the complexity of findings may prove overwhelming to the educator to be of any practical use. To equate this to the analogy used previously, the rough assessment of learners' Van Hiele levels would be like taking stock of the topography before analysing the soil structure. Indeed the nature of the levels or stages of reasoning and the degrees of acquisition or types of reasoning provide for a depth of discourse beyond the scope of this study. So too are the intricacies of human conceptual development with regard to the complexities of geometry thinking. For more on these aspects the reader is referred to Battista (2007).

The third troublesome characteristic as mentioned previously is how the model has been applied in various studies (Senk, 1989). In considering just a few studies, the Van Hiele model has been used as to measure the learners' levels of thinking and the relevance of these findings to predict future success in geometry and even to investigate whether the levels can be operationalised or described by student behaviour (Burger & Shaughnessy, 1986; Mayberry, 1987; Usiskin, 1982). It is precisely the wide and varied use of the Van Hiele model that legitimizes it as a good yardstick with which to survey the geometry terrain of this case study.

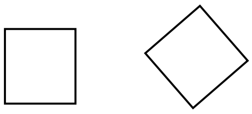
Later in his life, Van Hiele (1996) reduced the number of the levels to three and referred to them as levels of argumentation, these being: the visual, descriptive, and theoretical levels, thus perhaps implicitly addressing the contentions regarding the nature of the levels of thinking. Although Van Hiele elaborated on the types of reasoning learners manifest in each level, accurately assigning learners even to these three levels may prove difficult to do since the nature of learning is dynamic and idiosyncratic. The three levels of argument do not nullify the five levels of thinking still widely used and because no reliable instrument to assign learners to the three levels has been developed, this investigation makes use of the five levels of thinking presented below.

For the purposes of this study, the model simply tabulated by Van de Walle is adapted to the Level 1 – Level 5 numbering.

Level 1 – Visualisation

Thinking subject: shapes and what they look like

The learner recognizes a shape by its appearance and can group together shapes that look more or less alike. Thus a square is a square because it looks like a square.



Is the second figure still a square?

Thinking result: shapes can be grouped according to what they look like

Level 2 – Analysis

Thinking subject: classes /groups of shapes rather than individuals

The learner can think about what makes a rectangle a rectangle and not a cube: the reasons for which certain shapes are grouped together become clearer as the learner identifies the properties of a shape type.

Thinking result: Shapes can be grouped according to their properties

Level 3 – Informal deduction

Thinking subject: the properties of shapes

The learner is now able to make deductions about the properties and can follow a logical argument in simple deductive reasoning. Therefore, he can now figure out that if two adjacent sides of a parallelogram are equal, and at least one corner is a right angle, it is a square.

Thinking result: relationships among properties of shapes

Level 4 – Deduction

Thinking subject: relationships among properties of shapes

The learner is now able to go beyond just the properties, placing them in a structure where given information can be used to derive further information. He can now use logic rather than intuition most of the time.

Thinking result: axiomatic systems for thinking deductively

Level 5 – Rigor

Thinking subject: axiomatic systems for thinking deductively

The learner now thinks about axiomatic systems themselves, making comparisons and appreciating their differences. Thinking result: comparisons and contrasts between different axiomatic systems of geometry.

Thinking result: comparisons and contrasts between different axiomatic systems of geometry

In contrast to the broad and general application of Piaget and Vygotsky's work, the growth in geometry understanding is very specific in the Van Hiele model.

The inherent objects at one level become the objects of study at the next level. For example, at Level 1 only the form of the figure is perceived. Of course, the figure is determined by its properties, but it is not until Level 2 that the figure is analysed and its components and properties discovered. Crowley calls this the *Extrinsic/Intrinsic* property of the model. P.M. van Hiele (1986) also identified this as a property of the model. Usiskin named it ***Adjacency*** (Usiskin, 1982). Crowley mentions four other properties of the model also recognized by other researchers, as particularly significant as they provide valuable guidance for making instructional decisions (Crowley, 1987).

Arguable the most significant of the properties is the sequential nature of the model. Usiskin (1982, p. 5) stated that, "A student cannot be at Van Hiele Level *n* without having gone through Level *n-1*" and he called it the ***Fixed Sequence*** of the levels.

Another property of the model is that of ***Distinction*** (Usiskin, 1982) or *linguistics* (Crowley, 1987). Each level has its own linguistic symbols and its own systems of relations connecting these symbols. Thus, a relation that is 'correct' at one level may be modified at another level. For example, a figure may have more than one name- a square is also a rectangle (and a parallelogram). A student at Level 2 does not conceptualise that this kind of nesting or class inclusion can occur. This type of notion and its accompanying language, however, are fundamental at Level 3 (Crowley, 1987).

It is therefore impossible for learners to progress in their understanding until they have mastered the knowledge and processes of their current level of thought. Carefully guided instruction or scaffolding affords the learners this opportunity but slower learners are at risk of missing the relevant instruction should their progress be out of step with the progression of the Mathematics curriculum taught in the class. With each passing curriculum cycle the chances of receiving instruction at the appropriate level becomes less and less, meaning that the learners are at risk of being 'stuck' at a particular level.

Up to this point, instructional practices and instructional materials in relation to the Van Hiele theory have been discussed; but what of the learners' part in progressing through the Van

Hiele levels? How does the learner negotiate the progression from the visual, to the descriptive, to the theoretical level according to the Van Hiele theory?

The levels of abstraction (Battista, 2007) describe a process whereby an individual can isolate or 'dis-embed' an item or object from the experience to the degree where the item or object can be operated on in the mind and applied to new and novel contexts. For example, at the first stage (*Perceptual abstraction*) the learner may not be able verbally to explain the idea of '*between'* but given a picture would be able to indicate the object between two others. At the second level of abstraction a learner may be able to draw a picture to show the meaning of between but may not be able to deduce that an object '*between'* will always be flanked by objects on either side. An item or object is fully abstracted when it can be operated on in the mind (without a concrete model or representation) and applied to new and novel contexts. This final level of abstraction is called *Interiorisation*. Researchers (Battista, 2007; Bransford *et al.*, 2000; Gerace, 1992) propose that experts unlike novices are able to transfer knowledge and concepts to new situations more successfully because, among other factors, their knowledge is organized around big ideas and not dependent on the context which originally was used to present the knowledge (see Section 2.3.4). The learners' role can be described as actively constructing and revising conceptual connections to develop a deeper or more sophisticated understanding of phenomena.

The idea that guided instruction helps learners to make the new information their own dovetails with Vygotsky's zone of proximal development. By starting with a conceptual schema that the learner already possesses, the educator can guide and support or scaffold the development of new concepts. Vygotsky emphasized the role of the learning environment including social aspects, agent and tools in the learners' growth of new knowledge. It is this social interaction and negotiation of meaning that learners personally construct and internalize knowledge (Bransford *et al.*, 2000; Webb *et al.*, 2009).

Pegg and Davey (1998, p. 110) summarise the content and the intent of the Van Hiele theory as follows:

> *In summary, the Van Hiele theory is directed at improving teaching by organizing instruction to take into account students' thinking, which is described by a hierarchical series of levels. According to the theory, if students' levels of thinking are addressed in the teaching process, students have ownership of the encountered material and the development of insight (the ability to act adequately with intention*

> *in a new situation) is enhanced. For the Van Hieles, the main purpose of instruction was the development of such insight.*

However, literature also reveals discussion about the most reliable and valid means of assessing learners' Van Hiele levels. Some researchers have made use of paper and pen tests whilst others are more in favour of using an interview assessment. The structure of the tests and the number of questions are other aspects that have come under scrutiny (Burger & Shaughnessy, 1986; Crowley, 1990; Usiskin, 1982; Usiskin & Senk, 1990; Wilson, 1990). However, no matter which method is used, creating and interpreting tasks that reliably and validly assess Van Hiele levels is extremely challenging (Battista, 2007).

2.5 LANGUAGE AND COMMUNICATION IN THE LEARNING OF GEOMETRY

The final property of the Van Hiele model to which Usiskin (1982) assigned the name ***Separation*** simply states, "two persons who reason at different levels cannot understand each other". Crowley (1987) refers to this as ***Mismatch***. In the case of *Mismatch*, instructional practices fall far beyond the learners' level of understanding. Essentially, the educator may just as well be speaking a foreign language. Atebe and Schäfer cite Van Hiele (2008, p. 50):

> *People reasoning at different levels speak different languages and the same term is interpreted differently. The mismatch between instruction and students' cognitive levels in geometry is caused largely by teachers' failure to deliver instruction to the learners in a language that is appropriate to students' thinking levels.*

The notion of cognitive dialogue which may be described as both the teacher and the learner together establishing the learners' level of understanding, serves to inform the teacher how to best scaffold the learners' construction of new ideas. This notion draws from the work of other researchers such as Bransford *et al.* (2000), Carpenter *et al.* (2000), Cobb *et al.* (1991), Gerace (Southern African Association for Research in Mathematics, Science and Technology Education (SAAMSTE) conference proceedings 1992, p. 39) and Webb *et al.* (2009).

Sfard (2007) introduces the idea of *commognition,* which is intimately related to the concepts of cognition and communication but just as a child bears a strong resemblance to its parents; it is still uniquely individual; so too the notion of commognition has its own distinct characteristics. Sfard (2007) uses this idea as a framework to make sense of classroom processes. She argues that thinking can be regarded as communicating with oneself and not necessarily in an audible or visible way. In other words thinking, or as she would have it self-communication, includes informing ourselves, arguing, asking questions and waiting for an answer. She acknowledges that this process is an individual activity that begins on the inside of the person, is biologically determined, and generally considered inaccessible to others by direct means. This is a point to note considering the need for educators to understand their learners' thinking to be able to teach within their learners' level of understanding and hence be effective (Carpenter *et al.*, 2000; Jones *et al.*, 2002; Ritter *et al.*, 2007).

The Van Hieles referred to the process whereby learners internalise and make personal meaning of geometry concepts and vocabulary, as learners having 'ownership' of what is presented to them. Sfard (2007) calls this *individualizing* and describes the process as "becoming able to have Mathematical communication not only with others, but also with oneself". She goes on to say that Mathematical discourse alters and extends the learners' spontaneously learned colloquial discourses rather than constructing new ones from a void, which is very much in line with Vygotsky and other socio-constructivists; re-emphasising the importance of establishing the learners' prior knowledge. Our understanding or discourses are generally in harmony with our experiences and a change in discourse is needed to become aware of new possibilities. She concludes that Mathematics learning at school "requires an active lead of an experienced interlocutor and needs to be fuelled by a learning-teaching agreement between the interlocutor and the learners" (Sfard, 2007, p. 565). Rudd *et al*. (2008) study on the types and frequency of Mathematical language used in early childhood classrooms indicated that although discussion relating to spatial concepts exceeded any other type of Mathematical concepts in frequency, there was a lack in higher-level Mathematics concepts observed. They acknowledge the importance of language as a tool for teaching Mathematics and the effectiveness of instructional practices that connect new Mathematical terms or phrases to ideas learners already have.

Clearly, Sfard's (2007) work dovetails with that of the socio-constructivists (Bransford *et al.*, 2000; Cobb *et al.*, 1991) and particularly to that of Gerace (1992) who said:

> *…teaching is essentially a communication process. Consequently, the problem of defining good teaching methods is rephrased as a problem of establishing effective communication channels with the student.*

According to Gerace, establishing a dependable means of communication consists of four steps:

1. Create or refine a "code" or language both the sender and the receiver can understand.
2. Create a mental image/model of the receiver.
3. Make inferences based upon this model when constructing the message.
4. Observe some effect of sending the message.

Translating this into the context of the geometry classroom would mean that an effective educator would need to have a good understanding of how their learners think (refer to Gerace's second point) – establishing what Van Hiele level the learners have mastered and currently in transition towards. The educator would need to choose instructional practices which are relevant to the learners (refer to Gerace's point three) choosing geometry experiences such as concrete examples or models within the learners conceptual grasp. They also need to communicate with their learners in a language that appropriately scaffolds their learning (refer to point 1 of Gerace) – use language and symbols that the learner can understand and span the gap or scaffold the learners current level of thinking to the next. Finally the educator should reflect on the effectiveness of the communication process (see Gerace point four 4) – re-assess the learners' geometry thinking.

Geometry understanding is indeed complex and, internal discourses or thinking processes are not always apparent or easily accessible, however, the key to gaining greater insight into these complexities may very well lie within the notion of cognitive dialogue.

In their study, Atebe and Schäfer (2008) found that Grade 10, 11 and 12 learners used imprecise terminology in describing geometry shapes. This may imply that students imitate the language used by their educators who reason at a different level to them, with no real understanding. Moreover, with no understanding, the concepts, with which these words are related, cannot be accessed or used (Bransford *et al.*, 2000; Gerace, 1992). The resulting misconceptions in learners' thinking built from sketchy and unsupported knowledge have been shown to be difficult to uproot and widespread (Atebe & Schäfer, 2008; Gerace, 1992), impeding further growth. These findings support earlier research, which builds a strong case for developing learners' understanding relationally by linking new ideas to the learners' existing network of concepts and hence teaching within the learners' level of understanding.

2.6 THEORETICAL FRAMEWORK

The decision to use the Van Hiele model of geometry thinking as a framework for this study lies in the fact that it is arguably the most conceptually coherent model, which enabled ease of use. By using this framework, this study aims to map the current geometry practices of an independent school. According to Battista (2007), "*A considerable amount of research has established the* Van Hiele *theory as a generally accurate description of the development of students' geometry thinking*" (Battista, 2007, p. 846).Van de Walle (2007) illustrates the five hierarchical levels that describe the thinking processes in geometry concepts.

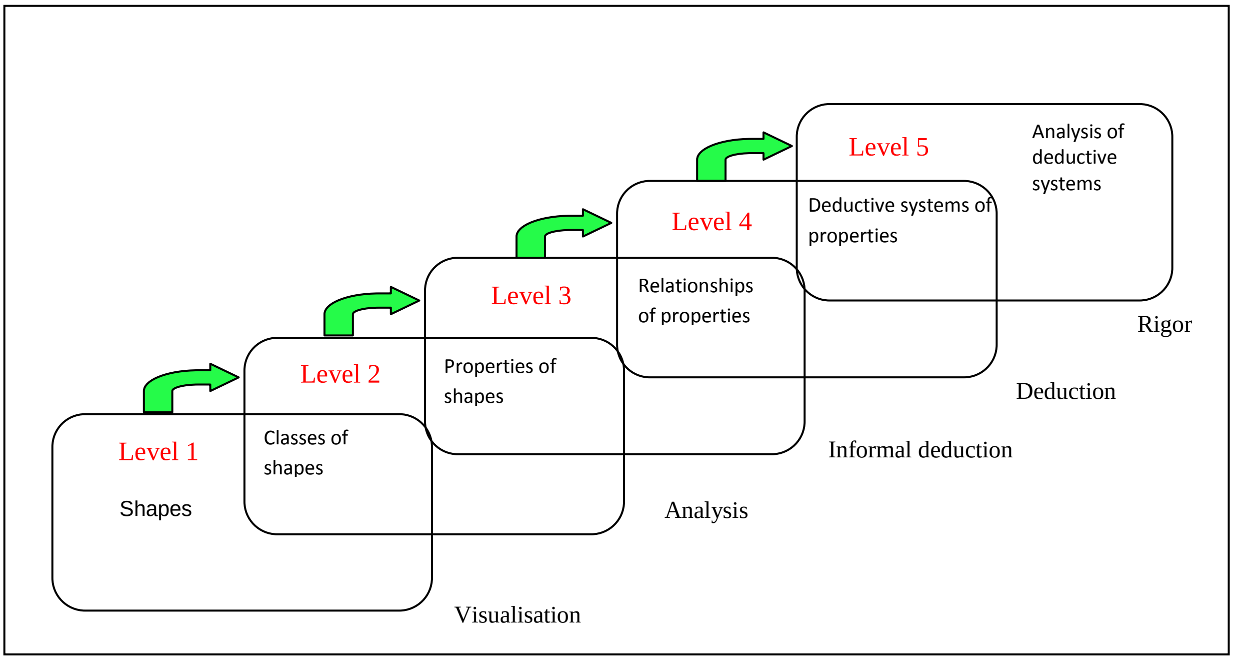


FIGURE 3: The Van Hiele Model of geometry thinking (Van de Walle, 2007 p.421)

In using the Van Hiele theory as the framework for this study, the generalities the Van Hieles used to characterize the model (Crowley, 1987; Usiskin, 1982) are applied in interpreting the findings. Firstly, the model is ***sequential***. The *Extrinsic/Intrinsic* property describes the process whereby the inherent objects at one level become the objects of study at the next level. This means that for a learner to operate successfully at a particular level the strategies of the preceding level need to have been acquired. Secondly, progress or lack thereof depends more on the content and methods of instruction than on age. This property is labelled ***Advancement***. Crowley (1987) provides a description of tasks appropriate to each Van Hiele level of thinking and this is used in interpreting data. Thirdly, each level has its own ***linguistic*** symbols and its own system of relations connecting these symbols. Finally, the property of ***Mismatch*** explains that if a learner is at one level and instruction is at another, the desired learning and progress may not occur. By slightly extending what Gerace (1992) called bi-directional communication on a foundation of socio-constructivism, the notion of cognitive dialogue can be introduced. This may be described as both the teacher and the learner together establishing where the learners' level of understanding lie and serves to inform the teacher how to best scaffold the learners' construction of new ideas. This notion of cognitive-dialogue is gleaned from the work of other researchers such as Carpenter *et al.* (2000) and Webb *et al.* (2009) and was also referred to by Van Hiele (1996, see also 1986, p. 56) as "mutual discussion".

Figure 4 presents a diagrammatic representation of the theoretical framework, which underpins this study. The blue rectangular shapes at the bottom of the picture represent the learners' levels of geometry thinking according to the Van Hiele theory. The pale orange double-sided arrow that connects the blue rectangles represents the Van Hiele test used to assess the learners' levels of thinking (Usiskin, 1982). The instructional practices, which promote progression from one Van Hiele level to the next, are represented in the large green arrow above the blue rectangles. The characteristics of the Van Hiele theory are listed on the left-hand-side of the green arrow. Only the S*equential property* of the theory is omitted since it is implicit in the A*dvancement* and the E*xtrinsic/Intrinsic* properties already present. The green arrow indicating instructional practices that promote progression through the levels is seen as double-sided. This is because evidence of whether or not a particular characteristic of the Van Hiele theory has been addressed would be found in the areas listed on the right-hand-side of the arrow. For example, the curriculum content would provide evidence of whether or not the geometry the learners were being taught followed a logical development and hence whether or not the *Extrinsic/Intrinsic* characteristic of the Van Hiele theory had been addressed. In the same way the *sequential* characteristic that states that the learners cannot progress to Level (*n*) until they have mastered Level (*n-1*), emphasizes the need for the correct type of geometry experiences. Progression is more experience (*Advancement* property) than age related and is therefore dependent on sufficient and appropriate experiences made available to the learners.

Embedded in the types of geometry experiences are the appropriate use of manipulatives and the allocation of sufficient time to progress through the phases of learning as outlined by Crowley (1987). Similarly, the property of *linguistics* is recognized in the language and level of instruction used to facilitate geometry lessons. Finally, the notion of cognitive dialogue supports the property of *Mismatch.* The pale green and blue arrow in the centre of the diagram, represent the objective of this study, that is, to investigate the relationship between instructional practices and the learners' levels of geometry thinking.

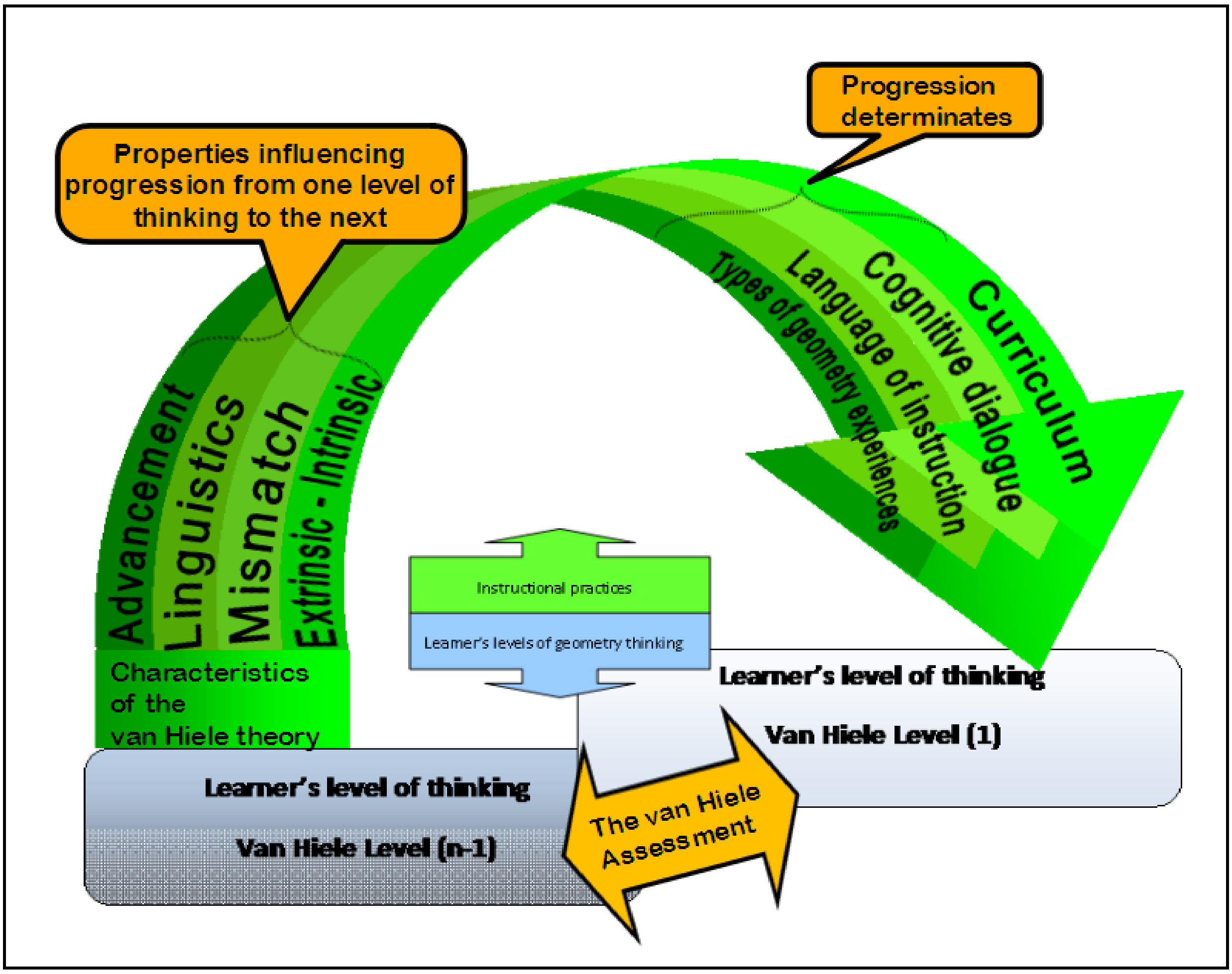


FIGURE 4: Theoretical Framework

2.6.1 TYPES OF GEOMETRY EXPERIENCES

The Van Hieles (Crowley, 1987; Van Hiele, 1996) proposed five *Sequential phases of learning* (emphasis added) or types of geometry experiences, which enable the learner to progress to a next level. These phases of learning are inquiry, directed orientation, explication, free orientation and integration.

Phase 1: Inquiry/information

The purpose of activities at this phase is twofold: firstly, the educator gains insight to the prior knowledge that the learners have about the topic, and secondly, the learners get an idea of the direction further study will take. Discussion and observations are characteristic of any activity in this phase. For example, the educator may ask the learners to identify the shapes presented and call for them to observe similarities and differences. At this phase, questions are raised on observations and level-specific language is introduced (Rudd *et al.*, 2008; Webb *et al.*, 2009).

Phase 2: Directed orientation

At this phase the educator uses a variety of carefully sequenced short tasks to help the learners explore the structures characteristic of the level currently being facilitated and to elicit specific responses (Chard *et al.*, 2008). For example, the educator may ask the learners to construct a triangle with three acute angles, then with two and then with only one. This sequence of activities could be repeated using right angles. All through these activities, the learners are being made aware of the properties of triangles (Hannafin, Vermillion, Truxaw & Lui, 2008).

Phase 3: Explication

The educator's role at this phase is minimal and is really to assist the learners in using appropriate and accurate language. At this phase, the learners verbalise and express their thinking and observations about the topic (Webb *et al.*, 2009). For example, continuing the triangle exercise used above, the learners may note that a triangle cannot be drawn using more than one right angle or more than one obtuse angle. It is here where the level's system of relations starts to become apparent.

Phase 4: Free orientation

The objective at this phase is for learners to explore relations within the level or "field of investigation" so that the relations between the objects of study become explicit to them. The educator facilitates this process by presenting the learners with: multi-step tasks, tasks with several means of solving them, and open-ended tasks (Cobb *et al.*, 1991). These more complex tasks could include something like investigating the lines of symmetry of different types of triangles or trying to establish the relationship between the size of the angle and the opposite side of a triangle.

Phase 5: Integration

At this phase, it is important that no new information be presented but that the learners summarise, and review what they have learnt to form an overview of the objects and relations they have investigated. The educator's role in this phase is to ensure that a 'complete' (relevant to the level) summary is formulated and the origin of this summary reviewed (Gerace, 1992). By the end of the fifth phase, learners ought to have obtained a new level of thinking that replaces the old and they are ready to begin the process again.

2.6.2 TEACHING ACCORDING TO THE VAN HIELE THEORY

Learners should be provided with experiences that will help them advance to a next level of geometry thinking. In planning for these experiences, geometry teachers should be considering the following:

- What would the ideal environment for progression look like?
- Roughly how much time would a learner need to master a level?
- What are the words the educator should use?
- What type of activities should the learner engage in?

What follows in this section are examples of activities that were established by other researchers as promoting the progression from one Van Hiele level to the next. These geometry experiences are gleaned from the work of Burger and Shaughnessy (1986), Crowley (1987) and Van de Walle (2007). These researchers provided sufficient examples to clarify the types of activities and language that should be used in Van Hiele-based experiences. Providing practical classroom examples provides a clearer picture of what to look for during classroom observation. In other words, using practical examples enables the researcher to move from the abstract ideas of good geometry practice to a more tangible measure or yardstick with which to gauge the geometry landscape within a particular setting. Since this study will involve only Grade 0-5 learners, examples beyond the initial stages of Level 3 will not be included.

Level 1 – Visualisation

Geometry shapes are recognized on their global, visual characteristics. Learners should be provided with opportunities:

1. to manipulate, colour, fold, and construct geometry shapes;

2. to identify a shape or a geometry relation
   - in simple drawing
   - in a set of cut outs or other manipulatives
   - in a variety of orientations
   - involving physical objects that are part of their everyday life
   - within and in relation to other shapes;
3. to create shapes
   - using cut outs, tracing paper, dot paper, grid paper, and geo-boards
   - by drawing figures
   - by constructing shapes with sticks, straws, pattern blocks etc.;
4. to describe geometry shapes verbally using appropriate standard and non-standard language
   - comparing shapes e. g. these shapes have the same number of "corners"
   - contrasting shapes and
5. to solve problems using shapes
   - investigating making squares and rectangles using triangles
   - covering a given area using different shapes.

In short, learners should have varied and sufficient experience with geometry shapes to allow them opportunity to become aware of the relevant features of shapes to the extent that they can begin to conceptualise classes of shapes.

Level 2 – Analysis

Awareness of the physical characteristics of shapes recedes as the properties of shapes become dominant. Learners should be provided with opportunities:

1. to measure, cut, fold, colour and tile shapes so that the properties of figures and geometry relations become apparent
   - finding the lines of symmetry of various shapes;
2. to describe a class of shapes by its properties
   - twenty questions, guess the shape e. g. Does it have three corners? / Are all its sides equal?
   - happy families, e. g. which shapes are related?
3. to sort and re-sort shapes according to a single attribute;

4. to identify and draw a shape given an oral or a verbal description of its properties
   - guess my name: the identity of a shape is revealed one property at a time;
5. to identify a shape from visual clues
   - gradually reveal a shape as learners guess at what it could be;
6. to derive rules and generalizations
   - by measuring and tiling many different rectangles, learners come to understand that length x breadth is a shorter way of finding the area than counting the tiles and
7. to identify properties which typify or contrast different classes of shapes.

At this level of geometry thinking, activities should be geared towards helping the learner understand the properties of shapes and that these properties can either include or exclude a shape from a certain family of shapes. Learners should use the correct names of shapes and learn that a shape can belong to more than one family e. g. a square is a rectangle as well as a quadrilateral.

Level 3 – Informal deduction

Relations between properties begin to emerge. Learners should be provided with opportunities:

1. to formalize relationships by looking for inclusions and implications
   - use property cards to compare and contrast shapes
   - on a geo-board or dynamic software programme, change one shape to another;
2. to develop and use definitions
   - identify the minimum set of properties to describe a shape;
3. to follow and present informal arguments;
4. to follow deductive arguments;
5. to provide alternative explanations or approaches
   - define a given shape in two ways;
6. to work with statements and their converse and
7. to solve problems where they use the interrelationship of properties to arrive at a solution.

