APPENDICES

SYNOPSIS

FURTHER REFINEMENTS AND A NEW EFFICIENT SOLUTION OF A NOVEL MODEL FOR PREDICTING INDOOR CLIMATE

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In the nineties and beyond energy conservation will play a dominant role in the design of new buildings. In the past two decades, many energy efficient buildings failed to provide adequate comfort. The main problem seems to have been the lack of a total approach to the design of buildings. Such an approach regards building and air–conditioning unit as a total thermal system, which together, in close co–operation, must ensure acceptable indoor climate. To facilitate a total design approach, a thermal analysis tool is required which is targeted at both the architect, designing the building shell, and the engineer, which installs the air–conditioning system. For such a tool to be successful, it must be simple, easy to use and powerful. This thesis contributes to the enhancement and establishment of such a design tool. In particular, the novel method originally proposed by Mathews and Richards, is enhanced and extended.

As point of departure, the derivation of simplified models from more, comprehensive models is discussed. Simplified thermo–flow models for buildings are usually based on apt reasoning about the main thermal storage– and transport– characteristics of buildings. Mathews and Richards base their model on the concept of *active thermal capacitance*, which is an attempt to model the salient features of energy storage in the massive parts of the building. The analysis here indicates that it is possible to derive the simplified model in a logical manner from a more comprehensive model, by suitable assumptions and simplifications. In this manner, the concept of active capacitance is clarified, and the assumptions of Mathews and Richards is illuminated. The theoretical foundation of the concept of active capacitance can be established. The outcome of this study proved fruitful in that a new enhanced simple network, with a better physical interpretation and theoretically more satisfactory treatment of the building shell, as well as internal mass, is suggested. In addition, it is found that the definition of the mean sol–air forcing function, as used by Mathews and Richards, probably causes the phase discrepancy they observe. The study further indicates that the high degree of lumping of the simplified model is acceptable. Theoretically, the accuracy of the model is limited by the assumption that the massive parts of the building are at the same temperature.

The utility of the simplified model is extended by the inclusion of capabilities for structural storage systems, variable parameter thermoflow and proportionally controlled active systems. A further enhancement is the development of a procedure for extending the method to multi–zone thermal analysis. Structural storage can be easily included in the thermal model, but necessitates a somewhat unusual definition of the heat transfer coefficient, since the model of Mathews and Richards does not provide for the prediction of surface temperatures. Inter–zone heat–flow is accomplished in a manner which forms a natural extension of the single–zone method. The proposed method first applies the single zone model to each zone individually and, thereafter, determines the heat–flow between the zones. The procedure requires a single matrix inversion, with the order of the matrix given by the number of zones. Unfortunately, the

procedure can only be implemented for time invariant systems.

This thesis investigated solutions for a time dependent system in order to make provision for time variable ventilation rates, and variable shell conductance etc. A very simple and highly efficient numerical solution procedure is presented. An explicit solution for the initial value is given so that the usual initial period of integration – to get rid of transients – is not required. An alternative, less efficient Fourier series method is also given. The solution method is quite general and can be extended to higher order systems and implicit discrete systems.

With this thesis, a firm foundation has been laid for the extension of simplified methods, which have been very successfully applied to passive thermal design, to active thermal systems. The novelty of these methods lies in their simplicity, which enables analysis and comprehension of their behaviour. A simple model is a prime requirement for the investigation of the behaviour of systems wich incorporate complicated non–linear control. The investigation of these systems is required for the successful design and optimization of comfortable and energy efficient buildings in future.

UITTREKSEL

VERDERE VERFYNINGS VAN 'n METODE VIR DIE VOORSPELLING VAN BINNENSHUISE KLIMAAT, INSLUITEND 'n NUWE EFFEKTIEWE OPLOSSING.

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In die negentigs, en daarna, sal energie besparing 'n baie belangrike invloed uitoefen op die ontwerp van nuwe geboue. In die vorige twee dekades het heelwat nuwe 'energie effektiewe geboue' gefaal, in die sin dat die binneklimaat nie altyd aanvaarbaar was nie. Die probleem is waarskynlik toe te skryf aan 'n gebrekkige ontwerpsmetodiek, wat die passiewe gebou en aktiewe lugversorgingstelsel apart hanteer. Om 'n geïntegreerde ontwerpsprosedure te bevorder, word 'n rekenaarprogram benodig, wat beide die argitek en die ingenieur kan gebruik, en wat die gebou as 'n termiese stelsel in geheel hanteer. Hierdie tesis is 'n bydrae tot die vestiging en bevordering van so 'n program. In besonder word die unieke, vereenvoudigde metode van Mathews en Richards verder verbeter en uitgebrei.

As vertrekpunt dien die afleiding van vereenvoudigde modelle uit meer omvattende modelle van termiese gedrag. Gewoonlik word vereenvoudige

UITTREKSEL

modelle gebaseer op redenasies oor die hoofbeginsels van die berg– en vloei van warmte in 'n gebou. Mathews en Richards baseer hulle model op die konsep van *aktiewe termiese kapasiteit*, wat die heveelheid warmte, geberg in die gebou, en wat 'n rol speel by die bepaling van die binne–temperatuur, weergee. Die studie toon aan dat dit moontlik is om die eenvoudige model te herlei uit 'n omvattende model, d.m.v. logiese beredenering en vereenvoudigende aannames. Op hierdie manier kan die idee van aktiewe kapasitansie, asook die aannames van Mathews en Richards, onder die soeklig geplaas word en 'n teoretiese fondament kan onder die beginsel van aktiewe kapasitansie geplaas word. Hierdie studie het verder daartoe gelei dat 'n teoreties meer gebalanseerde vereenvoudigde model voorgestel kon word, met bepaalde voordele. Hierbenewens word daar gevind dat die definisie van die gemiddelde sol–lug temperatuur, soos gebuik deur Mathews en Richards, waarskynlik verantwoordelik is vir die empiriese korreksie faktor – betreffende die faseverskuiwing van die binne temperatuur – wat hul model benodig. 'n Verdere resultaat van belang is; dat die hoë mate van vereenvoudiging, vanaf 'n akkurate verspreide elementmodel – na 'n lompmodel, nie die beperkende fator van die eenvoudige model is nie, maar eerder die aanname dat die massiewe strukture almal altyd dieselfde by temperatuur bly.

Die nuttigheid van die vereenvoudigde model word in hierdie studie verhoog deur die metode uit te brei om voorsiening te maak vir warmte berging in die struktuur, tydafhanklike elemente en proporsioneel beherde stelsels. Nog 'n toevoeging is die ontwikkelling van 'n metode vir die berekening van die termiese gedrag van meervoudige–gebousones. Struktuur berging is redelik maklik om te inkorporeer in die model, maar dit vereis 'n iewat ongewone definisie van die warmte oordrag koëffisiënt, aangesien die model van Mathews en Richards nie oppervlak temperature modelleer nie. Die tegniek vir die hantering van warmte vloei tussen verskillende sones is so ontwerp dat dit 'n natuurlike uitbreiding van die enkelsone–model vorm. Die voorgestelde metode gebruik eers die enkelsonebenadering vir elke sone afsonderlik, waarna die warmtevloed tussen die sones en die finale temperature bereken word. Dit geskied d.m.v. die omkering van 'n

UITTREKSEL

matriks waarvan die orde bepaal word deur die aantal sones teenwoordig. Ongelukkig kan die metode nie so maklik geïmplementeer word indien die elemente tydafhanklik is nie.

In hierdie tesis word 'n nuwe oplostegniek voorgestel wat voorsiening maak vir tydafhanklike elemente, sodat die model ook gebruik kan word by die termiese ontleding van geboue met variërende ventilasie, dopweerstand ens. 'n Hoogs eenvoudige en uiters effektiewe oplostegniek is ontwikkel met 'n eksplisiete vergelyking vir die beginwaarde. Dit is dus nie meer nodig om 'n lang aanvanklike integrasieperiode te gebruik om van oorgangs–verskynsels ontslae te raak nie. Die oplostegniek is algemeen bruikbaar. Dit kan uitgebrei word na hoër orde stelsel asook implisiet versyferde stelsels. 'n Alternatiewe, minder effektiewe metode, gebaseer op Fourier reekse, word ook aangebied.

Met hierdie tesis word 'n stewige fondament gelê vir die toepassing van die vereenvoudigde tegnieke ook op aktiewe stelsels. Die bruikbaarheid van die vereenvoudigde tegnieke is gesetel in die gemak waarmee hulle gemanipuleer kan word en hulle verstaanbaarheid. 'n Eenvoudige model is 'n voorvereiste vir verdere studies oor die gedrag van termiese geboue met nie–liniêre beheerstelsels. Die bestudering van hierdie stelsels is broodnodig vir die ontwerp en optimisering van suksesvolle, energie–effektiewe geboue in die toekoms.

Efficient, Steady State Solution of a Time Variable RC Network, for Building Thermal Analysis.

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Abstract

This paper introduces an efficient method for obtaining the steady state, periodic thermal response of a building with arbitrary, time dependant ventilation. The method is applicable to a single time–constant thermal model but can be extended to higher order models. A simple, elementary numerical method for integrating the governing differential equation is proposed which does not need Fourier analysis, convolution or even evaluating an exponential, as required by most other methods. The initial value is also obtained explicitly. Hence, the usual initial period of integration – to get rid of transients – is not required. By direct comparison of the numerical method with an exact analytical solution in a special case, it is proved that the method is sufficiently accurate, provided the sampling interval is not too large, compared to the thermal time–constant of the building. The method is further demonstrated by calculating the interior temperature of a building subjected to forced night cooling.

1 Introduction

The analogy between thermo–flow and electron–flow is often exploited to derive simple models for building thermal analysis [2–4]. In these models it is usual to lump [13] the distributed thermal conductance and capacitance so that instead of the partial differential equation for heat conduction, one is faced with an ordinary differential equation, the order of which is determined by the number of lumped resistances and capacitances (nodes) used to describe the distributed parameters. Other heat transfer phenomena, such as radiative exchange and convection, can be integrated in the electrical analogy by defining the appropriate thermal resistances. In the case of natural ventilation, the ventilation resistances is time dependant since ventilation rates vary appreciably with the hour of the day. To model radiative exchange requires a thermal resistance which is strongly dependant on the temperature, hence strongly non–linear [14]. However, in most methods it is assumed that the parameters (resistances and capacities) are time invariant, to enable the application of Fourier or Laplace techniques to obtain the solution. This limits the solution to cases where the ventilation rate is constant; a serious practical limitation.

In this paper we describe an elementary numerical method for solving the variable coefficient differential equation of the model of Mathews and Richards [1], without sacrificing functional accuracy or computation speed. Furthermore, the method is in no way restrictive regarding the magnitude or details of the variation of the parameters with time. The solution obtained is also stable and free of transients, even with discontinuous variations of the parameters. The method is generally applicable to the simple RC networks which many investigators use to model heat flow in buildings and other structures [2..4]. It can be extended to more complicated higher order networks by treating the higher order equations as a system of first order equations.

In the next section of this paper, §2, some relevant remarks on computer methods for thermal analysis are presented, In §3, the general governing equation for the network of [1] with time dependent parameters is given. In §4 a few available solutions and standard numerical techniques for solving variable RC networks are discussed. An efficient, approximate method is presented in §5 and the accuracy of this method is discussed in §6. The method can easily be extended to include linear feedback, active indoor climate control systems. In §7, the extension to include an active system, with indoor temperature controlled by a proportionally controlled thermostat is given. Finally, §8 concludes with an example of the application of the method to night cooling of buildings.

2 Computer Methods for Building Thermal Analysis

A large number of computer programs for building thermal analysis are available. According to Tuddenham [15] there are more than one hundred in the United Kingdom and many hundreds more elsewhere. Most of these methods are based on the 'admittance method' of the CIBS [17] or the 'response factor method' of ASHRAE [17]. The CIBS method was originally developed for manual calculations [18]. It employs pre-calculated tables of decrement- and other factors for building materials. The response factor method was originally developed for computer implementation [19]. It does seem, however, that the method was severely influenced by the very crude computer hardware and software available at the time and the central theme of the method appears to be an attempt to ease the evaluation of the convolution integral, as required to obtain the forced response [14]. In view of the undreamt of growth in computer technology and numerical techniques, both these methods appear outdated. The Fast Fourier Transform has made the evaluation of convolution integrals a very simple and computationally efficient exercise.

