

Chapter 1

Introduction and literature survey

1.1 Introduction

Gears and gearbox systems are vital components in many industrial mechanical applications. A gearbox failure in a large mechanical system could easily lead to production losses. Early detection of incipient failure in gearboxes is, therefore, of great practical and commercial importance. It permits the plant operators and maintenance personnel to schedule shutdown and repair of the gearbox instead of unscheduled catastrophic failure.

Different signal processing techniques have been employed by operators and engineers to gather information about the condition of gearboxes to schedule maintenance activities. These techniques include oil debris analysis, vibration analysis, visual inspection and various non-destructive testing techniques. In recent years, neural networks have been used with much success in pattern recognition and fault identification (Bishop, 1995; Zhong et al., 2003; Fidêncio et al., 2003). Vibration based analysis has been used with success in detection of damage in structures and rotating machinery. Vibration-monitoring techniques are based on the assumption that changes in the measured structural response can be linked to the deterioration in the condition of the structure (McFadden, 1987). Recent advances in integrated circuit technology and digital signal processing has allowed for real time analysis of vibration response, in both the frequency and the time domain to be performed. If permanent transducers can be mounted on the structures, online condition monitoring of the vibrating structure could be possible, resulting in a much safer working environment.

This is however not necessarily true when monitoring incipient failure on rotating machinery such as gearboxes, especially when they are operating under varying load conditions. Varying load conditions amplitude modulate the measured vibration signal

and cause the rotation speed of the system to change. The change in system speed results in frequency modulation of the gear mesh frequency (Stander et al., 2001; Stander et al., 2002^a; Stander et al., 2002^b).

When using vibration signatures for condition monitoring of gearboxes it is difficult to extract meaningful information from raw time domain vibration data. This is because the characteristic frequencies generated by newly developed faults in gearboxes can be very low in amplitude and are therefore often overshadowed, or masked, by other vibration components such as random noise and interference from additional vibration sources in the machine or neighbouring machines. To overcome this problem the vibration signal is sampled at a frequency that is synchronised exactly with the rotation of the gear of interest and the samples obtained for each singular position of the gear are then ensemble-averaged. When sufficient averages are taken, all the vibration from the gearbox, which is asynchronous with the vibration of the gear, cancels out, leaving only the vibration produced during one rotation of the gear of interest. Local variations in the meshing pattern and modulation in the gear of interest are therefore made visible (McFadden, 1987; McFadden et al., 1985; Stewart, 1977). This procedure is called time domain averaging or synchronous averaging.

The resulting time synchronously averaged signal obtained through the time domain averaging process indicates the vibration produced during one rotation of the monitored gear. The synchronous vibration signal can be related to the meshing stiffness of the gear being monitored. Variations in the meshing stiffness of the gear indicate wear and or incipient local defects that are related to a variation in gear teeth stiffness. Time domain averaging is an extremely effective technique, but it requires an enormous amount of vibration data to calculate. This problem makes time domain averaging less attractive on online gearbox condition monitoring system. The challenge remains to develop a synchronous time domain averaging filter that reduces the amount of vibration data that is required for direct synchronous time domain averaging of gear vibration data. A reduction in the amount of input gear vibration data required for synchronous time domain averaging of gear vibration brings us closer to the successful implementation of synchronous time domain averaging on an online gearbox condition monitoring system.

1.2 Literature survey

The objective of this research is to develop a filter for synchronous time domain averaging of gear vibration using computational intelligence. The purpose of the filter is to reduce the amount of gear vibration data that needs to be stored in the data acquisition system in order to calculate the time domain average of a gear vibration signal. The literature survey addresses the following topics:

- Signal processing techniques for early detection of gear failure through vibration measurements.
- Digital filtering.
- Application Artificial Neural Networks and Support Vector Machines in pattern recognition.

1.2.1 Signal processing techniques for early detection of gear failure through vibration measurements.

In this section, as background, different signal processing techniques for early detection of gear failure through vibration measurements are discussed. The underlying premise of vibration analysis is that changes in the mechanical condition of the system produce changes in the vibration that the system produces. In extremely simple systems, these changes take the form of an increase in the amplitude of the total vibration, which can be easily detected with simple instruments. For more complex systems, changes in the total vibration due to the deterioration of a single machine element are less significant and more sophisticated vibration-processing techniques are needed to detect the damage (McFadden, 1987).

One of the most popular techniques for early detection of gear failure through vibration measurements over the last four decades has been spectral analysis, in which the amplitude spectrum of the measured vibration spectrum is measured and displayed. Spectral analysis is a particularly powerful technique because different elements of a mechanical system generally produce vibration at different frequencies.

In 1977 Stewart presented some useful data analysis techniques for gear diagnostics. These techniques enhance the clarity of the changes on the time domain average using digital signal processing, by removing the normal vibration from the time domain

average. In one of these techniques all of the tooth meshing components and their harmonics are eliminated from the spectrum of the time domain averaging and the remaining time signal is reconstructed to produce the “residual” signal. Stewart showed that the residual signal often shows evidence of a defect long before it can be seen in the time domain average.

In 1985 McFadden and Smith applied modulation theory to a model of gear vibration and showed that band pass filtering the “residual” signal about the dominant meshing harmonics and developing the envelope produced a function that describes the amplitude and phase modulation present in the original averaged signal. The application of this technique to the vibration produced by a gear known to contain incipient fatigue cracks suggested that this method is highly effective, and demonstrate that the phase modulation of the vibration is a more important indicator of a crack than amplitude modulation.

McFadden (1986) illustrated that the signal average can be completely demodulated by simple signal processing techniques to produce separate approximations to the amplitude and phase modulation functions. He demonstrated the effects of both early and advanced fatigue cracks on the modulation functions using signal averages of vibration on spiral bevel pinion in a helicopter gearbox.

In another publication McFadden (1987) presented time domain averaging as an alternative approach for early detection of failure in gears. This author stated that, if a second signal is acquired which is synchronised with the rotation of the gear of interest, and the ensemble average of the vibration is calculated with the start of each frame being determined by the synchronised signal, all the vibration that is asynchronous with the rotation of the gear cancels out, leaving an estimation of the vibration of the gear of interest during one gear revolution. Time domain averaging therefore reduces a complex system such as a gearbox into a simpler system as it eliminates vibration from other system element (McFadden, 1987).

In his paper White (1991) demonstrated the use of a signal demodulator unit for extracting meaningful information from rolling element bearing and gearbox vibration for predictive maintenance.

Wang and McFadden (1993) examined the application of the spectrogram to calculate the time-frequency distribution of gear vibration signals. The spectrogram represents the energy distribution of the signal over the frequency and time. Their results suggest that the spectrogram may provide a powerful tool for the early detection of local gear damage.

