3 Chapter 3: Recent theoretical and empirical developments

3.1 Introduction

As noted in Chapter 1, economists have attempted to resolve the three puzzles in several ways. In all the puzzles, economists have tried to provide a new theoretical framework or a new empirical approach.

In the context of PPP, this chapter discusses recent developments associated with seminal contributions by authors such as Enders and Granger (1998), Berben and van Dijk (1999), Caner and Hansen (2001), Lo and Zivot (2001), Shin and Lee (2001), Kapetanios and Shin (2002), Bec, Ben Salem and Carrasco (2004), and Kapetanios, Shin and Snell (2003). These authors have developed various nonlinear tests of nonstationarity that tend to have better power than the PP and ADF tests.

In the context of half-lives, the chapter discusses seminal contributions by Kim, Silvapulle and Hyndman (2006), Norman (2007) and Rossi (2005a).

As far as the disconnect puzzle is concerned, the chapter briefly discusses the general equilibrium approaches.

As far as the exchange rate determination puzzle is concerned, the chapter discusses the market microstructure approach, a paradigm that attempts to explain exchange rate determination by paying attention to order flow — the difference between the buyer-initiated and seller-initiated orders in a securities market. In particular, Evans and Lyons (2005) argue that order flow might be able to anticipate future exchange rate movements.

3.2 Recent developments: PPP mean-reversion puzzle

At the theoretical level, economists are beginning to develop nonlinear models of exchange rate adjustment in which transaction costs play an important role. Dumas (1992) has demonstrated that for markets which are spatially separated, and feature 'iceberg' transactions costs, deviations from PPP should follow a non-linear mean-reverting process, with the speed of mean reversion depending on the size of the deviation from PPP. The upshot of this is that within the transaction band, deviations are persistent and take a considerable time to mean-revert. In this setting, the real exchange rate behaves like to a random walk. However, large deviations, those that occur outside the band, will rapidly dissipate and for them the observed mean reversion speeds up. A similar model is authored by Sercu, Uppal and Van Hulle (1995), and includes transport costs which create a band for the real exchange rate within which the cost of arbitrage is larger than the benefit at the margin, creating a no-trade corridor. This approach results in a two regime threshold model, whereby the real exchange rate is reset by arbitrage to an upper or lower inner threshold whenever it hits the corresponding outer threshold (Smallwood, 2005).

A more formal example is associated with Obstfeld and Taylor (1997), who develop a band transition autoregressive model using demeaned and detrended data. The model is of the following form:

If $y_{t-1} > c$, then $\Delta y_t = \phi^{out}(y_{t-1} - c) + \varepsilon_t^{out}$

If $c \geq y_{t-1} \geq -c$, then $\Delta y_t = \phi^{in} y_{t-1} + \varepsilon_t^{in}$

If $-c > y_{t-1}$, then $\Delta y_t = \phi^{out}(y_{t-1} - c) + \varepsilon_t^{out}$,

where errors, denoted ε_t^{out} and ε_t^{in} are normally distributed with mean zero and constant standard deviations. In this setting $\phi i^n = 0$ and ϕ^{out} is the speed of convergence outside the transaction cost band. Using the data set of Engel and Rogers (1995), Obstfeld and Taylor find that for inter-country CPI-based real exchange rates, the adjustment speed was only 12 months for the TAR model. When disaggregate price series were used to test the law of one price the B-TAR model produced evidence of mean-reversion which was well below 12 months.

3.3 Threshold and STAR approaches to the PPP puzzle

The STAR approach takes nonlinearities into account when testing for unit roots. The most referenced contributions in the context of threshold autoregressive (TAR) models are associated with Enders and Granger (1998), Gonzalez and Gonzalo (1998), Berben and van Dijk (1999), Caner and Hansen (2001), Lo and Zivot (2001), Shin and Lee (2001), Kapetanios and Shin (2002), Seo (2003), Bec, Ben Salem and Carrasco (2004), and Kapetanios, Shin and Snell (2003) in the context of an exponential smooth transition autoregressive specification. This has led to the employment of non-standard asymptotic theory and joint tests of nonlinearities and nonstationarity in which nonlinear methods tend to require transition autoregressive modelling. The difficulty with these models is that the model parameters are only defined under the alternative hypothesis, a problem identified by Davies (1987). An important feature of any nonlinear approach is that the parameter space must be clearly defined to achieve proper asymptotic null distributions, the critical values of which form the basis of inference. When the parameters are defined only under the alternative hypothesis, usually a truncated Taylor expansion of the transition function becomes the basis of an auxiliary regression that can be estimated using commercial software.

Following van Dijk, Terasvirta, and Franses (2002), the smooth transition autoregressive (STAR) representation requires the following descriptions.

Let y_t be a time series observed at $t = 1-p, 1-(p-1), \ldots -1, 0, 1 \ldots T-1, T$. Let $x_t = (1, y_{t-1}, \ldots y_{t-p})$. Denote $\Omega_{t-1} = \{y_{t-1}, y_{t-2}, \ldots, y_{1-(p-1)}, y_{1-p}\}$. Assume that $E[\varepsilon_t \mid \Omega_{t-1}] = 0$ and that $E[\varepsilon^2{}_t \mid \Omega_{t-1}] = \sigma^2$. Let the transition function be:

$$F(s_t;\gamma,c) = [1+\exp(-\gamma(s_t - c))]^{-1} \tag{3.1}$$

such that $F(s_t;\gamma,c)$ is continuous and is bounded between 0 and 1.

Consider the following representation of the STAR model:

$$y_t = \theta'_1 x_t (1 - F(s_t;\gamma,c)) + \theta'_2 x_t F(s_t;\gamma,c) + \varepsilon_t \tag{3.2}$$

Equation (3.2) can be written as:

$$y_t = (\theta_{1,0} + \theta_{1,1} y_{t-1} + ... + \theta_{1p} y_{t-p})(1 - F(s_t;\gamma,c)$$
$$+ (\theta_{2,0} + \theta_{2,1} y_{t-1} + ... + \theta_{2p} y_{t-p}) F(s_t;\gamma,c) + \varepsilon_t \tag{3.3}$$

In equation (3.3), s_t is a transition variable such that $s_t = y_{t-d}$ where d is an integer and represents a delay parameter. We note that the extreme values of the transition function are 0 and 1. So, for $F(s_t,\gamma,c) > 0$ and $F(s_t,\gamma,c) < 1$, the model exhibits a smooth regime-switching behaviour. When the transition function is represented by the first-order logistic equation (3.1), this gives rise to a logistic STAR (LSTAR) model. The parameter c denotes a threshold between the regimes, while γ determines the smoothness of the transition from one regime to another. For large values of γ and for $s_t = c$, there is an instantaneous change for $0 < F(s_t,\gamma,c) < 1$. Consequently, $F(s_t,\gamma,c)$ becomes an indictor function such that, say, for I =1, $s_t > c$ and $I = 0$, otherwise.

We note that, when the transition parameter is $s_t = y_{t-d}$, the model becomes a self-exciting smooth transition autoregressive (SETAR) model. When γ approaches zero, the logistic function becomes a constant, such that $F(s_t,\gamma,c) = 1/2$. When $\gamma = 0$, the LSTAR becomes a linear model.

