Chapter 4

A New-Keynesian DSGE Model for Forecasting the South African Economy

4.1 Introduction

The objective of this paper is to develop a New-Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) Model for forecasting growth rate of output, inflation, and a measure of nominal short-term interest rate, in our case the 91-days Treasury Bills rate, for South African economy. The model is estimated via maximum likelihood technique for quarterly data over the period of 1970:1-2000:4. Based on a recursive estimation using the Kalman filter algorithm, the out-of-sample forecasts from the NKDSGE model are then compared with the same generated from the Classical and Bayesian variants of the VAR models for the period 2001:1-2006:4.

During the last three decades, lot of work has gone into developing well-structured New-Keynesian-Macroeconomic (NKM) models in response to criticisms on the traditional, once-dominant, IS-LM framework of macroeconomic analysis. The NKM models incorporate the nominal (price and/or wage) rigidities into the traditional IS-LM framework to capture the time series properties of the data. More recently, the so called new generation NKM models (Goodfriend and King, 1997; Rotemberg and Woodford, 1997; McCallum and Nelson, 1999, Smets and Wouters, 2003) that are built on a dynamic stochastic general equilibrium framework, based on optimizing behavior of agents, has also gained tremendous prominence. However, this type of micro-founded NKM models have generally been used for policy analysis, few being used for forecasting purposes. One exception in this regard is the study by Smets and Wouters (2004). The authors develop and estimate a micro-founded NKM model with sticky prices and wages for the Euro area. The results indicate that the forecasting performance of NKM model is reasonably well comparable to the atheoretical VAR.

In a recent paper, Liu *et al.* (2007) develop and estimate a Hansen(1985)–type hybrid model for forecasting the South African economy. The hybrid model is based on a real business cycle (RBC) framework. Kydland and Prescott (1982) argue that in the basic RBC framework, the U.S. business cycle fluctuations are purely driven by real technology shocks. This one-shock assumption makes RBC models stochastically singular. In order to overcome this singularity problem, the authors augment the theoretical model with unobservable errors having a

VAR representation. This allows one to combine the theoretical rigor of a DSGE model with the flexibility of an atheoretical VAR model. The results indicate that the estimated hybrid DSGE model outperforms the Classical VAR, but not the Bayesian VARs in terms of out-of-sample forecasting performances. Having resorbed to a RBC framework, prevents Liu *et al.* (2007) from analyzing the role of nominal shocks. This is, in our opinion, inappropriate for the South African economy, since South African economy, just as other developing economies, is subject to nominal shocks.

In this paper, following Rotemberg and Woodford (1997) and Ireland (2004), we develop and estimate a NKDSGE model with sticky prices. The model consists of three equations, an expectational IS curve, a forward-looking version of the Phillips curve, and a Taylor-type monetary policy rule. Furthermore, the model is characterized by four shocks: a preference shock; a technology shock; a cost-push shock; and a monetary policy shock. Essentially, by incorporating four shocks, that generally tends to affect a macroeconomy, we attempt to model the empirical stochastics and dynamics in the data better, and hence, improve the predictions. In addition, using a NKDSGE model, allows us to model product market rigidities, which is also an important feature of the South African economy. Further allowing for explicit interest rate rules also helps in modelling the inflation targeting frame regime of the South African economy, understanding better in comparison to the RBC model for obvious reason.

The rest of the paper is structured as follows. Section 2 lays out the theoretical model, while Section 3 shows the solution of the model. Results are presented in Section 4 and Section 5 concludes.

4.2 The Model

4.2.1 The Representative Household

The economy consists of a continuum of infinitely-lived households. In each period $t = 0, 1, 2, ...$, a representative household makes a sequence of decisions to maximize the expected utility over a composite consumption good C_t, real money balance M_t/P_t, and leisure $1 - h_t$:

$$E\sum_{t=0}^{\infty}\beta^t\big[a_t log(C_t) + log(\frac{M_t}{P_t}) - (\frac{1}{\eta})h_t^{\eta}\big], \qquad 0 < \beta < 1, \qquad \eta \geq 1, \tag{1}$$

where β is the subjective discount factor and a_t is the preference shock which follows an AR(1) process as in Ireland (2004):

$$log(a_t) = \rho_a log(a_{t-1}) + \varepsilon_{at}, \qquad 0 \leq \rho_a < 1, \qquad \varepsilon_{at} \sim i.i.d.(0, \sigma_a^2), \tag{2}$$

The representative household carries money M_{t-1} and bonds B_{t-1} from the previous period into the current period t. In time period t, the household receives a lump-sum transfer T_t from the monetary authority and the nominal profit or dividend payment D_t from the intermediate good firms. In addition, the household also receives its usual labor income $W_t h_t$, where W_t denotes the nominal wage. Therefore, in each time period the representative household maximizes its expected utility (1) by choosing consumption, labor supply, money and bond, subject to the following budget constraint:

$$C_t + \frac{B_t}{r_t P_t} = \frac{W_t}{P_t} h_t + \frac{B_{t-1}}{P_t} + D_t + T_t - \frac{M_t - M_{t-1}}{P_t} \tag{3}$$

where r_t denotes the gross nominal interest rate and P_t denotes the nominal price.

In this version of NKDSGE model, capital accumulation decision is ignored. Christiano *et al.* (2005) assume that the household owns capital stock and makes capital accumulation and utilization decisions in each time period[1] . However, as noted that there exists little relationship between capital stock and output at business cycle frequencies (McCallum and Nelson, 1999; Cogley and Nason, 1995), the role of capital has been ignored here.