2.6.3 TEACHING AND LEARNING TIME

It has been established that progression through the Van Hiele levels of geometry thought is not as dependent on chronological age as it is on the type of geometry experiences with which the learner is provided. The current level of the learners' geometry understanding determines the type of the geometry experience the learner needs to develop a deeper understanding and master the Van Hiele level at which they are currently functioning. Through certain activities designed by researchers such as Usiskin (1982), Burger and Shaughnessy (1986) and Crowley (1987) the learners' Van Hiele level to some significant degree of accuracy can be established and the results used to inform the teacher with which type of activities the learners should be provided. Unfortunately, the number of a certain type of activity or the amount of time a learner needs to master a concept is more difficult to establish. This is possibly due to the fact that efficacy in the teaching and learning process is influenced by so many variables such as teacher competency, individual learning styles, resources and attitudes, to mention but a few (Adam, 2004; Balfanz & Byrnes, 2006; Dumay & Dupriez, 2007; Zvoch & Stevens, 2006). In a paper presented at a Southern African Association for Research in Mathematics, Science and Technology Education (SAAMSTE) conference, Atebe and Schäfer (2008) conjectured that it took between 20-30 lessons for a learner to progress from one Van Hiele level to the next. The relevance of their findings to this study is debatable in light of the fact that their sample consisted of high school learners, however, when one considers that the levels of thinking of the learners may be common to both studies, one may be tempted to regard their findings as applicable. The guidelines on time allocation for the facilitation of geometry concepts in the NCS is used as a yardstick in this study in order to plot the educational terrain in this school with reference to what is referred to in literature as sufficient geometry experiences.

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CHAPTER 3
RESEARCH DESIGN

3.1 INTRODUCTION

As researcher in this study, I had the unique privilege of being allowed into the personal domain of each of these teachers who form part of this study. Each of these teachers willingly let me into their classrooms. The purpose of this study is to explore the current instructional practices in relation to the learners' level of geometry as outlined in the literature discussed in Chapter 2. My intent was to get as clear a picture of the actualities of each case or grade without intruding and/or making the teachers involved in this study feel exposed or insecure.

3.2 RESEARCH PARADIGM

For the integrity of this study, it is important to acknowledge and elaborate on the assumptions that informed and influenced my research methods and the analysis of the data collected (McMillan & Schumacher, 2001). In the past twenty years of my teaching experience, I have observed that any meaningful remediation of incorrect Mathematical concepts begins by taking account of the learners' existing understanding and working from the learners' existing perception of what they think they understand. The notion that knowledge is constructed on existing conceptual schema and is facilitated through social interaction is referred to in literature as socio-constructivism. This perspective also recognizes that constructed realities do not exist in a vacuum, but are influenced by context (Maree, 2007). The idea that Mathematical understanding is constructed is based on the work of Piaget. These perspectives play an integral part of the lenses through which I, as researcher, construct my understanding of this particular educational context.

My assumption then is that Mathematical learning/understanding is constructed relationally and is assisted by the appropriate use of language, contexts, and manipulatives. Hence, it can be said that as researcher, I am working from a constructivist or socio-constructivist paradigm. Thus for me as researcher and as teacher, it is essential that the prior knowledge and prior experience of the learner is recognized and is used as a foundation upon which to build new knowledge, skills and attitudes.

From a constructivist perspective one accepts that multiple, subjective realities exist based on the idiosyncratic nature of developing understanding (Merriam, 1991; Niewenhuis, 2007). Hence, my ontological assumptions can be described as relativist (McMillan & Schumacher, 2001; Niewenhuis, 2007). As the primary data collection instrument (Niewenhuis, 2007), I acknowledge the influences of my assumptions on the method of inquiry and my analysis of the data. To this end, in this exploratory case study, I acknowledge that the distinctly descriptive information presented and the conclusions inductively drawn in this study are incomplete and tentative.

It may be argued that this study is worked from a post-positivist perspective in that it seeks to understand the subjective realities of the participants whilst quantifying the findings (Maree, 2007). This is an apt perspective for those interested in the pragmatic combination of quantitative and qualitative methods. Furthermore, post-positivists focus on searching for evidence that is valid and reliable in terms of the existence of a phenomenon rather than generalisation. The findings of this study may not have a wide transferability but have value in naturalistic generalisability (Lincoln & Guba, 1985).

3.3 METHODOLOGICAL PERSPECTIVE

A review on the research methods in Mathematics education done by Hart, Smith, Swars and Smith (2009) revealed that only 29% of the 710 reviewed articles used mixed methods combining quantitative and qualitative data. This discovery gave impetus to the notion of using a pragmatic combination of quantitative and qualitative methods for this investigation. Since the aim of this study is to describe the educational terrain of a school in terms of its geometry teaching and learning, a mixed method approach is most appropriate as rich descriptions tend to emerge from multiple sources and varied collection techniques. Furthermore, this study looks specifically at the relationship (the case) between levels of thinking and instructional practices within a bounded system (the school).

A case study design was selected as the most appropriate design for this study as the most significant characteristic of this type of research is the delimitation of the object of study (Cohen *et al.*, 2000; Henning, Van Rensberg & Smit, 2004; Merriam, 1998). This study may be classified as an exploratory case study since it seeks to chart the ‘geometry terrain’ within a particular school. Although this study may appear to be “examining a particular case in order to gain insight into an issue or a theory” and hence an instrumental case study (Cohen *et* *al.*, 2000, p. 183) it does not assume an issue or a research problem. This study strives to "portray what it is like in a particular situation" (Cohen *et al.*, 2000, p. 182). The purpose for providing a rich description of the 'geometry terrain' is to afford the relevant parties the opportunity to make effective decisions regarding future progress of geometry in this school.

3.4 CONTEXT

The purpose of this exploratory case study was to plot the educational landscape of an independent school in terms of the geometry that is being taught and learnt in the Foundation and Intermediate Phases. The school in which this research was conducted is a private Christian, co-educational school with approximately six hundred and fifty learners ranging from Grade 00 to Grade 12. The first formal year of schooling preceding Grade 1 is referred to by some as Grade R, but in this school most teachers refer to this year as Grade 0. Therefore, Grade 0 and Grade R are interchangeable in the context of this study. Grade 00 refers to pre-school children between the ages of 3-4 years of age; however, they are not included in this study.

The school is affiliated to an inter-denominational church and shares the facilities and premises with the church. The school is racially integrated and the medium of instruction is English. The school generally has two classes per grade, except in some grades where an extra class has been introduced to accommodate more applicants. The three-class grade or "bubble", a term coined by the management, is introduced at Grade 0 level and moves through the grades until Grade 12. At the time of the investigation there were two such "bubbles" in the school, one at Grade 4 and one in Grade 12.

The number of learners per class is limited to 25. The relatively small class size is regarded as advantageous as it allows the learners to enjoy attention that is more individual. In addition to the smaller class size, the classes in the lower grades (Grades 00 – Grade 2) have a teacher's assistant assigned to them whose purpose is to assist the teacher with administrative duties to allow the teacher to be able to engage more with their learners. The school also has a library of Mathematics manipulatives available for teachers' use.

The teachers at the school are required at the beginning of each year to submit a document that plots their intended course of curriculum facilitation. The year is divided into four terms where each term is roughly between nine to ten weeks depending on where public and school holidays fall. The teachers in each grade either plan together or divide the learning areas between them so that one teacher is responsible for the planning in a particular learning area. In most grades, the planning for Mathematics or numeracy is shared or done in consultation and then the typing and /or editing are left as the responsibility of one teacher. At the site, school Mathematics lessons are schedule daily. From Grade 0 to Grade 3, the duration of a Mathematics lesson is 30 minutes. When and where these lessons happen is left to the discretion of the class teacher once they have taken into account activities given by other specialist teachers, for example music, sport and recreation, and information technology.

3.5 SAMPLE AND PARTICIPANTS

The study population spanned six Grades (Gr. 0-5), 13 teachers, 6 teachers' assistants, and approximately 325 learners. It was not feasible for this study to include the Grade 0 learners, as they were still too young to participate in a paper and pencil test and would have needed to be part of a different more time consuming interview-type assessment. From the literature reviewed for this study, it is generally accepted that learners in the Foundation Phase usually operate at a level preceding Level 1 – *Visualisation* or at Level 1 where shapes are recognized by their appearance (Wu & Ma, 2006; Van de Walle, 2007). It is of no major consequence as to which of the two classes participated in the study as it is the custom in the school that both teachers from the grade plan their lessons together. The learners are also randomly assigned to these classes in the beginning of the year, ensuring a mix of gender, ethnicity and ability. The data would only be further enriched should both teachers within a grade wish to be part of the study. Although this purposive sample involves a large number of participants in the beginning, it really only entails six cases, those being the instructional practices in relation to the learners' level of thinking at each of the six grade levels.

3.6 DATA COLLECTION AND INSTRUMENTS

In this section, the sources and instruments used to gather the data for this investigation are elaborated. Firstly, the aim of the study is redefined and this is followed by Figure 5, which is a pictorial representation of how the research methods align with the theoretical framework. Finally, Table 1 presents the data sources and instruments used in this study as they relate to the research questions.

In this investigation mapping of these cases was dependent on the following:

1. Describing the current instructional practices. These instructional practices are listed below.
   - The type of geometry experience facilitated by the teacher and described by Crowley (1987) (elaborated on in Section 2.6.1) of which the use of manipulatives forms a part. This data was obtained from the teachers' Lesson Plans and questionnaires.
   - The language of instruction and data for this was also sourced from the teachers' Lesson Plans and questionnaires.
   - Cognitive dialogue, which describes the teacher engaging with the learner within the learner's conceptual schema. This data was gained during the classroom observations.
   - Curriculum content gleaned from the NCS (2003) and the teachers' Year Plans that give an indication of the time spent teaching geometry and the content covered in the grades.
2. Finding the learners' levels of geometry thinking to provide a measure with which to assess the appropriateness of the current instructional practices.
3. Investigating the current approach to teaching and learning geometry in relation to the suggested approach based on the Van Hiele theory and described in research.

Figure 5 is a representation of the data sourced in this study as it relates to the theoretical framework described in Chapter 2. The factors influencing progression from one Van Hiele level of thought to another (as elaborated in Section 2.4) appear on the left of the arrow, these are listed alongside the progression indicators. For example, the learners' level of geometry thinking would indicate whether the *sequential* characteristic of the Van Hiele model was recognized in the teaching and learning of geometry. It follows then that the data to provide evidence of this would be found in the assessment results of the Van Hiele test conducted at each grade and presented on the right hand side of the arrow. The source of the data for each progression indicator appears in the arrow on the right hand side.

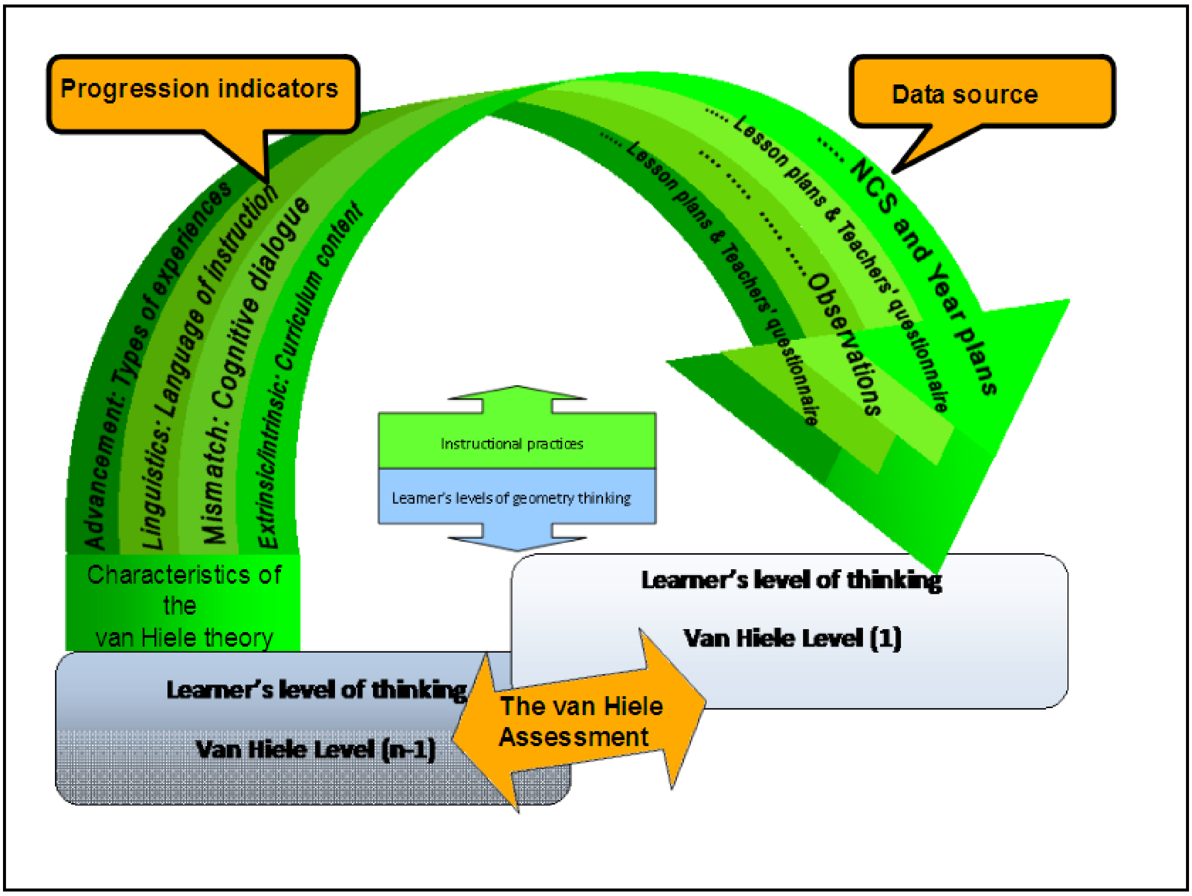

FIGURE 5: Data sources within the theoretical framework

Table 1 presents an overview of the data collection methods underpinning this study.

TABLE 1: Research instruments

Research question	Indicator	Data source	Instrument
Objective 1: Describe instruction practices which support the learning of geometry			
What does existing research say about teaching geometry?	Literature	Journal articles Books Papers Theses	Literature review
Objective 2: Describe the current instructional practices in the school			
1. What are the intended geometry instructional practices in the school (in terms of what the teachers planned to do)?	Curriculum content	NCS Year Plans and Lesson Plans Time tables	Document analysis Teachers' questionnaires

Research question	Indicator	Data source	Instrument
2. What are the actual geometry instructional practices in the school (as observed in the class visits)?	Type of geometry experiences (Crowley, 1987) Language of instruction Cognitive dialogue	Classroom observations Teachers' questionnaire	Observation schedule Teachers' questionnaire
Objective 3: Describe the learners' level of geometry thinking			
What are the learners' levels of geometry thinking?	Learners' level of geometry thinking	Van Hiele test results	Van Hiele Test (Usiskin, 1982)
Objective 4: Compare the current geometry instructional practices with practices suggested in literature			
To what extent may the teaching and learning of geometry in this school be described as in line with the Van Hiele model?	Instructional practices Learners' levels of geometry thinking	Data from objectives 1-3	Analysis procedures

3.6.1 DATA FROM THE TEACHER'S YEAR AND LESSON PLANS

It is asserted by the Van Hieles that progress through the levels is more dependent on the instruction received than on age or maturity (Crowley, 1987). In this light, it was important to see what types of experiences were planned for the learners in the course of the school year and the frequency and duration of the activities. The analysis of the teachers' Year and Lesson Plans allowed for an account of the time allotted to the teaching of geometry and type of activity planned for the learners' geometry development. In terms of describing the type of geometry activity evident in the teachers' planning documents, Crowley's elaboration of the five phases of learning (Section 2.6.1), those being *Inquiry/information, Directed orientation, Explication, Free orientation and Integration,* form the standard. The National Curriculum Statement (NCS) provided a reference for the time that should be allocated to developing an understanding of geometry in the Foundational and Intermediate Phases.

3.6.2 THE VAN HIELE TEST

The Van Hiele model although graceful in its simplicity is not without its challenges. A study by Burger and Shaughnessy (1986) found the levels not to be as discrete as originally presented by the Van Hieles. This is a discussion elaborated on by Battista (2007) but beyond the scope of this study. In his review of the developments regarding the Van Hiele model, he posits that there remains the issue of whether to use interviews or paper-and-pencil tests. The interviews yield more in-depth and valid data than the tests yet, are considerably more time-consuming to administer. Since this study aimed to describe the current instructional practices in geometry teaching and compare these practices with those described in the literature as promoting the progression of geometry understanding and that the learners' levels of thinking is a dimension to the study, it was found that a paper-and-pencil test (Appendix A) yielded data that was sufficiently valid (see Section 3.7) in terms of establishing the predominant levels of geometry thinking in a grade. It also highlighted any learners at any of the six grade levels who operated at levels beyond those which other researchers have described for children in this broad age group.

Leaning on what other researchers have reported as the general levels of thinking for the age groups in this study (Wu & Ma, 2006) and the fact that the National Curriculum Statement is structured around Van Hiele Levels 1 and 2 for Foundation and Intermediate Phases, the use of the CDASSG Van Hiele test was regarded as sufficient in establishing the dominant levels of thinking at a particular grade. The reliability of the Van Hiele test (Crowley, 1990) is discussed in Section 3.7. The Van Hiele test was administered to Grades 1-5 and, in order to maintain the validity of the test findings, steps were taken to deliver the assessment to the learners at their level of understanding and these are also discussed in Section 3.7.

3.6.3 OBSERVATION SCHEDULE

The purpose of the lesson observations (Appendix B) was to shed more light on the actual instructional practices that are happening in the classrooms at this school and to identify the nature of the teacher-learner interaction in terms of the geometry concepts being facilitated. It was important to create an opportunity to be able to identify the cognitive levels of activities and language occurring and relate that data to the learners' levels of geometry thinking. In other words, to find out if the teacher was or was not using activities and language/words and reasoning appropriate to the levels of geometry thinking of the learners she was teaching. Furthermore, the observations were used as an opportunity to gain insight as to whether the prior understanding of the learners was considered and to what extent cognitive dialogue was entertained. Although the nature and depth of the interaction recorded using this method would be superficial, the purpose of the observations was to provide evidence of the type of interaction actually happening in the classroom and corroborate with data from what the teachers said about interacting with their learners in the teachers' questionnaire.

A semi-structured observational schedule was most appropriate for this study as the categories for observation were predetermined. The most significant category was that of cognitive dialogue. In addition, to this end, the observation scheduled was structured to allow for 'teacher talk' and 'learner talk' to be written down. The predominant learners' levels of geometry thinking in that grade also appeared on the schedule providing a reference to evaluate the appropriateness of the teacher talk. The type of geometry experience as well as the use of resources was also noted during the observation and this data was used to substantiate data about the types of experiences recorded in the Year and Lesson Plans. The schedule allowed the researcher to list the activities observed, the resources used during the lesson, and if any geometry vocabulary was introduced, emphasized, or made particular mention of in the planning documents. The schedule contained a checklist of the five phases of learning (Section 2.6.1) so that the types of activity observed during the lesson could be easily classified later on in the study. The particular class, the time of the lesson and its duration were also recorded on the observation schedule. The lessons observed were digitally recorded to allow for reviewing and thus to enhance the reliability of the results.

3.6.4 TEACHER QUESTIONNAIRE

As soon as possible after the observation and transcription of field notes, an interview with the teacher observed was to be conducted (Maree & Pietersen, 2007). This was to allow the teacher to give greater clarity to interactions that were perhaps not clear in the recording or observation. Furthermore, it was considered that the interview might contribute additional information that may be used to triangulate other data. However, during the course of this study the teachers expressed a reluctance to participate in an interview process as they said that they felt a formal interview would be imposing and too time consuming. As I stated in the introduction to this chapter, I regard it a privilege to be allowed into the classrooms of these teachers and did not want to jeopardize the window of opportunity I have of gathering first-hand data and establishing a culture of accountability based on trust, by making these teachers feel insecure about their practices. The unwillingness of the teachers to participate in interviews posed limitations on the project; however, the dynamic nature of this explorative study forced the consideration of another approach to acquire sufficient data. Although the depth of data from these intended interviews could not be replaced, a questionnaire (Appendix C) was sent out to obtain an additional source of data.

3.7 DATA ANALYSIS PROCEDURES

In this exploratory case study, data was collected from seven different sources. The data was sourced from the NCS (Department of Education, 2003), the teachers' work schedules, Lesson Plans, timetables, the Van Hiele test results, the teachers' questionnaire, and from the observation schedules.

The data from the NCS and the teachers' work schedules, timetables and Lesson Plans were captured using EXCEL and juxtaposed in tables and graphs to compare the suggested, intended and actual time (Atebe & Schäfer, 2008; Usiskin, 1982) and type (Crowley, 1987) of geometry experiences evident in this school and those advocated in literature as sound practice (Chapter 2).

The dominant levels of geometry thinking at each grade level, was obtained using the Van Hiele test (Usiskin, 1982). The researcher using the two criteria described by Usiskin (Section 3.8) captured the test results. The raw data from the Van Hiele test was then also submitted to the Statistics Department at the University of Pretoria for statistical analysis. The Statistics Department provided a descriptive statistical analysis using SPSS.

The learners' levels of geometry thinking was juxtaposed with the data presented in the tables and graphs regarding the time spent facilitating geometry development and the type of geometry experiences offered to the learners at each grade level to establish the appropriateness of the instructional practices evident in this school. The data was then compared to the types of experiences advocated by the Van Hieles (Crowley, 1987) in terms of guiding the learners through the levels, to clarify the links between the levels of geometry thought and instructional practices. By comparing the two descriptions – one being what the Van Hieles describe as the ideal experiences (Objective 1, Table 5) and the other description of current geometry instructional practices (Objectives 2 & 3, Table 5) – one is given an opportunity to recognize the strengths and weaknesses of the current teaching methods in the school. An informed foundation is established from which to predict, to some extent, the success of the prevailing mode of teaching in this setting.

The classroom observations were focused on gleaning specific data regarding the topics of cognitive dialogue and the types of geometry experiences, including the use of manipulatives, being facilitated. A semi-structured schedule was used to capture incidences of these three predetermined focus areas. The analysis of this data consisted of recording whether the teachers engaged in cognitive dialogue with learners and whether manipulatives were used, or not. The type of activity used in the classroom at the time of the observation was used to triangulate data presented in documentation as to the types of geometry experiences afforded to the learners at this school.

3.8 RESEARCH INTEGRITY

Although there is a difference in opinion in research whether one should use the word “crystallisation” rather than “triangulation” in qualitative research, it is generally accepted that engaging in multiple methods of data collection and analysis enhances the trustworthiness of the results (Cohen *et al.*, 2000; Maree, 2007; Onwuegbuzie & Leech, 2007). It was the aim of this study to provide a rich, honest description of the case within its context and so, to add to the validity of the investigation, six different sources relevant to the case were consulted. The multiple sources clarified the case enhancing the overall reliability of the data.

Because I, the researcher, was a familiar face at the site school, the participants said that they felt comfortable allowing me to observe their lessons and perhaps then, it can be assumed that they did not feel the need to perform beyond their normal teaching manner. In the same vein, this may have jeopardized the credibility of the findings, as I was vulnerable to the danger of being biased during the observations and interviews. To address this threat, a video recording of the lessons was observed by another expert in the field and only the data where a consensus was reached was used. The participant minimizing the threat of selective reporting and misrepresentation checked the data from the observations.

The paper-and-pencil test was a test designed by Usiskin (1982) and used with his permission. This test was the result of the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project and, although the target group was high school learners, and hence the relevance of the test in Intermediate Phase is questionable, there remains no such test for primary school children except the time-consuming interview-type assessments. As Usiskin (1982) reports, although other geometry tests exist, not every question is assignable to a Van Hiele level. In the CDASSG report (Usiskin, 1982), the Van Hiele test scored low on reliability. The K-R formula reliabilities for the five parts in the fall and spring assessments are as follows: fall: .31; .44; .49; .13; and .10, and in spring: .39; .55; .56; .30 and .26. The low reliabilities are attributed to the small number of test items.

The test consists of 25 multiple-choice questions arranged in order of the levels of geometry thinking i.e. the first five questions correspond to the first Van Hiele level, questions six to ten correspond to Van Hiele Level 2 and so on. In the original study, students were assigned a Van Hiele level according to a weighted score on a three out of five questions correct criterion, or the stricter four out of five questions correct criterion. Since this study was to deal with a narrower span of possible Van Hiele levels, the weighted scores were not used and only the two criteria were used to assign learners to a level.

Despite the Van Hiele test being in Usiskin's (1982, p 30) own words "a rather crude device for classifying students, 70% of students were classifiable into a level on the three out of five criterion and 88% on the stricter (*4 out of 5)* criterion". Wu (2000) edited the Van Hiele test so that it was more age appropriate to the learners whose level of geometry thinking they wished to assess. In their adaptation of the Van Hiele test, they changed some of the mathematical symbolic language for example they changed "[$\overline{AB}$] to [edge AB]" (Wu, 2000). In the same vein when the researcher to the learners in the Foundation Phase administered the Van Hiele tests, the questions were read aloud to them so that neither the mathematical language nor their reading ability inhibited their understanding of the questions. The reading of the questions was not considered a serious threat to the reliability of the results as the paper-and-pencil test was intended to highlight those learners who may have needed further investigation as mentioned in Section 3.6.2. To enhance the reliability of these assessments in terms of their administration, they were administered by one person and the context in which they were administered was controlled as far as was possible to replicate each sitting in the various classes and to make it as less threatening as possible. To this end, the assessments took place in the learners' home classrooms during the learners' scheduled mathematics lessons and with both teacher and, where applicable, the teacher's assistant present. The assessment was introduced to the learners as an activity, not as a test, and they were encouraged to ask questions if they were unsure of what to do.

3.9 SCOPE AND LIMITATIONS

The internal validity or credibility of a study is the measure of the consistency between the research phenomenon and the interpretive research findings (Lincoln & Guba, 1985). It is therefore important to acknowledge that although efforts have been made to protect the reliability of the findings, the results do not include the Grade 6 year group, who fall within the governance of the Intermediate Phase. This was a deliberate choice as I, the researcher, was the Grade 6 mathematics teacher and to avoid any further threat to the credibility of the results, the scope of the study was restricted to the Grades 0-5 classes.

Development of Mathematics in general, and Geometry understanding in particular, is indeed complex and this study has looked at very specific domains for specific indicators in a particular context. These issues make the applicability or transferability of the research questionable and hence a threat to the external validity (Lincoln & Guba, 1985). In response to these concerns, it should be noted that the purpose of the study was to explore the nature of the teaching and learning of geometry within a specific context. Having once identified the factors affecting progression in geometry understanding as discussed in literature, the investigation of the case then becomes dynamic, relative to the educational terrain being explored. For example, if the school concerned followed an externally set curriculum, more emphasis may have been placed on document analysis or in the case where an impoverished school has little or no access to manipulatives, the nature of interaction between the learners and their teacher may have played a larger role in the study.

The dynamic nature of this explorative case study is evidenced in the use of a teachers' questionnaire to replace the interviews and influenced the depth of analysis of the same. It became apparent through the work schedules and Lesson Plans that the teachers were unaware of the Van Hiele model and the factors supporting progression from one level to the next. To gain data that would yield the depth of information needed for a thorough analysis regarding each teachers' understanding of geometry and how they should facilitate this development in their learners, would be very time consuming, relatively intrusive, and possibly provide more information than necessary for the purpose of this study. The findings in this study reveal many such opportunities for further, more in-depth research.

The context specific results of this study limit the transferability of the findings however, since many schools use the NCS, the methods of inquiry used in this study may be used to plot the educational terrain of other schools in terms of the teaching and learning of geometry. A comparison of these different cases would no doubt provide an interesting research opportunity.

3.10 ETHICAL ISSUES

This study did not involve experimental methods or an intervention of any sort, neither was it sensitive or intrusive in terms of the participants revealing personal information. It did require the learners' level of geometry thinking to be assessed but this was done whilst respecting the individual's dignity, and with their knowledge of the study and their consent. The study also involved an examination of the teachers' instructional practices. This did pose a challenge since I had recently been appointed head of mathematics at this school. However, I had been acting in an advisory capacity to the Foundation Phase Head of Department for the past two years. In this respect, the teachers and I had a good relationship and since my work with them previously, as well as in this study did not involve or threaten any chance of promotion or performance appraisal, they said that they felt safe in participating in this research. Their participation in the study was informed and voluntary. Their anonymity remains protected in this report. Consent from all the relevant parties was obtained before the commencement of the study.

Since Cohen, Manion and Morrison state "significance rather than frequency is a hallmark of case studies, offering the researcher an insight into the real dynamics of situations and people" (Manion & Morrison, 2005, p. 185), the choice of which class observation to analyse was based on the type of data which was yielded by the classroom observation and the corresponding teacher's questionnaire. Furthermore, presenting the instructional practices of only one teacher and not both or all three, respects the participants anonymity. In this analysis, every attempt has been made to focus on the objective of the study, which is to survey the 'geometry terrain' within a particular school, and the discussion that follows must be seen within its context and not as a criticism of any individual.

