APPENDIX A

CO-VARIANCE MATRIX OF MULTIPLE UNCORRELATED SOURCES

Consider the signals from D sources impinging on the elements of an array. The received signal of source d at the array elements is:

$$\mathbf{X}_d(t) = S_d(t)\mathbf{U}_d \tag{A1}$$

where $S_d(t)$ is the data transmitted by source d and $\mathbf{U}_d$ is the array vector of source d. The total received signal at the array is the sum of all the signals plus noise received at the array, or:

$$\mathbf{X}(t) = \sum_{d=1}^{D} S_d(t)\mathbf{U}_d + \mathbf{n}(t) \tag{A2}$$

where $\mathbf{n}$ is zero mean Gaussian noise at the antenna elements. The co-variance matrix of the received signals at the array elements is:

$$\mathbf{R} = E\left\{\mathbf{X}(t)\,\mathbf{X}^H(t)\right\} \tag{A3}$$

Inserting now (A2) in (A3) and assuming that the signals and noise are uncorrelated, the co-variance matrix becomes:

$$\mathbf{R} = E\left\{\sum_{d=1}^{D} S_d(t)\mathbf{U}_d \left(\sum_{d=1}^{D} S_d(t)\mathbf{U}_d\right)^H\right\} + \sigma^2\mathbf{I} \tag{A4}$$

where $\mathbf{I}$ is a unity matrix and σ^2 is the noise power. Expanding (A4) the following is obtained:

$$\mathbf{R} = E\begin{Bmatrix} S_1(t)S_1^*(t)\mathbf{U}_1\mathbf{U}_1^H + S_1(t)S_2^*(t)\mathbf{U}_1\mathbf{U}_2^H + \ldots + S_1(t)S_D^*(t)\mathbf{U}_1\mathbf{U}_D^H + \\ S_2(t)S_1^*(t)\mathbf{U}_2\mathbf{U}_1^H + S_2(t)S_2^*(t)\mathbf{U}_2\mathbf{U}_2^H + \ldots + S_2(t)S_D^*(t)\mathbf{U}_2\mathbf{U}_D^H + \\ + \ldots. + \\ S_D(t)S_1^*(t)\mathbf{U}_D\mathbf{U}_1^H + S_D(t)S_2^*(t)\mathbf{U}_D\mathbf{U}_2^H + \ldots + S_D(t)S_D^*(t)\mathbf{U}_D\mathbf{U}_D^H + \end{Bmatrix} + \sigma^2\mathbf{I} \tag{A5}$$

Now, if the sources are uncorrelated, then:

$$E\left\{S_1(t)S_2^*(t)\right\}=E\left\{S_1(t)S_2^*(t)\right\}=.....=E\left\{S_1(t)S_D^*(t)\right\}=...=E\left\{S_D(t)S_1^*(t)\right\}=...=0 \tag{A6}$$

Since the power P_d of each signal $d \in \{1,2,...,D\}$ is:

$$P_d = E\left\{S_d(t)S_d^*(t)\right\} \tag{A7}$$

and using (A6) and (A7), equation (A5) becomes:

$$\mathbf{R} = P_1\,\mathbf{U}_1\mathbf{U}_1^H + P_2\,\mathbf{U}_2\mathbf{U}_2^H + ... + P_D\,\mathbf{U}_D\mathbf{U}_D^H + \sigma^2\mathbf{I} \tag{A8}$$

which can be written as:

$$\mathbf{R} = \sum_{d=1}^{D} P_d\,\mathbf{U}_d\mathbf{U}_d^H + \sigma^2\mathbf{I} \tag{A9}$$

APPENDIX B

PROOF THAT THE OPTIMUM COMBINED SINR OF TWO ARRAYS WITH INDIVIDUAL BEAMFORMING IS EQUAL TO THE SUM OF THE SINR OF EACH INDEPENDENT BEAMFORMING ARRAY

It was shown in section 4.2.1 that the signal to interference plus noise ratio after optimum combining of two individual arrays, each with signals combined with independent optimum beamforming, is the sum of the individual array signal to interference ratios. However, it was done for the special case where $\psi_{12} = \psi_{22}$. The derivation will be extended to the general case ($\psi_{12} \neq \psi_{22}$) in this appendix.

The signal to interference plus noise ratio of optimum combining of the individual array signals is given in (116) as

$$\mathrm{SINR}_C = \mathbf{U}_{dc}^H \mathbf{R}_{nnC}^{-1} \mathbf{U}_{dc} \tag{B1}$$

with $\mathbf{R}_{nnC}$ the covariance matrix of the signals from the two arrays, given by:

$$\mathbf{R}_{nnC} = \begin{bmatrix} \mathbf{R}_{nnC,11} & \mathbf{R}_{nnC,12} \\ \mathbf{R}_{nnC,21} & \mathbf{R}_{nnC,22} \end{bmatrix} \tag{B2}$$

and $\mathbf{U}_{dc}$ is the array steering vector in the direction of the desired signal, given by:

$$\mathbf{U}_{dc} = [1\ 1]^T \tag{B3}$$

The inverse of a two by two matrix is [66]:

$$\mathbf{R}_{nnC}^{-1} = \left(R_{nnC,11} R_{nnC,22} - R_{nnC,12} R_{nnC,21}\right)^{-1} \begin{bmatrix} R_{nnC,22} & -R_{nnC,12} \\ -R_{nnC,21} & R_{nnC,11} \end{bmatrix} \tag{B4}$$