In a later publication Wang and McFadden (1995) investigated the use of the orthogonal wavelet transform to detect the abnormal transients generated by early gear damage from gearbox casing vibration. Orthogonal wavelets, such as Daubechies 4 and harmonic wavelets, are used to transform the time domain synchronous vibration signal into the time-scale domain. The orthogonal wavelet transform uses fast algorithms and decomposes the signal into the minimum number of wavelets series. These authors discovered through comparison with non-orthogonal wavelet transform for same length of discrete data, that the description of the signal in the 3-dimensional map of the wavelet transform is not sufficiently comprehensive due to limited scales.

McFadden et al. (1999), described the generalised S transform, a variant of the wavelet transform, which allows the calculation of the instantaneous phase signal, and its application to decomposition of vibration signals from gearbox systems for early detection of failure. They demonstrated the decomposition of a signal using the generalised S transforms and a new window function with a numerically generated test signal and experimentally measured gear vibration data.

Baydar and Ball (2000) used another time-frequency distribution called the Instantaneous Power Spectrum (IPS) in the detection of local faults in helical gears. Their paper describes the IPS and then examines its capability of extracting condition indicating information from gear vibration signals and also assessing the severity of the fault. The paper further examines the ability of the IPS to detect faults under varying load conditions. Their results show that the IPS can be used to detect faults both under constant and varying load conditions.

In a later study Baydar and Ball (2001) conducted a comparative study of acoustic and vibration signals in the detection of gear failure using Wigner-Ville distributions. Their

results suggest that acoustic signals are very effective for early detection of faults and may provide a powerful tool for indicating the various types of progressing faults in gears.

Staszewski and Tomlinson (1994) presented an application of the wavelet transform to fault detection in spur gears. In further work these authors use a moving window procedure for local fault detection in gearboxes (Staszewski and Tomlinson, 1997).

Staszewski et al. (1997) presented a study of the use of the Wigner-Ville distribution in gearbox condition monitoring. In contrast to other applications of the Wigner-Ville distribution, their paper reported on the application of statistical and neural network pattern recognition procedures to reliably detect gear tooth faults.

Wang and Wong (2000) developed a linear prediction method that is based on the assumption that the vibration caused by a sound pair of gears can be modelled as a stationary autoregressive process. These authors stated that the approach is independent of the operating conditions, but the precise influence of varying loads is not documented. The results of their paper indicate that the linear prediction method can be used effectively in the detection and diagnosis of gear failure.

Stander and Heyns (2001) noted the influence of varying loads on vibration monitoring of gears. Stander et al. (2002)^b conducted an experimental investigation to observe the influence of fluctuating load conditions on the measured acceleration signal. They concluded that the load variation manifests itself as a low-frequency modulation on the measured acceleration signal. In another publication Stander and Heyns (2002)^a investigated the use of the Instantaneous Shaft Speed (ISS) in condition monitoring of gearboxes. They postulated that the integrity of the gear tooth in the mesh could be monitored through the utilisation of the ISS measurement. The authors concluded that a natural separation between different levels of damage could be obtained by monitoring the instantaneous gear shaft speed under various fluctuating load conditions.

Paya et al. (1997) investigated the use of artificial neural networks based fault diagnostics of rotating machinery using wavelet transforms as a pre-processor. The real time domain vibration signal obtained from the gearbox transmission were pre-

processed by wavelet transforms for neural networks to perform fault detection and identify the exact kind of fault occurring in the transmission. They showed that by using multi-layer artificial neural networks on the set of data pre-processed by wavelet transforms, single and multiple faults could be successfully detected and classified into specific groups.

Zacksenhouse et al. (2000) conducted a series of tests on a helicopter transmission for the purpose of generating a database that can be used to evaluate general diagnostic tools, particularly neural networks. They demonstrated that the meshing vibrations induced by a large collector gear located on the quill shaft are significant and may interact with the vibrations induced by other elements attached to the same shaft. An appropriate model is developed and the effect of the collector gear, called cross-gear-pair interaction, is studied using different signal processing tools.

Decker (2002)^a conducted a survey of standard vibration diagnostic parameters for crack detection in spur gears used in the Health and Usage Monitoring Systems (HUMS). The results of his study indicated that detection methods used in HUMS are not robust or repeatable. The cracks actually progressed at a much faster rate than anticipated reducing the available time for detection.

In another study Decker (2002)^b proposed a new gear failure analysis feature and two new detection techniques. The time synchronous averaging concept was extended from being revolution-based to tooth-engagement based. The detection techniques were based on statistical comparison among the averages for the individual teeth. The results indicated that these techniques do not produce an indication of damage that significantly exceeds experimental scatter.

Dempsey et al. (2002) developed a diagnostic tool for detecting damage on a spiral bevel gear by integrating two different monitoring technologies, oil debris analysis and vibration analysis. Their results showed that combining vibration and oil debris measurement technologies improves the detection of pitting damage on spiral bevel gears.

The literature indicates that gear condition monitoring is now in its mature stages. Almost all the vibration analysis methods mentioned above require some form of pre-processing with synchronous time domain averaging to increase their diagnostic capabilities. It is because of this very reason that efficient methods for synchronous time domain averaging are required.

1.2.2 Digital filtering

Signal processing can be defined as the processing performed on signals to extract useful information. Of the many signal processing methods currently available, digital filtering is one of the most powerful. In order to understand the principles used in the development of the synchronous filter for time domain averaging it is necessary to understand of the fundamental theory and some of the recent developments of digital filter technology. In this section a brief history and some relevant applications of filter technology are presented.

Digital filters evolved from simulation of analog filters on the early digital computers of the 1940s. Their first application was in geological exploration of oil fields where the data was collected and stored for future processing. The seismologists found that analog signal-processing methods did not help them distinguish signal from noise. However, through discrete convolution and other noise elimination techniques, they were able to process the seismograms digitally to yield a filtered form that was much easier to interpret, and thus new oil sources were identified.

In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within a certain frequency range. This basic idea can be illustrated by the block diagram in Figure 1.

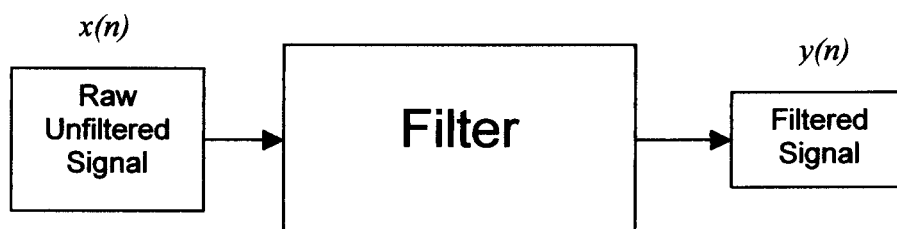


Figure 1 Basic filter concept

In general a filter takes an input sequence $x(n)$ and produces an output sequence $y(n)$ as shown in Figure.1. More recent developments in filter technology include the development of intelligent filters and adaptive filters. Some relevant filter developments are presented in the following paragraphs.

