



## 5 Chapter 5: Half-life deviations from PPP in the SADC

### 5.1 Introduction

This chapter discusses the half-life version of the PPP puzzle. According to the sticky price theories of international macroeconomics, the purchasing power parity (PPP) hypothesis is compatible with half-lives of real exchange rate of less than three years. However, economists are puzzled by the slow rate at which real exchange rates adjust to the PPP (Taylor and Taylor, 2004). This is an issue dealt with at length by Rogoff (1996), who points out that the high short-term volatility of real exchange rates is not compatible with the extremely slow rate at which shocks appear to die off.

Empirical analysis of the persistence of real exchange rate deviations from PPP is generally based on impulse response analysis. In this setting, the concept of a half-life is used to estimate how long it takes for the impulse response to a unit shock to dissipate by one half (Chortareas and Kapetanios, 2004). In this context, consider for instance that PPP holds continuously. This implies that the following relation should remain constant:

$$y_t = \ln \left[ \frac{S_t P_t}{P^*_t} \right], \quad (5.1)$$

where  $S_t$  represents the nominal exchange rate,  $P_t$  and  $P^*_t$  are the price levels in the domestic and foreign country, respectively. Following Rossi (2005a), suppose that the deviations of the real exchange rate,  $y_t$ , from its long-run value  $y_0$  follow a stationary autoregressive order one process such that

$$y_t = \phi y_{t-1} + \varepsilon_t. \quad (5.2)$$

Then the half-life of a process can be defined as the minimum value of  $H$  such that



$$E[y_{t+h} - y_0 \mid y_{t-s} - y_0] \leq \frac{1}{2}(y_t - y_0), \quad (5.3)$$

where  $E$  is the expectation operator and  $s \leq 0$ . Based on equation (5.2), estimates of  $H$  are obtained as follows:

$$\hat{h} = \ln(0.5) / \ln(\hat{\varphi}), \quad (5.4)$$

where  $\hat{\varphi}$  represents the estimate of  $\varphi$ . It is to be noted that for  $\hat{\varphi} \geq 1$ , the process has no half-life because it approaches infinity.

Kim, Silvapulle and Hyndman (2006) have identified the three main statistical properties of the half-life statistic as calculated using equation (5.4), namely, it has an unknown and possibly intractable distribution; it may not possess finite sample moments since it takes extreme values as the coefficient approaches one; that it is biased in small samples; and that it is a nonlinear function of  $\varphi$  which is also biased downward.

Empirical evidence is generally mixed when it comes to point estimates of half-life deviations. For instance, Parsley and Wei (1996) found that the half-lives for the European Monetary System countries were 4.25 years. Other studies on real exchange rates, such as Frankel (1990), found that the half-life of the dollar-pound real exchange rate was 4.6 years. In addition, Lothian and Taylor (1996) estimated that the corresponding numbers were 2.8 for the franc-pound and 5.9 for the dollar-pound real exchange rate.

In the context of panel data analysis, the evidence on point estimates is also mixed. Frankel and Rose (1995) found that for 150 countries the half-life averaged 4 years. Moreover, Cheung and Lai (2000) estimated that the half-lives ranged between 2 and 5 years for industrial countries but under 3 years for developing economies. A study by Manzur (1990) assessing seven industrial countries found that the half-lives of their real exchange rates were 5 years, while Fung and Lo (1992) put the half-lives at 6.5 years for the sample of six industrial countries they consider.



The main contribution of this chapter is that it follows Rossi's (2005a) to generate point estimates and confidence intervals for the SADC in which deviations from PPP are in some cases compatible with nominal price and wage stickiness. To the author's knowledge, no published article has ever produced these half-life confidence intervals for the SADC countries. The motivation for using Rossi's methodology is that she uses local-to-unity asymptotic theory in the presence, in most cases, of highly persistent data. As it is commonly observed, real exchange rates manifest themselves as processes with roots near-unity. This characteristic makes them provide no good small-sample approximation to the distribution of estimators and test statistics. In such cases econometricians use an alternative approach by modelling the dominant root of the autoregressive lag order polynomial as local-to-unity (Diebold, Killian, and Nerlove, 2008). This approach leads to an alternative asymptotic approximation that provides a better small-sample approximation than imposing the order of integration.