Given (1) and (2), the representative household's first order conditions are as follows:

$$\begin{aligned}
\frac{W_t}{P_t} &= a_t^{-1} C_t h_t^{\eta-1} && (4)\\
\frac{a_t}{C_t} &= r_t \beta E_t \Big[a_{t+1} \Big(\frac{1}{C_{t+1}} \Big) \Big(\frac{P_t}{P_{t+1}} \Big) \Big] && (5)\\
\frac{M_t}{P_t} &= a_t^{-1} C_t [r_t/(r_t - 1)] && (6)
\end{aligned}$$

[1]For further details on capital accumulation and utilization in NKDSGE models see Dostey and King (2001), and Smets and Wouters (2003) .

where (4) is the intratemporal optimality condition, capturing the consumption and leisure trade-off, i.e. the marginal rate of substitution between consumption and leisure equals to the real wage. Equation (5) represents the intertemporal allocation of consumption, whereas (6) is the money demand equation. It shows that the optimal condition of money holding requires that the marginal rate of substitution between money and consumption must equalize with the opportunity cost of holding money.

4.2.2 Final-Goods Production

In the final-goods sector, a representative firm produces the final good Y_t according to a constant elasticity of substitution (CES) production function as suggested by Dixit and Stiglitz (1977):

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta_t - 1}{\theta_t}} \, dj \right)^{\frac{\theta_t}{\theta_t - 1}}, \qquad \theta_t > 1, \tag{7}$$

where Y_{jt} denotes the output of intermediate good j which the representative final-goods firm uses as input to produce Y_t units of final goods.

Given the intermediate-goods price P_{jt},[2] firm maximizes its profits:

$$\max_{Y_{jt}} \left\{ Y_t - \frac{1}{P_t} \int_0^1 P_{jt} Y_{jt} \right\} \tag{8}$$

Solving the firm's profits maximization problem (8), yields:

[2] As explained in section 2.3, the representative intermediate-goods firm is assumed to sell its output in a monopolistically competitive market.

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta_t} Y_t \tag{9}$$

Since the final-goods firms operate in a perfectly competition, in equilibrium the representative firm's profit should equal to zero. Hence, the equilibrium market price for final good is given as follows:

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta_t} dj\right)^{\frac{1}{1-\theta_t}} \tag{10}$$

It is important to point out that the production function (7) implies a constant elasticity of substitution between intermediate goods. θ_t is a stochastic parameter determining the time-varying mark-up in the goods market (Smets and Wouters, 2003; Ireland, 2004). This is a convenient way to introduce the so called mark-up or cost-push shocks into the NKDSGE model as proposed by Clarida *et al.* (1999). The cost-push shock follows the following autoregressive process:

$$log\theta_t = (1-\rho_\theta)log\theta + \rho_\theta log\theta_{t-1} + \varepsilon_{\theta t}, \qquad 0 \leq \rho_\theta < 1, \qquad \varepsilon_{\theta t} \sim i.i.d.(0, \sigma_\theta^2), \tag{11}$$

where the serially uncorrelated innovation $\varepsilon_{\theta t}$ is normally distributed.

4.2.3 Intermediate-Goods Production

In the intermediate-goods sector, firms are monopolistically competitive and face a quadratic cost of price adjustment. In each time period, the representative intermediate-goods firm hires h_{jt} units of labor and produces Y_{jt} units of intermediate good j, according to the following technology:

$$Y_{jt} = Z_t h_{jt} \tag{12}$$

This is a standard constant-return-to-scale production function, but without capital. Z_t is the aggregate technology shock, which is assumed follow a random walk with a positive drift:

$$logZ_t = log\bar{Z} + logZ_{t-1} + \varepsilon_{zt}, \qquad \varepsilon_{zt} \sim i.i.d.(0, \sigma_z^2), \tag{13}$$

where $\bar{Z} > 1$ and the serially uncorrelated innovation ε_{zt} is normally distributed. In equilibrium, this supply-side disturbance acts as a shock to the Phillips curve in the NKDSGE model (Ireland, 2001).

As stated above, the representative intermediate-goods firm faces a quadratic cost of nominal price adjustment along the line of Rotemberg (1982). Mathematically, we have:

$$\frac{\phi}{2}\left[\frac{P_{jt}}{\pi P_{jt-1}}\right]^2 Y_t, \qquad \phi > 0, \qquad \pi > 1, \tag{14}$$

where ϕ is the parameter that governs the magnitude of the cost of price adjustment and π is the steady-state gross rate of inflation.

Since the representative intermediate-goods firm operates in a monopolistically competitive market, it chooses its own sale price P_{jt} taking as given a downward sloping demand curve in order to maximize its market value:

$$E\sum_{t=0}^{\infty}\beta^t(a_t/C_t)\Big\{\Big[\frac{P_{jt}}{P_t}\Big]^{1-\theta}Y_t-\Big[\frac{P_{jt}}{P_t}\Big]^{-\theta}\Big(\frac{W_t}{P_t}\Big)\Big(\frac{Y_t}{Z_t}\Big)-\frac{\phi}{2}\Big[\frac{P_{jt}}{\pi P_{jt-1}}\Big]^2Y_t\Big\} \tag{15}$$

where $\beta^t(a_t/C_t)$ measures the representative household's marginal utility of an additional unit of real profit generated in time period t. The first order condition is:

$$\begin{aligned}(\theta_t-1)\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta_t}\Big(\frac{Y_t}{P_t}\Big) &= \beta\phi E_t\Big[\Big(\frac{a_{t+1}}{a_t}\Big)\Big(\frac{C_t}{C_{t+1}}\Big)\Big(\frac{P_{jt+1}}{\pi P_{jt}}-1\Big)\Big(\frac{Y_{t+1}}{P_{jt}}\Big)\Big(\frac{P_{jt+1}}{\pi P_{jt}}\Big)\Big]\\ &+\Big[\theta_t\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta_t-1}\Big(\frac{W_t}{P_t}\Big)\Big(\frac{Y_t}{Z_t}\Big)\Big(\frac{1}{P_t}\Big)\Big]-\Big[\phi\Big(\frac{P_{jt}}{\pi P_{jt-1}}-1\Big)\Big(\frac{Y_t}{\pi P_{jt-1}}\Big)\Big]\end{aligned} \tag{16}$$

The representative intermediate-goods firm sets its markup price P_{jt} in such a way that the actual markup price will differ from, but tend to gravitate towards, the desired markup overtime (Ireland, 2004: 9).