Zhong et al. (2003) tackled the fault detection problem in dynamic systems with modelling errors and unknown inputs. In this paper the robust fault detection filter design problem for uncertain linear time-invariant systems with both unknown inputs and modelling errors is studied. The main results include the development of an optimal reference residual model, the formulation of robust fault detection filter design problem, the derivation of a sufficient condition for the existence of a robust fault detection filter and its construction based on the linear matrix inequality solution parameters, and the determination of adaptive threshold for fault detection.

Augustyn et al. (2003) presented a new method for filtering signals using the Modified Recursive Discrete Fourier Transform (MRDFT). The basic idea of this method is the application of the user-defined context to the recursive form of the Discrete Fourier Transform (DFT) and filtration data or signals. The context is defined in the frequency domain and the mathematical implementation of the context in a recursive DFT is presented. The method is controlled by an intelligent decision making system, which decides what context present in predefined base of context can be applied to the algorithm. This means that the filtration process extracts only the desired signal features.

Another important concept in the context of this work is that of averaging. Braun (1975) analysed the extraction of a periodically repeating signal from noise coherent averaging. He considered the averaging process as a filtering process and conducted most of his analysis in the frequency domain. He described a general approach for dealing with digital comb filters, enabling the design and analysis of related signal processors.

McFadden (1987) showed that the comb filter model for time domain averaging does not correctly describe the extraction of periodic waveforms from additive noise because

it assumes knowledge of the signal over an infinite time and the result it produces is not exactly periodic. He presented a revised model which requires only a finite number of samples of the signal and which produces a result that is periodic. He also demonstrated that the rejection of periodic noise of a known frequency could be optimised by the appropriate selection of the number of averages.

Moczulski (1987) described the digital synchronous filtering technique. The digital synchronous filtering technique makes it possible to estimate the time history of periodic components of the signal being analysed and the corresponding frequencies which are integer multiples of some triggering frequency. The signal components, which are not synchronous with the triggering signal, are simultaneously attenuated. The digital synchronous filtering technique is based upon the time domain averaging technique. The bank of filters obtained by Moczulski makes it possible to estimate the averaged time courses of the periodic components of the signal and the amplitude and phase characteristics of the filters are given. Only simple fixed-point arithmetic operations were used in order to prepare the necessary software for signal processing.

McFadden (1989) presented an interpolation technique for time domain averaging of gear vibration by digital computer. This technique provides an alternative to the phase-locked frequency multiplier for the calculation of the time domain average of gear vibration signals. Higher-order interpolation techniques produce flatter pass bands and lower side lobes in the stop band but require longer calculation times. Aliasing errors are introduced into the result by replication of the side lobes during interpolation, but in general are attenuated by time domain averaging.

1.2.3 Application of artificial neural networks and support vector machines in pattern recognition

Artificial Neural Networks (ANNs) can be defined as an information-processing paradigm inspired by the way the densely interconnected, parallel structure of the human brain processes information. They are also referred to by other names, such as connectionism, parallel distributed processing, neuro-computing, natural intelligent systems and machine learning algorithms. The key element of the ANN paradigm is the novel structure of the information processing system. It is composed of a large number

of highly interconnected processing elements that are analogous to neurons and are tied together with weighted connections that are analogous to synapses.

There are numerous neural networks that have been investigated. Some of the more popular neural networks include the Perceptron, Multi-Layer Perceptron (MLP), Learning Vector Quantization (LVQ), Radial Basis Function (RBF), Hopfield networks, and Kohonen's self-organizing feature maps (SOM). ANNs are also classified as feedforward, or recurrent (implement feedback) depending on how data are processed through the network. Sometimes ANNs are classified by the method of learning or training they use. Some ANNs such as MLP and RBF employ supervised training, in which the network's error function minimisation involves both the input and the target values. Other ANNs such as the SOM networks employ unsupervised learning, which only involves the input during the training. ANNs are attractive in digital signal processing for the following reasons:

- ANNs can form arbitrary decisions so that any complex mapping from a set of noise-contaminated signal to a noise free signal can be realized.
- ANNs can easily be implemented as software or in specialized hardware.
- ANNs are quite resilient against distortions in the input data and have a capability to learn and generalize when properly trained.
- ANNs are often good at solving problems that are too complex for conventional technologies and are often well suited to problems that people are good at solving, but for which traditional methods are not suitable, such as character recognition.

Neural networks have found extensive application in pattern recognition, signal classification, and image processing. In this work the idea is to apply ANNs and SVMs in a pattern recognition or predictive task. The idea is to train the ANN to predict the ensemble average (time domain average) of a large input matrix (rotation synchronised gear vibration signals) without using the entire input matrix. The successful application of ANNs and SVMs in this sense holds potential of massive reduction in the amount of data required in the synchronous time domain averaging task. In this application, this means a reduction in the amount of data that needs to be collected and stored in the data

acquisition systems before the synchronous time domain averaging can be calculated. Some of the more popular ANN formulations for applications related to this work include the MLP, RBF networks and more recently the Support Vector Machines (SVMs). Some of the most relevant applications to this work include work by the following researchers:

Gaudart et al. (2002) compared the performance of MLP and linear regression (LR) with regards to the quality of prediction and estimation and the robustness to deviations from underlying assumptions of normality, equality of variance and independence of errors. The comparison between connectionist and linear models was achieved by graphic means including prediction intervals, as well as by classical criteria including goodness-of-fit and relative errors. The empirical distribution of estimations and the stability of MLP and LR were studied by re-sampling methods. MLP and LR comparable performance and robustness despite the flexibility of the connectionist models.

Gardner and Dorling (1999) trained MLP neural networks to model hourly NO_x and NO_2 pollutant concentrations in Central London from basic hourly meteorological data. Their results show that the models perform well when compared to previous attempts to model the same pollutants using regression based models. Their work also illustrates that MLP neural networks are capable of resolving complex patterns of source emissions without any explicit external guidance.

Walde et al. (2003) investigated the impact of sample size and sample randomness on the predictive accuracy of MLP. The MLP proved to be useful for classification problems although they are dependent on the sample size and the non-linearity of the underlying problem. A saturation curve describes the dependency of the network performance on the sample size used. This function enables the user to evaluate the achieved network performance and the usefulness of additional data. It is demonstrated that the network leads to narrower confidence intervals of the performance measures in comparison to classical methods even for small sample sizes. The experimental results show the validity of the law, for even relatively small sample sizes, that the standard error of the hit ratio decreases by one over the square root of the sample size.

Taurino et al. (2003) showed the capability of a sol-gel based electronic nose to be used in qualitative and quantitative analysis with the aim to recognize common volatile compounds usually present in the headspace of foods. They showed how linear technique, such as the Principal Component Analysis (PCA) algorithm can be used for inspecting data distribution in simple cases like cluster discrimination. They also used MLP and RBF networks for difficult non-linear regression problems. Their results showed that the MLP gives better performance for their application.