The second reason for following Rossi (2005a) is that she derives a measure of the half-life for a general  $AR(p)$  process that allows for better asymptotic approximations in the presence of a root close to unity.

The rest of the chapter is organised as follows: section 5.2 discusses Rossi (2005a) approach to measuring half-life deviations. Section 5.3 covers econometric issues and empirical evidence. Section 5.4 concludes.

## 5.2 Rossi (2005a) methodology for general $AR(p)$ processes

The approach followed is based on the factorization of the data generating process (DGP) of the following form:

$$y_t = d_t + u_t \quad t = 1, 2, \dots, T \quad (5.5)$$

$$u_t = \rho u_{t-1} + v_t \quad (5.6)$$

$$\rho = e^{c/T} \approx 1 + c/T \quad (5.7)$$

where  $d_t$  is a deterministic component,  $v_t$  is a zero mean, stationary and ergodic process, with finite autocovariances. Equation (5.7) represents local-to-unity asymptotics in the spirit of Stock (1991). The factorisation process produces:

$$(1 - \lambda_1 L)(1 - \lambda_2 L) \dots (1 - \lambda_p L)(y_t - d_t) = \varepsilon_t$$

where  $\lambda_j$  are eigenvalues of an  $AR(p)$  process. The half-life statistic for an  $AR(p)$  process has been suggested by Rossi (2005a) and takes the following form:

$$\hat{h} = \text{Max} \left\{ \frac{\ln(0.5)b(1)}{\ln(\hat{\phi})}, 0 \right\}, \quad (5.8)$$

where  $b(1) = (1 - \lambda_2)(1 - \lambda_3) \dots (1 - \lambda_p)$  is the correction factor of an  $AR(p)$  process, whereby  $p$  denotes the number of lags. Rossi(2005a) treats a unit root process as having an infinite half-life. The author points out that the data generating process (5.5), can be rearranged to generate the following ADF regression:

$$y_t = \tilde{\mu}^o + \alpha(1)y_{t-1} + \sum_{j=1}^{p-1} \alpha^*_{j-1} \Delta y_{t-j} + \varepsilon_t \quad (5.9)$$

$$\text{where } \alpha(1) = 1 + \frac{c}{T} b(1), \tilde{\mu}_0 = -\frac{c}{T} \mu_0 b(1), \alpha^*_j = -\sum_{i=j+1}^{p-1} \alpha_i \quad (5.10)$$

The half-life associated with the above regression is of the form:

$$H_a = \max \left\{ \frac{\ln(0.5)}{\ln \alpha(1)}, 0 \right\} \quad (5.11)$$

A conventional 95 per cent confidence interval associated with the above half-life statistic is of the following form:

$$\hat{H}_a \pm 1.96 \hat{\sigma}_{\hat{\alpha}(1)} \left[ \frac{\ln(0.5)}{\hat{\alpha}(1)} [\ln(\hat{\alpha}(1))]^{-2} \right]. \quad (5.12)$$



To construct confidence intervals, this chapter follows Rossi (2005a) by relying on Stock (1991), Elliott and Stock (2001), and Hansen (1999). The details of the strengths and weaknesses of these methods have been discussed at length by Rossi (2005a). At this point it is worth pointing out that when the data are highly persistent, a bootstrap method that is valid is Hansen's (1999) grid- $\alpha$  bootstrap method, which has the range-preserving property. This method is supposed to ensure that the calculated half-life is nonnegative. In the latter context, negative half-lives are treated as invalid and cannot be interpreted meaningfully.

The biggest pitfall associated with the calculation of half-lives using Elliot and Stock (2001) and Stock (1999) is that the confidence intervals for half-lives are too wide and their upper bounds can approach infinity. The excessively wide confidence intervals are associated with a high degree of uncertainty in the magnitudes of point estimates. Thus, deviations from the parity condition may represent the absence of mean-reversion, calling to question the empirical validity of the PPP hypothesis in the case in point.

### 5.3 Empirical results

Table 4 presents the results of confidence intervals using standard asymptotic theory. MAIC was used to determine the lag length based on the demeaned data. According to the results, half-lives of less than 36 months would be compatible with the PPP hypothesis. Due to their lower power as discussed in chapter 4, the ADF and ADF-GLS results cannot be interpreted with high degree of confidence. It is better not to focus too much on them. According to the results appearing on Table 4, point estimates of half life deviations less than 36 months depend on the method used. Such cases include all countries except Tanzania, Zambia, and Malawi.