3.11 SYNOPSIS

The framework within which this study was conducted is given account of in this chapter. The bounded system is defined as the specific instructional practices, identified in the progression indicators in terms of the Van Hiele model, of the teachers in Grades 0-5, in a private, independent, co-educational primary school. This explorative case study is described as making use of two types of data drawn from six various sources collected through documents, observations and assessments. The data collected in this multi method investigation is analysed inductively underpinned by the researcher's socio-constructivist perspective. The chapter closes with a discussion regarding the integrity, transferability and ethical considerations of the research.

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CHAPTER 4
RESULTS

4.1 INTRODUCTION

In this chapter, the results are presented as six cases, those being the cases at each grade level. Because the two, and in one case three, teachers teaching at each grade level either plan together or work from the same Year Plan, the instructional practices described by the documentation occurring in that grade are discussed as the case for that grade. The level of the learners' understanding of geometry is also discussed for that particular grade. Each case will be presented in the following format:

1. Introduction and context
2. The learners' levels of geometry thinking
3. Curriculum guidelines from the NCS
4. Instructional practices

4.1 *Curriculum*
- *Time allocated to the development of geometry understanding*
- *Types of geometry experiences*

4.2 *Language of instruction*

4.3 *Cognitive dialogue*

4.2 GRADE 0 TEACHING AND LEARNING OF GEOMETRY

4.2.1 GRADE 0 INTRODUCTION AND CONTEXT

The average number of years teaching experience for the Grade 0 teachers is 10 years. Both of these teachers have taught mathematics at two different grade levels for five years before the commencement of this study. Both of the Grade 0 teachers said that they felt capable of teaching at this grade level and both regarded geometry as an important part of a learners' education saying that geometry should not be omitted from the curriculum. The following excerpts from their questionnaires indicate a positive disposition toward teaching mathematics:

> *I just want to give children a proper foundation because it is vital to their future – I lacked it as a child.*

I enjoy teaching it (referring to the teacher's feelings regarding teaching mathematics).

In the recent past, the teachers in the Foundation and Intermediate Phases followed the guidelines for the teaching and learning of geometry as set out in the NCS. These teachers had teaching and learning resources at their disposal but were expected to formulate their own Year Plans. In 2008 some of the teachers in the Foundation Phase expressed their insecurities concerning teaching mathematics. A new literacy programme had just been introduced at the beginning of 2008 that clearly defined the progression of skills to be taught, and prescribed how the teacher should go about teaching the said skills. In response to the implementation of the literacy programme, teachers broached the lack of a mathematics programme with the same sort of clear-cut directions as those of the literacy programme in a Phase meeting in 2009. To support these teachers, an existing curriculum designed in the USA for Christian schools was introduced in 2010 at the Grade 0 and Grade1 level as a tool to give these teachers the guidance they wanted. The newly implemented curriculum moves away from the NCS and aligns itself with the NCTM standards. The curriculum was presented through a learner's workbook, a teachers' resource file, and a teacher's guide that outlines the lesson content and concepts. The Grade 0 and Grade 1 teachers have spent 2010 working through the curriculum and adjusting it to suit the context of this school and in their own words, the documentation that they submitted is "incomplete". The validity of the adopted curriculum is acknowledged in the fact that it was the product of the cumulative knowledge of experts in the field and operates from a paradigm shared by the management of the school and as such, the effects of this curriculum will only be evident in the coming years.

4.2.2 GRADE 0 LEVELS OF LEARNERS' GEOMETRY THINKING

The Grade 0 learners were not assessed in terms of their levels of geometry thought. The means of gaining this information proved to be very tedious and time consuming and due to existing commitments on both the Grade 0 teachers and my part, it was extremely difficult to acquire sufficient data for the results to be trustworthy. Nonetheless, the value of this study is not eroded. It is assumed that learners at this grade level without prior instruction would normally operate, either at a level of geometry thinking that precedes Van Hiele Level 1 or at Level 1 *Visualization*. Studies such as Wu (2006) would seem to confirm this assumption, furthermore the results of the learners' levels of geometry thought in Grade 1, of whom most are yet to operate on Van Hiele Level 1, rationalize this assumption. Moreover, the activities recommended in the NCS as seen in Tables 4.1 and 4.2 describe those sorts of experiences posited by researchers as most appropriate for learners working toward a Van Hiele Level 1.

4.2.3 GRADE 0 AND THE NCS

The allotted time for Mathematics/Numeracy within the Foundation Phase in terms of the National Education Act, (1996) is 35% of the 22 hours of formal teaching time per week. This works out to be 462 minutes (7.7 hours) for Grade 0. This means that the teaching and learning of numeracy should be happening for 90 minutes each day in Grade 0. The NCS recommends that 30% of the time allotted for the teaching and learning of mathematics in Grade 0 be allocated to the facilitation of spatial and geometry concepts. This means 139 minutes, (roughly 2 hours) per week is suggested by the NCS for the teaching and learning of LO3. By rough calculation, working on the assumption that each of the four terms in this school consists of ten weeks, the amount of time as per the NCS for the teaching and learning of geometry should be in the region of 20 hours per term.

Figure 6 is a graphical representation of the formal instructional time allocated to the instruction of mathematics, spatial/geometry development and other learning areas as suggested in the NCS. From Figure 6 it can be seen that almost 50% of the instructional time in Grade 0 should be spent on developing mathematical concepts including spatial and geometry understanding.

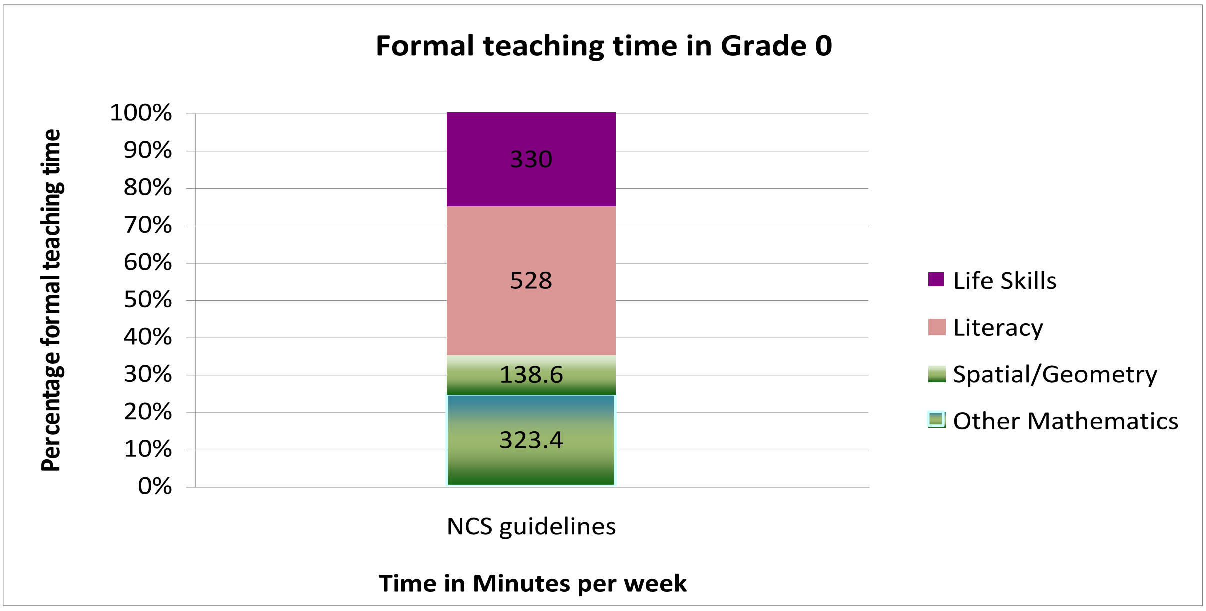


FIGURE 6: Time allocation for Learning Areas in the NCS

The focus of the NCS in the Foundation Phase in the study of space and shape is a practical experience. The learner should be given the opportunity to handle objects and be involved in cut out and drawing activities. The object at this phase is to enable learners to recognize and describe objects and shapes in their environment whilst using appropriate language and expanding their vocabulary. The NCS Assessment Standards for Grade 0 matches the activities described by Crowley (1987) as those, which would facilitate the acquisition of the first Van Hiele level, that being, Visualization. In the table below the Van Hiele type experiences advocated by Crowley (1987) for Level 1 are juxtaposed with the Assessment Standards in the NCS to show the correlation

Level 1 – Visualization

Geometry shapes are recognized on their global, visual characteristics

TABLE 2: Alignment of Van Hiele type experiences with the NCS

Van Hiele type experiences (Crowley, 1987)	Assessment Standards NCS Grade 0
1. To manipulate, colour, fold, and construct geometry shapes	1. Builds three-dimensional objects using concrete materials (e. g. building blocks)
2. To identify a shape or a geometry relation ▪ In simple drawing ▪ In a set of cut outs or other manipulatives ▪ In a variety of orientations ▪ Involving physical objects that are part of their everyday life ▪ Within and in relation to other shapes	2. Recognizes, identifies and names three-dimensional objects in the classroom and in pictures, including: ▪ boxes (prisms) ▪ balls (spheres) 3. Describes one three-dimensional object in relation to another (e. g. 'in front of' or 'behind')
3. To create shapes ▪ By using cut-outs, tracing paper, dot paper, grid paper, and geo-boards ▪ By drawing figures ▪ By constructing shapes with sticks, straws, pattern blocks etc.	4. Builds three-dimensional objects using concrete materials (e. g. building blocks)
4. To describe geometry shapes verbally using appropriate standard and non-standard language ▪ Comparing shapes e. g. these shapes have the same number of "corners" ▪ Contrasting shapes	5. Describes, sorts and compares physical three-dimensional objects according to: ▪ size ▪ objects that roll ▪ objects that slide

Van Hiele type experiences (Crowley, 1987)	Assessment Standards NCS Grade 0
5. To solve problems using shapes ▪ Investigating making squares and rectangles using triangles ▪ Covering a given area using different shapes	6. Follows directions (alone and/or as a member of a group or team) to move or place self within the classroom (e. g. 'at the front' or 'at the back').
	7. Recognizes symmetry in self and own environment (with focus on front and back)

It is evident from Table 2, that both Crowley (1987) and the NCS advocate that learners should have varied and sufficient experience with geometry shapes to allow them opportunity to become aware the relevant features of shapes to the extent that they can begin to conceptualize classes of shapes.

4.2.4 INSTRUCTIONAL PRACTICES

The results that follow are discussed in three sections namely: the curriculum, the language use in the instruction of geometry, and cognitive dialogue. Firstly, the *Extrinsic/ Intrinsic property* of the Van Hiele model, which states that the thinking results of the preceding level become the thinking subject for the next, is addressed in the curriculum followed by the teachers. Embedded in this curriculum is the time allocated for the development of geometry understanding and the types of geometry experiences (Crowley, 1987) facilitated by the teachers. Both the time and type of activity correspond to the *Advancement property* of the Van Hiele model. Secondly, the language used by the teachers as they facilitate geometry lessons correlates to the *Linguistics property*, and finally the property of *Mismatch* is addressed by considering the evidence of cognitive dialogue that describes the teacher engaging the learners at their level of understanding.

4.2.4.1 Curriculum

▪ **Time allocated to the development of geometry understanding**

Mathematics lessons of 30 minutes apiece are scheduled in the timetable at the discretion of the individual teachers. The class timetables from each Grade 0 teacher were collected and the time set aside for mathematics lessons is presented here. It is virtually impossible to verify the exact amount of time spent doing mathematics/ numeracy in the classes since the lessons may vary depending on the progression of a theme or the intrusion of an event such as an outing or a whole school activity. However, the figures used in this section represent the time intended by the class teacher for mathematics/ numeracy instruction.

According to the timetables submitted by both Grade 0 teachers, an average of 370 minutes (6.1 hours) per week is spent on the teaching of mathematics. This falls short of the 462 minutes (7.7 hours) recommended by the NCS. This discrepancy of 92 minutes (roughly 1.5 hours) per week may be compensated by the smaller class size and the presence of a teacher's assistant. Projected over 40 weeks the difference of 60 hours in a school year is a substantial amount of time in terms of opportunities to facilitate learning and therefore contributes a significant discrepancy between the recommended and the implemented curriculum.

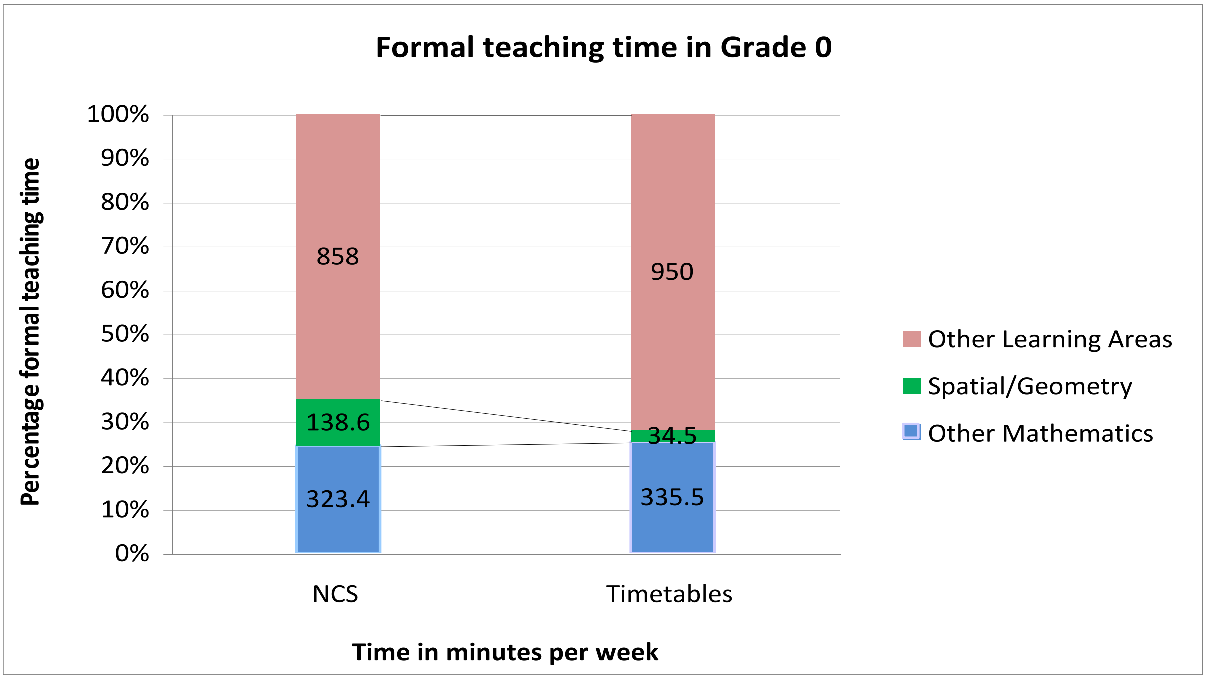


FIGURE 7: Comparison of the recommended and the implemented time allocation for mathematics and geometry concepts

The NCS recommends that 30% of the time allocated for the teaching of mathematics is used to facilitate the development of spatial and geometry understanding. As shown (in Section 4.2.3, Figure 6) this works out to approximately 2 hours per week, a projected 80 hours of spatial and geometry facilitation for the school year. Using the Grade 0 teachers' Work Schedule as a reference, the number of lessons allocated for the teaching of spatial and geometry concepts amounts to 23 lessons. Assuming that the lesson time remains consistent, this means that only 23 hours as opposed to the 80 recommended by the NCS is set aside for the development of spatial and geometry reasoning. This is a significant disparity. Calculated on the time for mathematics teaching scheduled in the teachers' own timetables 30% (370 minutes or 6.1 hours) amounts to 1 hour and 50 minutes per week that should be used for the facilitation of geometry concepts. Once again projected over 40 weeks the time spent on geometry should fall in the region of 73 hours. It is clear that even in the new programme being currently implemented, that not enough time is being afforded to the learners to develop their spatial and geometry understanding.

- **Types of experiences for the development of geometry understanding**

A new more structured American based curriculum was introduced in Grade 0 at the beginning of 2010. As a result, the teachers briefly listed the proposed topic to be covered each week of each term as their Year Plan or Work Schedule. Of the 141 lessons planned for numeracy in the Grade 0 year only 23 of these deal with spatial and geometry reasoning. Of these 23 lessons, only three focus on plane figures. Concepts establishing spatial reasoning are planned to take place in the first 3 weeks of the first term (excluding the very first week of school). Investigating solids and plane shapes happens in the last 3 weeks of the second term.

In the very first week of the new school year of 2010, the Grade 0 learners spend their days being acquainted with the culture of the school and a new routine. The second week begins their formal lessons. The mathematics planned for the first three weeks covers the skills of classifying and sorting. In four lessons in the first week of formal lessons the learners are given the opportunity to develop their spatial reasoning through activities that focus on the idea of top, middle, bottom, before, after, between, above, below, left and right.

According to Battista (2007), spatial reasoning forms the foundation of geometry reasoning and therefore, it can be assumed that establishing a good foundation in the above-mentioned concepts is important for further development in understanding the relationships between the properties of shapes. Knowing what '*between'* means would help the learners to 'see' the angle that is formed between two lines for example and without knowing the meaning of '*middle'*, learners could not understand that the diagonals of a kite bisect the angles. These concepts are central to learners developing an understanding of the space in which they live. They are also relative to the context in which they are being used and this 'relativity' should be part of the instructional programme. In essence, the contextual vehicles used to facilitate these spatial concepts of between, middle, etc. should be relevant to the learners but also allow the learners to progress through the levels of abstraction.

The four lessons in the second week of the first term in Grade 0 form the foundation of spatial reasoning for the learners. On closer examination of the lesson content and activities copied from the teachers' guide, the lessons did not seem to follow the phases of learning described by Crowley (1987). Each concept was introduced through activities, which could only be described as falling into the D*irected Orientation Phase* (Phase 2) of the five phases of learning through which the learners must progress in order to acquire a new level of geometry thought. This means that the learners were presented with the idea, albeit through an activity and discussion, and were not engaged in the *Inquiry Phase*, which not only considers the learners' prior knowledge but also prepares the way for cognitive dialogue. Although the Lesson Plans are sound in that they involve the learners in activities that use physical objects, centre on discussion, and use various contexts to which the learners can relate, they do not encourage the learners to take ownership because the learners themselves have not been considered. This is a tentative general statement based on the Lesson Plans analysed. Not all of these lessons were observed and the teachers may have presented the lesson in a different way to what was presented in the guide. There is also no evidence in the plans of the fourth and fifth phases *(Free Orientation and Integration)* being included in the instructional process.

Lessons six through eleven make use of different activities to facilitate the learners' understanding of shape, size and colour as attributes of objects and consolidate the concepts of position from the previous lessons. These lessons seem to follow the same pattern as the previous ones, omitting the I*nquiry Phase* and, as in the other preceding lessons; the last two phases are absent. The twelve lessons in the second term dealt more specifically with geometry understanding and were presented in a similar way.

The following quotes from the teachers' questionnaire show that the teachers in this grade felt that to use concrete objects and physical models was the best way to teach spatial and geometry concepts.

> *Practically – looking at shapes around us and then classifying them*
>
> *Let learners explore the shapes (use concrete manipulatives) and teach the correct Maths terms and language.*

They both also said that they felt that they had adequate support from the school and easy access to Math-models and manipulatives.

> *I use manipulatives often- every concept is first introduced using concrete manipulatives and then it is written down.*
>
> *Yes, we have easy access. I use manipulatives often (at times there are not enough or some classes keep the manipulatives too long). The children enjoy using them and they are very beneficial to our teaching.*
>
> *Yes, I do feel we get enough support but we need to decide on a particular programme and everyone must do it well and do it properly so that we are working together towards a common goal.*
>
> *I would appreciate more workshops or ideas/games when teaching different concepts.*

4.2.4.2 Language and level of instruction

It is of course impossible for any curriculum to prescribe or predict the precise dialogue most suited for a particular learner or context to best facilitate the development of abstract concepts and therefore such documents can only offer guidelines. There is thus incredible scope for further research into how teachers interpret Mathematics curricula and how to produce a document that guides and encourages teachers to consider such notions as cognitive dialogue, the five phases of learning described by Crowley (1987) and a socio-constructive approach. A vignette of one Grade 0 class observation is given below as an example of the activities and language used by the teacher during a Mathematics lesson to develop geometry thinking.

A vignette of Grade 0 Annie

The teaching and learning of Mathematics in this class is facilitated through differentiated group work. The class of 24 learners is arranged into three groups according to their perceived Mathematical ability. Two of these groups were busy with a worksheet activity with the teacher’s assistant as I arrived, while the third group was sitting on the floor in the front of the classroom with their teacher ready to begin the lesson.

The teacher used familiar objects from the learners’ home or school environment and questioned the learners about what shapes they could see in the object. The teacher then used a flashlight to cast a shadow of the object onto a piece of white paper on the floor in front of the learners. There was discussion about the shapes of the shadows that were cast from the various objects. The teacher encouraged the learners to use the correct names of the shapes and objects for example, *"sphere"* instead of *"ball"*.

The teacher then produced a set of prepared polyhedral made from plasticine, which were briefly discussed and named, and then she invited various learners from the group to cut the objects in half to *"see what shape it makes?"* This part of the lesson proved to be a little tricky as the plasticine models tended to change shape with the pressure and excitement of the little hands awkwardly wielding a plastic knife to cut them in half. This activity produced irregular shapes to which one learner responded that it was a "*square thingy".* The teacher coped with this new challenge by continuing the group's discussion in terms of the various characteristics of the shapes produced. The following excerpt is an example:

> *Learner: That's not a triangle thingy but a triangle*
>
> *Teacher: Hmmm… a triangle has a point at the top, same as a cone*

The teacher demonstrated a shape printing activity that the learners were to complete at their own desks and the group session ended. The teacher repeated the entire procedure with the other two groups before the lesson ended. The duration of the lesson was approximately 50 minutes.

4.2.4.3 Cognitive dialogue

To explore the notion of cognitive dialogue and to investigate the existence of *Mismatch* (Crowley, 1987), as discussed in 2. 5, the teachers were asked two questions. The first was if they always understood, what their learners were thinking or asking. The purpose of the question was to illicit data that showed either the teacher and learners share a language that facilitates understanding or they are mismatched in their communication. The second question was used to gauge the perceived value of establishing what the learners' level of understanding was and then using this knowledge to inform their own instructional practices. The teachers were asked how helpful it is to understand their learners. In response to these questions, both of the Grade 0 teachers regarded the ability to understand their learners as very important and essential to developing their learners' understanding, however, one of these teachers responded that she did "*not always*" understand her learners' thinking.

> *It is essential to understand where they are at, to answer them correctly.*
>
> *It is wonderful to understand my learners because then they can express where they need help and problems can be sorted out.*

4.2.5 GRADE 0 OVERVIEW

From the Grade 0 Year Plans and Lesson Plans, there is an inadequate amount of time allocated to spatial and geometry concepts as recommended by the NCS. In light of Battista's (2007) description of the progression of abstraction and that research has found the development of geometry concepts to develop sometimes over an extended period of time (Gutiérrez *et al*., 1991), it appears that insufficient time is allocated to the development of spatial and geometry concepts. Furthermore, the geometry experiences planned for the learners do not accommodate all of the five learning phases described by Crowley (1987) as instrumental in enabling the learners to progress from one Van Hiele level of thinking to the next. The importance of these phases of learning, lies in the fact that they provide a platform for each of the properties of the Van Hiele model to be negotiated by the learner. For example, Phase 1 attracts the learners' attention to the thinking subject of a Van Hiele level and activities in the *Directed Orientation* and *Explication* phases assist the learners in establishing relationships between the characteristics or properties or relationships of shapes, depending on which Van Hiele level they are operating, which is the thinking result of each Van Hiele level. The last two phases of learning are instrumental in helping the learners to accomplish what the Van Hieles refer to as having ownership and developing insight through activities that make relationships explicit and informing the learner of how the new knowledge relates to and can be appropriately applied to new contexts. The activities planned for the Grade 0 learners were introduced in the second phase of learning (*Directed Orientation),* meaning that their focus was on establishing some sort of relationship among the shapes they were shown. This implies a presumption on the part of the teachers that the learners have already been able to recognize certain characteristics of the shape with which they were presented. Consequently, the instructional practices at this grade can be described as at a Van Hiele Level 1. The properties that the learning phases accommodate, in particular the property of *Fixed Sequence, posits that progression to the next level* is dependent on the mastery of the previous level, are now magnified. Since more than half the learners are yet to operate at Van Hiele Level 1, and the language used by the teachers presumes an understanding that is not reached by all the learners, the expectation for learners to progress to the next Van Hiele level is misplaced and describes the property of *Mismatch* (Crowley, 1987; Usiskin, 1982).

Consistently evident in the Grade 0 planning and as observed in the class visits, is the use of concrete objects and models to facilitate geometry conceptual development. The use of manipulatives in conjunction with group work evident in this grade is indicative of sound teaching practices advocated by research (Arzarello *et al.*, 2005; Bransford, Brown & Cocking, 2000; Van de Walle, 2007).

It does appear that the Grade 0 teachers value a constructivist approach to facilitating conceptual development in that they use relevant and appropriate concrete examples to grow their learners' understanding. In addition, as evident in the vignette, the use of differentiated groups indicates that the teacher is attempting to teach within the learners' level of thinking and creating more opportunity to be able to engage in cognitive dialogue by working with smaller groups of learners at a time.

4.3 GRADE 1 TEACHING AND LEARNING OF GEOMETRY

4.3.1 INTRODUCTION AND CONTEXT

The combined teaching experience of both Grade 1 teachers is twenty-six years. Both teachers have been employed at the school for approximately four years and both have more than eight years of experience in the teaching of Mathematics. The following excerpts from the teachers' questionnaire seem to indicate that both teachers have a positive attitude towards teaching Mathematics:

> *Willing and able, enjoyment, motivated, enthusiastic*
>
> *I always really loved Math as a child – love the logic*
>
> *Enjoy it, but don't know if I'm always doing it the way I should.*
>
> One of the Grade 1 teachers defined geometry as *"Pictures of the world around us brought down to the basics".*

4.3.2 LEARNERS' LEVELS OF GEOMETRY THINKING

The Van Hiele geometry test developed by Usiskin (1982) was used to establish the predominant levels of thinking in each grade as discussed in Section 3.6.2. The questions were read aloud to the learners in the Foundation Phase where it was felt that the learners' reading ability or the Mathematical symbolic language used in the test may mask the geometry thinking of these learners. The question regarding the reliability of the results is discussed in Section 3.7.

The learners completed the assessment towards the end of June 2010 after six months of formal teaching in Grade 1. More than ninety per cent of theses learners were promoted from the Grade 0 classes in the same school. Only two of the 47 learners entered the school in 2010. Whilst twenty-two of the 47 learners in Grade 1operated on the first Van Hiele level, twenty-five of them are yet to operate on the *Visualisation* level. These results reflect that in Grade 1 at least two Van Hiele levels of thinking are evident. By implication, instructional practices to develop geometry understanding should encompass the range of learners' levels in that grade. Not only should the activities for Level 1 be continued, but activities that will enable learners to progress to Van Hiele Level 2 should also be occurring during the Grade 1 year. The Grade 1 Year Plans should include activities that are geared towards helping the learner understand the properties of shapes and that these properties can either include or exclude a shape from a certain family of shapes. During the Grade 1 year, learners should also be encouraged to use the correct names of shapes and learn that a shape can belong to more than one family e.g. a square is a rectangle as well as a quadrilateral. A more detailed description of these types of activities can be found in Section 2.6.1.

Figure 8 presents the Grade 1 results as they relate to the first five questions in the Van Hiele test.

