

**Efficient Gradient-Based Optimisation of Suspension
Characteristics for an Off-Road Vehicle**

by

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Abstract

The efficient optimisation of vehicle suspension systems is of increasing interest for vehicle manufacturers. The main aim of this thesis is to develop a methodology for efficiently optimising an off-road vehicle's suspension for both ride comfort and handling, using gradient based optimisation. Good ride comfort of a vehicle traditionally requires a soft suspension setup, while good handling requires a hard suspension setup. The suspension system being optimised is a semi-active suspension system that has the ability to switch between a ride comfort and a handling setting. This optimisation is performed using the gradient-based optimisation algorithm Dynamic-Q.

In order to perform the optimisation, the vehicle had to be accurately modelled in a multi-body dynamics package. This model, although very accurate, exhibited a high degree of non-linearity, resulting in a computationally expensive model that exhibited severe numerical noise. In order to perform handling optimisation, a novel closed loop driver model was developed that made use of the Magic Formula to describe the gain parameter

for the single point driver model's steering gain. This non-linear gain allowed the successful implementation of a single point preview driver model for the closed loop double lane change manoeuvre, close to the vehicle's handling limit.

Due to the high levels of numerical noise present in the simulation model's objective and constraint functions, the use of central finite differencing for the determination of gradient information was investigated, and found to improve the optimisation convergence history. The full simulation model, however, had to be used for the determination of this gradient information, making the optimisation process prohibitively expensive, when many design variables are considered. The use of carefully chosen simplified two-dimensional non-linear models were investigated for the determination of this gradient information. It was found that this substantially reduced the total number of expensive full simulation evaluations required, thereby speeding up the optimisation time.

It was, however, found that as more design variables were considered, some variables exhibited a lower level of sensitivity than the other design variables resulting in the optimisation algorithm terminating at sub-optimal points in the design space. A novel automatic scaling procedure is proposed for scaling the design variables when Dynamic-Q is used. This scaling methodology attempts to make the n-dimensional design space more spherical in nature, ensuring the better performance of Dynamic-Q, which makes spherical approximations of the optimisation problem at each iteration step.

The results of this study indicate that gradient-based mathematical optimisation methods may indeed be successfully integrated with a multi-body dynamics analysis computer program for the optimisation of a vehicle's suspension system. Methods for avoiding the negative effects of numerical noise in the optimisation process have been proposed and successfully implemented, resulting in an improved methodology for gradient-based optimisation of vehicle suspension systems.



Keywords : gradient-based mathematical optimisation, vehicle suspension,
spring and damper characteristics,
Dynamic-Q, ride comfort, handling.

Doeltreffende Gradiëntgebaseerde Optimering van die Suspensiestelsel van 'n Veldvoertuig

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Opsomming

Voertuigvervaardigers besef al hoe meer hoe belangrik doeltreffende optimering van suspensiestelsels is. Die hoofdoel van hierdie proefskrif is om 'n metode te ontwikkel om 'n veldvoertuig se suspensiestelsel vir ritgemak en hantering te optimeer, deur gebruik te maak van gradiëntgebaseerde wiskundige optimering. Tradisionele voertuigsuspensiestelsels het 'n sagte suspensiekarakteristiek nodig vir goeie ritgemak, en 'n harde suspensiekarakteristiek vir hantering. Die suspensiestelsel wat geoptimeer word is 'n semi-aktiewe suspensiestelsel, wat die vermoë het om te skakel tussen 'n ritgemak- en 'n hanteringskarakteristiek. Dié karakteristieke word bepaal deur gebruik te maak van die gradiëntgebaseerde wiskundige-optimeringsalgoritme Dynamic-Q.

Om die suspensiestelsel te optimeer is die voertuig akkuraat gemodelleer in 'n multi-liggaamdinamika sagteware pakket. Dié model het, as gevolg van sy lewensgetrouheid, hoë geraasvlakke en is berekeningsintensief as gevolg van die nie-lineariteit van die stelsel. Vir die suksesvolle optimering van hantering, moes 'n unieke stuurbeheermodel geïmplementeer word. Hierdie model maak gebruik van die towerformule, normaalweg gebruik vir die modellering van bande, om die stuurinsetaanwinstfaktor vir die enkelpuntstuurbeheerder te verkry. Deur die stuurinset met 'n nie-lineêre aanwinst te modelleer, kon 'n enkelpuntstuurbeheerder suksesvol

geïmplementeer word, om die voertuig naby aan sy hanteringslimiete deur die dubbelbaanverandering te stuur.

