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A SIMPLIFIED FINITE ELEMENT MODEL FOR TIME-DEPENDENT DEFLECTIONS OF FLAT SLABS

by

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SAMEVATTING

'n VEREENVOUDIGDE EINDIGE ELEMENT MODEL VIR DIE BEREKENING VAN TYD-AFHANKLIKE DEFLEKSIES VAN PLAT BLAAIE

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Die doel van hierdie verhandeling is om 'n vereenvoudigde eindige element model te ontwikkel vir die analise van tyd-afhanklike defleksies van plat blaaie.

Nuwe materiale en ontwerpbenaderings het tot gevolg dat diensbaarheidsfalings meer algemeen voorkom. Hierdie tipe van faling is geneig om eers lank na die voltooiing van konstruksie voor te kom en kan dus duur wees om te herstel.

Verskeie gesofistikeerde metodes, gebaseer op gelaagde elementmodelle, nie-lineêre materiaalwette en reologiese benaderings tot krimp and kruip probleme is al deur navorsers voorgestel. Hierdie metodes het die nadele van kompleksiteit en uitermatige rekenaar verwerkingstyd. Gesofistikeerde modelle is in baie gevalle nie geoorloof nie omdat die fisiese prosesse betrokke nie altyd goed verstaan óf akkuraat voorspel kan word nie.

Ontwerpers besef nie altyd die erns van sulke falings nie en ignoreer soms tyd-afhanklike defleksies van blaaie. 'n Eenvoudige metode, wat maklik inskakel by bestaande ontwerp-metodiek, sal dus 'n gaping vul in huidige praktyk.

Die veld van studie sluit , in breë trekke, die volgende in:

- Die keuse van eindige element formulering vir die model. 'n 8-node Serendipity element, gebaseer op 'n Mindlin plaatanalise, vertoon stabiele resultate in 'n groot verskeidenheid van toepassings.
- Kraak van blaaie en die trek-verstywingseffek: Branson se effektiewe traagheidsmoment en die Bilineêre metode is in dié opsig vergelyk.
- Die gekombineerde invloed van krimp en kruip: Die bekende „Effective Modulus Method" is gebruik om hierdie invloed te modelleer.

Die voorgestelde metode is saamgevat in 'n rekenaarprogram, waarvan die bronkode saamgevat word in die aanhangsels, en word vergelyk met ander metodes en eksperimentele resultate.

Die program vertoon goeie resultate in vergelyking met eksperimentele werk, veral in verband met kraak en trek-verstywing. Branson se benadering tot hierdie probleem lewer beter resultate as die bilineêre metode, veral waar wapening verhoudings laag is. Kruip en krimp resultate vergelyk ook goed met die balk wat ondersoek is, asook die handberekening van 'n bladpaneel se langtermyn defleksies.

Die program maak gebruik van parameters wat algemeen voorkom in betonontwerp en vereis dus nie spesialis kennis van materiaal eienskappe nie. Die metode, soos toegepas in 'n eindige element analise, maak voorsiening vir die plaat gedrag van blaaie en 'n meer realistiese skatting van langtermyn defleksies kan dus gemaak word.

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LIST OF SYMBOLS

$\{\delta\}$	Nodal deflection vector
$\{\delta^e\}$	Element deflection vector
$\{\delta_i\}$	Nodal deflection vector at node i
$\{\epsilon\}$	Strain vector
$\{\phi\}$	Nodal shear rotation vector
$\{\psi\}$	Curvature vector
α	Reduction factor to account for cracking
α_e	Modular ratio
α_{ae}	Age adjusted modular ratio
β_1	CEB-FIP parameter for reinforcement type
β_2	CEB-FIP parameter for duration of loading
δ	Deflection
$\Delta\epsilon_s$	Difference of the mean steel strain and the steel strain at condition 2
$\Delta\psi$	Change in curvature
ϵ_1	Concrete strain at steel level of a singly reinforced concrete section subjected to uniform shrinkage
ϵ_2	Concrete strain at the compression face of a singly reinforced concrete section subjected to uniform shrinkage
$\epsilon_c(t)$	Concrete strain at time t
ϵ_{cs}	Free shrinkage strain
ϵ_e	Instantaneous elastic concrete strain
ϵ_{s1}	Tension reinforcement strain at condition 1
ϵ_{s2}	Tension reinforcement strain at condition 2
ϵ_{sm}	Mean strain of the tension reinforcement of a partially cracked concrete member
ζ	Cracking parameter in the Bilinear Method
θ	Plate flexural rotation
κ	Creep modification factor
κ_c	Creep curvature coefficient
κ_s	Interpolated crack curvature coefficient
κ_{s1}	Crack curvature coefficient at condition 1
κ_{s2}	Crack curvature coefficient at condition 2
ν	Poisson's ratio
ξ, η	Axes of the natural coordinate system

