CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

> *Successful exposition may take many different forms but the following are some of the qualities which should be present: it challenges and provokes the pupils to think; it is reactive to pupils' needs and so it exploits questioning techniques and discussions; it is used at different points in the process of learning and so, for example, it may take the form of pulling together a variety of activities in which pupils have been engaged; and it uses a variety of stimuli* (DES, 1985: 38).

In the last decade issues of disadvantage and mathematics achievement have moved to the centre of policy-makers' agenda and academic debate. Underachievement in mathematics is particularly recognised as a major problem in schools serving disadvantaged communities in South Africa (Mkhabela, 2004).

As explained, in Chapter 1, mathematics is a pillar of almost all the streams in academic sectors. Given the important role mathematics plays in tertiary education and most careers, it is not only beneficial but also essential to establish some of the factors that facilitate achievement in mathematics in disadvantaged schools. This study will hopefully facilitate the quest to improve achievement in mathematics in disadvantaged schools and establish what schools can learn from one another. Although there are studies conducted in other countries regarding factors that facilitate achievement in mathematics, few such studies were conducted in disadvantaged secondary schools in South Africa.

According to Hughes (1999) the most important conclusions from qualitative research on factors related to achievement in schools are that (a) teachers are critical resources; (b) the composition of the student body matters; (c) schools make a difference, and (d) physical facilities, class size, curriculum, instructional strategies and other resources influence student learning indirectly through their effect on the behaviour of teachers and students.

In an effort to identify the causes for low achievement in mathematics, some researchers (Attwood, 2001; Brodie, 2004; Maree, 1997; Moyana, 1996; Murray, 1997; Malcolm *et al.*, 2000) have suggested that achievement in mathematics in secondary schools is influenced by a number of variables. These variables include learners' abilities, attitudes and perceptions, family and socio-economic status, parent and peer influences, school-related variables such as poor learning environment, learning cultures, past racial discrimination and low expectations by principals and teachers.

According to Singh *et al.* (2002) many of these variables are home and family-related and thus are difficult to change and beyond control of educators. Such factors alone cannot account for the lack of mathematics achievement and persistent differences among traditionally disadvantaged learners. In particular these explanations fail to account for intra-group achievement differences and the success of some South African disadvantaged learners in spite of these background factors. Some well-achieving disadvantaged learners come from the same communities and share similar socio-economic backgrounds, schools and classrooms.

In investigating factors that facilitate achievement in mathematics, variables related to school, learners and teachers were reviewed**.** In this regard Malcolm *et al.* (2000) in their literature review suggest that when investigating factors that facilitate achievement in science and mathematics, a more extensive investigation should consider learner, teacher and school variables. The chapter concludes with some learning theories relevant to secondary school mathematics learning and teaching.

2.2. SCHOOL-RELATED VARIABLES

Several studies have shown a positive correlation between a disadvantaged school environment and learners' achievement at school. For example learners in the Western and Northern Cape provinces, which have large white populations and well-endowed communities and schools, lead in pass rates in grade twelve examinations whereas those in Limpopo Province with its black population majority rank last in this regard (Murray, 1997). Attwood (2001) also found that grade eight learners in schools situated in economically depressed areas of the Cape Flats have a lower mathematics achievement than those who come from families with high socio-economic status. Furthermore, a comprehensive study on the status of mathematics and science teachers in South Africa found that black learners were underprovided for and performed worse than their white peer group in these subjects (Arnott *et al.*, 1997).

Although the above findings show in general that disadvantaged learners tend to achieve less well than other learners, some studies indicate that some disadvantaged learners perform better than advantaged learners. In this regard a September 2002 issue of the *Sunday Times* spotlights some successful rural schools. All the schools were from disadvantaged communities (*Sunday Times*, 2002).

From the United States of America there are studies that show some successful programmes for disadvantaged learners (Hilliard, 1988). Hilliard (1988) cited a programme for disadvantaged students in which the conditions in the community surrounding the schools were unchanged. The poor conditions surrounding the schools did not change the expected educational outcomes. Parental involvement, though desirable, was not a necessity. Deprivation in learners' lives outside the school was not eliminated. No special pedagogies were employed. According to Hilliard (1988) results were obtained because learners were offered excellent quality instruction. In this regard a review of 377 studies by Hanushek (1989) shows no consistent pattern between school resources (in terms of money spent on schools) and achievement. Hanushek (1989) concluded that no strong or consistent relationship exists between school resources and learners' performances and that more resources would not yield performance gain for learners.

2.2.1 Learning environment

According to Smith and Ragan (1993) a learning environment comprises teacher, existing curriculum, instructional equipment as well as the institutional and larger learner community. In this regard Shields (1991) stated that the school environment is the broader climate or context of the school that either facilitates or constrains classroom instruction and learning. Ross, Farish and Plukett as cited in Zaaiman (1998) describe the learning environment that is considered disadvantageous for Australian schools by using detailed census-based social profiles of school catchment areas. A learning environment, particularly the school, was considered as disadvantaged if a high proportion (of the enrolment) of learners came from neighbourhoods having certain characteristics known to be associated with a low capacity to take advantage of educational facilities. These characteristics include, among others, a high percentage of persons in low status jobs with low income or with lack of formal educational qualifications. Furthermore, many families with single parents and more non-fluent English speakers (English as a second language) were found in the low socio-economic areas. The homes of the disadvantaged tend to be more crowded, lacking in magazines, newspapers, and other objects that are likely to help in the development of the learner. Parents of the disadvantaged learner give little language encouragement to their children, have less direct interaction with them and take less interest in their learning.

For a number of interrelated reasons many of these disadvantaged learners' families and schools in South Africa are concentrated in the rural areas and in the outskirts of our great metropolitan areas known as townships and squatter camps. One can therefore state that learners who attend under resourced South African schools have been educationally disadvantaged through a lack of opportunities to access quality educational services. The most underresourced schools are the ones that belonged to the previously black-only educational system (Zaaiman, 1998). According to the Department of Education (1997) Limpopo Province, where this study was conducted, as well as the Eastern Cape and Kwazulu Natal are worst off in terms of the need for physical facilities, services, and equipment and teaching resources.

2.2.2 Curriculum

According to Pinar, Reynolds, Slattery and Taubman (1995) the concept of curriculum is highly symbolic; it is what the older generation chooses to tell the younger generation. Beggs (1995) states that curriculum traditionally means a list of content topics in a national or school syllabus and examination prescription, generally referred to as course outline. According to Beggs (1995: 97-106) a mathematics curriculum includes:

- Mathematical content (what mathematicians know)
- Mathematical processes (what mathematicians do)
- Mathematical thinking and logical reasoning, problem-solving making connections and using computational tools
- Contexts in which the topics are set
- Assessments strategies that are used, and appropriate teaching methods (DoE, 2002).

The International Mathematics Study (Schmidt *et al.*, 1996), sponsored by the International Association for the Evaluation of Educational Achievement (IEA), consider the study of mathematics at three levels:

- The *intended* curriculum is that which is reflected in curriculum guides, course outline, syllabi and textbooks, adopted by the educational systems. In most countries a nationally defined curriculum emanates from a ministry of education or similar national body. In South Africa the National Government develops intended goals or specifications of curricula content.
- The *implemented* curriculum is that which is actually taught in the classroom by the teacher. The difference between the intended and implemented curriculum could be due to many factors: the curriculum is too long, the learners do not master some of the competencies and the teacher does not have time to cover all of them. The teachers have different priorities regarding what should be taught; the teachers did not master some of the competencies to be taught and thus do not include them in the classes; or teachers do not have some of the educational materials needed to teach some competencies. From their review of literature Chen *et al.* (1988) acknowledge that content coverage is related to learners' achievement.

- The *attained* curriculum is a measure of what students have learned, and is reflected in the students' achievements and attitudes. According to Schmidt *et al.*, (1996) achievement is referred to as the attained curriculum.

The attained curriculum, and not the intended curriculum, explains mathematics achievement of the learners. The author of this study is of the opinion that learners have to have the opportunity to learn all the mathematical topics on which they have to be evaluated.

2.2.3 School and class size

School size and class size have been shown to have an impact on achievement. Lee, Smith and Croninger (1997) observed that larger schools had a negative influence on academic achievement in high school mathematics and science. In contrast, Rutter (1983) found no relationship between the size of the school and scholastic achievement; effective schools can be very small, very large or somewhat in-between. Rutter (1983) further observed that the relationship between the class size and a learner's achievement is not well defined for classes with 20 to 40 learners. Class sizes of below 20 learners have been found to be advantageous for disadvantaged learners. In this respect Rutter (1983) argued that small school size facilitates social interaction and inhabits teacher specialisation.

