References


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Appendix A  Rotor-Circumferential ELDV

A.1 Introduction

Whereas rotor-axial ELDV provides information on mainly the flap-wise on torsional vibration behaviour of the blade, the question arises whether any useful information is available from sideways vibration behaviour. To investigate this, it is necessary to align the LDV to measure along the blade leading edge in the plane of rotation as given in Figure 74 for a single-blade rotor. This will henceforth be referred to as Rotor-Circumferential (RC) ELDV. Jacobs and Grady (1977) present a system that allows RC TLDV using a scanning mirror directing the laser beam on a parabolic mirror. However no literature could be found on RC ELDV.

Evaluating Figure 74, it is clear that the circumferential speed of the blade will affect the measurements. Also the incidence angle of the laser beam relative to the blade edge normal will change continuously. Furthermore it is important to establish whether the ELDV scanning speed remains constant over the blade leading edge at a constant rotation speed. To address these issues, it is necessary to employ vector-loop calculations.

Figure 74: Effect of blade rotation on LDV measurement position
A.2 RC ELDV mathematical definition

A.2.1 Vector-loop equations

An equivalent mechanical system of the RC ELDV measurement technique is shown in Figure 75 along with its corresponding vector-loop diagram and the coordinate system used. It is important to define here the Mean Blade Leading Edge Curve (MBLEC). The MBLEC is obtained by drawing a straight line through the mean of the actual Blade Leading Edge Profile (BLEP) as demonstrated in Figure 76 for an arbitrary BLEP.

![Figure 75: a) Equivalent mechanical system b) Vector-loop diagram](image)

The vector notation used given by Equation 47:

\[
\vec{R}_h = \left| \vec{R}_h \right| e^{i\eta_h} = R_h e^{i\eta_h}
\]

Equation 47

with \( h \) the vector number.
The four vectors shown in Figure 75 can be defined as follows:

1. $\mathbf{R}_1$ describes the MBLEC offset. It starts from the rotation axis and terminates at its intersection to the MBLEC, perpendicular to that curve. This results in

   $\eta_2 = \eta_1 - 90^\circ$

   \textbf{Equation 48}

   which leads to the trigonometric relationships

   $\cos \eta_2 = \sin \eta_1$
   and $\sin \eta_2 = -\cos \eta_1$

   \textbf{Equation 49}

2. $\mathbf{R}_2$ defines the instantaneous measurement position of the laser beam on the MBLEC.

3. $\mathbf{R}_3$ gives the laser orientation and reaches from the laser beam intersection with the ZY plane to the measurement point location, which is at the end of $\mathbf{R}_2$.

4. $\mathbf{R}_4$ is defined as the laser offset vector, starting from the rotation axis along the ZY plane to the start of $\mathbf{R}_3$. 

**Figure 76: Mean Blade Leading Edge Curve**
Along with the vector definitions, the following assumptions are made:

1. RC blade vibration is ignored along with any motion of the centre of rotation of the shaft from the coordinate system origin. This means that $\mathbf{R}_1$ and $\mathbf{R}_2$ remain perpendicular and that $\mathbf{R}_4$ and $\mathbf{R}_1$ are constant.

2. The LDV is furthermore assumed to be perfectly stationary. As a result, $\eta_3$ is constant as well.

Using the vector-loop method, Equation 50 is obtained:

$$\mathbf{R}_1 + \mathbf{R}_2 - \mathbf{R}_3 - \mathbf{R}_4 = 0$$
$$R_1 e^{i\eta_i} + R_2 e^{i\eta_2} - R_3 e^{i\eta_3} - R_4 e^{i\eta_4} = 0$$

Equation 50

Rearranging for the real part of Equation 50:

$$R_1 \cos \eta_1 + R_2 \cos \eta_2 - R_3 \cos \eta_3 - R_4 \cos \eta_4 = 0$$
$$R_1 \cos \eta_1 + R_2 \sin \eta_1 - R_3 \cos \eta_3 = 0$$
$$R_2 \sin \eta_1 - R_3 \cos \eta_3 = -R_1 \cos \eta_1$$

Equation 51

Rearranging for the imaginary part of Equation 50:

$$R_1 \sin \eta_1 + R_2 \sin \eta_2 - R_3 \sin \eta_3 - R_4 \sin \eta_4 = 0$$
$$R_1 \sin \eta_1 - R_2 \cos \eta_1 - R_3 \sin \eta_3 - R_4 = 0$$
$$-R_2 \cos \eta_1 - R_3 \sin \eta_3 = R_4 - R_1 \sin \eta_1$$