Π	Potential energy functional
ρ_c	Percentage compression reinforcement
ρ_t	Percentage tension reinforcement
σ_{s1}	Tension reinforcement stress at condition 1
σ_{s2}	Tension reinforcement stress at condition 2
ϕ	Plate shear deformation
$\chi(t,\tau)$	Age adjustment factor at time t for loading applied at time τ
ψ	Curvature
ψ_1	Curvature at condition 1
ψ_2	Curvature at condition 2
ψ_e	Elastic curvature at time τ
ψ_{sh}	Shrinkage curvature
$\phi(t,\tau)$	Creep coefficient at time t for loading applied at time τ
τ	Age at loading

[B]	Element strain matrix
[D]	Total elasticity matrix
$[D_f]$	Flexural elasticity matrix
$[D_s]$	Shear elasticity matrix
[J]	Jacobian matrix
$[K^e]$	Element stiffness matrix
{M}	Bending moment vector
$\{P^e\}$	Vector of element nodal forces
$\{P^e{}_p\}$	Vector of element nodal forces due to distributed loads
$\{P_{sh}\}$	Vector of nodal forces due to shrinkage
{Q}	Shear force vector
A_c	Effective concrete area
d	Distance from the compression face of a section to the centroid of the tension reinforcement
d_t	Distance from the neutral axis of a section to the level of tension reinforcement.
E_c	Short term concrete elastic modulus
$E_e(t,\tau)$	Effective elastic modulus at time t for loading applied at time τ
E_s	Reinforcement modulus of elasticity
e_s	Eccentricity of the steel centroid with respect to the centroid of the transformed section
f_{c1}	Concrete tensile stress at the tension reinforcement level due to shrinkage

f_{c1}	Concrete tensile stress at the top fibre of the section due to shrinkage
f_r	Modulus of rupture
f_s	Steel stress due to shrinkage
G	Shear modulus
h	Full depth of the concrete section
$\bar{I}$	Moment of inertia of the age adjusted transformed section about an axis through its centroid
I_1	Moment of inertia of the concrete section at condition 1
I_2	Moment of inertia of the concrete section at condition 2
I_c	Moment of inertia based on the cracked, transformed concrete section
I_e	Effective moment of inertia
I_g	Moment of inertia based on the uncracked, untransformed concrete section
I_t	Moment of inertia based on the uncracked, transformed concrete section
Δl	Change in length
l	Length
M	Bending moment
M_r	Cracking moment, based on the modulus of rupture
N_i	Shape function at node i
p	uniformly distributed load
w	Lateral plate deflection
y_c	y-coordinate of the centroid of A_c, measured from the centroid of the age-adjusted, transformed section
Δy	y-coordinate of the centroid of the age adjusted transformed section, measured downwards from the centroid of the transformed section at time τ.

SUMMARY

A SIMPLIFIED FINITE ELEMENT MODEL FOR THE CALCULATION OF TIME-DEPENDENT DEFLECTIONS OF FLAT SLABS

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The aim of this dissertation is to develop a simplified finite element model for the analysis of time-dependent deflections of flat slabs.

New materials and design approaches have caused an increase in the incidence of serviceability failures. This type of failure tends to occur long after the completion of construction and can thus be quite expensive to repair, if at all possible.

Various sophisticated methods based on layered element models, non-linear constitutive laws and rheological creep and shrinkage models have been proposed by various authors. These methods suffer from a high degree of complexity and become prohibitive in terms of computer memory storage requirements and processing time. In many cases sophisticated models are uncalled for due to a lack of understanding of the physical processes involved or an inability to accurately predict the influence of these processes.

Broadly, the field of study includes the following:

- The choice of a finite element formulation for the model. An 8-noded Serendipity element, based on a Mindlin plate analysis has proven itself as a good performer in a wide range of problems and is the formulation of choice for this dissertation.
- Cracking and the tension-stiffening effect: Branson's Effective Moment of Inertia method and the bilinear method are compared in this regard.
- The combined effect of shrinkage and creep: The Effective Modulus method is used to model creep and shrinkage is included using a free shrinkage parameter.

The proposed method was implemented in a computer program, the source code of which is reproduced in the appendices, and compared to other methods and experimental results.

The program achieved good results when compared to experimental work, especially as far as cracking and tension stiffening are concerned. The bilinear method was found to produce results inferior to Branson's approach to tension stiffening, particularly when reinforcement ratios are low. Creep and shrinkage results compared well with the beam considered as well as the hand-calculation of the long-term deflections of a slab panel.

The program utilizes parameters commonly used in routine design, such as the creep coefficient and free shrinkage, and therefore does not require specialist knowledge of material properties. The method, as applied to plates in the finite element analysis, includes the plate behaviour of slabs and thus allows a more realistic estimate of deflection than the equivalent frame method.