Since $R_{nnC,12} = R_{nnC,21}^*$, the inverse in (B4) becomes:

$$\mathbf{R}_{nnC}^{-1} = \left(R_{nnC,11} R_{nnC,22} - R_{nnC,12} R_{nnC,12}^*\right)^{-1} \begin{bmatrix} R_{nnC,22} & -R_{nnC,12} \\ -R_{nnC,12}^* & R_{nnC,11} \end{bmatrix} \tag{B5}$$

Inserting (B5) in (B1), the SINR is:

$$\mathrm{SINR_C} = \frac{\left(R_{nnC,11} + R_{nnC,12} + R^{*}_{nnC,12} + R_{nnC,22}\right)}{R_{nnC,11}\,R_{nnC,22} - R_{nnC,12}\,R^{*}_{nnC,12}} \tag{B6}$$

If it cans now be shown that $R_{nnC,11}R_{nnC,22} >> R_{nnC,12}R^{*}_{nnC,12}$, then the SINR in (B7) becomes:

$$\mathrm{SINR_C} = \left(\frac{1}{R_{nnC,11}} + \frac{1}{R_{nnC,22}}\right) \tag{B8}$$

insert the components of (93) in (B8), the SINR becomes:

$$\mathrm{SINR_C} = \left(\frac{1}{\mathbf{W}_1^H \mathbf{R}_{nn11} \mathbf{W}_1} + \frac{1}{\mathbf{W}_2^H \mathbf{R}_{nn22} \mathbf{W}_2}\right) \tag{B9}$$

and using (88) and (89) the following is obtained:

$$\mathrm{SINR_C} = \left(\frac{\left(\mathbf{U}_d^H \mathbf{R}_{nn11}^{-1} \mathbf{U}_d\right)^H \left(\mathbf{U}_d^H \mathbf{R}_{nn11}^{-1} \mathbf{U}_d\right)}{\left(\mathbf{R}_{nn11}^{-1} \mathbf{U}_d\right)^H \mathbf{R}_{nn11} \mathbf{R}_{nn11}^{-1} \mathbf{U}_d} + \frac{\left(\mathbf{U}_d^H \mathbf{R}_{nn22}^{-1} \mathbf{U}_d\right)^H \left(\mathbf{U}_d^H \mathbf{R}_{nn22}^{-1} \mathbf{U}_d\right)}{\left(\mathbf{R}_{nn22}^{-1} \mathbf{U}_d\right)^H \mathbf{R}_{nn22} \mathbf{R}_{nn22}^{-1} \mathbf{U}_d}\right) \tag{B10}$$

After cancellation of the components in (B10), the following simplified equation is obtained:

$$\mathrm{SINR_C} = \left(\mathbf{U}_d^H \mathbf{R}_{nn11}^{-1} \mathbf{U}_d + \mathbf{U}_d^H \mathbf{R}_{nn22}^{-1} \mathbf{U}_d\right) \tag{B11}$$

comparing this to (108) and (109) it can be seen that:

$$\mathrm{SINR_C} = \left(\mathrm{SINR}_1 + \mathrm{SINR}_2\right) \tag{B12}$$

Using (93), (102) and (105), the product $R_{nnC,11}R_{nnC,22}$ in (B6) is:

$$R_{nnC,11}R_{nnC,22} = \frac{(2+\sigma^2)\sigma^4}{4((\cos(\pi \sin\psi_{12}) - 1 - \sigma^2)(\cos(\pi \sin\psi_{22}) - 1 - \sigma^2))} \tag{B13}$$

The relation between the angles ψ_{12} and ψ_{22} is:

$$\psi_{22} = \arctan\left(\frac{\sin\psi_{12}}{\xi - \cos\psi_{12}}\right) \tag{B14}$$

where ξ is the proportion of the range from array 1 to the mobile relative to the distance between the two arrays. Inserting this angle relationship in (B13) into (B14), the following results:

$$R_{nnC,11}R_{nnC,22} = \frac{\sigma^4}{\cos\left\{\frac{\omega}{\Gamma}\right\}\cos(\omega) - \cos(\omega) - \cos\left\{\frac{\omega}{\Gamma}\right\} + 1} \tag{B15}$$

where

$$\omega = \pi \sin \psi_{12} \tag{B16}$$

and

$$\Gamma = \sqrt{\xi^2 - 2\xi\cos\psi_{12} + 1} \tag{B17}$$

Let $\alpha = \frac{\omega}{\Gamma}$, then (B17) can be written as:

$$R_{nnC,11}R_{nnC,22} = \frac{\sigma^4}{\cos(\alpha)\cos(\omega) - \cos(\omega) - \cos(\alpha) + 1} \tag{B18}$$

