4 Chapter 4: Testing for PPP using SADC real exchange rates

4.1 Introduction

Recent literature recognizes the need to assess nonlinearities in the adjustment dynamics of real exchange rates. The main reason for this paradigm shift is that, due to lower power in small sample sizes, the standard Dickey-Fuller-type tests do not provide a solid foundation for an inference that reduces the probability of committing a Type 2 error. This has been the case even when stronger versions of Dickey-Fuller tests – such as the one suggested by Elliot, Rotenberg and Stock (1996) – were used.

The focus on nonlinearities has been reinforced by Taylor, Peel and Sarno (2001) who provide strong evidence that four major real bilateral dollar exchange rates were characterized by nonlinear mean-reversion. One influential study that has also corroborated nonlinear mean-reversion is by Michael, Nobay and Peel (1997). In the nonlinear models, an equilibrium level of the real exchange rate in the regime in which the log-level of the real exchange rate is close to a random walk becomes increasingly mean reverting as the absolute size of the deviation from equilibrium increases. This is consistent with the recent theoretical literature on the nature of real exchange rate dynamics in the presence of transaction costs (See Sercu, Uppal and van Hulle (1995)).

This chapter presents hypothesis testing in respect of joint tests of nonlinearity and stationarity associated with the seminal contribution by Kapetanios, Shin and Snell (2003), henceforth denoted KSS. It also presents the results of ADF tests and Bayesian unit root tests at conventional levels.

In addition to non-ESTAR alternative unit root testing, the chapter provides a background description of the Bayesian unit-root testing framework.

4.2 The ESTAR testing framework

In the context of nonlinearities, the testing framework is the smooth transition autoregressive modelling. In particular, we focus on the exponential version of the model, which is often used when the economic agents can have arbitrage opportunities by facing some deviation from the long-run equilibrium. In such a case, the unit root regime becomes an inner regime, and the mean-reverting regime becomes the outer regime.

In this setting, let y_t be real exchange rate time series observed at $t = 1 - p, 1-(p-1), \ldots -1, 0, 1 \ldots T-1, T$. Denote $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_{1-(p-1)}, y_{1-p}\}$. When time series is stationary we can assume that $E[\varepsilon^2{}_t \mid \Omega_{t-1}] = \sigma^2$.

Consider the following representation of the exponential STAR model:

$$y_t = a_1 y_{t-1} + a_2 y_{t-1} F(\gamma y_{t-d}) + \varepsilon_t \qquad (4.1)$$

where $F(\gamma y_{t-d}) \equiv 1 - \exp(-\gamma y^2{}_{t-d})$. (4.2)

In equation (4.1), y_{t-d} is a transition variable, making this ESTAR model a self-exciting one. The delay parameter d is an integer, which can be fixed or be determined endogenously by means of a supremum LM test in the spirit of Norman (2006a). In equation (4.2), $F(y_{t-d})$ represents the exponential transition function. We note that the extreme values of the transition function are 0 and 1. So, for $F() > 0$ and $F() < 1$, the model exhibits a smooth regime-switching behaviour. The parameter γ determines the smoothness of the transition from one regime to another. We note that as $y_t \to \pm\infty$, then the transition function $F() \to 0$. In addition, as $\gamma \to 0$ or $\gamma \to \infty$, then $F(s_t, \gamma, c) = 0$.

The null hypothesis of a unit root or no long-run equilibrium implies:

$H_0 : a_1 = 1$

This leads to an AR(1) model:

$$y_t = a_1 y_{t-1} + \varepsilon_t \quad (4.3)$$

where $a_1 = 1$.

Under the alternative hypothesis, the model becomes

$$y_t = \{a_1 + a_2 y_{t-1}[1 - \exp(-\gamma y^2{}_{t-d})]\} y_{t-1} + \varepsilon_t \quad (4.4)$$

and

$H_1 : \gamma > 0$.