Both the CIBS and the ASHRAE method (and others e.g. [21,23]) employ the exact analytical solution of the diffusion equation in the form of matrices. These matrices are then used to obtain the response\decrement factors from which the thermal response is obtained. In the admittance procedure the factors are used to obtain amplitude and phase-shift values for the internal temperature. The response factor method obtains the solution by superimposing the response to a series of triangular pulses. Both methods only apply to linear time-invariant thermal systems. (In [20] invariability is erroneously stated as a requirement for using the method of superposition. Linearity is sufficient. Invariability is only required for the Laplace transform method to be tractable. See [6].)

Since the exact matrix solution for heat conduction is simple and amendable to computer implementation, and standard two port theory can be used to combine the matrices of various elements, it is inexplicable that computer programs still employ response factors. Certainly, the matrices are limited to time invariant systems, but so are the response factors since they are derived from the matrices. For the time invariant case it seems that an elegant method would compute the matrices for the various elements at a number of frequencies, combine the matrices with two port theory, and compute the exact solution via the superpositioning of the responses to the various frequencies [7]. Computationally this is certainly feasible on modern computers. The method can be used to obtain both transient and steady state solutions by employing either the Laplace or the Fourier domain.

In [19] some emphasis is placed on the response factor method being applicable to non-steady state conditions. To simulate non-steady conditions one would have to specify initial conditions, which can only be guessed or become known by measurements. Therefore, simulation of non-steady conditions seems rather academic. In practice one is confronted with continuous variables which can be described as quasi-periodic. One should therefore rather attempt to find the quasi-periodic solutions which are often sufficiently accurately represented by periodic solutions [27]. Especially if the principle of a design day is adhered to.

In view of the availability of this practical, exact solution for the heat conduction equation with steady excitation, it is also disconcerting to find many programs employing finite difference or finite element techniques. It is often stated that these numerical methods facilitate solution of a time variable, fully non-linear model. In fact the powerful but often tedious and sometimes unpredictable numerical techniques are essentially employed to solve the time invariant, linear, heat conduction equation. The time variability and non-linearities in thermal models of buildings do not arise from heat conduction but from other heat transfer modes. Certainly, one could use the matrix solution for the conduction equation and combine it with time variable non-linear two ports. The complete solution is then obtained from non-linear network methods which were extensively developed in recent years [7].

But the important point we wish to make is that thermal analysis methods can either attempt to be extremely accurate, with emphasis on exact simulation – and be of academic value only, or sufficiently accurate with a design philosophy (e.g. design day) in mind – and practical.

To create a viable design tool the following points should be borne in mind:

a) A complete description of the thermo-flow in real buildings requires many details, some of which (e.g. ventilation rates) are wholly unknown or only partly known. This is even more true if the objective is a design tool, since the thermal properties of a building are largely determined in the very early design stages when no details at all are available.

b) Many studies and empirical models indicate that a few essential parameters of the buildings such as heat storage capacity and shell admittance are crucial and should be emphasized rather than details. It is crucial that the assumptions be clear, easy to explain and to understand.

c) A successful computer design tool must employ a very simple model with a straightforward physical explanation to allow the designer's good judgment and experience to play its essential part. Highly refined models and exact solutions are better employed in research laboratories for verification purposes and the extension of knowledge and understanding.

d) For a design tool, extreme numerical accuracy – at the cost of computing time – is detrimental. Certain essential parameters of thermo–flow in buildings, such as ventilation rates, are largely unknown. The emphasis should be on establishing the relative merit of various designs, rather than absolute accuracy.

e) A design tool should allow innovation and easy extension to cater for creative ideas and new techniques.

f) A computer procedure should not be based on 'established' techniques which were developed for hand calculations and which are overly simplified or unnecessarily rigid.

g) A design tool should follow a total approach. Both the passive response of the building and the HVAC system must receive due attention. Many available programs place too much stress on load prediction and the energy audit. This may lead to unnecessary efforts to optimize the HVAC system while much more could be gained by improving the shell.

It was shown by Mathews and Richards [1] that a simple, single time–constant RC network, as given in Figure 1, can be a very useful aid for determining the thermal performance of a building. This extraordinary simple approach has a number of important advantages; the thermal response analysis can be implemented on inexpensive, generally available computers (PC's), and still provides fairly accurate answers, quite rapidly. In addition, the simple thermal network has a very clear physical interpretation, which is easily explained to designers. The simplicity also enables easy extension to include new ideas and building materials. This facilitates a design process where various options can quickly be evaluated. The data input requirements are modest and no expert knowledge of thermal analysis is required. Furthermore, an extremely user friendly interface can relieve the burden of the specification of construction details. The result is a streamlined and highly efficient program. Complete temperature and load simulations are obtained in a few seconds.

The main limitations of the method are the single zone approach, the assumption of isothermal interior surface temperature and the assumption of well mixed interior air. These assumptions are difficult to establish on theoretical considerations but are vindicated by the excellent results obtained from verification experiments in many buildings [1].

The details of the method appear in [1]. Briefly, in figure 1 the resistances are R_o: conductive shell resistance including exterior film coefficients, R_a: film resistance of the interior surfaces and R_v: ventilation resistance. The ventilation resistance is obtained from: $R_v = 3.6/Vol \cdot \rho \cdot ach \cdot C_p$ where Vol [m^3] is the interior volume, ρ [kg/m^3] is the density of air, C_p [kJ/kg·K] is the specific heat and ach [/h] is the air change rate. The sources in figure 1 are T_{sa}: averaged sol-air temperature of the external surfaces, Q_r: mean radiation on the interior surfaces, Q_v: interior air convective sources and T_o: temperature of the ventilating air. The circuit can easily be extended to also include structural cooling, evaporative cooling etc. by adding more sources. The heat storage of the massive elements of the construction is represented by the capacitor C. The value of C, which is critical, includes only the active part of the total heat capacitance of the zone and is determined in a heuristic, experimentally well proven method [22]. Typical values for the network elements are given in table 1. The dependent quantities of interest are firstly; the interior air temperature T_i, and secondly; the sensible load required to maintain a specified interior temperature. (Latent loads present no problem, but for simplicity's sake this paper will be restricted to dry-bulb temperatures. A new version of the program is commercially available which includes evaporative cooling, latent loads and structural cooling.)

It was extensively validated and has already, in practice, proved a valuable design aid for architects, and for establishing norms for the thermally efficient design of buildings. Presently, the implementation is restricted to periodic, diurnal forcing functions. The philosophy is to calculate the response for a typical hot and cold design day as though every other day is exactly similar. In practice, the method will be grossly erroneous only when the building has a very long time constant (very massive with high shell isolation) and the thermal energy consumption differs radically on some days from the norm e.g. on weekends, or if the weather pattern is drastically different on a single day. This approach is in line with the philosophy of a design day, where days of typical extreme weather are used. There is no fundamental reason why the method is not applicable to periods of e.g. 7 or 365 days.

The past method of solution of the network was to obtain the convolution between forcing functions and system response in the frequency domain. This method assumes the parameters R_{o}, R_{a}, R_{v} and C are constants in time and the governing equation of the network is a linear, constant coefficient, differential equation. This is rather restrictive; with this assumption the program cannot treat situations where e.g. the ventilation rate varies with the hour of the day. Consequently, the method is only applicable to situations where the windows remain closed or open during all hours and\or a constant rate of forced ventilation and infiltration is maintained throughout the day. Most other thermal analysis programs suffer from the same limitation [18–21]. To relax this restriction, it is necessary to solve the governing equation of the network of figure 1, with circuit elements which are assumed functions of time; a considerably more difficult problem.

3 Governing Differential equation

The equation for the interior temperature of the network of figure 1, *with resistors and capacitance assumed functions of time*, is:

$$T_i = \frac{T_c \cdot R_v + T_x \cdot R_a}{R_v + R_a} \tag{1}$$

where: $$T_x = T_o + R_v \cdot Q_c. \tag{2}$$

T_c is the temperature at the structure node (across the capacitor in figure 1) and is given in terms of the stored heat q

$$T_c = q/C \tag{3}$$

which is found from the governing differential equation:

$$\dot{q} + \beta_T \cdot q = f_T. \tag{4}$$

In (4) the subscript T refers to the solution for the interior temperature. The forcing function f_T is:

$$f_T = \frac{T_x}{R_a + R_v} + \frac{T_y}{R_o} \tag{5}$$

with $$T_y = T_{sa} + R_o \cdot Q_r$$

and the time dependent coefficient $\beta_T(t)$ is the inverse of the time–constant τ_T of the building:

$$\beta_T = \frac{R_a + R_o + R_v}{C \cdot R_o \cdot (R_a + R_v)} = 1/\tau_T. \tag{6}$$

It is possible to substitute (2) to (6) in equation (1) to obtain an equation which directly delivers T_i in terms of the sources. In practice, it is a great advantage to write the governing equation in terms of the amount of stored heat, and not in terms of the primary quantities of interest. The stored heat is a fairly smooth function of time (provided mass is not added to or removed from the structure), while the temperatures are subject to sharp discontinuities when the values of the circuit elements suddenly change. Since we are especially interested in sudden, large changes, e.g. when windows are opened and closed or forced cooling is switched on and off, it is important to be able to solve the equation accurately for discontinuous coefficients.

The equations were derived for the general case where any of the elements, and not just R_v, may be subject to variation, although R_v is the most important. There is little formal distinction between changes in R_v and changes in the other elements. Hence the only type of variation the equations do not cater for is, when the storage capacity of the building is varied by introducing or removing mass. This is of more than theoretical importance; the other parameters of the circuit may be subject to various time dependent influences, e.g. the interior film resistance is definitely also affected by the time dependant air circulation rate.

Analogous to equations (1) to (6), the sensible convective load Q_{cr}, to obtain a prescribed interior temperature T_{ir}, can be found by substituting T_{ir} for T_i in (1) to (6) and solving for the convective load. While this load calculation is highly theoretical – practical thermostats normally include dead bands and/or hysteresis – the calculation is useful for estimating required system capacities, without troublesome iterative procedures. Q_{cr} is given by:

$$Q_{cr} = \frac{R_a + R_v}{R_a \cdot R_v} \cdot T_{ir} - \frac{T_{cr}}{R_a} - \frac{T_o}{R_v} \tag{7}$$

with T_{cr} the required structure temperature. T_{cr} is determined from the amount of stored heat:

$$T_{cr} = q_r / C \tag{8}$$

which satisfies the differential equation

$$\dot{q}_r + \beta_E \cdot q_r = f_E. \tag{9}$$

In this case the forcing function f_E is

$$f_E = \frac{T_v}{R_o} + \frac{T_{ir}}{R_a} \tag{10}$$

and the coefficient β_E is

$$\beta_E = \frac{R_a + R_o}{R_a \cdot R_o \cdot C} = 1/\tau_E. \tag{11}$$

In both instances, (4) and (9), the solution of a linear first order differential equation with time dependent coefficient $\beta(t)$ must be found. The equation is of the form

$$\dot{y}(t) + \beta(t) \cdot y(t) = x(t). \tag{12}$$

4 Existing Methods of Solutions

The general solution of (12) is well known and can be found in any text book on elementary differential equations. It is

$$y(t) = exp[-\Gamma(t)] \cdot \int_{-\infty}^{t} exp[\Gamma(t)] \cdot x(t) \; dt \tag{13}$$

with $$\Gamma(t) = \int \beta(t) \; dt. \tag{14}$$

In (13) the lower limit of integration is taken at minus infinity to indicate that the steady state response is required. The integral can be evaluated by numerical integration (either in the form of (13), or in the form of the original differential equation (12)) by a standard procedure such as Runge–Kutta. Numerical integration is, unfortunately, relatively inefficient. The initial condition is unspecified, or stated more precisely: is assumed to have occurred far back in history. The integration must continue until the transient response is extinct. Since the time constant of a building can be quite long (30 hours or more is not uncommon), and the transient response can be regarded as sufficiently extinct only after 5 time constants, integration may have to continue for a considerable period to ensure sufficient accuracy of the answer. Since a very high premium is attached to the speed of computation, it is desirable to find a quicker method.

Various methods for treating systems with variable parameters exist in the literature. It was shown by Carson [5] (see also [6]) that the solution can be expressed in the form of a Volterra integral equation. The solution of this integral equation is given in terms of an infinite progression. Alternatively, solutions in terms of a series expansion of Bessel functions can be found when the coefficient varies sinusoidally [6]. These and other similar expansions [8,9], converge rapidly when the variation of the coefficient $\beta(t)$ is slow, compared to the variation of the forcing function or when $|\dot{\beta}/\beta| \ll 1$. For sudden jumps in the value of β, i.e. when $\dot{\beta}$ is very large, they are of little practical value.