Using (93), (103), (104) and (B14) the product $R_{nnC,12}\,R^*_{nnC,12}$ in (B6) becomes:

$$\begin{aligned}
&\frac{1}{4}\sigma^4\Bigg(\cos\left(-\frac{1}{2}\omega + \frac{1}{2}\alpha\right) - \cos(\omega)\cos\left(-\frac{1}{2}\omega + \frac{1}{2}\alpha\right) + \sin(\omega)\sin\left(-\frac{1}{2}\omega + \frac{1}{2}\alpha\right) \\
&\quad + \sigma^2\cos\left(-\frac{1}{2}\omega + \frac{1}{2}\alpha\right) + \cos\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right) + \cos\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right)\sigma^2 \\
&\quad - \cos\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right)\cos(\omega) - \sin\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right)\sin(\omega)\Bigg)^2 \Big/ (1 + 8\cos(\alpha)\sigma^2\cos(\omega) \\
&\quad - 2\cos(\alpha)\cos(\omega)^2\sigma^2 - 2\cos(\alpha)^2\cos(\omega)\sigma^2 + 4\cos(\alpha)\sigma^4\cos(\omega) - 2\cos(\alpha) \\
&\quad + \cos(\alpha)^2 + 4\sigma^2 - 2\cos(\omega) + \cos(\omega)^2 + 6\sigma^4 + \sigma^8 + 4\sigma^6 + 4\cos(\alpha)\cos(\omega) \\
&\quad + \cos(\alpha)^2\cos(\omega)^2 - 2\cos(\alpha)\cos(\omega)^2 - 2\cos(\alpha)^2\cos(\omega) - 6\cos(\alpha)\sigma^4 \\
&\quad + \cos(\alpha)^2\sigma^4 - 6\cos(\alpha)\sigma^2 - 2\cos(\alpha)\sigma^6 + 2\cos(\alpha)^2\sigma^2 - 6\sigma^4\cos(\omega) \\
&\quad + \sigma^4\cos(\omega)^2 + 2\sigma^2\cos(\omega)^2 - 6\sigma^2\cos(\omega) - 2\sigma^6\cos(\omega))
\end{aligned} \tag{B19}$$

Using (B18) and (B19), the ration of $R_{nnC,11}\,R_{nnC,22}$ to $R_{nnC,12}\,R^*_{nnC,12}$ can be calculated. Once (B18) is divided in (B19), the fact that $\sigma^2 << 1$ is used to simplify the result, given by:

$$\frac{R_{nnC,11}R_{nnC,22}}{R_{nnC,12}R^{*}_{nnC,12}} = \frac{A}{BC} \tag{B20}$$

where

$$A = \cos(\alpha)^2\cos(\omega)^2 - 2\cos(\alpha)^2\cos(\omega) - 2\cos(\alpha)\cos(\omega)^2 + 4\cos(\alpha)\cos(\omega) + \cos(\alpha)^2 - 2\cos(\alpha) + \cos(\omega)^2 - 2\cos(\omega) + 1 \tag{B21}$$

$$B = \left(\cos\left(\frac{1}{2}\omega - \frac{1}{2}\alpha\right) - \cos(\omega)\cos\left(\frac{1}{2}\omega - \frac{1}{2}\alpha\right) - \sin(\omega)\sin\left(\frac{1}{2}\omega - \frac{1}{2}\alpha\right) + \cos\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right) - \cos\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right)\cos(\omega) - \sin\left(\frac{1}{2}\omega + \frac{1}{2}\alpha\right)\sin(\omega)\right)^2 \tag{B22}$$

$$C = \cos(\alpha)\cos(\omega) - \cos(\alpha) - \cos(\omega) + 1 \tag{B23}$$

Applying the following trigonometry properties to (B23):

$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b) \tag{B24}$$

and

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \tag{B25}$$

the equation becomes zero (i.e. B = 0), which results in a large value for the ratio $\frac{R_{nnC,11}R_{nnC,22}}{R_{nnC,12}R^{*}_{nnC,12}}$ and thereby proving that the SINR of the arrays combined with optimum combining is equal to the individual array SINRs.

APPENDIX C

BER OF ARRAY WITH MULTIPLE NON-UNIQUE EIGENVALUES (MULTIPLICITY > 1)

In the case of one eigenvalue with multiplicity equal to one and M-1 eigenvalues with multiplicity equal to M-1, the characteristic function of the probability density function can be written as:

$$\Psi(z)=\frac{\langle\lambda_1\rangle^{M-1}\langle\lambda_M\rangle}{\left(z+\langle\lambda_1\rangle\right)^{M-1}\left(z+\langle\lambda_M\rangle\right)} \tag{C1}$$

A partial fraction expansion of (C1) is:

$$\Psi(z)=\frac{-C}{(z+\lambda_1)^{M-1}(\lambda_1-\lambda_M)}+\frac{-C}{(z+\lambda_1)^{M-2}(\lambda_1-\lambda_M)^2}+\ldots\ldots\ldots \\ \frac{+C}{(z+\lambda_5)(\lambda_1-\lambda_M)^{M-1}} \tag{C2}$$

where

$$C=\langle\lambda_1\rangle^{M-1}\langle\lambda_M\rangle \tag{C3}$$

Equation (C2) can also be written as:

$$\Psi(z)=C\left\{\frac{\Omega_1}{(z+\lambda_1)^{M-1}}+\frac{\Omega_2}{(z+\lambda_1)^{M-2}}+\ldots\ldots+\frac{\Omega_M}{(z+\lambda_5)}\right\} \tag{C4}$$

where

$$\Omega_1=\frac{-1}{(\lambda_1-\lambda_M)} \tag{C5}$$

$$\Omega_2=\frac{-1}{(\lambda_1-\lambda_M)^2} \tag{C6}$$

and

$$\Omega_M = \frac{1}{(\lambda_1 - \lambda_M)^{M-1}} \tag{C7}$$

The inverse Laplace transform of (C4) is given by:

$$\begin{aligned} p(\eta) &= L^{-1}\{\Psi(z)\} \\ &= C\left\{\Omega_{M-1} e^{-\lambda_1} + \eta\Omega_{M-2} e^{-\lambda_1\eta} + ...+ \eta^{M-2}\Omega_1 e^{-\lambda_1\eta} + \Omega_M e^{-\lambda_M \eta}\right\} \end{aligned} \tag{C8}$$