Table 5 presents the empirical results based on Stock (1991) approach. The main weakness of the approach is that it does not guarantee non-negative half lives. So, the Stock (1991) method can be seen as not reliable in respect of confidence intervals. It is noteworthy, however, that the median unbiased point estimates appear quite reasonable in the context of PPP.



In Table 5 the results associated with Mauritius results remain incomplete and those of Tanzania are invalid, while those associated with Zambia and Malawi have infinite half-lives. The reason for these problems is that the exchange rates of those countries may not be compatible with local-to-unity asymptotic theory used.

It is seen from table 6 that Hansen's method is supposed to guarantee non-negative numbers. However, Tanzania is a serious problem. The Tanzania results are invalid owing to the negative numbers. The large numbers associated with Zambia under the Elliot and Stock (2001) method should be treated as infinity.

Taken together, the current approaches used in this analysis are not informative in terms of confidence intervals. However, point estimates in the case of Hansen's method may be biased.

**Table 4 Confidence intervals based on standard asymptotics and ADF tests**

Countries	$\hat{\alpha}(1)$	ADF	ADF-GLS	$\hat{h}_a$	$(\hat{h}_a^l, \hat{h}_a^u)$	$\hat{h}_a^*$	$\hat{\gamma}$
Angola	0.972	-2.37	-0.53	24.10	3.84; 44.30	24.10	0.10
Botswana	0.961	-2.29	-1.18	17.40	2.18; 32.60	16.20	0.06
Madagascar	0.966	-1.99	-1.16	20.20	0; 40.40	18.10	0.01
Malawi	0.991	-0.79	0.01	81.00	0; 284.00	86.00	0.00
Mauritius	0.861	-3.95	-2.61	4.63	1.99; 7.30	4.91	0.20
Mozambique	0.949	-2.13	-0.48	13.30	0.76; 25.90	14.10	0.20
SA	0.964	-1.72	-0.68	19.00	0; 41.10	20.10	0.10
Swaziland	0.882	-2.73	-2.13	5.51	1.29; 9.73	4.66	0.01
Tanzania	0.96	-1.40	0.53	16.90	0, 40.90	-28.80	-0.30
Zambia	0.99	-1.29	1.51	70.50	0, 178.00	70.50	0.03

Note: For each bilateral real exchange rate the chapter reports the estimated test statistic of the demeaned ADF regression, the estimated coefficient of the lagged regressor  $\hat{\alpha}(1)$  as defined in (5.9) and the DF-GLS test proposed by Elliott, Rothemberg and Stock, 1996 (ADF-GLS). The lag lengths are selected by the modified AIC criterion based on the OLS and on the GLS detrending methods proposed by Ng and Perron (2001).



**Table 5 Confidence intervals based on Stock (1991)**

Countries	$\hat{c}_l; \hat{c}_u$	$c_l^{0.05}$	$h^{a}_{0.05}$	$h^{a}_{median}$	$\hat{b}(1)$	$h^*_{0.05}$	$h^*_{median}$
Angola	-19.0; 2.62	4.79	5.29	11.43	1.00	4.7	5.29
Botswana	-18.0; 2.77	19.02	21.07	47.75	0.4	17.69	19.60
Madagascar	-14.5; 3.31	20.71	23.35	70.74	0.37	18.59	20.96
Malawi	-4.15; 4.62	45.89	67.26	$\infty$	0.72	48.71	71.38
Mauritius	*	*	*	*	*	*	*
Mozambique	-16.10; 3.02	8.25	9.21	23.48	0.81	8.71	9.72
South Africa	-11.6; 3.78	15.05	17.25	128.83	0.79	15.94	18.26
Swaziland	-23.8; 1.64	4.47	4.86	9.04	1.25	3.77	4.11
Tanzania	-8.53; 4.18	3.79	4.49	$\infty$	2.98	6.49	-7.68
Zambia	-7.65; 4.28	17.86	21.61	$\infty$	0.99	17.87	21.62

Note. As in the previous Table, for each real exchange rate we ran a demeaned ADF regression. The two-sided and one-sided median unbiased confidence intervals for  $c$ , denoted respectively by  $(\hat{c}_l; \hat{c}_u)$  and  $\hat{c}_{0.05}$  were obtained as discussed in Rossi (2005a, Table 3). We report one-sided lower bounds for the median unbiased confidence intervals for the half-life ( $h$ ) with 95 per cent coverage. Upper bounds were infinity for all currencies.  $h_{median}$  is the median unbiased estimate of the half-life (based on the median unbiased estimate of  $c$ ).