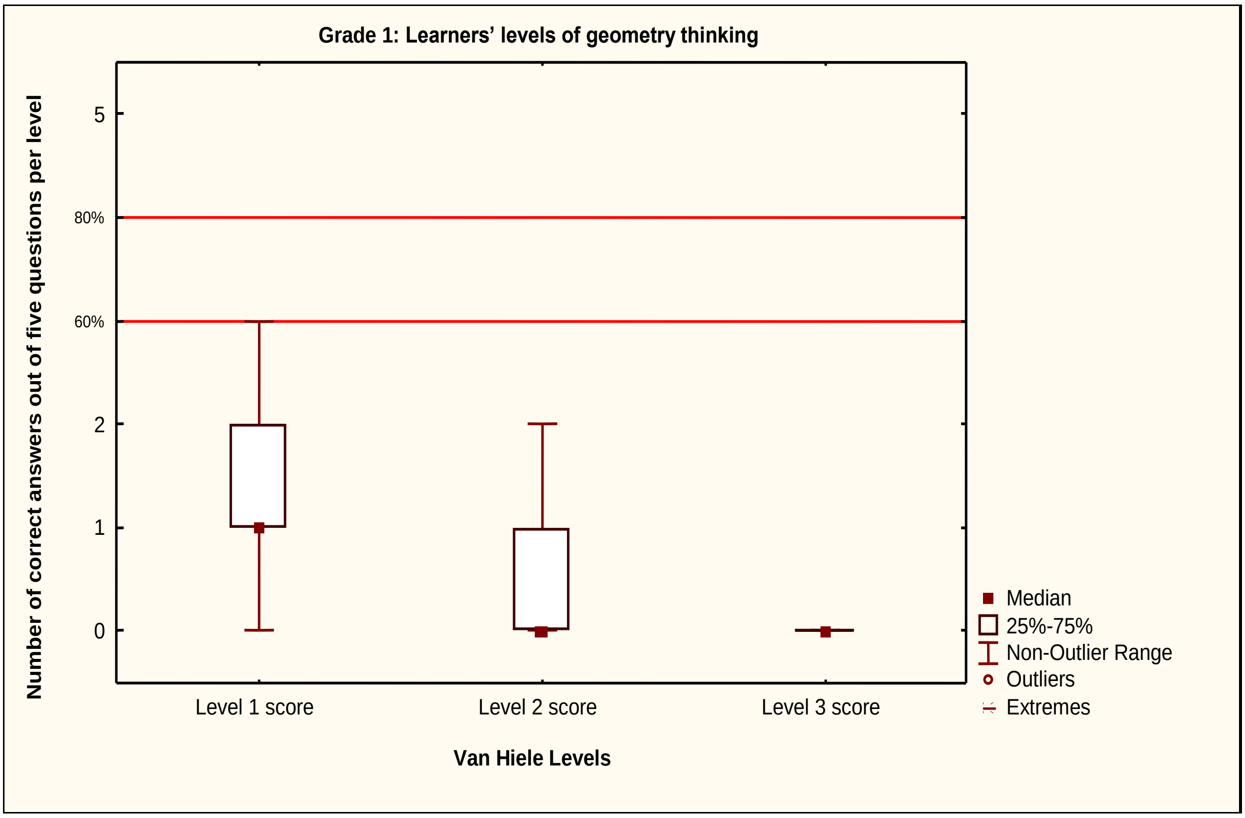


FIGURE 8: Levels of geometry thinking in Grade 1

4.3.3 GRADE 1 AND THE NCS

The time allocation for the facilitation of spatial and geometry concepts in Grade 1 as recommended by the NCS are the same as for Grade 0 (Section 4.2.3). In addition, the teaching and learning of spatial and geometry concepts remain focused on providing the learners with practical hands-on experiences.

The Assessment Standards for Grade 1 are an extension of those for Grade 0, which thus builds on the learners' conceptual understanding developed the preceding year. Once again, these Assessment Standards suggest that the learners are to be involved in activities described by Crowley (1987) for Van Hiele Level 1, to enable the learners to progress to the next Van Hiele level. The Grade 1 Assessment Standards in italics are shown as a continuation of Grade 0 in the following table:

TABLE 3: Van Hiele type experiences and the NCS

Van Hiele type experiences (Crowley, 1987)	**Assessment Standards NCS** Grade 0 *(Grade 1)*
1. To manipulate, colour, fold, and construct geometry shapes	1. *Observes and* builds three-dimensional objects using concrete materials (e. g. building blocks *and construction sets*)
2. To identify a shape or a geometry relation: ▪ in simple drawing ▪ in a set of cut outs or other manipulatives ▪ in a variety of orientations ▪ involving physical objects that are part of their everyday life ▪ within and in relation to other shapes	2. Recognizes, identifies and names *two-dimensional and* three-dimensional objects in the classroom and in pictures, including: ▪ boxes (prisms) ▪ balls (spheres) ▪ *triangles and rectangles* ▪ *circles* 3. Describes one three-dimensional object in relation to another (e. g. 'in front of' or 'behind')
3. To create shapes ▪ by sing cut-outs, tracing paper, dot paper, grid paper, and geo-boards ▪ by drawing figures ▪ by constructing shapes with sticks, straws, pattern blocks etc.	4. *Observes and* builds three-dimensional objects using concrete materials (e. g. building blocks *and construction sets*)

Van Hiele type experiences (Crowley, 1987)	**Assessment Standards NCS** Grade 0 *(Grade 1)*
4. To describe geometry shapes verbally using appropriate standard and non-standard language: ▪ comparing shapes e.g. these shapes have the same number of "corners" ▪ contrasting shapes	5. Describes, sorts and compares physical *two-dimensional and* three-dimensional objects according to: ▪ size ▪ objects that roll ▪ objects that slide ▪ *shapes that have straight or round edges*
5. To solve problems using shapes: ▪ by investigating making squares and rectangles using triangles ▪ by covering a given area using different shapes	6. Follows directions (alone and/or as a member of a group or team) to move or place self within the classroom *or three-dimensional objects in relation to each other.*
	7. Recognizes symmetry in self and own environment (with focus on 'front', 'back' *and 'left' and 'right'*)

Maintaining a focus on practical activities and extending the curriculum only slightly in the Grade 1 year, implies a need for the learners to be involved in many different hands-on tasks that would facilitate a progression through the levels of abstraction described by Battista (2000) towards a level of thinking akin to Van Hiele Level 1.

4.3.4 INSTRUCTIONAL PRACTICES

4.3.4.1 Curriculum

- **Time allocated to the development of geometry understanding**

According to the timetables submitted by both Grade 1 teachers, an average of 255 minutes (4.25 hours) per week is spent on the teaching of Mathematics. This falls short of the 462 minutes (7.7 hours) recommended by the NCS by 207 minutes (roughly 3.5 hours) per week. Projected over 40 weeks the difference of 138 hours in a school year is substantial. Working from the NCS recommended time of 7.7 hours in a week, a deficit of 138 hours calculates to a staggering 18-weeks' worth of Mathematics' lessons not being accommodated in the Grade 1 school year. This fact begs the question whether the argument presented in the Grade 0 case of smaller class size and the presence of a teacher's assistant as compensation for time on Mathematics remains justified in this case.

In order to establish the approximate amount of time the Grade 1 teachers planned to facilitate the development of spatial/geometry concepts, the number of lessons, which included relevant topics in their Year Plan, was counted. The activities to develop spatial and geometry reasoning appeared along with other concepts and content in four weeks out of the 26 weeks that make up the first three terms. This was used as a reference to project the possible amount of time spent facilitating spatial and geometry understanding for the year. In other words, four out of twenty-six weeks calculates to 0.1538 or 15%. It was then assumed that the teachers in this grade would spend 15% of their Mathematics/numeracy lessons developing geometry understanding. These findings are juxtaposed alongside the recommendations of the NCS and presented in Figure 9.

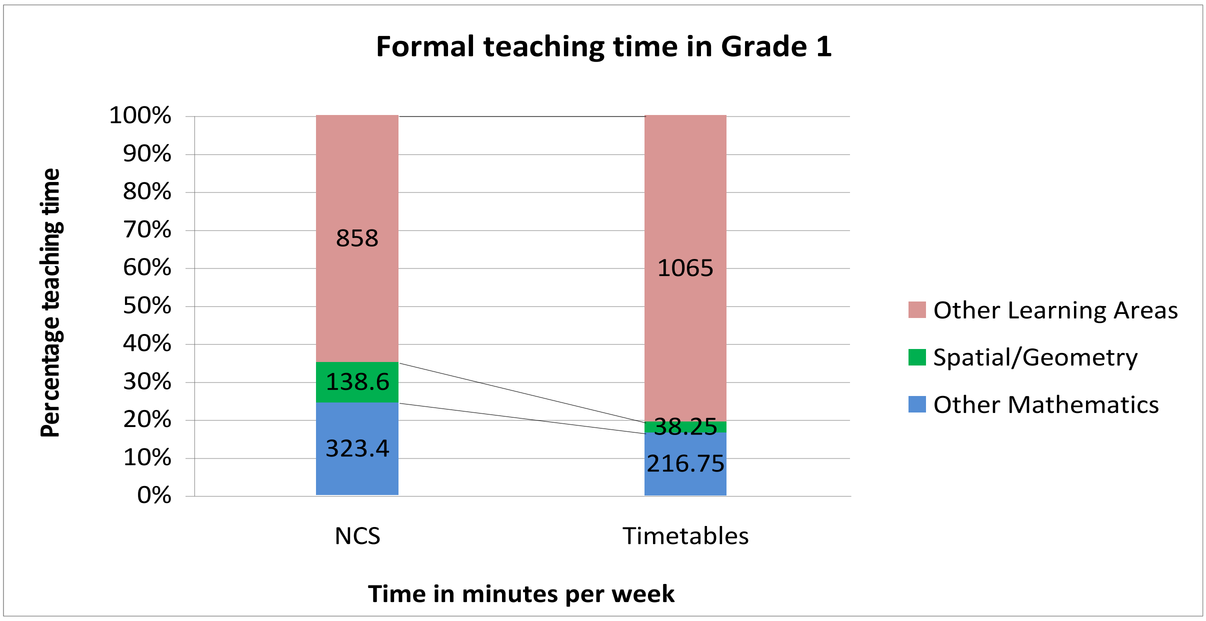


FIGURE 9: Comparison of the recommended and the implemented time allocation for Mathematics and geometry concepts

- **Types of experiences for the development of geometry understanding**

As discussed (Section 4.2.1), the Grade 1 teachers, like the teachers in Grade 0, are in the process of adapting an American-based numeracy programme to fit the NCS requirements and the school context. The Grade 1 Year Plan focuses on the content of each Learning Outcome that needs to be covered within the Grade 1 year as set out by the National Curriculum Statement (NCS) 2002. A checklist of the Learning Outcomes and the relevant assessment standards is preceded by a plan that lists the type of activity planned; the resources that will be used or the content to be covered. The checklist is followed by a weekly plan of when each assessment standard will be facilitated; however, there is no indication as to how many lessons during the week are set aside for any particular concept. Therefore, there is no reliable means to calculate how much time is dedicated to the development of spatial and geometry reasoning. There is also no evidence in this planning of the language or vocabulary that will be used during those lessons allocated to Learning Outcome 3. Copies from the teachers' guide of the new curriculum were submitted as the Grade 1 Lesson Plans.

One week in the first term (that being the second week) is allocated for teaching patterns (Learning Outcome 2) and 2-D shapes- shapes on my body (Learning Outcome 3). According to the checklist the learners were given the opportunity to "recognize, identify and name two-dimensional shapes and three-dimensional objects in the classroom and pictures, including: boxes and balls, triangles and rectangles, and circles" (ibid, NCS, 2003). Building three-dimensional objects and the concept of '5^{th}' were also covered in this week. A basic introduction to symmetry using the theme of insects as a vehicle to relate the concept to the learners' life experience is the topic of the last week of the second term. This week was also scheduled for revision. The assessment standards for term two include: "recognizes symmetry in self and own environment (with focus on 'left', 'right', 'front' and 'back'); describes one three-dimensional object in relation to another (here the checklist uses ordinals instead of 'in front of' or 'behind' as stipulated in the NCS); and follows directions (alone and/or as a member of a group or team) to move or place self within the classroom or three-dimensional objects in relation to each other" (NCS, 2003).

Although listed under the heading of 'measurement' (Learning Outcome 4), it was planned that the learners build a prism and a box/cube and construct a picture of a bird using two-dimensional shapes in the third term. These activities were planned for the last two weeks of the third term along with sharing/grouping, measurement and assessment. All the assessment standards as stated in the NCS for Learning Outcome 3 were checked by the close of the third term, however, the Grade 1 teachers are in the process of assimilating a new numeracy curriculum and the assessment standards set by the NCS and in delivering their planning emphasized that it was a work in progress. An additional checklist for the fourth term featured arrows next to 'symmetry', 'position' and 'following directions' presumably an indication that these areas will be revisited.

From the data collected through the teachers' questionnaires, these teachers professed that the best way to teach and learn geometry is:

by discovery and by being engaged in the learning process and
by getting practical.

Both acknowledged the value of using manipulatives and one teacher described the use of concrete aids as being able to:

show the picture of what you are saying and
help link the idea/concept to the understanding/meaning.

4.3.4.2 Language of instruction

A vignette of Grade 1 Busi

The lesson started with a whole class discussion about the three different cut out shapes the teacher was holding. The teacher referred to a skill of which the learners had previously made use. She asked the learners if they could remember what it meant to *compare* things. There was consensus that it meant to look at '*what was the same'* and *'what was different'*. The teacher held brightly coloured cut outs of a triangle, a circle and a rectangle. She did not name the shapes but merely asked the learners to compare them. The learners offered comments, which included the number of sides and the number of corners. The teacher paid careful attention to what each learner offered making sure that they all agreed with what was said. Although this was a whole class discussion, the teacher made a sincere and consistent effort to engage in cognitive dialogue with the learners in her class. I include the following exerts to substantiate this statement:

Teacher: *When you talking about sides, do you mean these things here?* (Teacher indicated the edge of the shape with her finger).
Teacher: *Why don't you think a circle has any sides?*
Learner: *A circle has no sides because it is not straight*.
Teacher: *So does a circle have sides? Yes, a circle has one side but it is curved.*
Teacher: *What are these?* (Teacher touched the three angles of the triangle)
Learner: *Corner is sharp pointy*.
Teacher: *What happens at a corner?*
Learner: *Two sides join!*
Teacher: *Can a circle have a corner if it means where two sides join?*
Teacher: *Face is the flat side* (Learner looks unhappy)
Teacher to unhappy learner: *Why don't you like that, don't you understand?*

Learner: *I don't know why it is the face*

Teacher: *What is confusing you?*

Learner: *There are no eyes or nose*

The teacher then produced a model of a cylinder, a cuboid and a triangular prism and repeated the discussion. This was followed by a demonstration on the interactive whiteboard to remind the learners how to draw a table. The learners were then set a task where they had to compare the two-dimensional shape with its corresponding three-dimensional object and record their findings on their table. The learners were divided into three different groups, it is unclear whether these groups were mixed ability or of similar ability, and each group were given concrete models to work with. Each of the three groups was given a different shape and prism. The learners moved quickly into their groups and began working together without too much fuss, which seems to suggest that working together is a regular occurrence in this class. The learners explained their answers to other members of the group when there seemed to be a difference of opinion about the number of edges or the number of corners. The teacher and teacher's assistant moved around the class providing a verbal prompt where needed. While the learners were still sitting in their groups with the models in front of them, the teacher drew their attention and asked a volunteer to report their groups' findings. This was unfortunately interrupted by the schools intercom system announcing housekeeping trivia and the commencement of break.

4.3.4.3 Cognitive dialogue

One of the Grade 1 teachers stated that she has been able to understand what her learners were thinking at the grade levels she has taught in, but was not sure she would if she taught in the *"upper grades".* The following quote from the other Grade 1 teacher indicates an intention to engage in cognitive dialogue with her learners and suggests that this occurs habitually in her classroom:

> *I try to, but obviously don't always. I usually ask many questions to determine where they are.*

4.3.5 GRADE 1 OVERVIEW

Although it is impossible to analyse with certainty the types of geometry experiences planned for the Grade 1 learners due to the incomplete documentation submitted by the teachers, there is certainty about the insufficient time allocated to the facilitation of Mathematics concepts. The timetables submitted by the Grade 1 teachers, show an inadequate time allocation to Mathematics per week and this would in no doubt impact on the teaching and learning of geometry.

The learners in Grade 1 operated on a *Pre-visualisation* and a *Visualisation* level according to the Van Hiele model. This means that the learners require multiple and diverse activities described in table 4.2 to enable them to be able to progress through the levels of abstraction (Battista, 2007) and hence identify the characteristics of certain shapes for them to begin to conceptualise classes of shapes. In other words, a learner needs to be able to identify what sides and corners (angles) are before they can sort shapes into those with three sides and those with four. The teaching of spatial and geometry understanding at this grade level presumes a level of understanding which not all the learners are operating at. For example in linking the two-dimensional shapes with three-dimensional objects the teachers presume that the learners are able to identify a triangle and a square in any orientation, which is indicative of Van Hiele's Level 1 (*Visualisation*) thinking. This is another example of *Mismatch* (Section 2.5).

Both of the Grade 1 teachers profess to value the use of Math manipulatives and both said that they believe getting the learners involved in the learning process is the best way to learn geometry. The classroom observation analysed in this chapter shows a deliberate intention of the teacher creating the opportunity for cognitive dialogue. The teachers also made use of examples of shapes and objects that were familiar to the learners. Using words like corner/edge and clarifying the meaning of them indicates that these teachers are trying to work within the learners' level of understanding. However, since the learners' level of geometry thinking is not being addressed, the learners are prevented from having ownership of the material and developing insight as described by the Van Hieles (Pegg & Davey, 1998). The frequent use of group work seems to indicate that these teachers incorporate an element of socio-constructivism in their teaching approach.

4.4 GRADE 2 TEACHING AND LEARNING OF GEOMETRY

4.4.1 GRADE 2 INTRODUCTION AND CONTEXT

Both Grade 2 teachers have experience at two grade levels and have a combined teaching experience of 33 years. They also have been employed in the school for more than four years.

In the questionnaire, they both responded that they felt relatively confident in teaching Mathematics and that the study of geometry is very valuable as indicated in these quotes:

> *No, I don't think (geometry) it should be optional because it is essential for understanding the world around us. It also requires complex, abstract thinking, which is important.*
>
> *I feel that I teach my Grade 2s quite well with many different strategies but I still think that I spoon feed them too much rather than making them think which is important.*

These teachers described geometry as two-dimensional and three-dimensional shapes, the measurement of corners/sides, and the relationship between lines, surfaces, apex and angles adding that they consider using manipulatives and group work as the best approach to teaching and learning geometry. Their responses follow:

> *With apparatus.*
>
> *All research says that the best learning happens when you think it through yourself and are able to teach others. Therefore, group work where one group teaches another group would be great.*

Both Grade 2 teachers professed to see the educational value in using manipulatives and made use of them in their classes.

4.4.2 GRADE 2 LEVELS OF LEARNERS' GEOMETRY THINKING

Currently there are 49 learners in Grade 2. Twenty-three of the learners were unable to recognize a square when presented in a different orientation or identify a triangle accurately or distinguish a square from a rectangle. This is as indication that they are not yet functioning on the Visualization level where a figure is identified by what it looks like. Twenty-six of the forty-nine learners in Grade 2 were able to do this and therefore seem to operate at Van Hiele Level 1.

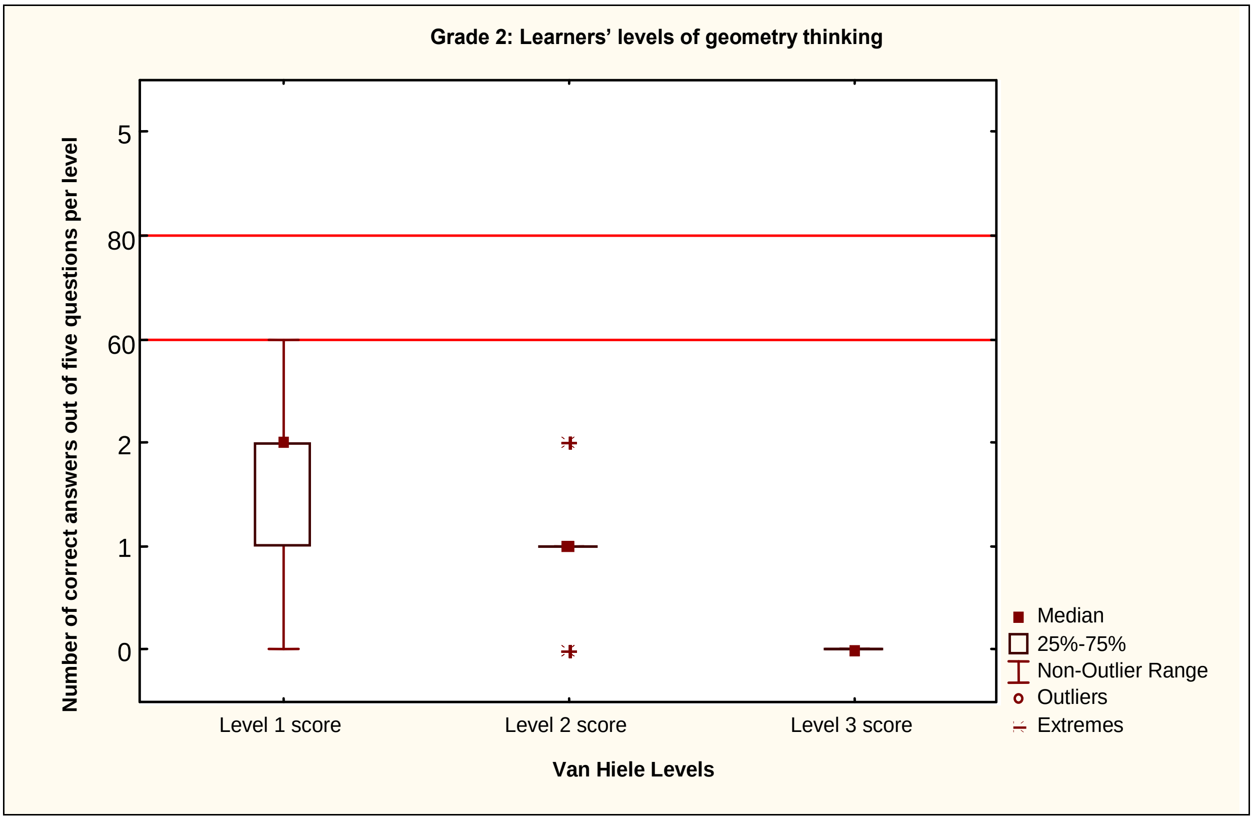


FIGURE 10: Learners' levels of geometry thinking

4.4.3 GRADE 2 AND THE NCS

The time allocation and the focus of the NCS concerning the teaching and learning of geometry concepts in Grade 2, remains the same as for Grade 0 and Grade 1. Once again, the Assessment Standards for Learning Outcome three in the NCS for the Grade 2 year are built on the foundation of the preceding two years content. A notable difference in the Grade 2 Assessment Standards is the inclusion of pictures as well as physical objects. This would encourage a move toward the abstract where pictures become representations of the abstract concept. Using pictures in conjunction with the physical objects scaffolds the learner towards greater abstraction (Section 2.4).

4.4.4 INSTRUCTIONAL PRACTICES

4.4.4.1 Curriculum

- **Time allocated to the development of geometry understanding**

The average number of hours for the teaching and learning of Mathematics in the Grade 2 year is 5. 25 hours per week. Again, this is less that the recommended time stated in the NCS.

On the assumption that there are approximately forty weeks in the school year and that Mathematics lessons happen once a day, it is possible to have 200 Mathematics lessons in the Grade 2 year. By counting the number of lessons planned for teaching geometry in the Grade 2 Year Plan and comparing that figure to the total number of Mathematics lessons in the year, the Grade 2 teachers are planning to spend six per cent of their time facilitating the development of geometry concepts. This is remarkably less than the thirty per cent recommended by the NCS. From the data received, Grade 2 learners have 12 lessons planned for the facilitation of spatial and geometry reasoning.

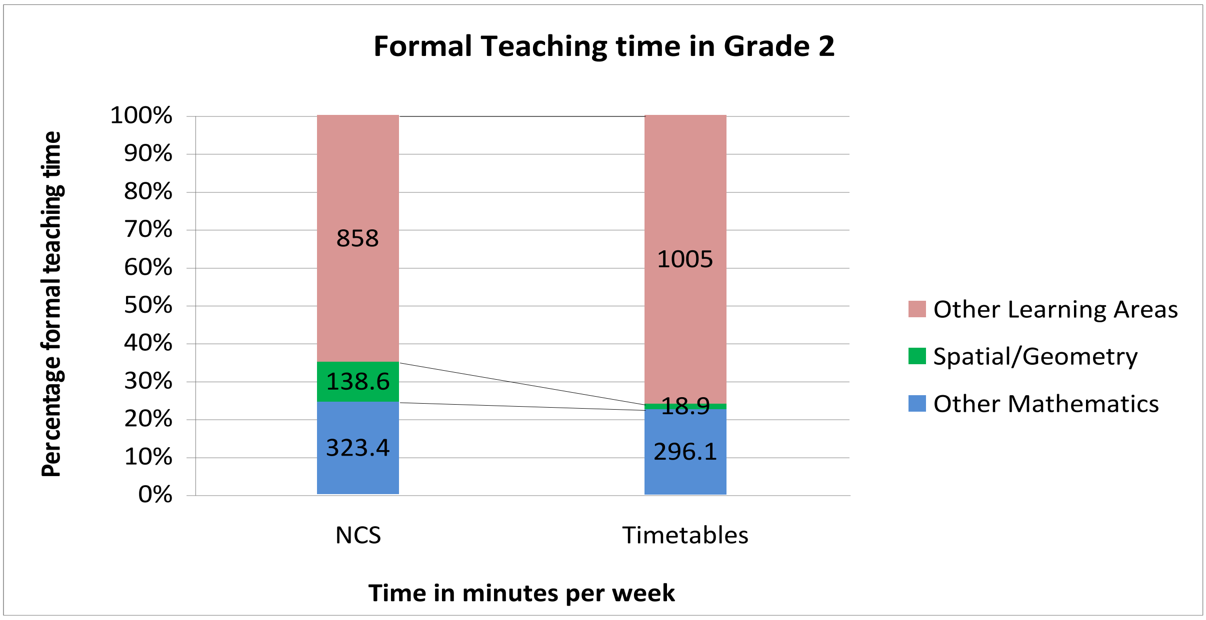


FIGURE 11: Comparison of the recommended and the implemented time allocation for Mathematics and geometry concepts

- **Types of experiences for the development of geometry understanding**

The Grade 2 Year Plan lists the topic that will be covered each week of each of the four terms. Two-dimensional and three-dimensional shapes along with the bonds of 11 are covered in weeks four and five of the first term and again in weeks four and five of the second term but with the bonds of 15. Week seven of term two is set aside to cover parallel lines, bonds of 16, and revision work. The remainder of the year is dedicated to the development of number, patterns, division and measurement.

The weekly plans for the Grade 2 year clearly set out the Assessment Standards for each lesson that week, the resources that will be used and the class learning intended for each lesson. In the planning for weeks four and five of the first term, four lessons each week are entirely set aside for the investigation of 2D shapes and 3D objects. There is clear direction as to what type of activity should take place during the lesson and these tasks involve the learners either in discussion or in some physical activity for example drawing or guessing a shape by clues given or building a shape with straws/boxes. The planning for the fourth week of the second term is as detailed. Lesson one, week four, term two starts by reviewing the previous term's learning and involves the children in an activity where a learner must guess a shape from a description given by another learner. The lesson ends with the learners having to record this activity in their workbooks. The second lesson of the same week begins with a discussion of train tracks as an introduction to parallel lines. The learners are asked which shapes of those they are shown have parallel lines and the lesson is concluded with an activity where the learners are to draw or cut out of magazines examples of parallel and not parallel. The vocabulary highlighted for the next lesson (lesson three, week four, term two) is: 'line of symmetry, fold, mirror image, reflection and symmetrical' and the learners are asked to identify the line of symmetry of various pictures on a worksheet. The last lesson of this week continues the idea of symmetry. There is no indication that the symmetry of plane figures is part of the learning planned, although this may have been an oversight in the writing of the plan – there is no way to be sure. Unfortunately, the planning for week five of term two was blank.

4.4.4.2 Language of instruction

A vignette of Grade 2 Craig

The Grade 2 lesson started with the teacher calling all the learners to bring their whiteboards and markers and join her on the carpet at the back of the classroom. She started the whole class session with a few minutes of mental Maths questions and counting exercises. The teacher then had the learners draw a picture on their whiteboards according to certain criteria. For example, she asked them to draw a house using only two shapes. The learners showed the teacher and each other what they had drawn and the teacher prompted discussion by asking what shapes could be seen in each learner's picture and how the pictures were the same or different. During this activity, the teacher would produce either a model of a regular polyhedron or a polygon and use it to prompt the next drawing task. This activity continued for a little over fifteen minutes.

Throughout the drawing activity, the teacher would use the learners' responses to draw attention to the properties of the various shapes being used. The following excerpts are from the class discussion, which happened spontaneously although in a respectful manner.

> Teacher: *What are the rules for a square?*
>
> Learner1: *It has to have four sides.*
>
> Learner 2: (Quickly added to the previous statement as the teacher tried to respond) *The sides have to be the same.*
>
> Teacher: *What are the rules for a triangle?*
>
> Learner 3: *It has to have three sides* (Showed his picture of what looked like a scalene triangle to which another learner said that a triangle has to have sides that were the same)
>
> Learner 4: *No! They don't have to be the same size, it can have two long and one short.*
>
> Teacher: *But can they all be the same size? … Yes.*
>
> Teacher: *What are the rules for a rectangle?*
>
> Learner 5*: It must be two long and two short.*

The teacher ended the class discussion by assigning different tasks to the three differentiated groups. These groups seemed to be chosen on perceived Mathematical ability. Two groups had to complete a worksheet at their desks, while the third built objects with geo-structa pieces at the back of the classroom. The worksheets required the learners to observe and record the different properties of shapes either by counting the number of sides/corners or by drawing. The lesson lasted for thirty minutes.

4.4.4.3 Cognitive dialogue

The teachers agreed that it was valuable to understand their learners' thinking in order to help their learners understand better.