The average bit error rate (BER) of phased shift keyed signals is given by [11]:

$$BER = \frac{1}{2}\int_{-\infty}^{\infty} p(\eta)\ \mathrm{erfc}(\sqrt{\eta})\, d\eta \tag{C9}$$

Inserting (C8) in (C9) the following is obtained:

$$BER = \frac{C}{2}\int_{-\infty}^{\infty}\left\{\Omega_{M-1} e^{-\lambda_1} + \eta\Omega_{M-2} e^{-\lambda_1\eta} + ...+ \eta^{M-2}\Omega_1 e^{-\lambda_1\eta} + \Omega_M e^{-\lambda_M\eta}\right\}\mathrm{erfc}(\sqrt{\eta})\, d\eta \tag{C10}$$

The following general integral formula [67] is used to solve (C10)

$$\frac{1}{2(K-1)!}\int_0^{\infty} x^{K-1} e^{(-ax)}\ \mathrm{erfc}(\sqrt{bx}\ dx = \left(\frac{\sqrt{1+\frac{a}{b}}-1}{2a\sqrt{1+\frac{a}{b}}}\right)^K \sum_{k=0}^{K-1}\left(\frac{K-1+k}{k}\right)\left(\frac{\sqrt{1+\frac{a}{b}}+1}{2\sqrt{1+\frac{a}{b}}}\right)^k \tag{C11}$$

where

$$\left(\frac{K-1+k}{k}\right) = \frac{(K-1+k)!}{k!(K-1)!} \tag{C12}$$

Using (C11) in (C10) the bit error rate is

$$BER = -C\sum_{m=2}^{M-1}\left\{\Omega^{M-1}\ \zeta_1^m\right\}\sum_{i=0}^{m-1}\left\{\frac{(m-1+i)!\,\mu^i}{i!(m-1)!}\right\} + C\Omega^{M-1}\zeta_M \tag{C13}$$

where

$$\zeta_1 = \frac{\sqrt{1+\lambda_1}-1}{2\lambda_1\sqrt{1+\lambda_1}} \tag{C14}$$

$$\zeta_M = \frac{\sqrt{1+\lambda_M} - 1}{2\lambda_5 \sqrt{1+\lambda_5}} \tag{C15}$$

and

$$\mu = \frac{\sqrt{1+\lambda_1} + 1}{2\sqrt{1+\lambda_1}} \tag{C16}$$

APPENDIX D

OPTIMUM COMBINING WEIGHT VECTOR

The average output signal to interference plus noise power ratio (SINR) is given in (44) as:

$$\text{SINR} = \Gamma = \frac{\mathbf{W}^H \mathbf{U}_{des} \mathbf{U}_{des}^H \mathbf{W}}{\mathbf{W}^H \mathbf{R}_{nn} \mathbf{W}} \tag{D1}$$

where $\mathbf{R}_{nn}$ is the interference plus noise co-variance matrix, equal to [54]:

$$\mathbf{R}_{nn} = \mathbf{U}_{int} \mathbf{U}_{int}^H + \sigma_N^2 \tag{D2}$$

The optimum weight vector, $\mathbf{W}_{opt}$, which maximizes the SINR is now required. The derivation given here is from [58]. Since the interference plus noise co-variance matrix $\mathbf{R}_{nn}$ is positive definite, i.e.

$$\mathbf{W}^H \mathbf{R}_{nn} \mathbf{W} > 0 \tag{D3}$$

(positive definiteness is ensured since it has an uncorrelated noise component included) it can be factored into the product of two Hermitian ($\mathbf{G} = \mathbf{G}^H$) matrices:

$$\mathbf{R}_{nn} = \mathbf{G}\mathbf{G} \tag{D4}$$

and

$$\mathbf{R}_{nn}^{-1} = \mathbf{G}^{-1}\mathbf{G}^{-1} \tag{D5}$$

Matrix $\mathbf{G}$ can now be used to transform the weight vector $\mathbf{W}$ into a new vector $\mathbf{V}$ and visa versa:

$$\mathbf{V} = \mathbf{G}\,\mathbf{W} \tag{D6}$$

and

$$\mathbf{W} = \mathbf{G}^{-1}\,\mathbf{V} = (\mathbf{G}^{-1})^H\,\mathbf{V} \tag{D7}$$

Substituting equations (D6) and (D7) in (D1) yields:

$$\Gamma = \frac{\mathbf{V}^H \mathbf{G}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^H \mathbf{G}^{-1} \mathbf{V}}{\mathbf{V}^H \mathbf{G}^{-1} \mathbf{G} \mathbf{G} \mathbf{G}^{-1} \mathbf{V}} \tag{D8}$$

which is equal to:

$$\Gamma = \frac{\mathbf{V}^H \mathbf{G}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^H \mathbf{G}^{-1} \mathbf{V}}{\mathbf{V}^H \mathbf{V}} = \frac{\mathbf{V}^H \mathbf{B} \mathbf{V}}{\mathbf{V}^H \mathbf{V}} \tag{D9}$$

where

$$\mathbf{B} = \mathbf{G}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^H \mathbf{G}^{-1} \tag{D10}$$

