

# Chapter 2

## Train model

### 2.1 Introduction

According to the literature, there are two models in the study of train handling: a mass-point model and a cascade-mass-point model. For the study of in-train forces, it is natural to choose the cascade-mass-point model in this thesis.

In section 2.2 of this chapter, the longitudinal dynamics of a heavy haul train is modelled as a cascade of mass points connected with nonlinear couplers. Considering the practical application in COALink line trains, the states and input constraints are given. This model is used throughout the thesis.

The second part of this chapter describes the calculation of the emergency stop distance for a collision. A mass-point model and a cascade-mass-point model are employed respectively. The result can be used to compare the difference between the two models.

### 2.2 Cascade mass-point model

A heavy haul train, composed of locomotives and wagons (both referred to as cars), can be modelled as a cascade of mass points connected with couplers. In the following model, only the longitudinal dynamics of the train is analyzed.

### 2.2.1 Car model

A car is running on the track while it is subjected to aerodynamic force, the adjoining cars' internal forces, the gravity force and its own traction or brake force. The forces experienced by a car in the longitudinal direction are shown in Fig. 2.1.

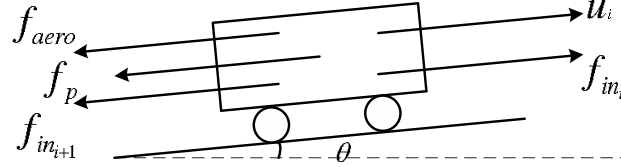


Figure 2.1: Longitudinal model of car

The aerodynamics of a train can be divided into two parts [22] [23]: mechanical drag and aerodynamic drag. The former includes the sliding forces between the train's wheels and the track and the rolling forces of wheels. Aerodynamic drag is dependent on the cross-sectional area of train body, train length, shape of train fore- and after-bodies, surface roughness of the train body, and geographical conditions around the proceeding train.

It has been reasonably assumed that the aerodynamic drag is proportional to the square of the speed, while the mechanical drag is proportional to the speed. Compared with the mechanical drag, the portion of the aerodynamic drag becomes larger as the train speed and length increase (see details from [22] and [23]).

In the open air without any crosswind effects, the total drag on a travelling car can be expressed by the sum of the aerodynamic and mechanical ones:

$$f = D_M + D_A = mc_0 + mc_1v + mc_2v^2, \quad (2.1)$$

where  $D_A$  and  $D_M$  are the aerodynamic and mechanical drags, respectively,  $c_0$ ,  $c_1$  and  $c_2$  are constants determined by experiments,  $v$  is the car speed and  $m$  is the car mass under discussion.

The variables  $f_{in_i}$  and  $f_{in_{i+1}}$  are the in-train forces between the neighbouring cars. Only one in-train force is experienced by the front and rear car. The variable  $u_i$  is the car's traction or brake force. For a wagon it refers to the brake force, which must be no more than zero, while for a locomotive it refers to traction force or brake force, whose quantity depends on the locomotive's power notch and speed and its dynamic brake capacity.

In Fig. 2.1, the resistance force  $f_p = f_g + f_c$ , a function of the position, is composed of the gravity force  $f_g = mg \sin \theta \approx mg\theta$  in longitudinal direction and the curvature resistance force  $f_c$  [24]. Generally, the track information is known, so it is convenient to yield the function of  $f_p$ . In the following simulation study, the curvature resistance force is ignored, which will not affect the simulation result.

## 2.2.2 Coupler model

The coupler between two cars is modelled as Fig. 2.2. When the draft gear is in its natural length, the in-train force is zero. Considering the coupler's slack length, the coupler can be regarded as a composition of the two gears plus the slack length. Assuming the sum of the length of two gears is  $L_0 - \frac{1}{2}L_{slack}$  while the in-train force is zero, the displacement of the coupler is defined as  $x = L - L_0$  in which  $L$  is the coupler length. The variable  $f_{in_i}$  is the in-train force between the  $i$ th and  $(i + 1)$ th cars, which is a function of  $x_i$ , the relative displacement between the two neighbouring cars, and the difference of the neighbouring cars' velocities (damping effect). A typical relationship between the static in-train force  $f_{in}$  (without damping) and  $x$  is depicted in Fig. 2.3, which is simplified from the data of Spoornet.