2.2.4 Culture

South Africa is a country of diverse cultures as manifested by the notion of "rainbow nation". Fantini and Weinstein (1968) point out that culture is the aggregate of attitudes, traditions, and ethical codes peculiar to the particular society. Internationally much has been written about the relationships between culture, mathematics learning and teaching. The topics range from cultural bases for mathematics, mathematics development from different cultures, the effect of culture on mathematics learning and political effects of societies on mathematics. Maree (1994) cites Scholnick as stating that people's cultural backgrounds can influence aspects of mathematics that different cultures may stress. Maree (1994:145) quotes Scholnick as follows:

> *There are strong biases about who can learn mathematics, and pervasive differences in learning skills…. There is a hot debate about whether there are generic differences in mathematical capacities…Similarly, there may be cultural differences in the patterning of skills that reflect attitudes and values about the role of mathematics in daily life …Although mathematics is not a Rorschach blot that every society and family within a society can interpret, nevertheless, there may be fundamental differences in aspects of mathematics that different cultures may stress … that may account in part for the difference in mathematics achievement.*

Research in mathematics education has sought to understand how cultural differences affect learners' performance in mathematics. Analyses of studies on culture and mathematics reveal some general cultural factors that affect mathematics performance, namely:

- Societal influence
- Parental attitudes, values and beliefs
- Teacher attitudes, values and beliefs
- Learners' perceptions and beliefs
- Language

Johnson (1984) states that blacks in the United States of America underachieve greatly in mathematics and as a reason for this underachievement he blames cultural-related factors such as:

- An inability to see the usefulness of mathematics to their lives, both present and future;
- a lack of success in previous mathematics courses;
- a failure to receive positive career counselling;
- an absence of role models;
- a lack of significant others, such as parents, who show an interest in mathematical achievements;
- a view of mathematics as a subject appropriate for white males only (Maree, 1994).

2.2.5 Effectiveness of schools

Effective school characteristics are what help to create a fertile school culture that facilitates learners' achievement. Several researchers (Henson & Eller, 1999; Berliner, 1990, and Rutter, 1983) have identified such characteristics. Their findings indicate that learners excel when the following factors are present (Henson & Eller, 1999; Berliner, 1990, and Rutter, 1983):

- Strong leadership is provided by a principal who works with the staff to communicate the mission of the school; provide reliable support for staff; and meet with teachers and other members of the staff frequently to discuss classroom practices.
- High learner achievement is the foremost priority of the school, and the school is organised around this goal as shown by teachers who demonstrate high expectations for learners' achievement and make learners aware of and understand these expectations.
- Parents are aware of, understand, and support the basic objective of the school and believe they have an important role to play in their children' education.
- Teachers work together to provide an orderly and safe school environment.
- Schools use evaluation to measure learners' progress and promote learning.

2.3 LEARNER-RELATED VARIABLES

2.3.1 Attitudes and beliefs

According to McLeod (1992) factors such as attitudes and beliefs play an important role in mathematics achievement. The general relationship between attitude and achievement is based on the concept that the better the attitude a learner has towards a subject or task, the higher the achievement or performance level in mathematics.

Stuart (2000) argues that teacher, peer and family attitudes toward mathematics may either positively or negatively influence learners' confidence in mathematics. The findings are that learners who have positive attitudes towards their teachers have high achievement levels. Newman and Schwager (1993) found that at all grades a sense of personal relatedness with the teacher is important in determining a learner's frequency in seeking help from the teacher. They further state that this aspect of the classroom climate has been shown to be related to good academic outcome. In the same vein Dungan and Thurlow (1989) state that the extent to which learners likes their teacher, influence their liking of the subject.

2.3.1.1 Career choice and mathematics achievement

Research on attitudes towards career choice and towards mathematics teachers is extensive. Eccles and Jacobs (1986) found that self-perceptions of mathematics ability influence mathematics achievement. Norman (1988) concluded from a wide review of literature that there is a positive correlation between career choice and mathematics achievement. Subsequently Trusty (2002) reported that learner attitudes impact on later career choices in mathematics. Ware and Lee (1998) found that mathematics attitudes during high school had a positive effect on choosing science careers. Accordingly, Armstrong and Price (in Pedersen *et al.*, 1986) found that the career aspirations of high school learners influence their participation in mathematics, which in turn influenced their mathematics achievement. Trusty and Ng (2000) studied learners' self-perceptions of mathematics ability and found that positive self-perception mathematics ability has relatively strong effects on later career choices.

2.3.1.2 Enjoyment and ability

Ma (1997) observed that in the case of trigonometry learners, the attitude that mathematics was important and enjoyable was significantly associated with achievement in mathematics. According to Ma (1997) learners who have more enjoyable experiences while learning mathematics achieve higher scores. In a study of Grade 10 to 12 geometry classes, Schoenfeld (1989) explores aspects of the relationship between learners' beliefs about mathematics, their sense of mathematics as a discipline and their relationship with it, and their mathematics performance. Schoenfeld (1989: 346) found the following:

> *Learners who think less of their mathematical ability tend more to attribute their mathematical successes to luck and their failures to lack of ability, whereas those who consider themselves to be good at mathematics attribute their success to their ability.*

Learners studied by the researcher made a distinction between the mathematics they know and experience in the classrooms and abstract mathematics, the discipline of creativity, problem-solving, and discovery, about which they are told but have not yet experienced (Schoenfeld, 1989). According to Schoenfeld (1989), the learners tend to think of classroom mathematics as requiring memorisation of equations and formulas and knowing the rules.

In mathematics education most of what is known about beliefs and attitudes of the learners towards mathematics is based upon large-scale survey data (Martin, 2000). For example the National Assessment of Education Progress in the United States of America has shown that African American learners constantly express the most positive attitudes towards mathematics among all learner groups. Other studies show that many African American learners identify mathematics as their favourite subject. Similar studies in South Africa show that most of the learners have positive attitudes towards mathematics (Howie, 2001). According to Howie (2001) the high rate of absenteeism reported among learners indicates that the problem lies more with learners not being motivated enough to attend school. Molepo (1997) found that the rural communities regard mathematics as an important subject that can play a role in developing them socio-economically.

2.3.1.3 Peer pressure

Peer pressure in mathematics affects all learners, successful ones as well as those who are less successful. The effect of negative peer pressure has been recorded in numerous articles (Dungan & Thurlow, 1989; Reynolds & Walberg, 1992; Stuart, 2000). In this regard Stuart (2000) argues that peer and family attitudes towards mathematics may either positively or negatively influence learners' confidence in the subject. In their review of literature Dungan and Thurlow (1989) found that learners' attitudes towards mathematics have been associated with peer group attitudes. Accordingly, Reynolds and Walberg (1992) identified peer attitudes as one of the most influential factors in learners' mathematical achievements. According to Harris (1995) learners are ridiculed by their peers for taking challenging mathematics while others are encouraged by their peers to pursue academic excellence in mathematics.

2.3.1.4 Peer support

Evans, Flower and Holton (2001) define peer support or tutoring as that part of the teaching process that involves learners teaching other learners. Griffiths, Houston and Lazenbatt (in Evans *et al.*, 2001: 161) state that:

> *Peer tutoring is a structured way of involving students in each other's academic and social development. As a learning experience it allows students to interact and to develop personal skills of exposition while increasing their knowledge of a specific topic.*

Tutors may be high-ability learners or learners in higher grades. Tutors may also be low-ability learners who assist other low-ability learners. Abrami, Chambers, D' apollonian and Farrell (1992) report that learners may benefit motivationally from being in groups which provide peer encouragement and support. As a result their achievements can be improved. The view of this researcher is that since peers can encourage one another to view mathematics positively or negatively, a major task for teachers is to understand the nature of peer relationships so that this influence can be directed towards positive engagement.

2.3.2 Effort and recognition

2.3.2.1 Self-esteem and mathematics anxiety

Research on attitudes towards mathematics has focused on two major dimensions, namely **mathematics self-concept** or **self-esteem** and **mathematical anxiety**. According to Fiore (1999) reinforcing effort in mathematics begins with helping learners to develop a positive self-concept. Michell *et al.* (2003: 42) states that:

> *Mathematics self-concept refers to a person's perception of their ability to learn new topics in mathematics and to perform well in mathematics classes and tests.*

Fennema and Sherman (1978) find that the mathematics self-concept is correlated with achievement in mathematics. They further find that mathematics self-concept is higher in males than in females at high school. In contrast Maqsud and Khalique (1991) find that there is a significant positive relationship between self-concept and attitude towards mathematics for female groups, but no significant correlation between these variables for male groups was found. Maqsud and Khaqlique (1991) go on to report that self-concept measures for both males and females do not reveal any significant association with their mathematics achievement.

Visser (1989: 38) defines mathematics anxiety as follows:

> *Maths anxiety may be defined as an irrational and impedimental dread of mathematics. The term is used to describe the panic, helplessness, mental paralysis and disorganization that arise among some individuals when they are required to solve a problem of mathematical nature.*

This definition has been supported by Mitchell *et al.* (2003) when they state that mathematics anxiety refers to a person's feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary and academic situations. According to Visser (1989) this phenomenon could occur at any time during a learner's career and it usually does not disappear spontaneously. Mathematics anxiety has been found to be consistently related to lower achievement in mathematics at all ages (Betz, 1978 & Hembree, 1990). Nimer (1990) establishes a significant relationship between low achievement in mathematics and high levels of mathematics anxiety. In support of this finding, a consistent, negative relationship between mathematics anxiety and performance is reported by Wong (1992).

