2.3 Impedance Matching

This section will present a brief review of some of the more important impedance matching network design techniques. The strengths and weaknesses of each type of approach will be considered to justify the development of a new algorithm.

Graphical impedance matching is considered in Section 2.3.1, and analytic approaches are reviewed in Section 2.3.2. The real-frequency techniques are presented in Section 2.3.3 and represent the best techniques available today. The application of genetic algorithms to impedance matching network design is considered in Section 2.3.4.

2.3.1 Graphical Techniques

A number of graphical techniques for impedance matching network design are available in the literature. This section will consider some of these techniques.

The vast majority of graphical techniques use the Smith Chart which is described in most electromagnetics and high frequency design textbooks such as Gonzalez [3], Pozar [11], and Marshall and Skitek [49]. There are however some graphical techniques that do not use the Smith Chart such as the technique proposed by Hamid and Yunik [50].

The simplest of these graphical techniques are the single and double stub matching techniques considered by a large number of textbooks including Gonzalez [3], Marshall and Skitek [49], and Pozar [11]. These techniques are usually only considered for single matching problems, but they can be generalised to the double matching case. The most significant limitation of these techniques is that the impedances of the transmission lines used are fixed, removing one degree of freedom from the design procedure for each section of transmission line. This can however be an advantage where transmission lines are only available with a few values of characteristic impedance.

Another well-known graphical technique is the design of lumped element matching networks using an immitance Smith Chart (a Smith Chart with both impedance and admittance circles). This technique is considered by many authors including Gonzalez [3]. The process is essentially to move along a constant resistance circle when a reactive element is added in series, and along a constant conductance circle when a reactive element is added in parallel. This technique can be used for double matching problems, but is usually only explicitly presented for the single matching case. The bandwidth of these matching networks can be estimated from the transformation-Q values obtained during synthesis.

The paper by French and Fooks [51] deals with the design of a matching network with two transmission lines in series. The problem considered is that of single matching. The characteristic impedances of the transmission lines is fixed in advance, removing two degrees of freedom from the design algorithm. As before this does have the advantage of allowing transmission lines with readily available characteristic impedances to be used. The major advantage of this technique is that it is possible to plot the impedances that can be matched using two series transmission lines of given characteristic impedances on the Smith Chart, allowing insight into the problem to be gained.

The problem of matching two impedances using a series transmission line has been considered by a number of authors. The techniques are all based on the fact that the locus of the input impedances of a series transmission line as the length varies is a circle and the characteristic impedance is given by the geometric mean of the intercepts with the real axis. French and Fooks [52] describe a single matching technique that uses a Smith Chart normalised to the desired impedance to find the characteristic impedance of the series transmission line. Somlo [53] describes a more general double matching technique where a circle with its centre on the real axis of the Smith Chart is constructed through the two impedances to be matched to find the characteristic impedance of the series transmission line. Both these techniques then re-normalise the Smith Chart to the transmission line's characteristic impedance to find the line length. Arnold [54] proposes a technique that can find the line length without re-normalisation, but the geometric constructions are more complex and often have to be repeated to obtain a good result. Day [55] combines Somlo's method of determining the characteristic impedance with Arnold's method of determining the line length to obtain a double matching technique that does not require re-normalisation. The major advantage of this technique is the impedances that can be matched to a given impedance using a series transmission line can be plotted on the Smith Chart allowing a greater understanding of the problem to be gained.

The major advantages of graphical techniques are that they require very few calculations and they provide insight into the matching procedure that cannot be obtained with other techniques. The major disadvantages are that these techniques only consider a single frequency explicitly, the geometric constructions on the graph can be complex, and inaccuracies in reading values from the graph can limit accuracy.

2.3.2 Analytic Methods

This section will consider impedance matching methods that use analytic theory. These methods allowed the formulation of gain-bandwidth theory that sets theoretical limits on the quality of match that can be obtained for a given load. Analytic impedance matching methods were essential before the advent of low cost, high performance computers. This section will give a brief history of analytic impedance matching.

The first significant work on impedance matching was published by Bode [56]. Bode analysed RC and RL loads and proved the existence of gain-bandwidth limits. Fano [57, 58] extended Bode's work on gain-bandwidth theory and matching network design to consider more complex cases including RLC loads. Youla [59] simplified Fano's work by using the principle of complex normalisation. Later a number of approaches to consider the case where both the source and load are networks were developed, including those proposed by Carlin and Yarman [8], and Chen and Satyanarayana [60].

These results were still extremely complex and difficult to use, so a number of papers containing simplified results were published. The approach used by most early papers, including Bode's [56] classic work, was to include graphs that allowed the design parameters to be read from the graph. Another approach to the problem was to publish design tables. This approach was used by Matthaei [61] and Cristal [62] to publish optimum networks for matching two resistances using Chebyshev and Butterworth responses respectively. The last approach involved deriving formulae for the values of the circuit elements required to design an impedance matching network. As an example, Chen [63], and Chen and Kourounis [64] have published formulae to match a load consisting of a RLC network to a resistor.