4.2.4 The Monetary Authority

The model is closed by assuming that the monetary authority follows a modified Taylor (1993) rule. That is, the monetary authority adjusts its instrument, the nominal short-term interest rate, in response to deviations of inflation and output from their steady-state levels, as well as lagged deviations of interest rate and deviations of current growth rate.

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + \rho_\pi \hat{\pi}_t + \rho_g \hat{g}_t + \rho_x \hat{x}_t + \varepsilon_{rt}, \qquad \varepsilon_{rt} \sim i.i.d.(0, \sigma_r^2), \tag{17}$$

r_t is the nominal short-term interest rate, g_t output growth, and x_t output gap[3] . The ε_{rt}'s represent exogenous monetary policy shocks, which are assumed to be serially uncorrelated.

Monetary policy rules are often preferred over discretionary decisions. A formal rule is the desire for governance"by laws, not by means", as well as, the way to overcome"dynamic inconsistency" (Barro and Gordon, 1983; Rogoff, 1985). From a monetary transmission mechanism point of view, monetary policy affects the target variable(s) and the economy mainly through the private-sector expectations of the future interest rates, inflation, and output. Since growth rate of output is public knowledge, besides output gap, we include output growth in our interest rate rule as well. Moreover, output growth can be one of the most important and observable indicator, as apposed to the more elaborated output gap, that the monetary authority responds to.

The measure of output gap associated with NKM model differs from the empirical (statistical) approach. The empirical approach essentially involves de-trending output from its smooth trend. It requires using either a univariate technique like the Hodrick-Prescott filter or a multivariate technique like adapted

[3]A letter with a hat above indicates its deviation.

multivariate filter to determine the smooth trend – potential output [4] . However, the main properties of the resulting series, the potential output, do not seem to hinge critically on the exact techniques used. Moreover, the use of de-trended output as a proxy for the output gap has been criticized due to the lack of theoretical justification (Gali, 2002). Using a simple estimated linear model, Smets (1998) shows that output gap uncertainty can have a significant effect on the efficient response coefficients in Taylor-type rules for the US economy.

We define the output gap in the following way as proposed by Ireland (2004). Under the structure of our model, suppose there is a benevolent government that seeks to maximize the representative household's welfare:

$$E\sum_{t=0}^{\infty}\beta^t\Big[a_t logY_t - \frac{1}{\eta}\Big(\int_0^1 N_{jt}dj\Big)^{\eta}\Big] \tag{18}$$

that is, in each time period N_{jt} units of labor are allocated to the representative intermediate firm to produce Y_{jt} units of intermediate good j, which will then be used as input goods to produce Y_t units of final goods.

This optimization problem is subject to the following economy-wide constraint:

$$Y_t = Z_t\Big(\int_0^1 N_{jt}^{\frac{\theta_t-1}{\theta_t}}dj\Big)^{\frac{\theta_t}{\theta_t-1}} \tag{19}$$

[4] For detailed discussion of different techniques for computing potential output, see Nelson and Plosser (1972), Hodrick and Prescott (1997), Laxton and Tetlow (1992).

The first order condition implies that the optimal level of output in the final-goods sector is given by[5] :

$$Y_t = a_t^{\frac{1}{\eta}} Z_t \tag{20}$$

The model's output gap x_t is then defined by dividing the actual output by the optimal level of output:

$$x_t = \left(\frac{1}{a_t}\right)^{\frac{1}{\eta}} \frac{Y_t}{Z_t} \tag{21}$$

4.3 Solution of the Model

In equilibrium, markets must clear. A symmetric equilibrium is characterized by the following conditions: $Y_{jt} = Y_t$, $P_{jt} = P_t$, $h_{jt} = h_t$, for all $j \in [0,1]$ and $t = 0, 1, 2, \ldots$. In addition, market clearing conditions require $M_t = M_{t-1} + T_t$, $B_t = B_{t-1} = 0$.

These market clearing conditions imply that $Y_t = C_t$; households are homogeneous with respect to consumption and bond holdings (Woodford, 1996; Erceg et al., 2000); intermediate-goods firms are identical with respect to price and production decisions, and; money and asset markets are clearing for all $t = 0, 1, 2, \ldots$.

[5] It is clear that the optimal level of output responds positively to the preference shock a_t and the technology shock Z_t.

We then log-linearize the model around its steady-state. The log-linearized model contains two main equations of our NKDSGE model, the expectational IS curve (B.12) and the New Keynesian Phillips curve (B.13): [6]

$$\hat{x}_t = E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + \left(1 - \frac{1}{\eta}\right)(1-\rho_a)\hat{a}_t \qquad \text{((B.12))}$$

$$\hat{\pi} = \beta E_t\hat{\pi}_{t+1} + \psi\hat{x}_t - \hat{\theta}_t/\phi \qquad \text{((B.13))}$$

These two main equations (B.12) and (B.13) imply that in a NKDSGE model the presence of nominal rigidities (the cost-push shock $\hat{\theta}_t/\phi$ here) is a potential source of nontrivial real effects of monetary policy shocks (Gali, 2002). Without the cost-push shock, the monetary authority can simply set the real interest rate equal to its natural rate $\left(1 - \frac{1}{\eta}\right)(1-\rho_a)\hat{a}_t$ in order to stabilize both the inflation rate and the output gap.