The maximization of the SINR has now been reduced to a generalized eigenvalue problem, that of maximization of (D9), whose solution is:

$$\mathbf{V} = \mathbf{E}_{max} \tag{D11}$$

and

$$\mathbf{B}\mathbf{E}_{max} = \lambda_{max} \mathbf{E}_{max} \tag{D12}$$

where λ_{max} is the largest eigenvalue of $\mathbf{B}$ and $\mathbf{E}_{max}$ is the associated eigenvector. Inserting (D11) and (D12) in (D9), the following is obtained:

$$\Gamma_{max} = \frac{\mathbf{E}_{max}^H \lambda_{max} \mathbf{E}_{max}}{\mathbf{E}_{max}^H \mathbf{E}_{max}} = \frac{\lambda_{max} \mathbf{E}_{max}^H \mathbf{E}_{max}}{\mathbf{E}_{max}^H \mathbf{E}_{max}} = \lambda_{max} \tag{D13}$$

Inserting (D11) in (D7), the equivalent weighting vector is:

$$\mathbf{W}_{opt} = \mathbf{G}^{-1} \mathbf{E}_{max} \tag{D14}$$

Let now:

$$\mathbf{C} = \mathbf{G}^{-1} \mathbf{U}_{des} \tag{D15}$$

then equation (D10) becomes:

$$\mathbf{B} = \mathbf{C}\mathbf{C}^H \tag{D16}$$

inserting (D16) in (D12), the following is obtained:

$$\mathbf{B}\mathbf{E}_{max} = \mathbf{C}\mathbf{C}^H \mathbf{E}_{max} = \lambda_{max} \mathbf{E}_{max} \tag{D17}$$

and therefore the maximum eigenvalue is:

$$\lambda_{max} = \mathbf{C}\mathbf{C}^{H} \tag{D18}$$

B is a rank one matrix and $\mathbf{E}_{max}$ is the only eigenvector with non-zero eigenvalue. Substitution of (B17) and (D18) in (D14) and also using (D15) and (D16), the weight vector that will optimize the signal to interference ratio is:

$$\mathbf{W}_{opt} = \mathbf{G}^{-1}\mathbf{G}^{-1}\mathbf{U}_{des} = \mathbf{R}_{nn}^{-1}\mathbf{U}_{des} \tag{D19}$$

APPENDIX E

WEIGHT VECTOR WITH INTERFERENCE CO-VARIANCE MATRIC

In this appendix a derivation will be shown for re-writing the optimum weight vector containing the full co-variance matrix in terms of only the interference plus noise co-variance matrix. It will also be shown how the constant in the optimum weight vector cancels out when estimating the SINR. The optimum weight vector using the received co-variance matrix is given in [54] as:

$$\mathbf{W}_{opt} = \mu \mathbf{R}^{-1} \mathbf{U}_{des} \tag{E1}$$

where μ is a constant give as:

$$\mu = \frac{1}{\mathbf{U}_{des}^{H} \mathbf{R}^{-1} \mathbf{U}_{des}} \tag{E2}$$

and

$$\mathbf{R} = \mathbf{U}_{des} \mathbf{U}_{des}^{H} + \mathbf{R}_{nn} \tag{E3}$$

is the full received signal co-variance matrix. Using the matrix inversion lemma, the inverse of the full received signal co-variance matrix can be written as:

$$\mathbf{R}^{-1} = \mathbf{R}_{nn}^{-1} - \frac{\mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1}}{1 + \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \mathbf{U}_{des}} = \frac{\Omega \mathbf{R}_{nn}^{-1} - \mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1}}{\Omega} \tag{E4}$$

where Ω is a scalar given by:

$$\Omega = 1 + \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \tag{E5}$$

Inserting now (E4) in (E1) the following is obtained:

$$\mathbf{W}_{opt} = \frac{\left(\frac{\Omega \mathbf{R}_{nn}^{-1} - \mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1}}{\Omega} \right) \mathbf{U}_{des}}{\mathbf{U}_{des}^{H} \left(\frac{\Omega \mathbf{R}_{nn}^{-1} - \mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1}}{\Omega} \right) \mathbf{U}_{des}} = \frac{\left(\Omega \mathbf{R}_{nn}^{-1} - \mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \right) \mathbf{U}_{des}}{\mathbf{U}_{des}^{H} \left(\Omega \mathbf{R}_{nn}^{-1} - \mathbf{R}_{nn}^{-1} \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \right) \mathbf{U}_{des}}$$

$$= \frac{\mathbf{R}_{nn}^{-1} \left(\Omega - \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \right) \mathbf{U}_{des}}{\mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \left(\Omega - \mathbf{U}_{des} \mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \right) \mathbf{U}_{des}} = \frac{\mathbf{R}_{nn}^{-1} \mathbf{U}_{des}}{\mathbf{U}_{des}^{H} \mathbf{R}_{nn}^{-1} \mathbf{U}_{des}}$$

$$= \frac{}{\mathbf{U}_{des}^{H}\,\mathbf{R}_{nn}^{-1}\,\mathbf{U}_{des}}\,\mathbf{R}_{nn}^{-1}\,\mathbf{U}_{des} = \mu_{nn}\,\mathbf{R}_{nn}^{-1}\,\mathbf{U}_{des} \qquad \text{(E6)}$$