As wideband circuits began to gain popularity, the need to level the gain of a circuit over a frequency band became apparent. The is necessary because an active device's gain decreases with increasing frequency at high frequencies. Pitzalis and Gilson [65] used a computer to generate tables of networks with 4, 5, and 6 dB per decade frequency slopes to compensate for active device gain rolloff. Ku and Peterson [66] extended this work by publishing an algorithm for obtaining matching networks with specified frequency slopes without the use of a computer.

Transmission lines are considerably more complex to analyse than lumped elements, so much less work has been done on the analysis of impedance matching networks with distributed elements, and results tend to be incremental improvements of previous work. Most of the work has concentrated on the use of a number of series transmission lines of the same length (commensurate lines). An equiripple frequency response gives much wider bandwidth than the maximally flat case, so the equiripple frequency response has been extensively considered in the literature. The first results were published by Collin [67] and Cohn [68], with a more rigorous formulation being developed by Riblet [69], and later, extensions by Young [70] were made available. Young [71] used these results to publish tables of quarter-wave series matching networks. Gledhill and Issa [72], and Chang and Mott [73] derived simpler equations for determining the characteristic impedance of each transmission line. All these techniques have the limitation of only being applicable to the case where two resistances that do not vary with frequency are matched.

Another approach to designing distributed impedance matching networks is to transform a lumped circuit to a distributed approximation. This has the advantage that all the theory relevant to the design of lumped matching networks can be applied to distributed networks. The major result in obtaining distributed equivalents to lumped networks was published by Richards [74] and allows lumped elements to be replaced by open or short circuited transmission line stubs. Richards' transform is usually used in conjunction with Kuroda's identities and Norton's identities [2] to obtain a practically realisable circuit. Carlin and Kohler [75] have used Richards' transform to derive transfer functions that can be realised by transmission lines, and Kuroda's identities to eliminate series stubs. Richards' transformation and Kuroda's identities were also used by van der Walt [76] to obtain very small distributed matching networks. Other possibilities for approximation of lumped elements exist such as using a transmission line with a high impedance to approximate a series inductor and a transmission line with a low impedance to approximate a parallel capacitor. This approach was used by Matthaei [77] to produce tables of matching networks using transmission lines shorter than a quarter wavelength.

The approach described by Matthaei [77] reduces the length of each transmission line, but increases the range of characteristic impedances required. This problem led Levy [78,79] to the develop a class of mixed lumped and distributed matching networks. Levy's approach differs from Matthaei's in that the parallel capacitors are not replaced by series transmission lines. The capacitances can be adjusted to compensate for discontinuities before being implemented. Possible implementations for the capacitances include capacitors, irises, and parallel stubs.

Recently, Drozd and Joines [80] have published a paper which considers parallel stubs as resonators. This technique has the advantage of allowing the design of transmission line impedance matching networks that use stubs without requiring transformations from lumped circuits.

The main advantage of analytic impedance matching techniques is that fundamental limits have been derived in the form of gain-bandwidth theory. The other advantage of analytic methods is that a computer is not required to compute the results – an essential requirement of any design technique before the advent of low cost, high performance computers. The main disadvantage of analytic impedance matching techniques is that they only consider cases that can be analysed analytically. This means that most of these methods only consider very simple cases where the load and source are simple networks, the frequency response is either flat or rolls off at some constant rate, the resulting network is either low-pass or high-pass even if a band-pass structure would be more suitable, and discontinuities are often not adequately compensated for.

The development of the real-frequency techniques has meant that analytic techniques are hardly ever used today because the real-frequency techniques use the measured data of the system rather than an approximation, and the results are significantly better than analytic methods [5, 8].

2.3.3 Real-Frequency Techniques

Real-frequency techniques are those impedance matching techniques that use the actual data of a system to design a matching network. This is in contrast to analytic techniques which rely on some model of the system rather than the system data itself, and optimisation techniques which cannot be considered as design techniques because they require a good starting point. This section will give a brief overview of some of these techniques.

The first real-frequency technique was proposed by Carlin [4]. This is a single-matching technique which can design for specified transducer gains. The basic principle is that the transducer gain of the matching network is derived in terms of the load and the input impedances of the network. Only the input resistance or conductance as a function of frequency has to be considered because the imaginary part of the input impedance or admittance can be found from the real part using a Hilbert transform. A piecewise linear approximation of the input resistance or conductance is used to simplify the implementation, but other approximations such as the Wiener-Lee mapping can also be used [10]. The optimisation of the resistance or conductance function is then achieved using a standard optimisation technique. There are many possibilities for initialising the algorithm. Carlin [4] suggests that the gain could be levelled at its DC value over the frequency band and Abrie [2] suggests using the load resistance value, with good results being obtained in both cases. Carlin and Komiak [81] show how this technique can be expanded to consider other factors such as amplifier stability by including constraints during optimisation. This technique is compared to analytic impedance matching algorithms by Carlin and Amstutz [5] and even this simple technique is seen to be significantly better than analytic techniques.