To estimate the model, we apply the method proposed by Blanchard-Kahn (1980) to the log-linearized model. Specifically:

$$f_t = As_t \qquad (22)$$

and

$$s_{t+1} = Bs_t + C\varepsilon_{t+1} \qquad (23)$$

[6] Appendix B describes the symmetric equilibrium and the log-linearization of the model.

where

$$f_t = [\hat{g}_t,\ \hat{\pi}_t,\ \hat{r}_t]' \tag{24}$$

$$s_t = [\hat{y}_{t-1},\ \hat{\pi}_{t-1},\ \hat{r}_{t-1},\ \hat{x}_{t-1},\ \hat{g}_{t-1},\ \hat{a}_t,\ \hat{e}_t,\ \hat{z}_t,\ \hat{\varepsilon}_{rt}]' \tag{25}$$

$$\varepsilon_{t+1} = [\varepsilon_{at+1},\ \varepsilon_{et+1},\ \varepsilon_{zt+1},\ \varepsilon_{rt+1}]' \tag{26}$$

The empirical model consisting of (22) and (23) has three observable variables, output growth, inflation, and the nominal short-term interest rate, and two unobservable variables namely the de-trended output and the output gap. The model also consists of four different shocks, the preference shock $\hat{a}_t$, the cost-push shock[7] $\hat{e}_t$, the technology shock $\hat{z}_t$, and the monetary policy shock ε_{rt}. All the shocks are assumed to be serially uncorrelated. In other words, the covariance matrix of ε_{t+1} is diagonal:

$$E\varepsilon_{t+1}\varepsilon_{t+1}' = \begin{bmatrix} \sigma_a & 0 & 0 & 0 \\ 0 & \sigma_e & 0 & 0 \\ 0 & 0 & \sigma_z & 0 \\ 0 & 0 & 0 & \sigma_r \end{bmatrix} \tag{27}$$

The empirical model is in state-pace form and can be estimated via maximum likelihood approach. The model is estimated based on quarterly data on real Gross Domestic Product (GDP), GDP deflator, and 91-day Treasury Bills rate (TBILL) as the nominal short-term interest rate over the period of 1970:1-2000:4.

[7] $\hat{e}_t = \hat{\theta}_t/\phi$ is the transformed cost-push cost.

Before calculating the output (GDP) growth, GDP is converted into per-capita form by dividing it with the size of population aged between 15-64. The data for seasonally adjusted real GDP, GDP defletor, and the 91-days TBILL rate are obtained from the South African Reserve Bank Quarterly Bulletin. Note the base year is the year of 2000. Series for population aged between 15-64 is obtained from World Bank database.

4.4 Results

In this section, we compare the out-of-sample forecasting performance of the NKDSGE model with the VARs, both Classical and Bayesian, in terms of the Root Mean Squared Errors (RMSEs). At this stage, a few words need to be said regarding the choice of the evaluation criterion for the out-of-sample forecasts generated from Bayesian models. As Zellner (1986: 494) points out "the optimal Bayesian forecasts will differ depending upon the loss function employed and the form of predictive probability density function". In other words, Bayesian forecasts are sensitive to the choice of the measure used to evaluate the out-of-sample forecast errors. This fact was also observed in a recent study by Gupta (2006). However, Zellner (1986) points out that the use of the mean of the predictive probability density function for a series, is optimal relative to a squared error loss function and the Mean Squared Error (MSE), and, hence, the RMSE is an appropriate measure to evaluate performance of forecasts, when the mean of the predictive probability density function is used.

But, before we proceed to the discussion of the forecasting performance of the alternative models, it is important to lay out the basic structural differences and advantages of using BVARs over traditional VARs for forecasting.

4.4.1 Classical and Bayesian VARs

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

$$\chi_t = C + \lambda(L)\chi_t + \varepsilon_t \tag{28}$$

where χ is a $(n \times 1)$ vector of variables being forecasted; $\lambda(L)$ is a $(n \times n)$ polynominal matrix in the backshift operator L with lag lenth p, i.e., $\lambda(L) = \lambda_1 L + \lambda_2 L^2 + ... + \lambda_p L^p$; C is a $(n \times 1)$ vector of constant terms; and ε is a $(n \times 1)$ vector of white-noise error terms. The VAR model, thus, posits a set of relationships between the past lagged values of all variables and the current value of each variable in the model.

A crucial drawback of the VAR forecasts is "overfitting" due to the inclusion too many lags and too many variables, some of which may be insignificant. The problem of "overfitting" results in multicollinearity and loss of degrees of freedom, leads to inefficient estimates and large out-of-sample forecasting errors. Thus, it can be argued the performance of VAR forecasts will deteriorate rapidly as the forecasting horizon becomes longer.

A forecaster can overcome this "overfitting" problem by using Bayesian techniques. The motivation for the Bayesian analysis is based on the knowledge that more recent values of a variable are more likely to contain useful information about its future movements than older values. From a Beyesian perspective, the exclusion restriction in the VAR is an inclusion of a coefficient without a prior probability distribution (Litterman, 1986a).

