CHAPTER 9: INTERPRETATIONS AND CONCLUSIONS

In this chapter, the results for the investigation into students' learning difficulties involving VSOR are interpreted in terms of the five skill factors of knowledge, consisting of 11 elements, as discussed in the conceptual framework in Chapter 3. The research question for this study: ***Why do students have difficulty when learning about volumes of solids of revolution?****, is answered based on performance in the five skill factors of knowledge, being*

- *the graphing skills and translation between visual graphs and algebraic equations/expressions*
- *translation between two-dimensional and three-dimensional diagrams*
- *translation between continuous form and discrete form*
- *general manipulation skills and*
- *the consolidation and general level of cognitive development*

The interpretations are extended to categorising the skill factors in terms of procedural knowledge and conceptual knowledge, how VSOR is taught and assessed, how the different elements correlate and related to the studies discussed in Chapter 2. Finally, the recommendations, the limitations and the conclusions are discussed.

9.1 OVERVIEW OF THIS RESEARCH

The aim of this study was to investigate students' learning difficulties with VSOR. From the motivation of this study, discussed in Chapter 1, I highlighted the problem I experienced in teaching VSOR for eight years and marking the N6 examinations for three years. I argued that one of the problems experienced in VSOR might be as a result of the lack of in-depth knowledge of the Fundamental Theorem of Calculus (FTC) and its application to calculus. The question that I established was: **Why do students have difficulty when learning about volumes of solids of revolution?** In an attempt to address the research question for this study, I focused on three issues. I identified what the VSOR content entailed from the textbooks and the previous examination papers to aid in the development of the assessment instrument under five skill factors which were subdivided into 11 elements, comprising 23-items; observed how the content was taught (own teaching and that of others), learnt and assessed so as to establish where the difficulties emanate from and to suggest possible ways of improving learning of VSOR.

The three modes of representation of knowledge, visual/graphing; algebraic/symbolical and numerical (Tall, 1996; Habre & Abboud, 2006) were used to develop the conceptual framework for this study, discussed in Chapter 3 under the five skill factors that became the subquestions for this study. In Chapter 4 the methods of data collection and analysis for this study were discussed using a mixed method approach. In Chapter 5, the results for the preliminary and the pilot studies were discussed and modified in order to design the main data collection instrument for this study.

For the main data collection, six investigations, on why students have difficulty when learning about VSOR were carried out as follows:

- Questionnaire 1st run with 37 respondents for 23 questions (Chapter 6).
- Questionnaires 2nd run with 122 and 54 respondents for 16 questions and 7 questions respectively (Chapter 6).
- Examination response analysis with 151 respondents (Chapter 6)
- Detailed selected written examination responses with seven students (Chapter 6).
- Classroom observations of one class with about 40 students (Chapter 8).
- Interview with a previous N6 student (Chapter 8).

The average rank scores obtained from the 11 elements, were correlated to determine if they were associated with one another (discussed in Chapter 7). With the results from the preliminary and the pilot studies, the six investigations and the correlations, the results were triangulated.

9.2 ADDRESSING THE RESEARCH QUESTIONS FOR THIS STUDY

The aim of this study was to investigate students' learning difficulties with VSOR, resulting in the main research question as follows: **Why do students have difficulty when learning about volumes of solids of revolution?**

In order to address the main research question in this study, the following subquestions were established under the five formulated skill factors, also relating to conceptual understanding and procedural understanding as well as how VSOR was taught and assessed.

1. **Skill factor I: How competent are students in graphing skills? How competent are students in translating between visual graphs and algebraic equations/expressions in 2D and 3D?**
2. **Skill factor II: How competent are students in translating between two-dimensional and three-dimensional diagrams?**
3. **Skill factor III: How competent are students in translating between discrete and continuous representations visually and algebraically in 2D and in 3D?**

4. **Skill factor IV: How competent are students in general manipulation skills?**
5. **Skill factor V: How competent are students in dealing with the consolidation and the general cognitive demands of the tasks?**
6. **Teaching and assessment: How is VSOR taught and assessed and how does that impact on learning?**

In the section that follows the main research question in this study is addressed under the six headings. The results for the six different investigations as well the preliminary and the pilot studies are compared and contrasted where they are applicable under each heading in terms of the different performance levels. The performance levels discussed are for the different skill factors and/or elements where applicable. The result of an interview with a previous N6 student is discussed only under the subquestion that focuses on how VSOR is taught and assessed and how that impacts on learning. Finally the correlations are discussed.