> *I often understand what they are asking but I don't always know how to answer them to help them understand what they need.*

4.4.5 GRADE 2 OVERVIEW

The Grade 2 teachers only planned for approximately six per cent of their Mathematics lessons to facilitate the development of geometry concepts, which is a fifth of the time recommended by the NCS. Evident in their planning is clear direction as to what type of activity should take place during the lessons and these tasks involve the learners either in discussion or in some physical activity. For example, drawing or guessing a shape by clues given or building a shape with straws or boxes. The types of activities documented in the planning reflect four out of the five phases of learning described by Crowley (1987). The Lesson Plans in the Work Schedule indicate that geometry lessons start with a review of previous work done and an introduction to the current lesson, which falls within the characteristics of the *Inquiry/Information* phase. This is followed by activities, which align well with the second phase that Crowley calls *Directed Orientation*. The frequent use of group work and the indication of specific vocabulary to focus on in that particular lesson, suggests that Crowley's third phase of learning, *Explication*, is also included in Grade 2 geometry lessons. In the planning, the learners are often required to record what they have learnt in the lesson and this may be viewed as the final phase of learning called *Integration.* There is, however, no indication in the documents or in the classroom observations of the fourth phase of learning being accommodated. *Free Orientation*, where the learners are provided with open-ended, multi-step tasks, was not planned for and was not observed. This type of activity is precisely the kind of task that will enable learners to **think**, which one Grade 2 teacher expressed as a need in her learners. Both of the Grade 2 teachers expressed understanding of the value of manipulatives in teaching geometry and their use of concrete aids was evident in their planning and classroom observations.

Although differentiated teaching and group work appeared in their planning, it was inconsistent and unclear as to whether this allowed opportunity for cognitive dialogue and for the teachers to address the learners at their level of thinking. It seemed rather to serve as a guide as to which worksheet or level of activity the group was to do based on a perceived notion of the learner's ability. This does indicate though, that the teachers are trying to meet the learners' needs, and hence suggests a constructivist approach to teaching geometry.

The teachers gave thought to the language they were to use in the geometry lessons in that the specific vocabulary is listed, and sometimes briefly explained, in their planning. This may seem to indicate an awareness of the property of *Distinction,* where each level has its own symbols and system of relations, but since a level of understanding is presumed by presenting the curriculum at a higher Van Hiele level at which most of the learners operate, it is more indicative of *Mismatch.* From the classroom observation the vocabulary was presented to the learners with no discussion about the meanings that the learners attached to the word, hence it cannot be concluded with certainty that cognitive dialogue is a regular occurrence in the Grade 2 classrooms.

4.5 GRADE 3 TEACHING AND LEARNING OF GEOMETRY

4.5.1 GRADE 3 INTRODUCTION AND CONTEXT

The Grade 3 teachers have a considerable 34 years of combined teaching experience. Both teachers have taught at this grade level before this year and have experience at other grade levels, one of whom has taught Grades 8 – 11 in the Senior Phase. In response to the questionnaires both teachers said that they felt confident to teach Mathematics and that they viewed the study of geometry as "*valuable*" and "*imperative*" for thinking skills and a requirement for many careers. One teacher did comment that although she liked teaching Maths it was not her passion. They similarly defined geometry as the *"relationships between space and shape – spatial awareness".*

The Grade 3 teachers shared the view that the best way to teach and learn geometry was through using practical, hands-on experiences. They both affirmed their use of manipulatives often adding that they had easy access to the equipment and that their learners, as well as they themselves, enjoyed using the physical resources the school had. The teachers said that they felt that they had support from the school in terms of their Mathematics teaching. I include two excerpts to substantiate this analysis:

> (How do people learn? What is the best way to teach geometry?)
> *Continue the instinctive learning pattern started at birth i.e. learn by experiencing:*
> *DO TALK RECORD CONCRETE to ABSTRACT*
> (Access, value and use of manipulatives)
> *Yes, I love it and it is good to see how they think. They learn from each other when they pack a problem with manipulatives.*

4.5.2 GRADE 3 LEVELS OF LEARNERS' GEOMETRY THINKING

Twenty-one of the fifty Grade 3 learners do not yet operate at a Van Hiele Level 1. A little more than half the grade (29 learners) is able consistently and accurately to recognize a shape based on its physical properties. The results do indicate that there are two different levels of geometry thinking operating in this grade and hence instructional practices should span the range of learners' levels of acquisition of geometry understanding.

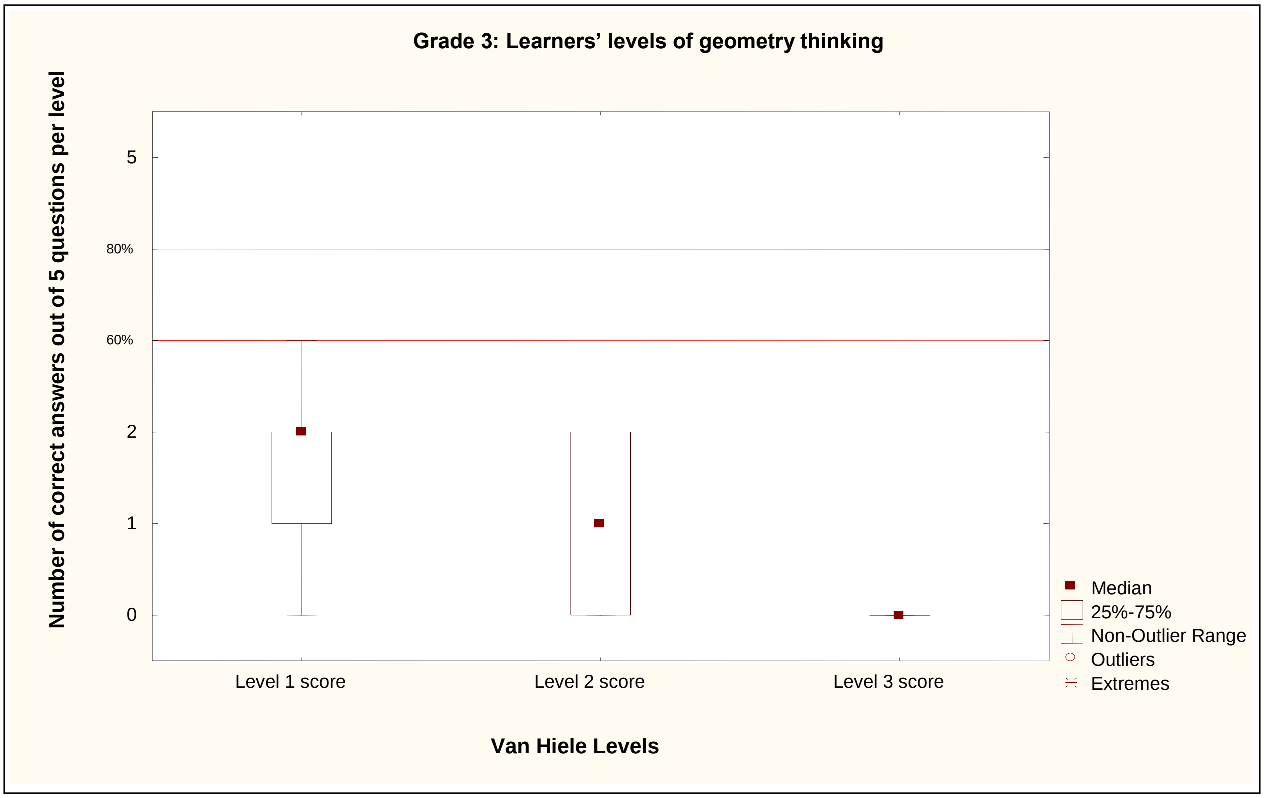


FIGURE 12: Learners' levels of geometry thinking in Grade 3

4.5.3 GRADE 3 AND THE NCS

Grade 3 is the final year of the Foundation Phase. The NCS recommends that 35% of the 25 hours per week of formal teaching time be allocated to the instruction of Mathematics. This calculates as 525 minutes (8. 75 hours) per week, which is approximately 1 hour and 45 minutes daily. Of the 105 minutes allocated per week for Numeracy, the NCS suggests that 30% of this time be used in the facilitation of spatial/geometry concepts, which roughly calculates to a single lesson a week.

The Assessment Standards listed in the NCS for Grade 3 are an extension of those listed for Grade 2 (see Table 4). The extensions reflect a logical and natural progression in line with the Van Hiele theory and other research. The activities described by Crowley *et al.* (Section 2.6.1) to enable learners to operate on the first two Van Hiele levels would provide the right kind of geometry experiences for learners to be able to achieve the Assessment Standards for Grade 3 as listed by the NCS.

4.5.4 INSTRUCTIONAL PRACTICES

4.5.4.1 Curriculum

- **Time allocated to the development of geometry understanding**

Together both Grade 3 classes spend a total of 675 minutes (11. 25 hours) per week teaching and learning Mathematics. On recommendation of the NCS, each class should plan for 525 minutes (8.75 hours) per week, which means that together both classes should have over 17 hours of Mathematics in a week compared to the 5. 625 hours per class reflected on the teachers' timetables. The argument, that smaller class size and differentiated group teaching are more effective methods of teaching and therefore compensates for time spent on Mathematics, may still hold in this case. Yet, it is threatened by the fact that the Grade 3 teachers do not have the assistance of a teachers' aide, which has been part of the argument presented in the previous grades. According to the Grade 3 Year Plan, some thirty-one lessons, feature geometry concepts. This is approximately 16% of the Mathematics lessons as opposed to the 30% recommended in the NCS.

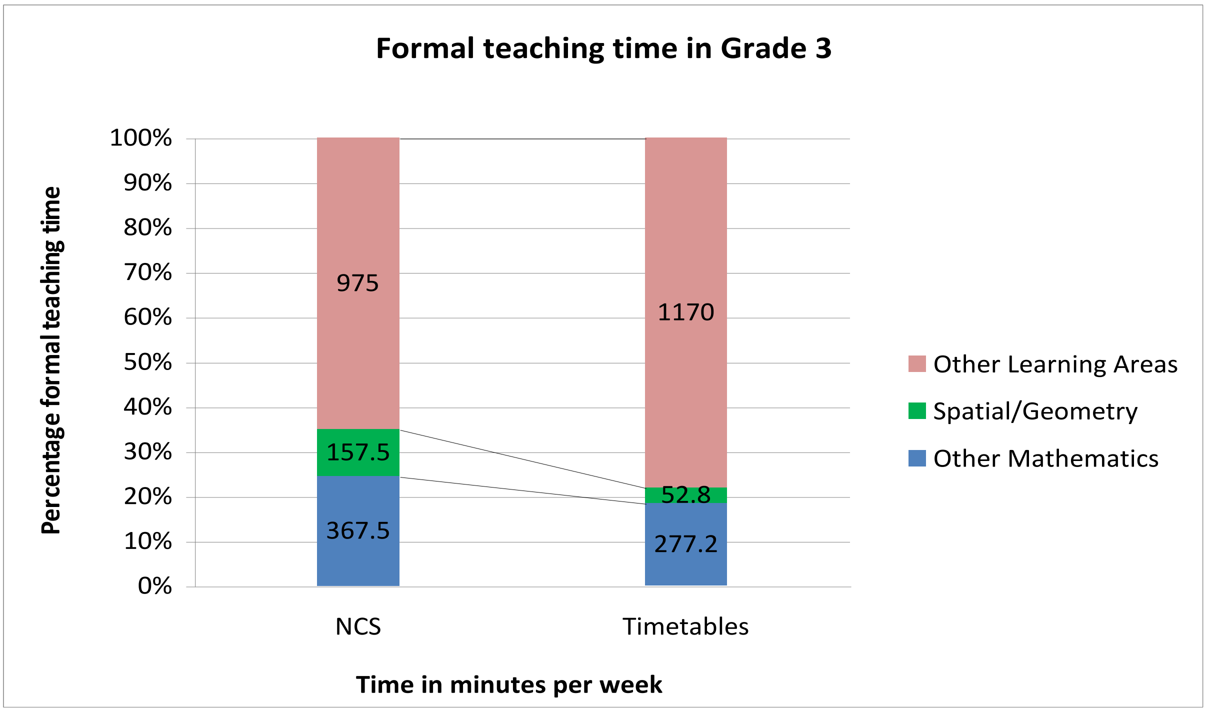


FIGURE 13: Comparison of the recommended and the implemented time allocation for Mathematics and geometry concepts

- **Types of experiences for the development of geometry understanding**

The teachers in this grade try work on a three-day cycle within a given week. It is their habit to introduce a topic or review a section as a whole class on Mondays and follow with group work on Tuesday through Thursday. The learners are sorted into three groups according to their Mathematical ability. Three different activities are planned around a concept or topic and each group gets a turn to complete the activities on a particular day. In the three-day rotation, each group would have completed all three of the activities. The three activities include a group teaching time with the teacher, a worksheet or textbook type task, and a task involving manipulatives or a Maths game. Friday's lesson is used to facilitate consolidation of a concept or skill, integration and assessment.

The Grade 3 teachers were reluctant to submit their plan for the year as they said that they were in the process of developing a new learning programme since recent changes in literacy and themes affected all areas of their teaching. They did submit the old plan along with an example of a previous year's Lesson Plan. The Grade 3 Year Plan reflects the same format as the Grade 0 and Grade 2 plans by listing the topic to be covered each week of each term. According to the plan submitted, the concept of symmetry is dealt with in the second week of the first term and revisited in the last week of the third term. Two-dimensional shapes and three-dimensional objects are covered in the third week of the first term and revisited in the eighth week of the third term when the learners do activities using the nets of three-dimensional objects. Patterns using Cuisenaire tiles are explored in week six of the first term and again in week eight of the third term when they are also used to continue the development of the concept of symmetry.

Position in space and in relation to other objects is covered in weeks six and seven of the third term using map work, grids and an eight-point compass. The Lesson Plan format is very structured drawing attention to the assessment standards, the skills, the attitudes and values, the learning style, the intelligence type and the learning activities to take place during the week. The activities to be done are listed but there is no direction in terms of how to do them or what language/vocabulary should be introduced or emphasized.

4.5.4.2 Language and level of instruction

A vignette of Grade 3 Dion

The lesson described here occurred early in the morning on a Wednesday, meaning that the class was mid-way in a rotation of activities planned for the week. The lesson started with all the learners seated on the floor around the teacher and a whiteboard. About five minutes were spent doing mental Maths and number facts. The teacher then asked the learners to review what they had learnt during the previous two days. She asked if there was any advice that any one group could give to the group about to start the tasks, which they had done the day before. This enabled the learners to reflect on *what* they had learnt and *how* they had gone about their task.

In this lesson there was no group teaching time. The three activities are recorded below.

1. Learners had to construct a three-dimensional object, in this case a house with a square base and a triangular prism for a roof with straws and Prestik. The learners were given a task sheet with the written objective and dimensions clearly typed. There was no diagram on the task sheet.
2. The learners were given a task sheet asking them to design a floor tile of given dimensions using Cuisenaire pattern blocks. They were asked to ensure that their design had at least two lines of symmetry and they had to repeat the pattern three times.
3. The learners worked together in pairs against the clock to complete a Tangram puzzle.

The learners organized themselves into their groups quickly and without much fuss and set about their tasks. The teacher moved from group to group checking on progress, prompting learners when necessary maintaining discipline. The learners seemed comfortable and used to working in groups and there was consistent interaction among them concerning their tasks.

4.5.4.3 Cognitive dialogue

The Grade 3 teachers seemed to be aware of the notion of cognitive dialogue and commented that they did not always understand their learners' thinking. The following quote is included here as it captures one teacher's understanding of cognitive dialogue most honestly.

> *No I definitely don't!* (Understand learners thinking) *It would help a lot if I did. To see things from (especially) a young child's perspective, and to be able to pre-empt the misconceptions that arise from my assumptions of what they already know/have experienced, would help. Sometimes when I finally understand* <u>*what*</u> *they're asking, (which sounds like a dumb question), I realize it's because I assumed there was a 'hook' to hang an idea on – and there wasn't. They hung it somewhere else – dah!*

4.5.5 GRADE 3 OVERVIEW

Although the Grade 3 time allocation for the teaching and learning of geometry falls beneath the recommended time stated in the NCS, the way the teachers have structure their teaching seems to be ideally suited to accommodate the five phases of learning described by Crowley (1987). The lesson on Monday could be used to facilitate the first phase of learning that of *Inquiry/Information*. The activities planned for Tuesday, Wednesday and Thursday could facilitate *Directed Orientation, Explication and Free Orientation* (Phases 2-4). Friday's lesson could facilitate the final phase of *Integration*.

The Grade 3 teachers made use of manipulatives in their lessons and seemed to use differentiated group work regularly. Due to the nature of the lesson observed it was difficult to capture data of cognitive dialogue since it was too difficult to follow the teacher or get close enough to the learners without being intrusive. An overt awareness of my presence may not only have hindered learning but could also possibly threatening the integrity of the data in that the learners or the teacher may have felt the need to perform or conversely shied away. Although evidence of cognitive dialogue is inconclusive in this case, the frequent use of group work allows greater opportunity for the learners' levels of thinking to be addresses and hence for cognitive dialogue to occur. The sequential nature (*Fixed Sequence Property)* of the Van Hiele theory plays an ever-increasing role in the learners' opportunity to develop greater insight into geometry concepts as the instructional practices of the teachers presume a level of understanding established in the preceding academic years. An interesting point for further investigation is the significant number of learners that move up a Van Hiele level from Grade 3 to Grade 4. Whether the structure of the Mathematics lessons in Grade 3 supports geometry insight due to a greater degree of differentiated teaching is worth further investigation.

4.6 GRADE 4 TEACHING AND LEARNING OF GEOMETRY

4.6.1 GRADE 4 INTRODUCTION AND CONTEXT

The average years of teaching experience of the Grade 4 teachers is approximately 10 years. All of these teachers have taught at different grade levels before and all have taught Mathematics at either the Foundation or Intermediate Phase during their careers. The responses of the Grade 4 teachers in the questionnaire indicated that they saw geometry as the study of space, shape, surfaces, measurement and angles and that geometry was indeed valuable as part of the school curriculum.

Grade 4 teachers professed to using manipulatives in their Mathematics lessons and stated that they had easy access to the equipment. They also said that they felt that the school gave them good support although they would appreciate more opportunity to talk with other teachers about the Mathematics curriculum. In terms of the best way to teach and learn geometry, their responses were as follows:

> *People all learn differently (learning styles). Try to start with core knowledge and teach as many varieties as possible. People see things differently.*
>
> *Practically /??And with as much visual aids as possible and loads of practice.*

4.6.2 GRADE 4 LEVELS OF LEARNERS' GEOMETRY THINKING

There are 58 learners currently at Grade 4 level in this school. Ten of these Grade 4 learners do not yet consistently operate on Van Hiele Level 1. Thirty-three learners can be described as operating on the first Van Hiele level (Visualisation) and fifteen learners seemed to be able to operate on the second level – Analysis. There were a small number of learners (three in Grade 4 and two in Grade 5) who scored below the benchmark 60% at Level 1, but then scored 60% or higher for the next five questions at Level 2. Although difficult to interpret reliably, these test results do not significantly change the levels of geometry thinking predominant in these grades to warrant further investigation in terms of the focus of this study. The focus being to explore the instructional practices currently employed in relation to practices advocated by the Van Hiele theory and other literature that promote the progression of geometry thinking.

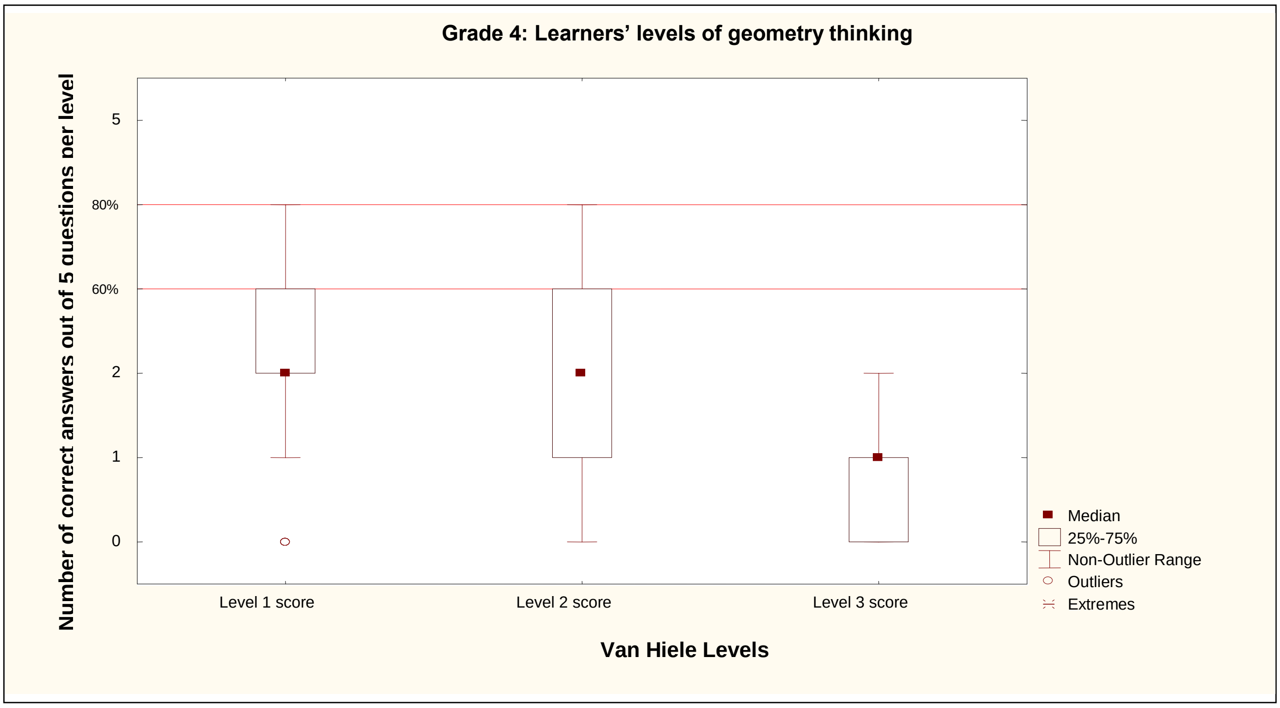


FIGURE 14: Learners' levels of geometry thinking

4.6.3 GRADE 4 AND THE NCS

The NCS recommends that 18% of the 26 hours per week of formal instruction time be used for the teaching of Mathematics. Of the 4.68 hours of Mathematics instructional time per week, 30% should be allocated to the teaching and learning of Space and Shape (geometry) in the Intermediate Phase. Geometry instruction, according to these figures should be happening in the Intermediate Phase classrooms for a little less than 1.5 hours per week.

The focus of geometry instruction in this Phase as stated in the NCS, "moves from recognition and simple description to classification and more detailed description of features and properties of two-dimensional shapes and three-dimensional objects" (NCS, p. 36). The progression from concrete to abstract is scaffolded in the NCS Assessment Standards by the natural extension of content and skills from one grade level to the next. For example, in Grade 3 learners are required to recognize and identify two-dimensional shapes and three-dimensional objects in their environment, whereas the same Grade 4 Assessment Standard requires learners also to be able to visualize and name these shapes and objects.

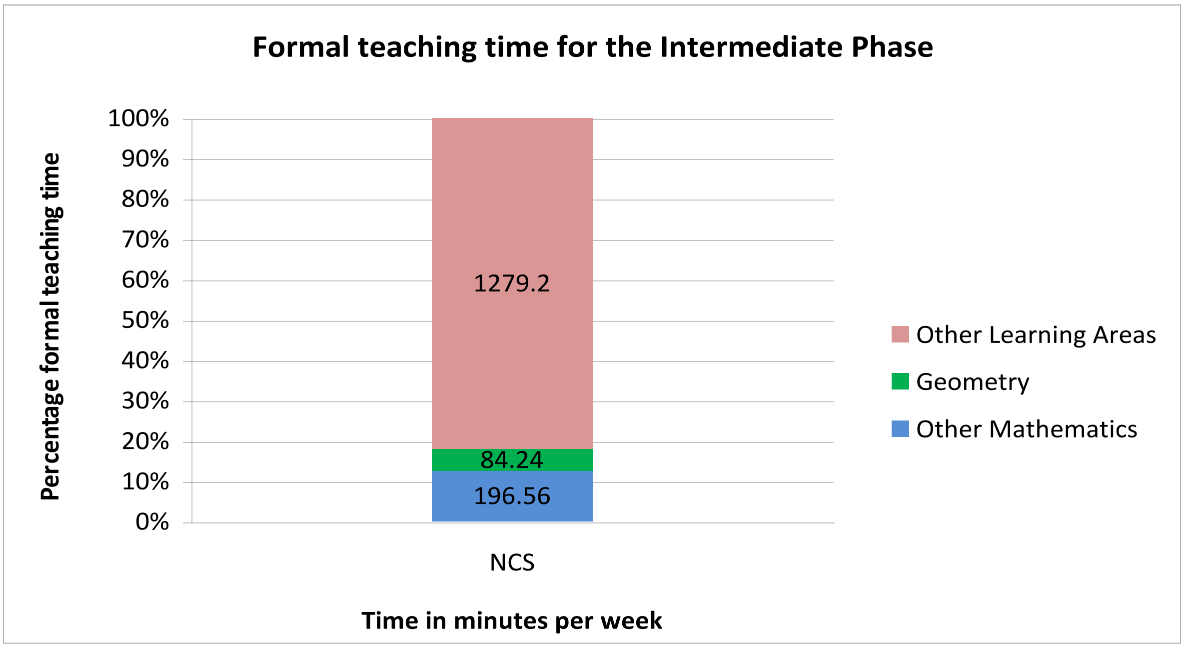


FIGURE 15: Teaching time in the Intermediate Phase recommended by the NCS

4.6.4 INSTRUCTIONAL PRACTICES

4.6.4.1 Curriculum

- **Time allocated to the development of geometry understanding**

There are currently three Grade 4 classes in the school and they all have 240 minutes (4 hours) of Mathematics instruction time allocated on their timetables. The lesson time in the Intermediate Phase is 40 minutes, which means that Mathematics happens once a day except on one day of the week when there are two periods of Mathematics. The Grade 4 time allocation for Mathematics instruction is closer to the NCS recommendation than any other previous grade described in this study. Unfortunately from the documentation submitted for analysis, it is impossible reliably to determine how much of this time is set apart for the instruction of geometry.

- **Types of experiences for the development of geometry understanding**

There are currently three Grade 4 classes in the school and the Mathematics planning for the year is the elected responsibility of one of these teachers. There is a Macro-plan, which is an overview of the work to be covered in the Grade 4 year. There is a Term Plan which lists the topic and relevant textbook page to be covered in a specific week. There is a Work-Schedule, which elaborates on the content covered, resources needed/used and assessment scope and type for a module/topic within a given period. In addition, there is a Daily Plan which includes what homework is to be set.

It may be that these various plans are to accommodate the different working styles of each of these three teachers. According to the Macro-plan, Learning Outcome 3 (listed as geometry) along with other topics is taught in the first three weeks of the third term and revised in the last week of the same term. Geometry with other topics is planned for the first three weeks of the final term. The Work-Schedule and the Daily Plan reflect that 2-D and 3-D shapes are taught in the last week (week 10) of the first term. No daily planning for the geometry to be taught in the fourth term was submitted. The geometry planned for term one is accommodated in the last three days of the term. This includes activities such as discussion, sorting and completing worksheets. Mention is made of the vocabulary to be discussed. For example, ‘face, edge, vertex (corner)’.

4.6.4.2 Language and level of instruction

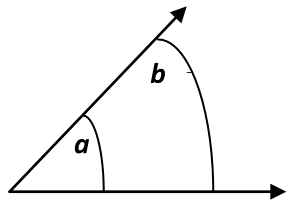
A vignette of Grade 4 Erica

The Grade 4 lesson started with a group quiz. The learners were arranged into four groups and sat around their tables in these groups. The timekeeper, scribe and leader for each group were identified and a quick, verbal revision of group-work rules was discussed. The teacher proceeded with a multiple-choice type quiz by posing the question and placing a choice of four possible answer cards onto the chalkboard. The learners were given a minute to discuss and record their answers. The questions were all related to geometry. This type of questioning continued for about ten minutes when the teacher presented a worksheet to each group and asked them to match the shape with its name. This worksheet concluded the quiz and the learners then tallied their points and the winning group was congratulated.

The groups put away their whiteboards and the teacher asked the learners to make a right angle using their arms. The teacher looked around to make sure that the learners had made an angle with their arms, which resembled a right angle and then asked them similarly to show an 180˚ angle. Whilst the learners were still standing, the teacher revised the meaning of acute angles and obtuse angles and had the learners use their arms to show these angles. The learners were then seated and the teacher called on a volunteer to help her demonstrate the meaning of a revolution and half a revolution being 180˚. This was done by the having the volunteer indicate a starting point with her outstretched arm touching the teacher's outstretched arm and the teacher then moved her arm as the hand on a clock would, until the other learners called out stop. This action was repeated using examples of acute and obtuse angles as well as right angles and straight angles.

The teacher then drew a circle on the board to represent a 360˚ protractor and defined an angle as "*the space between two lines that meet".* Whilst repeating this statement the teacher traced the line between the markings on the circumference of the circle she had drawn on the board. Arguably, the teacher may have a clear understanding of the concept of angle but I feel that this action may result in the learners developing a misconception of angle and associating the size of the angle with the length of the arms that form it.