Fidêncio et al. (2003) used the RBF and MLP networks for non-parametric regression of organic matter content in soils determined by conventional chemical measurements and by diffuse reflectance spectra in the near infrared region. The observed results using RBF were better than those obtained by Partial Least Squares (PLS) regression and MLP feedforward networks with a back-propagation learning algorithm. These authors concluded that RBF is a suitable tool for their application, with additional advantages over MLP, since the training procedure is less dependent on the initial conditions.

Alsing et al. (2002) introduced a multinomial selection problem procedure as an alternative to classification accuracy and receiver operating characteristic analysis for evaluating competing pattern recognition algorithms. The multinomial selection problem procedure demonstrates increased differentiation power over traditional classifier evaluation methods when applied to three “toy” problems of varying difficulty. The multinomial selection problem procedure is also used to compare the performance of statistical classifiers and artificial neural networks on three real-world classification problems. The results provide confidence in the multinomial selection problem procedure as a useful tool for distinguishing between competing classifiers and providing insights on the strength of conviction of a classifier.

Another promising method for tackling regression and prediction problems is Support Vector Machines (SVMs). Yang et al. (2002) applied the SVMs in financial prediction of noisy, time-varying financial data. Their experimental results showed that the use of standard deviation to calculate a variable margin results in a good predictive result in the prediction of Hang Seng Index.

Ramesh et al. (2003) presented a hybrid Support Vector Machines–Bayesian Network (SVM-BN) model that seeks to address the issue that most error models developed thus far generally employ neural networks to map the input to the output and give no account for the specific conditions that apply to the process being modelled. In their model the experimental data is first classified using a Bayesian Network model. Once the classification has been effected, the error is predicted using a SVM model. Their hybrid error model thus predicts the error according to the specific operating conditions. This concept leads to a more generalised prediction model.

Gunn (1998) wrote a comprehensive technical report on the support vector machines for classification and regression. Another similar publication is a tutorial by Burges (1998) entitled Tutorial on Support Vector Machines For Pattern Recognition in which the author presents in the light of regression and classification problems.

1.3 Research objectives

The literature indicates that pre-processing gear vibration data with synchronous time domain averaging can increase the diagnostic capabilities of the measured gear vibration data. This is important to the engineer because it bears the potential of increasing the reliability, repeatability and the diagnostic capability of gearbox condition monitoring strategies. Time Domain Averaging (TDA) is an extremely effective technique for the extraction of periodic data from the vibration signals of rotating machinery. TDA, however, requires an enormous amount of gear vibration data to calculate. This makes it unattractive for on-line gearbox condition monitoring systems.

The literature also indicates that ANNs and SVMs can be successfully used in the non-linear mapping of some input space to an output space. This observation is important to this work in that it bears the potential of reducing the amount of input gear vibration that is required for calculating the TDA of the gear vibration. The time domain averaging process can itself be viewed as a broadband noise filter that eliminates all the vibration that is asynchronous to the vibration of the gear of interest.

The purpose of this study is therefore to investigate and develop a synchronous filter for time domain averaging of gear vibration data using of ANNs and SVMs.

- The developed filter should provide a considerable reduction in the amount of vibration data that needs to be collected and stored in the data acquisition system in order to calculate synchronous time domain average of a gear vibration signal.
- The developed synchronous time domain averaging filter model should retain the diagnostic enhancement capabilities of the TDA calculated by direct averaging.
- The filtering and diagnostic capabilities of the developed filter should be validated for both constant and varying load conditions on experimental gear vibration data.
- Comparison should be made between the performance of the developed filter model and direct time domain averaging.

1.4 Document overview

The theory and mathematics of existing time domain averaging models is presented in Chapter 2. This chapter also presents some simulations that highlight the strengths and weaknesses of some of the popular time domain averaging models.

In Chapter 3 the theory of the MLP neural network, the RBF neural network and SVMs in the context of this work is presented. Simulations on experimental gear vibration data are conducted to investigate the suitability of these formulations for application in the synchronous time domain averaging filter model.

Chapter 4 presents the development process of the synchronous filter for time domain averaging of gear vibration data. Two different synchronous filtering models are developed. Gear vibration data from previous tests is used to investigate the influence of different model parameters on the prediction capability for each of the developed synchronous filtering models.

In Chapter 5 the developed synchronous filtering models are tested on experimental data from accelerated gear life test rig for constant and varying load conditions. The results confirm the suitability of the developed synchronous filtering models for time domain averaging of gear vibration. A comparative study of these models is presented

In Chapter 6 the conclusion to the research and recommendations for further work are presented

Chapter 2

Time domain averaging models

2.1 Introduction

Time domain averaging is a signal processing technique that may be used to extract the synchronous periodic content of a measured vibration signal from the measured vibration signal. This process requires either accurate knowledge of the repetition frequency of the desired frequency, if periodic, or else a second signal that is synchronous with the first signal, but free of noise. Using either the repetition frequency or the synchronous signal, successive blocks of the noisy signal may be sampled and ensemble averaged. When sufficient averages are taken, it is found that the noise in the ensemble averaged signal cancels out, leaving an improved estimate of the desired repetitive signal (McFadden, 1989). One important application in which the periodic signal must be extracted from the noise is the mechanical engineering problem of analysis of vibration from gearboxes. When analysing gearbox vibration, it is sometimes necessary to extract a periodic signal such as the tooth meshing vibration of a single gear from the vibration of the machine. Some understanding of the time domain averaging technique is required for the analyst to appreciate its limitations and successfully optimise its performance for a particular application. This chapter, therefore, presents some of the most commonly used time domain averaging models.

2.2 Existing models

2.2.1 Comb filter model

For many years, time domain averaging has been modelled by the convolution of the noisy signal with a finite train of impulses in which the time between the impulses is equal to the period of the desired signal. It has been shown that this process is equivalent in the frequency domain to the multiplication of the Fourier transform of the noisy signal by a comb filter, thus passing only the frequency components which fall at the fundamental and harmonic frequencies of the desired signal (Trimble, 1968). In this

section the application of the comb filter model for time domain averaging in the extraction of periodic signals is presented.