**Table 6 Confidence intervals based on Elliot and Stock (2001) and Hansen (1999)**

Countries	Elliot and Stock (2001)		Hansen (1999)	
	$h^{a}_{0.05}$	$h^*_{0.05}$	$h^{a}_{0.05}$	$h^*_{0.05}$
Angola	89.45	68.28	6.07	6.79
Botswana	42	17.54	6.44	7.19
Madagascar	46.44	31.8	5.45	6.09
Malawi	117.01	94.79	6.45	7.2
Mauritius	1.53	1.16	6.44	7.2
Mozambique	22.31	17.2	6.42	7.17
South Africa	34.39	19.14	6.43	6.61
Swaziland	4.25	2.74	6.43	5.74
Tanzania	95.04	-124.14	-10.4	-11.61
Zambia	2092.56	1598.29	6.43	6.79

Note: We report the one-sided confidence intervals for the alternative half-lives (as described in Table 4) with 95 per cent coverage. Upper bounds are not reported because they diverted to infinity.



## 5.4 Conclusions

The objective of this chapter has been to utilise various methods to obtain better approximations to the half-life for highly persistent processes in the presence of small samples and to generate confidence intervals for half-life deviations from PPP. The robust methods of Stock (1991), Elliott and Stock (2001), and Hansen (1999) imply that point estimates of less than 36 months exist, making them compatible with PPP. Specifically, the Elliott and Stock (2001) approach point to Mauritius, Mozambique, South Africa, and Swaziland as the main examples. However, the results of ADF and ADF-GLS tests render the SADC real exchange rates as nonstationary processes, a result that is patently at odds with mean-reversion, implying at the same time the possibility of infinite half-lives. Therefore the empirical results appearing in this chapter do not convincingly resolve the half-life puzzle of the PPP hypothesis. It has been pointed out by Kim, Silvapulle and Hyndman (2006) that the half-life statistic has an unknown and possibly intractable distribution and that it may not possess finite sample moments since it takes extreme values as the coefficient  $\hat{\rho}$  approaches one. Another implausible characteristic is that the half-life statistic is biased in small samples. Moreover, the authors indicated that a tiny estimation error in  $\hat{\rho}$  could result in extreme variability of  $\hat{h}$ , which makes it uncertain and unreliable.

When the results of this chapter are compared with those appearing in chapter 4, in which PPP was found to hold in some countries, one is inclined to believe that a different and improved framework with regard to the calculation of half-lives is necessary to achieve consistent results that include tighter confidence intervals. For future research, the most promising approach seems to be in the spirit of Kim, Silvapulle, and Hyndman (2006). They authors propose a non-parametric method for point and interval estimation of half-life. This method relies on the use of the bias-corrected bootstrap to approximate the sampling distribution of the half-life. It estimates the kernel density of the above bootstrap distribution, by adopting the transformed kernel density method and the data-based bandwidth selection method. It uses the highest density region method for point and interval estimation of half-life.





Besides the latter work, recently Pesavento and Rossi (2005) have developed a method that provides median unbiased confidence intervals with accurate coverage properties regardless of whether the root is unity or close to unity.

The results of this chapter should be taken as suggestive and not definite, because of the problems mentioned above.



## 6 Chapter 6: Tests of long memory regarding the PPP puzzle

### 6.1 Introduction

This chapter employs “a class test for fractional integration” associated with the seminal contribution of Hinich and Chong (2007) to appraise the possibility that Southern African Development Community (SADC) country real exchange rates can be treated as long memory processes. The justification for considering fractional integration arises from the general failure to reject the unit-root hypothesis in real exchange rates when standard Dickey-Fuller unit-root tests are used. In allowing for only integer orders of integration in the series dynamics, the linear tests of nonstationarity were found by authors such as Diebold and Rudebusch (1991) to have low power against fractional alternatives.