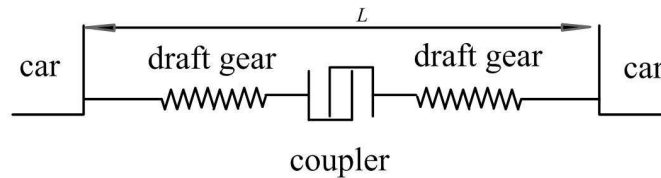


Figure 2.2: Longitudinal model of coupler

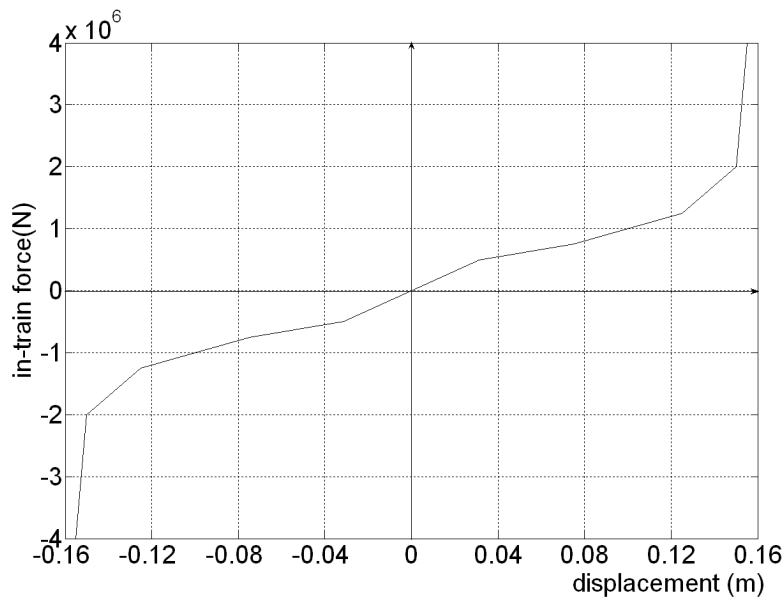


Figure 2.3: Coupler force vs. displacement

## 2.2.3 Train model of a cascade of mass points

Fig. 2.4 is a sketch of the longitudinal motion of a train. Assuming the train consists of  $n$  cars and the locomotives are located at positions  $l_i$ ,  $i = 1, 2, \dots, k$ , where  $k$  is the

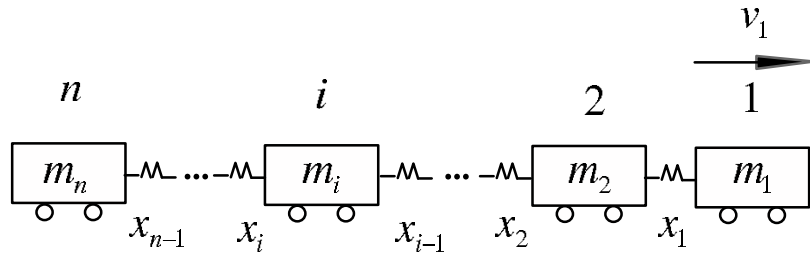


Figure 2.4: Longitudinal model of train

number of locomotives, the train model is described by the following equations.