Equation (E6) gives the weight vector in terms of a constant and co-variance matrix of the interference signals alone (as well as desired signal array vector). Using now (E6) in the average SINR as given in equation (41), the average SINR becomes:

$$\text{SINR} = \frac{P_{des}(\mu_{nn}R_{nn}^{-1}\mathbf{U}_{des})^{H}\mathbf{U}_{des}\mathbf{U}_{des}^{H}(\mu_{nn}R_{nn}^{-1}\mathbf{U}_{des})}{(\mu_{nn}R_{nn}^{-1}\mathbf{U}_{des})^{H}\left(P_{int}\mathbf{U}_{int}\mathbf{U}_{int}^{H}+\sigma_{N}^{2}\right)(\mu_{nn}R_{nn}^{-1}\mathbf{U}_{des})}$$

$$= \frac{(\mu_{nn})^{2}P_{des}(R_{nn}^{-1}\mathbf{U}_{des})^{H}\mathbf{U}_{des}\mathbf{U}_{des}^{H}(R_{nn}^{-1}\mathbf{U}_{des})}{(\mu_{nn})^{2}(R_{nn}^{-1}\mathbf{U}_{des})^{H}\left(P_{int}\mathbf{U}_{int}\mathbf{U}_{int}^{H}+\sigma_{N}^{2}\right)(R_{nn}^{-1}\mathbf{U}_{des})}$$

$$= \frac{P_{des}(R_{nn}^{-1}\mathbf{U}_{des})^{H}\mathbf{U}_{des}\mathbf{U}_{des}^{H}(R_{nn}^{-1}\mathbf{U}_{des})}{(R_{nn}^{-1}\mathbf{U}_{des})^{H}\left(P_{int}\mathbf{U}_{int}\mathbf{U}_{int}^{H}+\sigma_{N}^{2}\right)(R_{nn}^{-1}\mathbf{U}_{des})} \qquad \text{(E7)}$$

It can be seen in (E7) that the constant μ_{nn} cancels out of the SINR

REFERENCES

[1] S.C. Swales, M.A. Beach, D. J. Edwards, J.P. McGeehan, "The Performance Enhancement of Multibeam Adaptive Base Station Antennas for Cellular Land Mobile Radio Systems", IEEE Trans. Veh. Tech., Vol. 39, No. 1, Feb. 1990, pp. 56-67.

[2] J.H. Winters, "Optimum combining in digital mobile radio with co-channel interference", IEEE Journal on Selected Areas in Comm., Vol. SAC-2, 4 July 1984, pp. 528-539.

[3] M.C. Wells, "Increasing the Capacity of GSM Cellular Radio using Adaptive Antennas", IEE Proc. Comm., Vol. 143, No. 5, Oct. 1995, pp. 304-310.

[4] S. Talwar, A Paulraj, M Viberg, "Reception of Multiple Co-Channel Digital Signals using Antenna Arrays with Applications to PCS", IEEE Int. Conf. on Humanity Through Comm., SUPERCOMM/ICC '94, Vol. 2, May 1994, pp. 790-794.

[5] P. Zetterberg, "The Spectrum Efficiency of a Base Station Antenna Array System for Spatially Selective Transmission", IEEE Trans. Veh. Tech., Vol. 44, No. 3, Aug. 1995, pp. 651-660.

[6] J.H. Winters, J. Salz, R.D. Gitlin, "The impact of Antenna Diversity on the Capacity of Wireless Communications Systems", IEEE Trans. on Comm., Vol. 42, No. 2/3/4, Feb./Mar./ Apr. 1994, pp. 1740-1750.

[1] S. Zürbes, W. Papen, W. Schmidth, "A new architecture for mobile radio with macroscopic diversity and overlapping cells", Proc. 5th IEEE Int. Symp. on Pers., Indoor and Mobile Radio Comm. (PIMRC '94), Den Haag, Sept. 1994, pp. 640-644.

[8] A. Turkmani, "Performance evaluation of a composite microscopic plus macroscopic diversity system", IEE Proc.-1, Vol. 138, No. 1, Feb. 1991, pp. 15-20.

[9] A. Turkmani, "Probability of error for m-banch macroscopic selection diversity", IEE Proc. 1, Vol. 139, No. 1, Feb. 1992, pp. 71-78.

[10] S. Loyka, "MIMO Channel Capacity: Electromagnetic Wave Perspective", 27th URSI General Assembly, Maastricht, The Netherlands, Aug. 2002, pp. 1-12.

[11] T.D. Pham, "Multipath performance of adaptive antennas with multiple interferers and correlated fadings", IEEE Trans. Veh. Tech., Vol. 48, No. 2, Mar. 1999, pp. 342-352.

[12] A.F. Naguib, A. Paulraj, "Effects of multipath and base-station antenna arrays on uplink capacity of cellular CDMA", IEEE Global Comm. Conf., GLOBECOM'94, Vol. 1, Nov. 1994, pp. 395-399.

[13] U. Martin, I. Gaspard, "Capacity enhancement of narrowband CDMA by intelligent antennas", The 8th IEEE International Symp. on Personal, Indoor and Mobile Radio Comm. PIMRC '97, Vol. 1 , Sept. 1997, pp. 90-94.

[14] D.G. Gerlach, A.Paulraj, "Base-station transmitting antenna arrays with mobile to base feedback", Proc. 27th asilomar conference on signals, systems and computers, 1993, pp. 1432-1436.

[15] M. da Silveira, J.W. Odendaal, J. Joubert, "The range increase of adaptive vs. phased arrays in mobile radio systems: Interference included", URSI, July 2001, Boston, p. 85.