The concept of long memory is gaining popularity in econometrics, because econometricians wish to ensure that a stationary process is not mistaken for a non-stationary or a fractionally integrated process. The introduction of the concept of long memory in time series econometrics is associated with the seminal work on fractional integration by Granger and Joyeux (1980) who, according to Smallwood (2005: 4), observed that “the spectral density function of the differenced process appeared to be overdifferenced, while the level of the series exhibited long-run dependence that was inconsistent with stationary ARMA dynamics”. In short, Granger and Joyeux (1980) developed the concept of long memory or fractional integration to fill the gap between a covariance stationary process and a linear autoregressive moving average (ARMA) process. The model of fractionally integrated time series allows for a fractional exponent in the differencing process of the time series, thereby avoiding the unit-root distinction while admitting persistence, or long memory, which characterises many macroeconomic time series. Some major works on fractional integration in the behaviour of exchange rates include Cheung (1993), Baillie (1996), Kapetanios and Shin (2003), Baillie and Kapetanios (2004), Robinson (2003), and Smallwood (2005). A more general approach to long-range dependence is associated with Guégan (2005).



The rest of the paper is organised as follows: Section 2 describes the concept of long memory. Section 3 describes the Hinich-Chong testing methodology used. Section 4 presents the results of the testing algorithm and Section 5 concludes.

## 6.2 Concept of fractional integration

According to the definition in Baillie (1996), a time series process  $y_t$  with autocorrelations  $\rho_k$  possesses a long memory if:

$$\lim_{n \rightarrow \infty} \sum_{k=-n}^n |\rho_k| \rightarrow \infty. \quad (6.1)$$

In addition, a process is integrated of order  $d$ , denoted  $I(d)$ , if

$$(1-L)^d y_t = u_t, \quad (6.2)$$

where  $u_t$  is ergodic and stationary when  $-0.5 < d < 0.5$ . It is noted that when  $u_t$  is  $I(0)$  the process is covariance stationary. In addition, the series  $y_t$  has an invertible moving average representation when  $d > -0.5$ . When  $d > 0$ ,  $y_t$  has long memory, and the autocovariances of  $y_t$  are not absolutely summable.

The autoregressive fractionally integrated moving average representation for a time series process  $y_t$  can be written as:

$$\varphi(L)(1-L)^d (y_t - \mu) = \theta(L)\varepsilon_t \quad (6.3)$$

where the characteristic polynomials  $\varphi(z) = 0$  and  $\theta(z) = 0$  have all their roots outside the unit circle. In this setting,  $\{\varepsilon_t\}$  is a martingale difference sequence,  $d$  is a real number, and  $(1-L)^d$  is the generalised factorial function of the form



$$(1-L)^d = \sum_{k=0}^{\infty} \Gamma(k-d) \{\Gamma(-d)\Gamma(k+1)\}^{-1} L^k$$

$$\text{where } \Gamma(k-d) = \{(k-d-1)(k-d-2)\dots(2-d)(1-d)(-d)\}\Gamma(-d) \quad (6.4)$$

According to Baillie (1996), the Wold representation of a fractional white noise process is given by the following equivalent relations:

$$y_t = \sum_{k=0}^{\infty} w_k \varepsilon_{t-k} \quad (6.5)$$

$$y_t = (1-L)^{-d} \varepsilon_t \quad (6.6)$$

$$y_t = \{1 + dL + d(d+1)L^2 / 2! + d(d+1)(d+2)L^3 / 3! + \dots\} \varepsilon_t \quad (6.7)$$

The autocorrelation function of the process appearing in equation (6.3) decays at a hyperbolic rate. In the frequency domain, the spectral density function of  $y_t$ , denoted  $f_y(\lambda)$ , is of the following form:

$$f_y(\lambda) = \frac{\sigma^2}{2\pi} (2 \sin \lambda / 2)^{-2d} \frac{|\theta(e^{i\lambda})|^2}{|\phi(e^{i\lambda})|^2} \quad (6.8)$$

where  $\lambda$  is the frequency. In the context of long-range dependence, the spectral density approaches infinity as frequency approaches zero.

It should be pointed out that there are several concepts of long memory in the frequency domain and time domain. For instance, Guégan (2005) discusses several concepts, one of which is Parzen's (1981) concept of long memory. In this setting, Guégan (2005) shows that there are processes that are long memory in the covariance sense but may not be so in a spectral density sense. In addition, he shows that there are processes that are long memory in both the covariance sense and spectral density sense. Furthermore, there are processes that are long memory only in the spectral density sense.