9.2.1 Skill factor I: *How competent are students in graphing skills? How competent are students in translating between visual graphs and algebraic equations/expressions in 2D and 3D?*

In order to address the research question, students' performance in graphing skills as well as translating between visual graphs and algebraic equations/ expressions were explored. Graphing skills require procedural understanding while translating between visual graphs and algebraic equations/expressions require conceptual understanding. The results of the Questionnaire 1st run revealed that the students' performance in graphing skills and in translating between visual graphs and algebraic equations/expressions in 2D and 3D was satisfactory. This performance implies that the students have some ability in graphing skills and in translating between visual graphs and algebraic equations/expressions in 2D and 3D. However, in both runs of the questionnaire, the students' performance in drawing graphs was not satisfactory. Failure to draw graphs could imply that the students might have forgotten what they learnt in previous levels or that they were not properly taught or that they have learnt it without understanding.

Under Skill factor I, most of the students were struggling when translating from algebraic to visual in 3D, especially in the Questionnaire 2nd run, where the performance was poor and the students were not competent. The results of the questionnaire runs, the preliminary study and pilot study, revealed that the majority of the students were only able to draw simple graphs. The students were only able to draw graphs that they were familiar with, but struggled to draw the graphs that they were unfamiliar with or given as word problems. The students struggled more when the questions from the questionnaire runs involved the symbolic representation of integrals, like drawing a graph(s) or object(s) that represents $V = \pi\int_0^1 (1-x)^2\ dx,$ where they had to translate from algebraic to visual. Students' failure in responding to questions similar to this one revealed that the students did not comprehend the disc method conceptually. They could only translate a given visual graph to represent a formula for a disc but could not revert to the original graph if an equation representing a disc was given. The students do not have conceptual understanding of the integral formula for area and volume; they can only use it as given from the formula sheet, without interpreting it critically.

Similar to the results of my study, a study by Samo (2009) gives evidence that students' difficulties in algebra could be related to their difficulties and misinterpretation of symbolic notations. In Montiel's (2005) study, similar to the results from all the questionnaire runs, most of the students were not able to interpret the given integral notation. They failed to translate from algebraic to visual, except in cases where simple or familiar equations were given. When asked to draw the 3D solids from the equations of volume, given as an integral for a disc or a shell, students were seen to draw 2D diagrams. These results reveal that perhaps the students were not familiar with drawing solids of revolution or that they operate with symbols without relating to their possible contextual meanings (White & Mitchelmore, 1996). The results of this study supports the findings by Maharaj (2005) that many students perform poorly in mathematics because they are unable to handle information given in symbolic form adequately.

Despite the difficulties encountered, a large majority of students were seen, from the questionnaire runs and the pilot study, to succeed in translating from visual graphs to algebraic equations in 2D, especially if a Δx strip was appropriate and in 3D if the rotation resulted in a disc or a washer. Most of the students avoided using a Δy strip as well as the shell method. For those students who tried to use a shell method, errors were found as the students failed to use correct limits of integration, when applying the Fundamental theorem of Calculus. The results also revealed that most of the students were more able to translate from visual to algebraic representation than from algebraic to visual representation, especially in 3D. This means that the students were more competent in interpreting the drawn graphs, resulting in a formula for volume, rather than drawing the graphs from an equation of a 3D representation (volume) of a certain graph. Similar results were found when working in 2D. Contrary to the above results the results of the preliminary study revealed that the students were, in most cases, unable to translate from visual graphs to algebraic equations, when they calculated volume, since they abandoned the drawn graphs.

The results from the examination response analysis revealed that students' performance in drawing graphs was satisfactory. From the detailed selected written examination responses, students' performance was good. In contrast, during the classroom observations, it appeared as if in some instances, students' performance in drawing graphs was satisfactory, whereas in other cases they had difficulties, evident from their reliance on the table method when drawing graphs. The students' performance in drawing graphs was satisfactory (examination responses analysis) and good (detailed examination responses) probably because simple graphs were tested. However, the performance was not satisfactory when the students were required to translate from the drawn graphs to algebraic representation, when using the selected strip to give rise to the algebraic formula for volume. From the classroom observations it appeared that the lecturer did not teach the students how to draw graphs as it was expected that they had done them in-depth at previous levels, but focussed more on the translation of the drawn strip to algebraic equations for volume (disc, washer or shell), which students were seen to be partially competent in during the classroom discussions. When working on their own, most of the students had difficulties when translating from visual to algebraic, especially in 3D.

There were no instances during the examinations and the classroom observations where students were expected to translate from algebraic equations to visual graphs where integral equations were used. It appears as if these students were not adequately competent in Skill factor I, probably because they could not interpret their drawn graphs, or at times abandon them, since they lacked spatial abilities involving three-dimensional thinking. If students have spatial abilities they will be able to "generate, visualize, memorize, remember and transform any kind of visual information such as pictures, maps, 3D images" (Menchaca-Brandan, Liu, Oman, & Natapoff, 2007, p. 272), without any problem. The result of a study by Dettori and Lemut (1995) concurs with my results. Their study also revealed that students could not use diagrams and they attributed that to some blockages, referred to as *cognitive obstacles*.