The next major advance in real-frequency impedance matching techniques came with the extension of the real-frequency principle to the double matching problem. The first double matching real-frequency technique was published by Yarman and Carlin [7]. The basic premise of the technique is that the transducer gain can be calculated from the input reflection coefficient. Further, it can be shown that the denominator of the reflection coefficient can be calculated from the numerator of the reflection coefficient, so the technique only explicitly considers the numerator of the input reflection coefficient. The major advantages of this technique over the technique considered above are that it can do double matching problems and the Hilbert transform is not required to calculate the reactance. The most important problem with this technique is that it requires a good starting point. Yarman and Carlin [7] suggest that an initial point could be obtained from the results of Carlin's [4] single matching technique applied to the case where one of the impedances is assumed to be purely real.

Another double matching technique was proposed by Carlin and Yarman [8]. This technique is based on the single matching technique considered above. The basic principle is to transform the double matching problem into an equivalent single matching problem and then to apply Carlin's [4] single matching technique. The major advantage over the double matching technique considered above is that an initial point can be generated, but this technique is more computationally intensive. An analytic double matching technique is also derived by Carlin and Yarman [8], and as before, the results of the real-frequency technique are significantly better than those obtained with the analytic technique.

More recently, Yarman and Fettweis [9] have proposed a simplified version of the Carlin and Yarman's [8] double matching technique. The basic formulation is the same, but Brune functions are used in this case to simplify the numerical calculations by avoiding the factorisation of polynomials. Numerical stability is also improved. Carlin's [4] single matching technique is again used to initialise this algorithm.

Abrie [2] has proposed a technique based on the transformation-Q of an element. The transformation-Q is defined as the ratio of the imaginary part of an impedance of susceptance to the real part of that impedance or susceptance at any point in a ladder network. The transformation of the impedance or susceptance by each element can be related to the transformation-Q. The transformation-Q is only accurate at one frequency, but broadband matching can be accomplished by using an optimisation algorithm. The algorithm is initialised using an exhaustive search, so this algorithm can be considered to be an optimisation algorithm applied to impedance matching. This technique has the advantages that it can have series and parallel resonant sections, and it can be generalised to consider transmission lines.

Many of the published computer-based methods of impedance matching network design are little more than general purpose optimisation algorithms applied to this problem. A good example of this is the paper by Dedieu *et al.* [82] where a new optimisation algorithm was used to optimise the component values of a given network. The review papers by Bandler [12] and Charalambous [13] give an overview of some of the more important optimisation theory that has been applied to circuit design and supply some examples. The most significant disadvantage of this approach is that the circuit configuration must be supplied to the algorithm. The most important benefit of this approach is that optimisation methods specifically related to impedance matching network design have been developed. An example of this is the work on minimax optimisation that has been done by Madsen *et al.* [83], Bandler *et al.* [84, 85], and Bandler and Charalambous [13], among others.

As in the analytic case, the problem of using transmission lines for impedance matching has not been as well addressed as the lumped problem. However, useful results do exist.

One approach is to start with a lumped prototype, transform some of the elements to transmission lines using a technique such as Richards' transform [74], and optimise the result. Mahdi and Macnee [86] used this approach to transform low pass lumped prototypes to distributed networks, and then optimise the results. Yarman and Aksen [87] applied a similar approach with new transformations that consider more than one frequency. The biggest problem associated with these transformation techniques is that the resonant effects of transmission lines are not accounted for in the lumped prototype, so the final distributed network is often suboptimal.

Another approach to the design of distributed networks is to modify the frequency variable to account for the frequency response of a transmission line. Richard's transform is used to modify lumped techniques by Gibson [88], and by Pandel and Fettweis [89]. Gibson recommends adjusting the normalising frequency (the frequency where the lines are 90° long) to obtain realisable transmission line impedance values, while Pandel and Fettweis allow the normalisation frequency to be adjusted by their algorithm. Both these approaches require all the transmission lines to have the same length, and the number of parallel elements to be specified.

The real-frequency techniques listed above have a number of advantages over analytic and graphical techniques. The first major advantage is that the real data of the systems to be matched over the entire frequency range of interest are used. The analytic techniques use some approximation to the real data and graphical techniques only consider one frequency. This means that problems which cannot be solved using other techniques are possible with the real-frequency techniques. The second major advantage is that the circuit configuration does not have to be selected, it is generated by the algorithm. The last major advantage of the real-frequency techniques is that they produce much better results than analytic techniques [5, 8].