$$m_s \dot{v}_s = u_s + f_{in_{s-1}} - f_{in_s} - f_{a_s}, \quad s = 1, 2, \dots, n, \quad (2.2)$$

$$\dot{x}_j = v_j - v_{j+1}, \quad j = 1, 2, \dots, n-1, \quad (2.3)$$

where the variable  $m_i$  is the  $i$ th car's mass; the variable  $v_i$  is the speed of the  $i$ th car; the variables  $f_{a_i} = f_{aero_i} + f_{p_i}$ ,  $i = 1, 2, \dots, n$ ; the variable  $f_{aero_i} = m_i(c_{0_i} + c_{1_i}v_i + c_{2_i}v_i^2)$  is the cars' aerodynamic force; the variable  $f_{p_i} = f_{g_i} + f_{c_i}$  is the force due to the tracking slope and curvature where the  $i$ th car is running; and the variable  $f_{in_i}$  is the in-train force between the  $i$ th and  $(i+1)$ th cars. In (2.2), one has  $f_{in_0} = 0, f_{in_n} = 0$ .

This model is the same in nature as that in [16], which is validated in [17] with the data from Spoornet.

## 2.2.4 Input constraint

For a heavy haul train, the control inputs are the efforts of the locomotives and the wagons. The efforts of locomotives can be traction forces or dynamic braking forces, and the efforts of wagons are braking forces. The dynamic brake power is also called regenerative brake power, which can be fed back to the system and could conceivably be saved. All these inputs are constrained. For a locomotive, the effort is governed by the current velocity and the current notch setting, which is depicted in Fig. 2.5 for the 7E1 locomotive used in the COALink trains. The locomotives in this study are assumed to be electric and it is also convenient to formulate the problem of a train with diesel–electric locomotives in a similar way.

In the practical operation of 7E1 locomotives, any notch change requires an interval delay for the field changes. When it changes from dynamic braking to traction or the other way round, the time delay requires a longer interval.

The braking forces of the wagons are also limited by the braking capacities of the wagons.

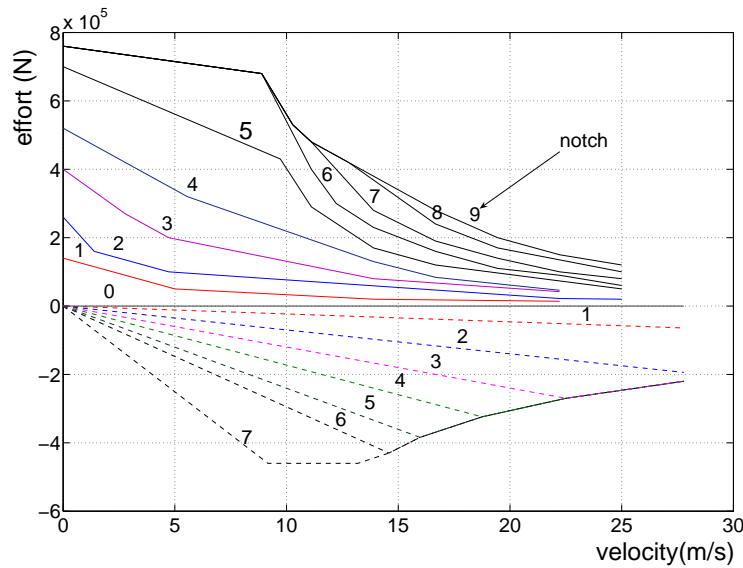


Figure 2.5: Locomotive group effort vs. notch and velocity

### 2.2.5 In-train force constraints

The quantity of the in-train forces is related to the safe running of the train. In practice, the safety range of the in-train forces for COALink trains is  $\pm 2,000$  kN.

## 2.3 Stop distance calculation – a model-comparison

During the study, the (emergency) stop distance of a train in a collision is requested to be calculated. The stop distance is calculated with a mass-point model and a cascade-mass-point model, respectively. From the calculation, the difference between the two models can be seen. As indicated in chapter 1, the iDP (DP) operation cannot be considered in a mass point model as well as the internal dynamics in the train. Even without considering the iDP operation, the cascade-mass-point model is still more accurate in modelling a train longer than 1.8 km. To demonstrate this, the calculation procedure is reported in the following.