From the above discussions, a conclusion can be made that students have difficulty in learning VSOR in part because they are not fully competent in graphing skills and translating between visual graphs and algebraic equations/expressions in 2D and 3D. Overall from all the investigations, generally students have difficulty in interpreting the drawn graphs. They also have difficulty especially when translation is between visual and algebraic in 3D.

9.2.2 Skill factor II: *How competent are students in translating between two-dimensional and three-dimensional diagrams*?

Translating between 2D and 3D diagrams requires conceptual understanding. From the Questionnaire 1st run, it appears that students were able to translate between 2D and 3D when simple diagrams were given, but struggled to rotate when given diagrams that required that they imagine the rotations, which requires conceptual skills. For example most students failed to draw a torus that could result after rotation of a circle that was a certain distance away from the *x*-axis and from the *y*-axis, involving translation from 2D to 3D. The ability to translate properly depends on the students' visualisation ability, a kind of mathematical reasoning activity based on the use of spatial or visual elements (Gutiérrez, 1996) that can assist a student to imagine the rotations. The results from the study conducted by Gorgorió (1998) revealed that students used 2D drawings to represent 3D objects when interpreting 2D representations of 3D objects.

From the Questionnaire 1st run, most of the students' performance was not satisfactory in translating from 2D to 3D when they were expected to draw solids of revolution. When the tasks involved translation from 3D to 2D, in cases where the students were expected to represent 2D diagrams from the given 3D diagrams, the performance was satisfactory. In contrast, the results from the Questionnaire 2nd run reveal that the students' performance was poor when translating from 3D to 2D. As for the pilot study, the results reveal that the students' performance in translating between 2D and 3D was poor. The results of the preliminary study revealed that only after using Mathematica, more students were able to rotate the strip correctly, even though its 3D representation was not drawn. Overall, from Questionnaire 1st run, the performance in three-dimensional thinking, where the students were required to translate between 2D and 3D was not satisfactory.

The classroom observations did not include problems where 2D diagrams were rotated to form 3D diagrams or where 3D diagrams were used to determine which 2D diagrams they originated from. The rotations that were demonstrated were when representative strips were rotated about a particular axis, resulting in a shell, washer or disc, without drawing the exact solids of revolution. The lecturer gave a clear explanation about the strip being parallel or perpendicular to a certain axis and the shape it would generate after rotation, hence enhancing visual skills. Alias et al., (2002) point out that if spatial activities are emphasised during teaching, then students' spatial visualisation ability is enhanced. From the solutions for the written examination and most of the examination analysis, in most cases, strips were indicated on the diagram but they were not drawn after rotation as representing a disc, a washer or a shell. The solids of revolution formulated were never drawn. Failure to draw the translated diagram from 2D to 3D in this case interferes with the ability to develop visual and imaginative skills, necessary if mental images are made about rotations (Dreyfus, 1995), and to derive the formula for volumes from the diagrams and not from the formula sheet.

From the above discussions, a conclusion can be drawn that students' performance was not satisfactory in Skill factor II, which involves translating between two-dimensional and three-dimensional diagrams. Even if at times the students drew the strip correctly, they could not rotate it correctly as a result of failing to translate between 2D and 3D, or even interpret a 3D diagram to determine which 2D diagram it originated from. If students have difficulty in solving problems that involve three-dimensional thinking, then learning VSOR can be problematic.

9.2.3 Skill factor III: *How competent are students in translating between continuous and discrete representations visually and algebraically in 2D and in 3D?*

The performance on translating between continuous and discrete representations visually and algebraically in 2D and in 3D (both requiring conceptual understanding) in the first and the second run of the questionnaire and the pilot study was poor, revealing that the students lacked competency in this area. It was revealed that almost all of the students struggled with the translation from the algebraic expressions to the discrete approximation of area and volume, as a result of students not being familiar with the concept of Riemann sums. Only a few students managed to approximate the area by using rectangles and volume by using discs. Similar to the results of my study, students in Montiel's (2005) study were also seen to use the rectangles for approximation of the area inappropriately. The results from a study by Orton (1983) also concur with the results from my study. It was revealed that most students had little idea of the procedure of dissecting an area or volume into narrow sections, summing the areas or volumes of the sections. From a study by Camacho and Depool (2003), some students were seen to be in a position to use the Riemann sums. The lack of competency in translation between continuous and discrete representations visually and algebraically in 2D and 3D, as it was the case in my study, can be as a result of an absence of proper concept images (Harel et al., 2006).