A number of researchers have suggested the following methods to facilitate the processes that are likely to lead to improvement in learners' attitudes towards mathematics and other subjects. Some of these address the feeling of mathematics anxiety or negative mathematics self-concept and they are as follows:

- Teaching learners to praise others (Henson and Eller, 1999).
- Assigning tasks that challenge learners and yet are within their ability range and give them the autonomy required to complete the task (Henson and Eller, 1999).
- The content of mathematics must be taught so that learners understand. The more they understand, the less anxiety learners will exhibit (Reyes, 1990).

- Learners increase their mathematical understanding and reduce their anxiety when they work collaboratively in small groups using examples to demonstrate concepts (Sherard, 1995).
- Teachers should encourage learners to ask questions while creating a comfortable class environment that emphasizes learning through class participations (Tobias & Weissbrod, 1980).
- Teachers should encourage learners to understand their own personal learning style by discussing the relationship of their learning style to math concepts (Fiore, 1999).
- Incorporating co-operative learning in the classroom was shown to have a positive effect on learners' achievements and attitudes towards mathematics (Walmsley & Muniz, 2003).
- Writing an essay about prior math experiences, both positive and negative, also helps learners understand that their success or failure may be more related to environmental factors than their own intelligence (Fiore, 1999).
- Teaching learners to develop self-evaluation skills and to make realistic evaluations of themselves (Henson and Eller, 1999).

2.3.2.2 Interest

Some research has suggested that, compared to other subjects, there is a relatively strong relationship between interest and achievement in mathematics (Schiefele, Krapp & Winteler: 1992). In this regard, Maree (1994) stated that the following factors are of significance in the learners' interest in mathematics:

- Learners' feelings play an important role in mathematics interest.
- Learners' interest and ability are positively related. According to Maree (1994) the better a learner performs in mathematics, the more he/she will like the subject and vice versa.

2.3.3 Language

Mother tongue is very important to the clear formulation of mathematical concepts as all ideas are communicated between the teacher and the learner, either through oral or written material (Cocking & Mestre in Kaphesi 2004). In this regard, the question of whether language proficiency is related to learning ability in mathematics and general academic achievement has been debated for many years. Much of the debate has centred on the performance of disadvantaged learners with limited proficiency in English. Factors in language proficiency and academic achievement include bilingualism, ethnicity, socio-economic status, use of non-standard dialects and other social and cultural variables. The problem posed by a foreign language when learning mathematical concepts, is confirmed by Orton (1992:141) when he states:

> *Communicating mathematical ideas so that the message is adequately understood is difficult enough when teacher and learner have a common first language, but the problem is more acute when their preferred languages differ.*

Emphasising the importance and relevance of language in mathematics teaching, Orton (1992) further indicates that:

> *Language is important not only for communicating but also because it facilitates thinking. The language used for thinking is most likely to be the first language, thus mathematics communicated in one language might need to be translated into another to allow thinking, and would need to be translated back in order to converse with the teacher.*

In this regard Berry (1985) has contrasted the mathematical progress of a group of university mathematics students in Botswana and a similar group of Chinese University students in Canada. The former group claimed they had to do all their thinking in English, because their own language did not facilitate mathematical proof, and they found this difficult. The Chinese students, on the other hand, daimed that they carried out their proofs in Chinese and then translated back to English, and that they were able to do this quite successfully. Berry concludes that the more severe problems would be likely to lie with learners trying to learn mathematics through the medium of an unfamiliar language, which is very different from their own.

In this regard Berry as cited in Orton (1992: 143) has summarised the language problems as follows:

> *In general it is likely to be easier for a student to function effectively in a second language which is semantically and culturally close to his mother tongue than in one which is remote…[for]…the structure of a person's language has a determining influence on that person's cognitive processes…such as classification and recognition of equivalences - processes which are central to the understanding of mathematical concepts.*

According to Dlamini (2008: 11) there are certain considerations that need to be noted in relation to language and mathematics. These are as follows:

- English language is a poor predictor of future performance for learners in Swaziland in the mathematics field of study.
- High achievers in mathematics may not be high achievers in the English language.

2.3.4 Learner motivation

The question of how to motivate learners in the classroom has become a leading concern for teachers in all disciplines, let alone in mathematics. A learner is motivated by a desire for knowledge. Stimulation of this desire is one of the basic tasks of a teacher. In this regard Wlodkowskin (1986: 6) defines motivation as:

> the word used to describe those processes that can (a) arouse and instigate behaviour; (b) give direction or purpose to behaviour; (c) continue to allow behaviour to persist; and (d) lead to choosing or preferring a particular behaviour.

Motivation deals with the reasons why learners become interested and react to those events that catch their attention. There are two distinct types of academic motivation that are interrelated in most academic settings, namely intrinsic and extrinsic motivation.

According to Piek (1984:22):

> *Extrinsic motivation stems from outside the subject matter area, but is in some way analogous to it. One thinks here of favourable circumstances, an exemplary teacher, the subject matter and the method of instruction, competition, prizes, allocation of marks, promotion and various other rewards.*

In this type of motivation, learners tend to centre on such performance goals as obtaining favourable judgement of their competence from teachers, parents, and peers or avoiding negative judgements of their competence (Ames & Archer, 1988; Duda & Nicholls, 1992).

Academic intrinsic motivation is the drive or desire of the learners to engage in learning **for its own sake.** Hunter (as cited in Molepo, 1997:63) describes the meaning of intrinsic motivation as follows:

> *When the activity itself is rewarding (enjoying reading or swimming) we have a situation where motivation is intrinsic, that is the activity will achieve its goal.*

The implication of this type of motivation is that learners who are intrinsically motivated engage in academic tasks because they enjoy them. Learner's motivations tend to focus on learning goals such as the understanding and mastery of mathematical concepts. When learners engage in tasks in which they are motivated intrinsically, they tend to exhibit a number of pedagogically desirable behaviours including time spent on tasks and persistence in the face of failure (Duda & Nicholls, 1992). In order to increase achievement in mathematics for traditionally disadvantaged learners' proper use of both intrinsic and extrinsic motivation is important. The author of this study is of the opinion that one of the tasks of the teacher working with disadvantaged learners is to ensure that the confidence and interest of these learners in mathematics is cultivated.
According to Alderman (1990) there are many aspects of learner motivation but four seem to be particularly important, namely:

- *The learning culture* – this is mainly about peer pressure and the ethos that exist at schools.
- *Intrinsic purpose of learning* – this shifts the emphasis from extrinsic motives for learning to intrinsic motives.
- *Self- efficacy* – this concerns learners' belief in themselves and their ability to learn.
- *Task value* – this is about the learners' interest in the task and the importance and value

Alderman (1990: 28) points out that:

> *Teachers who are successful in reaching disadvantaged learners combine a high sense of their own efficacy with high realistic expectations for learner achievement.*

Teachers with a high sense of efficacy or confidence in their ability to influence learner motivation are more likely to view disadvantaged learners as teachable (Ashton & Webb, 1986).

2.3.5 Learners' academic involvement

The concept of learners' involvement in learning is closely related to that of academic achievement. According to Sigh, Granville and Dika (2002) academic engagement is defined as active involvement and commitment as opposed to apathy and lack of interest. For instance, doing homework, coming prepared for classes, regular attendance and not skipping classes reflect learners' engagement and motivation. Astin (cited in Steyn, 2003: 91) points out that:

> *The theory of learner involvement encourages educators to focus less on what they do and more on what learners do.*

2.3.5.1 Homework

Homework is one of the instructional tools used by teachers to determine a learner's academic engagement. Homework here is defined as any subject work completed outside the regularly scheduled class. According to Grouws (2001) the purpose of homework includes the following:

- Developing skills
- Increasing understanding
- Demonstrating application
- Developing connections

A synthesis of research by Cooper (1994) and Pezdek, Berry and Renno (2002), shows that homework could have both positive and negative effects. Cooper (1994: 1) reports the positive effects of homework to include: *better retention of factual knowledge; increased understanding; better critical thinking; concept formation and information* *processing*. Cooper (1994) also noted that positive long-term academic effects include improved attitude towards school and better study habits and skills. Cooper (1994) also reports significant negative effects of homework, namely loss of interest on academic material, pressure to compete and perform well; parental interference, confusion regarding instructional techniques, copying homework from other learners and physical and emotional fatigue. Mullis (1990: 351) found that in some states in the United States of America, learners who reported doing homework had low achievement in mathematics. Research has shown, however, that the positive effects of homework outweighed the negative effects.

2.3.5.2 Time on task

According to Schoenfeld (1985) high school learners who perform poorly in mathematics tend to believe that mathematics problems should be solved in 10 minutes or they will never be solved. Schoemer, Calvert, Garglietiti and Bajaj (1997) observe that the more learners believe in fixed ability or quick learning, the lower their mathematics scores.

2.3.6 Learning approaches

Learning approaches (strategies) are defined as the behaviours and thoughts that learners use to select, organise, and integrate new information with their existing knowledge (Weinstein & Mayer: 1986). According to Biggs and Telfer (1987) there are three types of learning approaches, namely, **deep approach, surface approach and achieving approach.** These approaches have been widely used in learning. Table 2.1 identifies and compares characteristics of a learner who follows a deep approach and one that follows a surface approach.