Equation 52

To calculate velocities, Equation 50 is differentiated with respect to time:

$$\frac{d}{dt} (R_1 e^{i\eta_i} + R_2 e^{i\eta_2} - R_3 e^{i\eta_3} - R_4 e^{i\eta_4}) = 0$$
$$i\dot{\eta}_1 R_1 e^{i\eta_i} + i\dot{\eta}_2 R_2 e^{i\eta_2} + \dot{R}_1 e^{i\eta_i} - \dot{R}_4 e^{i\eta_4} = 0$$
$$i\dot{\eta}_1 (\cos \eta_1 + i \sin \eta_1) + i\dot{\eta}_2 R_2 (\cos \eta_2 + i \sin \eta_2) + \dot{R}_2 (\cos \eta_2 + i \sin \eta_2) - \dot{R}_3 (\cos \eta_3 + i \sin \eta_3) = 0$$

Equation 53

The real part of Equation 53 can be rearranged as:

$$-R_1 \dot{\eta}_1 \sin \eta_1 - R_2 \dot{\eta}_2 \sin \eta_2 + \dot{R}_2 \cos \eta_2 - \dot{R}_3 \cos \eta_3 = 0$$
$$-R_1 \dot{\eta}_1 \sin \eta_1 + R_2 \dot{\eta}_1 \cos \eta_1 + \dot{R}_2 \sin \eta_1 - \dot{R}_3 \cos \eta_3 = 0$$
$$R_2 \dot{\eta}_1 \cos \eta_1 + \dot{R}_2 \sin \eta_1 - \dot{R}_3 \cos \eta_3 = R_1 \dot{\eta}_1 \sin \eta_1$$
While the imaginary part Equation 53 is rearranged as:

\[ R_1 \dot{\eta}_1 \cos \eta_1 + R_2 \dot{\eta}_2 \cos \eta_2 + R_2 \sin \eta_2 - \dot{R}_3 \sin \eta_3 = 0 \]

\[ R_1 \dot{\eta}_1 \cos \eta_1 + R_2 \dot{\eta}_1 \sin \eta_1 - \dot{R}_2 \cos \eta_1 - \dot{R}_3 \sin \eta_3 = 0 \]

\[ R_2 \dot{\eta}_1 \sin \eta_1 - \dot{R}_2 \cos \eta_1 - \dot{R}_3 \sin \eta_3 = -R_i \dot{\eta}_1 \cos \eta_1 \]

Combing Equation 51, Equation 52, Equation 54 and Equation 55 in matrix form:

\[
\begin{bmatrix}
\sin \eta_1 & -\cos \eta_3 & 0 & 0 \\
-\cos \eta_1 & -\sin \eta_3 & 0 & 0 \\
\dot{\eta}_1 \cos \theta_1 & 0 & \sin \eta_1 & -\cos \eta_3 \\
\dot{\eta}_1 \sin \theta_1 & 0 & -\cos \eta_1 & -\sin \eta_3 \\
\end{bmatrix}
\begin{bmatrix}
R_2 \\
R_3 \\
\dot{R}_2 \\
\dot{R}_3 \\
\end{bmatrix}
=
\begin{bmatrix}
-R_i \cos \eta_1 \\
R_4 - R_4 \sin \eta_1 \\
R_i \dot{\eta}_1 \sin \eta_1 \\
-R_i \dot{\eta}_1 \cos \eta_1 \\
\end{bmatrix}
\]

Equation 56 can then be solved as:

\[
\begin{bmatrix}
R_2 \\
R_3 \\
\dot{R}_2 \\
\dot{R}_3 \\
\end{bmatrix}
=
\begin{bmatrix}
\frac{R_i \sin(\eta_1 - \eta_3) - R_4 \cos \eta_3}{\cos(\eta_1 - \eta_3)} \\
\frac{R_1 - R_4 \sin \eta_1}{\cos(\eta_1 - \eta_3)} \\
\frac{2\dot{\eta}_1 R_1 - \dot{\eta}_1 R_4 \sin \eta_1 - \dot{\eta}_1 R_4 \sin(\eta_1 - 2\eta_3)}{\cos(2\eta_1 - 2\eta_3) + 1} \\
\frac{2\dot{\eta}_1 R_1 \sin(\eta_1 - \eta_3) - 2\dot{\eta}_1 R_4 \cos \eta_3}{\cos(2\eta_1 - 2\eta_3) + 1} \\
\end{bmatrix}
\]

Equation 57 thus allows the calculation of the instantaneous measurement position of the laser beam on the MBLEC (\( R_2 \)) along with its instantaneous scanning speed \( \dot{R}_2 \) as functions of the laser beam orientation (defined by \( \overline{R}_3 \) and \( \overline{R}_4 \)) as well as the rotation angle \( \eta_1 \).