During the classroom observations and the examinations, the concept of Riemann sums was never dealt with as it was assumed that the students had done them on the previous levels. However, from the responses that students produced, it seems that even if the concept of

Riemann sums had been done, it might have been at a more procedural level, without relating the continuous to the discrete representations. From the examination response analysis, the detailed selected written examination responses and during the classroom observations, many students had problems when they had to select the representative strip. They did not know when the strip should be vertical or when should it be parallel to a certain axis. For that reason, the students were unable to translate between continuous and discrete representations. In all three investigations, the students' performance in selecting the representative strip was not satisfactory. Similarly to Montiel's (2005: 101) study, as during the classroom observations, some students did not know when to use a Δx strip and when to use a Δy strip. In that case the difficulties in learning VSOR arise from the fact that the students cannot translate between the continuous and the discrete representation. Based on the overall poor performance, it means that the concept of Riemann sums, which is crucial to the learning of VSOR, is lacking. In order for the students to do well with VSOR, the concept of Riemann sums should be dealt with at a level where conceptual understanding is reinforced starting from N4 to N6.

9.2.4 Skill factor IV: *How competent are students in general manipulation skills?*

General manipulation skills require procedural understanding. The results from the questionnaire runs, the examination response analysis, the classroom observations, the pilot study and the preliminary study showed that the students performed satisfactorily in problems that required general manipulation skills. Even if the performance was satisfactory, errors were made when students solved problems that involved evaluation of integrals using integration techniques. From the classroom observations, it was observed that generally, the students struggled with integration by parts and the substitution method involving a square root. Even if the integral to be evaluated is not difficult to calculate, students already had a negative attitude towards integration. Gonza 'Lez-marti'n and Camacho (2004) assert that even 'simple' calculation of integrals causes problems for the students since most of them perceive integration as cognitively demanding to the extent that they even develop a negative attitude even towards the simple exercises.

Errors were also made when calculating the point of intersection of graphs, where in some instances students were unable to interpret the square root of negative numbers. The nature of errors made may be as a result of the students' lack of the mathematics register and probably because their knowledge in mathematics rules is superficial. The problem with general manipulation skills was also evident in Montiel's (2005) study. Similar to the results of my study, some students in Montiel's (2005) study had problems expressing the functions "in terms of *x*" or "in terms of *y*". Students encounter difficulties in problems requiring general manipulation skills because their mathematics content knowledge is too low. They lack the necessary mathematics register. However, in a study by Haciomeroglu et al. (2010), students succeeded without difficulties with procedural tasks that involved the computation of an integral.

In contrast, the results from the detailed selected written examination responses reveal that the students' performance in general manipulation skills was good. Not so many errors were made, as it was the case during the classroom observations.

9.2.5 Skill factor V: *How competent are students in dealing with the consolidation and general cognitive demands of the tasks?*

This skill factor involves problems that require both procedural and conceptual understanding included in Skill factors I, II, III and IV. The results from all the investigations as well as the pilot study, reveal that students lack the cognitive skills required to solve problems under Skill factor V, since they lack conceptual understanding of the VSOR content required in Skill factor II and Skill factor III. Students' failure in identifying the strip correctly and drawing the 3D diagram of the strip resulting after rotation to show the solid of revolution formulated, may hamper success in dealing with the problems that require the consolidation ability and general level of cognitive development, hence leading to poor performance when tested on the threshold concepts (Pettersson et al., 2008) in learning VSOR.

The results reveal that from the other skill factors, students' performance was poor in Skill factors II and III, and satisfactory in Skill factors I and IV. If the students' performance in the Skill factors II, and III is so poor, how do they then manage to solve problems that require application using these skill factors? The students have difficulties in Skill factor II and Skill factor III, since they require conceptual understanding, which they lack. The students also have difficulties since they have to start by calculating the important points on the graphs and drawing the graphs, which they perform satisfactorily in and at times make errors; select the representative strip and continue to rotate the selected strip which they are not competent in (in cases where the volume is to be calculated) and to interpret it so as to come up with the formula to calculate volume. These results reveal that the main reason that students struggle with Skill factor V is because they cannot select the correct representative strip as well as rotating it properly. The fact that they do not usually draw the solid of revolution generated reduces their chances of interpreting what is visual accordingly. The students' failure in this case is because they have not developed cognitively to the correct level.

The results of the examination response analysis revealed that in most cases, students' performance deteriorated from the first question, where they were asked to draw graphs, to its subquestions, when they were asked to interpret the drawn graphs, as in the performance from the written examination responses. Students managed in some instances to draw graphs but failed to interpret those graphs after the selection of a rectangular strip. These students tended to abandon the drawn graphs and the drawn strip when selecting the formula for area or volume. The general conclusions that could be made from the examination analysis are that students' performance in drawing graphs was satisfactory, but were unable to interpret the region bounded by such graphs, as is evident from their incompetency in selecting the representative strip, translating from visual to algebraic in 2D and in translating from visual to algebraic in 3D, which was performance that was not satisfactory.