The Bayesian model proposed by Litterman (1981), Doan, *et al.* (1984), and Litterman (1986b), imposes restrictions on those coefficients by assuming they are more likely to be near zero. The restrictions are imposed by specifying normal prior[8] distributions with zero means and small standard deviations for all the coefficients with standard deviation decreasing as lag increases. One exception is that the mean of the first own lag of a variable is set equal to unity to reflect the assumption that own lags account for most of the variation of the given variable. To illustrate the Bayesian technique, suppose the "Minnesota prior" means and variances take the following form:

$$\begin{aligned} \beta_i &\sim N(1, \sigma^2_{\beta_i}) \\ \beta_j &\sim N(0, \sigma^2_{\beta_j}) \end{aligned} \tag{29}$$

where β_i represents the coefficients associated with the lagged dependent variables in each equation of the VAR, while β_j represents coefficients other than β_i. The

[8]Note Litterman (1981) uses a diffuse prior for the constant, which is popularly referred to as the "Minnesota prior" due to its development at the University of Minnesota and the Federal Reserve bank at Minneapolis.

prior variances $\sigma^2_{\beta_i}$ and $\sigma^2_{\beta_j}$, specify the uncertainty of the prior means, $\beta_i = 1$ and $\beta_j = 0$, respectively.

Doan *et al.* (1984) propose a formula to generate standard deviations as a function of a small number of *hyperparameters*[9] : w, d, and a weighting matrix $f(i,j)$. This approach allows the forecaster to specify individual prior variances for a large number of coefficients based on only a few *hyperparameters*. The specification of the standard deviation of the distribution of the prior imposed on variable j in equation i at lag m, for all i, j and m, defined as $S(i,j,m)$:

$$S(i,j,m) = [w \times g(m) \times f(i,j)]\frac{\hat{\sigma}_i}{\hat{\sigma}_j} \tag{30}$$

where:

$$f(i,j) = \begin{cases} 1 & \text{if } i = j \\ k_{ij} & \text{otherwise, } 0 \leq k_{ij} \leq 1 \end{cases}$$

$$g(m) = m^{-d}, \qquad d > 0$$

The term w is the measurement of standard deviation on the first own lag, which indicates the overall tightness. A decrease in the value of w results a tighter prior. The parameter $g(m)$ measures the tightness on lag m relative to lag 1, and is assumed to have a harmonic shape with a decay of d. An increasing in d, tightens

[9]The name of *hyperparameter* is to distinguish it from the estimated coefficients , the parameters of the model itself.

the prior as lag increases. [10] The parameter $f(i,j)$ represents the tightness of variable j in equation i relative to variable i. Reducing the interaction parameter k_{ij} tightens the prior. $\hat{\sigma}_i$ and $\hat{\sigma}_j$ are the estimated standard errors of the univariate autoregression for variable i and j respectively. In the case of $i \neq j$, the standard deviations of the coefficients on lags are not scale invariant (Litterman, 1986b: 30). The ratio, $\frac{\hat{\sigma}_i}{\hat{\sigma}_j}$ in (30), scales the variables so as to account for differences in the units of magnitudes of the variables.

The BVAR model is estimated using Theil's (1971) mixed estimation technique, which involves supplementing the data with prior information on the distribution of the coefficients. For each restriction imposed on the parameter estimated, the number of observations and degrees of freedom are increased by one in an artificial way. Therefore, the loss of degrees of freedom associated with the unrestricted VAR is not a concern in the BVAR.

4.4.2 Forecast accuracy

Table 13 to 15 report the RMSEs from the NKDSGE model along with the VARs. When compared to the VAR and BVAR, the NKDSGE model does a better job in predicting inflation than it does in predicting output growth and the nominal short-term interest rate (TBILL). To be more precise, for inflation, the NKDSGE model outperforms both the unrestricted VAR and the optimal

[10]In this paper, we set the overall tightness parameter (w) equal to 0.3, 0.2, and 0.1, and the harmonic lag decay parameter (d) equal to 0.5, 1, and 2. These parameter values are chosen so that they are consistent with the ones that used by Liu and Gupta (2007), and Liu *et al.* (2007).

Table 13: **RMSE (2001Q1-2006Q4): Output Growth**

QA	1	2	3	4	AVE
NKDSGE	0.726	0.787	0.888	0.961	0.840
VAR (1)	0.756	0.700	0.797	0.851	0.776
BVAR (w=.1, d=1)	0.633	0.701	0.797	0.863	**0.748**

QA: quarter ahead; RMSE: root mean squared error (%).

BVAR [11], while for output growth and TBILL the RMSEs generated from the NKDSGE model are larger than those generated from the unrestricted VAR and the BVAR.

As far as the forecasting performances of the BVARs are concerned, except for inflation, the optimal BVAR outperforms both the NKDSGE model and the unrestricted VAR. For inflation, the optimal BVAR only outperforms the unrestricted VAR. As shown in Table 13 to 15, for output growth and inflation a BVAR with a relatively tighter prior ($w = 0.1, d = 1$) produces smaller forecast errors, whereas for TBILL the opposite holds. Interestingly, this finding is different from Liu *et al.* (2007), in which a BVAR with a relatively loose prior produces smaller forecast errors. Specifically, Liu *et al.* show that for all four variables forecasted, namely output, consumption, investment and hours worked, a BVAR with the most loose prior ($w = 0.3,\ d = 0.5$) outperforms the estimated Hansen(1985)–type DSGE model and a Classical VAR.

[11] Here we only report the RMSEs from the optimal BVAR, i.e. a BVAR with a specific set of "hyperparameters" for which we obtain the lowest RMSEs for each quarter.

Table 14: **RMSE (2001Q1-2006Q4): Inflation**

QA	1	2	3	4	AVE
NKDSGE	0.280	0.349	0.422	0.439	**0.373**
VAR (1)	0.364	0.409	0.462	0.519	0.439
BVAR (w=.1, d=1)	0.312	0.402	0.467	0.520	0.425

QA: quarter ahead; RMSE: root mean squared error (%).

Table 15: **RMSE (2001Q1-2006Q4): TBILL**

QA	1	2	3	4	AVE
NKDSGE	0.914	1.586	2.067	2.406	1.743
VAR (1)	0.813	1.464	1.962	2.334	1.643
BVAR (w=.3, d=.5)	0.688	1.365	1.901	2.306	**1.565**

QA: quarter ahead; RMSE: root mean squared error (%).