The traditional method for isolating the steady state response is through Fourier series methods. In essence there is little fundamental difference in the application of this method to systems with variable parameters – as opposed to constant parameters – except that it must be assumed that the Fourier coefficients of the output are functions of time [8]. The method leads to a mixed time–frequency domain description. The Fourier technique is so prevalent in the literature that it warrants some further discussion.

Since the differential equation is linear (although time variant) it will be sufficient to determine the solution for the phasor $x(t) = X \cdot e^{j\omega t}$ ($X = X(\omega)$ a complex number independent of time). The solution for general periodic inputs can be obtained by superpositioning the phasor components of each constituent frequency component. Assuming the response $y(t)$ to the phasor input $x(t)$ is of the form $y(t) = Y(t,\omega) \cdot e^{j\omega t}$ and substituting these assumed values for x and y in (12) furnishes:

$$\frac{\partial Y(t,\omega)}{\partial t} + [j\omega + \beta(t)] \cdot Y(t,\omega) = X(\omega). \qquad (15)$$

This is the modulation function equation (MFE) of the system as discussed in [9]. In [8] and [9] methods are presented for directly transforming (12) into (15) for more general systems. See also [24 , 25]. It is customary to define the system transfer function:

$$H(t,\omega) = \frac{Y(t,\omega)}{X} \qquad (16)$$

which, from (15), satisfies:

$$\frac{\partial H}{\partial t} + [j\omega + \beta] \cdot H = 1. \qquad (17)$$

Equation (17) is of exactly the same form as (12), however, it involves complex functions and the input in (17) is a constant. But obviously, the Fourier series expansion is not beneficial regarding computation time. Instead of the initial value, $y(0)$ in (12), the initial frequency response $H(0,\omega)$ is required, and furthermore, (17) must be solved for each frequency component of the forcing function. However, if the time–constant is sufficiently long at all hours, only the first few frequency components will be significant in the solution. Alternatively, an approximate solution for a few components can be obtained by the method of Galerkin [6].

Another approach is to assume the circuit parameters are constant in small intervals and then to solve the equation exactly for each interval [10]. This approach requires matching the final conditions of each interval to the initial conditions of the next interval. For a steady, periodic solution, the initial condition of the first interval must match the final condition of the last interval. It is seen that the process requires the simultaneous solution of a large number of equations. In the next section we obtain an approximate method following this approach with the additional assumption that all forcing functions and output variables are constant between sampling points, in which case an explicit formulation for the initial value is possible.

5 Approximate Numerical Solution

The advantage of writing the governing equation in terms of the amount of stored energy is; it reduces the sensitivity of the solution of T_i to errors in the solution of the differential equation (12). A large part of the variation in T_i is accurately included in the final calculation of T_i and Q_{ri} from q, in (1) and (7). This is easily demonstrated by noting that in the limit, when the ventilation rate is very large ($R_v \sim 0$), the sole contributor to T_i in equation (1) is T_x. On the other hand, when R_v approaches infinity, the sole contributor is T_c. This is in accordance with the network of figure 1.

To obtain a numerical solution for (12) we follow the standard procedure of rewriting the equation in the form of an integral equation:

$$\begin{aligned} y(t) &= \int_{-\infty}^{t} x(t_1) - \beta(t_1)\cdot y(t_1)\ dt_1 \\ &= \int_{-\infty}^{0} x(t_1) - \beta(t_1)\cdot y(t_1)\ dt_1 + \int_{0}^{t} x(t_1) - \beta(t_1)\cdot y(t_1)\ dt_1 \\ &= y(0) + \int_{0}^{t} x(t_1) - \beta(t_1)\cdot y(t_1)\ dt_1 \qquad (18) \end{aligned}$$

For periodic, steady state solutions with period T, it is required that the initial value of every period equals the final value of the previous period:

$$y(0) = y(T) = y(0) + \int_{0}^{T} x(t_1) - \beta(t_1)\cdot y(t_1)\ dt_1$$

and therefore

$$\int_0^T x(t_1) - \beta(t_1) \cdot y(t_1)\; dt_1 = 0. \qquad (19)$$

The steady, periodic solution of (12) is given by (18) with initial condition stipulated by (19). For discrete data at $t = t_i = i \cdot \Delta T$, $T = N \cdot \Delta T$, these equations take the form:

$$y_k = y_0 + \sum_{i=0}^{k-1} \Delta T \cdot (x_i - \beta_i \cdot y_i) \qquad (20)$$

and

$$\sum_{i=0}^{N-1} \Delta T \cdot (x_i - \beta_i \cdot y_i) = 0. \qquad (21)$$

It is assumed all variables are constant between sampling points and $x_i = x(t_i)$ and $\beta_i = \beta(t_i)$ are tabulated functions. Equation (20) can be written in the following corresponding, iterative form:

$$y_k = \Delta T \cdot x_{k-1} + (1 - \Delta T \cdot \beta_{k-1}) \cdot y_{k-1} \qquad k = 1,2,3 \ldots N-1 \quad (22)$$

If one value of y is known the other values are easily found from (22), provided the iteration is stable. A closed form solution for the initial value y_0 is obtained by starting with

$$y_N = \Delta T \cdot x_{N-1} + (1 - \Delta T \cdot \beta_{N-1}) \cdot y_{N-1} = y_0 \qquad (23)$$

and substituting previous values of y_k.

$$\begin{aligned} y_N &= \Delta T \cdot x_{N-1} + (1 - \Delta T \cdot \beta_{N-1}) \cdot (\Delta T \cdot x_{N-2} + \\ &\quad (1 - \Delta T \cdot \beta_{N-2}) \cdot (\Delta T \cdot x_{N-3} + (\;\ldots\; \Delta T \cdot x_0) \ldots))) + \\ &\quad (1 - \Delta T \cdot \beta_{N-1}) \cdot (1 - \Delta T \cdot \beta_{N-2}) \cdot \ldots \cdot (1 - \Delta T \cdot \beta_0) \cdot y_0 \\ &= y_0. \end{aligned}$$

This can be rewritten compactly:

$$y_0 = \frac{\left[\sum_{k=0}^{N-2} \Delta T \cdot x_k \cdot \prod_{j=k+1}^{N-1} (1 - \Delta T \cdot \beta_k) \right] + \Delta T \cdot x_{N-1}}{1 - \prod_{k=0}^{N-1} (1 - \Delta T \cdot \beta_k)}. \qquad (24)$$

When programming equation (24), advantage can be taken of the fact that the product expression occurs both in the numerator and the denominator, by starting with the highest value of k and counting down instead of up. By storing at each step k the partial sum and the partial product, the product expression can be evaluated successively for every term in the sum. The product expression in the denominator is found by multiplying the final product of the numerator with $(1 - \Delta T \cdot \beta_0)$.

The complete approximate solution is given by (22) with initial value given explicitly by (24). In effect the iteration is carried out twice through one period. The first iteration is used to determine the initial value, and the second calculates the results at all hours. We maintain that the method is efficient; the initial value is determined after a single iteration, instead of after iterating through five time–constants. It will be shown in the next section that sufficient accuracy can be obtained from a fixed step size.

Firstly we need to establish the stability of (22). The question is important despite the fact that physical constraints ensure the absolute stability of the original differential equation. It is well known that the sampling interval has an influence on the stability of a sampled data system [12]. To investigate the stability of (22) the $\mathcal{Z}$ transform method [12] will be used under the assumption that β is independent of time. The transfer function of

(22) in the z domain is then:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-1} \cdot \Delta T \cdot \beta}{1 - z^{-1} \cdot (1 - \Delta T \cdot \beta)}. \tag{25}$$

The time invariant system is stable if the pole of the transfer function lies inside the unit circle on the complex z-plane, i.e.

$$|1 - \Delta T \cdot \beta| \leq 1. \tag{26}$$

It must be noted that this condition will not guarantee stability in the time variable case, but it is indicative of stability [24]. In practice we have found the iteration stable under the condition (26) even for sudden large changes in the value of β. According to (26) the sampling interval is chosen so that $\Delta T \cdot \beta < 2$. For most practical buildings a sampling period of 1 hour suffices. For extremely light constructions we have found it necessary to decrease the sampling rate to 15 min.

6 Accuracy of the method

The method of §5 is essentially Euler's method for the numerical integration of a differential equation. An upper bound for the total propagated error is [11]:

$$|e| \leq \frac{\Delta T \cdot |\ddot{y}|}{2 \cdot \beta} \cdot [exp(\beta \cdot T) - 1]. \tag{27}$$

The error grows rapidly when $\beta \cdot \Delta T = \Delta T / \tau > 0.1$. Since we have written the governing equation in terms of the stored heat, which is a slowly varying quantity, one would normally expect $|\ddot{y}|$ to be quite small. In fact, the derivative of (12) gives:

$$\ddot{y} = \dot{x} - \beta \cdot x + (\beta^2 - \dot{\beta}) \cdot y. \tag{28}$$

$|\ddot{y}|$ contains a term proportional to $\dot{\beta}$ which might be large for large variations of $\beta(t)$. Actually, the numerical integration technique is rigorously exact if all variables assume constant values between sampling points, even with discontinuous derivatives at the sampling points. Equation (27) must not be taken too seriously, it is derived under the assumption of continuous functions. To decrease error propagation and accuracy for continuous signals, one can use a higher order numerical approximation technique. Beginning with the trapezoid rule, for instance, an exactly similar scheme with local error theoretically proportional to the third derivative and square step size results. These and a host of other higher order approximation techniques [7] are not as advantageous for discontinuous input functions. Unless the integration is done over continuous subintervals, they tend to smooth the discontinuities and to make the solution appear non–causal, since they pre–empt the sudden change. The effect is easily explained by noting that the higher order techniques in effect interpolate between the sampling points so that values in the immediate future will influence the present result. In practice, we have found the Euler algorithm sufficiently accurate and quick. Further reduction in computation time is possible from implicit and higher order methods [26], but the simple method is already so fast that the matter is of academic importance only. The implicit methods have the advantage that they are generally stable, but they do not readily, explicitly provide the initial condition, in a form similar to (24).

To obtain a practical evaluation of the accuracy of the method, the approximate solution can be compared with the exact solution in a special case. We take the case where $\beta(t)$ is constant everywhere, except at two points where the value jumps discontinuously i.e. $\beta(t)$ given by:

$$\beta(t) = \begin{cases} \beta_0 & \text{when} \quad 0 \le t < T_1 \\ \beta_1 & \phantom{\text{when}} \quad T_1 \le t < T \end{cases} \tag{28}$$

The exact solution for a constant β and a sinusoidal input function given by $x = X\cdot(1 + m\cdot cos\omega[t+t_o])$ is from the Laplace transform:

$$y(t) = y(0)\cdot e^{-\beta t} + A(t) \tag{29}$$

where

$$A(t) = \frac{X}{\beta}\cdot\left[1 - e^{-\beta t} + \frac{m\beta}{\sqrt{\beta^2+\omega^2}}\cdot\alpha(t)\right]$$

$$\alpha(t) = cos(\omega[t+t_o]-\varphi) - cos(\omega t_o-\varphi)\cdot e^{-\beta t}.$$

and $y(0)$ is the initial value, $tan\varphi = \omega/\beta$. Next, apply this solution to the intervals in (28) and set $y(0) = y(T)$, and $y(T_1)$ continuous. The solution for $y(0) = y_o$ is:

$$y_0 = \frac{A_1 + A_0\cdot e^{-\beta_1(T-T_1)}}{1 - e^{-\beta_0 T_1 - \beta_1(T-T_1)}} \tag{30}$$

with

$$A_0 = A_o(T_1) \quad \text{and} \quad A_1 = A_1(T-T_1).$$

The subscripts 0 and 1 of A and β in (30) refer to the first and second intervals in (28) respectively. Figure 2 shows the error between analytic solution (29), (30), and approximate numerical solution (24), (22), for a low-mass building. The building (an agricultural shed) has a time-constant of about 5 hours (see Table 1) with closed windows. This is a very short thermal time–constant for a building, and a practical sampling rate would be 15 min but to show the robustness of the method, a sampling period of 1 h is used in the calculation. The air change–rate varies between 0.1 and 30 /h resulting in a time–constant jump from 5.7 to 3.8 h, the jump occurring at $T_1 = 11$ h. The forcing functions used for the calculation are:

$$T_{sa} = 20+10\cdot cos(2\pi/24\cdot t) \ ^{o}C,$$

$$T_o = 20+5\cdot cos(2\pi/24\cdot t) \ ^{o}C \quad \text{and}$$

$$Q_c = Q_r = 0 \text{ kW} .$$

The figure shows the error obtained by a sudden increase in the number of air changes as well as a sudden decrease of similar strength. In this worst case, $\beta\cdot\Delta T = 0.25$, the temperature error is less than 1°C. The error is decreased to insignificant levels by decreasing the sampling period to 15 min, with linear interpolation between sampling points. Figure 3 shows the resulting interior temperatures. Note the sharp discontinuity. In practice, the heat capacitance and the finite mixing time of the interior air (both neglected in the model) will tend to smooth the discontinuity so that a smooth transition will be measured. (We have also found that with sudden changes in the ventilation rate the time constant of the thermograph can often not be neglected.)