There are special cases that can be convenient in the analysis of macroeconomic variables. Suppose the threshold parameter value is 0, that is, $c = 0$ and that y_t represents a country's GDP growth rate. Then for $s_t = y_{t-d}$, the model depicts periods of positive and negative growth rates. When the model is applied to exchange rates the transition function becomes an exponential function, such that

$$F(s_t,\gamma,c) = 1 - e^{\{-\gamma(s_t-c)^2\}} \quad \text{where } \gamma > 0. \tag{3.4}$$

This leads to what is called the exponential smooth transition autoregressive (ESTAR) model. We note that as $s_t \to \pm\infty$, then the transition function $F(s_t, \gamma, c) \to 0$. In addition, as $\gamma \to 0$ or $\gamma \to \infty$, then $F(s_t, \gamma, c) = 0$. This leads to a linear model.

Luukkonen, Saikkonen and Teräsvirta (1998), Teräsvirta (1994), Saikkonen and Luukkonen (1988), Gonzalez-Rivera (1998), Escribano and Jorda (2000), and others have truncated the transition function around $\gamma = 0$ as a means to overcome the nuisance parameter problem, which is normally accompanied by nonstandard asymptotic distribution theory (Hill, 2004).The Taylor expansion approximation leads to a simple auxiliary regression. Tests on subsets of coefficients can be used to infer whether the process is linear or not.

From Luukkonen, Saikkonen and Teräsvirta (1998), the nature of the auxiliary regression from (3.1) and (3.2) is of the following form:

$$y_t = a_0 + \sum_{j=1}^{p} a_{1j} y_{t-j} + \sum_{j=1}^{p} b_{1j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} b_{2j} y_{t-j} y^2{}_{t-d} + \sum_{j=1}^{p} b_{3j} y_{t-j} y^3{}_{t-d} + \xi_t \qquad (3.5)$$

where ξ_t are the white noise residuals with zero mean and constant variance under the null hypothesis of linearity. Under the null, all the $b's$ are equal to zero, whereas under the alternative, at least one b is not equal to zero.

The test statistic required, denoted LM_{LST}, is of the following form:

$$LM_{LST} = \frac{T(SSR_1 - SSR_0)}{SSR_1}, \qquad (3.6)$$

where T is the sample size, SSR_1 and SSR_0 are residual sum of squares of the restricted and unrestricted regressions, respectively.

The LM_{LST} statistic has an asymptotic χ^2 distribution with $3p$ degrees of freedom. Large values of the statistic lead to the rejection of the null of linearity, suggesting that linear $AR(p)$ specification is inadequate in characterizing the process under consideration.

Applications of these threshold regime switching models can be found in Obstfeld and Taylor (1997) and Michael, Nobay and Peel (1997), and Bec, Ben Salem and Carrasco (2004).

Recently, Kapetanios, Shin and Snell (2003) have proposed a new testing procedure for the null hypothesis of a unit root against an alternative of a nonlinear stationary ESTAR process. In particular, the authors have shown that their suggested test is more powerful than the Dickey-Fuller test against the stationary STAR alternative. They call this test the nonlinear augmented Dickey-Fuller (NADF) test statistic. The result is based on the univariate exponential smooth transition autoregressive model of order 1:

$$y_t = a_1 y_{t-1} + a_2 y_{t-1}\Phi(\theta; y_{t-d}) + \varepsilon_t \quad (3.7)$$

where $\varepsilon_t \sim iid(0,\sigma^2), d \geq 1$.

The transition function is of the form: $\Phi(\theta; y_{t-d}) = 1 - e^{(-\theta y^2{}_{t-d})}$.

To test the null hypothesis of a unit root in the above case implies that $a_1 = 1$ and that $a_2 = 1$. Because of the Davies (1987) problem mentioned earlier, the hypothesis testing requires an auxiliary regression of the form:

$$\Delta y_t = \delta y^3{}_{t-1} + error \, . \quad (3.8)$$

In the presence of serial correlation, the auxiliary regression takes the form:

$$\Delta y_t = \sum_{j=1}^{p} \varphi_j \Delta y_{t-j} + \delta y^3{}_{t-1} + error \quad (3.9)$$

KSS developed a NLADF t-test of the form:

$$NLADF = \frac{\hat{\delta}}{s.e(\hat{\delta})}, \tag{3.10}$$

which is accompanied by the asymptotic distribution of the following form:

$$NLADF \Rightarrow \frac{\{\frac{1}{4}B(1)^4 - \frac{3}{2}\int_0^1 B(r)^2 dr\}}{\sqrt{\int B(r)^6 dr}}, \tag{3.11}$$

where $B(r)$ is the standard Brownian motion defined on $r \in [0,1]$

Another paper distinguishing a nonstationary linear process from a stationary nonlinear ESTAR process is Kilic (2004). The author develops a supremum or $\sup - t$ test for unit roots against a globally stationary exponential STAR model, simultaneously allowing for the presence of a drift term and trend term. The distribution is found to be nuisance parameter free, allowing for the calculation of critical values. The t-test is found to have a substantial power compared to the ADF and Phillip-Perron test.

Kilic relies on the ESTAR framework defined as:

$$y_t = \phi y_{t-1} + \phi^* y_{t-1} F(\gamma, c, z_t) + u_t, \tag{3.12}$$

where $u_t \sim NID(0, \sigma^2)$ and z_t is stationary and can take the form $z_t = \Delta y_{t-d}$.

The $\sup - t$ statistic is defined as:

$$\sup - t = \sup_{(\gamma,c)\in \Gamma XC} \quad t_{\phi^*} = \sup_{\gamma,c\in \Gamma XC} \{\frac{\hat{\phi}^*(\gamma,c)}{s.e(\hat{\phi}^*(\gamma,c))}\}\phi^* \tag{3.13}$$

Its asymptotic distribution was found to be:

$$\sup -t \Rightarrow \sup_{(\gamma,c)\in\Gamma XC} \frac{\{\frac{1}{2}[1-C_0(\gamma,c)][B(1)^2-1]\}}{1-2C_0(\gamma,c)+C_1(\gamma,c)\}^{1/2}\{\int_0^1 B(r)^2\,dr\}^{1/2}}, \quad (3.14)$$

where the parameter space is defined as $\Gamma = [\underline{\gamma}, \overline{\gamma}]$ and $C = [\underline{c}, \overline{c}]$ such that $0 < \underline{\gamma} < \gamma < \overline{\gamma}$ and $0 < \underline{c} < c < \overline{c}$. Also, $C_0(\gamma,c) = E(\exp(-\gamma(\Delta y_{t-1} - c)^2))$ and $C_1(\gamma,c) = E(\exp(-2\gamma(\Delta y_{t-1} - c)^2))$.

3.4 Recent developments: Half-lives

In this subsection we take a selective overview of suggested ways to calculate half-lives. Some of the methods take nonlinearities into account. Traditional half-life calculation of half life is generally based on an autoregressive model of order one, $y_t = \varphi y_{t-1} + \varepsilon_t$, with concomitant regularity conditions on the structure of errors, as explained by Rossi (2005a). As demonstrated by Chortareas and Kapetanios (2004), the calculation of the half-life $\hat{H}$ of the process is based on the following:

$$\hat{H} = \ln(0.5)/\ln(\hat{\varphi}), \quad (3.15)$$

where $\hat{\varphi}$ represents the estimate of φ. Based on the sticky price theory, estimates of $\hat{\varphi}$ leading to an estimated half-life of less than 3 years would be deemed acceptable.