The null hypothesis cannot be tested directly due to the fact that a_2 and γ are not identified under the null hypothesis. This is called the Davies (1987) problem. According to KSS, testing for nonlinearity in the context of possible nonstationarity requires an auxiliary regression of the form:

$$\Delta y_t = \delta y^3{}_{t-1} + error\ . \quad (4.5)$$

In the presence of serial correlation, the auxiliary regression takes the form:

$$\Delta y_t = \sum_{j=1}^{p} \varphi_j \Delta y_{t-j} + \delta y^3{}_{t-1} + error\ . \quad (4.6)$$

KSS developed a nonlinear ADF t-test, denoted $NLADF - t$, of the form:

$$NLADF - t = \frac{\hat{\delta}}{s.e.(\hat{\delta})}, \quad (4.7)$$

where $\hat{\delta}$ is the estimator of δ in (4.4) or (4.5) and $s.e.$ is the standard error of regression. The $NLADF-t$ statistic is accompanied by a nuisance-parameter-free asymptotic distribution of the following form:

$$NLADF \Rightarrow \frac{\{\frac{1}{4}B(1)^4 - \frac{3}{2}\int_0^1 B(r)^2 dr\}}{\sqrt{\int B(r)^6 dr}}, \qquad (4.8)$$

where $B(r)$ is the standard Brownian motion defined on $r \in [0,1]$.

In essence, the unit root test is for testing the null hypothesis of non-mean reverting time series against the alternative hypothesis of a globally ergodic nonlinear process.

4.3 Bayesian unit root testing

Over and above the ESTAR testing, for comparative purposes we employ Bayesian unit root tests in conjunction with the ADF tests. Bayesian unit root testing was introduced by Zellner and Siow (1980) but was popularised by Koop (1992) and Ahking (1997, 2004), among others. Under the null hypothesis, a times series is an autoregressive model of order p with a linear time trend. In short, hypothesis testing takes the following form:

$$H_1 : y_t = \beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i} + \beta_{p+1} t + \varepsilon_{1t} \qquad (4.9)$$

$$H_2 : y_t = \beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i} + \varepsilon_{2t} \ , \ \beta_{p+1} = 0$$

$$H_3 : \Delta y_t = \beta_0 + (\sum_{i=2}^{p} \beta_i) \Delta y_{t-i} - (\sum_{i=3}^{p} \beta_{p+i}) \Delta y_{t-i} - \beta_p \Delta y_{t-(p-1)}) - \ldots - \beta_p \Delta y_{p-1} + \varepsilon_{3t}$$

$$\sum_{i=1}^{p} \beta_i = 1, \beta_{p+1} = 0 \, ,$$

$$H_4 : \Delta y_t = \beta_0 + (\sum_{i=2}^{p} \beta_i)\Delta y_{t-i} - (\sum_{i=3}^{p} \beta_{p+i})\Delta y_{t-2} - \beta_p \Delta y_{t-(p-1)} - \ldots - \beta_p \Delta y_{p-1} + \varepsilon_{3t} , \sum_{i=1}^{p} \beta_i = 1,$$

where t denotes a linear deterministic time trend; and ε_{jt}, $j = 2,3,4$ represents a serially uncorrelated error process with zero mean and constant variance. H_1 represents the null model, hypothesising a trend-stationary auto-regressive process of order p. The first alternative against the null, hypothesises a stationary $AR(p)$ process, while the second alternative hypothesises an $AR(p)$ process with a unit root. It is to be noted that H_2 and H_3 are special cases of the null hypothesis, with linear restrictions imposed on the null model. The trend-stationary hypothesis is included because it is the leading alternative to unit-root non-stationarity in macroeconomic time series. According to Ahking (2004) the stationary alternative is included to appraise the extent to which the Bayesian test can distinguish between nonstationary series and a stationary one with a high degree of persistence, as is frequently encountered in time series econometrics.