The calculations were repeated for a massive building (office block) where the time constant jumped from 144 to 38 h when the ventilation rate was increased from 0.1 to 30 air changes per hour. The error between the analytic and approximate solutions in this case, $T = 1$ h, $\beta \cdot \Delta T = 0.025$, was less than 0.1 °C.

When evaluating the numerical error it must be borne in mind that the accuracy of the modeling is definitely limited. It makes no sense to strive for infinite accuracy in the calculation procedure when both the assumptions inherent in the model, and uncertainties in the detail of the construction and ventilation rates limit the practical, attainable accuracy.

7 Proportional Feedback, Active Systems

The method is very easy to extend to active indoor convective systems. If another system convective load, Q_s, given by:

$$Q_s = \alpha \cdot (T_i - T_t) \qquad (31)$$

with α the proportional feedback gain [W/K] and T_t the thermostat temperature, is added to the convective load Q_c in (2), it is found that the behaviour of the system is again governed by (4) but with parameter $\beta(t)$ now given by:

$$\beta_S(t) = \beta_T - \frac{\alpha \cdot R_v^2}{C \cdot (R_a + R_v) \cdot (R_a + R_v - \alpha \cdot R_a R_v)}. \qquad (32)$$

In the limit $\alpha \rightarrow \infty$, $\beta_S \rightarrow \beta_E$ in accordance with the root–locus theorem for closed loop systems. It is seen that by this simple redefinition of β the method can be extended to proportionally controlled systems. Practical thermostats include non–linearities such as dead bands and hysteresis. These effects can also be included in the model, but the solution of the model becomes arduous; the initial value must be found by succesive iteration [7].

8 Demonstration of the Method

To conclude the paper, we demonstrate the application of the method to predicting the interior temperature in buildings subjected to night-time forced ventilation. Figure 4 gives the interior temperature obtained for a building of medium (shop) and heavy mass (office); with a ventilation rate of 1/h during the hours 17h00 to 09h00 and a daytime ventilation rate of 2/h, and secondly, with a ventilation rate of 20/h from 20h00 to 07h00 and 2/h during the rest of the day. The outside air temperature is also shown. Both buildings are modeled with an interior load of 2 persons and 0.5 kW during office hours. (More details of the buildings are supplied in Table 1.) Both buildings respond favourably. The peak temperatures drop nearly 4 °C in the office and 2 °C in the shop, bringing them close to the comfort range. The authors will attempt to verify predictions like these in the near future by actual measurement in test huts and real buildings.

9 Conclusion

The method of §4 has been implemented in a Pascal routine which calculates the solution of the variable network in a fraction of a second on an ordinary PC. The method is simple, straightforward, efficient and sufficiently accurate. It does not require Fourier analysis or even the evaluation of exponentials.

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SYMBOLS

C	Heat storage capacitance of massive structures [kJ/K].
Q_c	Convective load [kW].
Q_r	Radiative load [kW].
q	Heat energy stored in the massive structure [kW].
R_a	Mean film resistance from interior surface of shell to interior air [K/kW].
R_o	Conductive shell resistance [K/kW].
R_v	Equivalent ventilation resistance [K/kW].
T	Diurnal period of 24 hours [h].
T_c	Mean structure temperature [°C].
T_{cr}	Required structure temperature for comfort [°C].
T_{ir}	Required interior comfort temperature [°C].
T_i	Zone interior air temperature [°C].
T_{sa}	Effective sol–air external temperature [°C].
T_o	Temperature of ventilating air [°C].
T_x, T_y	Effective forcing temperatures [°C].

T_t Thermostat temperature [°C].

ΔT Time interval between sampling points [h].

β Coefficient of differential equation, equal to inverse of time–constant [/h]. Subscript T refers to interior temperature, E to energy loads and S to active systems.

τ Thermal time–constant of building [h].

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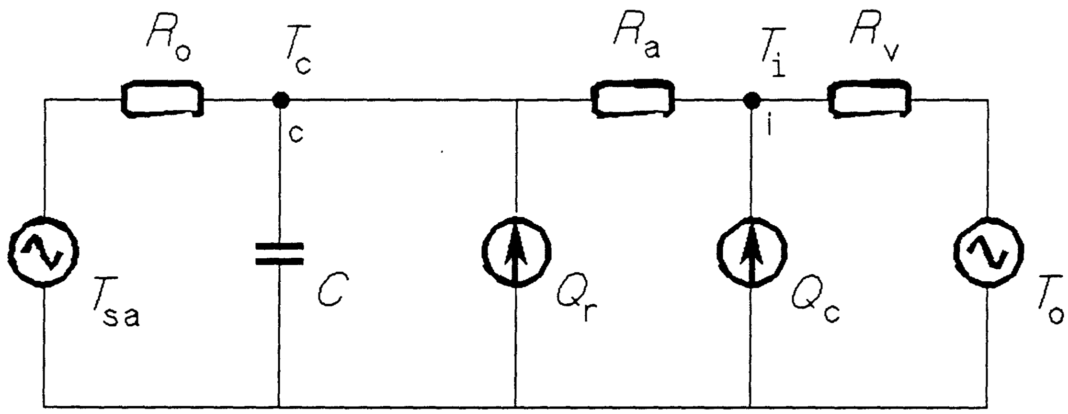

Figure 1

Figure 2

Numerical Predicted Temperature Error

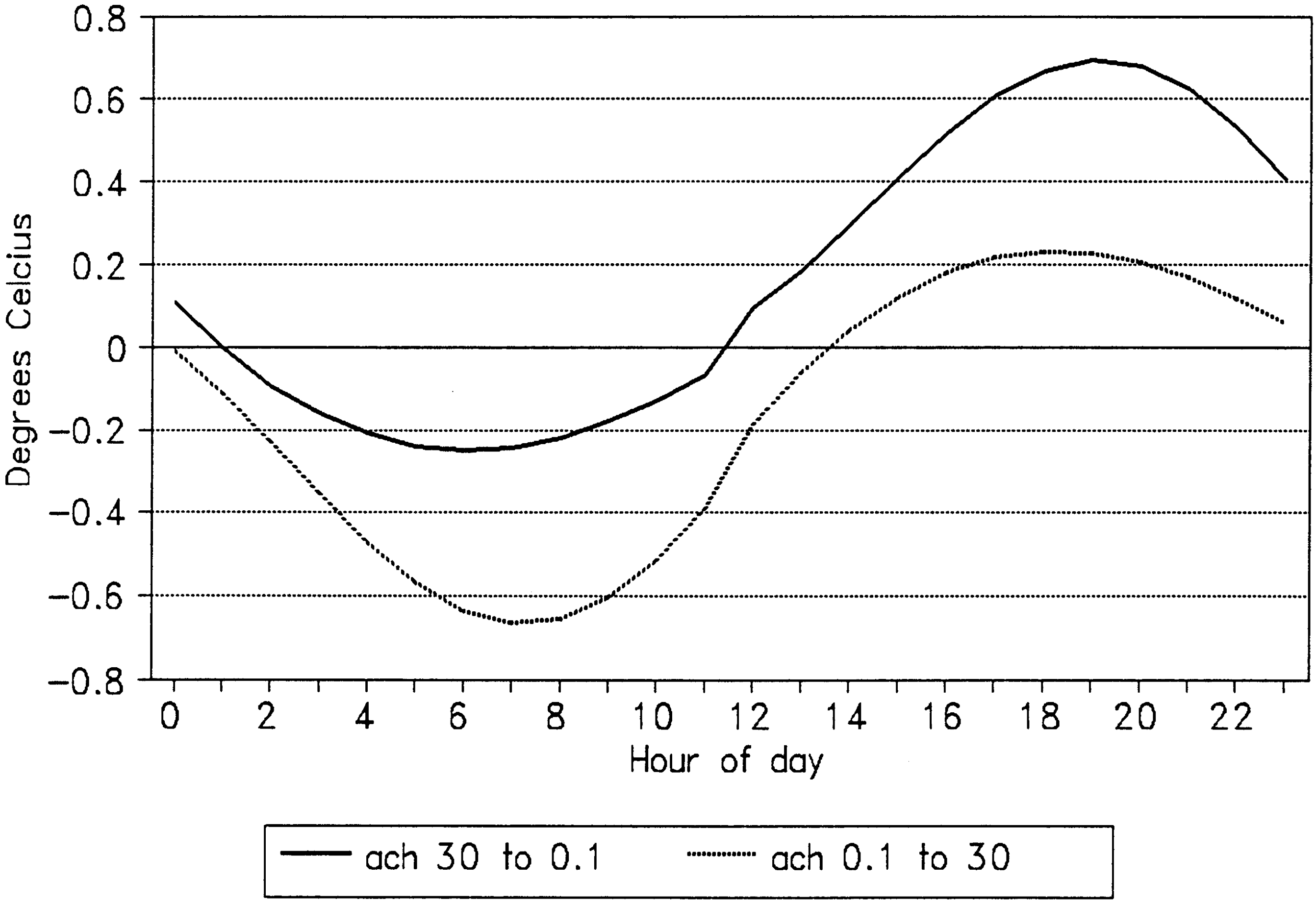

Figure 3

Predicted Temperature (Numerical)