It is understood that the above-mentioned approach has severe limitations and not applicable to $AR(p)$ processes. In addition, several authors have found the estimate $\hat{\varphi}$ to biased downward. Also, according to Kim, Silvapulle and Hyndman (2006), the statistic appearing in equation (3.15) suffers from the weakness that it is biased in small samples, that it has unknown and possibly complicated distribution and that it may not possess finite sample moments since it takes extreme values as the estimated coefficient approaches one.

3.4.1 Kim, Silvapulle and Hyndman (2006) approach to half-lives

Kim, Silvapulle and Hyndman (2006) propose a bias-corrected bootstrap procedure for the estimation of half-life deviations from PPP by adopting Hyndman (1996) highest density region (HDR) approach to point and interval estimation. The authors' approach necessitates the use of the Kilian (1998) bias-corrected bootstrap to approximate the sampling distribution of the half-life statistic. In addition, the kernel density of the bootstrap distribution is estimated by adopting the transformed kernel density method of Wand, Marron, and Ruppert (1991).

As indicated earlier, due to software constraints, this promising approach is left for future research.

3.4.2 Chortareas and Kapetanios (2004) half-life approach

Chortareas and Kapetanios (2004) provide an alternative half-life measure. They define the half-life $h*$ as a point in time at which half the absolute cumulative effect of the shock has dissipated. In this setting, $h*$ solves by means of numerical methods the following equation:

$$2\sum_{j=1}^{p}\frac{c_j\lambda_j^{h*}}{\ln(\lambda_j)}=\sum_{j=1}^{p}\frac{c_j}{\ln(\lambda_j)}, \qquad (3.16)$$

where λ_j are eigenvalues of an $AR(p)$ process and c_j is given by:

$$c_j=\frac{\lambda_j^{p-1}}{\Pi_{k=1,k\neq i}(\lambda_j-\lambda_k)}. \qquad (3.17)$$

It is to be noted that (3.16) is not an easy equation to solve. For instance, in the case of an $AR(2)$ process, when simplified, (3.16) takes the following form:

$$2[x_1^{h*}+x_2^{h*}]=z. \qquad (3.18)$$

Hence, numerical methods are required and more so for higher order lags. The application of this method is left for future research.

3.4.3 **Rossi (2005a) approach to half-life deviations from PPP**

Rossi (2005a) introduces a half-life measure for an $AR(p)$ process that produces improved asymptotic approximations in the presence of a root close to unity. Thus the analysis is based on the local-to-unity asymptotic theory. In this context, a half-life can diverge to infinity at the rate of the sample size.

In chapter 6 we provide a detailed exposition of Rossi (2005a) approach.

3.4.4 **Nonlinear approach to half-life deviations**

Another alternative approach to the calculation of exchange rate half-lives in the context of nonlinearities is associated with the work of Koop, Pesaran and Potter (1996) and Norman (2007). In the nonlinear frameworks, impulse response functions have been used to assess the dynamic nature of the effects of shocks on the behaviour of time series in both the univariate and multivariate contexts. By definition, an impulse response function is a change in the conditional expectation of the variable or vector Y_{t+s} as a result of an exogenous shock ε_t :

$$IRF_Y = E[Y_{t+s} \mid \Omega_{t-1}, \varepsilon_t] - E[Y_{t+s} \mid \Omega_{t-1}] , \tag{3.19}$$

where Ω_{t-1} represents the history of the process. In linear models impulse response functions are based on the Wold representation:

$$y_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \tag{3.20}$$

Consider a univariate case of a stationary variable y_t such that it is represented by an autoregressive model:

$$y_t = \phi y_{t-1} + \varepsilon_t \tag{3.21}$$

where $|\phi| < 1$. The associated impulse response function takes the following form:

$$IRF(y_{t+n}) = \theta \frac{1-\phi^{n+1}}{1-\phi}, \tag{3.22}$$

where θ is the size of the shock and $n = 1,2,3,\ldots$.

It has been observed by Beaudry and Koop (1993), Potter (1995), and Pesaran and Potter (1994) that linear models are restrictive in that their symmetry property implies that shocks occurring in one regime are as persistent as the shocks occurring in another regime. Furthermore, linear models cannot adequately capture asymmetries that may exist in the various stages of the business cycle, which is problematic in the light of the evidence that the degree of persistence varies over the business cycle.

Moreover, according to Koop, Pesaran, and Potter (1996), the nonlinear impulse response functions depend on the size of the shock, the sign of the shock, and the history of the system. This has led to the development of the concept of generalised impulse response functions (GIRF). By definition similar to the one appearing above, a generalised impulse response function for an n-period horizon, for multivariate models is of the following form:

$$GIRF_Y(n, \Phi, \Omega_{t-1}) = E[Y_{t+n} \mid \Phi_t, \Omega_{t-1}] - E[Y_{t+n} \mid \Omega_{t-1}], \tag{3.23}$$

where Φ is a vector of shocks and Ω_{t-1} is the history of the system. The generalised impulse response function is a function of Φ and Ω_{t-1}. In this setting, future shocks are averaged out.

In the threshold framework, consider an ESTAR bivariate model:

$$Y_t = AY_{t-1} + BY_{t-1}1_{(\Delta X_{t-1} \geq 0)} + U_t , \tag{3.24}$$

where A and B are 2X2 matrices and U_t and Y_t are vectors or variables. The shock to the $j-th$ variable of Y_t occurs in period 0, and responses are computed for l periods thereafter. The shock is a one or two standard deviation shock, consistent with the Cholesky factorisation framework. Under these circumstances, Koop, Pesaran and Potter (1996) and Atanasova (2003) recommend the following bootstrap-based algorithm:

a. Pick a history $\Omega^h{}_{t-1}$ where $h = 1,2,...,H$. Pick a sequence of (m-dimensional) shocks $\varepsilon^b{}_{t+l}$, $b = 1,2,...,B$ and $l = 0,1,2,...,L$.
b. The shocks are drawn with replacement from the estimated residuals of the model. If one does not want to make any assumptions about the form of dependence but has some knowledge of conditional heteroskedasticity, then one can draw weighted shocks from the joint empirical distribution.
c. Using $\Omega^h{}_{t-1}$ and $\varepsilon^b{}_{t+l}$, simulate the evolution of Y_{t+k} over $l+1$ periods. The resulting path is denoted $Y_{t+n}(\Omega^h{}_{t-1}, \varepsilon^b{}_{t+k})$.
d. Substitute ε_{j0} for the j_0 element of $\varepsilon^b{}_{t+k}$ and simulate the evolution of Y_{t+k} over $l+1$ periods. Denote the resulting path $Y_{t+n}(\varepsilon_{j0}, \Omega^h{}_{t-1}, \varepsilon^b{}_{t+k})$.
e. Repeat steps a to d B times.
f. Repeat steps a to e H times and compute $[Y_{t+n}(\varepsilon_{j0}, \Omega^h{}_{t-1}, \varepsilon^b{}_{t+k})$- $Y_{t+n}(\Omega^h{}_{t-1}, \varepsilon^b{}_{t+k})$]/HB for the average impulse response function.