### 2.3.1 Information on the collision

When a stop distance is to be calculated, the following data are required:

- 1) Train composition: locomotive type, wagon type, mass and length of the locomotives and wagons, characteristics of the couplers connecting the locomotives

and wagons, braking system characteristics (pneumatic braking system or ECP braking system, and its parameters).

- 2) Track data.
- 3) Operation code.
- 4) The initial speed of the train before brake is applied.

Limited information is known from the collision report:

- 1) The train locomotive was numbered E 7092.
- 2) There were 40 wagons pulled by two locomotives, one at the front and one at the rear. The weight of the locomotive is unknown; the wagons and their load weighed 66 ton each, but the wagon type is unknown.
- 3) One of the locomotives was not functional.
- 4) During the collision, the maximum sight distance from the approaching path of the train was 728 m. The departure path shows the end position of the train at 250 m from the area of impact.
- 5) The initial speed of the train before applying brake is unknown.

### 2.3.2 Calculation of stop distance

The emergency stop distance is considered. So the control sequence of the train is assumed to be in the emergency operation. From the collision report, this assumption is justified. As is known from [3], the emergency application of freight equipment provides an extremely rapid transmission of the application throughout the train, as well as developing higher brake cylinder pressure than is obtained during service braking. By the movement of the brake valve on the locomotive to the emergency position, the brake pipe is vented unrestricted to the atmosphere, which results in a rapid drop in brake pressure, causing the succeeding valves to go to emergencies and vent brake pipe pressure to the atmosphere at the location. The transmission rate of an emergency is about 930 feet per second or 635 miles per hour. During the emergency, the AB or ABD control valves will apply rapidly causing approximate 15psi brake cylinder pressure to occur on each car within 1.5 seconds after the car brake valve is at the emergency position. This action applies the brake shoes to the wheel quickly and minimizes severe slack changes. During an emergency application, the locomotive throttle should be reduced to idle [3].

From the above description, it is clear that there are time delays for the wagons to start braking after the application of the emergency brake. The time delays are different

from wagon to wagon. The longer the distance from the wagon to the locomotive from where the emergency command originates, the longer the time delay. However, the time delays are limited, since the train is composed of only 40 wagons with one locomotive at the front and one locomotive at the rear. The length of the train is about 524 m and the longest time delay is about 2 seconds. From the collision report, the wagon type is unknown and the locomotive type, from the number plate, could be 7E. The calculation of the braking distance can be done by two methods respectively employing a simple mass-point model and a cascade-mass-point model. The simple mass-point model takes the whole train as a single mass point, while the cascade-mass-point model take it as mass points connected by couplers. The cascade-mass-point model is a more accurate model that takes into consideration the couplers' energy consumption. It could be seen from the results that the braking distances are different from two different calculations. Because of lack of information, the calculation of the braking distance is done on the basis of the following set of technical assumptions:

- (1) The locomotives are idle when the emergency operation is initiated.
- (2) The track is flat.
- (3) The train is running in its steady state with a velocity that is unknown. However, calculation is done with a number of initial speeds of worst case scenarios.
- (4) The train is equipped with a pneumatic braking system. All the brake equipment works as designed.
- (5) The air dynamics is ignored.
- (6) In the analysis using a simple model, the in-train dynamics is ignored, too.
- (7) The locomotive is thought to be of the 7E1 type, whose length is 20.47 m and whose mass is 126 tons.
- (8) The loaded mass of the wagon is known to be 66 tons.
- (9) The wagon types are unknown. Spoornet has a number of different wagon types in operation: CCL1, CCL2, CCL3, CCL5–9 and CCR1, CCR2, CCR3 and CCR5-9. Calculations are done for all these wagon types.

The characteristics of the wagon types in emergency application are indicated in Table 2.1, where BBF is the brake block force, EBCP is the emergency brake cylinder pressure and COF BB is the coefficient of friction. All these terms of the railway are referred to [3] or [25].