The most important disadvantage of these techniques is that they all (except for the transformation-Q technique) actually design a transfer function rather than the circuit itself. The circuit then has to be extracted from the transfer function using algorithms such as those described by Yarman and Fettweis [9]. This means that these techniques do not allow the element values to be constrained between minimum and maximum values, and often require coupled coils and ideal transformers. A minor problem with most of these real-frequency techniques is that initialisation is obtained from another algorithm, so two algorithms have to be implemented.

2.3.4 Genetic Algorithms

This section will consider the application of genetic algorithms to impedance matching. Significantly, none of the review papers that consider genetic algorithms in electromagnetics give any examples of impedance matching applications [90–92]. However there has been some work done in the field and it will be reviewed here.

The main application of genetic algorithms in impedance matching to date has been the optimisation of tapered transmission line matching sections. Günel [93] derived equations for the input impedance of a non-reciprocal and non-symmetric exponential tapered transmission line and then used a genetic algorithm to optimise the result. Later Günel [94] expanded the procedure to consider lossy parabolic and exponential tapered transmission lines, and to use a hybrid algorithm to speed convergence. Bornholdt [95] considered the more general case where the tapered line can be represented by B-spline curves. The resulting tapered transmission line is then simulated using a three-dimensional method of moments algorithm. The main problem associated with tapered transmission lines is that analysis is complex and time consuming in all but the simplest case. Günel [93] takes the approach of considering comparatively simple shapes that can be analytically analysed to shorten simulation time, whereas Bornholdt [95] considers potentially much more complex shapes, but has to run long simulations in each case.

Raychowdhury *et al.* [96] overcome this difficulty by using an impedance matching network consisting of series transmission lines. The transmission lines are all a quarter of a wavelength long at the centre frequency and a genetic algorithm is used to determine the

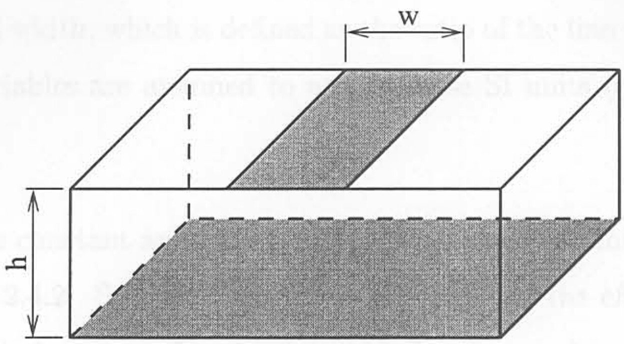


Figure 2.6: Microstrip line.

characteristic impedance of each transmission line. The structure used is very simple and can easily be analysed.

Sun and Lau [97, 98] consider an even simpler situation, a Π-section matching network. The values of the elements are tuned to match an antenna to a resistance using a genetic algorithm.

Genetic algorithms have tremendous potential for use in impedance matching network design algorithms. The papers presented above have not realised this potential because they either take too long to run or they only consider special cases which can be solved with simpler techniques.

2.4 Discontinuity Models

A number of discontinuity models have been published and this section will as give a brief overview of some of these. The models used here were obtained from the references given in the books by Hoffmann [99] and Gupta *et al.* [100], and the manuals for Agilent's ADS microwave design software [101].

A diagram of a microstrip line is shown in Figure 2.6 with the symbols for the width of the line and the height of the substrate indicated. In this document the symbol u is used to denote the normalised width, which is defined as the ratio of the line width to the substrate height (w/h). All variables are assumed to use the base SI units (e.g. m and Hz) unless otherwise stated.

The effective dielectric constant and characteristic impedance of a microstrip line are given in Sections 2.4.1 and 2.4.2. Section 2.4.3 gives equations for the effect of an open-ended microstrip stub. The inductance of a circular via hole to ground is given in Section 2.4.4. A model for the effect of a step in microstrip line width is given in Section 2.4.5. A microstrip T-junction model is given in Section 2.4.6 and the microstrip cross is considered in Section 2.4.7.

2.4.1 Effective Dielectric Constant

The effective dielectric constant is a very important factor that is used in a number of the models developed in the following sections. The first part of this section will present the formula for relative dielectric constant proposed by Hammerstad and Jensen [102]. Then the corrections proposed by Kirschning and Jansen [103] to account for high frequency dispersion will be covered.

At low frequencies the effective dielectric constant of a microstrip line can be given by the following equation proposed by Hammerstad and Jensen [102]:

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + \frac{10}{u}\right)^{-ab} \tag{2.41}$$

where

$$a = 1 + \frac{1}{49}\ln\left[\frac{u^4 + (u/52)^2}{u^4 + 0.432}\right] + \frac{1}{18.7}\ln\left[1 + \left(\frac{u}{18.1}\right)^3\right], \tag{2.42}$$

$$b = 0.564\left(\frac{\varepsilon_r - 0.9}{\varepsilon_r + 3}\right)^{0.053} \tag{2.43}$$

and ε_r is the relative dielectric constant of the substrate.