3.4.5 Nonlinear impulse response functions by means of MCIM

According to Gallant, Rossi and Tauchen (1993) and Norman (2007) the following algorithm can be used to generate nonlinear impulse response functions:

- With the j initial conditions set to zero, use the estimated model to generate observations based on innovations distributed as a mean zero normal distribution with variance, denoted $\hat{\sigma}^2$ where the latter represents the estimated variance of the error term.
- After the first 200 observations are generated, each observation, y^*_t, produced must satisfy $(\mu - \xi) \leq y^*_t \leq \mu + \xi$, where ξ is a small number.
- After 5000 such observations have been found, no additional data are generated. The 5000 observations and their lags form the basis for the initial conditions, denoted $(y_{-p+1},..., y_0)$. These are used to calculate the impulse response function. For each set of initial conditions, 2 time series of 120 observations each are generated from the initial conditions $(y_{-p+1},..., y_0)$ and $(y_{-p+1},..., y_0 + \theta)$ where θ is the shock used.
- The innovations are distributed as a mean zero normal distribution with variance $\hat{\sigma}^2$. The average difference between these two series among the 5000 replications is taken as the impulse response function.

3.4.6 Norman (2007) ESTAR-related half-lives

In the context of nonlinear mean reversion of an exponential smooth transition type, Norman (2007) makes the assumption that "the question of how long it should be expected for a process to return to its long-run equilibrium is more relevant than how persistent are one period innovations" (p.6). This leads to the following definition of a shock, denoted θ_t:

$$\theta_t = E[y_t] - y_t \quad (3.25)$$

In the context of purchasing power parity analysis, y_t can define an exponential smooth transition model of the form:

$$y_t - \mu = \alpha_1(y_{t-1} - \mu) + (\alpha_2 - \alpha_1)(y_{t-1} - \mu)F(y_{t-d} - \mu) + \varepsilon_t \,, \tag{3.26}$$

where the mean of the process is denoted μ and the transition function is of the form:

$$F(y_{t;}\gamma, \mu) = 1 - \exp[-(\gamma / \hat{\sigma} y_t)(y_t - \mu)^2] \,. \tag{3.27}$$

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Norman (2007) uses the definition of a half-life appearing in Gallant, Rossi, and Tauchen (1993), denoted H, which is :

$$\min[H] \text{ such that } E[y_{t+h} \mid y_{t-1} = \mu + \theta] - E[y_{t+h} \mid y_{t-1} = \mu] \leq \frac{\theta}{2} \,. \tag{3.28}$$

Norman (2007) uses the following algorithm for the calculation of half-lives:

- Select the initial condition such that it equals the mean of the process.
- Specify and estimate the ESTAR model.
- For $t \in [1 \ldots T]$, calculate the shock associated with each observation y_t as $\theta_t = y_t - \hat{\mu}$, where $\hat{\mu}$ is the estimated mean of the ESTAR process.
- Use the Monte Carlo integration method to calculate the impulse response function associated with each shock.
- The half-life corresponding to each shock is then calculated according to equation (3.28).
- Draw with replacement from the set of shocks and associated half-lives.

3.5 Testing for long memory in respect of the PPP puzzle

Another new approach to resolving the purchasing power parity puzzle is through fractional integration. The concept of long memory is gaining popularity in econometrics, because econometricians wish to ensure that a nonlinear stationary process is not mistaken for a nonstationary process or a fractionally integrated process. In this context, it is well-known that the presence of unit roots in a time series implies the autocorrelation function of the time series process does not die out and that the variance of the process is unbounded and model innovations will have permanent effects on the level of the process. In equilibrium terms, the process will not revert to a long-run mean. In addition, the presence of unit roots implies that the regressors will have nonstandard asymptotic distributions, thereby invalidating standard tools of inference.

In the STAR framework long memory was introduced by van Dijk, Franses, and Paap (2000). Other works on fractional integration in the behaviour of exchange rates include Cheung (1993), Baillie (1996), Ballie and Kapetanios (2004), Robinson (2003), Smallwood (2005), Kapetanios (2006).

In Smallwood (2005), the tests of nonlinearity utilise the following model of fractional integration:

$$(1-L)^d y_t = \{\varphi_{1,0} + \sum_{i=1}^{p} \varphi_{1,i}(1-L)^d y_{t-i}\} + \{\varphi_{2,0} + \sum_{i=1}^{p} \varphi_{2,i}(1-L)^d y_{t-i}\}F(y_{t-d};\gamma,c) + \varepsilon_t \quad (3.29)$$

The associated auxiliary regression is given by:

$$(1-L)^d y_t = \{\varphi_{1,0} + (\sum_{i=1}^{p} \varphi_{1,i}(1-L)^d y_{t-i}) + \sum_{i=1}^{p} \varphi_{2,i}(1-L)^d y_{t-j} y_{t-d}\} + \{\sum_{i=1}^{p} \varphi_{3,i}(1-L)^d y_{t-i}\} y^2{}_{t-d} + e_t \quad (3.30)$$

To test the null hypothesis of linearity – that the time series process is a long memory ARFIMA(p,d,0) – is the same thing as testing as follows:

$Ho: \varphi_{2,i} = \varphi_{3,i} = 0 \qquad i = 1,\ldots, p$

$Ha: \varphi_{2,i} \neq 0 \qquad or\ \varphi_{3,i} \neq 0$ for at least one i.

In this setting, hypothesis testing is based on an LM-type statistic, which is derived using the following algorithm:

Estimate the ARFIMA(p,d,0) model and store the residuals $\hat{\varepsilon}_t$;

Obtain an optimal estimate of d and denote it $\hat{d}$;

Construct the restricted sum of squared errors, denoted SSR_R;

To obtain the unrestricted squared sum of errors, denoted SSR_{UR}, regress $\hat{\varepsilon}_t$ on

$1, (1-L)^{\hat{d}} y_{t-1}, \ldots, (1-L)^{\hat{d}} y_{t-p}, \quad -\sum_{i=1}^{t-1} \frac{\hat{\varepsilon}_{t-i}}{i}$

$(1-L)^{\hat{d}} y_{t-1} y_{t-d}, \ldots, (1-L)^{\hat{d}} y_{t-p} y_{t-d}$ and

$(1-L)^{\hat{d}} y_{t-1} y^2{}_{t-d}, \ldots, (1-L)^{\hat{d}} y_{t-p} y^2{}_{t-d}$.

The chi-squared version of the LM statistic is calculated as:

$$LM_{\chi^2} = T(SSR_R - SSR_{UR}) / SSR_R \tag{3.31}$$

and is distributed as a $\chi^2(2p)$.

The F version of the statistic is calculated as:

$$LM_F = \frac{(SSR_R - SSR_{UR})/2p}{SSR_{UR}/(T-3p-1)} \; . \tag{3.32}$$

In chapter 6 we use the latest techniques to test for long memory. In particular, we utilise a class test for fractional integration developed by Hinich and Chong (2007). The benefit of this test is that it is able to determine whether or not a time series falls under a class of fractionally integrated processes.

3.6 Recent developments: Exchange rate disconnect puzzle

There are currently two strands of research trying to explain the exchange rate disconnect puzzle. There is currently no survey of the models proposed in respect of the disconnect puzzle. The first strand of research is theoretical in that it attempts to explain the conditions under which “the disconnect” between the economic fundamentals and exchange rate movements is expected to exist. Such studies include Devereux and Engel (2002), Xu (2005), Duarte and Stockman (2005), Evans and Lyons (2005), and Bacchetta and van Wincoop (2006). The second strand is the market microstructure approach that attempts to find reliable short-run determinants of exchange rates.