We compare the four hypotheses, based on both prior and sample information, by calculating the posterior odds ratios:

$$K_{1j} = \frac{[P(H_1)P(H_j)]P(H_1 \mid \tilde{y})}{P(H_j \mid \tilde{y})} \qquad (4.10)$$

where $\tilde{y}$ is the sample data; $P(H_1)/P(H_j)$ denotes the prior odds ratio, and $P(H_i \mid \tilde{y})$ is the posterior probability that H_2, H_3 and H_4 were true given the sample data. We note that the posterior odds ratio gives the ratio of the probabilities of the two hypotheses holding given the sample data. On the assumption that all three hypotheses have equal prior probability, then the posterior odds ratio becomes:

$$K_{1j} = \frac{P(H_1 \mid \tilde{y})}{P(H_j \mid \tilde{y})}, j = 2,3,4 . \qquad (4.11)$$

According to Ahking (1997), the Zelner-Siow posterior odds ratio is approximated by the following:

$$K_{1j} = \frac{\sqrt{\pi} / \Gamma[(r+1)0.5](0.5v)^{0.5r}}{(1 + rF/v)^{(1/v)/2}}, \tag{4.12}$$

where

$\Gamma[.] =$ the Gamma function, which, in mathematics, is an extended factorial function to complex and non-integer numbers;

$T =$ the number of observations;

$v = T - K$ number of observations less the number of linear restrictions;

$k =$ the number of regressors in the null model;

$r =$ the number of linear restrictions to be tested, and

$F =$ the F-statistic for testing the set of linear restrictions.

The following are the particularly relevant values of the gamma function:

$$\Gamma[1/2] \approx 1.772;\ \Gamma[1] = 1;\ \Gamma[1,5] \approx 0.886;\ \Gamma[2] = 1;\ \Gamma[5/2] = 1.329.$$

The calculated posterior odds ratios are used to compute the posterior probability for each of the four hypotheses. The results of this analysis appear in the next subsection.

4.4 Empirical evidence

SADC dollar-based real exchanges were chosen on the basis of adequate data availability. We used the International Financial Statistics database of the International Monetary Fund. Real exchange rates were derived from the relative form of the purchasing power parity hypothesis, namely:

$$y_t = \ln Y_t = \ln S_t - \ln \overline{P}_t + \ln P_t, \tag{4.13}$$

where $\ln Y_t$ is the logarithm of a real exchange rate (domestic price of foreign currency) at time t; $\ln \overline{P}_t$ and $\ln P_t$ are the logarithms of foreign and domestic price levels, respectively. The United States consumer price index (CPI) inflation is the all-item CPI inflation and the foreign CPI inflation rates are the general CPI inflation rates of the chosen countries. Sample periods varied according to data availability in respect of CPI inflation and nominal exchange rate series.

4.4.1 The results of Bayesian unit root testing

According to the results appearing in Table 2, nonstationarity hypothesis receives small posterior probability relative to other hypotheses. In this setting, the Bayesian results strongly support the hypothesis that all the real exchange rates are trend-stationary autoregressive processes.

It is necessary to point out that the Bayesian unit root test results are sharply at odds with the ADF results in that the hypothesis of a unit root does not receive significant posterior probability in all cases. Instead the results seem to support the hypothesis of trend-stationarity for all cases. That been said, Ahking (2004) found that that the Bayesian test used in this paper could not distinguish between a trend-stationary autoregressive model from a stationary autoregressive one, especially when the time trend effect was relatively small, and the time series was highly persistent. The latter author found that the bias was in favour of finding a trend-stationary model. Thus, the results should be treated with caution.

Table 2 The results of Bayesian unit root tests

Country	AR(p)	Trend stationary	Stationary	Unit root	Two trends
Angola	1	0.53	0.22	0.03	0.22
Madagascar	2	0.58	0.20	0.03	0.20
Botswana	1	0.62	0.18	0.02	0.18
Malawi	1	0.62	0.18	0.02	0.18
Mauritius	6	0.62	0.18	0.02	0.18
Mozambique	3	0.57	0.20	0.03	0.20
South Africa	6	0.62	0.18	0.02	0.18
Swaziland	5	0.61	0.18	0.02	0.18
Tanzania	2	0.55	0.21	0.03	0.21
Zambia	1	0.62	0.18	0.02	0.18

4.4.2 **Results from nonlinear tests of nonstationarity**

In the context of nonlinear analysis, we used partial autocorrelation function to determine the optimal lags. This approach is recommended by Granger and Terasvirta (1993) and Terasvirta (1994). The usage of PACF over that of information criteria represents an effort to avoid possible bias when choosing lag length. The delay parameter was fixed at 1. In general, the appropriate choice of the delay parameter is the one associated with the highest test statistic. However, fixing the delay parameter is generally of little consequence since economic intuition would suggest that smaller values of the delay parameter were to be preferred.