In the following calculation, the simple model is used to estimate the possible velocities and the cascade-mass-point model is used to simulate the emergency operation of the train.

Table 2.1: Wagon types and parameters

|            | CCL1&3  | CCL2    | CCL5–9  | CCR1&3  | CCR2    | CCR5–9  |
|------------|---------|---------|---------|---------|---------|---------|
| EBCP (kpa) | 472     | 472     | 472     | 240     | 240     | 240     |
| COF BB     | 0.25    | 0.25    | 0.33    | 0.25    | 0.25    | 0.33    |
| BBF(N)     | 218,951 | 220,921 | 215,205 | 266,782 | 220,921 | 215,205 |

### 2.3.3 Simple model

In this model, a train is simplified to a mass point. The train mass is

$$M = (126 \times 2 + 66 \times 40) \times 1,000 = 2,892,000 \text{ kg}.$$

#### When the wagon's type is CCL1&3

The brake force acting on the train is

$$F = 40 \times \text{COF BB} \times \text{BBF} = 40 \times 0.25 \times 218,951 = 2,189,510 \text{ N}.$$

The deceleration velocity of the train is

$$a = \frac{F}{M} = \frac{2,189,510}{2,892,000} = 0.7571 \text{ m/s}^2.$$

The first point from where the train driver is able to see to the collision point is 728 m. The first simple calculation is the maximum initial speed of the train that the collision could have been avoided.

For the maximum possible braking distance  $s_1 = 728$  m, the admitted maximum initial velocity is

$$v_0 = \sqrt{2as_1} = \sqrt{2 \times 0.7571 \times 728} = 33.2 \text{ m/s} = 119.5 \text{ km/hour}.$$

The distance from the point (where the train driver is able to see the collision point) to its full stop is 978 m. The next simple calculation is the maximum initial speed of the train if the driver had applied the emergency brake immediately at this point. For the real braking distance  $s_1 = 978$  m, the velocity before the application of emergency braking is

$$v_0 = \sqrt{2as_1} = \sqrt{2 \times 0.7571 \times 978} = 38.48 \text{ m/s} = 138.54 \text{ km/hour}.$$

This calculation is done without considering the time delay of the application of the emergency. If one considers the worst case, considering the longest time delay for each wagon and the operational time between the emergency operation of the brake valve and



the full application of the emergency brake, one allows 2 seconds for transmission delay and 2 seconds for the application delay. This yields  $T_d = 4$  s and  $s_1 = T_d v_0 + v_0^2 / (2a)$ . The solution is  $v_0 = 30.31$  m/s = 109.12 km/hour for  $s_1 = 728$  m. When  $s_1 = 978$  m, the initial velocity is  $v_0 = 35.57$  m/s = 128.06 km/hour. That is, if the train's velocity before emergency is no more than 109.12 km/hour (128.06 km/hour), the train can be stopped within the braking distance 728 m (978 m).

It is normal that a reaction time is needed for the train driver to apply the emergency brake. A normal reaction time is 5 seconds. When 5 seconds is accepted for the decision of the operation of the emergency brake, then  $T_d = 9$  s. The initial velocity is solved with  $v_0 = 27.08$  m/s = 97.5 km/hour and  $v_0 = 32.27$  m/s = 116.16 km/hour for  $s_1 = 728$  m and  $s_1 = 978$  m, respectively.

The similar calculation steps are applied in other wagon types.

### When the wagon's type is CCL2

The brake force acting on the train is

$$F = 40 \times COF \ BB \times BBF = 2,209,210 \text{ N.}$$

The deceleration velocity of the train is  $a = 0.7639$  m/s<sup>2</sup>.

For the maximum possible braking distance  $s_1 = 728$  m, the admitted maximum initial velocity is  $v_0 = 33.35$  m/s = 120.1 km/hour.

For the real braking distance  $s_1 = 978$  m, the velocity before the application of emergency braking is  $v_0 = 38.65$  m/s = 139.16 km/hour.