Below a survey of general equilibrium approaches to the disconnect puzzle is undertaken. Below we begin by discussing in detail the Deveroux and Engel (2002) model. However, the discussion of Bacchetta and van Wincoop (2004), Duarte and Stockman (2005), Xu (2005), and Evans and Lyons (2005) will be more descriptive, with emphasis on the main results rather than the mathematical structure of the model. With the exception of Evans and Lyons (2005), the approach used by the above-mentioned authors is similar to the one appearing in Chapter 10 of Obstfeld and Rogoff (1999).

3.7 A survey of GE models in respect of the disconnect puzzle

We begin with one of the “older” models, which laid the foundation for subsequent studies.

3.7.1 The Devereux-Engel (2002) model

Devereux and Engel (2002) develop a general equilibrium model of the exchange rate that is in line with the view espoused by Krugman (1989) that the volatility of the exchange rates is high because ordinary fluctuations in the exchange rate generally do not matter much for the economy. The authors explain that a combination of local currency pricing, heterogeneity in international price setting and goods distribution, as well as biases in expectations in international financial markets may produce very high exchange rate volatility without significant repercussions for the volatility of other macroeconomic variables. The authors stress that "there ought to be a greater disconnect when the degree of local-currency pricing is high and the wealth effects of exchange rate changes are small."

Devereux and Engel (2002) develop static and dynamic versions of the general equilibrium model. Below we present the dynamic model. In this context, households trade in non-contingent nominal domestic and international bonds in incomplete markets. Households are assumed to trade in domestic currency denominated bonds. Home country trading is carried out by foreign exchange dealers who buy and sell foreign currency denominated bonds to maximise profit.

More formally, a representative consumer in the home country maximises expected utility as follows:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \frac{M_t}{P_t}, L_t) \qquad \beta < 1 \tag{3.33}$$

where $U = (1-\rho)^{-1} + \chi \ln\left(\frac{M_s}{P_s}\right) - \frac{\eta}{1+\psi} L_s^{1+\psi} \qquad \rho > 0$; (3.34)

C_t denotes consumption;

$\frac{M_t}{P_t}$ are real money balances;

L_t is the labour supply.

In this setting, C_f and C_h are consumption indexes that are CES function of goods produced at home and in the foreign country.

$$C = \left(n^{1/\omega} C^{1-i/\omega} + (1-n)^{1/\omega} C_f^{\ 1-1/\omega}\right)^{\omega/(1-\omega)} \tag{3.35}$$

We note that ω denotes the elasticity of substitution between home and foreign consumption aggregates. The model assumes that there are n identical households in the home country, such that $0 < n < 1$. C_h and C_f are defined as:

$$C_h = \left[n^{-1/\lambda} \int_0^n C_h(i)^{\lambda-1/\lambda} di \right]^{\lambda/\lambda-1} \qquad C_f = \left[(1-n)^{-1/\lambda} \int_n^1 C_h(i)^{\lambda-1/\lambda} di \right]^{\lambda/\lambda-1} \tag{3.36}$$

The price index, P, is defined by:

$$P = \left[nP_h^{1-\omega} \_(1-n)P_f^{1-\omega} \right]^{1/1-\omega}$$

where $$P_h = \left[\frac{1}{n} \int_0^n P_h(i)^{1-\lambda} di \right]^{1/1-\lambda} \qquad P_f = \left[\frac{1}{(1-n)} \int_n^1 P_h(i)^{1-\lambda} di \right]^{1/1-\lambda} \tag{3.37}$$

We note that optimal behaviour of households is dictated by the following equations:

$$d_t = E_t \beta \frac{P_t C_t^{\rho}}{P_{t+1} C^{\rho}_{t+1}}, \tag{3.38}$$

$$\frac{M_t}{P_t} = \frac{\chi C_t^{\rho}}{(1 - E_t q_t)} \tag{3.39}$$

where q_t is the discount factor.

The home country household budget constraint is given by:

$$P_t C_t + d_t B_{t+1} + M_t = W_t L_t + \Pi_t + \Pi^f{}_t + M_{t-1} + T_t + B_t, \tag{3.40}$$

where d_t is the price of bonds, B_t is the number of domestic currency denominated bonds in the hands of home country household, Π_t denotes profit income from domestic firms, and $\Pi^f{}_t$ income from foreign exchange dealers, T_t are government transfers.

In this model, firms set prices to equal marginal costs:

$$p_{ht} = E_{t-1}w_t \qquad p^*_{ht} = E_{t-1}(w_t - s_t), \tag{3.41}$$

The home country goods market clearing condition is given by the following relation:

$$L_t = n\left(\frac{P_{ht}}{P_t}\right)^{-\omega} C_t + (1-n)\theta\left(\frac{P_{ht}{}^{P*}}{P^*_{ht}}\right)^{-\lambda}\left(\frac{P^*_{ht}}{P^*_t}\right)^{-\omega} C^*_t$$

$$+(1-n)(1-\theta)\left(\frac{P_{ht}{}^{D*}}{P^*_{ht}}\right)^{-\lambda}\left(\frac{P^*_{ht}}{P^*_t}\right)^{-\omega} C^*_t \tag{3.42}$$

where θ is a proportion of home country firms selling directly to foreign households and $1-n$ is the of home firms who distribute foreign products.

Other details are as follows:

Incomplete goods market and local distribution

- Foreign firms are owned by foreign-owners and local firms by the locals. In each country there are producers and distributors. Producers sell directly to the local residents. When the producers market their products to foreign market, they have the option of either selling directly or relying on foreign-owned distributors. In the case the home producer sells directly to foreign households, the prices are set in foreign currency. When trade takes place through foreign-owned distributors, the pricing is in home currency, making the distributor the absorber of the exchange-rate risk because it buys at prices set in the home currency, but it sets prices for foreign consumers in foreign currency.
- The authors avoid using the PPP relation because the “expenditure-switching" effect of exchange rate changes will lead to substitution between domestically-produced goods and internationally-produced goods, leading to the conclusion that that the exchange rate volatility could be transferred to macroeconomic fundamentals. They instead eliminate any expenditure-

switching role for exchange rates to highlight the role of the contribution of local-currency pricing to exchange-rate volatility.

- Production firms operate as monopolists and set prices in advance to maximize expected discounted profits. The authors assume that distributors sign binding contracts in advance to distribute the composite good.

Noise trading

- At home the foreign exchange dealers buy or sell foreign-currency denominated bonds to maximize the discounted expected returns. The authors assume that foreign exchange dealers exhibit bias in their conditional forecasts of the future exchange rate, making them noise traders. This suggests the following representation of conditionally biased expectations:

$$E_t^n s_{t+1} = E_t s_{t+1} + u_t , \tag{3.43}$$

such that $\text{var}_t^{\ n}(s_{t+1}) = Var_t(s_{t+1})$ and the conditional expectation of the random error u_t is $E_{t-1}(u_t) = 0$.