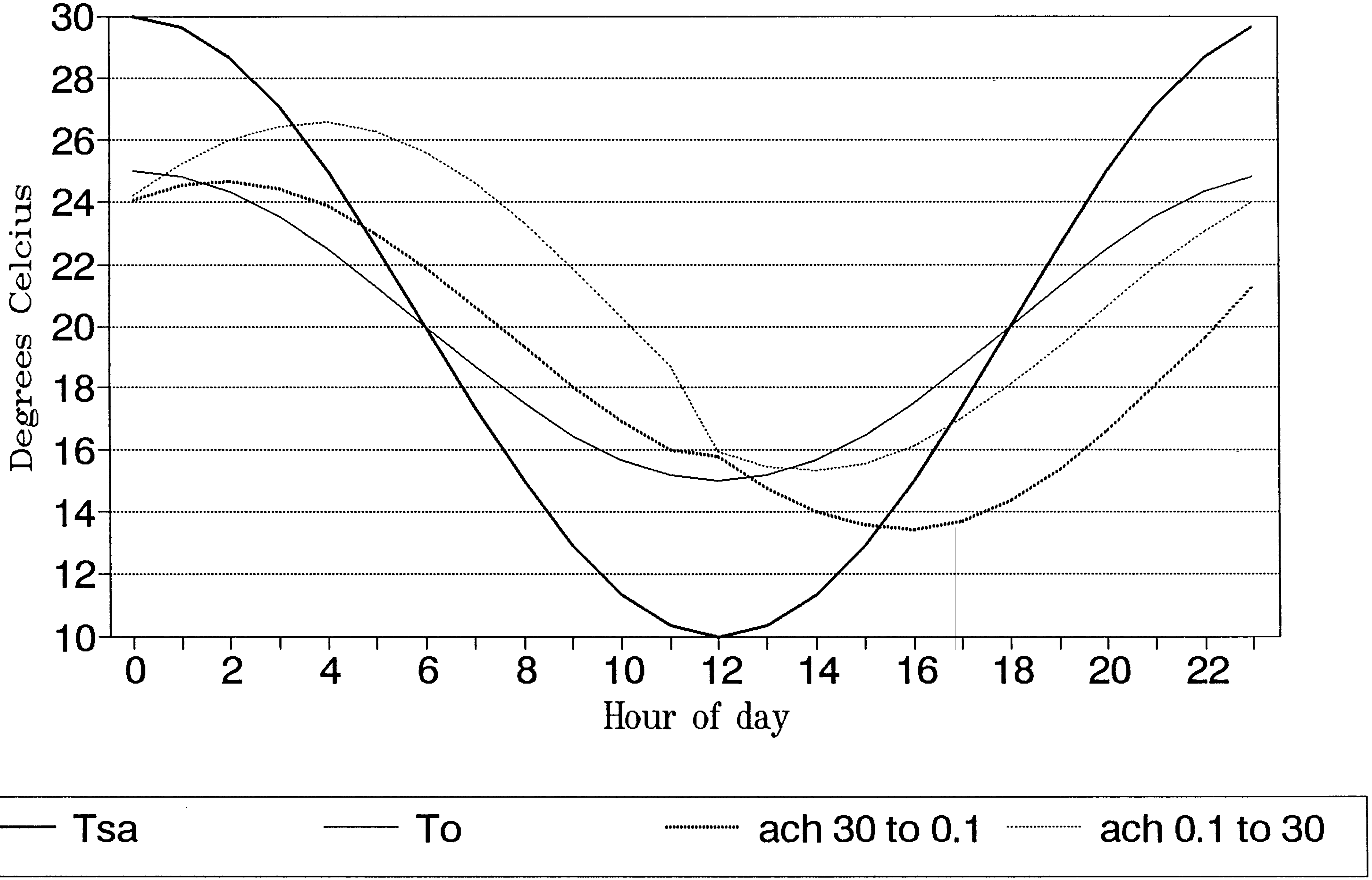

Figure 4
Night Cooling

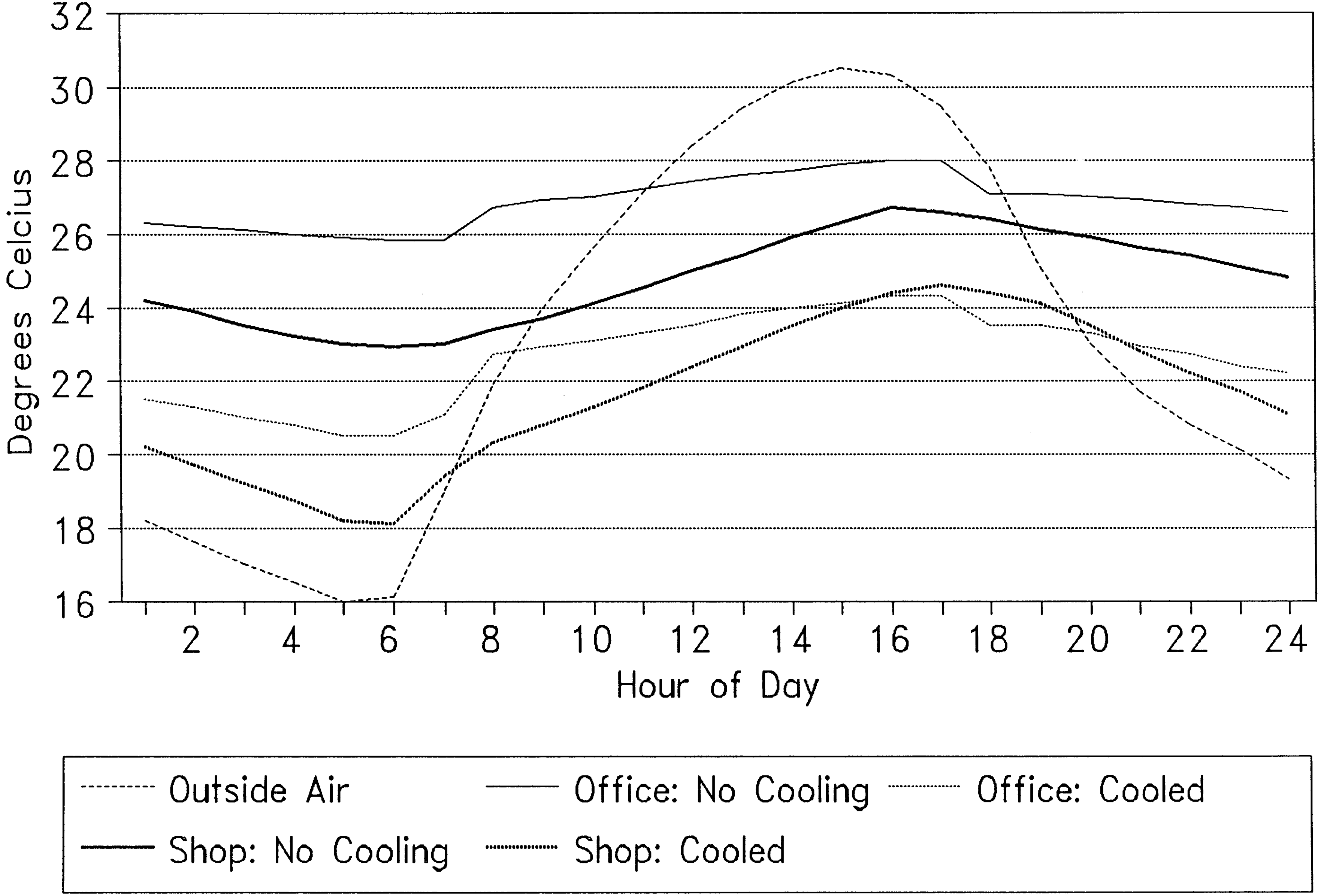

Table 1

Type	C	Ro	Ra	ach	Rv	TauT	TauE
	[kJ/K]	[K/kW]	[K/kW]	[/h]	[K/kW]	[h]	[h]
shop	85114	0.0015	0.0003	0.5	0.0057	32	5
office	44459	0.0118	0.0012	0.6	0.1463	136	14
factory	2.7E+0	2.5E-05	6E-06	1.2	2.8E-05	11	3.7
room	7304.7	0.0164	0.0017	0.57	0.2339	31	3.2
shed	529674	3.9E-05	4.6E-05	5	0.0002	5	3.1

Captions

Figure 1. Electrical analogue for modelling thermal response of buildings. T_{sa} mean sol–air temperature, T_c structure effective temperature, T_i internal air temperature, T_o external air temperature. R_o shell resistance, R_a mean internal surface film resistance, R_v ventilation resistance. C is the effective heat storage capacity of the structure.

Figure 2. Difference between analytically derived exact prediction of interior temperature, for sinusoidal forcing functions, and numerical algorithm. The sampling period is 1 h. Building: low–mass agricultural shed, τ = 5 h. The upper and lower traces show the error when the air–change–rate jumps from 0.1 to 30 /h, and from 30 to 0.1 /h respectively.

Figure 3. Predicted interior temperature for the low–mass building when the air–change rate jumps between 0.1 and 30 /h. The thick–solid line is the assumed sol–air temperature and the thick–broken line the assumed outside air temperature.

Figure 4. Night cooling of a shop and a massive office. The traces marked 'no cooling' are with an air–change–rate of 1 /h during the night from 17h00 to 09h00 and 2 /h during 09h00 to 17h00. The other traces show the effect of increasing the ventilation during the night to 20 /h.

Table 1. Circuit parameters of some typical buildings. The floor area , air–change–rate (*ach*), time–constant for interior temperature (τ_T) and time–constant for load calculation (τ_E) are also given.

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A Procedure to Estimate the Effective Heat Storage Capability of a Building

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Building designers usually prefer simplified thermal analysis procedures because these provide short simulation times and are usually easy to use. However, it is difficult to incorporate all the complex heat transfer phenomena for buildings of diverse design into a simplified method. It is especially true for the heat storage of a building. This paper presents a novel method to calculate this aspect. Features of the proposed method, some of which are often omitted in simplified methods, include the following: the modelling of different multi-layered elements in the exterior shell, different multi-layered high mass objects inside the building e.g. high mass suspended floors, floors in ground contact as well as high mass multi-layered partition walls. The method is derived inter alia *from physical interpretation and empirical data.*

The single value for heat storage obtained with the proposed procedure is incorporated in a thermal network which is solved numerically to determine hourly air temperatures and sensible energy loads. The single value of heat storage ensures a very fast and efficient building thermal analysis. Verification for 62 cases showed that for 80% of the time temperature predictions were within 2°C of measurements. This correlation provides an acceptable level of confidence in the use of the new calculation procedure for heat storage.

NOMENCLATURE

- A area of surface (m^2)
- ach flow rate (air change per hour)
- $\overline{C}$ thermal capacity per unit area (kJ °C^{-1} m^{-2})
- ΣC total active thermal capacity of building per exposed shell area (kJ °C^{-1} m^{-2})
- c_p specific heat capacity for air at constant pressure (kJ °C^{-1} kg^{-1})
- h combined indoor air convection and radiation heat transfer coefficient (W °C^{-1} m^{-2})
- R thermal resistance (°C W^{-1})
- R_k thermal resistance of layer k (thickness/conductivity) (°C m^2 W^{-1})
- R_o total thermal resistance of shell calculated from sol–air to indoor surface node (°C m^2 W^{-1})
- R_s total thermal resistance of shell calculated from sol–air to indoor air node (°C m^2 W^{-1})
- T temperature (°C)
- t time of day (hours)
- Vol volume of zone (m^3)
- $\overline{\beta}$ correction factor
- ρ density (kg m^{-3})
- τ thermal time constant (hours)

Subscripts

- a air
- b building
- c convective
- i indoor air
- j counter for any layer
- k counter for external enclosing element
- o outdoor air
- r radiative
- s shell
- sa sol–air
- v ventilation

Notation

- ∥ resistances in parallel

1. INTRODUCTION

A GOOD thermal design for a building originates during the sketch design stage. The lack of accessible design tools for this early stage, comprising acceptable analysis methods and user-friendly computer programs based thereon, has contributed to neglect in this field [1]. Part of the problem may be that the scientists have failed to develop efficient tools which designers want to use. Scientists often believe that true science is only where all the basic differential equations are solved without approximations [2]. Such a belief may not help to achieve an efficient tool for designers. A fundamental change is thus needed in this outlook. To develop efficient tools, problem complexity must be reduced without sacrificing too much accuracy. Such a simplified tool must still be able to analyse fairly accurately all types of real buildings [1]. Other criteria for a good design tool [3], which are not discussed in detail in this paper, include the following: user-friendly software; minimum time to become proficient in its use; input of only the necessary data; and quick simulation times.

Adhering to the philosophy for an efficient tool, a new tool to analyse all the important and complex heat transfer phenomena related to buildings was developed,

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using an interplay between empirical, analytical and numerical techniques. The tool *inter alia* predicts hourly air temperatures and sensible energy loads. In this paper only the treatment of the heat storage will be discussed in detail. It will be shown that with certain well-considered simplifications only one mass mode is needed rather than hundreds as for finite element or finite difference methods. Computer running times are thus drastically reduced. The difficult heat storage problem of buildings with floors not insulated from the indoor air is also efficiently addressed.

Ultimately any simulation method must stand or fall based on its agreement or disagreement with experiments. For this reason the tool, unlike many others, was extensively verified in more than sixty passive, naturally ventilated buildings comprising a wide range of thermal properties. It was found that for 80% of the time temperature predictions were within 2°C of measurements, showing that the heat storage effect was efficiently simulated.

2. THERMAL SIMULATION OF A BUILDING ZONE

The thermal simulation was described in detail in a previous article [3]. For the sake of completeness important aspects of the simulation will again be highlighted in this section.

All the important thermal interactions in a building should preferably be simulated by the simulation model. Important building characteristics that should be modelled are *inter alia* the fact that a multi-layered roof or wall can either be exposed to the outdoor environment or to adjacent zones. Furthermore, a multi-layered floor can be suspended as a division between upper and lower zones, or it can be in direct contact with the ground. High interior mass such as multi-layered partition walls should be included. The following should also be simulated: the thermal effects of the outdoor air temperature; solar radiation as well as shading, colour and orientation of different exterior surfaces; solar penetration and window shading devices; internal convective and radiative heat sources as well as varying natural and mechanical ventilation rates. The proposed tool models all the above-mentioned thermal interactions by an extremely simplified thermal network of a single building zone. The network is solved numerically. Detail of the solution technique is given in references [3] and [4].

The network in its original form was developed by Mathews [5, 6] for completely passive buildings, since which time it has been refined by adding convective heat generation or extraction [3, 7, 8]. The latest version of the thermal network for a single building zone is presented in Fig. 1. Various thermal properties can be identified in the network. The time-dependent forcing inputs are the radiative heat source ($Q_r(t)$) acting on the internal structure, the sol–air temperature ($T_{sa}(t)$) acting on the exterior, the convective heat source ($Q_c(t)$) and the outdoor air temperature ($T_o(t)$) both acting on the internal air volume. The radiative source includes solar gains through windows while the convective source includes air-conditioning. Heat gains due to machines, lighting and occupancy can also be included in these two forcing inputs. Natural ventilation and infiltration are dependent on the outdoor air temperature, while the sol–air temperature accounts *inter alia* for the colour of and radiation on and from exposed surfaces.