While generally useful in economics and finance, the concept of fractional long memory is not without controversy. For instance, Ashley and Patterson (2007) argue that long memory is “an artefact of unmodelled nonlinear serial dependence and/or structural shifts in the generating mechanisms for these time series.” Other works that espouse similar views include Granger and Hyung (1999), and Diebold and Inoue (2000). Guégan (2005) points out that there is a possibility that long memory may be confused with structural change. Furthermore, Charfeddine and Guégan (2007) point out that observing a long memory in a specific data set does not necessarily mean that the data generating mechanism is a long memory process. It is for the latter reason that we employ the Hinich-Chong test for fractional integration to avoid spurious long memory.

As far as the testing strategy is concerned, the Hinich and Chong (2007) methodology is followed to test for long memory using what the authors call “a class test of fractional integration”, a test that is able to distinguish fractionally integrated processes from other time series processes. In this context, the null hypothesis is that  $\{y_t\}$  is a long memory process, while the alternative is that  $\{y_t\}$  does not follow an  $I(d)$  process, where  $-0.5 < d < 0.5$ .

In the context of long memory and the purchasing power parity (PPP) puzzle as discussed by Rogoff (1996), an interesting question is whether deviations from the PPP are transitory or permanent. Thus, the empirical tests generally take the form of testing for stationarity of the real exchange rate. If deviations from PPP are transitory, the time series of the real exchange rate is a stationary series, meaning it is  $I(0 < d < 0.5)$ . By contrast, if deviations from PPP are permanent, then the time series of the real exchange rate is nonstationary, which implies that the process is  $I(d \geq 1)$ . It should be noted that for  $I(0.5 < d < 1)$  the process is mean-reverting, even though it is not covariance stationary, as there is no long-run impact of innovations on future values of the process.



### 6.3 Testing methodology

The development of the Hinich-Chong fractional integration test begins by supposing a regression of  $y_t$  on  $y_{t-1}, y_{t-2}, \dots, y_{t-n}$ . In this context, the following is considered:

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix}, \text{ and } Z_n = \begin{pmatrix} 0 & 0 & \dots & 0 & 0 \\ y_1 & 0 & \dots & 0 & 0 \\ y_2 & y_1 & \dots & 0 & 0 \\ \vdots & y_2 & & & 0 \\ \vdots & \vdots & & & 0 \\ \vdots & \vdots & & & y_1 \\ \vdots & \vdots & & & \vdots \\ y_{T-1} & y_{T-2} & \dots & y_{T-n} \end{pmatrix} \quad (6.9)$$

$$\hat{\beta}(n) = (Z_n' Z_n)^{-1} Z_n' Y = (\hat{\beta}_{n1}, \hat{\beta}_{n2}, \dots, \hat{\beta}_{nn-1}, \hat{\beta}_{nn})' \quad (6.10)$$

According to the authors, a long memory process has a unique feature in the sense that if it is approximated by an  $AR(n)$  model via a regression, then the probability limits of the autoregressive coefficient estimates are functions of  $d$  and  $n$ . It follows that if  $T \rightarrow \infty$ , then

$$\beta_{n,j} \xrightarrow{p} \binom{n}{j} \frac{\Gamma(j-d)\Gamma(n-d-j+1)}{\Gamma(-d)\Gamma(n-d+1)} \quad (6.11)$$

where  $\xrightarrow{p}$  represents convergence in probability and  $\Gamma(d)$  is the Euler gamma function.

It is indicated that the estimated coefficients of  $y_{t-1}$  and  $y_{t-n}$  converge in probability to  $nd(n-d)^{-1}$  and  $d(n-d)^{-1}$ , respectively, making it likely that the first estimate will be about  $n$  times the previous one. Therefore, to test whether the process follows an  $I(d)$  process, a researcher can construct a test based on the relationship  $\beta_{n,n}$  and  $\beta_{n,1}$ , such that



$$n\hat{\beta}_{n,n} - \hat{\beta}_{n,1} \xrightarrow{p} 0$$

The authors recommend that autoregressions  $AR(2), AR(3), \dots, AR(n)$  be run to generate a test statistic of the form