- Foreign exchange dealers are assumed to form accurate expectations of the households state contingent discount factor q_t. In addition, there is the assumption that new foreign exchange dealers continue to exhibit biased expectations, driving the expected returns to zero. This suggests that

$$d_t^* = E_t^{\ n} \frac{q_t S_{t+1}}{S_t} . \tag{3.44}$$

Solution of the model

- The authors utilise log-linearisation to solve for the unanticipated movement in the exchange rate as:

$$\hat{s}_t = \frac{(1+\frac{\sigma}{r})(\hat{m}_t - \hat{m}_t^{\ *}) + \frac{\sigma}{r} u_t}{\frac{\sigma}{r} + \rho(\theta - (1-\theta^*))} , \tag{3.45}$$

where the variables with hats are of the form: $\hat{s}_t = s_t - E_{t-1}s_t$. The results derive from a relationship between the consumption differential and the initial net foreign asset condition:

$$E_{t-1}(c_t - c_t^{*}) = \frac{1-\beta}{\sigma}\frac{dB^{*}{}_{ht}}{(1-n)PC}, \qquad (3.46)$$

where $\sigma \equiv (1 - \frac{(1-\omega)\rho}{(1+\psi\omega)}$.

The conditional variance of the exchange rate is given by:

$$Var_{t-1}(\hat{s}_t) = \frac{(1+\frac{\sigma}{r})^2 \operatorname{var}_{t-1}(\hat{m}_t - \hat{m}_t^{*})}{\left[\frac{\sigma}{r} + \rho(\theta - (1-\theta^{*}))\right]\left[1 - \left\{\frac{\kappa\sigma}{r\left[\frac{\sigma}{r} + \rho(\theta - (1-\theta^{*}))\right]}\right\}^2\right]} \qquad (3.47)$$

- In this setting, the volatility of the conditional bias in noise traders' expectations is generated by exchange rate volatility, which depends only on the volatility in relative money supplies. We note that when $\theta + \theta^{*} \rightarrow 1$ the conditional volatility of the exchange rate rises without bound, with no associated unbounded volatility in the fundamentals/money supplies.

Stochastic deviations from uncovered interest parity are obtained from the log-linearization of equations (3.43), (3.44) and (3.46). The result is:

$$\rho E_t(c_{t+1} - c_t) + E_t(p_{t+1} - p_t) =$$
$$\rho E_t(c^{*}{}_{t+1} - c^{*}{}_t) + E_t(p^{*}{}_{t+1} - p^{*}{}_t) + E_t s_{t+1} - s_t + v_t \qquad (3.48)$$

Equation (3.48) shows that the presence of conditionally biased expectations of future exchange rate introduces a stochastic deviation from uncovered interest rate parity.

As it is clear from the above information, Deveroux and Engel combine local currency pricing, asymmetric marketing, and the presence of noise-trading liquidity premiums in foreign exchange markets to show the ‘disconnect’ between exchange rates and fundamentals. The final conclusion is that the “combined presence of local currency pricing, asymmetric marketing, and `noise-trader’ conditionally-biased expectations in foreign exchange markets generates the possibility for a degree of short-term exchange rate volatility that is completely out of proportion to all shocks impacting on the economy.”

3.7.2 **The Xu (2005) model**

Xu studied under Deveroux and her model is not that different in structure from that of Deveroux and Engel (2002). Xu (2005) develops a welfare-based model which can explain exchange rate volatility and its relationship with macroeconomic fundamentals and provides a well-defined framework for policy evaluations regarding policies that are designed to control non-fundamental exchange rate volatility.

As explained above, the Deveroux-Engel model included, among other components, a well-defined structure of international pricing and product distribution to minimize the wealth effect of exchange rate changes, incomplete international financial markets for asymmetric risk sharing, and stochastic deviations from the uncovered interest parity. Xu (2005), in addition to these components, puts more emphasis on the micro-structural aspects of noise trading. In this setting, noise traders and rational traders are assumed to be risk-averse, utility-maximising agents, allowing for the analysis of Tobin tax ─ an international transaction tax on the purchases and sales of foreign exchange ─ to appraise the feasibility of reducing non-fundamental exchange rate volatility.

Rational traders and noise traders

Xu models traders as overlapping generations of investors who decide how many one-period foreign nominal bonds to buy in the first period of their lives. Traders who are able to form accurate expectations on risk and returns are called rational traders, and those with inaccurate expectations about future returns are called noise traders. The informed trader is denoted by a superscript I and the noise trader is denoted by a superscript N .

There are two specifications of the model. In the first case the number of incumbent noise traders is exogenously determined, while in the second specification the traders have to pay a fixed entry cost to trade on the foreign exchange market, making it possible to endogenise the noise component of the market.

To trade in the foreign exchange market, traders face entry costs such as tax, information costs for investment in the foreign bond market, and other costs when investing abroad. Rational traders are assumed to have a superior knowledge of the economy, enabling them to minimise the cost of acquiring information to zero. Noise traders, by contrast, have to pay an entry cost that is greater than zero because they are assumed to have a limited innate ability to acquire and process the information about the economy.

Additional details in Xu (2005)

The following are the main results:

- The consumption-based interest parity condition is of the form:

$$E_t(c_{t+1} - c^*_{t+1}) = (c_t - c^*_t) - \frac{1}{\rho}[s_t - (1 - N_I)v_t ,$$
$$+ \frac{a(1+r)S}{P} \text{var}_t(s_{t+1}) dB^*_{h,t+1} \quad (3.49)$$

where $(1 - N_I)$ is the number of noise traders.

- The deviation of the exchange rate from expectations depends on the expectation error of the noise traders. The exchange rate equation for the exogenous entry by traders is of the following form:

$$\hat{s}_t = E_t(\hat{s}_{t+1}) - \beta(dr_{t+1} - dr^*_{t+1}) + (1 - N_I)v_t - a\frac{(1+\bar{r})\bar{S}}{\bar{P}} \text{var}_t(s_{t+1}) dB^*_{h,t+1} \quad (3.50)$$

For the endogenous trade, the equation becomes:

$$\hat{s}_t = E_t(\hat{s}_{t+1}) - \beta(dr_{t+1} - dr^*{}_{t+1}) + \frac{1}{N_I} n_t v_t - a\frac{(1+\bar{r})\bar{S}}{\bar{P}N_I}\operatorname{var}_t(\hat{s}_{t+1})dB^*{}_{h,t+1} \qquad (3.51)$$

where $n_t = \frac{E_t(\hat{s}_{t+1}) - s_t - \beta(dr_{t+1} - dr^*{}_{t+1})}{2a\operatorname{var}t(s_{t+1})}\left(\frac{(1-N_I)}{\bar{c}}\right)$ (3.52)

is the number of incumbent noise traders.

When Tobix tax, denoted τ, is imposed, for the exogenous case the exchange rate equation takes the form:

$$\hat{s}_t = E_t(\hat{s}_{t+1}) - \beta(dr_{t+1} - dr^*{}_{t+1}) + (1-N_I)v_t$$
$$- \frac{P\tau dB^*{}_{h,t+1}}{\bar{S}(1+\bar{r})} - a\frac{(1+\bar{r})\bar{S}}{\bar{P}}\operatorname{var}_t(s_{t+1})dB^*{}_{h,t+1} \qquad (3.53)$$

For the endogenous case the exchange rate equation takes the form:

$$\hat{s}_t = E_t(\hat{s}_{t+1}) - \beta(dr_{t+1} - dr^*{}_{t+1}) + \frac{1}{N_I} n_t v_t$$
$$- \frac{\bar{P}\tau B^*{}_{h,T+1}}{N_I\bar{S}(1+r)} - a\frac{(1+\bar{r})\bar{S}}{\bar{P}N_I}\operatorname{var}_t(\hat{s}_{t+1})dB^*{}_{h,t+1.} \qquad (3.54)$$

3.7.3 The Duarte and Stockman (2005) model

The second sub-strand of research related to theoretical explanations does away with the notion of the purchasing power parity but retains the covered interest parity condition. This work is associated with Duarte and Stockman (2005). The authors focus on the effects of rational speculation in the foreign exchange markets. They argue that as new information comes becomes public, the risk premia associated with exchange rates adjust in such a way that the changes take place in asset markets but not in the goods market. The premise is that international market segmentation coupled with incomplete risk sharing can invalidate the fundamental equilibrating condition, namely, the equality between relative prices and the marginal rate of substitution. This break-down of the link between product markets and foreign exchange market allows the asset markets to determine the changes such that expectations and premia change the exchange rates without changing the fundamental variables such as GDP growth rates.