At a given time during the day, $T_{sa,k}(t)$ is determined for each exposed building element k. A single value for $T_{sa}(t)$ at that time is then calculated by means of the following weighting equation where A_k denotes the area and R_k the thermal resistance from the sol–air node to the interior surface node for each exposed building element k:

$$T_{sa}(t) = \Sigma_k(T_{sa,k}(t)A_k/R_k)/\Sigma_k(A_k/R_k). \quad (1)$$

There are three resistances in the network, namely R_o, R_a and $R_v(t)$. The weighted total resistance (R_o) for the exterior shell, calculated from the sol–air node through the shell to the indoor surfaces in terms of the shell area ($\Sigma_k A_k$), is given by

$$R_o = \Sigma_k A_k/\Sigma_k(A_k/R_k). \quad (2)$$

Note that, unlike the other resistances in the network, R_o is expressed in terms of the total exposed shell area. This is done to simplify comparisons between the thermal resistances of exterior shells of building zones with different dimensions.

The second resistance in the network is the time-varying ventilation resistance ($R_v(t)$) which is given by $3.6/(\text{Vol}\cdot\rho\cdot\text{ach}(t)\cdot c_p)$, where Vol is the internal volume of the zone, ach(t) the varying air change rate per hour, ρ the air density and where c_p is the specific heat of the air at constant pressure. The resistance $R_v(t)$ can change during the day as infiltration and natural or mechanical ventilation flow rates change.

The remaining resistance (R_a) is the resistance between the indoor air and the interior surfaces. Previously [5] this resistance was incorporated in R_o to give a single value of the total shell resistance (R_t) between the outdoor and indoor air. This single value, together with a single value for the active thermal capacity of the building zone, was needed to conform to another semi-empirical method widely used in South Africa at that time [9]. However, for more detailed and theoretically more rigorous analyses, two resistances R_o and R_a are needed.

The heat capacity of the indoor air can be simulated by adding a capacitor between the indoor air node (i) and the reference node (r). This capacitor was, however, omitted from the thermal network as its effect is usually negligible [6].

The capacitor (ΣC) in the network represents the "effective" thermal capacity of the building zone and is expressed per total shell area ($\Sigma_k A_k$) to simplify comparisons between building zones with different exterior dimensions. The "effective" thermal capacity is that part of the building zone's thermal capacity that is "effective" in storing heat for the indoor air. It consists of contributions from the exposed shell and internal mass which includes interior walls and floors. It is difficult to incorporate all these effects in an appropriate single value for heat storage capability [10]. A procedure to calculate such a value is the main purpose of this paper and will be discussed in detail in the next section.

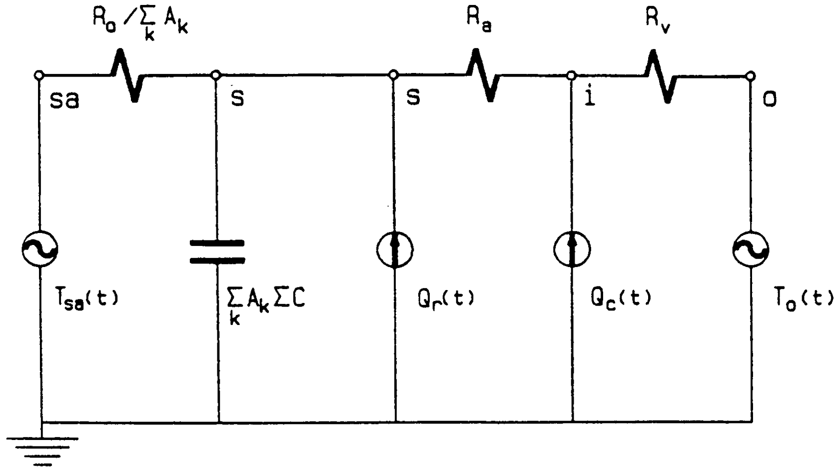

Fig. 1. Simplified thermal network for a building zone.

3. SIMULATING HEAT STORAGE EFFECT

3.1. *Preamble*

It is important that the heat storage effect of the following elements should be accounted for in any simulation of building thermal performance: an unlimited amount of different multi-layered enclosing surfaces; high mass multi-layered floors which are not insulated from the indoor air and high interior mass such as multi-layered partition walls. Furthermore, it must be possible to simulate large variations in and distribution of thermal mass and resistance of the different building elements. All the above-mentioned must preferably be simulated by means of a simplified technique if an efficient tool for designers is to be developed.

A fairly well-known simplified method using a single time constant to describe the heat storage capability of a building is the total thermal time constant method for the complete building (TTTCB) as reported by Givoni [11]. The method as described by Givoni is only applicable to building zones where all the elements of the external shell are nearly identical in construction, placing a restriction on its applicability. Indoor partitions, dividing floors and other interior mass may furthermore only consist of a single material layer.

In two recent papers by Athienitis *et al.* [12, 13], limitations of current simplified tools were highlighted. The noteworthy method proposed by Athienitis *et al.* [12] for eliminating some of these shortcomings is, like all methods, still subject to limitations. Only the heat storage of external surfaces is treated and it is assumed that all the opaque surfaces are made up of an inner lining of storage mass of uniform transport properties and an outer massless insulation.

A very useful thermal design tool was developed by Delsante *et al.* [14] at the CSIRO in Australia. However, Delsante agrees that there is scope for improvement in modelling multi-layered floors which are in contact with the ground and not insulated from the indoor environment. Most simplified methods, e.g. TTTCB [11], ELAN [15], etc. [12] and even the comprehensive TTTC [16] and DEROB [17, 18] methods do not efficiently model the heat storage capability of such floors. Yet, these floors for low mass, well insulated buildings, e.g. many factories, can significantly reduce the indoor air temperature swing [3]. In climatic regions with large diurnal temperature swings the accurate modelling of ground contact is thus very important as most low-rise buildings can effectively use the ground for heat storage purposes.

Wentzel *et al.* [9] derived equations for the heat storage contribution of a floor in ground contact and for exterior, interior and partition walls. A semi-empirical trial-and-error approach was followed to establish these equations. Through a vast amount of measurements, a correlation equation for the indoor air temperature swings for buildings of varying thermal properties was derived. The procedure was called the CR method and it was shown to be fairly accurate in predicting temperature swings. However, this procedure has certain disadvantages which are described elsewhere [5]. The equations further differ vastly for different mass contributing elements, making them difficult to comprehend. The success, however, of these simplified empirical equations prompted the authors to use the CR equations for the effective heat storage of the building in previous versions of the proposed electrical analogue [3–8]. However, a single empirical constant was always needed to ensure good comparison with measurements. In this paper a single, more cohesive equation for all mass elements, without the need for the above-mentioned empirical constant, is developed.

3.2. *New equation for heat storage calculations*

It can be shown [14] that the thermal time constant (τ_i) for layer *i* of a multi-layered building element is given by the following:

$$\tau_i = (CR_{out})_i, \tag{3}$$

where C_i is the thermal capacity of layer *i* and $(R_{out})_i$ the thermal resistance from the centre of gravity (node) of the layer up to the sol–air node as shown in Fig. 2.

The total thermal time constant for the whole building zone (τ_b) cannot be obtained by simply averaging the above-mentioned thermal time constants of all the layers across all the areas of its elements. The reason is that the

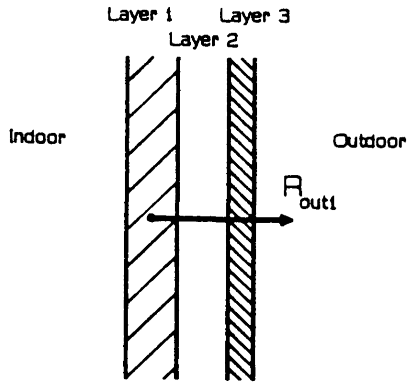

Fig. 2. Multi-layered wall exposed to the outdoor environment.

thermal resistances of the exposed elements may differ considerably, e.g. a brick wall versus a window pane. A simple averaging procedure will only be valid if all the elements of the external shell are nearly identical in construction. As the TTTCB method [11] uses this approach, its predictions may fail when simulating a building with a large variation in and distribution of thermal resistance of exterior elements. It can be shown that the following equation for the total thermal time constant (τ_b) of the whole building zone, consisting of n layers inside exposed and interior elements, is the correct one:

$$\tau_b = \Sigma_{i=1,n}(CR_{out})_i\beta_i \quad (4)$$

where β_i is a correction factor which differs for exterior and interior building elements. The value of β_i for each type of element will be discussed in detail in the rest of this section under the special headings of "exterior" and "interior" elements.

When the simplified thermal network in Fig. 1 is used, the total thermal time constant for the whole building, where no ventilation is present, is given by

$$\tau_b = \Sigma CR_o \quad (5)$$

where ΣC is the "effective" heat storage capability of the whole building and R_o the exterior resistance calculated from the sol–air node through the shell to the indoor surface.

From Eqns (4) and (5) the following equation for ΣC is found:

$$\Sigma C = \Sigma_{i=1,n}(CR_{out})_i\beta_i/R_o. \quad (6)$$

The application of Eqn. (6) to exposed, interior and floor elements will be discussed in more detail in the rest of this section.

Exposed elements. It can be shown that the value of β_i for exterior elements, which present parallel heat flow paths between the indoor and outdoor air nodes, is given by $\beta_i = R_o/R_k$. R_k is the thermal resistance of exterior element k, calculated from the sol–air node up to the interior surface as shown in Fig. 2. With β_i known, Eqn.(6) can now be applied without any difficulty. The contribution of layer 1 in Fig. 2 is, for example, simply given by $(CR_{out})_1/R_k$.

Equation (6) clearly shows an important physical aspect, namely that the mass nearest to the indoor environment will have the most pronounced effect on it. Furthermore, it is evident from Eqn. (6) that if the thermal resistance (R_k) of one of the exterior elements is much larger than the resistance of the rest of the exterior shell, the contribution of that element would be reduced considerably due to the small value of β_i. If this was not the case a very large time constant would result which is not physically correct. This problem is encountered when using the TTTCB method of Ref. [11]. On the other hand, if all the exterior surfaces are identical, β_i will be equal to one and all the elements will contribute fully to the total time constant of the zone. For such a case, simulations by the TTTCB method of Ref. [11] will be correct.

For the CR method [9] the node for R_{out} was chosen at the interface between the layer under investigation and the next layer between it and the indoor air. However, it is more correct to use a single node at the centre of gravity of a layer as proposed in Eqn. (6). The CR method further does not divide by R_k but rather by the resistance calculated from the sol–air node up to the interface of the investigated layer and the adjacent layer between it and the outdoor air. This may be the main reason why an empirical constant was needed when using the CR equations for exterior elements in the thermal network.

Interior elements. Contributions include intermediate floors in multi-storeyed buildings, interior walls and other high interior mass. At a first glance it may seem that Eqn. (6) and Fig. 3 can be used without adjustment for interior elements. However, when using this approach for the intermediate floor in the well-insulated building shown schematically in Fig. 3, it was found from measurements that the "effective" heat storage was overestimated by a factor of 2. The reason is that in practice only half of the floor mass is available for storing heat for zone A, while the above-mentioned procedure predicts that all the mass is available for zone A. It must therefore be decided what part of an element between two zones contributes to the "effective" heat storage for the zone under consideration.

This aspect is explained with reference to Fig. 4(a) which shows a multi-layered division floor/wall with three layers between zones A and B. As a single zone approach is used in the thermal network, no heat may be transferred between the zones across the dividing floor/

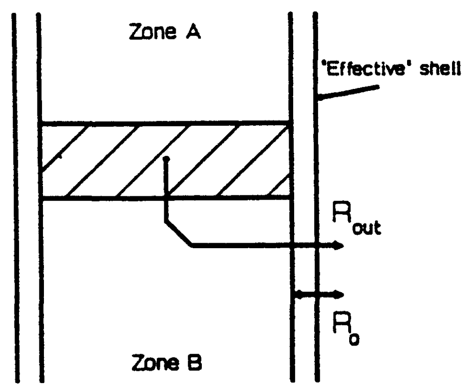

Fig. 3. Multi-layered division wall/floor between zones A and B in a well insulated building.

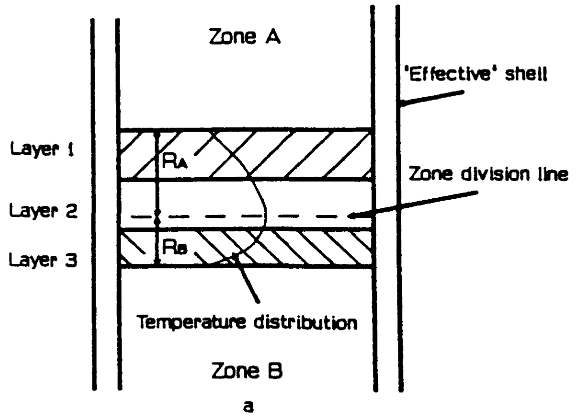

Fig. 4(a). Temperature distribution and zone division line in a multi-layered division wall/floor between zones A and B.