As gevolg van die hoë geraasvlakke wat in die doel- en begrensingsfunksies teenwoordig is, is sentrale eindige verskille ondersoek vir die berekening van gradiëntinligting. Daar is vasgestel dat die optimeringsgeskiedenis verbeter kan word deur gebruik te maak van sentrale eindige verskille vir gradiëntinligting. Die berekening van hierdie gradiënte is duur, omdat die volledige berekeningsintensiewe simulasiemodel gebruik word. Die gebruik van goed gekose nie-lineêre vereenvoudigde modelle vir die bepaling van gradiëntinligting is ondersoek. Daar is bevind dat die vereenvoudigde modelle goed werk vir die verkryging van gradiëntinligting en dat die optimeringstyd heelwat verminder kan word.

Wanneer meer ontwerpveranderlikes in ag geneem word by die optimering, word bevind dat die doel- en begrensingsfunksies nie dieselfde sensitiviteit het vir al die veranderlikes nie. Die gevolg is dat die optimeringsproses termineer op suboptimale punte in die ontwerp ruimte. 'n Unieke skalingsmetode is voorgestel vir gebruik met Dynamic-Q om aan die veranderlikes gelyke sensitiviteit te gee. Die skalingsmetode maak die n-dimensionele ontwerp ruimte meer sferies van aard. Dynamic-Q, wat gebruik maak van sferiese subprobleme, kan dus beter benaderings tot die sub-probleme maak, en as gevolg daarvan is die optimering meer suksesvol.

Die resultate van hierdie werk, bevestig dat gradiëntgebaseerde optimeringsalgoritmes suksesvol met 'n multi-liggamdinaamika analise-program geïntegreer kan word om die suspensiekarakteristieke van 'n veldvoertuig te bepaal. Metodes om die nadelige effekte van numeriese geraas te oorkom, is voorgestel en suksesvol geïmplementeer, vir gradiëntgebaseerde optimering van voertuigsuspensiestelsels.



Sleutelwoorde : wiskundige optimering, voertuigsuspensie,
veer- en demperkarakteristieke,
Dynamic-Q, ritgemak, hantering.

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Preface

This thesis is part of a group effort at the University of Pretoria, to improve the safety, comfort and handling of Sport Utility Vehicles (SUV's), through the use of an intelligent suspension system. I have included here an article I wrote for the Saturday Star, which in a light hearted manner introduces the reader to the project.

I am your status, favourite toy, your freedom and your recreation. But while I am good for all these things, I still have many faults, just like you! You might not think so! My design gives you a sense of security and power. While you are really dying to know who I am, I have to keep you in suspension a little longer, as this is the key.

The secret is almost out! We are relaxing in the veld, and you are marvelling at my abilities, spraying mud everywhere, ploughing the sand, climbing the boulders. Like mountain goats nothing can stop us.

Party-time is over. Back in the week, running late as always. The cursed fat passenger, stubborn as an axe, will not buckle up. We go faster than a cheetah about to kill, along the motorway, my big powerful horses really galloping. BUT, just in front, out pops a ghostly pedestrian. No time to even curse, your arms frantically turning the wheel. Wow, just missed him! But my height catches you out and, roll, roll, roll your pride, roughly down the roadside.

Both looking as though battered by a ram, you emerge to the frozen motorway, gawkers everywhere. The vultures like sumo wrestlers, fighting for the first feed. The pedestrian? Well, gone like the wind. And finally the sound of sirens, help on it's way.

This is the fate of many a SUV (Sport Utility Vehicle) on the roads. Many owners of these vehicles are fooled into thinking they are safe as they are so high above the other road users. It is this height coupled to a soft suspension for good off-road manœuvrability, which leads to a dangerous package when performing sports car manœuvres, typically occurring during accident conditions.

What are the solutions? First of all always wear your safety belt. Then buy a sports car for the road and a 4 × 4 for off-road only. No this is too

expensive, what else can be done? Remove the pedestrian. We would love to, but the pedestrian could be anything in the road, an animal, a pothole, or a tree if you're drunk. No, the vehicle needs to be driven with less steam. And in the mean time, some more work is being done on the vehicle's suspension, which is exactly what we are busy doing. We at the University of Pretoria, are developing a controllable suspension system, for this type of vehicle. See, I told you! Keeping you suspended is the key.

