TECHNICAL APPENDIX

THE CEPSTRUM

Speaking mathematically, the cepstrum is an integral transform with a long history. The use of the cepstrum dates back to Poisson (1823) and Schwarz (1872), who applied cepstra to problems involving potential functions with real parts fixed on the unit circle. Later, Szegő (1915) and Kolmogorov (1939) used cepstra in the extraction of stable causal systems by factoring power spectra of random processes. However, it is the application of Bogert et al. (1963) in engineering that most coincides with our interest here.

Bogert et al. (1963) considers a time-varying function $f(t)$ which is made up of another function $f_1(t)$ and its additive “echo,” $f_1(t - \tau)$, lagged by $\tau$ periods. Formally, we have:

$$f(t) = f_1(t) + af_1(t - \tau)$$  \hspace{1cm} (A.1)

The power spectrum of $f(t)$ is

$$|F(\omega)|^2 = |F_1(\omega)|^2 \left\{ 1 + a^2 + 2a \cos(\omega \tau) \right\},$$  \hspace{1cm} (A.2)

where $F_1(\omega)$ is the complex Fourier transform of $f_1(t)$. The “echo”, in turn, manifests as a cosine function riding on the envelope of the power spectrum. The period of the cosine function is the reciprocal of the lag $\tau$. In studies involving autocorrelation analysis in hydroacoustic problems, Griffin et al. (1980) has shown that if the spectra of $f(t)$ and its “echo” differ, i.e., they are not perfectly correlated, then the parameter $a$ in Equation (A.2) varies with frequency. Thus, $f(t)$ is better represented by:

$$f(t) = f_1(t) + f_2(t - \tau), \quad f_1(t) \neq f_2(t)$$  \hspace{1cm} (A.3)

so that the power spectrum of $f(t)$ is given by:

$$|F(\omega)|^2 = |F_1(\omega)|^2 + |F_2(\omega)|^2 + 2 |F_1(\omega)| |F_2(\omega)| \cos\left\{ \phi_1(\omega) - \phi_2(\omega) + \omega \tau \right\},$$  \hspace{1cm} (A.4)

where $\phi_1(\omega)$ and $\phi_2(\omega)$ are the phase spectra of $f_1(t)$ and $f_2(t)$, respectively.

The modulating cosine is phase-shifted by an amount which is equal to the differences in the phases of functions that composes $f(t)$. Thus, if the function $f_1(t)$ and $f_2(t)$ are not close.

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78 This section relies heavily on the discussion available in Cunningham and Vilasuso (1994).
copies of one another, the argument of the modulating cosine is not constant, which, in turn, has important consequences as we shall see below.

We proceed under the assumption that \( f_1(t) = f_2(t) \) and return to the more general case later. The function \( f(t) \) can be represented as the linear convolution of \( f_1(t) \) with a train of impulses. Bogert et al. (1963) argue that if the envelope of \( |F(\omega)|^2 \) could be made optimally white, this would be equivalent to making \( |F_1(\omega)|^2 \) into a “boxcar” function, whose Fourier transform would be a sinc function at the origin. In the limit, as the envelope of the power spectrum becomes uniform at all frequencies, the sinc function tends towards a Dirac delta function. The transform of the modulating cosine would be an impulse whose delay is related to the frequency of the modulating cosine which, in turn, is equal to the lag length between \( f_1(t) \) and its “echo.” Therefore, under ideal conditions and when scaled properly, this resulting series yields a time domain function with a global maximum, or “peak” at the origin, and the local maximum or “peak” indicating the arrival time of the “echo.”

In practice, the whitening of the power spectrum is performed by first applying the natural logarithm and then the inverse Fourier transform (IFT). Because it ignores the phase spectrum, and is calculated directly from the log power spectrum, the resultant function is referred to as the power spectrum.