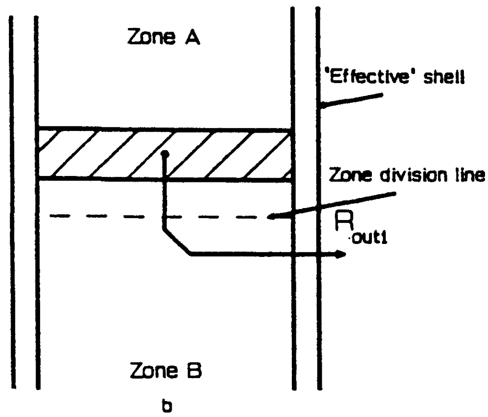

Fig. 4(b). Definition of R_{out} for division wall/floor between different zones.

wall. This means that heat stored will always be transferred from the floor/wall to both zones. The zero gradient point on the temperature distribution graph given in Fig. 4(a) will determine up to which point heat will be transferred to either zone A or B. The "effective" thermal capacity for such a floor/wall is therefore calculated up to a zone division line situated at the zero gradient point, which is assumed to be fixed for steady and transient state.

The zero gradient point and therefore the zone division line is positioned where the thermal resistance (R_A) from the indoor surface of zone A up to the division line is equal to the resistance (R_B) from the line up to the indoor surface of zone B as shown in Fig. 4(a). Note that only the mass above the division line contributes to the "effective" thermal capacity for zone A, as heat stored in this area will only be transferred to zone A and not to zone B. This contribution will differ from that for zone B when analysing an asymmetrical division floor/wall.

At this stage it is important to note how $(R_{out})_i$ in Eqn. (6) is calculated for interior elements. If $(R_{out})_i$ is taken as the thermal resistance from node i through the interior mass layers and the zone under investigation (zone A) up to the sol–air node, a layer further away from the indoor air would have a larger "effective" capacity than a layer closer to it. As this is not physically correct, the following method is proposed for the calculation of $(R_{out})_i$ for interior elements: once the position of the zone division line is established, all mass contributions belonging to zone B should be completely ignored as shown in Fig. 4(b). The value of $(R_{out})_i$ in Eqn. (6) is then calculated as the thermal resistance from the node under investigation through the adjacent zone B up to the sol–air node [see Fig. 4(b)]. Using the path through zone B rather than through zone A ensures that layers further away from the indoor air in zone A will have a less pronounced effect on it. As a single zone approach is used, it is assumed that the surface coefficients and exterior shell of zone B is the same as that for zone A. No extra information than the information for zone A is therefore needed to calculate $(R_{out})_i$.

For certain cases there may be a further complicating factor which warrants further attention. A thermal network, which describes a multi-layered interior wall more correctly than the very simplified network given in Fig. 1, is presented in Fig. 5(a). Typical temperature swings at the nodes in the wall are also shown in Fig. 5(a), while the wall is schematically given in Fig. 5(b). From the temperature swings ($\mathrm{d}T/\mathrm{d}t$) in Fig. 5(a) it can be seen that the swing at a node in a layer far removed from the indoor air may become small, *inter alia* due to the high thermal resistance of layers closer to the indoor air. The heat flow due to storage $C\,\mathrm{d}T/\mathrm{d}t$ [3] in the heat transfer equation will thus be negligible ($\mathrm{d}T/\mathrm{d}t$ becomes small). As this effect can not be simulated by the simplified single node network in Fig. 1 the "effective" heat capacity is used to compensate for it. A layer where $\mathrm{d}T/\mathrm{d}t$ is very small will thus not contribute to the "effective" heat capacity of the building. To account for this the correct value for the correction factor β_i incorporating relevant resistances must be specified in Eqn. (6).

It was reasoned that the above-mentioned effect would *inter alia* depend on how readily heat can be transferred inside the material of the layer under investigation in relation to how easily it is transferred to the indoor surface. Therefore the correction factor for layer 1 in Fig. 5(b) is given by $(R_1/2)/(R_1/2)$ and for layer 2 by $(R_2/2)/(R_2/2+R_1)$. Equation (6) for each layer should now be multiplied by the correction factor for that layer as well as the correction factors for all the layers between the present layer and the indoor surface. This will ensure that mass layer 3 will under no circumstance contribute to the effective heat storage if mass layer 2 did not contribute due to high thermal resistance. The following general equation for the correction factor for a multi-layered interior mass is therefore proposed:

$$\beta_i = \Pi_{j=1,i}(R_{mat}/R_{in})_j \qquad (7)$$

where $(R_{mat})_j$ is half of the resistance of the material of layer j and $(R_{in})_j$ is the resistance from node j up to the indoor surface of zone A [see Fig. 5(b)].

It should be noted that the magnitude of the indoor air temperature swing also "influences" the "effective" heat capacity of interior elements. The heat flow due to storage ($C\,\mathrm{d}T/\mathrm{d}t$) inside the interior mass will become smaller when the temperature swing ($C\,\mathrm{d}T/\mathrm{d}t$) in the interior element becomes smaller. As the indoor air temperature influences the temperature swing inside the

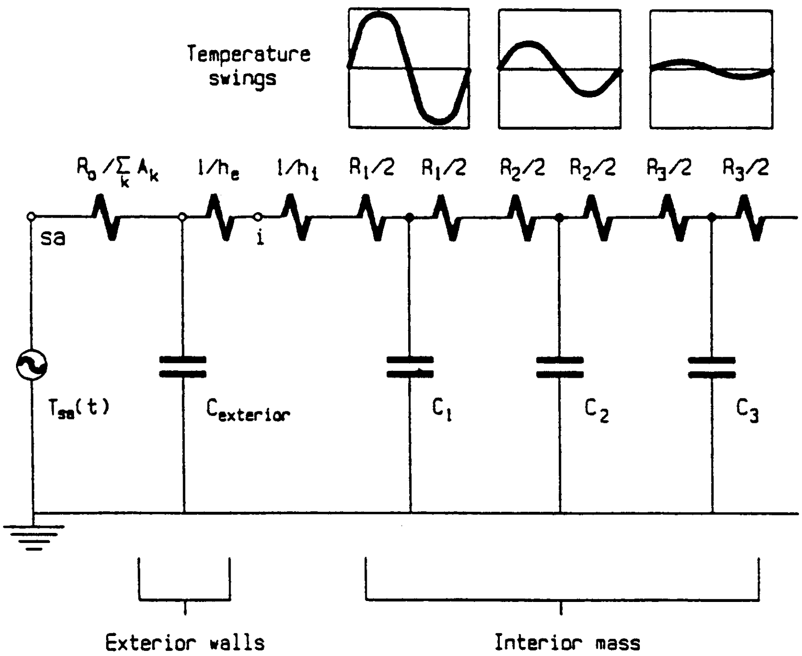

Fig. 5(a). Thermal network for multi-layered interior wall/floor showing typical temperature swings at the different interior mass nodes.

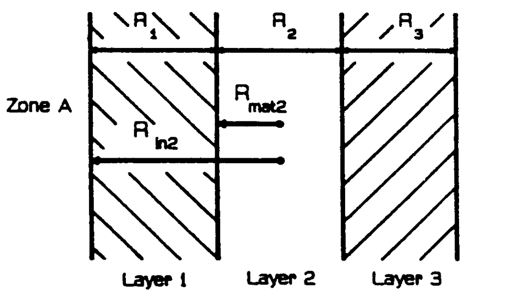

Fig. 5(b). Definition of R_{in} and R_{mat} for material layer 2 in multi-layered division wall/floor between two different zones.

element, the heat flow due to storage will become smaller when the indoor air temperature swing becomes smaller. This effect was not incorporated in the present method for passive analysis. It should definitely be considered when the system start-up period of an air-conditioned building is investigated. The interior mass for such a case will progressively become less "effective" as the indoor air temperature swing becomes smaller. More theoretical work is definitely needed regarded the correction factor for interior elements.

The contribution of high mass, such as a partition wall completely inside a zone, is also calculated by means of Eqn. (6). The only difference being that R_{in} in Eqn. (7) is calculated as the parallel resistance from the node up to the left and right indoor surfaces as shown in Fig. 6, namely $R_{left} \| R_{right}$. Also note that R_{out} in Eqn (6) is found by adding the parallel resistance (R_{in}) to the resistance from the indoor surface of the partition up to the sol–air node.

Although the CR method [9] does not account for multi-layered interior elements, it can estimate the storage effect of a single layer element. For such an element the thermal resistance to the outside (R_{out}) is approximated as the effective shell resistance (R_s) and not as the total resistance from the interior material layer through the indoor air and shell to the sol–air node on the outside of the building [9]. It was found in practice that Eqn. (6) provides a better approximation than the equation used in the CR method and no empirical constant is needed.

The TTTCB method as reported by Givoni [11] may have difficulty in treating multi-layered interior elements where high mass is separated from the indoor air by means of high resistance elements such as carpets and acoustic panels. This fact becomes very notable when the exterior shell resistance is small. Equation (30) for the total thermal time constant (TTTCB) of interior elements, given in Ref. [11], is used to explain the above. The relevant part of this equation is given here as Eqn. (8):

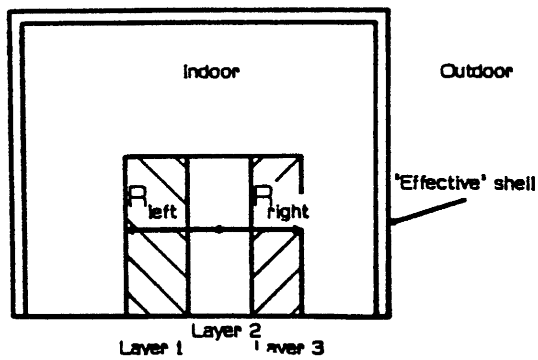

Fig. 6. High mass multi-layered partition wall with R_{left} and R_{right} defined.

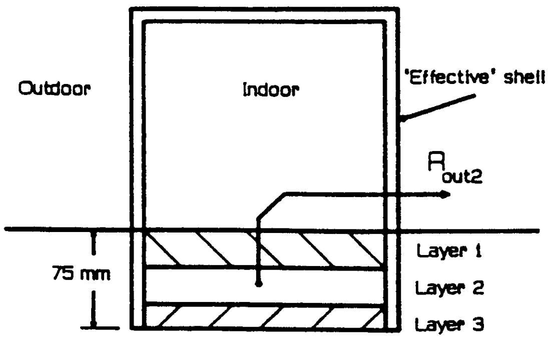

Fig. 7. Multi-layered floor in contact with the ground.

$$\mathrm{TTTCB} = 0.5C(\Sigma_k(A_k R_k)/\Sigma_k A_k + R_s + 0.25R_{in}). \quad (8)$$

When the exterior shell resistance is small and the floor is insulated from the indoor air by carpets, R_{in} is very large and dominates the other terms in parentheses. This will result in a large predicted time constant (TTTCB). However, in practice it was found that the time constant for variations in indoor air temperature is much smaller due to the mass being insulated from the indoor air, therefore the need for the correction factor proposed in Eqn. (7). Furthermore, half of the lumped capacity is always used for TTTCB instead of only the "effective" part as given by the zone division concept.

Another aspect which should be pointed out is that the first term in parentheses in Eqn. (8) is a weighted external resistance, rather than the effective external resistance which is calculated from the relevant resistances in parallel. If, for example, a building zone is extremely well insulated on four sides but very poorly insulated on the others, Eqn. (8) will predict a large time constant (TTTCB) which is not true in practice. Equation (8) is therefore only valid for cases where the exterior elements are nearly identical in thermal resistance. If a building with a large variation in and distribution of thermal properties of the different elements is investigated, the first term should be replaced by an equation for parallel resistances similar to Eqn. (2). Note that the relevant indoor surface coefficients for the different exterior elements should also be accounted for.

Ground floor elements. For a building in contact with the ground, the floor and ground contribution is discussed with reference to Fig. 7 which shows a multi-layered floor. For such floors Eqn. (6) can be applied without difficulty if the depth up to which the floor elements contribute to the "effective" heat storage is known. This depth will *inter alia* depend on the resistances of the layers. Based on temperature measurements inside South African buildings, Wentzel *et al.* [9] reasoned that the ground underneath such buildings can contribute to the heat storage capacity of a structure down to a depth of 300 mm below ground level. This depth is deeper than the depth of 200 mm used by the ELAN method [15].