Table 2.1 Approaches to learning according to Marton and Saljo

A learner who follows a deep approach	**A learner who follows a surface approach**
• tries to understand the 'big picture' • tries to relate content in a broader context • tries to synthesise aspects and integrate them within the 'big picture' • identifies core principles and distinguish detail • seeks for deeper meaning of course contents and • is committed to and enjoys the learning task	• focuses on aspects and not on the 'big picture' • sees the learning task as a short-term activity, for example to pass a test • cannot synthesise aspects and integrate them into a 'big picture' • invariably masters content through memorization • is focused on time and immediate completion of the task and not on a possible deeper meaning thereof and • is not necessarily committed to the learning task or enjoyment of the process

Adapted from Steyn (2003: 39)

In this regard Entwistle and Ramsden (1983) state that learners choose a specific learning strategy depending on their cognition of the learning task. They (Entwistle & Ramsden, 1983) go on to say that learners adapt their learning strategies through contact with other learners. In addition to deep and surface approaches there is the achieving approach. This is a learning approach motivated by the need to achieve success, in particular through obtaining high grades (Entwistle & Ramsden, 1983). This approach to learning has the following characteristics:

- putting consistent effort into studying;
- finding the right condition and material for effective study;
- managing time and effort efficiently.

Because the intention of learners using an achieving approach is to obtain high grades, they perceive the task as a medium to achieve the end and not as a learning opportunity. According to Biggs and Telfer (1987) learners may combine the achieving approach with either the deep or the surface approach in the form of deep-achieving approach or surface-achieving approach. The opinion of the author of this study is that the deep-achieving approach makes learners learn and achieve with understanding.

2.3.7 Learners' poor achievement in mathematics

A limited number of researchers have cited reasons for poor mathematics achievement among disadvantaged learners in the classroom situation. The following paragraphs will outline some of these reasons. According to Gourgey (1992) the following reasons were stated as poor achievement by many learners, let alone among disadvantaged learners.

- Feeling of being powerless when mistakes are made and not knowing how to correct them.
- Distrust of own intuition.
- Maths is emotionally charged, evoking strong feelings of aversion and fear of failure.
- Maths is seen as a subject to be performed by applying algorithms dictated by higher authority, rather than understanding underlying logical principles.

The researcher of this study agrees with the opinion of Russell (1995) who remarks that learning requires personal effort and learners need to understand that teachers, parents and peers cannot do the work required of learners.

2.4 TEACHER-RELATED VARIABLES

Meyer and Koehler (1990) state that one of the most important factors in developing learners' mathematics ability is the attitude of their teacher of mathematics. According to Meyer and Koehler (1990) knowledge of the learners' thinking is important while teachers' knowledge of mathematics content and pedagogy is also critical to the culture of the learning environment. According to Lubinski (1994) knowledge of the content and pedagogy in conjunction with learners' thinking, allows a teacher to design blueprints for worthwhile mathematics tasks.

In this respect it is reasonable to expect that teachers will feel successful when their learners perform well in mathematics, irrespective of whether or not they come from a historically disadvantaged school situation. It should also be expected that teachers would feel frustrated and unsuccessful when the learners perform badly. What is not clear, is who should be blamed, the teachers or the learners? Fennema and Franke (1992: 176) observe that:

> *If teachers attribute success or failures of students to themselves, then they will do something to alleviate the problem. If, on the other hand, the reason that learners succeed or fail lies within learners, then the teachers do not feel as much responsible for the failure.*

2.4.1 Attitudes and beliefs

2.4.1.1 Attitudes towards mathematics

In mathematics research, one area of focus has been on teachers' beliefs and attitudes towards mathematics. Ernest (1989) observes that the practice of teaching mathematics depends on a number of key elements, such as the teachers' mental contents and schemes, particularly the system of beliefs concerning mathematics and its teaching and learning; the social context of the teaching situation, particularly the constraints and opportunities it provides and the teachers' level of thought processes and reflection.

Fennema and Romberg (1999: 174) have made similar observations that teachers' beliefs influence the way teachers teach and talk about mathematics to their learners. She observes that:

> *If teachers believe that mathematics is useful, it seems reasonable to assume that they will work harder to ensure that their learners learn mathematics.*

Mudeliar (1987) also asserts that teachers' attitudes towards mathematics have a strong bearing on learners' attitudes and achievement in mathematics. In a review of related literature on learners' attitudes towards mathematics, Dungan and Thurlow (1989) conclude that learners' attitudes towards mathematics are derived from teachers' attitudes towards the subject. These attitudes in turn affect learners' mathematics achievement.

O'Laughlin (1990) found that novice teachers maintain definite beliefs regarding knowing, learning and teaching, which usually lead them to endorse didactic approaches with the teacher acting as the primary conveyer of knowledge. A teacher's beliefs about learners' abilities greatly influence the decisions the teacher makes about the learning environment (Lubinski, 1994). Lubinski (1994) also feels that teachers, who believe that the content of the mathematics in their classroom is guided by the textbook, make decisions that differ from those of teachers who believe that learner' interest and ability guide the content of the mathematics. In this regard Leder and Gunstone (cited in Ethington, 1990) referred to Colburn who recommends that the textbook should not be followed slavishly, but should be adapted to suit the needs of both the teacher and learners.

Research suggests that teachers' beliefs and teachers' knowledge are related to the instructional decision-making process (Fennema & Franke, 1992; Thompson, 1992). Consequently what a teacher believes about the content, methods, and materials available to teach mathematics influences the teacher's instructional decisions.

Schmidt (1999: 81) also observes that:

> *What teachers teach and how they teach it are affected by their subject matter belief and preferred pedagogical approaches, things that are consequences of their training and experiences.*

Fennema and Franke (1992) further indicate that the way teachers teach is not only affected by their own beliefs and by their conception of subject matter discipline in mathematics, but also by their beliefs about their learners and by their understanding of appropriate pedagogy.

In their survey of teachers' beliefs Schmidt (1999) classifies teachers' beliefs into four categories:

- Discipline-oriented teachers: These teachers indicate that it was important to remember formulas, and that mathematics was essentially abstract, and that mastering algorithms and basic computation was more important. They also indicate that the **real-world use** of mathematics was less important. They more often indicate that success in mathematics learning was more a matter of natural talent than other factors.

- Process-oriented teachers: Teachers in this group indicate that it was relatively important to remember formulas, to focus on algorithms, or to emphasise basic computation. They hold that mathematics was not abstract, and that its real world use was important. They also tend to emphasise creativity and thinking about mathematics conceptually.
- Procedure-oriented teachers: This group though having more common beliefs with the first group are more concerned with emphasising the real-world use of mathematics. They regard algorithms as only modestly important and indicate that subject matter should be present conceptually. To them mastering mathematics is just a talent.
- Eclectic-teachers: these teachers essentially emphasise nothing and do not possess a distinctive character. They are both somewhat discipline-oriented and somewhat real-world oriented.

2.4.1.2 Attitudes towards learners in mathematics

Research related to the issue of attitude towards learners in mathematics is extensive. One of the most important factors in developing learners' mathematics ability is the attitude of the teacher towards learners. Fennema and Romberg (1999) state that it is not only the teachers' beliefs about mathematics and its usefulness that are important, but also that the teachers' beliefs about their learners' ability to do mathematics have an influence on how they teach and subsequently on how learners learn.

In a report of a study of Japanese classrooms intended to acquaint American educators with mathematics teaching and learning in Japan, Becker, Silver, Kantowski, Travers and Wilson (1990: 13) make the following comment:

> *Even a casual observer realizes that all students are regarded as capable of learning mathematics and other subjects…The Japanese assume that learning is a product of effort, perseverance, and self-discipline rather than ability. Consistent with this philosophy, the schools have no ability grouping in elementary and junior high schools and virtually no individualised classroom instruction.*

The above suggests that for learners to learn mathematics effectively, teachers need to regard the learners as capable of learning and expose them to quality experiences that enhance learning.

Chen, Clark and Schaffer (1988) establish in their literature review that teachers positively influence learning and achievement through high expectations in relation to learners' learning. Cheung (1998) found that if a learner believes a teacher has a low opinion of him/her, it is possible that the learner will perform accordingly.

2.4.2 Teacher quality

Sarason (1993) maintains that if one wants to change the education of learners, one needs to first change the education of the teachers. According to Sarason (1983) it is necessary to prepare educators for what life is like in classrooms, schools, school systems and society. The preservice and continuing education of teachers of mathematics should provide them with the opportunity to examine and revise their assumptions about how mathematics should be taught, and how learners learn mathematics (National Council Teachers of Mathematics, 1989:160).

2.4.2.1 Teacher's role

The National Education Policy Act of 1996 (DoE: 2000) lists seven roles for educators, namely educators as

- ***Learning mediators***

  Educators have to mediate learning in a manner which is sensitive to the diverse needs of learners, including those with learning impediments and demonstrate sound subject content knowledge.

