Learning difficulties involving volumes of solids of revolution: A comparative study of engineering students at two colleges of Further Education and Training in South Africa

by

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Submitted in partial fulfilment for the Degree
Philosophiae Doctor

in the Department of Mathematics and Applied Mathematics in the Faculty of Natural and Agricultural Sciences

University of Pretoria
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September 2011

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DECLARATION

I, the undersigned, declare that the thesis which I hereby submit for the degree Philosophiae Doctor to the University of Pretoria contains my own, independent work and has not previously been submitted by me for any degree at this or any other tertiary institution.

Signature:

Name: Batseba Letty Kedibone Mofolo-Mbokane

Date:
This study investigates learning difficulties involving volumes of solids of revolution (VSOR) at two FET colleges in Gauteng province, in South Africa. The research question for this study was: **Why do students have difficulty when learning about volumes of solids of revolution?** In order to answer the research question five skill factors were identified as the conceptual framework, subdivided into 11 elements. The five skill factors are: I. Graphing skills and translating between visual graphs and algebraic equations/expressions, II. Three-dimensional thinking, III. Moving between discrete and continuous representations, IV. General manipulation skills and V. Consolidation and general level of cognitive development.

Before collecting the main data for this study, a preliminary study and a pilot study were conducted. The data for the main study were then collected in six different investigations. The investigations consisted of two runs of a questionnaire, classroom observations, examination analysis; detailed examination responses and an interview with one student.

The results from the questionnaire runs as well as the pilot study reveal that students performed poorly in tasks involving three-dimensional thinking (Skill factor II), moving between discrete and continuous representations (Skill factor III), and consolidation and general level of cognitive development (Skill factor V). Students’ performance was satisfactory in tasks involving graphing skills and translating between visual graphs and algebraic equations/expressions (Skill factor I) and general manipulation skills (Skill factor IV). Students were also more competent in solving problems that involved procedural skills than those that required conceptual skills. The challenges that students were faced with in class, evident from the classroom observations allude to the fact that the topic of VSOR is difficult to teach and to learn.

It is recommended that VSOR be taught and assessed more conceptually in line with the five skill factors; that curriculum developers must communicate with other stakeholders like industries and other institutions of higher learning and that the Department of Education must provide adequate training for these teachers and liaise with industry in this regard. It is also recommended that the suitability of this topic for the particular cohort of students be reconsidered as it appears to be of too high cognitive demand.
DEDICATION

This study is dedicated to my family for walking this long path with me. I dedicate this study to my husband Majagaodwa Mbokane, my daughter Mmamonkwe, my son Umalusi, my mother Mmamonkwe Mofolo and in soul my father Malebye Mofolo who inspired me throughout this journey. Perseverance is what kept me going. I can now sing and praise GOD, who lifted me up when I was tripping.

“Praise the LORD with the harp; make melody to Him with an instrument of ten strings. Sing to Him a new song; play skilfully with a shout of joy”.

PSALM 33: 2-3
ACKNOWLEDGEMENTS

I wish to thank my supervisor Prof Johann Engelbrecht and my co-supervisor Prof Ansie Harding for their hard work in continuously guiding and supporting me in the writing of this thesis. Their constructive criticism and feedback led to the improved versions of this report. Thank you for your patience, your encouragement and your motivation when I felt that the road was difficult.

I wish to thank the following people and organisations for their support during my research journey.

- All the lecturers and the students at the three FET colleges who were participants in this study for the time they spent to make this report possible.
- Many thanks to the national department of education for allowing me to analyse the examination scripts.
- I wish to thank my SMTE HODs Prof Onwu, Prof Braun and all my colleagues in the mathematics department for encouraging me to carry on with my studies and sharing my work load. Thank you Dr Gaigher and Mrs Randall for your sweet words of motivation when I was downcast. I wish to thank Mr Mnguni also for helping me with technical aspects when I got stuck, my colleague Mrs Kazeni for her words of encouragement and Mrs Alison Kitto for sharing my working load.
- Thank you Dr Lizelle Fletcher for the statistical analysis and interpretation of the data in this thesis.
- I wish to thank the language editor A.K. Welman for ensuring that the correct standards are maintained.
- I appreciate the financial support I received from the University of Pretoria and the NRF in terms of funding my research fully.
- Many thanks to the Almighty for making this possible.
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LIST OF ACRONYMS

ABET - Adult Basic Education and Training
ACE - Advanced Certificate in Education
ARIRE - Average Ranking for Individual Responses per Element
CAS - Computer Algebra System
DoE - Department of Education
DoL - Department of Labour
FET - Further Education and Training
FTC - Fundamental Theorem of Calculus
GET - General Education and Training
HE - Higher Education
MMA - Mixed methods approach
NC(V) - National Certificate (Vocational)
NQF - National Qualification Framework
SAQA - South African Qualifications Authority
TIMSS - Third International Mathematics and Science Study
VSOR - Volumes of solids of revolution (VSOR)
ZPD - Zone of Proximal Development
2D - Two-dimensional
3D - Three-dimensional