It can be shown that calculation of the synchronous time domain using a trigger signal having a frequency f_t is equivalent to the convolution

$$y(t) = c(t) * x(t) \quad (2.1)$$

where $c(t)$ is a train of N impulses of amplitude $1/N$, spaced at $T_t = 1/f_t$, given by

$$c(t) = \frac{1}{N} \sum_{n=0}^{N-1} \delta(t + nT_t). \quad (2.2)$$

The convolution of $c(t)$ and $x(t)$ is given by

$$\begin{aligned} c(t) * x(t) &= \frac{1}{N} \sum_{n=0}^{N-1} x(t + nT_t) \\ &= [x(t) + x(t + T_t) + \dots + x(t + (N-1)T_t)] / N. \end{aligned} \quad (2.3)$$

The time domain average $y(t)$ of the signal is then defined by

$$y(t) = \frac{1}{N} \sum_{n=0}^{N-1} x(t + nT_t). \quad (2.4)$$

This equation has the same form as that of the existing comb filter model (Braun, 1975; McFadden, 1987). In the frequency domain this is equivalent to the multiplication of the Fourier transform of the signal $X(f)$ by the Fourier transform of the impulse signal $C(f)$. This operation is represented by

$$Y(f) = C(f) \cdot X(f). \quad (2.5)$$

The Fourier transform of $c(t)$ is $C(f)$, which is a comb filter function of the form

$$C(f) = \frac{1}{N} \frac{\sin(\pi N T_i f)}{\sin(\pi T_i f)}. \quad (2.6)$$

Increasing the value of N in Equation (2.6) narrows the teeth of the comb. For very large values of N , only frequencies at exact multiples of the trigger frequency f_i are passed. Equation (2.6), therefore, implies that in the frequency domain, for large values of N , synchronous time domain averaging can be viewed as a complete removal of all components within the signal that occur at integer multiples of the trigger frequency f_i .

Figures 2.1 (a) to 2.1 (d) show a form of the amplitude spectrum $|C(f)|$ for $N = 1, 2, 4$ and 8 of a comb filter plotted against the normalised Nyquist frequency. The spectrum takes the form of a comb with the teeth of the comb spaced at intervals $f_i = 1/T_i$. The teeth of the comb have unit amplitude regardless of N .

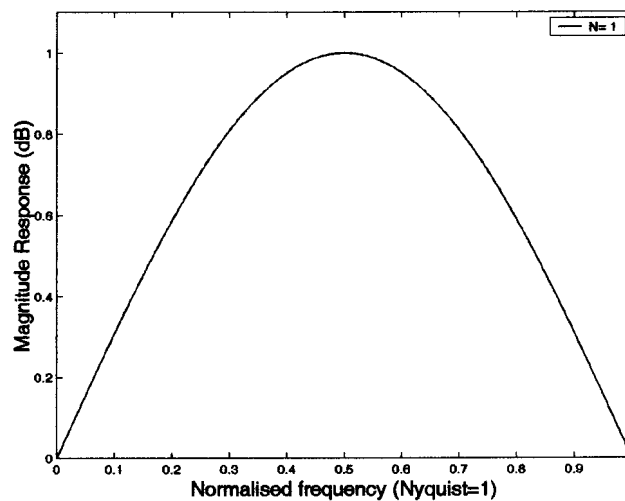


Figure 2.1 (a) Amplitude response of comb filter vs. normalised Nyquist frequency for $N = 1$.

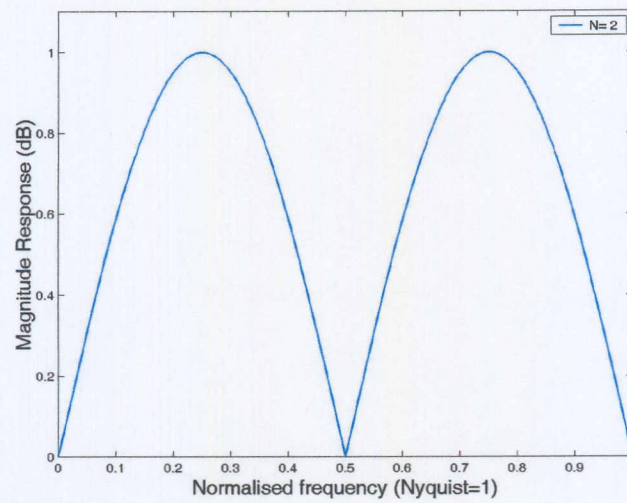


Figure 2.1 (b) Amplitude response of comb filter vs. normalised Nyquist frequency for $N = 2$.

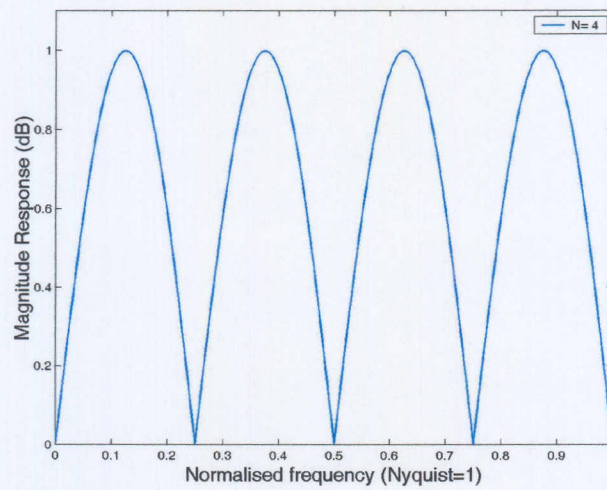


Figure 2.1 (c) Amplitude response of comb filter vs. normalised Nyquist frequency for $N = 4$.

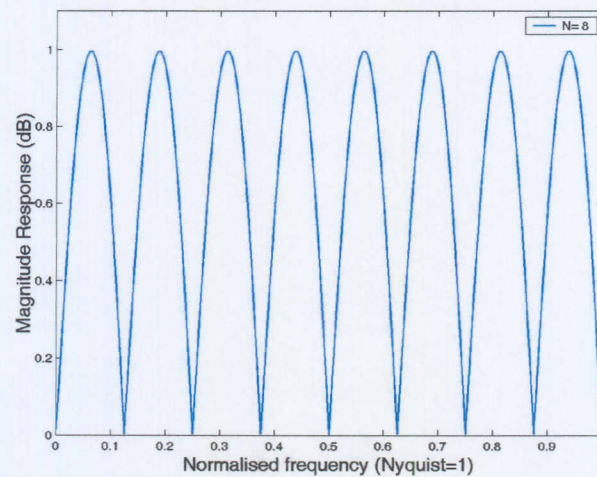


Figure 2.1 (d) Amplitude response of comb filter vs. normalised Nyquist frequency for $N = 8$.

There are two features of the comb filter model that restrict its application to the extraction of periodic waveforms using digital computers. The first of these factors is that in the comb filter model there are bounds placed on the time signal. The model assumes that the signal $x(t)$ is known over infinite time t and that the time domain average $y(t)$ is defined over all time t , even though only a finite number of averages are calculated. In practice, the signal $x(t)$ can only be defined over a finite time. Noise components that are not harmonically related to the repetition frequency f_i may be passed by the comb filter, therefore, the estimate of the time domain average will not be exactly periodic.