The problem is that if a student fails to draw the graphs correctly and select the strip correctly and fails to interpret it correctly in selecting the correct formula, then the rest of the answers that follow from the drawn graphs are likely to be incorrect. In that way the general manipulation skills performed thereafter would also be incorrect as it depends on the preceding steps. The majority of these students were unable to deal with the cognitive demands of the task, hence lacked the skills in the consolidation and general level of cognitive development. The performance in the examination response analysis was not satisfactory.

From the classroom observations, one can conclude that students were not mathematically adequately proficient (Kilpatrick et al., 2001), as they struggled to draw graphs, failed in most cases to select the proper strip, had problems in translating from a visual representation to an algebraic representation and made many errors in their calculations, but succeeded only through scaffolding. These students struggled because they lacked conceptual understanding, had limited procedural fluency, limited strategic competence and no adaptive reasoning. The results from the classroom observations also revealed how problematic VSOR can be in terms of the consolidation and general level of cognitive development required. This was evident during the classroom observations as even the lecturer could not solve a problem that required the use of the substitution method in evaluating an integral derived from an equation of a hyperbola. VSOR requires more time for students to conceptualise the cognitively demanding aspects. Eisenberg (1991) argues that the abstraction of the new mathematical knowledge and the pace with which it is presented often become the downfall of many students. The content at the N6 level seem to be too abstract for students, hence they cannot meet the cognitive demands of the VSOR content. It seems as if a reason why students have difficulty is that their background level is not at an appropriate level, or that the VSOR content is too abstract for them. The results discussed above make explicit the fact that competencies in all other four skill factors hinder or prohibit success in solving VSOR problems. As a result students encounter difficulties in learning VSOR. Students' difficulty may also be as a result of the lack of the recognition and the realisation rules or failing to realise the speciality of the context that one is in (Bernstein, 1996), because VSOR is above the students' cognitive abilities.

The difficulty in VSOR can as well be explained from the average mark that the students scored in Question 5, from the examinations analysis, falling under the consolidation and general level of cognitive development. It appears that students' performance, with a mean of 15.4 (38.5%) for Question 5 which was marked out of 40 was not satisfactory. This mean percentage for question 5 is less than the mean percentage (45.5%) of the whole examination, implying that question 5 was difficult.

The results of this study raise questions about the level of difficulty of content that the N6 students must study and the career paths they must follow. If most of these students did not meet the requirements of being at the university of technology, then why do they have to study this difficult content at the FET college, which does not even qualify them to be accepted at universities. Only a few of these students who get above 50% per subject including mathematics qualify to write the government certificate of competency which qualifies them as certificated engineers.

The mathematics content learnt at the FET colleges from N5 to N6 levels is more advanced and more abstract as compared to the school mathematics learnt in Grade 11 to 12. They do complex differentiation and integration which should qualify them to be accepted at universities, as they come with a proper background, provided they pass it with high marks. In this regard, students who completed N6 mathematics with high marks are more competent than those students with a matric qualification who even fail the National Benchmark Test (NBT) at universities for entrance into engineering and science fields because of the lack of in-depth knowledge about basic engineering concepts which the FET college students have. However, the fact that there was no instance during this study where the students' performance was good or excellent in any of the five skill factors, imply that most of the students at the FET colleges lack the necessary mathematics background, only a few may qualify to be at universities.

9.2.6 Teaching and Assessment: *How is the VSOR content taught and assessed and how does that impact on learning?*

The question on how VSOR is taught has already been addressed in the six subquestions above when classroom observations were discussed.

The way in which the VSOR content is assessed create a huge burden for lecturers who are expected to teach new concepts required for the application of areas and volumes including centroids and centre of gravity. The burden here is that these lecturers are compelled to start by reinforcing concepts that were done previously including drawing graphs and using the Riemann sums (based on the prior knowledge that these students have) in a very short space of time. The new concepts that are to be taught can only be done properly if the students draw correct graphs, which they normally manage to draw if they are familiar with them and to select the representative strip with which they struggle extremely.

When assessing VSOR in the final N6 examinations, the VSOR content assessed in Question 5 focuses on five elements only namely: general manipulation skills; graphing skills; translation from continuous to discrete (visually); translation from visual to algebraic in 2D and translation from visual to algebraic in 3D. It is clear from the N6 examination question paper and the memorandum that questions where translating between two-dimensional and three-dimensional diagrams, where students are given marks for drawing a solid of revolution generated are not assessed. For that reason, the lecturers do not normally teach this section properly. From the classroom observations it was evident that the lecturer did not emphasise that students should draw the solids of revolution generated, even if it is shown in textbooks. In that way the rotation of the strip is not learnt in-depth and used visually as a starting point to generate the formula for volume. Students are influenced to rely on the formula sheet instead of visualising the formula for the disc, washer or shell from the drawn diagram, also the one for area.