The suspension is the link between the tyres and the vehicle body. By controlling this link intelligently, with the help of Newton's laws of physics, the accident could have been better avoided. Yes, you read right, the laws of physics apply to everything especially vehicles. Our unique suspension has two different spring settings and two different damper settings per wheel, in order to better eliminate the compromise between off-road and on-road driving.

With the help of intelligence we switch between the spring and damper settings while the driver is doing his normal task of driving. We thus have a setting for severe handling manoeuvres, and one for comfort and off-road driving. The controller uses it's knowledge of physics to switch the suspension within 50 milliseconds.

Let's take a quick look at the accident again. Driver driving fast, suspension in comfort mode, pedestrian appears in road. Driver frantically turns steering wheel, suspension switches to handling spring and adjusts the damping every 50 milliseconds. Driver misses the pedestrian, overcorrects a little, vehicle stays upright, and driver continues driving. The intelligent suspension did not panic, and kept the vehicle upright better than the driver did. This is because it remembers the laws of physics at all times.

List of Abbreviations

$4S_4$	4 State Semi-Active Suspension System
AD	Automatic Differentiation
ADAMS	Advanced Dynamic Analysis of Mechanical Systems
admsgrad	ADAMS Model Gradient
ANN	Artificial Neural Network
apg	Aberdeen Proving Ground
Ascl-DynQ	Automatic Scaling Dynamic-Q
ascl	Automatic Scaling
BFGS	Broyden-Fletcher-Goldfarb-Shanno
BS	British Standard
CCD	Central Composite Design
CFD	Computational Fluid Dynamics
cfid	Central Finite Difference
cg	Center of Gravity
DFP	Davidon-Fletcher-Powell
dof	Degree(s) of Freedom
dpsf	damper scale factor
ETOP	Euler-Trapezium Optimiser
ETOPC	Euler-Trapezium Optimiser for Constrained Problems
FEM	Finite Element Method
ffd	Forward Finite Difference
GA	Genetic Algorithm
gvol	static gas volume
lf	Left Front
inf	Infeasible Starting Point



lr	Left Rear
ISO	The International Organisation for Standardisation
LFOP	Leap-Frog Optimiser
LFOPC	Leap-Frog Optimiser for Constrained Problems
MAM	Multipoint Approximation Method
MDOG	Multi-disciplinary Design Optimisation Group
min	Minimum
matgrad	Matlab Model Gradient
max	Maximum
NRF	National Research Foundation
ode	Ordinary Differential Equation
PID	Proportional Integral Derivative
PSD	Power Spectral Density
RMS	Root Mean Square
RSM	Response Surface Methodology
SLP	Sequential Linear Programming
SQP	Sequential Quadratic Programming
std	Standard
SUMT	Sequential Unconstrained Minimisation Technique
SUV	Sports Utility Vehicle
VDV	Vibration Dose Value

List of Symbols

A	Dynamic-Q Approximated Hessian Matrix
a	Curvature
a	Pseudo Arctangent Coefficient
a_i	Quadratic Constant of Polynomial Approximation
a_{RMS}	Root Mean Square Acceleration
a_{zRMSd}	Driver Root Mean Square Vertical Acceleration
a_{zRMSd}	Passenger Root Mean Square Vertical Acceleration
a_w	Weighted Acceleration
a_{0-13}	Magic Formula Coefficients
B	Stiffness Factor
b	Pseudo Arctangent Coefficient
b_i	Linear Constant of Polynomial Approximation
C	Shape Factor
C_d	Drag Factor
C	Damping Matrix
c_t	Tyre Damping
D	Peak Factor
$dpsf$	Damper Scale Factor
$dpsff$	Damper Scale Factor Front
$dpsfr$	Damper Scale Factor Rear
dx_k	Perturbation in Design Variable k
E	Curvature Factor
F	Force
F_{dmp}	Damper Force
$F(\mathbf{x})$	Multi-Variable Function