Considering 2 seconds for transmission delay and 2 seconds for the application delay, one has  $T_d = 4$  s and the solution is  $v_0 = 30.43$  m/s = 109.56 km/hour for  $s_1 = 728$  m while  $v_0 = 35.72$  m/s = 128.59 km/hour for  $s_1 = 978$  m.

When 5 seconds is considered for the decision of the operation of the emergency brake,  $T_d = 9$  s. The initial velocity is solved with  $v_0 = 27.18$  m/s = 97.84 km/hour and  $v_0 = 32.39$  m/s = 116.59 km/hour for  $s_1 = 728$  m and  $s = 978$  m, respectively.

### When the wagon's type is CCL5–9

The brake force acting on the train is

$$F = 40 \times COF \ BB \times BBF = 2,840,700 \text{ N.}$$

The deceleration velocity of the train is  $a = 0.9827$  m/s<sup>2</sup>.

For the maximum possible braking distance  $s_1 = 728$  m, the admitted maximum initial velocity is  $v_0 = 37.8$  m/s = 136.2 km/hour. For the real braking distance  $s_1 = 978$  m, the velocity before the application of emergency braking is  $v_0 = 43.84$  m/s = 157.83 km/hour.

Considering 2 seconds for transmission delay and 2 seconds for the application delay, the solution is  $v_0 = 34.01$  m/s = 122.76 km/hour for  $s_1 = 728$  m while  $v_0 = 40.09$  m/s = 144.32 km/hour for  $s_1 = 978$  m.

When 5 seconds is considered for the decision of the operation of the emergency brake,  $T_d = 9$  s. The initial velocity is solved with  $v_0 = 30.00$  m/s = 108.01 km/hour and  $v_0 = 35.88$  m/s = 129.17 km/hour for  $s_1 = 728$  m and  $s = 978$  m, respectively.

### **When the wagon's type is CCL1&3**

The brake force acting on the train is  $F = 2,667,820$  N. The deceleration velocity of the train is  $a = 0.9225$  m/s<sup>2</sup>.

For the maximum possible braking distance  $s_1 = 728$  m, the admitted maximum initial velocity is  $v_0 = 36.65$  m/s = 131.9 km/hour.

For the real braking distance  $s_1 = 978$  m, the velocity before the application of emergency braking is  $v_0 = 42.48$  m/s = 152.92 km/hour.

Considering 2 seconds for transmission delay and 2 seconds for the application delay, the solution is  $v_0 = 33.14$  m/s = 119.32 km/hour for  $s_1 = 728$  m while  $v_0 = 38.95$  m/s = 140.21 km/hour for  $s_1 = 978$  m.

When 5 seconds is considered for the decision of the operation of the emergency brake,  $T_d = 9$  s. The initial velocity is solved with  $v_0 = 29.28$  m/s = 105.39 km/hour and  $v_0 = 34.98$  m/s = 125.93 km/hour for  $s_1 = 728$  m and  $s = 978$  m, respectively.

### **When the wagon's type is CCR2**

The result is the same as that of CCL2.

### **When the wagon's type is CCR5-9**

The result is the same as that of CCL5-9.

### 2.3.4 Cascade mass-point model

The simple model is used to estimate the initial velocity of the train. Here a cascade-mass-point model connected with nonlinear couplers is used to simulate the running of the train. In this model, the locomotive is thought to be 7E1 and the wagons are thought to be the above different types of wagons. The coupler characteristic is as indicated in Fig. 2.5.

The operational procedure of the emergency brake is thought to be as follows.

First the train driver notices the environment and makes a decision to initiate the emergency brake operation. In the following simulation, the time is denoted as  $T_1$ . The train driver begins to move the brake valve to the emergency position. During this operation, the braking pipe needs time to release its air pressure. This time is denoted as  $T_2$ . Actually there is another time delay mixed in this time delay, which is the transmission delay of the braking signal. In simulation, the transmission velocity of the emergency brake is about 300 m/s.