We employed an auxiliary regression appearing in (4.5) or (4.6). In this setting, the null and alternative hypotheses are of the form:

$H_0 : \delta = 0$,

$H_1 : \delta < 0$.

Failing to reject the null implies that the real exchange rate should be treated as nonstationary. By contrast, the rejection of the null hypothesis in favour of the alternative implies that the exchange rate is mean-reverting and nonlinear.

KSS provide the simulated asymptotic critical values for the nonlinear unit root tests. For both the nonlinear Dickey-Fuller test and nonlinear ADF tests, the 1, 5 and 10 per cent critical values for the demeaned series are –3.48, -2.93 and –2.66 respectively, whereas for the demeaned and detrended series are, in the same order, –3.93, –3.40 and –3.13.

In the context of linear analysis, our dataset has log-levels of real exchange rates and demeaned series. In the case of non-demeaned data, we apply the ADF test. When testing for unit roots, we allow for a constant but no deterministic time trends in the test regression.

The above being said, the pitfalls of the tests should be noted. As was noted by Hall (1994) and Ng and Perron (1995), the ADF tests suffer from low power when the lag length is too small. In some cases, lag selection alone may be responsible for the difference in rejections rates.

Table 3 summarises key inferences that can be made from the above estimation. The results from KSS nonlinear unit root and linear ADF tests are based on the demeaned series and suggest that the null hypothesis of nonstationarity should be rejected at 1 per cent significance level for 4 real exchange rates (Mauritius, South Africa, Swaziland, and Tanzania) out of 10 country exchange rates under study. This suggests that a linear specification for these countries would be inappropriate. In addition, these real exchange rates are mean-reverting but in a nonlinear fashion.

At 1 per cent significance level, all the series were nonstationary. However at 10 per cent significance level the real exchange rates of 6 countries were stationary: Mozambique, Madagascar, Mauritius, South Africa, Swaziland, and Tanzania were found to be stationary.

Table 3 ADF and Nonlinear ADF test results

	Sample periods	**Critical values, ADF (10%)**	**Linear ADF**	**Nonlinear ADF**
Angola	1995M9-2006M6	-1.62	-0.55	-1.98
Botswana	1990M1-2006M6	-1.62	-1.04	-1.41
Madagascar	1990M1-2003M3	-1.62	-1.05	-2.55
Malawi	1990M12006M6	-1.62	-0.39	-0.50
Mauritius	**1990M1-2006M6**	**-1.62**	**-3.71**	**-6.25**
Mozambique	1993M7-2006M5	-1.62	-2.60	-2.15
South Africa	1990M1-2006M6	-1.62	-2.43	**-6.89**
Swaziland	1990M1-200M6	-1.62	-2.06	**-5.42**
Tanzania	1994M12-2006M6	-1.62	-2.26	**-5.95**
Zambia	1994M1-2006M5	1.61	-1.09	-2.08
NADF Demeaned data significance levels				
10%				-2.66
5%				-2.93
1%				-3.48

4.5 Conclusions

This chapter has sought to present evidence indicating that the PPP puzzle is becoming less of a puzzle. It presented the results of Bayesian unit root tests, ADF test and nonlinear tests of nonstationarity. The Bayesian tests were found to be biased in favour of a trend stationary model in all cases.

It is argued that nonlinear approaches to exchange rate adjustments are likely to provide a firmer basis for inference and stronger support for the PPP in the long-term. This is more so at 1 per cent and 5 per cent significance levels.

The results obtained from the KSS tests suggest that the behaviour of 4 dollar-based real exchange rates should be treated as nonlinear rather than linear. This finding of nonlinear behaviour provides statistical evidence in support of a smooth transition mean-reverting behaviour in 4 out of 10 real exchange rates. As such, any deviation from the PPP, either over- or under-appreciation of real exchange rates is temporary.