In order to evaluate the models' forecast accuracy, we perform the across-model test between the NKDSGE model and the VAR and BVAR models in pairs. The across-model test is based on the statistic proposed by Diebold and Mariano (1995). The test statistic is defined as the following. For instance, let $\{e_t^v\}_{t=1}^T$ denote the associated forecast errors from the unrestricted VAR(1) model and $\{e_t^k\}_{t=1}^T$ denote the forecast errors from the NKDSGE model. The test statistic is then defined as $s = \frac{l}{\sigma_l}$, where l is the sample mean of the "loss differentials" with $\{l_t\}_{t=1}^T$ obtained by using $l_t = (e_t^v)^2 - (e_t^k)^2$ for all $t = 1, 2, 3, ..., T$, and where σ_l is the standard error of l. The s statistic is asymptotically distributed as a standard normal random variable and can be estimated under the null hypothesis of equal forecast accuracy, i.e. $l = 0$. Therefore, in this case, a positive value of s would suggest that the NKDSGE model outperforms the unrestricted VAR(1) model in terms of out-of-sample forecasting. Results are reported in Table 16. In

Table 16: **Across-Model Test Statistics**

Quarters Ahead	1	2	3	4
(A) Output Growth				
BVAR vs. NKDSGE	-1.573	-1.271	-1.332	-1.257
BVAR vs. VAR(1)	-0.913	3.143*	0.002	1.433
NKDSGE vs. VAR(1)	0.976	1.710	1.310	1.270
(B) Inflation				
BVAR vs. NKDSGE	0.760	0.541	0.358	1.078
BVAR vs. VAR(1)	-1.145	-0.533	0.052	0.747
NKDSGE vs. VAR(1)	-0.889	-0.588	-0.355	-1.019
(C) TBILL				
BVAR vs. NKDSGE	−2.226*	-1.542	-0.896	-0.547
BVAR vs. VAR(1)	-1.377	-1.009	-0.769	-0.576
NKDSGE vs. VAR(1)	2.463*	1.010	0.577	0.371

Note:∗ indicates at the 5% level significant.

general, the NKDSGE model does a better job in predicting inflation than it does in predicting output growth and the nominal short-term interest rate (TBILL). The differences between RMSEs generated from the NKDSGE model and the VARs are minor, since most of the test statistics are insignificant.

4.5 Conclusion

In this paper, we show that, besides its usual usage for policy analysis, a small-scale NKDSGE model has a future for forecasting. We show that the NKDSGE model outperforms both the Classical and Bayesian variants of the VARs in forecasting inflation, but not for output growth and the nominal short-term interest rate. However, the differences of the forecast errors are minor. The indicated success of the NKDSGE model for predicting inflation is important, especially in the context of South Africa — an economy targeting inflation.

As suggested by Smets and Wouters (2004), a NKDSGE model estimated by Bayesian techniques can become an useful tool in the forecasting kit for central banks. In this backdrop, further research will concentrate on developing an estimated NKDSGE model based on Bayesian techniques. In addition, future research in this area will aim to extend the current framework into that of a small open economy.

Optimization

Household

In the NKDSGE model, the representative household chooses $\{C_t, h_t, \frac{M_t}{P_t}, \frac{B_t}{P_t}\}$ to maximize the utility function (1):

$$E\sum_{t=0}^{\infty}\beta^t\big[a_t log(C_t) + log(\frac{M_t}{P_t}) - (\frac{1}{\eta})h_t^{\eta}\big], \qquad 0<\beta<1, \qquad \eta\geq 1, \qquad ((1))$$

subject to the budget constraint (3):

$$C_t + \frac{B_t}{r_t P_t} = \frac{W_t}{P_t}h_t + \frac{B_{t-1}}{P_t} + D_t + T_t - \frac{M_t - M_{t-1}}{P_t} \qquad ((3))$$

The resulting Bellman's equation is as follows:

$$V(M_{t-1}, B_{t-1}, a_t, Z_t, \varepsilon_{rt}) = \max[a_t log(C_t) + log(\frac{M_t}{P_t}) - (\frac{1}{\eta})h_t^{\eta} + \beta E_t V(M_t, B_t, a_{t+1}, Z_{t+1}, \varepsilon_{rt+1})] \qquad (A.1)$$

Substituting C_t from (3) into (A.1) and solving this problem yields the following first order condition (FOC) for hours worked:

$$\begin{aligned}
\frac{\partial V(M_{t-1}, B_{t-1}, a_t, Z_t, \varepsilon_{rt})}{\partial h_t} &= 0 && (A.2)\\
\frac{a_t}{C_t}\frac{W_t}{P_t} - h_t^{\eta-1} &= 0 && (A.3)\\
\frac{W_t}{P_t} &= a_t^{-1}C_t h_t^{\eta-1} && (A.4)
\end{aligned}$$

The FOC for bond holdings is given as follows:

$$\begin{aligned} \frac{\partial V(M_{t-1}, B_{t-1}, a_t, Z_t, \varepsilon_{rt})}{\partial B_t} &= 0 & \text{(A.5)} \\ \frac{a_t}{C_t}(-\frac{1}{r_t P_t}) + \frac{\partial \beta E_t V(M_t, B_t, a_{t+1}, Z_{t+1}, \varepsilon_{rt+1})}{\partial B_t} &= 0 & \text{(A.6)} \end{aligned}$$

The associated envelope condition is:

$$\frac{\partial V(M_{t-1}, B_{t-1}, a_t, Z_t, \varepsilon_{rt})}{\partial B_{t-1}} = \frac{a_t}{C_t}\frac{1}{P_t} \tag{A.7}$$