F_{drive}	Driving Force
F_{ztyre}	Vertical Tyre Force
F_y	Lateral Force
F_z	Vertical Force
f	Front
f_{4S_4}	Force in $4S_4$ Suspension
$f(\mathbf{x})$	Objective Function
$\tilde{f}(\mathbf{x})$	Approximated Objective Function
$f^*(\mathbf{x})$	Optimum Objective Function Value
$f^*(\mathbf{x})_{hand}$	Optimum Handling Objective Function Value
$f_{hand}(x)$	Ride Comfort Objective Function
$f_{ride}(x)$	Ride Comfort Objective Function
G_{do}	Road Roughness Coefficient
g	Gravity
$gvol$	Static Gas Volume
$gvol_f$	Static Gas Volume Front
$gvol_r$	Static Gas Volume Rear
$g_j(\mathbf{x})$	Inequality Constraint Functions
$\hat{g}_{il}(\mathbf{x})$	Design Variable Lower Limit Inequality Constraint
$\check{g}_{iu}(\mathbf{x})$	Design variable Upper Limit Inequality Constraint
h	Finite Difference step Size
h_{cg}	Height of Center of Gravity
$h_j(\mathbf{x})$	Equality Constraint Functions
H_k	Hessian Matrix at Iteration k
\mathbf{I}	Identity Matrix
I_x	Mass Moment of Inertia About x-Axis
I_y	Mass Moment of Inertia About y-Axis
I_z	Mass Moment of Inertia About z-Axis
i	Index
j	Index



K	Stiffness Matrix
k	Constant
k	Index
k_t	Tyre Vertical Stiffness
\hat{k}_i	Design Variable Lower Limit
\check{k}_i	Design variable Upper Limit
l	Counter
l	Left
m_A	Mass of Axle
m_b	Mass of Body
m_t	Mass of Tyre
m_v	Mass of Vehicle
M	Mass Matrix
M_z	Moment About z-Axis
n	Number of Design Variables
O	Zero Matrix
O	Zero Vector
P [i]	Approximated Sub Problem
r	Right
r	Rear
sf	Scale Factor
S_h	Horizontal Shift
S_v	Vertical Shift
t	Time
t_i	Design Variable Scale Factor
t_s	Start Time
t_s	Track Width Suspension
t_{total}	Total Time
T	Transposed
u	Counter



v	Velocity
$v_{current}$	Current Variable Value
v_{low}	Variable Lower Limit
v_{high}	Variable Upper Limit
x	Relative Displacement
x	Magic Formula Slip Term
x_i	Design Variable i
x_k	Design Variable k
x_n	Design Variable at Iteration n
x_{1-14}	Design Variables
\dot{x}_{act}	Instantaneous Speed
\dot{x}_d	Desired Speed
\dot{x}	Vehicle Speed
\mathbf{x}	Vector of Design Variables
\mathbf{x}^*	Vector of Optimum Design Variables
X	Tyre Slip Angle
\mathbf{X}	Vector of Rescaled Design Variables
X_i	Rescaled Design Variable
y	Vehicle Lateral Displacement
\ddot{y}_v	Lateral Acceleration
Y	Tyre Lateral Force
wb	Wheelbase
z	Vertical Displacement
\mathbf{z}	Vector of Scaled Design Variables
z_i	Scaled Design Variable Between Zero and One
z_{rf}	Road Disturbance Input Front
z_{rr}	Road Disturbance Input Rear
α	Tyre Slip Angle
δ	Move Limit Magnitude
δ	Steering Angle



$\dot{\delta}$	Steering Rate
δ_{stat}	Static Spring Displacement
δt	Time Step
∂	Partial Derivative
γ	Camber Angle
∇	Gradient Vector
μ	Penalty Multipliers
μ_j	Penalty Multiplier
ω	Terrain Index
ψ	Vehicle Yaw Angle
ψ_d	Desired Yaw Angle
ψ_a	Actual Yaw Angle
$\dot{\psi}_a$	Actual Yaw Rate
$\ddot{\psi}$	Yaw Acceleration
τ	Preview Time
θ	Pitch Angle
$\dot{\theta}$	Pitch Rate
$\ddot{\theta}$	Pitch Acceleration
φ	Roll Angle
$\dot{\varphi}$	Roll Rate
$\ddot{\varphi}$	Roll Acceleration
ϑ_0	Current Rotational Angle
ϑ_p	Predicted Rotational Angle
$\dot{\vartheta}$	Current Rotational Velocity
$\ddot{\vartheta}$	Rotational Acceleration