- ***Interpreters and designers of learning programmes and materials***

  Educators have to provide, understand and interpret learning programmes, identify the requirements for a specific context of learning and select and prepare suitable textual and visual resources for learning. They should select the sequence and pace of the learning in a manner sensitive to the differing needs of the subject/learning area and learners.

- ***Leaders, administrators and managers***

Educators have to make decisions appropriate to the requirement level, manage learning in the classroom administrative duties efficiently and participate in school decision-making structures. This has to be done in ways that are democratic, which support learners and colleagues, and which demonstrate responsiveness to changing circumstances and needs.

- ***Scholars, researchers and lifelong learners***

Educators have to achieve ongoing personal, academic, occupational and professional growth through pursuing reflective study and research in their learning area, in broader professional and educational matters, and other related fields.

- ***Community, citizenship and pastoral role***

Educators have to practise and promote a critical, committed and ethical attitude towards developing a sense of respect and responsibility towards others. In the school the educators have to demonstrate an ability to develop a supportive and empowering environment for the learner and respond to the educational and other needs of the learners and fellow educators

- ***Assessors***

Educators have to understand that assessment is an essential feature of the teaching and learning process and know how to integrate this into the process of learning. The educator must have an understanding of the purposes, methods and effect of assessment, and also be able to provide helpful feedback to learners. They should design and manage both formative and summative assessment in ways that are appropriate to the level and purpose of the learning and meet the requirements of accrediting bodies.

- ***Learning specialists***

Educators have to be thoroughly familiar with the knowledge, skills, values, principles, methods, and procedures relevant to the discipline, subject, learning area, phase of study, or professional or occupational practice. They should know the different approaches to teaching and learning, and how these may be used in ways which are appropriate for learners and context. Educators should have a well-developed understanding of the knowledge appropriate to their specialisation.

2.4.2.2 Pedagogical content knowledge

Learners' achievement in mathematics is also likely to be affected by the teacher's pedagogical content knowledge (Vatter, 1992). According to Shulman (1987) pedagogical content knowledge is the capacity of a teacher to transform the subject knowledge that he or she possesses into forms that are pedagogically powerful and yet adaptable to the variations in ability and background presented by the learners. Shulman (1987:9) further argues that a teacher must:

> *Understand the structure of subject matter, the principles of conceptual organisation, and the principles of inquiry that help answer two kinds of question in each field: what are the important ideas and skills in this domain? How are new ideas added and deficient ones dropped by those who produce knowledge in this area? That is, what are the rules and procedures of good scholarship or inquiry?*

According to Ethington (1990) much of the literature suggests that it is critical for secondary mathematics teachers to have strong mathematical knowledge, a positive attitude towards mathematics and teaching, as well as an alignment with proper pedagogical beliefs. For instance Ma (1999) describes what she calls *profound understanding of fundamental mathematics* in terms of the depth, breadth and thoroughness of the knowledge that the teachers need. **Depth**, according to Ma (1999) refers to the ability to connect ideas to the larger and powerful ideas of the domain, whereas **breadth** has to do with connections among ideas of similar conceptual power. According to Ma (1999) **thoroughness** is essential in order to *weave this into a coherent whole*. Ma (1999: 123) further argues that:

> *Teachers' knowledge of mathematics for teaching must be like an experienced taxi driver's knowledge of a city, whereby one can get to significant places in a wide variety of ways, flexibly and adaptively.*

2.4.2.3 Teacher experience and in-service training

According to Ball and Wilson (1990) mathematics education majors have not been exposed to enough alternative teaching methods to be capable of teaching mathematics with an emphasis on meaning. Ball and Wilson (1990) go further and mention that preservice secondary mathematics teachers often lack sufficient mathematical understanding to teach the subject effectively.

In 1991 the National Council of Teachers of Mathematics together with the Association for Supervision and Curriculum Development published A *Guide for Reviewing School Mathematics Programs.* In this document they state that in order to have high-quality mathematics programs, teachers of mathematics must be well-prepared, process and demonstrate positive attitudes, continue to grow professionally, and be actively involved in educational issues that affect the quality of their learners' learning (NCTM & ASCD, 1991). Mullis (1991) in his assessment of the state of mathematics achievement in the USA found some modest evidence of a positive relationship between the extent of in-service education and learners' achievement in Grade 8. However, in Grade 4, in-service education did not seem to be significantly related to mathematics achievement.

The lack of adequate in-service training opportunities for some teachers is a barrier to learners' academic achievement in mathematics. In "Learning without Limits: An agenda for the Office of Postsecondary Education" (2000), it is reported that experienced teachers do not have adequate opportunities to improve their knowledge and skills, and that in-service training opportunities for teachers are "second rate" (2000:32). The report cites the following problems regarding the in-service training of teachers:

- In-service training remains largely short-term and non-collaborative.
- In-service training is often unrelated to the teachers' needs and the challenges faced by their learners.

- Teachers are offered in-service training opportunities that last for a few hours (less than eight).

Lockheed and Komenan (1989) show a significant positive relationship between teacher experience and learner achievement in some developing countries, for instance in Nigeria and Swaziland. In contrast Chen at al (1988) establishes no significant relationship between teacher experience and learners mathematical achievement.

2.4.2.4 Competence

Georgewill (1990) in his review of literature shows that professional education is necessary, particularly for secondary school teachers in a special field like mathematics. Shaveson, McDonnell and Oakes (1989) go further and state that what teachers actually do, depends not only on their competence, but also on the conditions under which they must provide instruction. They noted that a fully competent teacher might perform less than adequately in the classroom, if he or she is working in a disorganised and unsupportive environment for teaching and learning. On the other hand, teachers with only minimal competence can perform quite adequately, given supportive and favourable working conditions.

Peng and Hill (1995) in their assessment of the state of mathematics achievement in the United States of America find that teachers of learners from disadvantaged schools, namely Hispanic, Black, and American Indian learners were not necessarily under-prepared in terms of certification, education level or number of years of teaching although their learners continue to attain lower average achievements in science and mathematics. In South Africa Maqsud and Khalique (1991:379) note that in spite of the availability of well-qualified mathematics teachers in some Bophuthatswana (North-West Province) schools, Grade 12 results of such schools remain far below acceptable standards.

Several studies (Henson & Eller, 1999; Nelson & Prindle, 1992) document the characteristics of effective teachers. Although the majority of these studies do not focus specifically on mathematics, the opinion of the writer is that the characteristics they identify are applicable to teachers in all subject areas. According to Henson and Eller (1999) the following are the characteristics of effective teachers working with learners in regular classroom. They:

- set high goals and communicate these goals to learners;
- appreciate creativity and enjoy the unpredictability of working with divergent thinkers;
- are flexible in their thinking and willing to admit mistakes or change their positions or opinions when evidence warrants this;
- are willing to be flexible in terms of time of the task during the school day, and they devote extra time after school to working with their learners;
- are well organised and flourished in classrooms where there were multiple activities running concurrently.

In studies focusing on mathematics classrooms, expert teachers were identified as having a strong background in mathematics, demonstrating expertise in and enjoyment of problem-solving and being able to engage in deep mathematical thinking (Nelson & Prindle 1992; Shefield 1994). These teachers also had general knowledge, interest in non-mathematical ideas and concern about problems facing their communities.

2.4.3 Mathematics lesson structure

A number of researchers have investigated the building blocks of mathematics lesson structure, because contributes to effective teaching and learning. In their comparison of a typical lesson structure in the United States of America and Japan, Schmidt *et al*. (1999) find that most lesson structures focus on five global behaviours, namely:

- Reviewing the content covered in a previous lesson (5 min).
- Reviewing homework assignment in a previous lesson (10 min).
- Providing instruction on new subject matter (20 min).
- Having students work on in-class exercises that were either used in the lesson development or otherwise discussed in the lesson (15 min).

- Having students work on homework that would not be discussed until a later lesson. (15 min).

However, what was different was the extent to which certain teachers used these activities. In the United States they found that mathematics instruction for both nine and thirteen year olds seems to be dominated by class work and reviewing homework. Teachers spent some time teaching new material but it was not the dominant feature of lessons. Less than forty percent of United States teachers provided twenty minutes or more of instruction on new material during a class period. Japanese teachers by contrast spent most of their time on the combination of instruction on new material and class work, which was, for the most part, actively tied to the instruction of the new material during the course of the lesson. In this regard, Stevenson and Stigler (1992) attributed some of Japan's leading mathematics achievement to mathematics lesson structure.

In examining the way how mathematics has been taught, Wood, Cobb and Yackel (1992: 179) observe that:

> *Teaching mathematics in schools is characterised by heavy reliance on the textbook by teachers both as a source of activities and for explanations of procedures to use in completing the task.*

In this context the role of the teacher is that of an instructor whose instructions are followed in order to arrive at a given product. The learners are then expected to follow the teacher's example carefully and answer the questions that the teacher asks without necessarily engaging in dialogue when information is exchanged. Thomas (1986) has investigated achievement levels for learners learning mathematics in different situations.

This includes:

- Whole-class lecture – demonstration situations; here the teacher instructs the whole class as one group for the greater part of the lesson.
- Co-operative learning when the teacher divides the class into small heterogeneous groups.