Updating (A.7) and combining with (A.6) yields

$$\frac{a_t}{C_t} = r_t \beta E_t \Big[a_{t+1} \Big(\frac{1}{C_{t+1}} \Big) \Big(\frac{P_t}{P_{t+1}} \Big) \Big] \tag{A.8}$$

FOC for money holdings can be derived as follows:

$$\begin{aligned} \frac{\partial V(M_{t-1}, B_{t-1}, a_t, Z_t, \varepsilon_{rt})}{\partial M_t} &= 0 & \text{(A.9)} \\ \frac{a_t}{C_t}(-1)\frac{1}{P_t} + \frac{1}{M_t} + \frac{\partial \beta E_t V(M_t, B_t, a_{t+1}, Z_{t+1}, \varepsilon_{rt+1})}{\partial M_t} &= 0 & \text{(A.10)} \end{aligned}$$

The associated envelope condition is:

$$\frac{\partial V(M_{t-1}, B_{t-1}, a_t, Z_t, \varepsilon_{rt})}{\partial M_{t-1}} = \frac{a_t}{C_t}\frac{1}{P_t} \tag{A.11}$$

Updating (A.11) and combining with (A.10), we have:

$$\frac{a_t}{C_t} = \frac{P_t}{M_t} + \beta E_t\left(\frac{a_{t+1}}{C_{t+1}}\frac{P_t}{P_{t+1}}\right) \tag{A.12}$$

$$\frac{P_t}{M_t} = \beta E_t\left(\frac{P_t}{P_{t+1}}\right) - \frac{a_t}{C_t} \tag{A.13}$$

Using (A.8):

$$\frac{M_t}{P_t} = a_t^{-1} C_t[r_t/(r_t - 1)] \tag{A.14}$$

Final goods firm

A representative firm produces the final good Y_t using intermediate goods Y_{jt} according to the CES production function:

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\theta_t - 1}{\theta_t}} dj\right)^{\frac{\theta_t}{\theta_t - 1}} \tag{A.15}$$

The firm maximizes its profit:

$$\max_{Y_{jt}} \left\{Y_t - \frac{1}{P_t}\int_0^1 P_{jt} Y_{jt}\right\} \tag{A.16}$$

Alternatively, the firm minimizes its expenditure given the production constraint. The Lagrangean for the firm is given by the following expression:

$$L = \int_0^1 P_{jt} Y_{jt} dj - P_t\left[Y_t - \left(\int_0^1 Y_{jt}^{\frac{\theta_t - 1}{\theta_t}} dj\right)^{\frac{\theta_t}{\theta_t - 1}}\right] \tag{A.17}$$

Setting $\frac{\partial L}{\partial Y_{jt}} = 0$, yields:

$$P_{jt} = P_t \frac{\partial Y_t}{\partial Y_{jt}} \tag{A.18}$$

where:

$$\frac{\partial Y_t}{\partial Y_{jt}} = \frac{\theta_t}{\theta_t - 1}\left(\int_0^1 Y_{jt}^{\frac{\theta_t - 1}{\theta_t}} dj\right)^{\frac{1}{\theta_t - 1}} \left(\frac{\theta_t - 1}{\theta_t}\right) Y_{jt}^{-1/\theta_t} \tag{A.19}$$

$$= \left(\frac{Y_t}{Y_{jt}}\right)^{1/\theta_t} \tag{A.20}$$

Substituting (A.20) into (A.18), yields:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\theta_t} Y_t \tag{A.21}$$

Here, given Euler's theorem, profits in this sector must equal to zero in equilibrium:

$$P_t Y_t = \int_0^1 P_{jt} Y_{jt} dj \tag{A.22}$$

Solving for the optimal price of the final-goods Y_t, yields:

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta_t} dj\right)^{\frac{1}{1-\theta_t}} \tag{A.23}$$

Intermediate goods firm

A representative firm produces Y_{jt} according to the following production function:

$$Y_{jt} = Z_t h_{jt} \tag{A.24}$$

given a quadratic cost of price adjustment:

$$\frac{\phi}{2}\Big[\frac{P_{jt}}{\pi P_{jt-1}}\Big]^2 Y_t, \qquad \phi > 0, \quad \pi > 1, \tag{A.25}$$

The firm maximizes its market value:

$$E\sum_{t=0}^{\infty}\beta^t(a_t/C_t)\Big\{\Big[\frac{P_{jt}}{P_t}\Big]^{1-\theta}Y_t - \Big[\frac{P_{jt}}{P_t}\Big]^{-\theta}\Big(\frac{W_t}{P_t}\Big)\Big(\frac{Y_t}{Z_t}\Big) - \frac{\phi}{2}\Big[\frac{P_{jt}}{\pi P_{jt-1}}\Big]^2 Y_t\Big\} \tag{A.26}$$

The first order condition for this problem:

$$\begin{aligned}(\theta_t - 1)\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta_t}\Big(\frac{Y_t}{P_t}\Big) &= \beta\phi E_t\Big[\Big(\frac{a_{t+1}}{a_t}\Big)\Big(\frac{C_t}{C_{t+1}}\Big)\Big(\frac{P_{jt+1}}{\pi P_{jt}} - 1\Big)\Big(\frac{Y_{t+1}}{P_{jt}}\Big)\Big(\frac{P_{jt+1}}{\pi P_{jt}}\Big)\Big] \\ &+\Big[\theta_t\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta_t-1}\Big(\frac{W_t}{P_t}\Big)\Big(\frac{Y_t}{Z_t}\Big)\Big(\frac{1}{P_t}\Big)\Big] - \Big[\phi\Big(\frac{P_{jt}}{\pi P_{jt-1}} - 1\Big)\Big(\frac{Y_t}{\pi P_{jt-1}}\Big)\Big]\end{aligned} \tag{A.27}$$