As highlighted under Skill factor V, in order to do well in Question 5 one must be competent in graphing skills. Students must draw graphs first and then interpret them based on the requirements of the questions that follow. The high correlation of students' performance in graphing skills to the other elements reveals that the way in which the students are assessed in Question 5 is problematic as it starts by requiring the students to draw graphs which they may fail to draw. Hence the students may fail to respond to the questions that follow, based on the drawn graphs and how they interpret them. So the question is, do we teach the drawing of graphs and interpretations of the drawn graphs properly again as these students' capabilities are very low, or do we change the way of assessment that can accommodate the students who cannot draw or interpret graphs? Advice for teachers is that they must assist students to link new knowledge to their prior knowledge and develop instructional techniques that would facilitate cognitive growth and change (Kotzé, 2007), and not to focus on general manipulation skills. A study by Pettersson et al. (2008:781) reveal that students should be taught threshold concepts of calculus like the concepts of limit and integral in order to develop conceptual understanding. In in this study the threshold concepts involve the interpretation of graphs, based on three-dimensional thinking and the Riemann sums.

Another aspect that may impact on learning difficulties is the attitude that students have towards VSOR, the fifth strand of mathematical proficiency called *productive disposition* (Kilpatrick et al., 2001). The interviewed student showed a productive disposition. This student really enjoyed learning VSOR and appreciated the way in which it applies to other subjects and its meaning. She pointed out that that the other students should not be discouraged that VSOR is difficult, because if they agree to that, they will never succeed. The student interviewed highlighted that it is important that the students be able to draw a graph properly, to select a correct strip and to rotate it properly in order to do well in VSOR. She mentioned that without a sketch (graph) you are lost, as the formula that one uses is derived from the sketch. According to the student interviewed, one must visualise the bounded region and interpret it. However, she also mentioned that if questions were given where graphs were drawn, without students having to start by drawing graphs, then VSOR would be easy, especially for those students who struggle to draw graphs. The results from the interview reveal that students have difficulty with VSOR because they struggle to draw graphs and mainly to interpret them. The student also stated that students' attitude may also hamper success in learning VSOR.

9.2.7 Correlations

Most of the correlations from the questionnaire runs were not statistically significant. From the Kendall tau correlation coefficient, the correlations that were highly significant at 1% level, were those correlating the consolidation and general level of cognitive development to other six elements, namely: translation from continuous to discrete (visually); translation from discrete to continuous and from continuous to discrete algebraically; graphing skills; translation from visual to algebraic in 3D; translation from 2D to 3D and the translation from

3D to 2D. Such an association between the consolidation and general level of cognitive development to these six elements points out the strong association between consolidation and general level of cognitive development and performance in these elements. This performance is related to how graphs are drawn and interpreted, for example, selecting a strip and translating it to an equation for area or volume, also involving three-dimensional thinking. Such an association between the consolidation and general level of cognitive development and for example the translation from continuous to discrete representations, points out how important the selection of representative strip is to the ability to perform better in a question that requires consolidation and general level of cognitive development. However, the element involving general manipulation skills shows correlations that are not significant in relation to the rest of the elements. What this implies is that being capable of solving problems requiring general manipulation skills has no association with how one performs in the other elements.

In contrast, from the examination analysis, also using Kendall tau correlation coefficient, the correlations of the five elements, general manipulation skills; graphing skills; translation from continuous to discrete (visually); translation from visual to algebraic in 2D and translation from visual to algebraic in 3D are all highly significant at 1% level. These results reveal that the way in which Question 5 is assessed is problematic, compared to the 23-item instrument, since all other five elements correlate significantly to each other. For example, as performance in graphing skills, increases, then the performance in all other elements also increases, which is affected by the way in which question 5 has been assessed. The correlation of 0.852, using Pearson correlation coefficient, alludes to the significance of these correlations from the examinations.

9.3 ANSWERING THE RESEARCH QUESTION FOR THIS STUDY

From the interpretations of the results discussed under the subquestions in Section 9.2 above, I attempt to answer the research question for this study: **Why do students have difficulty when learning about volumes of solids of revolution?**

It was found that students lack graphing skills. They only manage to draw simple graphs, many at times using the table method to plot the points. Hjalmarson et al. (2008) argue that graphing representations play a significant role in conceptual understanding within upper-level applied mathematics. They believe that students need to be able to interpret and generate graphs (which the students in my study struggled with) as part of their mathematical reasoning. In that way they may develop some cognitive skills. If students are not taught the characteristics of a graph and are rather taught to rely on the table method, then they waste time and will most probably fail in identifying the important characteristics of the particular graph. In this case, the use of a table method becomes procedural and a source of the difficulty for students. Due to the scope of the VSOR content, the use of the table method becomes too time consuming; hence students might not be able to finish answering all the questions.