- Individualised situations where the students are given the individual seatwork and ask for the teacher's help when they need it.
- Within-class ability groups where the teacher divides the class into small groups based on ability.

The findings from these (Slavish & Karweit, 1984; Thomas, 1992) studies suggest that there are significant increases in achievement levels, measured by differences in scores in a pre-test and a post-test, when learners learn in small groups (co-operative learning) as opposed to individual seatwork.

In their study of mathematics teaching in Grades 4 to 6, Slavish and Karweit (1984) investigated the achievement effects of three commonly used methods, whole class instruction, within-class ability grouping and co-operative learning. They found that there were significant differences in the overall effects on learner's achievements. Slavish and Karwait's (1984) overall finding from their study is that two of the three modes they investigated, within-class ability grouping and co-operative learning in small heterogeneous groups, led to higher achievement levels in mathematics as compared to the whole-class mode.

In a study of Asian and United States of America (U.S.A.) classrooms, Stevenson and Stigler (1992: 65) report that **group orientation** in Japanese and Chinese classrooms

> *promotes the feeling of group membership*. Accordingly, participation in groups *enables children to learn and judge each other and appreciate the variety of ways individuals can contribute to a group's success (1992: 66).*

In contrast, in United States of America classrooms, Stevenson and Stigler (1992) report that students are observed working at their own pace and continuing to struggle with the assigned problems that others have already completed. This denies the students *social interaction with those who finished early.* According to Stevenson and Stigler (1992), teaching in groups, as is done in the Asian classrooms, is more advantageous than the individual approach of United States of America classrooms. Some research evidence suggests that students who prefer to co-operate, learn best in co-operative programmes, while other students who prefer to compete, do best in competitive programs (Smey-Riechman, 1988).

2.4.4 Teaching methods and strategies

Robitaille and Garden (1989) point out some factors that influence effectiveness of teachers, namely their teaching strategies, beliefs about teaching, and the general classroom processes that provide an immediate learning environment for mathematics. In this regard Dreckmeyr (1994: 67) defines

> *a teaching strategy as an extensive teaching plan which includes all elements of the instruction-learning events, such as form, content, classification, principles and aids.*

Teaching strategies can be classified in several ways for example, teacher-centred or learner-centred. Teacher-centred strategies are those in which the teacher has direct control. Learner-centred strategies are those strategies that allow learners to play a more active role.

In this regard, Stein, Leinhardt and Bickel (1989) suggest some factors in providing effective instruction for disadvantaged learners, namely

- They argue that the most important factor is the teacher.
- Time on task. They suggest that learners must be engaged in appropriate instruction for sufficient time to master the academic skills.
- The presentation of the lesson. Successful lessons include appropriate expectations, frequent monitoring and helpful feedback.

Furthermore they argue that the entire school experience of the learner should be designed to produce the maximum learning success for each individual. The negative effects of disorganised home environment can be overcome by providing a safe and consistent school environment. The learners' feelings of alienation can be overcome by showing genuine care for them and by involving them and making the school their own.

Accordingly Ysseldyke, Spicuzza, Kosciolek and Boys (2003: 163) identify some of the instructional features that are related to improved learners' achievement in mathematics. Some of these features include:

- Direct and frequent monitoring of progress.
- Corrective and motivational feedback.
- Learner academic involvement.
- Total length of time allocated for instruction.

2.4.5 Indicators for effective classroom teaching

According to Perrott (1982) good teaching cannot be defined because it is so complex. However, he (Perrott, 1982) tabulates some observable indicators of effective classroom teaching as indicated in the table below. These indicators include Ryan's factors, Flanders' indicators of indirect teaching style and Rosenshine and Furst's correlates positively.

Table 2.2 Observable indicators of effective classroom teaching

Ryan's factors
1.Teacher is warm and understanding 2.Teacher is organised and businesslike versus unplanned and slipshod 3.Teacher is stimulating and imaginative versus dull and routine *Flanders' indicators of indirect teaching style* 1. Teacher asks questions 2. Teacher accepts learners' feelings 3. Teacher acknowledges learners' ideas 4. Teacher praises and encourages learners *Rosenshine and Furst's correlates* 1. Teacher is enthusiastic 2. Teacher is businesslike and task-oriented 3. Teacher is clear when presenting instructional content

4. Teacher uses a variety of instructional materials and procedures
5. Teacher provides the opportunities for learners to learn the instructional content

Adapted from Perrott (1982: 3)

Most of the research on effective teaching was found to be consistent with the findings of Ryan, Flanders and Rosenshine (Perrott, 1982:4). Although a detailed discussion of these indicators is beyond the scope of this study, the author feels that they, in essence, also underpin the factors that facilitate achievement in mathematics in traditionally disadvantaged schools.

In its new curriculum standards, the National Council of teachers of Mathematics (1989) recommended decreased emphasis on a number of traditional practices in the teaching of mathematics. The following are examples of beliefs which are consistent with these traditional practices:

1. The teacher should:
   - 1.1 adhere closely to the textbook;
   - 1.2 provide step-by-step instruction in the use of algorithms or procedures;
   - 1.3 provide for extensive practice by learners.

2. Learners should:
   - 2.1 spend significant length of time practising one-step problems and problems categorised by types;
   - 2.2 rely on the authority of the teacher or an answer key;
   - 2.3 learn isolated topics;
   - 2.4 memorise rules, algorithms, formulas, and procedures without understanding.

Following are some of the beliefs which are consistence with effective classroom teaching:

Table 2.3 Beliefs consistent with effective classroom teaching

STUDENTS should:
1. value mathematics
2. be confident in their mathematical ability
3. be allowed to challenge concepts traditionally presented as "facts"
4. be able to
a) work with others
b) reason and communicate mathematically
c) apply concept to real application
d) risk discussing their ideas
e) see mathematical connections
THE TEACHER should:
1. be a guide of learners' knowledge
2. engage learners in activities which promote high levels of understanding and encourage
3. watch and analyse learners learning
3. place less emphasis on the " right answer"
4. ask questions which encourage learners to make own connections
METHODS can include the following:
1. small-group work
2. peer instructions
3. whole-class discussion with the teacher as moderator
4. investigation of alternative problem-solving strategies
5. discussion of "mistakes"
6. text used only as one of many resources

Borich (1996) gives the following summary of teacher variables that may be necessary to obtain high achievement gains in (social) settings of low socio-economic status. These involve the following:

- Generating a warm and supportive effect by letting learners know that help is available;
- getting a response, any response, before moving on to the next bit of new material;
- presenting material in small bits, with a chance to practise before moving on;
- emphasising knowledge and applications before abstraction, putting the concrete first;
- giving immediate help (through use of peers perhaps):
- generating strong structure and well-planned transition.

In the following section some teachers' instructional strategies that apply in mathematics are discussed.

2.4.6 Co-operative learning

One of the methods of learning mathematics is co-operative learning. Co-operative learning, peer learning, or collaborative learning are all terms used to describe groups of learners, whether small or large, who are engaged in different learning environments to learn to solve problems, to understand the cause and effect, to develop and defend different perspectives, and grow as learners and decision makers. Felder and Brent (in Steyn, 2003) define cooperative learning as a subset of collaborative learning. According to them, in collaborative learning learners are interacting with one another while they learn actively and apply course material. In cooperative learning (Bruffee, 1999:1):

> *Learners discuss issues in small consensus groups. They plan and carry out long-term projects in research teams, tutor one another, analyse and work on problems together, unravel difficult lab instructions together, read aloud to one another what they have written, and help one another edit and revise research reports and term papers.*

According to Tinzmann, Jones, Fennimore, Blakker, Fine and Pierce (1990) collaborative learning has four general characteristics:

- Shared knowledge among teachers and learners
- Shared authority among teachers and learners
- Teachers as mediators
- Heterogeneous groupings of learners

Davidson (1990: 53) offers the following motivation for employing cooperative learning in mathematics classes:

- Small groups provide a social support mechanism for the learning of mathematics.
- Small-group learning offers opportunities for success for all learners.
- Mathematics problems are ideally suited for group discussion in that they (usually) have solutions that can be objectively demonstrated.
- Mathematics problems can often be solved by several different approaches. Learners in groups can discuss the merits of the different proposed solutions and perhaps different strategies.
- The field of mathematics is a field with exciting and challenging ideas that merit discussion. One learns by talking, listening, explaining and thinking with others.
- Mathematics offers many opportunities for creative thinking. Learners in groups can often handle challenging situations.

Dossey, McCrone, Giordano and Weir (2002: 502) express the importance of co-operative learning as follows:

> *Co-operative learning or small-group learning ideally provides students with a less threatening environment in which to work since they don't feel the pressure to perform in front of their peers. In addition and possibly more importantly, when students work with others, there is the possibility that students will share ideas, build on the ideas of others, justify their ideas, and, hence create a deeper understanding of the concept being explored. This is of course, the ideal outcome.*

Small-group collaborative learning provides an alternative to both traditional whole-class expository instruction and individual instructional systems.