The original paper states that the accuracy of this model is better than 0.2% for at least

$\varepsilon_r \leq 128$ and $0.01 \leq u \leq 100$. However Kirschning and Jansen [103] show that this model is only valid at low frequencies.

To overcome this limitation, Kirschning and Jansen [103] proposed the following set of equations that account for dispersion in the relative dielectric constant:

$$\varepsilon_{eff}(f) = \varepsilon_r - \frac{\varepsilon_r - \varepsilon_{eff}(f=0)}{1+P(f)} \tag{2.44}$$

where

$$P(f) = P_1 P_2[(0.1844 + P_3 P_4)10fh]^{1.5763} \tag{2.45}$$

with

$$P_1 = 0.27488 + [0.6315 + 0.525/(1+0.157fh)^{20}] \times u - 0.065683\exp(-8.7513u), \tag{2.46}$$

$$P_2 = 0.33622[1 - \exp(-0.03442\varepsilon_r)], \tag{2.47}$$

$$P_3 = 0.0363\exp(-4.6u) \times \{1 - \exp[-(fh/3.87)^{4.97}]\}, \tag{2.48}$$

and

$$P_4 = 1 + 2.751\{1 - \exp[-(\varepsilon_r/15.916)^8]\}, \tag{2.49}$$

and where f is the frequency in GHz and h is the substrate height in cm.

This model is accurate to better than 0.6% for $0.1 \leq u \leq 100$, $1 \leq \varepsilon_r \leq 20$, and $0 \leq h/\lambda_0 \leq 0.13$. Unfortunately, Kirschning and Jansen do not state which model they used for $\varepsilon_{eff}(f=0)$ in (2.44), but the model given above is the most accurate one referenced, so the accuracy claimed above should apply.

At this point the finite thickness of the conductor and losses in the line have been ignored because these effects would introduce a large amount of extra complexity for small accuracy gains. March [104] gives an equation which modifies the line width to account for finite conductor thickness and a comprehensive overview of microstrip losses is given by Denlinger [105].

2.4.2 Characteristic Impedance

A model for the characteristic impedance of microstrip lines is given in this section. The formulae presented here were proposed by Hammerstad and Jensen [102].

The following equation gives the value of the characteristic impedance for a microstrip line with a free space substrate:

$$Z_{0l} = \frac{\eta_0}{2\pi} \ln \left[\frac{f}{u} + \sqrt{1 + \left(\frac{2}{u}\right)^2} \right] \tag{2.50}$$

where

$$f = 6 + (2\pi - 6) \exp \left[- \left(\frac{30.666}{u} \right)^{0.7528} \right] \tag{2.51}$$

and η_0 is the free space wave impedance $(\sqrt{\mu_0/\varepsilon_0})$.

Hammerstad and Jensen [102] claim that this equation is accurate to less than 0.01% for $u \leq 1$, and to less than 0.03% for $u \leq 1000$.

When a substrate other than free space is used, the characteristic impedance is obtained by dividing Z_{0l} by the square root of the effective dielectric constant:

$$Z_0 = \frac{Z_{0l}}{\sqrt{\varepsilon_{eff}}} \ . \tag{2.52}$$

As in Section 2.4.1 the effects of the finite thickness of the conductor have been ignored, with correction factors for this effect being given by March [104].

2.4.3 Open End

This section will consider an open-ended microstrip line as shown in Figure 2.7. The model used here was proposed by Kirschning *et al.* [106] and consists of an extension of the line length.

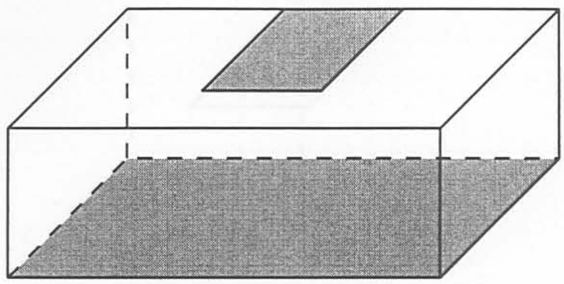


Figure 2.7: Open-ended microstrip line.

The length increase as a factor of the substrate height is given by

$$\Delta l/h = (\xi_1 \xi_3 \xi_5 / \xi_4) \tag{2.53}$$

where

$$\xi_1 = 0.434907 \frac{\varepsilon_{eff}^{0.81} + 0.26}{\varepsilon_{eff}^{0.81} - 0.189} \times \frac{u^{0.8544} + 0.236}{u^{0.8544} + 0.87}, \tag{2.54}$$

$$\xi_2 = 1 + \frac{u^{0.371}}{2.358\varepsilon_r + 1}, \tag{2.55}$$

$$\xi_3 = 1 + \frac{0.5274 \arctan[0.084 u^{1.9413/\xi_2}]}{\varepsilon_{eff}^{0.9236}}, \tag{2.56}$$

$$\xi_4 = 1 + 0.0377 \arctan[0.067 u^{1.456}] \times \{6 - 5\exp[0.036(1 - \varepsilon_r)]\}, \tag{2.57}$$

$$\xi_5 = 1 - 0.218 \exp(-7.5u), \tag{2.58}$$

and Δl is the line length extension.