[16] J.H. Winters, "The range increase of adaptive vs. phased arrays in mobile radio systems", IEEE Trans. Veh. Tech., Vol. 48, No. 2, Mar. 1999.

[17] M. da Silveira, J.W. Odendaal, J. Joubert, "Same cell co-channel interference reduction using multiple spatially distributed adaptive array systems", Signal Processing, Vol. 81, 2001, pp. 2059-2068.

[18] M.da Silveira, J.W. Odendaal, J. Joubert, " Cellular system capacity increase using spatially distributed adaptive array systems", URSI, July 2000, Salt Lake City, p. 137.

[19] J. Ylitalo, E. Tiirola, "Performance evaluation of different antenna array approaches for 3G CDMA uplink", IEEE Veh. Tech. Conf., May 2000, pp. 883-887.

[20] E. Tiirola, J. Ylitalo, "Comparison of beamforming and diversity approaches for coverage extension of WCDMA macro cells", 54^{th} IEEE Veh. Tech. Conf., Vol. 3, Oct. 2001, pp. 1274-1278.

[21] A. Kuchar, M. Tafener, "A robust DOA-based smart antenna processor for GSM base station", IEEE Int. Conf. on Comm., ICC '99, June 1999, pp.11-16.

[22] P. Zetterberg, "Mobile Cellular Communications with Base Station Antenna Arrays: Spectrum Efficiency, Algorithms and Propagation Models", Report Number TRITA-S3-SB-9712, ISSN 1103-8039, Dept. of Signals, Sensors and Systems, Royal Institute of Technology, Stockholm, Sweden, 1997, pp. 24-101.

[23] R. Cupo, G. Golden, "A four element adaptive antenna array for IS-136 PCS base stations", 47^{th} IEEE Veh. Tech. Conf., Vol. 3, May 1997, pp. 1577-1581.

[24] J.H. Winters, "Signal acquisition and tracking with adaptive arrays in the digital mobile radio system IS-54 with flat fading", IEEE Trans. Veh. Tech., Vol. 42, No. 4, Nov. 1993, pp. 377-384.

[25] T.D. Pham, "Statistical behavior and performance of adaptive antennas in multipath environments", IEEE Trans. On Microwave Theory and Tech., Vol. 47, No. 6, June 1999, pp 727-731.

[26] W. Mu, Y. Zhang, S. Park, M. Amin, "On the performance of CMA array in spatial macro-diversity antennas", 1998. Conference Record of the Thirty-Second Asilomar Conf. on Signals, Systems & Computers, Vol. 2 , Nov. 1998, pp. 1505-1509.

[27] A.F. Naguib, A. Paulraj, "Capacity improvement with base-station antenna arrays in cellular CDMA", IEEE Trans. on Veh. Tech., Vol. 43. No.3. Aug. 1994, pp. 691-698.

[28] A.F. Naguib, A. Paulraj, "Performance enhancement and trade-offs of smart antennas in CDMA cellular networks", 45th IEEE Veh. Tech. Conf., Vol. 1, July 1995, pp. 40-44.

[29] A.F. Naguib, B. Suard, "Performance of CDMA mobile communication systems using antenna arrays", IEEE Int. Conf. on Acoustics, Speech, and Signal Processing, ICASSP-93, Vol. 4, Apr. 1993, pp. 153-156.

[30] A.F. Naguib, A. Paulraj, "Performance of CDMA cellular networks with base-station antenna arrays: the downlink", IEEE Int. Conf. on Comm., SUPERCOMM/ICC '94, May 1994, pp. 795 -799.

[31] A.F. Naguib, "Power control in wireless CDMA: Performance with cell site antenna arrays", IEEE Global Telecommunications Conf., GLOBECOM '95, IEEE, Vol. 1 , Nov. 1995 , pp. 225-229.

[32] R.B. Ertel, S.V. Schell, "Comparative study of adaptive antenna arrays in CDMA communication systems", Nov. 1998, pp.1-9.

[33] I. Rivas, L.J. Ibbetson and L. Lopes, "Macro-cellular reception performance investigation in microcellular networks", 47th IEEE Veh. Tech. Conf., Vol. 3, May 1997, pp. 1503–1507.

[34] U. Weiss, "Designing macroscopic diversity cellular systems", 49th IEEE Veh. Tech. Conf., Vol. 3 , July 1999, pp. 2054 – 2058.

[35] M. Juntti, "Performance analysis of linear multi-user receivers for CDMA in fading channels with base station diversity", IEEE Trans. Veh. Tech., Sept. 1999, pp2845-2849

[36] M. Juntti, "Performance analysis of linear multi-sensor multi-user receivers for CDMA fading channels", IEEE Journal on Selected Areas in Comm., Vol. 18. No. 7, Jul. 2000, pp. 1221-1229.

[37] W. Papen, "Uplink performance of a macro-diversity cellular mobile radio architecture", Sixth IEEE Int. Symp. on Pers. Indoor and Mobile Radio Comm., PIMRC'95, Vol. 3 , 27-29, Sept. 1995, pp. 1118-1122.

[38] S. Zurbes, "Outage probabilities and handover characteristics of simulcast cellular mobile radio systems", 46th IEEE Veh. Tech. Conf., Vol. 1 , May 1996, pp. 522-526.