**A.2.2 Rigid Body Velocity Component**

Inherent to the RC ELDV measurement approach, is the presence of a Rigid Body Velocity Component (RBVC) in the measurements due to the circumferential velocity of the blade. To study this effect, vector-loop calculations are employed yet again. A new vector, \( \overline{R}_5 \), is defined stretching from the rotation centre to the end of \( \overline{R}_2 \) as seen in Figure 77:
Using the vector-loop, the following relationships are obtained:

\[
\vec{R}_5 = \vec{R}_1 + \vec{R}_2 = \vec{R}_3 + \vec{R}_4
\]

**Equation 58**

Solving for the X-components yield:

\[
R_5 \cos \eta_5 = R_3 \cos \eta_3 \\
\cos \eta_5 = \frac{R_3}{R_5} \cos \eta_3
\]

**Equation 59**

while solving for the Y-components give:

\[
R_5 \sin \eta_5 = R_3 \sin \eta_3 + R_4 \\
\sin \eta_5 = \frac{R_3}{R_5} \sin \eta_3 + \frac{R_4}{R_5}
\]

**Equation 60**
Referring to Figure 78, $v_{RB}$ can be calculated as follows:

$$v_{RB} = \dot{\eta}_1 R_5 \cos (\eta_5 - 90^\circ + 180^\circ - \eta_3)$$

$$= \dot{\eta}_1 R_5 \cos (\eta_5 - \eta_3 + 90^\circ)$$

$$= \dot{\eta}_1 R_5 \sin (\eta_3 - \eta_5)$$

$$= \dot{\eta}_1 R_5 (\sin \eta_3 \cos \eta_5 - \cos \eta_3 \sin \eta_5)$$

Equation 61

Substituting Equation 59 and Equation 60 into Equation 61:

$$v_{RB} = \dot{\eta}_1 R_5 \left( \sin \eta_3 \frac{R_3}{R_5} \cos \eta_3 - \cos \eta_3 \frac{R_3}{R_5} \sin \eta_3 - \cos \eta_3 \frac{R_4}{R_5} \right)$$

$$= -\dot{\eta}_1 R_4 \cos \eta_3$$

Equation 62

Since $\dot{\eta}_1 = 2\pi \psi$ is the rotor speed, $v_{RB}$ can be expressed as:

$$v_{RB} = -2\pi \psi \cdot R_4 \cos \eta_3$$

Equation 63

From Equation 63 is can be deduced that if $\psi$ is constant and the laser beam orientation is fixed, $v_{RB}$ will be constant and will be manifested in the measurements as a DC offset.
A.2.3 The influence of BLEP variance from the MBLEC

The variance of the actual BLEP from the MBLEC will affect the actual instantaneous measurement position as shown in Figure 79, and as a result the scanning speed as well. Also the incidence angle of the laser beam on the surface normal will be affected by the curvature of the BLEP and may thus affect the range of $\eta_1$ for which useful measurements can be recorded.

Since $v_{RB}$ is independent of measurement position, it remains unaffected by any variance of the BLEP from the MBLEC.

![Figure 79: Effect of BLEP variance from MBLEC](image)

A.3 Experimental verification

![Figure 80: RC ELDV experimental setup](image)
To verify the equations presented in Section A.2.2, experimental measurements were recorded on the single-blade rotor of Chapter 3. The experimental setup is demonstrated in Figure 80. To obtain the LDV orientation parameters, a tilt sensor was used to measure $\eta_3$ as well as the two values of $\eta_2$ for which $R_2 = 0.180$ m (i.e. when the blade tip enters and exits the laser beam). These two angles are labelled $\eta_{2,1}$ and $\eta_{2,2}$ respectively as shown in Figure 81. Since $\overrightarrow{R_1}$ and $\overrightarrow{R_2}$ are perpendicular, $\eta_{1,1}$ and $\eta_{1,2}$ is thus also known at these two positions. $R_4$ can then be obtained by rearranging the solution for $R_2$ of Equation 57 as shown in Equation 64 for $\eta_{1,1}$:

$$R_{4,1} = \frac{R_1 \sin(\eta_{1,1} - \eta_3) - R_2 \cos(\eta_{1,1} - \eta_3)}{\cos \eta_3}$$