Kirschning *et al.* [106] claim that this model is accurate to within 2.5% for $0.01 \leq u \leq 100$ and $\varepsilon_r < 50$ at 1 GHz when compared to numerical results listed in the paper. The effective dielectric constant (ε_{eff}) model used by Kirschning *et al.* is the one given by Hammerstad and Jensen [102], and is supplied in Section 2.4.1.

2.4.4 Via Holes

A via hole is present whenever a microstrip line is shorted to ground as shown in Figure 2.8. The model presented here is due to Goldfarb and Pucel [107].

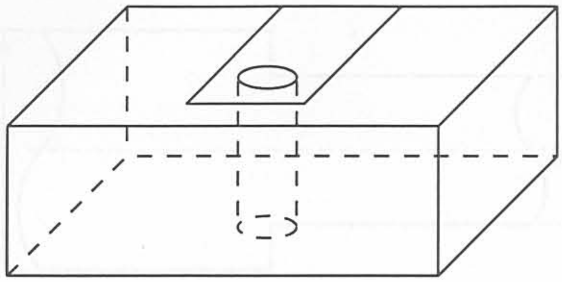


Figure 2.8: Via hole to ground.

The model consists of a parasitic inductance to ground with a value of

$$L_{via} = \frac{\mu_0}{2\pi}\left[h \cdot \ln\left(\frac{h+\sqrt{r^2+h^2}}{r}\right) + \frac{3}{2}\left(r - \sqrt{r^2+h^2}\right)\right] \tag{2.59}$$

where r is the radius of the via.

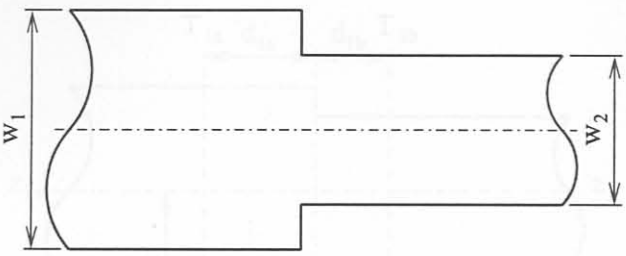
The accuracy of this model is not explicitly given by Goldfarb and Pucel [107]. All that is stated is that equation agrees very closely with simulations of vias in 100–635 μm substrates with $0.2 < d/h < 1.5$, $2.2 < \varepsilon_r < 20$, and $1 < u < 2.2$. The dependency of the via inductance of the width of the pad is specified as being less than 4% for a very large range of u ratios.

2.4.5 Width Step

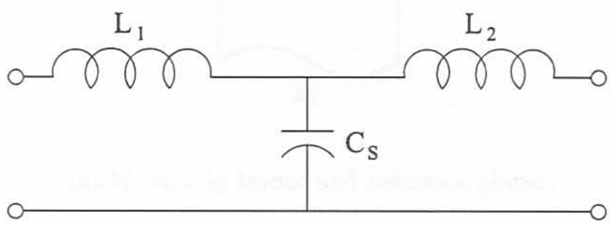
An abrupt junction between two microstrip lines leads to a step in the line width as shown in Figure 2.9(a). The model presented here is adapted from Hammerstad [102] and Gupta *et al.* [108]. The model is a T network which consists of inductors in series with each of the lines and a parallel capacitor between them as shown in Figure 2.9(b). Both symmetrical and asymmetrical steps are considered by Hammerstad [102], but only the symmetrical forms are given here.

The capacitance is given by

$$C_s = \left(\frac{\sqrt{\varepsilon_{eff1}}}{Z_{L1}c_0} - \varepsilon_0\varepsilon_r u_1\right)\frac{w_1 - w_2}{2} \tag{2.60}$$



(a) Microstrip layout.



(b) Equivalent circuit.

Figure 2.9: Microstrip width step.

where C_s is the capacitance value, Z_{Ln} is the characteristic impedance of line n, w_n is the width of line n, and the subscripts 1 and 2 denote the wider and narrower lines respectively.