The following tables show the simulation results. In these tables,  $S_1$  is the braking distance without any time delays considered;  $S_2$  is the braking distance with the transmission delays (part of  $T_2$ ) considered;  $S_3$  is the braking distance with the transmission delays and another 2-second delay for the application delay considered and  $S_4$  is the braking distance with the transmission delay, 2-second application delay ( $T_2$ ) and 5-second decision time delay ( $T_1$ ) considered.

Table 2.2: Wagon type CCL1&3

| $V_0$ (km/hour) | $S_1$ (m) | $S_2$ (m) | $S_3$ (m) | $S_4$ (m) |
|-----------------|-----------|-----------|-----------|-----------|
| 138.54          |           | 986.62    | 1,063.58  | 1,256.00  |
| 128.05          |           | 844.83    | 915.97    | 1,093.82  |
| 119.52          | 712.9     | 738.00    | 814.40    |           |
| 117.29          | 686.5     | 711.12    | 776.28    |           |
| 116.17          |           | 697.83    | 762.37    | 923.72    |
| 109.12          |           | 616.89    | 677.51    |           |
| 108.00          | 582.1     | 604.50    | 664.50    |           |
| 97.49           |           | 494.52    | 548.68    | 684.08    |

Some explanations of the above tables follow.

In Table 2.2, the wagon type of the train is CCL1&3 and the BBF is 54,738 N. If the driver applied emergency braking once the train reached the point of sight distance (728 m from the cross) and the train's velocity was no more than 109.12 km/hour, the braking distance would not be more than 677.51 m. If the driver applied emergency braking 5 seconds after the train passed the point of sight and the train's velocity was no more than 116.17 km/hour, the braking distance would not be more than 923.72 m. Even if the couplers connecting the locomotives and wagons were rigid, the above

Table 2.3: Wagon type CCL2(CCR2)

| $V_0$ (km/hour) | $S_1$ (m) | $S_2$ (m) | $S_3$ (m) | $S_4$ (m) |
|-----------------|-----------|-----------|-----------|-----------|
| 139.16          |           | 986.75    | 1,064.50  | 1,257.33  |
| 128.59          |           | 844.48    | 915.92    | 1,094.52  |
| 120.06          | 712.95    | 738.15    | 804.85    |           |
| 117.28          | 686.67    | 711.37    | 776.83    |           |
| 116.59          |           | 696.71    | 761.49    | 923.42    |
| 109.55          |           | 616.27    | 677.10    |           |
| 108.00          | 576.90    | 599.50    | 659.50    |           |
| 97.84           |           | 493.62    | 547.98    | 683.86    |

Table 2.4: Wagon type CCL5-9(CCR5-9)

| $V_0$ (km/hour) | $S_1$ (m) | $S_2$ (m) | $S_3$ (m) | $S_4$ (m) |
|-----------------|-----------|-----------|-----------|-----------|
| 157.77          |           | 990.48    | 1,078.12  | 1,297.25  |
| 144.32          |           | 830.91    | 911.09    | 1,111.53  |
| 136.08          |           | 740.90    | 816.50    |           |
| 133.38          |           | 712.35    | 786.45    |           |
| 129.17          |           | 668.72    | 740.48    | 919.89    |
| 122.76          |           | 605.00    | 673.20    |           |
| 108.00          |           | 471.00    | 531.00    |           |
| 108.01          |           | 471.10    | 531.10    | 681.11    |

Table 2.5: Wagon type CCR1&amp;3

| $V_0$ (km/hour) | $S_1$ (m) | $S_2$ (m) | $S_3$ (m) | $S_4$ (m) |
|-----------------|-----------|-----------|-----------|-----------|
| 152.92          |           | 989.42    | 1,074.38  | 1,286.77  |
| 140.21          |           | 834.45    | 912.35    | 1,107.09  |
| 131.94          |           | 710.45    | 813.75    |           |
| 129.28          |           | 711.59    | 783.41    |           |
| 125.93          |           | 676.02    | 745.98    | 920.88    |
| 119.32          |           | 607.86    | 674.14    |           |
| 108.00          |           | 500.50    | 560.50    |           |
| 105.39          |           | 477.12    | 535.68    | 682.05    |

two distances would not be more than 728 m and 978 m, respectively.