Figure 2.2 shows the performance of a comb filter in the extraction of a signal and its harmonics from a numerically generated signal $z(t) = x_p(t) + e(t)$ where $x_p(t)$ is known periodic component defined by:

$$\begin{aligned}
 x_p(t) = & \sin(2\pi 50t) + \sin(2\pi 100t) - 0.45 \cos(2\pi 200t) + 2.1 \sin(2\pi 150t) + \dots \\
 & \sin(2\pi 600t) - 2.5 \cos(2\pi 250t) + \sin(2\pi 300t) + \sin(2\pi 350t) + \dots \\
 & \cos(2\pi 400t) + \sin(2\pi 450t) + \sin(2\pi 500t) + 0.25 \cos(2\pi 550t)
 \end{aligned} \tag{2.7}$$

where $t = (1: 0.001: 6)$, and the additive noise component $e(t)$, the noise content of the signal is defined by:

$$\begin{aligned}
 e(t) = & 0.3 \text{randn}(6000) - 0.48 \text{randn}(6000) + 0.1 \text{randn}(6000) + \dots \\
 & 0.33 \text{randn}(6000) + 0.17 \text{randn}(6000),
 \end{aligned} \tag{2.8}$$

where *randn* is a MATLAB function that defines a set of normally distributed random numbers selected from a normal distribution with a mean of zero and variance of one.

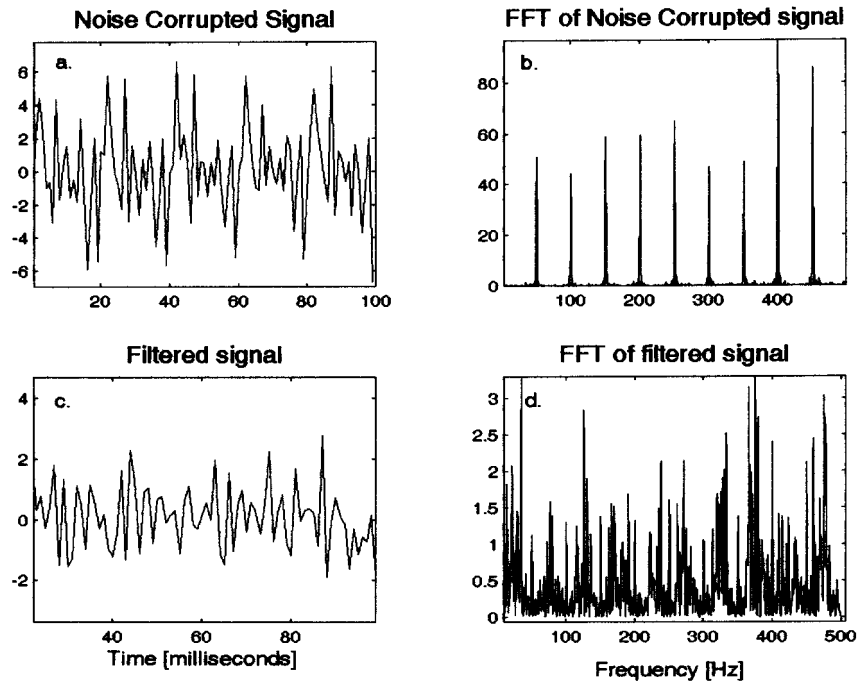


Figure 2.2 (a) Noise corrupted signal; (b) FFT of noise corrupted signal; (c) Signal filtered with a comb filter to remove 50 Hz and all the related harmonics; (d) FFT of filtered signal.

From Figure 2.2 (c) it is observed that the amplitude of the signal decreased after filtering but nothing much can be said about the frequency content of the resultant signal. Figure 2.2 (d) shows the frequency spectrum of filtered signal. It is observed that the amplitude of the frequency spectrum has decreased from 80 to 3 and 50 Hz and all its harmonics have been filtered out leaving only the noise content that does not coincide with 50 Hz.

2.2.2 Double comb filtering

Another attractive model for extracting the time domain averaging is the double comb filtering approach as documented by Braun and Seth (1980). In this model the time domain signal $x(t)$ is decomposed as follows:

$$x(t) = x_p(t) + x_n(t) + n(t) \quad (2.9)$$

where $y_p(t)$ is the periodic term and $x_p(t) = x_p(t+T)$; $x_n(t)$ is a random repetitive term and $n(t)$ is a residual term that is time-locked to a basic period T . A truly periodic

component can only be generated by a truly periodic mechanism. This can only exist in an ideal system because any process involving some sort of friction, liquid or gas flow, and some non-reversible fatigue processes would include to some extent a component like $x_n(t)$ (Braun and Seth, 1980). For a “more or less” periodic process this would show gross periodic character, but no exact repetition of the nature $x_p(t) = x_p(t+T)$ would occur. The parameter $x_n(t)$ thus describes a repetitive non-periodic process as opposed to $x_p(t)$. The periodic component $x_p(t)$ can be computed using the comb filter model as described in Section 2.2.1. The extraction of the random repetitive term $x_n(t)$ is based on the expression

$$y_n(t) = g(t)x_r(t). \quad (2.10)$$

In Equation (2.10) x_r denotes a continuous random process of no obvious time pattern, and $g(t)$ is a deterministic periodic function of period T , (i.e. $g(t) = g(t + T)$). For a case where $x_n(t)$ is derived from a narrow band continuous process, where narrow band refers to a band limited process whose bandwidth is small relative to its centre frequency, where $x_r(t)$ contains negligible energy above frequency f_{\max} such that

$$2f_{\max} \leq f_T = 1/T \quad (2.11)$$

a possible computation scheme for detecting the components of $x_n(t)$ consists of using a comb filter tuned to the narrow bands located around multiples of the fundamental frequencies. A schematic diagram of the computation scheme is shown in Figure 2.3.

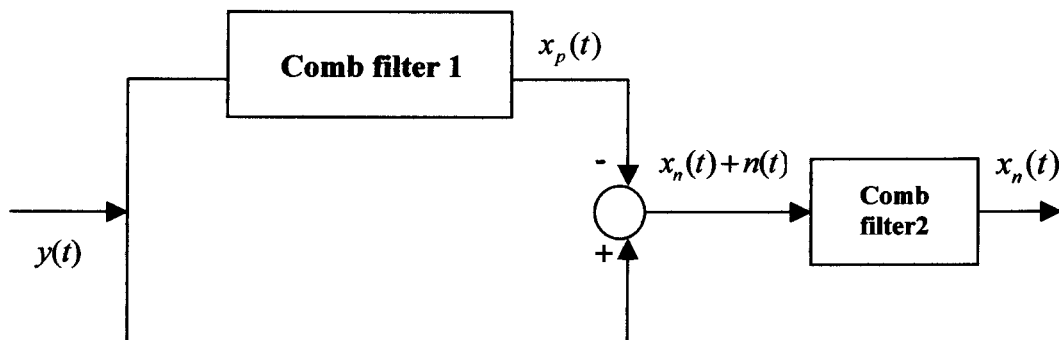


Figure 2.3 Double comb filtering model

The first stage is used to compute $x_p(t)$ and then $x_p(t)$ is subtracted from the original signal $x(t)$. After subtraction, a second comb filter extracts $x_n(t)$. Both these stages will be computed for the same period T .