Hammerstad [102] gives two models for the total series inductance in the model. The simpler of the two forms gives better results than the more complex form and is given by

$$L_s = \frac{\mu_0}{\pi} \ln \left[1/ \sin \left(\frac{\pi Z_{L1}}{2 Z_{L2}} \sqrt{\frac{\varepsilon_{eff1}}{\varepsilon_{eff2}}} \right) \right] h \, . \tag{2.61}$$

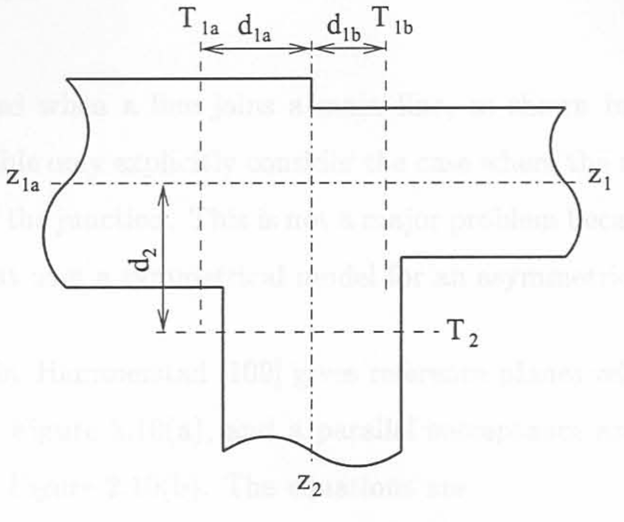
This inductance then has to be split to form the two series inductors. This is done using the following equation:

$$L_n = \frac{L_{wn}}{L_{w1} + L_{w2}} L_s \tag{2.62}$$

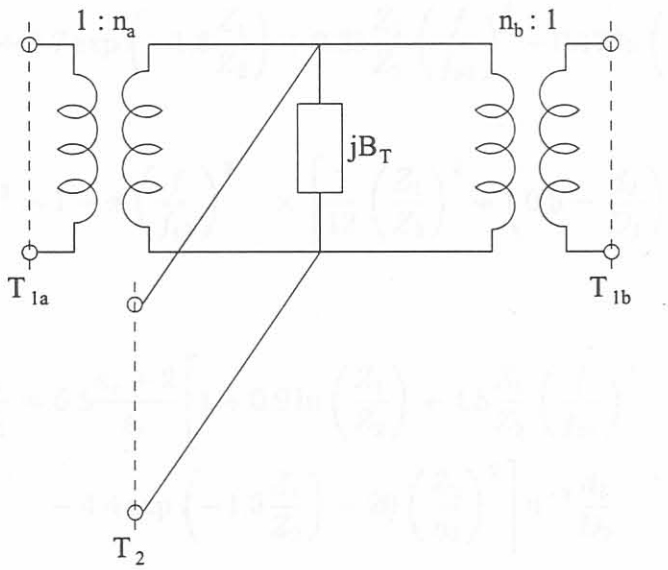
where L_n is the inductance in series with line n, and L_{wn} is the inductance per unit length of the transmission line given by

$$L_{wn} = \frac{Z_{0n} \sqrt{\varepsilon_{eff\ n}}}{c_0} \tag{2.63}$$

where c_0 is the speed of light in a vacuum.



(a) Microstrip layout and reference planes.



(b) Equivalent circuit.

Figure 2.10: Microstrip T-junction.

Unfortunately no error bounds are specified for these equations.

2.4.6 T-Junction

A T-junction is formed when a line joins a main line, as shown in Figure 2.10(a). The models that are available only explicitly consider the case where the main line has the same width on both sides of the junction. This is not a major problem because Hammerstad [109] suggests a method that uses a symmetrical model for an asymmetrical T-junction.

The model proposed by Hammerstad [109] gives reference planes relative to the middle of each line as shown in Figure 2.10(a), and a parallel susceptance and transformers in the main line as shown in Figure 2.10(b). The equations are

$$\frac{d_1}{D_2} = 0.055\left[1 - 2\frac{Z_1}{Z_2}\left(\frac{f}{f_{p1}}\right)^2\right]\frac{Z_1}{Z_2}, \tag{2.64}$$

$$\frac{d_2}{D_1} = 0.5 - \left[0.05 + 0.7\exp\left(-1.6\frac{Z_1}{Z_2}\right) + 0.25\frac{Z_1}{Z_2}\left(\frac{f}{f_{p1}}\right)^2 - 0.17\ln\left(\frac{Z_1}{Z_2}\right)\right]\frac{Z_1}{Z_2}, \tag{2.65}$$

$$n^2 = 1 - \pi\left(\frac{f}{f_{p1}}\right)^2 \times \left[\frac{1}{12}\left(\frac{Z_1}{Z_2}\right)^2 + \left(0.5 - \frac{d_2}{D_1}\right)^2\right], \tag{2.66}$$

and

$$\frac{B_T}{Y_2}\frac{\lambda_1}{D_1} = 5.5\frac{\varepsilon_r + 2}{\varepsilon_r}\left[1 + 0.9\ln\left(\frac{Z_1}{Z_2}\right) + 4.5\frac{Z_1}{Z_2}\left(\frac{f}{f_{p1}}\right)^2\right. \tag{2.67}$$