An angle is the space between two lines that meet



Does this mean that the angle represented by the smaller arc '*a'* is smaller than the angle represented by the larger arc '*b'*?

FIGURE 16: Representation of a misconception from a description observed

4.6.4.3 Cognitive dialogue

The Grade 4 teachers acknowledged the importance of understanding their learners' thinking in being able to facilitate learning more effectively, which can be seen in the following excerpts:

> *I try to understand, and always ask if what I've said answers their question.*
>
> *Very important to understand their thinking, as this will influence your teaching.*
>
> *Very, and no. One sometimes struggles to grasp what it is they don't understand. It is always a good thing to help them put into words what they* **do** *know but there isn't always time.*

From the latter response above, it is assumed that what the teacher meant by "*very, and no*" is that to be able to understand one's learners is very important but that she doesn't always understand them. Her approach to helping learners develop understanding is interesting in that she does not start by dealing with a misconception but launches from the learners' existing understanding. Her response suggests that she initiates cognitive dialogue from a position where the learners feel secure and not when the learners try to assimilate a new idea into an existing schema.

4.6.5 GRADE 4 OVERVIEW

Although the Grade 4 timetables come close to meeting the NCS recommended time allocation for Mathematics instruction, it is uncertain how much of this instructional time is used to facilitate the teaching and learning of geometry. The documents submitted also do not provide conclusive evidence of the type of geometry experiences that are planned. Only the first two phases of learning described by Crowley (1987) were observed during the classroom visits. In describing the progression from concrete examples to abstract ideas, Battista (2000) posits that the learner must mentally build explicit relations between the parts and conceptualising these relationships requires “intentional reflection”. The final two phases of learning, those of *Free Orientation* and *Integration* allow for this type of meta-cognition. In limiting the activities in Grade 4 to only some of the phases of learning, this curriculum inhibits the learners from integrating the ideas presented to them with their existing schema and from developing greater insight into geometry concepts. Although there were some learners yet to function at the first Van Hiele level, the predominant levels of learners’ geometry thought are *Visualisation* and *Analysis*, meaning that the instructional practices at this grade level should accommodate a wide range of understanding. Unfortunately, the planned activities only partly reflect those suggested in research for Level 1. Evident in the planning and in the classroom observations were the use of different teaching strategies and the use of manipulatives. The use of group work in the observed lessons suggests a socio-constructivist approach but there is insufficient data to validate this statement and also to comment on the degree to which cognitive dialogue takes place in this grade.

4.7 GRADE 5 TEACHING AND LEARNING OF GEOMETRY

4.7.1 GRADE 5 INTRODUCTION AND CONTEXT

The combined teaching experience of the Grade 5 teachers is more than forty years. One of the teachers at this grade level approached me at the introduction of this study and expressed her discomfort at being observed. The same teacher said that she would rather not be a part of this study, however, she still wanted her learners’ levels of thinking assessed and during the course of the study, of her own accord, and she completed a questionnaire. Her right to withdraw from being observed was upheld. Geometry was defined as:

the using of nets to make 3-D objects, naming them, comparing, match 3-D objects with their nets. Find volume of 3-D shapes.

Practical work was described as the best way to teach and learn geometry. Geometry was described as "*a very important section in Math*" and the teachers said that they really enjoyed teaching Mathematics. The teachers stated that they had access to manipulatives and that they felt they have sufficient support from the school. The following excerpt expresses the value that one Grade 5 teacher places in the use of concrete teaching resources.

I have access to manipulatives.

I use them wherever I can.

I believe that the pupils learn best when they can build, touch and experience the objects.

4.7.2 GRADE 5 LEVELS OF LEARNERS' GEOMETRY THINKING

Forty-six Grade 5 learners were assessed in terms of their level of geometry thinking according to the Van Hiele theory. Seven of these Grade 5 learners were not able to consistently and accurately identify a triangle or differentiate between a square and a rectangle implying that they operate at a *Pre-visualisation* level of geometry thought. Twenty learners seem operate at Van Hiele Level 1 and thirteen were able successfully to answer three or more of the five questions posed at a Van Hiele Level 2 in the test designed by Usiskin (1982). Six out of the forty-six Grade 5 learners were also able to answer three of the five questions posed at Van Hiele Level 3 – *Informal Deduction*. As mentioned in Section 4.7.5, there were some anomalies in the results; however, the findings provide sufficient data of the predominant levels of geometry thought of the learners in the grade. In the light of this study, the results indicate that instructional practices for the development of geometry reasoning should predominantly focus on the activities described in research as those for Van Hiele levels one and two.

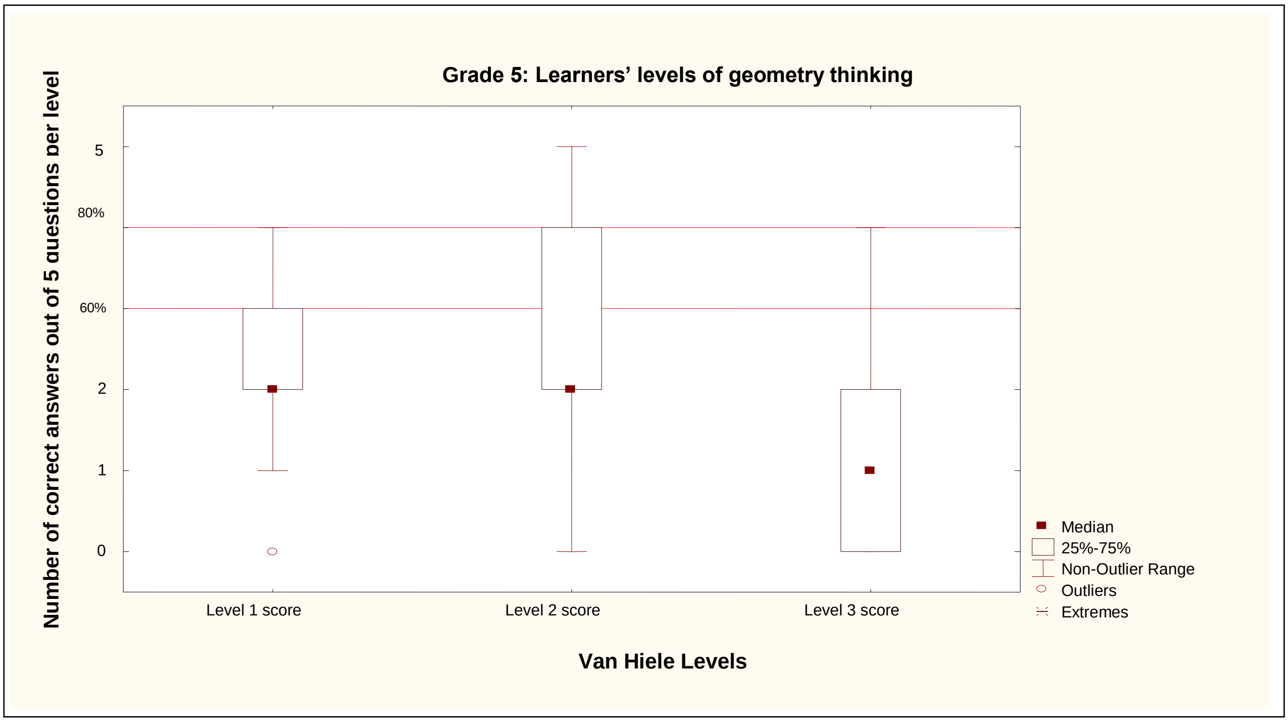


FIGURE 17: Learners' levels of geometry thinking

4.7.3 GRADE 5 AND THE NCS

The time recommended by the NCS for the teaching and learning of Mathematics in general and for geometry remains the same for Grade 5 and Grade 6 as it is for Grade 4. The focus, as stated in Section 4.6.3, is continued through the Intermediate Phase. Important to note, is not only the extension of content in the Grade 5 Assessment Standards, but a greater emphasis on the **properties** of two-dimensional shapes and three-dimensional objects. The type of activities suggested by Crowley (1987) and other researchers for Level 2 –*Analysis* are listed alongside the Grade 5 Assessment Standards to highlight the alignment of the NCS geometry focus with that of the Van Hiele theory.

TABLE 4: Alignment of the NCS to activities suggested by Crowley and other researchers

Van Hiele type experiences for Level 2 – Analysis (Crowley, 1987)	Assessment Standards NCS Grade 5
1. To measure, colour, fold, cut, model and tile in order to identify properties of figures and other geometry relationships 2. To empirically derive (from studying many examples) "rules" and generalizations	1. Investigates and compares ▪ by making models of geometry objects using polygons they have cut out ▪ by cutting open models or geometry objects to trace their nets

3. To discover properties of unfamiliar classes of objects	▪ by drawing shapes on grid paper
4. To describe a class of figures by its properties 5. Without using a picture, describe a [figure] to someone 6. To compare shapes according to their characterizing properties 7. To identify and draw a figure given an oral or written description of its properties 8. To identify a shape from visual clues	2. Recognizes, visualises and names two-dimensional shapes and three-dimensional objects focusing on: ▪ similarities and differences between cubes and rectangular prisms ▪ similarities and differences between squares and rectangles
9. To sort and resort shapes by single attributes 10. Sort cut-outs of quadrilaterals by number of parallel sides or number of right angles 11. To identify properties that can be used to characterize or contrast different classes of shapes.	3. Describes, sorts and compares two-dimensional shapes and three-dimensional objects according to properties including: ▪ number and/or shape of faces ▪ number and/or length of its sides

4.7.4 INSTRUCTIONAL PRACTICES

4.7.4.1 Curriculum

- **Time allocated to the development of geometry understanding**

The Grade 5 timetables reflect the same time allocation of 240 minutes (4 hours) per week for Mathematics instruction as the Grade 4 timetables. A Mathematics lesson of forty minutes is planned to take place daily except on one day during the week for which two lessons of Mathematics instruction are allotted. Reflected in the Grade 5 Year Plan, the total number of lessons set aside for the teaching of geometry is 20, excluding the 10 lessons on area, perimeter and volume. Twenty geometry lessons out of a possible two hundred Mathematics lessons in the year is 8% below the NCS recommended portion of time that should be afforded to learners to develop their geometry reasoning. The 10% of the Mathematics lessons allocated to the facilitation of geometry understanding reflected in the teachers' Year Plan was used to project the amount of time these teachers may allocated for the development of geometry understanding throughout the year.

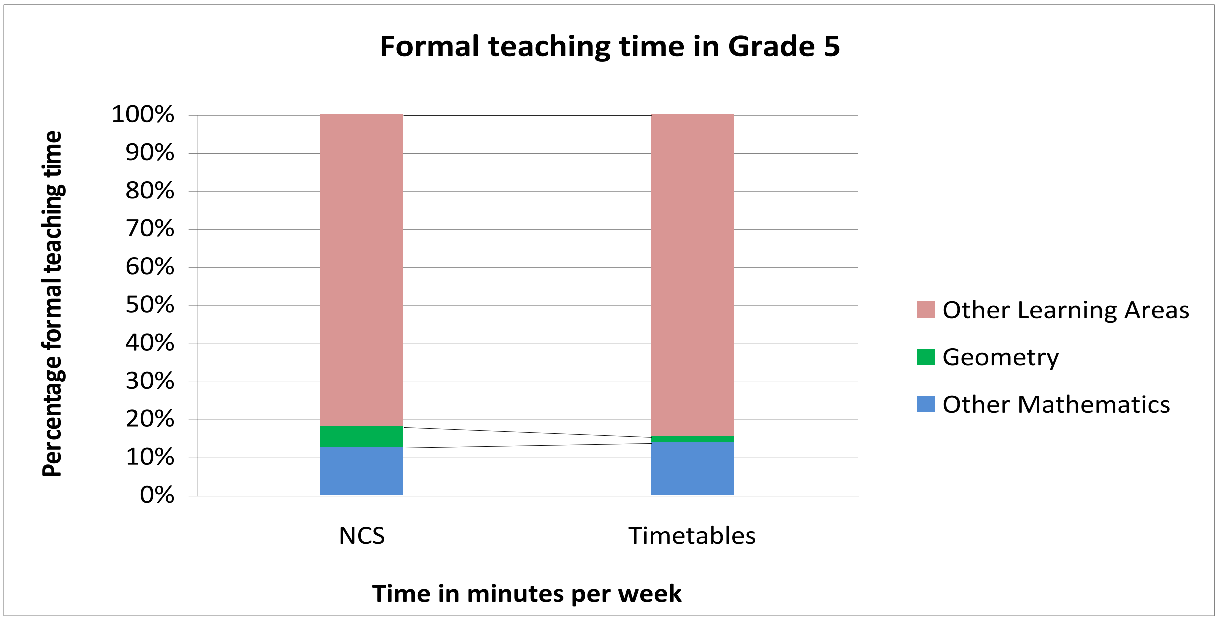


FIGURE 18: Comparison of the recommended and the implemented time allocation for Mathematics and geometry concepts

- **Types of experiences for the development of geometry understanding**

A weekly Year Plan listing the core knowledge taught each week of the term is accompanied by an assessment plan that lists what Learning Outcome is assessed and the type of assessment done. The daily Year Plan lists the core knowledge taught each day. Two-dimensional shapes and area and perimeter are the focus of weeks six and seven in the second term. The Grade 5 teachers plan discussion and activities around the relevant exercises in the textbook. The teaching in these two weeks begins with a discussion to revise names of “known shapes and introduce new shapes”. Textbook exercises on angles and tessellations as well as basic calculations of area and perimeter are also included during this time.

In weeks seven and eight of the third term the learners build three-dimensional objects from nets provided. Activities, discussions and exercises during this week are focused on three-dimensional objects and are used to introduce the next week’s topic, which is volume. Shapes and tessellations are revisited in the fourth week of the fourth term followed by the teaching of symmetry, including rotational symmetry, in week five. The geometry for the year ends with the teaching of transformations at the beginning of week six.

4.7.4.2 Language and level of instruction

A vignette of Grade 5 Francesca

The whole class was seated on the floor at the back of the classroom with their whiteboards and markers and the teacher started the lesson by asking the learners to write down all the shapes with straight lines that they knew. After approximately three minutes, the teacher called time. The learners counted the names of figures they had listed and after some discussion a winner was congratulated.

The learners remained where they were for the next activity. The teacher produced colourful posters of various polygons sans their names and asked the learners to name the shape giving a reason for their answer. During this activity, the teacher guided the learners to use the correct terminology and, on occasion, explained the meaning of words that were not familiar to all the learners, for example, the use of regular and irregular in connection to the lengths of the sides of a pentagon. The teacher also interrupted the activity to probe the learners' understanding in terms of the differences between a square and a 'diamond' (rhombus).

The class session ended when the teacher explained the group project, which was to be done and handed task cards to groups of five learners. Each group was commissioned to produce an informative poster and a mobile focusing on a particular shape and a three dimensional prism. The learners were permitted to use any resource at their disposal including their dictionaries and textbooks. A poster board and coloured paper was also made available to the groups. The learners seemed to attack the task with fervour and within minutes, the groups had organized who was doing what. Some discussion centred on the technicalities of the task and some discussion dealt with the definitions of the figures and prisms. The teacher moved from group to group maintaining a productive atmosphere and assisting learners where necessary.

4.7.4.3 Cognitive dialogue

There is insufficient data in the documentation and in the classroom observation to comment on the depth or frequency of the types of teacher-learner interaction indicative of cognitive dialogue in this case, however, one teacher response taken from the questionnaire is as follows:

> *most of the time I understand them. You need to think on their level to be able to explain the work so that* ***they*** *can understand.*

4.7.5 GRADE 5 OVERVIEW

The planning submitted by the Grade 5 teachers and the lesson observed demonstrated the first three phases of learning suggested by Crowley (1987) for the development of geometry reasoning. There is no convincing evidence if the last two phases of *Free Orientation* and *Integration* are included in the instructional practices at this grade level. The concepts and content planned for the Grade 5 year was based on the Assessment Standards recorded in the NCS therefore the instructional practices at this level start from an assumption that the Assessment Standards of Grade 4 have been covered in the previous academic year. A natural assumption for any teacher, however, the implications in terms of developing geometry insight are profound and once again, the property of *Fixed Sequence* is thrust to the fore. The time allocated for the teaching and learning of geometry is less than that recommended in the NCS.

Data from the teachers' questionnaires and the classroom observation acknowledge the use of Mathematics manipulatives. During the class visit, learners were actively engaged in the learning process and the group work observed allowed opportunity for learners to negotiate meaning and for teacher-learner dialogue.

4.8 TEACHING AND LEARNING GEOMETRY AS THE CASE FOR THIS SCHOOL

In this section, the results from each of the six grades discussed previously will be drawn together, to present the relationship between the learners' levels of geometry thinking and the instructional practices evident in this school. Firstly, the learners' levels of geometry thinking will be discussed followed by a discussion of the trends in instructional practices across the grades.

4.8.1 Learners' levels of geometry thinking

The Van Hiele test (Usiskin, 1982) was used to establish the learners' levels of geometry thinking from Grades 1-5. The Grade 0 learners were not assessed. The first five questions in the Van Hiele test correspond to the first level of geometry thinking in the Van Hiele model and questions 6-10 correspond to the second Van Hiele level of thinking. Usiskin (1982) worked on two criteria for establishing a level of thinking. The first, a stricter criterion, required the learner to get four or five out of the five questions at each level correct to be considered as thinking at that particular level. The second criterion that is applied in this study worked on a 60% pass rate. For example, if a learner got three, four or five of the questions out of questions 1-5 correct, they were regarded as thinking at the *Visualisation* level. If the same learner got less than three questions out of questions 6-10 correct, they maintained the *Visualisation* level of thinking. Learners whose results did not follow the criteria were marked for follow-up. The alpha reliability coefficients for the results are as follows: 0.27, 0.44 and 0.45. These compare favourably to corresponding the K-R formula reliabilities for the spring assessment (0.31; 0.44 and 0.49) in the CDASSG report. The results are presented in Table 5.

TABLE 5: 2010 Learners' levels of geometry thinking according to the Van Hiele test (Usiskin, 1982)

VHT 60%	Percentage learners: Pre. visualisation	Van Hiele Level 1	Van Hiele Level 2	Van Hiele Level 3
Gr. 1 (n= 47)	53. 2	46. 8	0. 0	0. 0
Gr. 2 (n=49)	46. 9	53. 1	0. 0	0. 0
Gr. 3 (n=50)	42. 0	58. 0	0. 0	0. 0
Gr. 4 (n=58)	17. 2	56. 9	25. 9	0. 0
Gr. 5 (n=46)	15. 2	43. 5	28. 3	13. 0

From Table 5 it seems that progression through the levels of thinking is occurring. The number of learners thinking at the *Visualisation* level increases from Grades 1-3 by 6.3 percentage points and 4.9 percentage points respectively. Figures 19 and 20, show the distribution of the learners across the grades in terms of Van Hiele Levels 1 and 2.

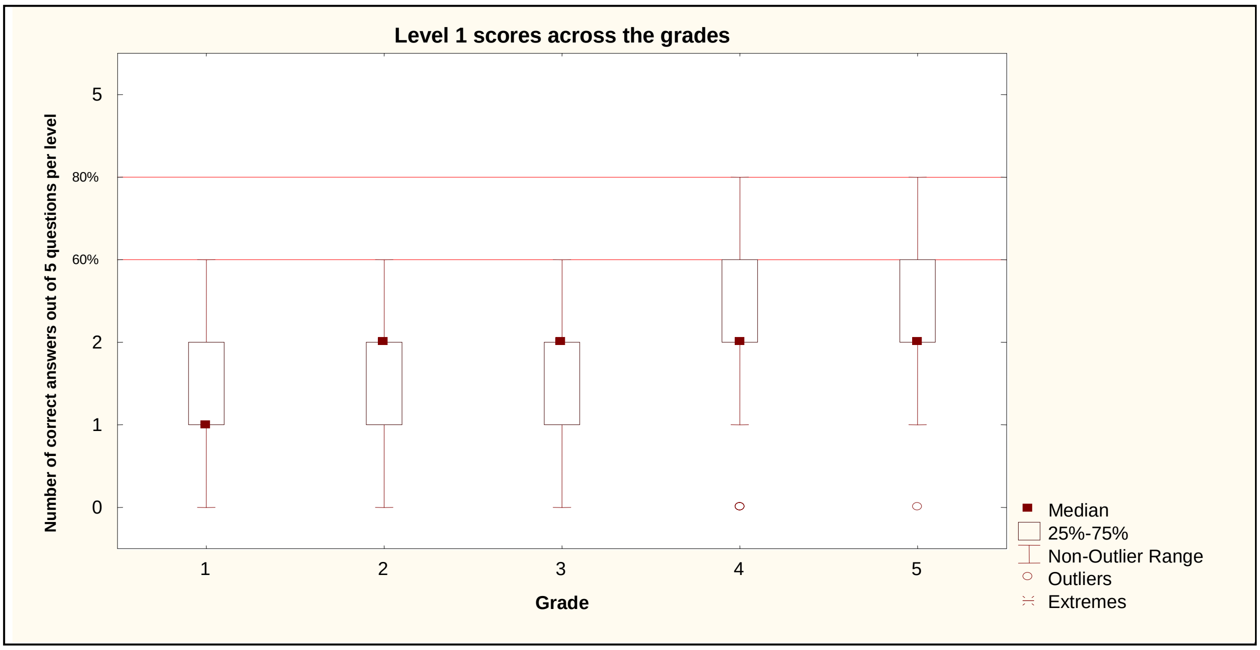


FIGURE 19: Distribution of learners across the grades at Van Hiele Level 1

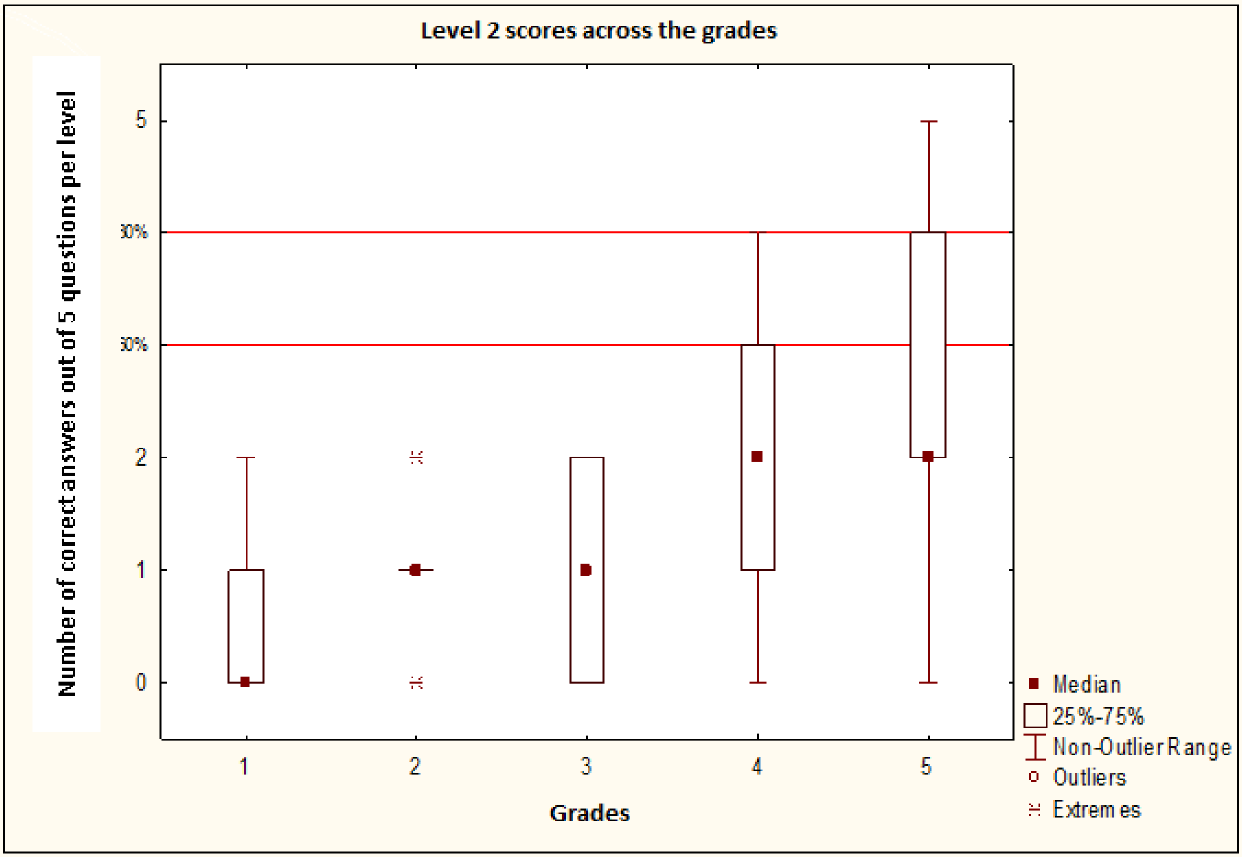


FIGURE 20: Distribution of learners across the grades at Van Hiele Level 2

4.8.2 Instructional practices

In this section, the results from the different grades are combined and discussed under the headings of the characteristics of the Van Hiele theory, which are recognized in the progression determinants of: types of geometry experiences, language of instruction, cognitive dialogue and curriculum (see Section 2.6).

4.8.2.1 Advancement – types of geometry experiences

The characteristics of *Advancement in* the Van Hiele theory is realised in the types of geometry activities the learners' experience. Crowley (1987) elaborates on the five phases of learning that a learner should be guided through in order to develop a new level of thinking. Implicit in these phases of learning is the use of concrete aids or manipulatives and adequate time for the learners to take ownership of their learning whilst navigating through these said phases.

Only 25% of the lesson observations during this study progressed to the third phase (*Explication*) of learning where the level's system of relations starts to become apparent. For example, in a Foundation Phase lesson where a group task required the learners to construct a house with a cube base and a pyramid roof from straws and Prestik, the learners began to cut a bunch of straws the same length at once instead of measuring each one as they placed it on the structure.

It would appear from the timetables and Year Plans submitted by each grade that the time allocated for the instruction of Mathematics in general is less than that recommended in the NCS. The time allocated for the development of spatial and geometry reasoning is significantly inadequate when juxtaposed with the NCS as can be seen in Figure 21.

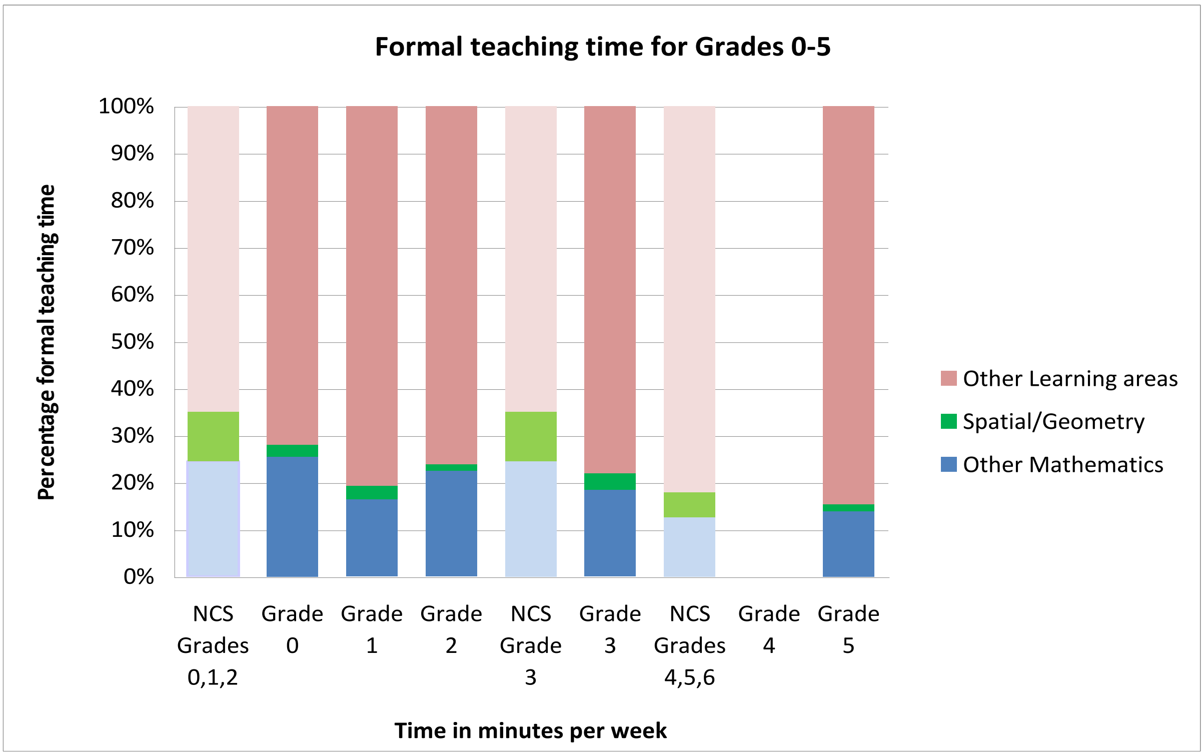


FIGURE 21: Allocation of Instructional Time as projected from timetables and Year Plans 2010

The results presented in Table 6 are projected from data obtained in the teachers' timetables and Year Plans and although tentative, they do highlight what seems to be the trend in that the time allocated to Mathematics in general and geometry across the grades falls short of the recommendations of the NCS. Table 6 indicates that a learner who attends this school from Grades 0-5 will have 455 hours (910 lessons) less of Mathematics than another learner whose curriculum meets the time allocations of the NCS.