[39] S. Zürbes, "Power control in simulcast digital cellular radio networks", 8th IEEE Int. Symp. on Pers., Indoor and Mobile Radio Comm., PIMRC '97, Vol. 3, Sept. 1997, pp. 887-991.

[40] D.C. Hastings, H. M. Kwon, "Soft Handoffs in code division multiple access systems with smart antenna arrays", IEEE Veh. Tech. Conf., Sept. 2000, pp. 181-188.

[41] D.C. Hastings, H. M. Kwon, "Optimization of sector orientation in CDMA communication architectures based on soft handoff", MILCOM 2000, 21st Century Military Communications Conference Proceedings, Vol. 2 , Oct. 2000, pp. 826- 830.

[42] W. Yung, "Direct sequence spread spectrum code division multiple access cellular systems in Rayleigh fading and log-normal shadowing channel", IEEE Int. Conf. on Comm., ICC 91, June 1991, pp. 871-876.

[43] D.G. Gerlach, A.Paulraj, "Adaptive transmitting antenna arrays with feedback", IEEE Signal Proc. Letters, Vol. 1, No. 10, Oct. 1994, pp. 150-152.

[44] S. Nagaraj, Y. Huang, "Downlink transmit beamforming with selective feedback", Conference Record of the Thirty-Fourth Asilomar Conf. on Signals, Systems and Computers, Vol. 2 , Nov. 2000, pp.1608-1612.

[45] R.B. Ertel, "Antenna array systems: propagation and performance", Preliminary review of initial research and proposal for current and future work towards PhD, Dec. 1998.

[46] R.B. Ertel, "Overview of spatial channel models for antenna array communication systems", IEEE Pers. Comm. Mag., Vol. 5, No.1. Feb. 1998, pp. 10-22.

[47] M. Larsson, "Spatio-temporal channel measurements at 1800 MHz for adaptive antennas", 49th IEEE Veh. Tech. Conf., 1999, Vol. 1 , July 1999, pp. 376-380.

[48] J. Laiho-Steffens, A. Walker, "Experimental evaluation of the two dimensional mobile propagation environment at 2GHz", 47th IEEE Veh. Tech. Conf., 1997 Vol. 3, May 1997, pp. 2070-2074.

[49] W.C.Y. Lee, *Mobile cellular telecommunications: analog and digital systems*, McGraw-Hill, Second Edition, 1995.

[50] T. S. Rappaport, *Wireless Communications: Principles and Practices,* Prentice-Hall, 1996.

[51] J. Ylitalo, M. Katz, "An adaptive antenna method for improving downlink performance of CDMA base stations", IEEE 5th Int. Symp. on Spread Spectrum Tech. and App., Vol. 2, Sept. 1998, pp. 599-603.

[52] A. Lopez, Performance predictions for cellular switched-beam intelligent antenna systems", IEEE Comm. Magazine, Oct. 1996, pp. 152-154.

[53] J. Litva, T. Lo, *Digital Beamforming in Wireless Communications,* Artech House, 1996, Ch. 3, pp. 42 - 43.

[54] L.C. Godara, "Applications of antenna arrays to mobile communications, Part II:, Beamforming and Direction of arrival Considerations", Proc. IEEE , Vol. 85, No. 8, Aug. 1997, pp. 1195-1245.

[55] R. Kohno, "Spatial and temporal communication theory using adaptive antenna array", IEEE Pers. Comm., Vol.5., No. 1, Feb 1998, pp. 28-35.

[56] J.C. Liberti, T. S. Rappaport, *Smart antennas for wireless communications: IS-95 and third generation CDMA principles,* Prentice Hall, New Jersey, 1999.

[57] R. Monzingo, T Miller, *Introduction to Adaptive Arrays,* John Wiley and Sons, 1980, Chapter 4, pp. 166-168.

[58] J.E. Hudson, *Adaptive Array Principles,* Peter Peregrinus, 1981, Chapter 3, pp. 58-75.

[59] P.A. Ranta, A. Lappetelainen, H. Zhi-Chun, "Interference cancellation by joint detection in random frequency hopping TDMA networks", 5th IEEE Int. Conf. on Universal Pers. Comm., Vol. 1 , 1996, pp. 428 -432

[60] M. R. Spiegel, *Mathematical handbook of formulas and tables*, NcGraw-Hill, 1968.

[61] S.M. Kay, Fundamentals of Statistical Signal Processing: Estimation Theory, Englewood Cliffs, NJ: McGraw-Hill, 1991.

[62] F.G. Stremler, *Introduction to communication systems*, Addison-Wesley, 1982, p. 446.

[63] W.C. Jakes, *Microwave mobile communications,* IEEE Press, New York, Jan. 1994.

[64] J. Gorricho, Paradells, J, "Evaluation of the soft handover benefits on CDMA systems", 5th IEEE Int. Conf. on Universal Pers. Comm., Vol. 1, Sept. 1996, pp. 305-309.

[65] W. Jianming, S. Affes, P. Mermelstein, "Forward-link soft-handoff in CDMA with multiple-antenna selection and fast joint power control" IEEE Trans. on Wireless Comm., Vol. 2., Issue 3 , May 2003, pp. 459 –471.

[66] G. Strang, *Linear algebra and its applications,* third addition, Harcourt Brace Javanovich, 1988.

[67] D.H. Johnson and D.E. Dudgeon, *Array signal processing: Concepts and Techniques,* Englewood Cliffs, NJ: Prentice-Hall, 1993.