$$\left. - 4.4\exp\left(-1.3\frac{Z_1}{Z_2}\right) - 20\left(\frac{Z_2}{\eta_0}\right)^2\right]n^{-2}\frac{d_1}{D_2} \tag{2.68}$$

where the subscripts 1 and 2 denote the main and side arms respectively, d_n is the offset from the centre line, n is the number of turns in the main arm transformer, B_T is the parallel susceptance, and Y_n is the inverse of the line characteristic impedance Z_n. The remaining parameters are:

$$\lambda_n = \frac{c_0}{f\sqrt{\varepsilon_{effn}}}, \tag{2.69}$$

$$f_{pn} = \frac{c_0 Z_n}{2\eta_0 h}, \tag{2.70}$$

and

$$D_n = \frac{\eta_0 h}{Z_n \sqrt{\varepsilon_{effn}}} \tag{2.71}$$

where λ_n is the wavelength in the line, f_{pn} is the first higher mode cutoff frequency of the equivalent parallel plate transmision line, and D_n is the line width of the equivalent parallel plate transmission line.

While these formulae are only valid for the case where the main arm has the same line width on both sides of the discontinuity, Hammerstad [109] proposes the following procedure to allow for an asymmetric main arm. Use the impedance of the main arm under consideration when calculating the main arm parameters. Use the geometric mean of the main line impedances when calculating the side arm parameters.

Unfortunately, no error bounds are supplied for this model.

2.4.7 Cross

This discontinuity is formed when two lines cross as shown in Figure 2.11. All the models that have been published are only valid for the case where the two lines that cross are the same width on both sides of the discontinuity. This dramatically limits these models' applicability and accuracy. Additionally, the model given by Garg and Bahl [110] only gives capacitance values for an alumina substrate ($\varepsilon_r = 9.9$). For these reasons, two parallel T-junctions will be used to approximate a cross as suggested by Abrie [2].

When a cross is approximated by two parallel T-junctions, the calculations for the side arms are performed in exactly the same way as for a T-junction by using the parameters of the side arm under consideration and the geometric mean of the main arm parameters. The calculations for the main arm parameters are accomplished using the parameters of the main arm under consideration and the geometric mean of the side arm parameters. The susceptance in the T-junction model is only used once. This is obviously not a particularly satisfying solution to the problem, but the other options are just as bad.

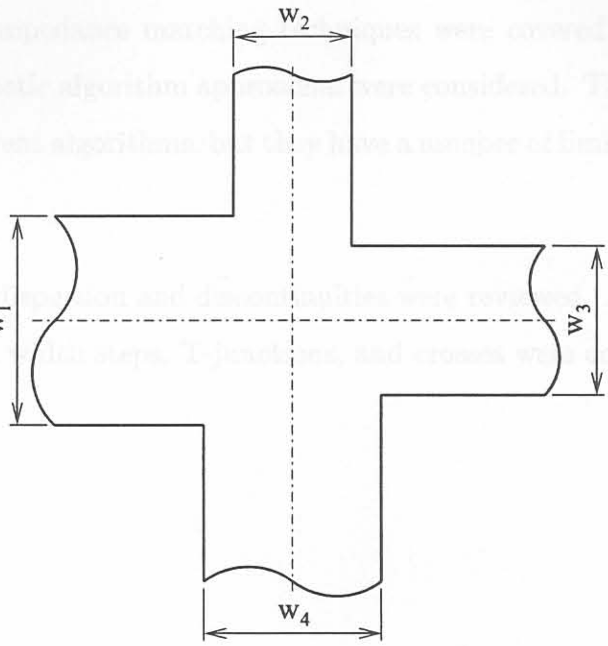


Figure 2.11: Microstrip Cross.

2.5 Summary

The theory used in this dissertation was briefly reviewed in this chapter. A reasonably detailed overview of each topic was given because the information in this chapter was used to motivate choices made later.

Optimisation algorithms form the basis of most CAD tools. The formulation of an optimisation problem and some special cases were presented. A number of local, global, and combinatorial optimisation techniques were considered. Hybrid algorithms that combine a number of different algorithms were briefly discussed.

An introduction to genetic algorithms, with the emphasis on the underlying principles rather than a specific paradigm, was given. The most important parts of a genetic algorithm were presented with selection schemes, representation, and genetic operators being considered in detail. Messy genetic algorithms were presented as a means to overcome some of the difficulties faced when applying genetic algorithms to practical problems.

The most important impedance matching techniques were covered. Graphical, analytic, real-frequency, and genetic algorithm approaches were considered. The real-frequency techniques are the best current algorithms, but they have a number of limitations that motivated this work.

Models for microstrip dispersion and discontinuities were reviewed. Microstrip lines, open-ended stubs, via holes, width steps, T-junctions, and crosses were considered.