The Duarte-Stockman model is a stochastic general equilibrium model that can be summarised as follows:

- Basic assumptions: there are two countries ─ called home and foreign. They specialise in the production of a composite good. There are segmented markets, with monopolistically competitive firms in each country. These firms set prices one period in advance in the currency of the buyer. Asset markets are incomplete and restrict the households to trade a risk-free, “no-Ponzi-game” discount nominal bond denominated in home currency and a risk-free nominal bond denominated in foreign currency.
- Households: the expected utility function of a representative household depends on consumption, labour effort, and real money balances. There is a continuum of domestic and foreign goods, which are imperfect substitutes.
- Budget constraints: The intertemporal budget constraint depends on the real transfers from government, profits of domestic firms, and nominal labour earnings.
- The risk premium at time t is defined as the covariance of expected exchange rate at period t+1, denoted e_{t+1}, and the nominal marginal utility of consumption of the home household λ:

$$rp_t = \frac{\text{cov}(e_{t+1}, \lambda_{t+1})}{E_t(\lambda_{t+1})}. \tag{3.55}$$

- The main exchange rate equation is given:

$$e_t = \frac{\lambda_t^* E_t(\lambda_{t+1})}{\lambda_t E_t(\lambda^*_{t+1})}(rp_t + E_t[e_{t+1}]), \quad (3.56)$$

where λ_t^* represents the nominal marginal utility of consumption of the foreign household. The equation shows that the exchange rate depends on the risk premium of holding bonds.

Duarte and Stockman utilise home representative household intertemporal budget constraint of the following form:

$$B_1 + \varphi_1 + Q\varphi_2 = 0,$$

such that $\varphi_t = P_t w_t l_t + m_{t-1} + \Pi_t + P_t T_t - M_t - P_t c_t$. The variables are described as follows:

P_t is the price index

B_1 is the price of a bond at time 1

c_t is the consumption index

M_t nominal balances

Π_t denotes profits of domestic firms

T_t represents transfers from the domestic government

$P_t w_t l_t$ denotes nominal labour wages.

Analogous conditions hold for the foreign country. The exchange rate equation is approximated by

$$e_1 = \Theta e_2$$

for some parameter Θ, the increase of which would signal a rise in the risk premium associated with holding a home-currency denominated bond.

When the exchange rate equation is solved by incorporating the foreign budget constraint, the final results is as follows:

$$e_2 = \frac{-(\varphi_1 + Q\varphi_2)}{\Theta\varphi^*_1 + Q\varphi^*_2}, \quad (3.57)$$

$$e_1 = \frac{-\Theta(\varphi_1 + Q\varphi_2)}{\Theta\varphi^*{}_1 + Q\varphi^*{}_2}. \qquad (3.58)$$

From the above equations, we note that a rise in the risk premium affects the exchange rate in both periods: the exchange rate rises in the first period and declines in the second period. "If the home country is a net international creditor at the beginning of the first period, …the extent to which an increase in Θ reduces the *future* exchange rate is proportional to the share of initial debt that the foreign country repays in he first period… so that the *current* exchange rate depends inversely on that share."

3.7.4 The Evans and Lyons (2005) model

Rather than make an effort to empirically link exchange rates directly to macro variables, Evans and Lyons (2005) attempt to describe the microeconomic mechanism by which information concerning macro variances is impounded in exchange rates by the market. They approach the problem through the present value relation in which the log spot exchange rate is expressed as the sum of the present value of measured fundamentals and the present value of unmeasured fundamentals.

Additional details unique to the model:

Financial intermediaries

Evans and Lyons provide a more realistic structure of financial markets. There are dealers who act as intermediaries in four financial markets: the home money markets and bond markets; the foreign money markets and bond markets. In this setting, dealers quote prices at which they stand ready to buy or sell securities to households and other dealers. They also have the opportunity to initiate transactions with other dealers at the prices they quote. In essence the behaviour of the exchange rates and interest rates is determined by the securities prices dealers choose to quote. An equilibrium in this setting is described by a set of dealer quotes for the prices of bonds and foreign currency, and consumer prices set by firms that clear markets, given the consumption and portfolio choices of households and dealers; and a set of consumption and portfolio rules that maximize expected utility of households and dealers, given the prices of bonds, foreign currency and consumer goods. It is to be noted that dealers quote bond prices without precise knowledge of household consumption plans, so the actual currency orders they receive may differ from what was initially planned. Usually dealers can offset the effects of any unexpected currency orders by trading with central banks, so they hardly find themselves with unwanted currency balances at the end of trading in each period.

Order flow

In this model, order flow depends upon the portfolio allocation decisions of domestic and foreign households, the level and international distribution of household wealth and the outstanding stock of foreign bonds held by dealers from last period. These elements suggest that order flow contains both backward-looking and forward-looking components. In particular, there will be positive order flow for foreign bonds if households are more optimistic about the future value of the exchange rate than home dealers.

Transaction flows and fundamentals

In the Evans and Lyons (2005) model spot rates are determined by dealer expectations regarding fundamentals, while order flow reflects the differences between household and dealer expectations regarding future spot rates.

The authors point out that if households have more information about the future course of fundamentals than dealers, and dealers are expected to assimilate at least some of this information from transactions flows each period, than order flow will be correlated with variations in the forecast differentials for fundamentals.

They point out that the household orders driving order flow are adjusted solely by the desire to optimally adjust portfolios. Households have no desire to inform dealers about the future state of the economy, so the information conveyed to dealers via transaction flows occur as a by-product of their dynamic portfolio allocation decisions. "The transactions flows associated with these decisions establish the link between order flow, dispersed information, and the speed of information…."

Data

The authors utilise a new data set that comprises end-user transaction flows, spot rates and macro fundamentals over six and a half years. By end users the authors refer to three main segments: non-financial corporations, institutional investors, and leveraged traders such as hedge funds. Empirical analysis also utilises new high-frequency real-time estimates of macro fundamentals for the US and Germany: specifically GDP growth, CPI inflation, and M1 money growth. 'Real time' implies the estimates corresponding to actual macroeconomic data available at any given time.

The main results

The main results are as follows:

- Order flows forecast future macro variables such as output growth, money growth, and inflation better than spot rates do.
- Order flows forecast future spot rates.
- Order flows appear to be the main driver in the process by which expectations of future macro variables are impounded into exchange rates.

3.7.5 **The Bacchetta and van Wincoop (2006) model**

Bacchetta and van Wincoop (2006) present a dynamic general equilibrium model that is premised on the heterogeneity of information in a monetary model of exchange rate determination, which consists of money market equilibrium, purchasing power parity, and an interest rate arbitrage equation. In this context, a continuum of investors has symmetrically dispersed information about future macroeconomic fundamentals but face different exchange rate risk exposure. To mitigate risk, investors rely on hedge trades. A unique characteristic of the Bacchetta-van Wincoop model is that order flow is modelled explicitly in a general equilibrium setup. Also, equilibrium is a result of auction market driven by orders.