Let us now go back to case presented in Equations (A.3) and (A.4). If the component functions are not close copies of one another, then the argument of the modulating cosine is not invariant, and, hence, cepstral peaks rapidly degenerate. Therefore, it must be realized that the impulse appears at the appropriate lag of the cepstrum only if the component functions are well correlated.

When we consider the discrete case, cepstra are a class of integral transform whose kernel is a function of the z-transform of a real sequence. The discrete power cepstrum of a data sequence \( x(nT) \) with z-transform \( X(z) \) is then given by:

\[
\tilde{x}(nT) = \frac{1}{2\pi i} \int \log |X(z)| z^{-n-1} dz
\]

(A.5)

Note that \( \text{sinc } x = \frac{\sin x}{x} \).
where \( n = 0, 1, 2, \ldots, N \), enumerates the samples, \( T \) is the sampling interval, and \( C \) is a closed contour inside the region of convergence of the power series and enclosing the origin. As discussed by Cunningham (1980), this can be extended to the cosine-squared cepstrum by addition of the signum function as follows:

\[
\hat{x}(nT) = \hat{x} \times \text{sgn} \left( \text{Re} \left[ \frac{1}{2\pi i} \oint_{C} \log |X(z)| e^{-nz} \, dz \right] \right)
\]  

(A.6)

The addition of the signum function allows the cepstrum to determine not only the arrival time of the secondary series, but also, and perhaps more importantly, its polarity relative to the original series making its interpretation analogous to the autocovariance function.\(^8\) Because in real-life applications the real sequences \( x(nT) \) are of finite length, the annulus of convergence of \( X(z) \) always includes the unit circle, the transforms may be computed by means of the fast Fourier transforms.

Because the cepstrum is essentially the spectrum of a spectrum, the cepstral domain is a time domain. The terminology easily becomes confusing, therefore, Bogert et al. (1963) suggested the following conventions: the term “cepstrum” is an anagram of the word “spectrum.” Likewise, periodicities in the cepstrum are discussed in terms of “quefrencies,” “gamnitudes,” and “repiods,” analogous to “frequencies,” “gain/amplitudes,” and “periods” in the time domain. “Filtering” in the cepstral domain is “liftering” and so on. Further, Bogert et al. (1963) refers to data analysis in the cepstral domain as “alanysis”. Finally, the term “saphe,” pronounced “safe” and related to “phase,” is used to refer to the displacement between the secondary, or lagged series, and the original. Thus, the detection and analysis of cepstral peaks and “phase” shifts of the lagged series is referred to as “saphe cracking.”

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\(^8\) The close relationship between the cosine-squared cepstrum and the autocovariance function becomes clear when we write the latter as a function of the \( z \)-transform of the same series: 

\[
x_{\text{acvf}}(r) = \frac{1}{2\pi i} \oint_{C} X(z) e^{rz} \, dz.
\]

Equivalently, the autocovariance function is the inverse Fourier transform of the power spectrum of a series.
9 Conclusion

Since the February of 2000, the sole objective of the SARB has been to keep the CPIX inflation rate within the target band of 3 percent to 6 percent, using discretionary changes in the Repo rate as its main policy instrument. Given this, this thesis aimed to evaluate inflation targeting regime of South Africa not only in term of welfare cost estimates, but also by comparing mean and volatility of inflation in pre- and post-inflation targeting regimes.

With the welfare cost of inflation lying between 0.70 percent of GDP to 1.33 percent of GDP for the target band of 3 percent and 6 percent, we show that there are considerable welfare gains to be had by reducing the lower level of the target band below the 3 percent mark. In addition, we show that mean and volatility of inflation is higher than it would have actually been had the SARB followed its earlier more eclectic monetary policy regime. We believe this is due to the width of the inflation targeting band. In line with the inflation targeting literature on focal points, we suggest a narrower and possibly lower target band could be of immense help in improving the central bank’s credibility and causing inflation expectations to converge to a focal point, and hence, bring down the mean and variance of the inflation rate.