The Log-linear Equilibrium

Symmetric Equilibrium

In a symmetric equilibrium, the model can be summarized as follows:

$$
\begin{aligned}
Y_t &= Z_t h_t && \text{(B.1)}\\
\frac{W_t}{P_t} &= a_t^{-1} C_t h_t^{\eta-1} && \text{(B.2)}\\
\frac{a_t}{C_t} &= r_t \beta E_t\Big[a_{t+1}\Big(\frac{1}{C_{t+1}}\Big)\Big(\frac{P_t}{P_{t+1}}\Big)\Big] && \text{(B.3)}\\
\frac{M_t}{P_t} &= a_t^{-1} C_t [r_t/(r_t-1)] && \text{(B.4)}\\
log(a_t) &= \rho_a log(a_{t-1}) + \varepsilon_{at} && \text{(B.5)}\\
log\theta_t &= (1-\rho_\theta) log\theta + \rho_\theta log\theta_{t-1} + \varepsilon_{\theta t} && \text{(B.6)}\\
logZ_t &= log\bar{Z} + logZ_{t-1} + \varepsilon_{zt} && \text{(B.7)}\\
0 &= (1-\theta_t)\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta_t}\Big(\frac{Y_t}{P_t}\Big) + \beta\phi E_t\Big[\Big(\frac{a_{t+1}}{a_t}\Big)\Big(\frac{C_t}{C_{t+1}}\Big)\Big(\frac{P_{jt+1}}{\pi P_{jt}} - 1\Big)\Big(\frac{Y_{t+1}}{P_{jt}}\Big)\Big(\frac{P_{jt+1}}{\pi P_{jt}}\Big)\Big]\\
&\quad + \Big[\theta_t\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta_t-1}\Big(\frac{W_t}{P_t}\Big)\Big(\frac{Y_t}{Z_t}\Big)\Big(\frac{1}{P_t}\Big)\Big] - \Big[\phi\Big(\frac{P_{jt}}{\pi P_{jt-1}} - 1\Big)\Big(\frac{Y_t}{\pi P_{jt-1}}\Big)\Big] && \text{(B.8)}
\end{aligned}
$$

Log-linearization

In our complete model, equations (B.1)-(B.8) together with the output gap equation (21) describe the behavior of the endogenous variables Y_t, C_t, π_t, r_t, and x_t, and the three exogenous shocks a_t, θ_t, and Z_t. Y_t, C_t, and Z_t are stochastically detrended so that $y_t = Y_t/Z_t$, $c_t = C_t/Z_t$, and $z_t = Z_t/Z_{t-1}$ are stationary.

In the absence of shocks, the economy converges to a steady-state growth path, in which $y_t = y$, $c_t = c$, $\pi_t = \pi$, $r_t = r$, $x_t = x$, $g_t = g$, $a_t = a$, $\theta_t = \theta$, and $z_t = z$ for all $t = 0, 1, 2, ...$ Therefore, in steady-state we have:

$$y = \left[a\left(\frac{\theta-1}{\theta}\right)\right]^{\frac{1}{\eta}} \tag{B.9}$$

$$r = \left(\frac{z}{\beta}\right)\pi \tag{B.10}$$

$$x = \left(\frac{\theta-1}{\theta}\right)^{\frac{1}{\eta}} \tag{B.11}$$

Using first-order Taylor approximation to rewrite all the equations of the model, we have:

$$\hat{x}_t = E_t\hat{x}_{t+1} - (\hat{r}_t - E_t\hat{\pi}_{t+1}) + \left(1 - \frac{1}{\eta}\right)(1-\rho_a)\hat{a}_t \tag{B.12}$$

$$\hat{\pi} = \beta E_t\hat{\pi}_{t+1} + \psi\hat{x}_t - \hat{\theta}_t/\phi, \qquad \psi = \eta\left(\frac{\theta-1}{\phi}\right) \tag{B.13}$$

$$\hat{x}_t = \hat{y}_t - \frac{1}{\eta}\hat{a}_t \tag{B.14}$$

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{B.15}$$

$$\hat{a}_t = \rho_a\hat{a}_{t-1} + \varepsilon_{at} \tag{B.16}$$

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \varepsilon_{\theta t} \tag{B.17}$$

$$\hat{z}_t = \varepsilon_{zt} \tag{B.18}$$

Chapter 5

Conclusions

The thesis is the first attempt in using alternative forms of Dynamic Stochastic General Equilibrium (DSGE) models for forecasting the South African economy. The out-of-sample forecast performances of these alternative forms of DSGE models are evaluated by comparing them with the same generated by the Classical and Bayesian variants of the VARs.

Compared to the VARs and the BVARs, the calibrated Hansen (1985)–type DSGE model produces large out-of-sample forecast errors. The results from the second paper suggest that the estimated hybrid DSGE (DSGE-VAR) model outperforms the Classical VAR, but not the Bayesian VARs. However, it does indicate that the forecast accuracy can be improved alarmingly by using the estimated version of the DSGE model. In the third paper, we show that, besides the usual usage for policy analysis, a small-scale NKDSGE model has a future for forecasting. The NKDSGE model outperforms both the Classical and Bayesian variants of the VARs in the case of forecasting inflation, but not for output growth and the nominal short-term interest rate. However, the differences of the forecasts errors are minor. The indicated success of the NKDSGE model for predicting inflation is important, especially in the context of South Africa — an economy targeting inflation.

As suggested by Smets and Wounters (2004), a NKDSGE model estimated by Bayesian techniques can become an useful tool in the forecasting kit for central banks. In this backdrop, further research will concentrate on developing an estimated NKDSGE model based on Bayesian techniques. In addition, future research in this area also will aim to extend the current framework into that of a small open economy.