3 IMPLEMENTATION OF THE PROPOSED METHOD

3.1 Cracked Sections

Polak's approach to the problem of tension stiffening was applied almost without change. The author modified the algorithm suggested by Polak to allow for iteration after each model update, figure 7-3.

Both the Bilinear and Branson's method were used in conjunction with Polak's approach and compared to experimental results in section 4.2. The bilinear method required further development before being utilised in a manner similar to Branson's method.

Assuming that elastic relations still hold on average for cracked sections:

$$\psi_1 = \frac{M}{EI_1} \tag{3.1}$$

$$\psi_2 = \frac{M}{EI_2} \tag{3.2}$$

where the subscripts 1 and 2 refer to conditions 1 and 2 as described in section 2.2.
Substituting equations (3.1) and (3.2) into equation (2.43) yields an effective moment of inertia

$$I_e = \frac{I_1 I_2}{(1-\zeta)I_2 + \zeta I_1} \tag{3.3}$$

I_e can then be used to calculate α_x and α_y as described in section 2.2.2. It should be noted that the procedure for instantaneous cracked deflection and long-term cracked deflection differs. For long-term deflections a shrinkage analysis should precede the crack analysis, as shrinkage normally causes additional member actions that contribute to cracking.

A very simple convergence check was used in the crack analysis as follows:

- Step 1 : Calculate deflections using $I_{(i)}$, where i denotes the iteration step. For the first iteration $I_{(i)}$ corresponds to I_1.
- Step 2 : Calculate $I_{e(i+1)}$ using either of the two tension stiffening methods.
- Step 3 : Average $I_{e(i+1)}$ and $I_{(i)}$ and calculate the reduction factors α_x and α_y.
- Step 4 : Loop back to step 1 and repeat until $I_{e(i+1)} \approx I_{(i)}$.

This approach converged very quickly when using Branson's method but the bilinear method often exhibited oscillating divergence. This phenomenon was model and loading dependent and the algorithm had to be modified on a case by case basis to achieve a convergent solution.

3.2 Creep

Using equation (2.64) the factors κ_x and κ_y can be calculated based on the reinforcement ratios in those two directions, similar to α_x and α_y in section 2.2.2. To account for the different creep characteristics of cracked and uncracked sections, a creep analysis must be preceded by a crack analysis as described in section 3.1.

The elasticity matrix in equation (2.19) can then be modified as follows for the calculation of creep deflection increments for a cracked element:

$$[D]=\begin{bmatrix} \dfrac{E_x h^3}{12(1-\nu_x\nu_y)} & \dfrac{E_y \nu_x h^3}{12(1-\nu_x\nu_y)} & 0 & 0 & 0 \\ \dfrac{E_x \nu_y h^3}{12(1-\nu_x\nu_y)} & \dfrac{E_y h^3}{12(1-\nu_x\nu_y)} & 0 & 0 & 0 \\ 0 & 0 & \dfrac{G_1 h^3}{12} & 0 & 0 \\ 0 & 0 & 0 & G_2 h & 0 \\ 0 & 0 & 0 & 0 & G_3 h \end{bmatrix} \tag{3.4}$$

where:

$$E_x=\alpha_x\frac{E_c}{\kappa_x\phi}, \qquad E_y=\alpha_y\frac{E_c}{\kappa_y\phi} \tag{3.5}$$

$$G_1=\alpha_x\alpha_y\frac{G}{\kappa_x\kappa_y\phi}, \quad G_2=\alpha_x\frac{G}{\kappa_x\phi}, \quad G_3=\alpha_y\frac{G}{\kappa_y\phi} \tag{3.6}$$

$$\kappa_x=\frac{I_{cx}+A_{cx}y_{cx}\Delta y_x}{\bar{I}_x}, \qquad \kappa_y=\frac{I_{cy}+A_{cy}y_{cy}\Delta y_y}{\bar{I}_y} \tag{3.7}$$

$$\nu_x=\alpha_x\nu, \qquad \nu_y=\alpha_y\nu \tag{3.8}$$

with $\alpha_x \leq 1$, $\alpha_y \leq 1$ and ϕ the creep coefficient.

The parameters α_x and α_y are based on short term properties and κ_x and κ_y are parameters smaller than unity that modify the creep coefficient to account for the presence of the reinforcement.

The variables in equations (3.4) to (3.8) apply to cracked and uncracked section parameters as needed. Uncracked elements and fully cracked sections pose little difficulty. Partially cracked sections, on the other hand, require the calculation of an effective neutral axis.

It is proposed that the neutral axis for partially cracked sections be calculated based on the assumption that since the parameter α provides a measure of the extent of cracking it can also be used directly to modify the depth of the neutral axis:

$$y_e = y_1 \alpha \tag{3.9}$$

where:

y_e = y-coordinate of the neutral axis of the partially cracked section, measured from the top of the section. This value should be larger than the cracked neutral axis coordinate and smaller than the uncracked value.

y_I = y-coordinate of the neutral axis for the uncracked section, measured from the top of the section.

The creep analysis algorithm is illustrated in figure 7-4.

3.3 Shrinkage

Equation (2.70) can be used to calculate x and y curvatures for each element, independent of loading. These curvatures need to be transformed into equivalent nodal loads in order to model the effect of boundary conditions on shrinkage in a finite element analysis.

Equivalent nodal loads are calculated simply from the following equation, utilising Gaussian numerical integration over the 4 sampling points:

$$\{P_{sh}\} = \int_A [B]^T [D] \{\varepsilon_{sh}\} \, dA \tag{3.10}$$

where $[B]$ is calculated from equation (2.26) and $[D]$ is calculated from equation (3.4).

$\{\varepsilon_{sh}\}$ is the vector of shrinkage strains:

$$\{\varepsilon_{sh}\} = \begin{bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \\ \phi_x \\ \phi_y \end{bmatrix} = \begin{bmatrix} \psi_{xsh} \\ \psi_{ysh} \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.11}$$

The vector of shrinkage forces for each element node is calculated as:

$$\{P_{sh}\} = \begin{bmatrix} P_z \\ M_x \\ M_y \end{bmatrix} = \begin{bmatrix} 0 \\ M_{xsh} \\ M_{ysh} \end{bmatrix} \tag{3.12}$$

All these forces are then assembled into a global force vector and the shrinkage deflections and forces are calculated with $[D]$ modified for creep.

The shrinkage analysis algorithm is illustrated in figure 7-5.