In Table 2.3, the wagon type of the train is CCL2 or CCR2 and the BBF is 55, 230 N. If the driver applied emergency braking once the train reached the point of sight distance (728 m from the cross) and the train's velocity was no more than 122.76 km/hour, the braking distance would not be more than 673.20 m. If the driver applied emergency braking 5 seconds after the train passed the point of sight and the train's velocity was no more than 116.59 km/hour, the braking distance would not be more than 923.42 m. Even if the couplers connecting the locomotives and wagons were rigid, the above two distances would not be more than 728 m and 978 m, respectively.

In Table 2.4, the wagon type of the train is CCL5-9 or CCR5-9 and the BBF is 71, 018 N. If the driver applied emergency braking once the train reached the point of sight distance (728 m from the cross) and the train's velocity was no more than 109.55 km/hour, the braking distance would not be more than 677.10 m. If the driver applied emergency braking 5 seconds after the train passed the point of sight and the train's velocity was no more than 129.17 km/hour, the braking distance would not be more than 919.89 m. Even if the couplers connecting the locomotives and wagons were rigid, the above two distances would not be more than 728 m and 978 m, respectively.

In Table 2.5, the wagon type of the train is CCR1&3 and the BBF is 66, 696 N. If the driver applied emergency braking once the train reached the point of sight distance (728 m from the cross) and the train's velocity was no more than 119.32 km/hour, the braking distance would not be more than 674.14 m. If the driver applied emergency braking 5 seconds after the train passed the point of sight and the train's velocity was no more than 125.93 km/hour, the braking distance would not be more than 920.88 m. Even if the couplers connecting the locomotives and wagons were rigid, the above two distances would not be more than 728 m and 978 m, respectively.

### 2.3.5 Conclusions of the calculation

Although some information is unknown about the train and the track, some conclusions can be drawn based upon the aforementioned assumptions and calculations:

- 1) The faster the train, the longer the braking distance.
- 2) For different braking systems, the braking distances are different.
- 3) The smaller the COF BB of the wagons is, the longer the braking distance is.
- 4) The later the driver initiated emergency operation, the longer the braking distance was.
- 5) If the braking equipment worked as designed, and the driver initiated the emergency operation once the train was at the point of sight distance from the cross,

the braking distance would not be more than 728 m if the velocity of the train was no more than 110 km/hour. Even if the driver applied emergency operation 5 seconds after the train passed the sight point, the braking distance would not be more than 728 m if the velocity of the train was no more than 97 km/hour.

- 6) If the braking equipment worked as designed, and the driver initiated the emergency operation once the train was at the point of sight distance from the cross, the braking distance would not be more than 978 m if the velocity of the train was no more than 128 km/hour. Even if the driver initiated the emergency operation 5 seconds after the train passed the sight point, the braking distance would not be more than 978 m if the velocity of the train was no than 116 km/hour.

From this calculation, it is also seen that at the same initial speed, the stop distance calculated with the cascade-mass-point model is shorter than that in the case of the mass-point model (simple model). This is because with a cascade-mass-point model, part of the kinetic energy is consumed by the couplers, which is ignored in the mass-point model. From this point of view, the cascade-mass-point model is more accurate when a train is longer than 1.8 km, which is the case in this study.

## 2.4 Conclusion

In this chapter, a cascade of mass points connected with nonlinear couplers is described as a model for a long heavy haul train. This model has been validated in [17] with data from Spoornet.

A stop distance calculation is also given in this chapter with a mass-point model and a cascade-mass-point model, respectively. The calculation result shows that the latter is more accurate when a train is longer than 1.8 km, which is the case in this study.