The CR method approximates the thermal resistance to the outside (R_{out}) only as the effective shell resistance (R_s) and not as the total resistance from the floor material layer through the indoor air and shell to the sol–air node on the outside of the building, as used in Eqn. (6). The CR method further does not divide by R_o in Eqn. (6) but rather by the resistance calculated from the mass node up to the indoor air node. The above-mentioned aspects coupled with the fairly large empirical depth of 300 mm necessitated the use of an empirical constant in previous versions of the proposed electrical network [3–8].

The empirical constant can be eliminated by employing Eqn. (6) and only calculating the heat storage of layers up to a depth of 75 mm. This depth was arrived at by comparing measured and predicted indoor air temperatures for more than forty cases with floor areas ranging from 9 up to 7700 m^2. At a certain depth, depending on the resistances of the floor material and the indoor air temperature swing, the temperature swing inside the floor will become negligible. This means that the mass below this point will not contribute to the "effective" heat capacity of the building as the heat storage term $C\,dT/dt$ will become negligible. As the simplified single node network in Fig. 1 cannot simulate the fact that dT/dt becomes zero inside the floor, the calculation of the "effective" heat capacity is used to compensate for it. The "effective" heat capacity is therefore only calculated up to a certain depth, in the present case 75 mm.

High thermal resistance on the floor, such as thick carpets, will reduce the depth where the temperature swing becomes negligible. This effect is accounted for by the correction factor in Eqn. (7). However, the problem of deeper layers contributing more to the effective heat capacity than more shallow layers, as discussed for interior elements, has not yet been fully addressed.

3.3. *Closure*

A single equation for the total "effective" heat capacity (ΣC) for a single building zone consisting of n exterior and interior mass layers is given by the following:

$$\Sigma C = \Sigma_{i=1,n}(CR_{out})_i / R_o \cdot \beta_i, \quad (9)$$

where $\beta_i = R_o/R_k$ for exterior elements and $\beta_i = \Pi_{j=1,i}(R_{mat}/R_{in})_j$ for interior elements. The value of C_i for interior division floors or walls is found by means of the zone division concept and C_i for floors in ground contact is found by calculating up to a depth of 75 mm.

Although the equation for the calculation of the "effective" heat storage capability for a building zone was not completely derived from first principles, it will be shown in the next section that it is successful in analysing typical building constructions. This means that the effect of relative position and thermal properties of mass and insulation as well as the fact that a temperature differential exists across the building elements [6] are effectively accounted for. The proposed procedure must not be viewed incorrectly [10] as the traditional lumped-capacity method with its assumptions that the entire mass of a material is equally and completely effective in storing heat and that all this mass is at a constant temperature throughout. The proposed procedure can be viewed as similar to a lumped distributed parameter model which does not suffer from the limitation of the lumped-capacity method, namely that the ratio of mass thermal resistance to external resistance should be small. Note that the limitation is on the above-mentioned ratio and not on the ratio of thermal inertia to external thermal resistance as often thought [10].

It was found that only one node per layer was needed for conventional building construction when using Eqn. (9). The thickest exterior layer for the cases investigated in the validation study was a 114 mm brick layer in a 360 mm thick wall element, consisting of two layers of bricks, a cavity and plaster on both sides. When using the traditional lumped capacity procedure [19], as most finite difference methods do, at least 10 nodes would be needed in each brick layer.

Necessary future work includes detailed derivation of more theoretically based equations which will more fully define the assumptions and limitations of the procedure.

4. VALIDATION

Ultimately the usefulness of a simulation model as a design tool can only be ascertained by comparing its predictions with experimental results from actual buildings. Predictions for the indoor air temperatures were compared to measurements in order to establish the level of confidence in heat storage calculations and in complete simulations with the simplified network.

It was found that temperature predictions for 62 cases representing a wide range of buildings were within 2°C of measurements for 80% of the time. It is important to note that the accuracy of the Thies Clima thermographs used for temperature measurements is ±1.5°C. Also note that the authors took great care to include all important types of buildings in the validation process [3]. Buildings included office blocks, shops, schools, residential buildings, townhouses, medium and high mass experimental buildings, low mass well insulated structures with ground contact such as factories, and low mass poorly insulated structures, e.g. agricultural buildings.

The applicability of the procedure for estimating the "effective" heat capacity can be determined by a comparison of predicted and measured indoor air temperature swings, defined as the difference between the maximum and minimum values. Figure 8 presents the measured and predicted swings for all the cases. From

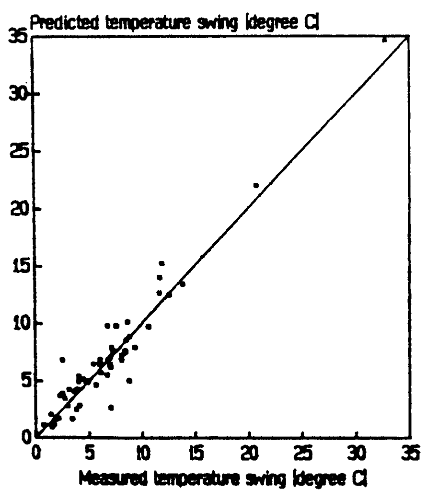

Fig. 8. Comparison between measured and predicted indoor air temperature swings for 62 cases.

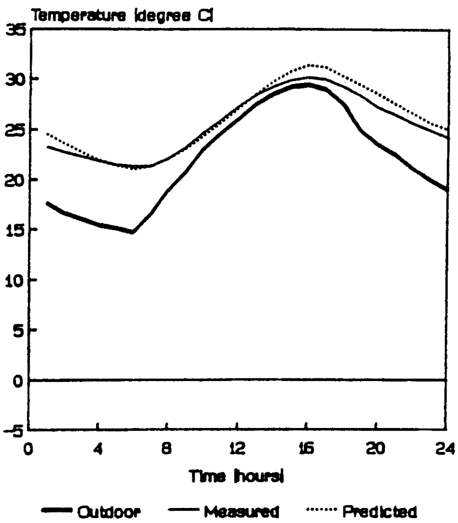

Fig. 9. Outdoor air temperature and measured and predicted indoor air temperatures for low mass, well-insulated factory with uninsulated floor in ground contact.

the large variations in swings, 0.7 to 20.8°C in Fig. 8, it can be deduced that buildings were used whicl varied significantly in "effective" heat storage capabil .y. For design purposes the correlation between measured and predicted swings should be acceptable. One of the important reasons for the difference between measurements and predictions can be related to the natural ventilation flow rates for buildings with open windows, which were not measured.

Extensive validation of hourly temperature predictions by the complete simulation procedure is given in another paper [3]. In the present paper it is only giver 'or the following cases: a low mass, well-insulated fact. y with uninsulated floor in ground contact (Fig. 9); a low mass, poorly insulated agricultural building with uninsulated floor in ground contact (Fig. 10); a school of intermediate mass with various interior mass elements (Fig.

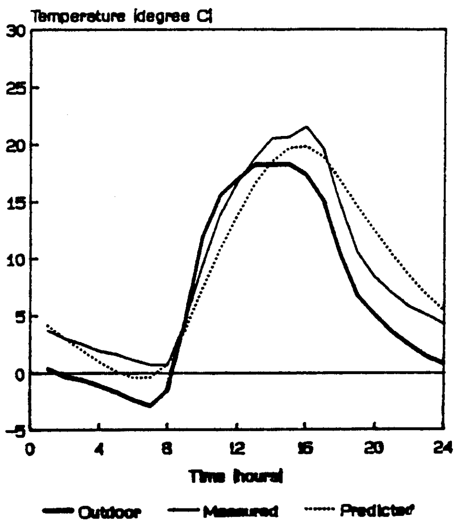

Fig. 10. Outdoor air temperature and measured and predicted indoor air temperatures for low mass, poorly insulated agricultural building with uninsulated floor in ground contact.

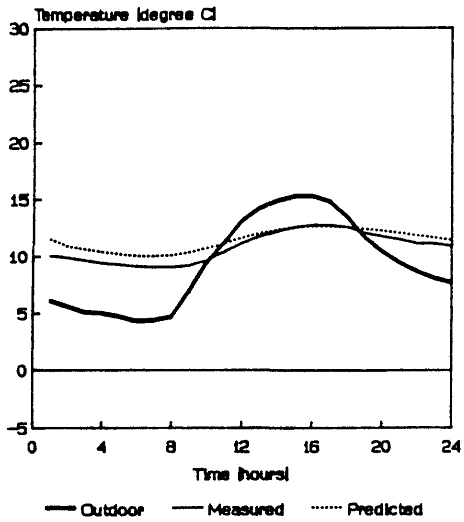

Fig. 11. Outdoor air temperature and measured and predicted indoor air temperatures for school of intermediate mass with various interior mass elements.

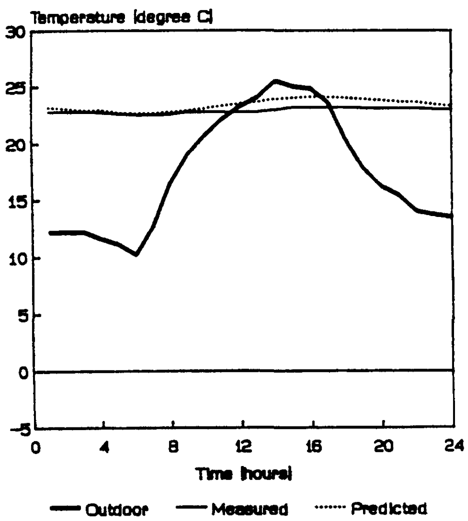

Fig. 12. Outdoor air temperature and measured and predicted indoor air temperatures for high mass office of which five enclosing surfaces are shared with adjacent zones.

11); and a high mass office of which five enclosing surfaces are shared with adjacent zones (Fig. 12). More details of the buildings are given in Table 1. From Figs 9–12 it can be seen that the comparison between predicted and measured indoor air temperatures is acceptable for design purposes, especially when considering that the IBM(XT) microcomputer running times for the simulations are less than 10 s.

The efficiency of the simulations, which included varying ventilation, internal loads, complex shading devices, direct solar gain and the calculation of sensible energy loads, can *inter alia* be attributed to the following features: the novel treatment of the heat storage of a building, a special treatment of the term describing varying ventilation rates and solving the relevant equations under the assumption that all variables are periodic. For example, if the simulation is performed for a design day, week, month or year, it is assumed that all the previous and following days, weeks, months or years are identical. A closed form solution for the initial value can be found from this periodic assumption. It is therefore not necessary to provide initial conditions or to repeat the simulation for several periods under consideration to eliminate the effect thereof. This fact contributes considerably to the efficiency of calculations.

As the computer program was developed for building designers, who may be computer illiterate, the program was made very user-friendly and data input is very straightforward. For example, the input for each of the above-mentioned buildings used in the validation took in the order of 15 min. The effect of changes to a design can be investigated in less than 30 s.

5 SUMMARY AND CONCLUSIONS

A simplified procedure was developed to lump the "effective" heat storage capability of a building zone. The procedure is not based on a rigorous theoretical derivation, but on sound reasoning about the physical problem and on empirical data. Features of the procedure include the ability to efficiently simulate ground contact and multi-layered exterior and interior elements.

The single value for the "effective" heat storage was employed in a simplified thermal network to complete an efficient simulation model of a building zone. Despite its simplicity, the procedure predicted indoor air temperatures within 2°C for 80% of the time for 62 buildings which included office blocks, shops, schools, residential buildings, low mass well-insulated structures in ground contact and low and high mass poorly insulated structures.

Although the thermal model is very simple, the law of

Table 1. Summary of information applicable to building zones used for validation purposes

Type	Factory	Agricultural building	School	Office
Location	Pretoria	Volksrust	Pretoria	Pretoria
Ground contact	Yes	Yes	No	No
Floor area (m^2)	7700	750	58	14
Exposed surfaces	4	5	3	1
Exterior colour	Medium/dark	Weathered steel	Medium	Light
ΣC (kJ °C^{-1} m^{-2})	204	411	654	3518
R_s (°C m^2 W^{-1})	0.327	0.050	0.196	0.117
Occupied	Yes	No	No	No

conservation of problems is also applicable here. In order to reduce the complex thermal problem to a simple one, new concepts had to be devised, for example the correction factor (β), the calculation of R_{out} for interior elements and the empirical depth of 75 mm for floors in ground contact. All these concepts are, however, based on physical interpretation and were shown to give acceptable results for all the cases that were investigated. It is the authors' view that in general building thermal models should only attempt to provide an economical description of empirical facts. A multitude of detail can easily clutter up the mind, diverting it from the essential.

Future work includes a detailed derivation of a more theoretically based model which will more fully define the assumptions and limitations of the complete model.

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