2.2.3 Revised window model

The revised window model was suggested by McFadden (1987) to address some of the problems encountered with the comb filter described in Section 2.2.1. This model overcomes the problems with the comb filter model in that it requires knowledge of only a finite block of the noisy data and it produces a result that is exactly periodic. This model includes the effect of the signal's sampling frequency f_s . In this section the revised window model is briefly discussed.

Consider a rectangular window $v(t)$ of unit amplitude and width T_R centred at $t = 0$ with Fourier transform $V(f)$ are defined by

$$V(f) = T_R \sin(\pi T_R f) / (\pi T_R f). \quad (2.12)$$

Shifting the window $v(t)$ to the positive direction by an amount $(T_R/2) - (T_s/2) = (T_R - T_s)/2$ where $T_s = 1/f_s$ is the period between the samples of the input signal therefore the edges of the window are located midway between the sampling of the impulses. This avoids the problem of an impulse occurring at the edge of the window. The shifted window $w(t)$ and its Fourier transform $W(f)$ are, respectively, defined by

$$w(t) = v(t - (T_R - T_s)/2) \quad (2.13)$$

$$W(f) = V(f) e^{-i\pi f(T_R - T_s)}. \quad (2.14)$$

Consider now the sampling of the signal $x(t)$ at a frequency f_s over the window of duration T_R . The window, defined by $w(t - nT_R)$, consists of the window $w(t)$ shifted by $t = nT_R$. The sampling of the signal is produced by multiplication of $x(t)$, $w(t)$ and $c(t)$, where $c(t)$ is the pulse signal defined by Equation (2.2). The result is convolved

with the unit impulse by $\delta(t + nT_R)$ located at $t = -nT_R$, thus performing a shift of the sampled signal by nT_R in the negative time direction. The result is given by

$$\delta(t + nT_R) * [x(t) \cdot w(t - nT_R) \cdot c(t)] = x(t + nT_R) \cdot w(t) \cdot c(t + nT_R). \quad (2.15)$$

Now the function $c(t)$ is periodic in T_S . If T_S is chosen such that an integral number M of samples is taken per repetition period T_R then $T_R = MT_S$. This implies that $c(t)$ will also be periodic to T_R , so that $c(t) = c(t + nT_R)$. By replacing $c(t + nT_R)$ in equation (2.15) one gets

$$\delta(t + nT_R) * [x(t) \cdot w(t - nT_R) \cdot c(t)] = x(t + nT_R) \cdot w(t) \cdot c(t). \quad (2.16)$$

An estimate of time domain average $a(t)$ is given by

$$a(t) = 1/N \sum_{n=0}^{N-1} x(t + nT_R) \cdot c(t) \cdot w(t), \quad (2.17)$$

which is equivalent to

$$a(t) = c(t) \cdot w(t) / N \sum_{n=0}^{N-1} x(t + nT_R), \quad (2.18)$$

therefore, the revised window model is defined by

$$a(t) = s(t) \cdot w(t) \cdot x(t). \quad (2.19)$$

Note that although $x(t)$ is not bound in time, $a(t)$ is bounded in time because of the effect of the window $w(t)$. This model therefore satisfies the requirement of knowledge of the signal over only a finite time. Analysis of $a(t)$ in the frequency domain by convolution theorem (Brigham, 1974) shows that $a(t)$ can be forced to be periodic by sampling its Fourier transform $A(f)$ in the frequency domain. This is achieved by multiplying $A(f)$ by an infinite train of ideal impulses $R(f)$, with the impulses spaced

at a repetition frequency f_R . An estimate of the revised window time domain averaging model is given by

$$h(t) = a(t) * r(t) \quad (2.20)$$

The revised window model remarkably changes the result that is predicted by the original comb filter model. Over and above requiring knowledge of the signal over finite time, it also ensures that the obtained result is periodic.

2.2.4 Using direct averaging

Again consider a signal $z(t)$ composed of a periodic signal $x_p(t)$ with known period T_R and an additive noise component $e(t)$

$$z(t) = x_p(t) + e(t). \quad (2.21)$$

The periodic component $x_p(t)$ of signal $z(t)$ can be extracted by direct time domain averaging. To calculate the direct time domain average of a vibration signal, a rotational signal from a sensor mounted on the input shaft or some other suitable location on the rotating machine is used. This rotational signal is used either to control the sampling of the total vibration signal or to determine the accurate period of the vibration of the component of interest and to separate out that vibration component. When the rotational signal is used to control the sampling of the total vibration, a phase locked frequency multiplier is used to convert the rotational signal to the required sampling control signal, which consists of a pulse train synchronised with the rotation of the required gear. When the rotational signal is used to determine the accurate period of the vibration of the component of interest, both the total vibration signal and the rotational signal are sampled simultaneously at fixed clock frequency. When monitoring a gearbox, the rotational signal can be obtained from a sensor like a shaft encoder mounted on the input shaft to the gearbox. The accurate period of the rotating signal can be easily obtained from the shaft encoder signal, and the accurate period of the vibration of the required gear can then be calculated using the transmission ratio.

After the correct period of the rotating signal has been obtained, there are two different approaches that can be followed to calculate the time-domain average of the vibration to separate out the required vibration component. The first approach is by directly averaging some segments of the total vibration (Braun, 1975, Braun and Seth, 1980). The second approach is to first interpolate the total vibration and resample it in the interval that can exactly divide the calculated period, and then averaging the interpolated signal (McFadden, 1989).

In direct time domain averaging, the rotational signal obtained from a sensor mounted on the input shaft to the component of interest is used to synchronise the measured vibration signal with the rotation of that component. This operation gives the vibration produced by that specific rotating component over each rotation. The vibration signals from the rotations are simply averaged to obtain the time domain average after k revolutions. Figure 2.4 illustrates the direct time domain averaging procedure using vibration data from the accelerated gear life test rig developed by Stander and Heyns (2002^a) for their work on gearboxes operating under fluctuation load conditions. The details of the accelerated gear life test rig are presented in Section 4.2.1.

Figure 2.4 (a) shows a plot of the once per revolution pulse signal that would typically be obtained from a shaft encoder to compute the period of each shaft rotation. This signal was measured over a period of 32 seconds in which time the shaft rotated 165 times therefore the shaft encoder gives 165 pulses. Each pulse represents the start of a new gear rotation. This signal is used to synchronise the vibration data measured from the gearbox casing with the rotation of the gear. In Figure 2.4 (a) only 2 seconds of the pulse signal are shown to enhance clarity. Figure 2.4 (b) shows the time domain representation of the measured acceleration signal from the gearbox casing. The acceleration signal was measured in the vertical direction over a period of 32 seconds at a sampling frequency of 51200 Hz, but to enhance the clarity of the figure only 0.5 seconds of measured vibration are shown. Figure 2.4 (c) shows the vibration signals produced by five rotations superimposed on the time domain average that is calculated from 160 gear rotations. From this plot it is observed that the amplitude of the time domain average is less than that of the original signals. This is because the broad-spectrum noise component $e(t)$ has been filtered out through the time domain averaging

process. Another way of looking at this is by observing the RMS value of the TDA as a function of the number of input rotations. Figure 2.4 (d) is a plot of the RMS of the TDA against the number of signals that are used to compute the TDA (number of averages). From this plot it is observed that the RMS value of the TDA decreases as the number of inputs (gear rotations) that are used to calculate the TDA is increased. This is because the non-synchronous component of the gear vibration is filtered out as the number of inputs is increased.