Equation 64

Setting Equation 64 into Equation 63, $v_{RB}$ can be expressed in terms of m/s/RPM as given for $\eta_{1,1}$:

$$v_{RB,1} = -(2\pi/60) \cdot [R_1 \sin(\eta_{1,1} - \eta_3) - R_2 \cos(\eta_{1,1} - \eta_3)]$$

Equation 65

The LDV orientation measurements and calculations are summarized in Table 10. Ideally $v_{RB,1}$ and $v_{RB,2}$ should be equal. However due to the resolution of the tilt sensor, $\eta_{2,1}$ and $\eta_{2,2}$ was measured with limited accuracy.
Table 10: LDV orientation measurements and calculations

<table>
<thead>
<tr>
<th>Position</th>
<th>η</th>
<th>°</th>
<th>R</th>
<th>m</th>
<th>υ_{RB}</th>
<th>mm/s/RPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>η₂₁</td>
<td>32.7</td>
<td>R₄₁</td>
<td>0.110</td>
<td>υ_{RB₁}</td>
<td>11.54</td>
</tr>
<tr>
<td></td>
<td>η₃</td>
<td>179.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>η₂₂</td>
<td>139.1</td>
<td>R₄₂</td>
<td>0.106</td>
<td>υ_{RB₂}</td>
<td>11.10</td>
</tr>
<tr>
<td></td>
<td>η₁₂</td>
<td>229.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

RC ELDV measurements were recorded on the blade during rotor run-up to 960 RPM as shown in Figure 82. From this figure a lower limit DC drift can be observed in the measurements. Although the lower limit of the measurements should be at 0 m/s (as seen at the start of the measurement), the LDV measurements start to drift at the lower rotor speeds and settles after about 13 s. This phenomenon is ascribed to the LDV measurement system. Feedback from the OEM was however not available.

To compare the measured and theoretical RBVCs, the experimental measurement range for each blade passage needs to be considered. The results are shown in Figure 83 and errors of 8.9 % and 4.7 % are observed for υ_{RB₁} and υ_{RB₂} respectively. Via optimization, υ_{RB,opt} was obtained as 10.60 mm/s/RPM which indicates angle measurement errors in η₂₁, η₂₂ and η₃ of about 1°. This corresponds to the errors that were measured during verification of the sensor.

From Figure 83, it is seen that a very good correlation exists between the experimental values of υ_{RB} and υ_{RB,opt} although there is some difference at the lower rotor speeds. This discrepancy however occurs during the non-stationary phase of the measurement lower limit DC drift and is therefore probably a manifestation thereof.
Appendix A

Figure 82: LDV drift

Figure 83: Comparison of measured and theoretical RBVCs
A.4 Response matrix interpolation for non-constant scanning speeds

In Equation 57, it was shown that the scanning speed along the blade edge during a single blade passage is not constant:

\[
c = \hat{R}_2 = \frac{2\dot{\eta}_1 R_1 - \dot{\eta}_4 R_4 \sin \eta_1 - \dot{\eta}_4 R_4 \sin (\eta_1 - 2\eta_3)}{\cos (2\eta_1 - 2\eta_3) + 1}
\]

Equation 66

In Section 2.2.3 it was shown that LVRM interpolation can be successfully employed for constant scanning speeds. To perform this for a non-constant \( c \), one approach is to construct the LVRM with a scan speed ratio \( k = 1 \) (i.e. \( c_{\text{ref}} = c = \hat{R}_2 \)). Another approach is to construct the LVRM for a constant \( c_{\text{ref}} \) chosen to obtain the required interpolation tolerance.

A.5 Conclusions

In this section, the peculiarities of the RC ELDV measurement approach were studied analytically using the vector-loop method. It was shown that RC ELDV introduces a DC offset or RBVC which is directly proportional to the rotor speed. This was verified experimentally.

Observed in the experimental measurements was a lower limit DC drift, which is contributed to the SLDV system. Further work is thus necessary to establish the feasibility of this measurement approach.

The work of this section is presented in the article titled “On the measurement of circumferential vibration on rotating blades using laser Doppler vibrometry” (Oberholster and Heyns, In progress).