$$W(d, n) = [B(n,1) - B(n, n)]\Lambda(n)\Omega(d)^{-1}[B(n,1) - B(n, n)]\Lambda(n)' \quad (6.12)$$

$$\text{where } B(n,1) = (\hat{\beta}_{2,1} \quad \hat{\beta}_{3,1} \cdots \hat{\beta}_{n,1})$$

$$B(n, n) = (\hat{\beta}_{22} \quad \hat{\beta}_{3,3} \cdots \hat{\beta}_{n,n})$$

$$\Lambda(n) = \text{diag}(2 \quad 3 \cdots n) \text{ and}$$

$$\Omega(d) = E[(B(n,1) - B(n, n)\Lambda(n)')(B(n,1) - B(n, n)\Lambda(n))]$$

Theorem 1 of Hinich and Chong (2007) establishes that

$$W(\hat{d}, n) \xrightarrow{l} \chi^2(n-1) \quad (6.13)$$

where  $\xrightarrow{l}$  represents weak convergence in distribution. To select a robust and consistent estimate  $\hat{d}$ , the authors recommend that a median of the following be taken:

$$\hat{d}_{j,1} = \frac{j\hat{\beta}_{j,1}}{j + \hat{\beta}_{j,1}} \quad (6.14)$$

$$\hat{d}_{j,j} = \frac{j\hat{\beta}_{j,j}}{j + \hat{\beta}_{j,j}} \quad (6.15)$$

whereby for  $j = 1, 2, 3, \dots, n$ ,  $\hat{d}_{j,1}, \hat{d}_{j,j}$  are arranged in an ascending order. In this setting,  $\hat{\beta}_{j,j}$  estimates the  $j^{\text{th}}$  order partial autocorrelation of a fractionally integrated process.

A Monte Carlo experiment regarding the power and size of the Hinich-Chong test established that for non-fractional alternatives, the null hypothesis that a process is an  $I(d)$  process was easily rejected. In addition, Hinich and Chong undertook 10 000

replications to assess the rejection rates of the test:

$$\Pr(W(d, n) > \chi^2_{\alpha=5\%}(n-1)) \text{ for } T = 50, 100, 200; n = 2, 3, 4, 5, 6, 7.$$

They found that for the fractional alternatives, the size of the test was, for large  $T$ , between 0.039 and 0.060. Also, for non-fractional alternatives, the null hypothesis is eventually rejected as the sample size increases.

## 6.4 Results

This section utilises data from the International Monetary Fund's International Financial Statistics database to test for dollar-based long memory in respect of the SADC countries. The real exchange rate is defined as

$$y_t = \ln Y_t = \ln S_t - \ln \bar{P}_t + \ln P_t \quad (6.16)$$

where  $\ln Y_t$  is the logarithm of a real exchange rate (domestic price of foreign currency) at time  $t$ ;  $\ln \bar{P}_t$  and  $\ln P_t$  are the logarithms of foreign and domestic price levels, respectively. We demeaned  $y_t$ . The sample period for each country appears in table 7.

The null and alternative hypotheses are formulated as follows:

$$H_0: y_t \sim I(d)$$

$$H_1: y_t \text{ does not follow } I(d)$$

for  $-0.5 < d < 0.25$ . Footnote one of the Hinich and Chong paper indicates that the authors allowed  $-0.5 < d < 0.5$  in the estimation of the differencing parameter, but for  $d > 0.25$  they point out that "the distribution of  $W(d, n)$  will no longer be Chi-squared but something related to the Rosenblatt distribution as found in Hosking (1996)."





**Table 7 Sample periods**

	<b>Sample periods</b>
Angola	1995M9-2006M6
Botswana	1990M1-2006M6
Madagascar	1990M1-2003M3
Malawi	1990M12-2006M6
Mauritius	1990M1-2006M6
Mozambique	1993M7-2006M5
South Africa	1990M1-2006M6
Swaziland	1990M1-2005M12
Tanzania	1994M12-2006M6
Zambia	1990M1-2006M5

In Table 8 the estimated values of  $d$  are reported in parentheses and the calculated values of the  $W(\hat{d}, n)$  statistic are also reported. In the same table (\*) and (\*\*) represent significance levels at 1 per cent and 5 per cent, respectively. In most cases the estimated value of  $d$  is not affected by the choice of  $n$ . However, there are some negative values of the estimated differencing parameter, implying that real exchange rates are not long memory processes. The negative values represent antipersistence and as observed by Hualde and Velasco (2006), are empirically unappealing. The negative values are associated with the following countries: Madagascar, Malawi, Swaziland, and Tanzania. Other results are as follows: At the 1-per-cent and 5-per-cent significance levels the real exchange rates associated with Mauritius and South Africa are not fractionally integrated. For  $n \geq 6$  Mozambique was found not be fractionally integrated. The currencies of Angola and Botswana, and Zambia were found to be fractionally integrated.