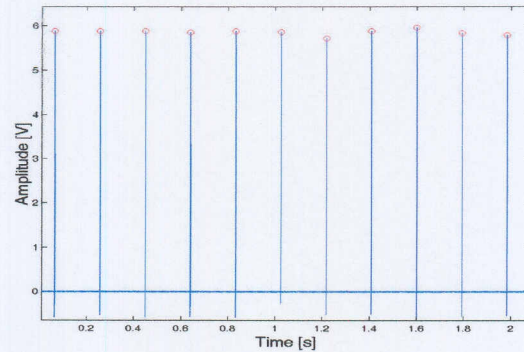


Figure 2.4 (a) One pulse per revolution shaft encoder signal used to synchronise the gear vibration with the gear rotation.

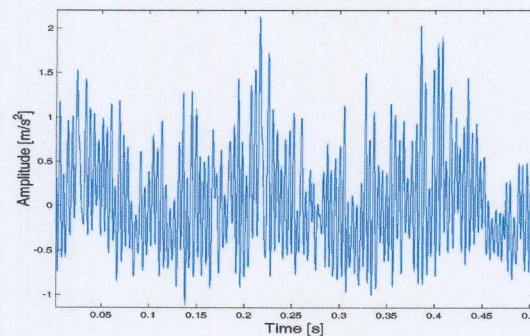


Figure 2.4 (b) Measured gear vibration signal over 0.5 seconds.

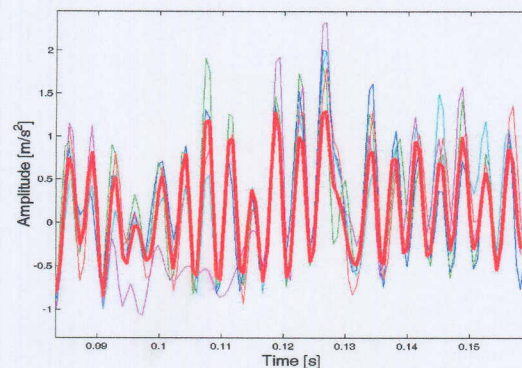


Figure 2.4 (c) Five rotation synchronised gear vibration signals superimposed on the TDA obtained after 160 gear rotations (red signal).

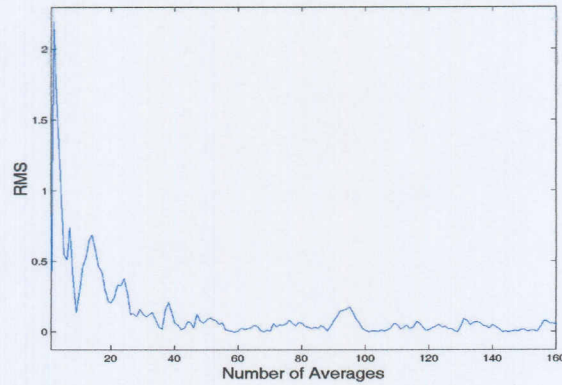


Figure 2.4 (d) TDA RMS vs. number of averages used to calculate TDA

Figure 2.5 illustrates the broad band filtering capabilities of TDA. The FFT spectrum of the TDA after 160 shaft rotations superimposed on the FFT spectrum of the original gear vibration signal. It is observed from Figure 2.5 that the noise content of the original signal $e(t)$ and the frequency content that is asynchronous to the rotation of the gear of interest have been filtered out. Only the gear mesh frequency (GMF) and its sidebands (SB.1 and SB.2) remain in the spectrum of the TDA. It is also observed that the amplitude of the GMF and SB.2 has increased. The amplitude of the spectrum of the TDA is generally less than the amplitude of the spectrum of the original signal at the frequencies that are not synchronous to the rotation of the gear of interest. These observations confirm the fact that calculating the TDA by direct averaging isolates the vibration produced by the rotation of a specific component, therefore the TDA calculated by direct averaging can be utilised to improve the diagnostic capability of a condition monitoring system.

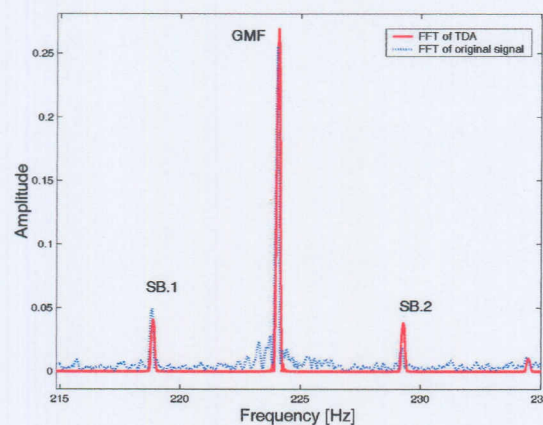


Figure 2.5 FFT of TDA after 160 gear rotation superimposed on the FFT the original gear vibration measured from the gearbox casing.

Another important observation is that calculating the TDA by direct averaging has filtered over an overlapping frequencies because it removed the noise over the entire frequency spectrum, while retaining all the frequency content that is related to the gear interest, in this case, the gear mesh frequency and its side bands. This capability gives calculating the TDA by direct averaging an advantage over other TDA models and linear filters, which can only retain or reject specific frequency bands.

2.3 Conclusion

In this chapter different approaches for calculating the TDA are presented. It is demonstrated that the comb filter model for time domain averaging is suitable for extracting specific frequencies and their harmonics from a signal when the period of the signal is known and constant. This model is, however, not effective when the frequency content of the noise coincides with that of the required signal as is commonly the case in many industrial applications. For gearboxes, a more suitable model for calculating the TDA is direct averaging. It is demonstrated that the direct averaging approach can filter out broadband noise over the entire spectrum of the signal leaving only the vibration content that is synchronous with the rotation of the gear of interest. This capability gives calculating the TDA by direct averaging an advantage over other TDA models and linear filters; therefore, in this study the TDA is calculated by direct averaging. Calculating the TDA by direct averaging requires an enormous amount of vibration data, and therefore, would still remain the main bottleneck in the development of an online gear condition monitoring system that utilises the TDA calculated by direct averaging to enhance its diagnostic capability. The TDA models developed later in this work seek to reduce the amount of vibration data that is required to calculating the TDA by direct averaging while retaining all the properties of the TDA.