Students do not learn about solving VSOR problems conceptually, hence perform better when they translate from visual graphs to algebraic equations (which was emphasised in class when they chose formulae from the formula sheet) and struggled to translate from algebraic equations/expressions to visual graphs (the reverse of the first process) because they were never taught to deal with such problems when they had to draw a diagram represented by an integral formula. The use of a formula sheet is an additional source of difficulty, since instead of students learning to develop the formula for area or volume from the drawn graphs or diagrams (conceptually) they rely on the formula sheet. In that case, the development of visual skills is compromised, serving as a source of learning difficulties. Students experience difficulties because instead of lecturers focusing on developing conceptual skills, they tend to concentrate on general manipulation skills, which students do not have major difficulty in, evident from all investigations.

Students struggle to solve problems that involve three-dimensional thinking. The main reason might be because three-dimensional thinking requires spatial visualisation abilities, which these students do not seem to have. Students had difficulty in learning about VSOR because they were not able to deal with problems in which they were required to imagine the rotations as well drawing solids of revolution. The most critical aspect that students struggled with was to identify the strip that would be best to approximate the region bounded by the graphs. This failure resulted in failing to use Riemann sums and applying the FTC, also failing in rotating the selected strip correctly and representing the rotated strip as a 3D diagram. If some students did not have a lecturer at N5 level, as it was stated during the interview and the classroom observations, then these students were not taught the basic concepts that involve three-dimensional thinking regarding the VSOR, and the basic knowledge of what a solid of revolution is, hence the difficulties. Seemingly, these students did also not learn the basic skills necessary for selecting a representative strip. The content that students do at lower levels before they do N5 and N6, does not prepare them adequately in three-dimensional thinking, to meet the challenges of the complex VSOR content. Due to the limited time that the N6 lecturers have to complete the syllabus in, the topic of VSOR is learnt procedurally, hence making it difficult for these students who come with the poor mathematics background, to cope. Such students are not given enough time to develop cognitively to be in a position to deal with the complex VSOR content.

With regard to general manipulation skills, many mathematical errors led to students' inability to obtain the final answer correctly, including drawing the correct graphs, which led to the steps that follow being incorrect. Students also at times encountered difficulties because they do not know the techniques of integration and do not possess a sufficient mathematics register, due to their poor mathematics background. The difficulties that students have with VSOR are as a result of the errors that they make when performing calculations and not being familiar with using correct integration techniques. The fact that lecturers also at times find it difficult to use some techniques of integration, add to the difficulties that students have. Such lecturers are unable to reach out to students and make the VSOR content accessible for students.

Perhaps the main problem that students experience is that the topic of VSOR is of too high a general cognitive level for these students. Students find it difficult to deal with questions in which all four other skill factors are consolidated in one question, possibly because the content that students do at lower levels before they do VSOR, does not prepare them adequately in three-dimensional thinking, critical and logical thinking, complex problem solving techniques and basic mathematics content to meet the challenges of the complex VSOR content.

9.4 RECOMMENDATIONS OF THE STUDY

Some recommendations resulting from this study are subsequently indicated for lecturers, examiners, curriculum developers and the department of education.

9.4.1 Teaching the VSOR content

Lecturers should teach the topic of VSOR more conceptually and design questions that encourage students to engage with problems more conceptually. Lecturers must encourage students to draw graphs or diagrams from the given equations/expressions, including formulae given as integrals representing areas and volumes of the shaded region and the limits of integration, that is, work in the opposite direction to the usual way of the examination paper. The focus should be on how the formulae derive from the diagrams (2D and 3D) rather than calculating areas or volumes. That is, translate the given graphs or diagrams from visual to algebraic as well as from visual to visual in 2D and in 3D to make students conceptually capable, not just procedurally.

Students should be encouraged to draw a solid of revolution after rotating a given diagram (region bounded by graphs) from 2D to 3D and to show a 3D diagram formed after rotating the representative strip, as it is in Skill factor II. Students should also be encouraged to draw graphs from the given formula in integral form that represent area and volume of a disc, washer or shell, as the students translate from 3D to 2D. Most importantly, students should be encouraged to use Riemann sums to approximate the bounded region to justify whether the rectangular strip should be vertical or horizontal as it is tested in Skill factor III. In that way, they are not just choosing any strip that they prefer to work with, but have to justify their choice. In teaching VSOR, emphasis should not be on performing the calculations for area and volume, but on development of conceptual skills, where mainly solids of revolution are drawn. As was evident from the correlations between the different elements, calculations performed are not associated with the skills required in the other elements. Even if a student performs excellently with general manipulation skills, it does not imply that such a student will perform well in any of the other elements, since the correlations found were non-significant.