TABLE 6: The cumulative deficit of time for the teaching and learning of Mathematics

Grade	Recommended time for year in minutes	Minutes timetabled by teachers for year	Shortfall In minutes	Cumulative deficit in minutes
0	17 556	370* 38wks = 14 060	3 496	
1	17 556	255* 38wks = 9 690	7 866	11 362
2	17 556	315* 38wks = 11 970	5 586	16 948
3	20 330	337. 5* 38wks = 12 825	7 505	24 453
4	10 670	240* 38wks = 9 120	1 550	25 803
5	10 670	240* 38wks = 9 120	1 550	27 353 (455 hrs.)

4.8.2.2 Linguistics – language and level of instruction

In terms of approaching Mathematics and particularly geometry teaching from a perceptuo-motor perspective, 77% of the teachers regarded using concrete examples/manipulatives and practical activities as the best way to teach geometry. Presenting the core knowledge in a variety of ways to accommodate the different learning styles of individuals, and group work where one group teaches the other were other opinions offered. In response to their access to and use of concrete aids or manipulatives, all the teachers said that they had easy access to equipment and made use of it. To quantify how frequently these teachers used manipulatives is difficult because responses such as ‘often’ and ‘loads’ are relative terms. One teacher wrote that she used manipulatives approximately four times per term.

All of the lessons observed involved the learners in some kind of activity using some kind of concrete aid. The contexts and the manipulatives evident in the planning and those that were used during the classroom observations were familiar and appropriate to the learners’ levels of understanding. The language used in the lessons observed was appropriate to the learners experience and corresponded to the activities and guidelines suggested in the NCS. Fifty-eight per cent of the lessons began with a class discussion of what the learners could remember in terms of what they have already learnt or know about shapes. The lesson on bearings and grids was a continuation of differentiated group work. One lesson began with a group quiz that was more of a summative assessment rather than a means to establish a baseline for the teaching of angles that followed. The lesson on symmetry in the Foundation Phase began with the three groups of learners giving feedback to their teacher on the previous day’s task and giving the group to follow them advice on how best to go about the impending activity. Although this was not clearly establishing the prior knowledge of the learners, it was a wonderful opportunity for the learners to reflect on what they had learnt. It was also a good opportunity for the teacher to assess her own instructional effectiveness and the appropriateness of the tasks chosen. One teacher put her learners into groups of about five and got them to brainstorm the collective knowledge of the members in terms of shapes and objects and record their statements on a large sheet of paper. These sheets were then pinned onto the board and discussed by the class. This activity gave the learners a chance to verbalise succinctly their understanding of some 2-D shapes and 3-D objects and refine their definitions.

4.8.2.3 Mismatch – cognitive dialogue

Although extremely difficult to record and capture all the occurrences of cognitive dialogue in terms of its frequency and depth during these classroom visits, the observations were used as an opportunity to identify moments where teachers did meet their learners on a cognitive level and to add to the trustworthiness of what the teachers wrote on their questionnaires. Crowley (1987) states, “Teacher questioning is a crucial factor in directing student thinking. At all levels, asking children how they “know” is important. ”

Although a large part of many of these lessons seemed to be about transferring knowledge albeit through an activity, there was evidence in seven lessons where the teacher probed a learner response to get a better grasp of what the learner’s understanding was.

4.8.2.4 Extrinsic/Intrinsic – curriculum

A tabulated summary (Table 7) of the geometry experiences planned for the Foundation and Intermediate Phases re-iterates that insufficient time is allocated for the development of geometry reasoning and can be used as an indication of where possible gaps in the curriculum may occur. There is a logical development of geometry understanding as one progresses through the grades in that the activities planned by the teachers are based on the guidelines in the NCS, which do recognize the *Extrinsic/Intrinsic* property of the Van Hiele theory.

TABLE 7: Content and distribution of geometry from Grades 0-5

DISTRIBUTION OF SPATIAL AND GEOMETRY CONCEPTS AND CONTENT COVERED THROUGHOUT GRADES 0-5				
GRADES	**TERM 1**	**TERM 2**	**TERM 3**	**TERM 4**
GRADE 0	Classify and Sort: Looking at attributes of objects (including shape) (approx. 11 lessons)	Solids, Shapes and Equal parts: Exploring and sorting solids finding shapes in solids exploring squares, rectangles, triangles and circles halves, symmetry slides, flips and turns. (approx. 12 lessons)	–	–
GRADE 1	Recognise and name 2D shape: circle, oval, square and rectangle House project using 3D objects Position (Ordinals) 5th (approx. 3 lessons)	Symmetry: left and right front and back (approx. 2 lessons)	Make a 3D prism Make a 3D cube (approx. 2 lessons)	Symmetry: Left/ right Front/ back Position (Ordinals) Following directions (approx. ? lessons)
GRADE 2	Investigation of 2D shapes and 3D objects exploring attributes naming drawing (approx. 8 lessons)	Revision of 2D and 3D work of term 1 Parallel and not parallel (approx. 4 lessons)	–	–
GRADE 3	Symmetry 2D shapes 3D objects Patterns using Cuisenaire pattern blocks (approx. 11 lessons)	–	Map work: grids 8pt compass 3D shapes using nets Patterns Symmetry (approx. 15 lessons)	Revision (approx. 5 lessons)
GRADE 4	Investigating 2d and 3D shapes: Naming Sorting Vocabulary (approx. 3 lessons)	–	Geometry (approx. ? lessons)	Geometry (approx. ? lessons)
GRADE 5	–	2D shapes: Revision Naming Perimeter area Angles Tessellations (approx. 10 lessons)	3D shapes: naming building drawing (approx. 4 lessons)	Symmetry Transformations (approx. 6 lessons)

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CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

5.1 OVERVIEW OF THE STUDY

The aim of this study was to investigate the relationship between the instructional practices of teachers and their learners' levels of geometry thinking. In order to frame the instructional practices that influence the development of geometry understanding and what is meant by learners' levels of geometry thinking, a review of relevant literature was conducted. The Van Hiele theory was selected as a theoretical framework to inform the study and other literature was used to augment concepts implicit in certain characteristics of the theory. For example, the *Advancement* property of the theory, which states that a learner's level of thinking can only progress to level (*n*) when level (*n-1*) has been mastered, implies that a learner must experience the appropriate geometry activities to allow for the said learner to take ownership (Van Hiele, 1986) or internalize his/her learning (Sfard, 2007) and hence master that particular level of thinking. In this regard, Crowley (1987) elaborates on the types of geometry experience that promote progression and implicit in her descriptions are the allocation of sufficient time (Atebe & Schäfer, 2008; Usiskin, 1982; Van Hiele, 1986) for learners to take ownership and appropriate the use of manipulatives (Arzarello *et al.*, 2005; Van de Walle, 2007).

Consideration of where in instructional practices the *Sequential Advancement, Linguistics, Mismatch, and Extrinsic/Intrinsic* characteristics of the Van Hiele theory may be evident enabled the research design. Quantitative and qualitative data was collected from multiple sources and analysed descriptively. The findings were compared to the instructional practices positively reported in literature to influence the development of geometry understanding and progression through the Van Hiele levels of thinking. The results of the comparison provided a description of the educational terrain in terms of the teaching and learning of geometry in this context. From this description conclusions about the nature of the teachers' instructional practices and their learners' levels of geometry thinking were drawn.

5.2 SUMMARY OF THE FINDINGS

5.2.1 OBJECTIVE 1: DESCRIBE INSTRUCTION PRACTICES WHICH SUPPORT THE LEARNING OF GEOMETRY

The van Hiele's proposed five *Sequential phases of learning* (emphasis added) or geometry experiences which enabled the learner to progress to the next level. These are *Inquiry, Directed Orientation, Explication, Free Orientation and Integration* (Section 2.6.1). Not only do teachers need to allocate adequate time to develop geometry understanding, but they should also provide the appropriate kind of instruction during this time. The kind of instruction advocated by the Van Hieles dovetails with teaching practices described by research on perceptuo-motor and cognitive approaches to learning (Adam, 2004; Arzarello *et al.*, 2005; Battista, 2007; Bransford *et al.*, 2000) as well as to relevant or realistic approach which enables the learners to identify with and relate to the concepts being facilitated (Barnes, 2004; Capraro & Capraro, 2006; Cobb *et al.*, 1991; Dye, 2001; Freudenthal, 1971; Hwang, Huang & Dong, 2009; Junius, 2008).

According to the Van Hiele theory, geometry experiences should begin in *Phase 1* with activities that elicit information regarding the learners' prior knowledge and prepare them to explore the concept to be presented. These activities should encourage the learner to become proactive in building their own understanding. *Phase 1* experiences should centre round getting the learner to engage in what Sfard (2007) calls self-communication. That is within oneself to ask questions and seek answers and hence think. *Phase 1* experiences should be followed in *Phase 2* by teacher-guided activities that direct the learners to search for answers or explanations of their responses in *Phase 1*. Crowley (1987) suggests that these tasks be short and designed to elicit specific responses. These tasks are meant to scaffold the learners in their construction of the geometry concept.

In *Explication*, which is the *third phase*, the teacher assists the learners in verbalizing their thinking and observations. It is in this phase that the learners are guided to use appropriate and accurate language. *Phase 4, Free Orientation*, should encourage learners to explore the parameters of their responses to this point. The final phase of learning, *Phase 5, Integration*, happens when the teacher is able to bring together all the learners' questions, observations and responses and enable the learners to formulate a summary of their learning.

The importance of these *phases of learning* lies, in the fact that they provide a platform for each of the properties of the Van Hiele model to be negotiated by the learner. For example, in *Phase 1*the learners' attention is directed to the "thinking subject" of a Van Hiele level and activities in the *Directed Orientation* and *Explication* phases then assist the learners in establishing relationships between the characteristics, properties or relationships of shapes, depending on which Van Hiele level they are operating. This is the "thinking result" of each Van Hiele level. The last two phases of learning are instrumental in helping the learners accomplish what the Van Hieles refer to as having ownership; developing insight through activities that make relationships explicit and inform the learner of how the new knowledge relates to and can be appropriately applied in new contexts.

For instance, consider the development of the concept of a triangle; a student may begin by describing a triangle as a 'pointy shape' that looks like an arrow. In the *Inquiry Phase* on a Van Hiele Level 1, a learner may identify all of the following as triangles.

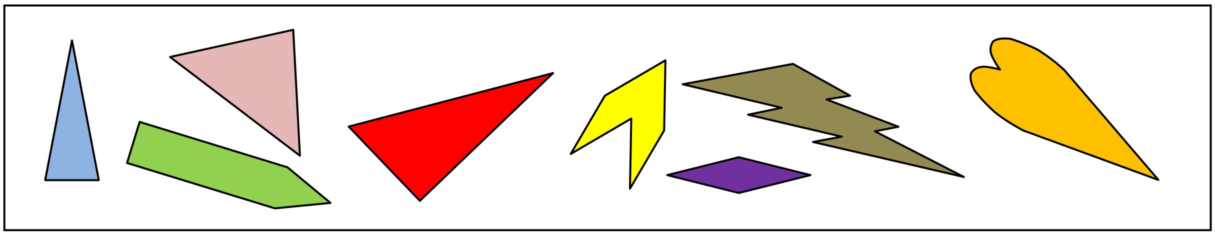

FIGURE 22: Development of the concept of 'Triangle' at the Inquiry Phase, Van Hiele Level 1

By using short sorting tasks, the teacher may help the learner to identify particular attributes of shapes, like size, colour and whether a shape is 'pointy' or 'rounded'. These *Directed Orientation* activities alert the learners to the properties of triangles. In the *Explication Phase,* the teacher would introduce level appropriate language by assisting the learners to verbalise that a triangle is a shape with three sides and three corners.

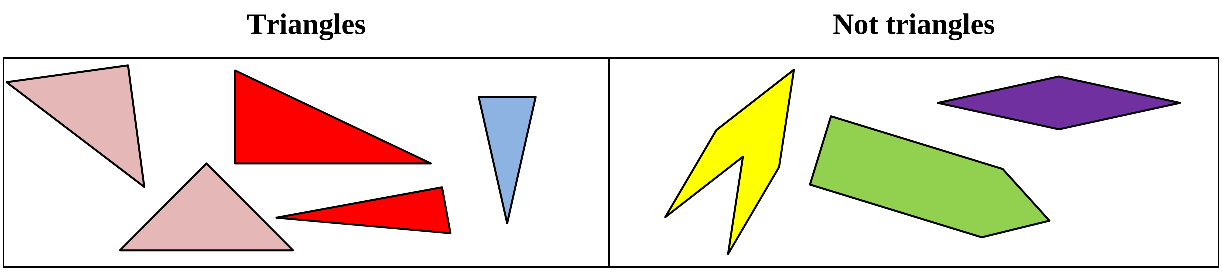

FIGURE 23: Development of the concept of 'Triangle' at the Directed Orientation Phase, Van Hiele Level 1

These sorting type activities can be extended in the *Free Orientation Phase* where the learners can be presented with different types of triangles and asked to investigate other properties like equal sides or right angles. These types of activities help to generalize what a triangle is and what it can look like in different orientations and contexts, and more importantly, lay a foundation for greater geometry understanding as in the case of the classification of triangles, surface areas of pyramids and triangular prisms.

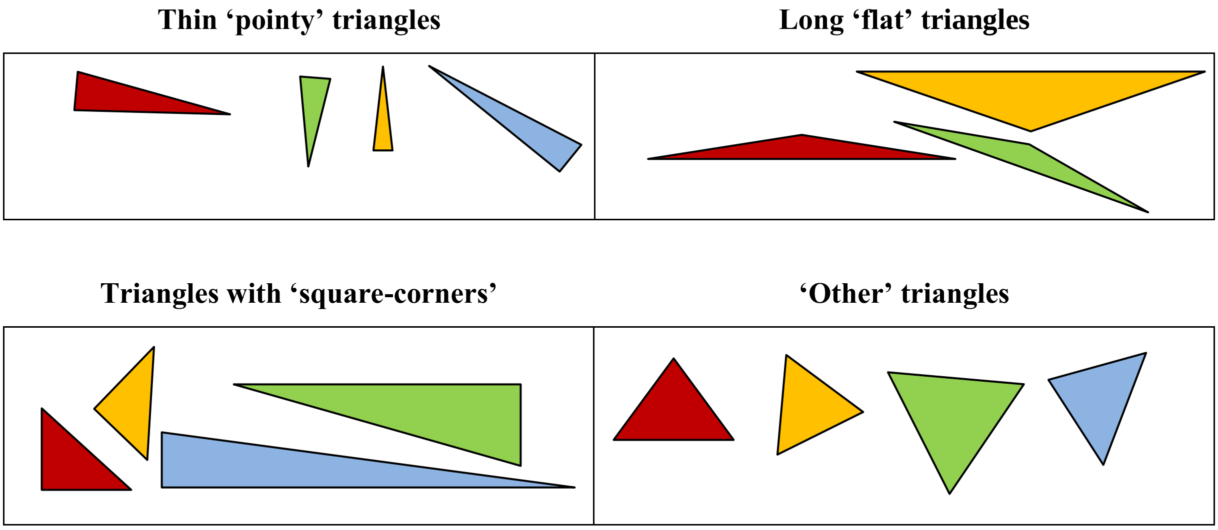

FIGURE 24: Development of the concept 'Triangle', Free Orientation Phase, Van Hiele Level 1

At a higher Van Hiele level, these triangles may be classified in terms of their properties, for example, equilateral, right-angled, obtuse, etc. However, at the first Van Hiele level, they are grouped together as shapes with three sides and three corners and hence are triangles. By omitting this phase of learning, the learners run the risk of only being able to identify a triangle if presented in a shape and orientation with which they were first presented. During the final phase of learning the teacher should allow time for 'intentional reflection' (Battista, 2000) and consolidate the level appropriate language. This phase may include a task where the learners use triangles to make a picture.

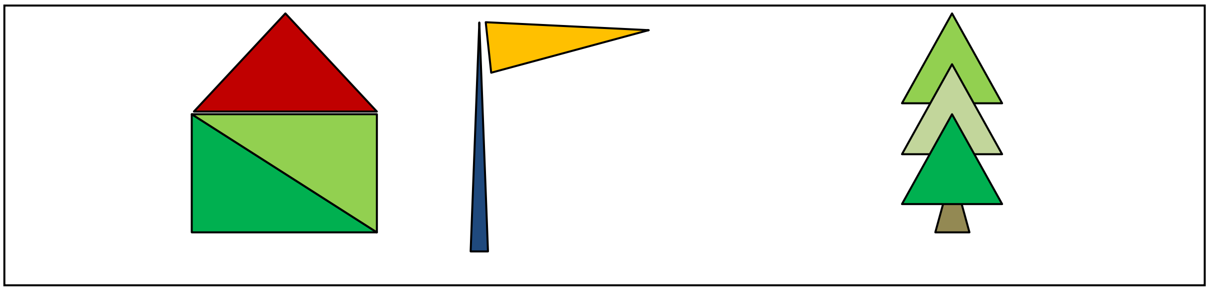

FIGURE 25: Development of the concept 'Triangle' Integration Phase, Van Hiele Level 1

The complexities of the process of abstraction elaborated on by Battista (2007) can be appreciated when one breaks down a simple concept such as that of 'triangle'. One can comprehend that it takes time for a learner to work through these levels, but this process is often neglected in the curriculum, either due to a lack of knowledge or time constraints. Gutiérrez *et al*. (1991) argue that learners develop several Van Hiele levels simultaneously and that acquisition of a specific level can take months and even years

The Van Hieles referred to the process whereby learners internalise and make personal meaning of geometry concepts and vocabulary, as learners having 'ownership' of what is presented to them. Sfard (2007) calls this *individualizing* and describes the process as "becoming able to have mathematical communication not only with others, but also with oneself" (p. 573). She goes on to say that mathematical discourse alters and extends the learners' spontaneously learned colloquial discourses rather than constructing new ones from a void, which is in line with Vygotsky and other socio-constructivists; re-emphasizing the importance of establishing the learners' prior knowledge. Our understanding or discourses are generally in harmony with our experiences and a change in discourse is needed to become aware of new possibilities.

The effectiveness of geometry experiences is influenced by the type and quality of the language (Roux, 2005; Rudd *et al.*, 2008) used to communicate the geometry concepts, which are being facilitated, and the teachers' ability to understand their learners and relate the new knowledge to the learners' existing understanding (Bransford *et al.*, 2000; Gerace, 1992; Ritter *et al.*, 2007). The idea that guided instruction helps learners make the new information their own, dovetails with Vygotsky's zone of proximal development. By starting with a conceptual schema that the learner already possesses, the teacher can guide and support or scaffold the development of new concepts. Vygotsky emphasized the role of the learning environment including social aspects, agents and tools in the learners' growth of new knowledge. It is from this social interaction and negotiation of meaning that learners personally construct and internalize knowledge (Bransford *et al*., 2000). From the framework of the Van Hiele theory, the property of *Distinction/Linguistics* posits that each geometry level of thinking has its own symbols and system of relations. This level specific 'language' refers to a presentation of concepts and activities that presume an understanding in as much as it does vocabulary.

The NCS (South Africa, DoE, 2003) indicates that 'Learning Programmes' should be organized in terms of whole Phase planning as well as for each year or grade within the Phase, called a 'Work Schedule' and also include activities, single or linked, called 'Lesson Plans'. The aim of the Learning Programme according to the NCS is "to design and sequence teaching, learning and assessment activities that will result in meaningful and relevant learning" (p. 5). Following a curriculum in which the geometry activities in the current year build on the geometry activities experienced the previous year addresses both the *Sequential* and *the Extrinsic/Intrinsic* properties of the Van Hiele theory Conference Board of the Mathematical Sciences, 2001; Chard *et al.*, 2008). The NCS provides guidelines on how to go about developing such a learning programme.

As reported in Section 2.6.3, the amount of time needed for learners to progress from one Van Hiele level to the next is difficult to ascertain with any reliable degree of accuracy. The NCS describes activities on Van Hiele Level 1 for Grades 0-4, whereas the activities suggested for Grade 5 align themselves more to the second Van Hiele level (see Tables 2, 3, and 4). Hence, the NCS allows five years from Grades 0-5 for a learner to progress from a *Pre-visualisation* level of thinking to the first Van Hiele level, which is the *Visualisation* level, and then to the *Analysis* level being Van Hiele Level 2. The early writings of Dina Van Hiele (Atebe & Schäfer, 2008; Usiskin, 1982) propose that it requires between 20-30 lessons to progress from one level to the next, the expected progress suggested by the NCS is reasonable. Should it take as long as fifty lessons, which Dina van Hiele (Usiskin, 1982, p. 6) suggested that 12-year-olds would require when progressing from Level 2 to Level 3, this would mean that a learner would need 150 lessons of geometry instruction from Grade 0-5 to be able to function at the *Analysis* level whilst in Grade 6. This roughly calculates to 30 lessons of geometry instruction per year and about 7 lessons each quarter. At the other end of the scale, if it took only 20 lessons to develop a greater level of geometry understanding, then progression to Level 2 would require a minimum of 60 geometry experiences, which equates to 12 lessons per year.

5.2.2 OBJECTIVE 2: DESCRIBE THE CURRENT INSTRUCTIONAL PRACTICES IN THE SCHOOL

In this private, independent, co-educational school, the class size is limited to 25 learners. There are usually only two classes per grade and the teachers of the Grades 0-2 classes have a teachers' assistant to assist them with administrative duties. The teachers plan the structure of their own timetables around specialist subjects such as Music or computer lessons and usually plan the curriculum for the year together or divide the drawing up of the Year Plans between them according to Learning Areas. Both teachers in the grade work from the same Year Plan. The NCS is used as a reference in determining the content and the time allocated to each Learning Area and each Learning Outcome within that Learning Area.

The time allocated for the development of geometry understanding as reflected in the teachers' timetables and Year Plans is consistently less than the time recommended for the same in the NCS from Grades 0-3 and in Grade 5.

The types of the geometry lessons facilitated by the teachers evident in their Year Plans, Lesson Plans and during classroom observations all involved the learners in some sort of group or class activity, implying a constructivist approach to learning. Seven of the nine teachers responded in the teachers' questionnaire that they regarded using concrete examples or manipulatives and practical activities as the best way to teach geometry. In response to the question about their access to and use of concrete aids or manipulatives, all the teachers said that they had easy access to equipment and made use of it. To quantify how frequently these teachers used manipulatives is difficult because responses such as 'often' and 'loads' are subjective terms. The contexts and the manipulatives evident in the planning and those that were used during the classroom observations were familiar and appropriate to the learners' levels of understanding.

Although 11 of the 13 the teachers who completed a questionnaire acknowledged the value of understanding their learners' level of thinking, there was insufficient data to establish with any certainty that these teachers engaged in cognitive dialogue regularly and consistently. It is not clear in this study to what extent the activities in the teachers' planning documents included the learners' conceptual understanding and to what extent this information informed their teaching practices.

5.2.3 OBJECTIVE 3: DESCRIBE THE LEARNERS' LEVELS OF GEOMETRY THINKING

The Van Hiele test (Usiskin, 1982) was used to assess the learners' levels of thinking from Grade 1-5. Figure 24 presents a summary of the results and shows that 53. 2% of the Grade 1 learners scored less than 60% for the first five questions on the test, placing their levels of thinking below the *Visualisation* level. The number of learners whose thinking remained below the *Visualisation* level was 38% lower in Grade 5. Figure 26 indicates a steady decrease in the percentage of learners' thinking at a *Pre-visualisation* level from Grades 1-3, however, the significant difference between the results at Grade 3 and Grade 4 warrants further investigation. The distribution of learners' levels of geometry thinking in this study is similar to those reported by Wu and Ma (2006). Furthermore it is of particular interest to note that in a later publication in which Wu and Ma used the Grey Relational Analysis to analyse the Wu–Ma Geometry Test (WMGT) of the Van Hiele levels, they concluded that the WMGT is a good measure of geometrical concepts and that "the GRA model demonstrated satisfactory prediction accuracy" (Wu & Ma, 2010, p. 113).

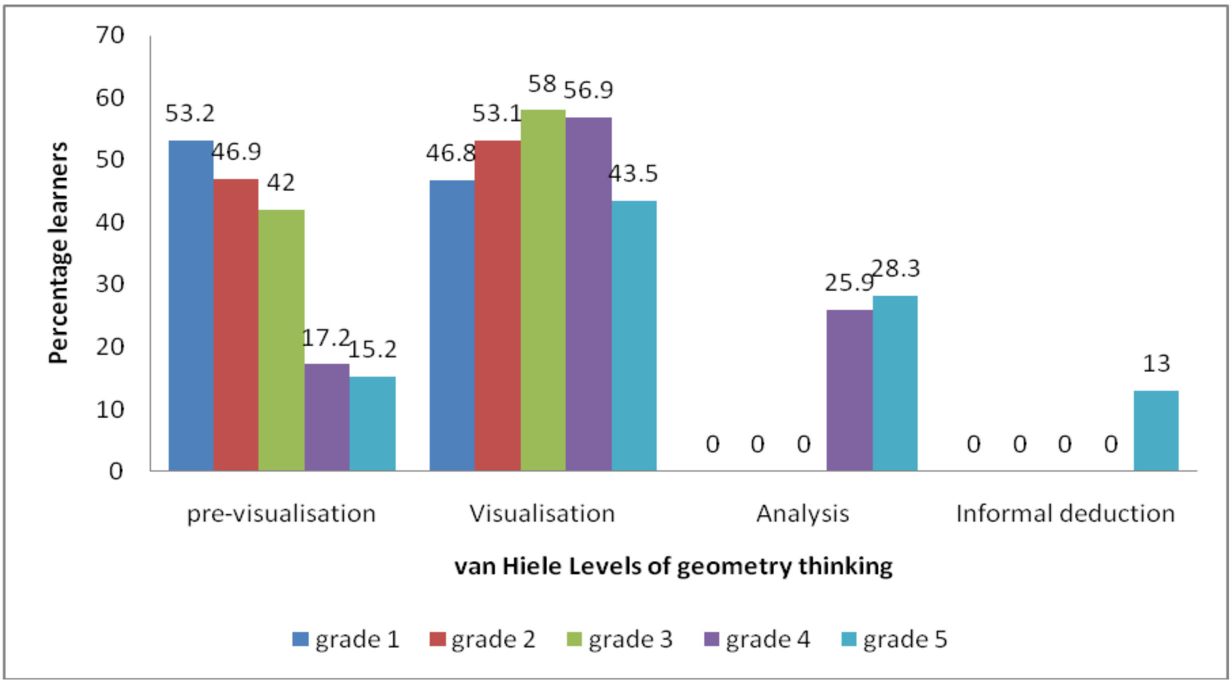

FIGURE 26: Distribution of learners' levels of geometry thinking 2010

5.2.4 OBJECTIVE 4: COMPARE THE CURRENT GEOMETRY INSTRUCTIONAL PRACTICES WITH PRACTICES SUGGESTED IN LITERATURE

The results in Figure 26 seem to confirm that the instructional practices at this school support the development of geometry understanding. When juxtaposed alongside research and the recommendations of the NCS, geometry instructional practices in this school, compare favourably to the teachers' professed and observed practice of using concrete aids (Arzarello *et al.*, 2005; Bransford *et al.*, 2000; Rudd *et al.*, 2008) and tasks that engage the learners actively in the development of geometry insight (Battista, 2007; Sfard, 2007). Furthermore, from their responses in the questionnaire, these teachers seem to have a positive disposition with respect to teaching Mathematics and seem to value the study of geometry.

The types of geometry activities described in the teachers' planning documents and lessons observed during the course of the study seemed to be appropriate to the learners' levels of understanding and were relevant to the learners' contexts (Carpenter *et al.*, 2000; Webb *et al.*, 2009; Bransford *et al.*, 2000). However, a lack of focus on geometry vocabulary and detail in the Lesson Plans implies that the language used in facilitating geometry understanding may be a point for further investigation and development (Atebe & Schäfer, 2008; Gerace, 1992; Rudd *et al.*, 2008; Sfard, 2007; Van Hiele, 1986). The cases presented in Chapter 4 show that the instructional practices at this school do not consistently include all five of the phases of learning (Crowley, 1987). Only 25% of the lessons observed progressed to the third phase of learning where the level's system of relations starts to become apparent.

5.3 REFLECTION AND DISCUSSION

5.3.1 REFLECTION ON THE THEORETICAL FRAMEWORK

The theoretical framework that informed this study was based on the Van Hiele theory. This theory has received criticism on three fronts. Firstly, the nature of the levels of thinking were first thought by the Van Hieles to be discrete, but further research showed the levels to be dynamic and continuous (Battista, 2007; Burger & Shaunessy, 1986; Gutiérrez, 1992; Gutiérrez & Jaime, 1998; Senk, 1989; Usiskin, 1982). Secondly, assigning a learner to a particular level of thinking proved problematic as research shows that learners develop several Van Hiele levels simultaneously over an unspecified amount of time (Gutiérrez *et al*., 1991; Wu & Ma, 2006). Finally, literature contains discourse regarding the application of the model (Burger & Shaughnessy, 1986; Mayberry, 1987; Usiskin, 1982).