While research shows positive effects on learners' achievement by learners engaged in co-operative learning, a number of authors have pointed out possible problems with co-operative small-group work, namely;

- Shared learners' misconceptions can be reinforced by group work (Good & Brophy, 1997).
- Learners might be tempted to engage in off-task social interaction (Good & Galbraith, 1996).
- Learners may also receive differential status in groups. Some may start to perceive themselves as having little to contribute to the group, or may find that their contributions are not greatly valued.
- Small group work may then favour high-ability learners more than lower-ability ones. Studies have found bwer ability learners to be less active in small groups, in part because they understand the task less well and in part because learners' talk can also express low expectancies of certain other learners (Good & Brophy, 1997).
- Small-group work requires far more classroom management skills from the teacher.

Brahier (2000: 158-159) states that in the co-operative learning environment:

> *The activity should be structured so that every student has no choice but to be actively involved in the problem-solving process. Also, each student has individual accountability, which means even though the work is done as a team, in the end, each student is required to individually demonstrate an understanding of the concepts through an interview, a written test or some other means.*

Dossey *et al*. (2002: 503) point out that:

> *In preparing for group work, the teacher must make decisions about the size, composition and how to arrange the groups. Many advocates of cooperative groups (Davidson, 1990; Foster & Theesfeld, 1999; Neyland, 1994) suggest creating diverse groups so that students of low and high ability, different genders and differing background are grouped together. This diversity will most likely offer a variety of ideas and opinions that may be valuable for exploring a problem situation.*

2.4.7 Problem-solving

According to Charles and Lester (1982:5) *a problem* is defined as a task for which:

- The person confronting it wants or needs to find a solution.
- The person has no readily available procedure for finding the solution.
- The person must make an attempt to find a solution.

According to Charles and Lester (1982) this definition emphasises three crucial components of a problem:

- A desire or need on the part of the problem-solver to attain a goal.
- The fact that the goal cannot be reached directly and immediately.
- The fact that a conscious effort is made to reach the goal.

The Cockcroft committee (in Backhouse, Haggarty, Pirie & Stratton, 1992: 137) describe problem-solving as *the ability to apply mathematics to the variety of situations*. They further state that:

> *However, the solution of a mathematical problem cannot begin until the problem has been translated into the appropriate mathematical terms. This first and essential step presents very great difficulties to many pupils - a fact, which is, too often appreciated.*

Polya (1971) defines problem-solving as searching for an appropriate course of action to attain an aim that is not immediately attainable. Schoenfeld (1998: 88) defines a problem in relation to its effect on the learner as a task,

- *in which the student is interested and engaged and for which he wishes to obtain a resolution, and*
- *for which the student does not have a readily accessible mathematical means by which to achieve that resolution.*

Schoenfeld's definitions suggest that a problem is not a problem unless the learner is interested in solving it.

The following are types of teacher behaviour that should be used at each of Polya's stages (Charles & Lester, 1982: 37):

- ***Understanding the problem.*** The teacher might ask questions to help the learner understand the problem. Learners should also be trained to ask themselves questions when they are confronted by a problem.
- ***Devising a plan.*** In this phase the teacher should direct learners' attention to related problems on previously used strategies where possible.
- ***Carrying out a plan.*** In this step learners should be encouraged to solve the problem on their own. If the selected plan does not work, the teacher can encourage the learner or group of learners to try an alternative plan suggested in phase 2.
- ***Looking back.*** This phase is essential for consolidating the knowledge gained from the solution and for developing in learners the processes needed for solving the problem. In this regard Schroeder and Lester (1989) describe three ways in which problem-solving is interpreted in the classroom, namely;
- ***Teaching for problem-solving***. In teaching for problem-solving, the goal is to first teach the concepts so learners will later apply that knowledge to a problem-solving situation
- ***Teaching about problem-solving***. Teaching about problem-solving is the teaching of strategies in order to solve problems.
- ***Teaching via problem-solving.*** Teaching via problem-solving is teaching mathematics in a problem-solving environment. Learning in this approach involves learning through a concrete problem and eventually moving to abstraction.

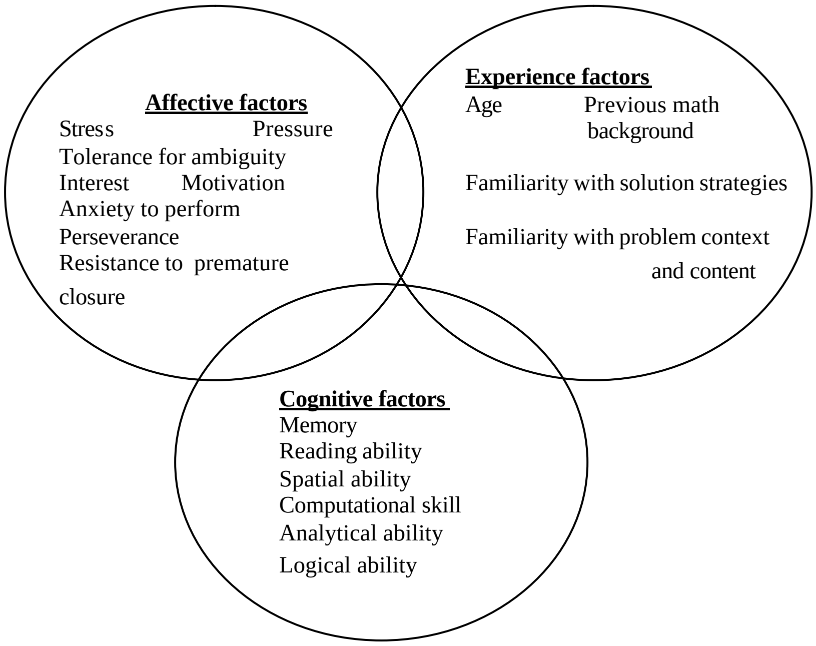
Erickson (1986) finds that a teacher teaching **for** problem-solving uses much direct instruction in her classroom while a teacher teaching **via** problem-solving facilitates learners' exploration. Hoffer and Gamoran (1993) studied the impact of various instructional approaches on learners' achievement. They find that one of the three main determinants was emphasis on problem-solving and this was particularly effective in the case of low- and middle- ability groups.

Bell (In Maree, 1997: 70) stresses the importance of problem-solving in mathematics as follows:

> *Problem-solving is an appropriate and important activity in school mathematics because the learning objectives which are met by solving problems and learning general problem-solving procedures are of significant importance in our society.*

According to Charles and Lester (1982: 37) there are three sets of factors that influence the problem-solving process, namely **affective factors**, **experience factors** and **cognitive factors**. Figure 2.1 is a summary of the contents of each of these factors. In the opinion of the author of this thesis, these factors are important to consider in order to facilitate mathematics achievement in secondary schools, let alone in Grade 12 disadvantaged learners' classrooms.

Figure 2.1 Some factors that influence the problem-solving process



Adapted from Charles and Lester (1982: 37)

Within this framework one can conclude that a learner who has the entire prerequisite to solve a problem may fail to do so through either lack of motivation, high degree of stress, or lack of familiarity with appropriate solutions.

2.5 EPISTEMOLOGICAL CONSIDERATIONS

According to Bigge and Shermis (1999: 3) learning theory is defined as

> *"a systematic integrated outlook in regard of the nature of the process whereby people relate to their environments in such a way as to enhance their ability to use both themselves and their environments in a most effective way".*

The intent of this section is to provide an overview of theories of learning suggested by psychologists in relation to the learning of mathematics. Theories help teachers to conceptualise learner communication, promote interpersonal relationships between teachers and learners, help teachers to implement professional ethics and have an impact on how teachers regard themselves.

The importance of learning theories is summarised by Ertmer and Newby (in Steyn, 2003:79) when they remark that:

> *"Learning theories provide instructional designers with verified instructional strategies and techniques for facilitating learning as well as a foundation for intelligent strategy selection."*

In this regard Owens (1995:158) states that *"No discipline can claim uniform agreement on the theoretical framework for teaching and learning".* Romberg (1988) alludes to the fact that there are many theories because the way humans learn is extremely complex. Romberg (1998: 23) states the following reasons in support of his remarks:

- Lack of general agreement on the definition of learning
- Different kinds of learning
- Different philosophical assumptions about the nature of the learning process.

In this study the author has chosen to concentrate on theories that are seemingly significant in influencing the way mathematics is currently studied in grade twelve, namely **behaviourism, gestalt learning theory** and **cognitive learning theories**. Each learning theory represents a particular view of knowledge. The author agrees with the opinion expressed by Maree (1997) that each theory is valid to a certain extent.

2.5.1 Behaviourism

Behaviourism focuses on the outcomes of learning rather than on how learning occurs. It assumes that learning occurs by passively, but rationally, reflecting on stimuli from the environment. According to Leonard (2002:38), in behaviourism *learners are placed in a controlled environment in order to be directed to a specific set of behavioural changes based on a set of predetermined, instructor-based objectives*. Learning is viewed as change in behaviour (or performance) and the changes in scores on some measure of performance are often used as evidence of learning.

In spite of the criticism against behaviourism, its underlining principles have influenced the teaching of mathematics in secondary schools to a large extent. For example, drill-and-practice routines such as factorizing trinomials and solving linear equations are influenced by behaviourism (Romberg, 1988).