9.4.2 Assessing the VSOR content

It is suggested that the five skill factors of VSOR content be assessed using two complementating modes of assessment. The first mode may follow the current format in which the first four skill factors are consolidated where students are firstly asked to draw the graph before solving the question based on the drawn graph, either to calculate area, volume, centroid or centre of gravity. I would advise that the marking memorandum be designed in such a way that a student who has drawn an incorrect graph is not penalised for the rest of the question, as is done presently. The marking memorandum should have all the possible alternative solutions taking into account the errors that students make. The selection of the strip and the rotation thereof must be marked based on students' drawn graphs. Further general manipulation skills, for example calculating the area, volume, centre of gravity and so on should as well be marked in the same way.

I suggest that in the second mode students should be given drawn graphs that they can interpret. In that way, the incorrect graph that the student might have drawn (similar to the current method of assessment as it is in this question in the examination paper) may not affect the solution to the problem. The main focus of these questions should therefore be on how students interpret the drawn graphs (which is the main focus at N6 level). The focus should also be on how students identify the representative strip for the bounded region, how they translate the visual graph (the bounded region) by drawing the solid of revolution formed and the diagram representing the 3D rotation of the selected strip. The questions must encourage students to derive an algebraic equation from the strip (in 2D or in 3D) that they have drawn in order to calculate area or volume. Another recommendation is that students should get credit for showing the location of the coordinates of the centroid of the strip and the centre of gravity of the solid from the drawn graphs (in 2D and in 3D) before they could calculate them, since that will help them to approximate the coordinates in relation to the limits of integration before performing the calculation of those points. In that way students are encouraged to come up with their formulae visually in a way of enhancing conceptual understanding instead of just relying on the formula sheet.

9.4.3 The role of curriculum developers

Curriculum developers must design the curriculum in line with the capabilities of the students registered at the FET colleges as this content seems to be too complex for these students and a hurdle for entrance into universities of technology. My experience in teaching at the universities of technology is that the VSOR content that the university of technology students learn is cognitively less demanding than what FET students are exposed to. This raises a concern as to whether there is any communication between the college curriculum developers and those from the universities of technology, in order to address the career paths that these students may follow.

I suggest that the VSOR content that is learnt at the FET colleges at N6 level be made less cognitively demanding by perhaps moving some of the concepts to the university of technology level, where these concepts can be addressed in-depth. As mentioned earlier, in the past, FET colleges (then technical colleges) were mainly training artisans as apprentices from companies, not students coming straight from school with no experience from the industry. The VSOR content at this stage is not in line with students' capabilities, since they come straight from schools and have no industrial background.

9.4.4 Duties of the Department of Education and the industry.

An alternative approach would be for the Department of Education to design programmes in which lecturers who teach mathematics at the FET colleges, especially from N4 to N6 level, are thoroughly trained in teaching VSOR as it requires that they teach more conceptually, an aspect that most of the lecturers lack, perhaps due the fact that most of them are not adequately qualified and specialists in their fields. Another aspect is that this section is more closely related to the industrial experience in engineering which most of the present lecturers at the FET colleges do not have. This topic is meaningful to mechanical and electrical engineering students since it is applicable in subjects such as Fluid Mechanics, Thermodynamics and Strength of Materials that deal with channel flow of fluids, heat transfer, beams, etc. These topics are applicable to e.g. civil engineering, hence cannot be removed from the curriculum. Perhaps computer programmes e.g. Mathematica, if used regularly, can be beneficial for students doing VSOR as it will enable them to visualise the rotations (from 2D to 3D).

According to Tall (1995: 52) computer programmes can also provide a rich interactive source of possible imagery, both visual and computational as well as allowing students to progress in their use of graphic and numerical aspects of the concept of definite integral (Camacho & Depool, 2003). Visualising these rotations is important since understanding and application of mathematical concepts using visually based representations and processes presented in diagrams, computer graphics programmes and physical models is essential (Rahim & Siddo, 2009). In developing curricula for FET colleges, the Department of Education must also involve employers. Young (2003:230) mentioned that the NQF offers opportunities to employers to have a bigger say in the kind of skills and knowledge that 16 to 18-year-olds are expected to acquire. Presently these 16 to 18-year-olds are studying at the FET colleges.

9.5 LIMITATIONS OF THE STUDY AND DIRECTIONS FOR FURTHER RESEARCH

This study has some limitations in that its results cannot be generalised since it was conducted in two colleges only, on a small sample of students. The lesson observations conducted were for one lecturer only, and one class only at one college. The classroom observations might have influenced teaching and learning styles. Other factors might have contributed to the results of this study since the students were taught by different lecturers and used different textbooks. However, the results can be transferrable to similar settings.

The strong positive correlation between the results in the questions on VSOR and the entire examination paper raises serious concerns about causation, which could be investigated further. The question is, does performance in these questions affect the performance in the whole examination paper or vice versa? This fact could be researched further.