**Table 8**  $W(\hat{d},n)$  based on the demeaned SADC real exchange rates

	T	$W(\hat{d},2)$	$W(\hat{d},3)$	$W(\hat{d},4)$	$W(\hat{d},5)$	$W(\hat{d},6)$	$W(\hat{d},7)$
Angola	130	1.79 (0.13)	1.84 (0.13)	2.45 (0.13)	2.74 (0.13)	3.23 (0.14)	4.18 (0.15)
Botswana	198	2.94 (0.03)	3.32 (0.03)	3.35 (0.03)	3.72 (0.03)	3.81 (0.03)	3.83 0.03
Madagascar	159	0.36 (-0.02)	4.92 (-0.02)	5.37 (-0.02)	5.40 (-0.02)	4.50 (-0.02)	6.25 (-0.01)
Malawi	198	0.01 (-0.01)	0.62 (-0.01)	3.74 (-0.01)	4.38 (-0.01)	5.23 (-0.01)	6.00 (-0.01)
Mauritius	198	0.36 (0.14)	15.77* (0.14)	24.52* (0.13)	25.83* (0.13)	30.30* (0.13)	30.32* (0.13)
Mozambique	155	0.84 (0.16)	5.63 (0.16)	6.37 (0.16)	6.77 (0.16)	12.53* (0.16)	12.64* (0.17)
South Africa	198	4.77** (0.09)	5.14 (0.1)	10.08** (0.1)	11.57** (0.1)	13.43** (0.1)	14.72** (0.09)
Swaziland	192	18.99* (-0.05)	19.36* (-0.05)	25.67* (-0.05)	25.89* (-0.06)	26.84* (-0.07)	27.38* (-0.08)
Tanzania	139	1.13 (-0.46)	1.91 (-0.46)	2.56 (-0.45)	4.87 (-0.45)	5.88 (-0.44)	6.12 (-0.45)
Zambia	197	1.18 (0.03)	1.19 (0.03)	1.20 (0.03)	1.34 (0.03)	2.06 (0.03)	6.44 (0.03)
$\chi^2_{n-1,1\%}$		6.63	9.21	11.34	13.28	15.09	16.81
$\chi^2_{n-1,5\%}$		3.84	5.99	7.81	9.49	11.1	12.60

Note: The numbers in parentheses are the estimated values of  $\hat{d}$ . The sample periods appear in Table 7.

A caveat regarding possible pitfalls of the above results is in order: In some studies, Angola's currency has been found to be nonstationary, which is likely to be the case, given its long history of war and high inflation. The real exchange rates of South Africa's and Swaziland's currencies were recently found by Mokoena (2007) to be nonlinear but ergodic. It is therefore likely that they are not long memory processes.

In chapter 4 we saw that Mauritius, South Africa, Swaziland, and Tanzania were mean-reverting in a nonlinear way. In the current chapter, Mauritius, Mozambique, South Africa and Swaziland have been reported as not having long memory. Mozambique is neither a stationary case nor a long memory. We must conclude that it is nonstationary.



## 6.5 Conclusions

This chapter sought to determine whether the SADC real exchange rates were generated by a long memory mechanism. A class test of fractional integration developed by Hinich and Chong (2007) was used. Antipersistence – an unappealing empirical result – was associated with Madagascar, Malawi, Swaziland, and Tanzania. At the 5 per cent significance level, the null hypothesis could not be rejected that the real exchange rate associated with Angola, Botswana, and Zambia were  $I(d)$  processes. In the case of Mozambique the null hypothesis could be rejected when  $n$  was either 6 or 7. In addition, at the 5 per cent significance level, the real exchange rates associated with Mauritius, Swaziland and South Africa were found not to be fractionally integrated.