2.5.2 Gestalt learning theory

Crowther (in Steyn, 2003:24) defines "*gestalt as an organised whole that is perceived as more than the sum of its parts*".

Steyn (2003) points out that the definition of *gestalt* has particular significance for the learning of mathematics at all levels. While behaviourism in many of its forms is an adequate model for many forms of learning (particularly low-level concept and skills), it was totally inadequate for explaining how one discovers a relationship, proves a theorem, and solves complex problems. According to Romberg (1988) repeated practice and reinforcement cannot make someone a creative mathematician; the invention of new ideas does not occur. Romberg further notes that learning is not simply change in performance. Therefore change in scores and an increase in the number of correct answers fail to capture changes in strategies or ways of thinking about a problem.

Gestalt theory posits that learning involves active construction rather than passive absorption from the environment and it implies that learners experience the world in meaningful patterns and then construct meanings from those patterns. This theory states that evidence of learning has been in terms of changes in the way persons said they thought about problems. Most early work on mathematical problem-solving (e.g. Polya, 1945, Hadamard, 1945) is rooted in this theory.

2.5.3 Information processing

Gagne (in Maree, 1997:32) defines learning as follows:

"Learning in human disposition or capability, which persist over a period of time, and which is not simply ascribable to processes of growth".

In developing his information processing model of teaching, Gagne first lists five major categories of learning capabilities, namely intellectual skills, cognitive strategies, verbal information, motor skills and attitudes (Bigge & Shermis, 1999). Gagne regards these categories as educational outcomes and as descriptive of possible different kinds of human performance. Gagne's theory implies that skills in mathematics are analysed according to learning hierarchies.

2.5.4 Cognitive learning theories

Behaviourism focuses mainly on the outcome of learning whereas the cognitive learning theories focus on the process of learning. Cognitive theories of learning emphasise that learners are ultimately responsible for their own understanding through active construction of meaning (Bigge & Shermis, 1999).

2.5.5 Constructivism

Constructivism is a theory of knowledge with roots in philosophy, psychology, and cybernetics (Jaworski, 1994). In the following section radical and social constructivism are defined.

(a) Radical constructivism

According to Glasersfeld (in Jaworski, 1994:15) radical constructivism asserts two main principles whose application has far-reaching consequences for the study of cognitive development and learning, namely

- Knowledge is not passively received by learners but actively built up by the teachers and learners in the classroom.
- Acquisition of knowledge is an adaptation process during which learners reorganise their experiential world.

(b) Social constructivism

The social constructivism view of learning is based on the theories of Piaget (1926) and Vygotsky (1978).

Ernest (1999) provides an account of social constructivism, identifying two key features:

- First of all there is the active construction of knowledge, typically concepts and hypotheses, on the basis of experiences and previous knowledge. These provide the basis for understanding and serve the purpose of guiding future actions.
- Secondly there is an essential role played by experience and interaction with the physical and social worlds, in both physical action and speech modes.

According to Balacheff (1990) the **constructivist hypothesis** states that knowledge is constructed by the learner, not passively received, and that one comes to know by an adaptive process of organising one's experiences rather than by perceiving some external reality.

2.5.6 Constructivism and epistemology

Balacheff (1990) characterises the learning and teaching of the mathematics process as a relationship between two hypotheses, namely Constructivism and Epistemology and two constraints namely the nature of mathematics knowledge and the nature of the classroom.
In relation to **epistemological hypothesis** Balacheff (1990) states that mathematical knowledge is developed through solving problems. Problems set the stage for construction of knowledge by establishing the need for mathematical knowledge and the context in which mathematics is learned. Under this hypothesis problems only generate mathematical knowledge to the extent that learners perceive the problem as their own.

Taken together, **constructivist** and **epistemological** hypotheses imply the centrality of the process of developing responsibility for learning from the teacher, where it has traditionally resided to the learner. They also suggest a view of learning and knowledge as a private domain intrinsic to the individual. According to Owen (1995) this view of learning and knowledge conflicts with two constraints inherent in the teaching process and in mathematics outside the classroom.

- Mathematical knowledge is social knowledge.
- The mathematics class exists as a community.

2.5.7 Constructivismand mathematics learning

According to Piaget (in Bigge & Shermis, 1999:18) the key processes in the stages of child development are ***assimilation*** and ***accommodation***. Assimilation consists of the modification of the input from the environment. In this process new knowledge meshes with the child's existing insight. Accommodation consists of the change in the child's internal patterns of understanding to fit reality. In this process existing internal insights are reconstructed to "***accommodate***" new information.

In this regard, Piaget (in Bigge & Shermis, 1999) argues that the mental development of any child consists of a succession of three periods: namely **sensorimotor** (birth to eighteen months or two years), **symbolic** (from eighteen months to age seven or eight), and **concrete-operational** (from ages seven or eight to twelve years). In the concrete-operational period children learn to do what they had learnt previously through physical action. Piaget's mental development refers to children from birth to twelve years. According to Piaget (Bigge & Shermis, 1999:19) the **formal operational stage** begins from ages twelve to fifteen years. Learners in the formal operational stage function on an abstract level that is an adult level of thinking and not bound by concrete experience. Piaget (cited in Maree, 1997) is of the opinion that insight into the basic structure of mathematics and an execution of mathematical operations are mastered by the learners when they construct their interactions within their physical, social and cultural environment. Piaget (in Maree: 1997) cannot accept that learning is subordinate to development.

Vygotsky (1978) maintains that culture, social instructions and customs are the dominant factors in a child's cognitive development, especially in the area of thought and language. The process of complexity and adaptation among children according to Vygotsky is influenced by such factors as the home environment, peer relationships, the food they eat, the clothes they wear and the mastery of language (Henson & Eller, 1999). This suggests that the changing social characteristics of disadvantaged learners can impact on their cognitive development.

Vygotsky (1978) emphasises the importance of the conceptual scaffolding for the gradual internalization of knowledge, obtained by social interaction between a novice and a more competent peer or adult who provides a model. Vygotsky (1978) notes the following elements of constructivism:

- The significant role of dialogue in learning
- The place of peers in instructional practices of various learning tasks
- The interactive structure of all learning tasks. Thus learning is the result of internalization of social interaction.

The instructional processes which Vygotsky emphasises teacher co-operation, teachers supporting their learners and language development of the learner.

In this regard Driver and Bell (1986) identify six main characteristics of the constructive approach, namely,

- Academic outcomes depend on knowledge, purpose and motivation brought by the learner into the learning situation.
- Learning involves personal construction of meanings.
- Meaning is constructed through an active, continuous process.
- Constructed meanings are evaluated by learners, who can reject or accept them.
- The learners hold final responsibility for learning.
- Types of meanings are constructed by learners based on experiences with real objects learnt and through natural language.

2.6 SUMMARY

The major aim of this chapter was to review relevant literature of factors that facilitates achievement in mathematics. This review of literature indicates some factors that facilitate achievement in mathematics. The first section of the literature review covers these factors that are related to school. Variables such as learning environment, societal influence, teacher attitudes and perception, language and culture were reviewed.

The second part of the review addresses the variables that are related to learners and mathematics achievement. Variables such as learners' motivation, learners' academic engagement, learning approaches, peer influence and support, influence of career choice and mathematics enjoyment were reviewed. The literature review shows a significant relationship between all peer group attitude variables considered in this study and achievement in mathematics. Almost all sources reviewed for the purpose of this study report a significant positive relationship between learners' mathematics self-concept, attitudes and study methods and learners' mathematical achievement.

The third section of the review shows some factors that are related to teachers' performance and learners' achievements in mathematics. Variables such as teacher pedagogical content knowledge, competence and qualification, teacher interest and commitment and teaching methods were discussed. Studies on teachers' attitudes and methods reported a significant relationship between these teacher variables and mathematics achievements. Inconclusive findings are reported for teachers' competence where this review reported poor performance of learners despite some better-qualified teachers. It was however, the purpose of this study to further investigate the attitudes and attributes of the mathematics teachers.

The final section reviewed literature pertinent to an overview of some theories and practices for teaching and learning mathematics. The notions from the general theories of learning often do not apply to the learning of mathematics in direct and obvious ways. The author of this study accepts the view that each theory has an influence and will continue to influence the way scholars and teachers view the learning of mathematics.

2.7 CONCLUSION

According to Mullis (1991) learners' achievement in mathematics is a function of learners' home environment, attitudes towards mathematics, mathematics curriculum, instructional contexts and practices and school variables that explain variations in learners' achievements. Ewen (2002) remarks that the question of how to motivate learners in the classroom has become a leading concern for teachers of all disciplines including mathematics. According to Ewen (2002) school teachers need to be well grounded in learners' motivation and learners' management (Wong, 2003) because this is relevant to mathematics achievement.

The review indicated that although South African learners from disadvantaged communities express positive attitudes towards mathematics, the failure rate in this subject is still high. The author of this study is of the opinion that there is a need to expand the knowledge based on factors that facilitate achievement in mathematics among South African learners, especially those from disadvantaged schools, beyond data that have been collected using survey methods