The model can be summarised by the following equations:

$$p_t = p^*_t + s_t, \tag{3.59}$$

where s_t is the log of the nominal exchange rate, and p_t and p_t^* are the logs of domestic and foreign prices. Thus equation (8.30) represents the purchasing power parity relation. The money demand equation of the form

$$m_t - p_t = -\alpha i_t$$
$$m*_t - p*_t = -\alpha i^*_t \tag{3.60}$$

where m_t and m_t^* are the domestic and foreign money supplies in logs.
The demand for foreign bonds takes the form:

$$b^i_{Ft} = \frac{E^i_t(s_{t+1}) - s_t + i*_t - i_t}{\gamma\sigma^2_t} - b^i_{it} \tag{3.61}$$

where i_t^* and i_t are foreign and domestic interest rates, and σ_t^2 is the conditional variance of s_{t+1}. Market equilibrium leads to the following interest rate arbitrage condition

$$\overline{E}_t(s_{t+1}) - s_t = i_t - i^*_t + \gamma b_t \sigma^2_t, \tag{3.62}$$

where the average expectation of individual investors is denoted $\overline{E}_t$. The observable fundamental is defined as a money supply differential $f_t = m_t - m^*_t$. The authors derive the following equilibrium exchange rate under higher order expectations:

$$s_t = \frac{1}{1+\alpha} \sum_{k=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^k \overline{E}_t^{\,k} (f_{t+k} - \alpha\gamma\sigma^2{}_{t+k} b_{t+k}) \tag{3.63}$$

where $\overline{E}^0{}_t (x_t) = x_t$, $\overline{E}^1{}_t (x_{t+1}) = \overline{E}_t (x_{t+1})$ and higher-order expectations are of the form:

$$\overline{E}^k{}_t (x_{t+k}) = \overline{E}_t \overline{E}_{t+1} \dots \ \overline{E}_{t+k-1}\, x_{t+k} \, . \tag{3.64}$$

Information structure

The information structure can be that of a common knowledge or heterogeneous information. In the context of common knowledge, a common signal is of the form, $v_t = f_{t+T} + \varepsilon^v{}_t$. In the model heterogeneous investors receive one signal about fundamentals. In this context, let i denote an investor. Then the signal is of the following form $v^{it} = f_{t+T} + \varepsilon^{vi}{}_t$ such that $\varepsilon^{vi}{}_t \sim N(0, \sigma^2{}_v)$ and $\varepsilon^{vi}{}_t \perp f_{t+T}$. Define $\beta^v \equiv 1/\sigma^2{}_v$ and let $D = \beta^v + \beta^f + \beta^s$. The authors conjecture that the equilibrium exchange rate is of the form:

$$s_t = (1+\alpha)^{-1} f_t + \lambda_f f_{t+1} + \lambda_b b_t \, . \tag{3.65}$$

From the signal takes the form:

$$\frac{\tilde{s}_t}{\lambda_f} = f_{t+1} + \frac{\lambda_b}{\lambda_f} b_t \, , \tag{3.66}$$

where $\tilde{s}_t = s_t - (1+\alpha)^{-1} f_t$, with the variance of the error being $(\lambda_b / \lambda_f)^2 \sigma_b{}^2$.

The equilibrium exchange rate is

$$s_t = (1+\alpha)^{-1} f_t + z\alpha(1+\alpha)^{-2} \frac{\beta^v}{D} f_{t+1} - z\alpha(1+\alpha)^{-1} \gamma\sigma^2 b_t \tag{3.67}$$

where the magnification factor is defined as

$$z = 1/(1-\alpha(1+\alpha)^{-2} (\beta^s / \lambda_f D) > 1 \tag{3.68}$$

Order flow

In the model there is a simple relationship between order flow and the exchange rate. For instance, aggregate order flow is defined as $\Delta x_t = \frac{\beta^v}{(1+\alpha)\gamma\sigma^2 D} f_{t+1} - b_t$ and equilibrium exchange rate is a function of order flow and an observable fundamental:

$$s_t = \frac{1}{1+\alpha} f_t + z \frac{\alpha}{1+\alpha} \gamma\sigma^2 \Delta x_t . \qquad (3.69)$$

As pointed out by the authors, the main implications of the above model are that in the short run, investor confusion leads to the disconnection of the exchange rate from observed fundamentals. At that point, investors do not know whether future fundamentals or an increase in hedge trades drive exchange rate changes. "This implies that unobserved hedge trades have an amplified effect on the exchange rate since they are confused with changes in average private signals about future fundamentals."

Model dynamics and numerical analysis

Bacchetta and van Wincoop make the following observations regarding the dynamics of the model:

- Transitory nonobservable shocks have a persistent effect on the exchange rate, due to the learning behaviour of investors.
- Hedge shocks are further magnified by the presence of higher-order expectations, but the overall impact on the connection between the exchange rate and observed fundamentals is ambiguous.
- In the common knowledge model, 1.3 per cent of the variance of a one-period change in the exchange rate is driven by the unobservable hedge trades, while in the heterogeneous model it is 70 per cent. In the short run

unobservable factors dominate exchange rate volatility, but in the long-run it is the observable fundamentals that dominate.

- At a one-period horizon 84 per cent of the variance of one-period exchange rate changes can be accounted for by order flow as opposed to public information.

3.8 Critical assessment of the models and conclusions

What has been central to the above models is the respective role of expectations, fundamental and nonfundamental factors such as risk premia and order flows. In the case of Deveroux and Engel, local currency pricing, asymmetric marketing, as well as rational and noise trading, play an important part in creating a disconnect between fundamentals and exchange rate movements. To the extend that reliable short run determinants of exchange rate movements can be established, it would appear that the Evans and Lyons model and Bacchetta and van Wincoop models are the front runners in the arena of general equilibrium models. Evans and Lyons and Bacchetta and van Wincoop have established that order flows play an important role in short run exchange rate dynamics.

It is therefore our judgement that Bacchetta-van Wincoop and Evans-Lyons models can explain the exchange rate determination puzzle.

The relevance of this chapter in relation to the rest of the current study is that it highlights the likely trajectories of future research. The Bacchetta-van Wincoop and Evans-Lyons models are seen as suitable for future research in that they can both explain the exchange rate determination puzzle and also provide meaningful insights in respect of the reliable determinants of exchange rates. In short, these models constitute a theoretical and empirical bridge for at least two strands of research in exchange rate economics. Moreover, the fact there exists a relationship among order flow, spot rates and fundamentals implies that short-term forecasting is likely to be reliable in the context of policy and corporate foreign exchange related strategies.

3.9 Recent developments: Exchange rate determination puzzle

The current literature in respect of the exchange rate determination puzzle attempts to find reliable determinants of exchange rates in the short run. Market microstructure theory, in particular, attempts to explain exchange rate determination by paying to order flow — the difference between the buyer-initiated and seller-initiated orders in a securities market. In particular, Evans and Lyons (2005) argue that order flow might be able to anticipate future exchange rate movements. Other variables taken into account are interest rate differentials.

The market microstructure approach is discussed in Chapter 7, where we discuss the short-run and long-run dynamics in respect of the determinants of exchange rates.