Despite the reported shortcomings of the theory, the model provided logical, well-structured guidelines, which allowed the researcher to construct a framework to fit the aim of the investigation. Although the theory describes instructional practices to promote the progression from one Van Hiele level to the next, not much is found in literature as to how these descriptions translate into the school context. The NCS provides guidelines on the content and examples of activities to be used but provides insufficient detail regarding the types of experiences and language that should be used at each grade. There is also a shortfall in literature on the subject of evaluating a geometry curriculum. To this end, the theoretical framework provides a perspective, albeit one, which may need refining through further research.

5.3.2 REFLECTION ON THE METHODOLOGY

The methodology used in this investigation was appropriate for the aim of the study in that it made use of qualitative and quantitative data to describe a phenomenon within a particular context. The data seemed relatively easy to collect, however, the analysis of it proved more challenging. Data from the Year Plans and the Lesson Plans were incomplete as in the case of the time allocated to geometry lessons in Grade 4. Trying to establish the different types of activities described in the *phases of learning* (Crowley, 1987) was also problematic in that many of the planning documents lacked sufficient detail confidently to classify them and that some of the documentation was incomplete.

Replacing the interviews with a questionnaire may have limited the depth and volume of the data gained. Nevertheless, this approach resulted in the consideration of how a tool for efficiently evaluating a school's geometry curriculum may be developed.

5.3 CONCLUSIONS AND RECOMMENDATIONS

1. The results of the learners' levels of geometry thinking seem to indicate that the instructional practices at this school support progression through the Van Hiele levels in that the number of learners below Level 1 decreases as one move from Grades 1-5 by 38%. Similarly, the number of learners on Level 1 in Grade 5 is 13.4% more than in Grade 4, and the percentage of learners who have advanced from Level 1 in Grade 4 is 15.4% greater in Grade 5. The types of geometry activities described in the NCS for Grades 5 and 6 are in line with those suggested by Crowley (1987) for Van Hiele Level 2 – *Analysis thinking*. This finding forces one to consider whether the progression indicated in the results is adequate since 58.7% of the Grade 5 learners' levels of thinking is reflected at being at Level 1 or below.

2. The *Sequential* characteristic of the Van Hiele theory is supported by the significant lack of difference between the Grades 4 and Grade 5 results at the *Pre-visualisation* level. The sequential characteristic states that a learner cannot progress to level *(n)* without having experienced level (*n-1*). The learner's progression is dependent on receiving the correct instruction (the *Advancement* property). Instruction incongruent

with the learner's level of thinking (*Mismatch*) impedes progression and places learners in the precarious position of possibly being stuck at a particular level if their development of geometry understanding is out of step with the curriculum being facilitated. The results of the Grades 4 and 5 learners at the *Pre-visualisation* level warrant further investigation.

3. The time allocated to the teaching of Mathematics and more specifically to the facilitation of geometry concepts from Grades 0-5 consistently falls short of the recommendation of the NCS and needs to be addressed in order to accommodate the types of geometry experiences described in literature.

4. The types of geometry experiences evident in this topographical survey of instructional practices lack the depth described in the five phases of learning proposed by Crowley (1987). Only 25% of the classroom observations reported activities, which included the third phase of learning. The learners cannot achieve a level of thinking without going through the processes described in these phases of learning. Learners may imitate the reasoning and language of a Van Hiele level presented to them, but may not successfully be able to appropriate or apply this reasoning to new contexts (Atebe & Schäfer, 2008). Once again, these learners are at risk of stagnating at a particular level with each academic cycle where the geometry curriculum is presented at a level incongruent to their level of understanding.

   In order to more accurately to determine the types of experiences the teachers are planning to facilitate, a more detailed Year Plan should be presented in each grade. It is recommended that these Year Plans centre on modules which reflect a concept or part of a larger concept, for example, 'angles'. Shifting the paradigm from a time-focused schedule to one that plans around a concept/module, places the concept at the centre of the decision-making process and allows the teacher greater opportunity to reflect on *what* is being taught and consequently *how* it should be taught.

5. There seemed to be no deliberate continuity of the geometry curriculum evident in the planning documents between the grades and between the Foundation and Intermediate Phases. The following excerpt from the teachers' questionnaire is used here to corroborate this conclusion:

Teacher 1: Yes, I do feel we get enough support but we need to decide on a particular programme and everyone must do it well and do it properly so that we are working together towards a common goal.
Teacher 2: To have more discussion around concepts. E.g. fractions you cover this, I do that. So that it's not just left to paper, but dialogue happens to open it up and make me aware of blind spots I have in an area.

In certain cases, adherence to the Assessment Standards presented in the NCS provides the only link from one grade to the next. In short, the types of geometry experiences and when these experiences are made available to the learners are at odds with the S*equential* and E*xtrinsic/Intrinsic* properties of the Van Hiele theory. It is recommended that the Year Plan within and across the Foundation and Intermediate Phases follow a similar format, so that gaps and redundancies in the conceptual development of this learning area would be easily identified.

6. The results of the learners' levels of geometry thinking and evidence of group work and the use of manipulatives as well as activities relevant to the learners' personal contexts imply that sound teaching practice is happening to some extent in this school.

5.4 IMPLICATIONS OF THE STUDY

This study is limited contextually but carries the following implications in terms of policy, practice and research:

- Changes in timetabling and the re-formatting of planning may cause internal policy pressures, however, this will highlight the time allocated to various learning areas and will reveal redundancies and overlaps in the planning.

- Reflect on time spent on learning areas, curricula cohesion, subject specific language, and curriculum content within and across the Foundation and Intermediate Phases will give the teachers a greater sense of security regarding what, when and how things are taught Establish curriculum cohesion within and between the Phases.

- Further research into creating an efficient reliable means of assessing the levels of geometry thinking of primary school learners is necessary.

A final thought: during the course of my research I not only found a gap within my own structure, but also within the department of which I am head. Just the fact of doing this research and asking the questions that were asked has made a difference to me and my colleagues. Implementing the ideas and innovation which has suggested themselves across this research can only promote the improvement in the level of understanding mathematics at this school.

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ADDENDUMS

ADDENDUM A:
CDASSG Van Hiele Geometry Test

ADDENDUM B:
Observation Schedule

ADDENDUM C:
Teacher Questionnaire

ADDENDUM D:
Letters of informed consent

ADDENDUM A:

CDASSG Van Hiele Geometry Test

Test Number: __________

VAN HIELE GEOMETRY TEST*

Directions

Do not open this test booklet until you are told to do so.

This test contains 25 questions. It is not expected that you know everything on this test.

There is a test number in the top right hand corner of this page. Write this number in the coo responding place on your answer sheet.

When you are told to begin:

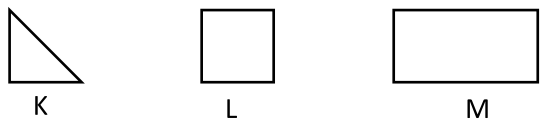
1. Read each question carefully.
2. Decide upon the answer you think is correct. These is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.
3. Use the space provided on the answer sheet for figuring or drawing. Do not mark in this test booklet.
4. If you want to change an answer, completely erase the first answer.
5. If you need another pencil, raise your hand.
6. You will have 20 minutes for this test.

Wait until the proctor says that you may begin.

*This test is based on the work of P.M. van Hiele.

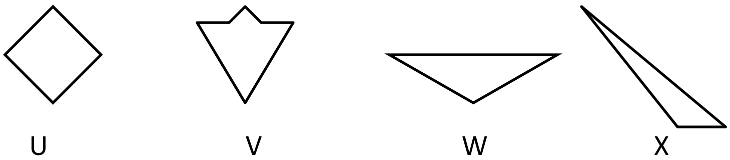
VAN HIELE GEOMETRY TEST

1. Which of these are squares?



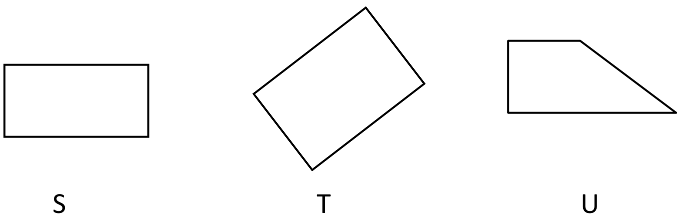
   A. K only
   B. L only
   C. M only
   D. L and M only
   E. All are squares

2. Which of these are triangles?



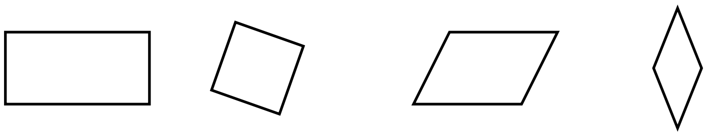
   A. None of these are triangles.
   B. V only
   C. W only
   D. W and X only
   E. V and W only

3. Which of these are rectangles?



   A. S only
   B. T only
   C. S and T only
   D. S and U only
   E. All are rectangles.

4. Which of these are squares?



F G H I

A. None of these are squares.

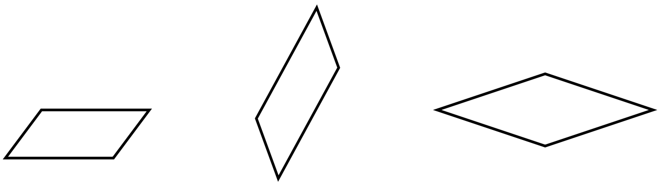
B. G only

C. F and G only

D. G and I only

E. All are squares.

5. Which of these are parallelograms?



J M L

A. J only

B. L only

C. J and M only

D. None of these are parallelograms.

E. All are parallelograms.

6. PQRS is a square.

Which relationship is true in all squares?

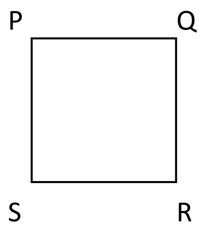
A. $\overline{PR}$ and $\overline{RS}$ have the same length.

B. $\overline{QS}$ and $\overline{PR}$ are perpendicular.

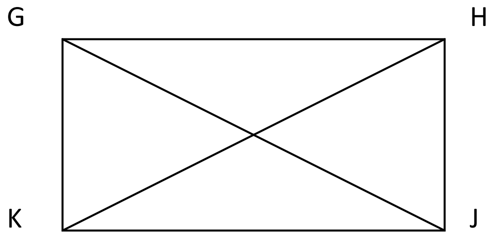
C. $\overline{PS}$ and $\overline{QR}$ are perpendicular.

D. $\overline{PS}$ and $\overline{QS}$ have the same length.

E. Angle Q is larger than angle R.



7. In the rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.



Which of (A)-(D) is not true in every rectangle?

A. There are four right angles.

B. There are four sides.

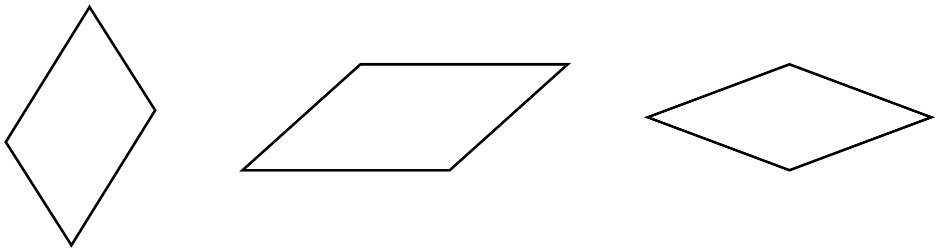
C. The diagonals have the same length.

D. The opposite sides have the same length.

E. All of (A)-(D) are true in every rectangle.

8. A rhombus is a 4-sided figure with all sides of the same length.

Here are three examples.



Which of (A)-(D) is not true in every rhombus?

A. The two diagonals have the same length.

B. Each diagonal bisects two angles of the rhombus.

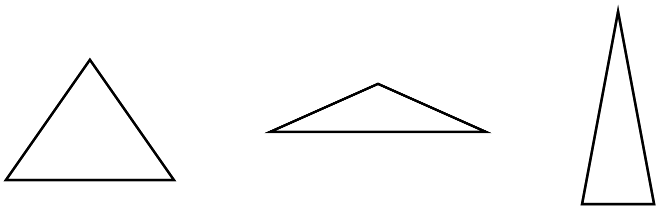
C. The two diagonals are perpendicular.

D. The opposite angles have the same measure.

E. All of (A)-(D) are true in every rhombus.

9. An isosceles triangle is a triangle with two sides of equal length.

Here are three examples.



Which of (A)- (D) is true in every isosceles triangle?

A. The three sides must have the same length.

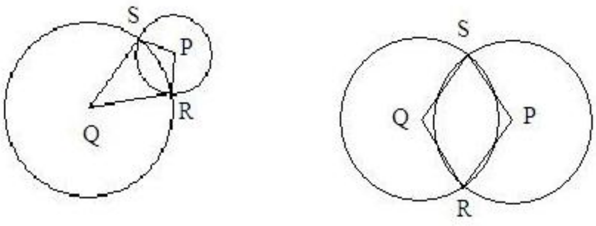
B. One side must have twice the length of another side.

C. There must be at least two angles with the same measure.

D. The three angles must have the same measure.

E. None of (A)- (D) is true in every isosceles triangle.

10. Two circles with centres P and Q intersect at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)- (D) is not always true?

A. PRQS will have two pairs of sides of equal length.

B. PRQS will have at least two angles of equal measure.

C. The lines $\overline{PQ}$ and $\overline{RS}$ will be perpendicular.

D. Angles P and Q will have the same measure.

E. All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.

Statement 2: Figure F is a triangle.

Which is correct?

A. If 1 is true, then 2 is true.
B. If 1 is false, then 2 is true.
C. 1 and 2 cannot both be true.
D. 1 and 2 cannot both be false.
E. None of (A)-(D) is correct.

12. Here are two statements.

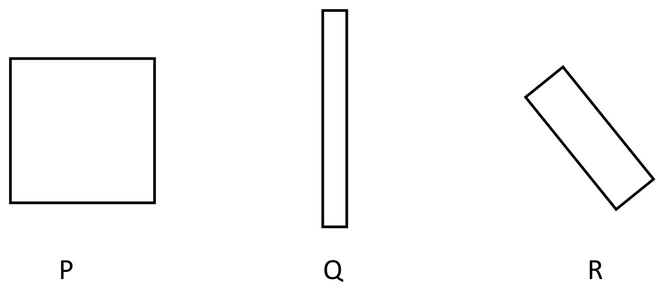
Statement S: ΔABC has three sides of the same length

Statement T: In ΔABC, ∠B and ∠C have the same measure.

Which is correct?

A. Statement S and T cannot both be true.
B. If S is true, then T is true.
C. If T is true, then S is true.
D. If S is false, then T is false.
E. None of (A)-(D) is correct.

13. Which of these can be called rectangles?

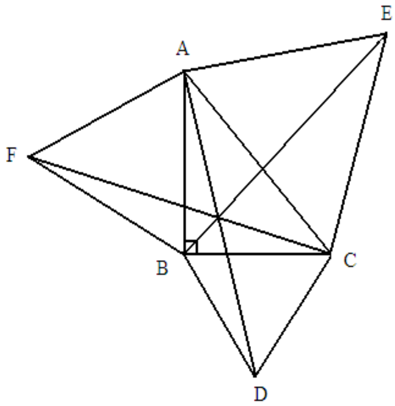


A. All can.
B. Q only
C. R only
D. P and Q only
E. Q and R only

14. Which is true?
    - A. All properties of rectangles are properties of all squares.
    - B. All properties of squares are properties of rectangles.
    - C. All properties of rectangles are properties of all parallelograms.
    - D. All properties of squares are properties of all parallelograms.
    - E. None of (A)-(D) is true.

15. What do all rectangles have that some parallelograms do not have?
    - A. Opposite sides equal
    - B. Diagonals equal
    - C. Opposite sides parallel
    - D. Opposite angles equal
    - E. None of (A)- (D)

16. Here is a right triangle ABC. Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that AD, BE, and CF have a point in common. What would this proof tell you?

- A. Only in this triangle drawn can we be sure that $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
- B. In some but not all right triangles, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
- C. In any right triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
- D. In any triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.
- E. In any equilateral triangle, $\overline{AD}$, $\overline{BE}$ and $\overline{CF}$ have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal length.

Property S: It is a square.

Property R: It is a rectangle.

Which is true?

A. D implies S which implies R.

B. D implies R which implies S.

C. S implies R which implies D.

D. R implies D which implies S.

E. R implies S which implies D.

18. Here are two statements.

I: If a figure is a rectangle, then its diagonals bisect each other.

II: If the diagonals of a figure bisect each other, the figure is a rectangle.

Which is correct?

A. To prove I is true, it is enough to prove that II is true.

B. To prove II is true, it is enough to prove that I is true.

C. To prove II is true, it is enough to find one rectangle whose diagonal bisect each other.

D. To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.

E. None of (A)- (D) is correct.

19. In geometry:

A. Every term can be defined and every true statement can be proved true.

B. Every term can be defined but it is necessary to assume that certain statements are true.

C. Some terms must be left undefined but every true statement can be proved true.

D. Some terms must be left undefined and it is necessary to have some statements which are assumed true.

E. None of (A)-(D) is correct.

20. Examine these three sentences.

Two lines perpendicular to the same line are parallel.

A line that is perpendicular to one of two parallel lines is perpendicular to the other

If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

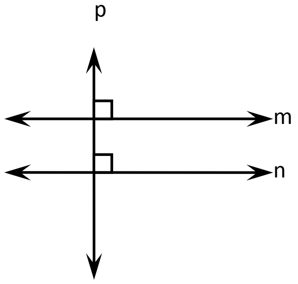
A. (1) only

B. (2) only

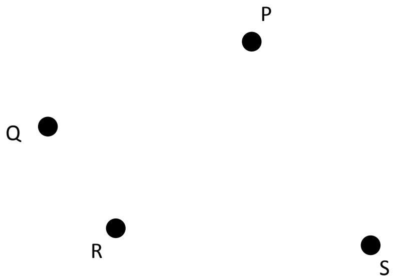
C. (3) only

D. Either (1) or (2)

E. Either (2) or (3)



21. In F-geometry, one that is different from the one you are used to, there are exactly four points and six lines. Every line contains exactly two points. If the points are P, Q, R and S, and the lines are {P, Q}, {P, R}, {P, S}, {Q, R}, {Q, S}, and {R, S}.



Here are how the words "intersect" and "parallel" are used in F-geometry. The lines {P, Q} and {P, R} intersect at P because {P, Q} and {P, R} have P in common.

The lines {P, Q} and {R, S} are parallel because they have no points in common.

From this information, which is correct?

A. {P, R} and {Q, S} intersect.

B. {P, R} and {Q, S} are parallel.

C. {Q, R} and {R, S} are parallel.

D. {P, S} and {Q, R} intersect.

E. None of (A)-(D) is correct.

22. To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?
    - A. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
    - B. In general, it is impossible to trisect angles using only a compass and a marked ruler.
    - C. In general, it is impossible to trisect angles using any drawing instruments.
    - D. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
    - E. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

23. There is a geometry invented by a mathematician J in which the following is true:
    The sum of the measures of the angles of a triangle is less than 180°.

    Which is correct?
    - A. J made a mistake in measuring the angles of the triangle.
    - B. J made a mistake in logical reasoning.
    - C. J has a wrong idea of what is meant by “true”.
    - D. J started with different assumptions than those in the usual geometry.
    - E. None of (A)-(D) is correct.

24. The geometry books define the word rectangle in different ways.

    Which is true?
    - A. One of the books has an error.
    - B. One of the definitions is wrong. There cannot be two different definitions for rectangle.
    - C. The rectangles in one of the books must have different properties from those in the other book.
    - D. The rectangles in one of the books must have the same properties as those in the other book.
    - E. The properties of rectangles in the two books might be different.

25. Suppose you have proved statements I and II.

I: If p, then q.

II: If s, then not q.

Which statement follows from statements I and II?

A. If p, then s.

B. If not p, then not q.

C. If p or q, then s.

D. If s, then not p.

E. If not s, then p.

ADDENDUM B:
Observation schedule

Observation Schedule.

Class code: Type here	Time of the lesson: Type here	Duration of the lesson: Type here

Activities: Type here

Resources used: Type here

Type of geometric experience:

Phase 1: Inquiry/information	Type here
Phase 2: Directed orientation	Type here
Phase 3: Explication	Type here
Phase 4: Free orientation	Type here
Phase 5: Integration	Type here

Language and concepts used in planning: Type here

Language and ideas used in execution: Teacher talk: Type here	Range of levels of learners' thinking (VHGT) Type here	Language and ideas used in execution: Learner talk: Type here

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ADDENDUM C:

Teacher Questionnaire

Teacher Questionnaire

Section 1: Biographical

Name:	1.1.	
Teaching experience: - General:	1.2.i.	
- Mathematics:	1.2.ii.	
Years in this school:	1.3.	
Grades taught:	1.4.	

Section 2: Experiential

Feelings regarding mathematics: - personal ability:	2.1.i.	
- teaching:	2.1.ii.	
Value of mathematics (geometry):	2.2.	
Define geometry:	2.3.	
Learning theory:	2.4.	
Cognitive dialogue:	2.5.	

Section 3: Context

Teaching resources:	3.1.	
Support:	3.2.	

Section 4: Comments

	4.1.	

Dear Colleague,

In light of the current demands on your time and energy, I have decided that a written questionnaire would be more convenient than the interviews that were planned. In addition, you may feel more comfortable writing some comments than telling me in person. Please know that I am available for you if you want to discuss anything concerning Mathematics.
Please answer as honestly and as comprehensively as you can. Writing your name on this document is optional. Please place your completed questionnaire in my pigeonhole before the close of school on Friday. Thank you.

Section 1: Biographical

Name: (optional)..

Teaching experience: – general..No. years

- Mathematics ....No. years

No. years in this school:..

Grades taught:..

Section 2: Experiential

Feelings regarding Mathematics: – personal ability

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Feelings regarding Mathematics: - teaching

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Value of Mathematics (geometry): (Do you think that it should be optional?)

……………………………………………………………………………………………

……………………………………………………………………………………………

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……………………………………………………………………………………………

Define geometry: (In your view what is geometry about?)

Learning theory: (How do people learn? What is the best way to teach geometry?)

Cognitive dialogue: (Do you always understand what your learners are thinking/ asking? How helpful is it to be able to understand your learners?)

Section 3: Context

Teaching resources: Do you have easy access to manipulatives? How often do you use manipulatives? How do you value manipulatives?

Support: Do you feel you have sufficient support from the school? In what way could the school support you better?

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Section 4: Any comment you would like to make.

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Thank you for your time and honesty.

ADDENDUM D:

Letters of consent

PO Box 33760
Glenstantia, Pretoria
0010, South Africa
E-mail: admin@hatfieldcs.co.za
PBO Reg. No. 18/11/13/4459

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8 February 2010

TO WHOM IT MAY CONCERN

I understand that Mrs C. Bleeker is to carry out a research study for her Masters of Education degree at the University of Pretoria. This study involves a group administration of an assessment designed to measure the level of geometric thinking. The learners in Grades 1-5 and their teachers will be involved in this process. The study may also include interviews with the staff concerned.

I am aware that the assessment will take place during school time and that the interviews will take place on the school premises. Mrs. Bleeker has taken steps to ensure the confidentiality of the respondents and will obtain parental consent.

I am convinced that Mrs Bleeker has the learners' best interests at heart and I support this study. I hereby grant her permission to conduct her research at Hatfield Christian School.

Kind regards

Graeme Holloway
EXECUTIVE PRINCIPAL

'Training leaders to serve nations'
Member of the Independent Schools' Association of Southern Africa

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Faculty of Education
Department of Science, Mathematics and Technology Education

8 February 2010

LETTER OF INFORMED CONSENT

Dear Parents/Guardians

As part of a research project I am to undertake the task of gathering data about levels of thinking in Mathematics and in particular Geometry.

The aim of the study is to establish what will help learners learn better.

To be able to do this project we need your consent for your child to participate.
By participating in the project, your child will be asked to complete a short, non-threatening assessment which will take place in their classroom during school hours and at no cost to you. Your child may also be videod during the assessment and in class as part of classroom observation. This digital recording will not be released to any-one and is for the sole purpose of the researcher to ensure that the assessment was accurately observed. Your child will in no way be harmed in this process, in fact, by doing this research I hope to find out how to improve your child's learning experience.

This study is an evaluation of the learning environment and levels of geometric thinking and not about an individuals' mathematical ability. Your child's identity will not be disclosed either at school or in any report.
Should you have any queries you may contact Mrs. Bleeker at 012 993 1285.

Thank you for helping me in this regard.

____________________________ ____________________ ____________________

Mr. G Holloway (Executive Principal) Mrs. C Bleeker Dr. G Stols

Please complete this form and send it back to school as soon as possible. Thank you.

I, ______________________________, parent/guardian of ___________________________,

in grade, _________ give permission for my child to participate in this study. I understand that should they wish to withdraw from the project, they may do so, and that their identity will not be disclosed either at school or in any report.

___________________________ ______________________

Signature of parent/guardian Date

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Faculty of Education

Department of Science, Mathematics and Technology Education

8 February 2010

LETTER OF INFORMED CONSENT TO A MINOR CHILD

Project title: 'Levels of geometry thinking"

To be read to children under the age of 18 years.

Why am I here?

Sometimes when we want to find out something, we ask people to join something called a project. In this project we will want to ask you about yourself and we will ask you to participate in activities focused on your own development and learning. Before we ask you to be part of this study, we want to tell you about it first.

This study will give us a chance to see how we, together with your school and teachers, can help you address career and learning challenges that you may have here at school. We also want to help you gain some skills in your learning here at school so that you can be better equipped to support yourself during your education and after leaving school. We are asking you to be in this study because your parents/guardians have agreed that you can be part of our study.

What will happen to me?

If you want to be part of our study, you will spend some time with us answering some questions. This will be done at school in some of your Math lessons. You will be asked to complete an activity about shapes. There are no right or wrong answers, only what you feel is best.

If you would like to say something about the activity, we may want to record your voice so that other people may hear what you have to say but they will not know who you are. We will not use your name on the recording or in any report.

Will the project hurt?

No, the project will not hurt. The activity will not take a long time but you can take a break if you are feeling tired or if you do not want to answer all the questions at one time. If you do not want to answer a question or say anything about the activity, you do not need to. All of your answers will be kept private. No one, not even someone in your family or your teachers will be told your answers.

Will the study help me?

We hope this study will help you feel good about yourself and learn more about what you can do in school and possibly in your job or career, but we do not know if this will happen.

What if I have any questions?

You can ask any questions you have about the study. If you have questions later that you do not think of now you can phone Mrs Bleeker at 012 348 2970 or you can ask her next time you see her at school.

Do my parents/guardians know about this project?

This study was explained to your parents/guardians and they said you could be part of the study if you want to. You can talk this over with them before you decide if you want to be in the study or not.

Do I have to be in the project?

You do not have to be in this project. No one will be upset if you do not want to do this. If you do not want to be in the project, you just have to tell us. You can say yes or no and if you change your mind later, you do not have to be part of the project anymore. It is up to you.

(a) Writing your name on this page means that you **agree to be in the project** and that you **know what will happen to you** in this study. If you decide to quit the project, all you have to do is tell the person in charge.

Signature of the learner **Date**

Signature of the researcher **Date**

Signature of the Supervisor **Date**

If you have any further questions about this study, you can phone the investigator, Mrs Bleeker. If you have a question about your rights as a participant, you can contact the University of Pretoria, Faculty of Education Ethics committee at 012 420 3751.

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LETTER OF INFORMED CONSENT

Dear Teachers/ teacher assistants

As part of a research project we are to undertake the task of gathering data about levels of thinking in Mathematics and in particular Geometry. The aim of the study is to establish what will help learners learn better. To be able to do this project we need you to consent to participate.

The project is about an evaluation of the learning environment and levels of geometric thinking and not about an individuals' mathematical ability. This project will involve an assessment of your learners' level of geometric thinking through an activity based group task. This may encroach upon your lesson time, however, it should yield valuable information which will be useful in your teaching.

It will be necessary for a lesson or two of geometry teaching in action to be observed. It will be necessary that the lesson be digitally recorded (video taped) to ensure that reliable data are captured. You will also be asked to participate in a one-on-one interview with the researcher. You will be given a chance to review what you have said in the interview and check that what has been recorded is an accurate account of the conversation.

The aim of the research project is to investigate the links between the levels of geometric thought and the learning environment and instructional practices. Please note that the project focuses on a very specific topic and therefore cannot in any way comment on your competencies as a teacher. This research is not to evaluate you as a teacher. Furthermore your name will not appear in any report and your anonymity will remain a high priority during the research and in the reporting process. You will in no way be harmed in this process.

Should you have any queries you may contact Mrs. Bleeker at 012 993 1285.

Thank you for helping us in this regard.

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Mrs. C Bleeker

Please complete this form and send it back to the school office as soon as possible. Thank you.

I, ____________________________, am willing to participate in the above mentioned research project. I understand that should I wish to withdraw from the project, I may do so, and that my identity will not be disclosed either at school or in any report.

_________________________ Signature

_____________________ Date