As stated in chapter 1, the main objective of this thesis is to develop an automated land cover change detection method. The EKF change detection methodology that is proposed in this thesis is based on the assumption that the parameters of the underlying EKF model (which will be discussed in detail in the following sections) is separable for the two classes involved in the land cover transition, which for the present case is natural vegetation and settlement. It follows that the more separable the EKF derived parameters are for natural vegetation and settlement classes, the easier it would be to detect when this type of land-cover transition occurs. In this chapter, the focus will be on evaluating the land-cover separability of the proposed method.

A recent method proposed in [56] shows that features estimated using Fourier analysis on NDVI signals provide very good land-cover class separability. The method proposed in this chapter extends on the method in [56] by modeling an NDVI time-series (section 2.6) as a triply modulated cosine function and updates the mean ($\mu$) amplitude ($\alpha$) and phase ($\phi$) parameters of the model for each iteration by means of an EKF (section 3.4). These parameters are then used to separate natural vegetation and settlement time-series more effectively. Another competitor, the sliding window FFT method, which is an extension proposed by the author to the method presented in [56] is also evaluated and the performance of the EKF method is compared to that of the aforementioned methods as validation of the increased land cover separability achievable using the proposed EKF formulation. The following chapter will show how the parameters, which will be shown to be adequately separable for the land cover classes considered in this study, can be used in a spatio-temporal context for land-cover change detection.
4.1 INTRODUCTION

Land-cover classification based on multi-temporal satellite data can capitalize on seasonal variation in land surface reflectance due to vegetation phenology to provide better classification than single-date imagery [15,87]. Multi-temporal coarse resolution satellite imagery such as MODIS and AVHRR have been widely used to map land-cover at regional to global scales [88–90]. Land-cover classification methods are often based on a series of secondary metrics derived from the NDVI time-series (section 2.6) and include Principal Component Analysis (PCA) [87, 91, 92], phenological metrics [93] or Fourier (spectral) analysis [56, 94].

Fourier (spectral) analysis expresses a time-series as the sum of a series of sinusoidal waves with varying frequency, amplitude and phase [77]. The frequency of each sinusoidal component is related to the number of completed cycles over the defined interval. The Fast Fourier Transform (FFT) is an effective and computationally efficient algorithm to compute the Discrete Fourier Transform (DFT) [77] and is often used when evaluating NDVI time-series data [56–58, 78].

In many applications where the FFT transformation of NDVI time-series data are used for classification and segmentation, only the first few FFT components are considered as they tend to dominate the spectrum [56–58]. The reason for this is because of the strong seasonal component and slow variation relative to the sampling interval of the time-series (8 days for the MODIS MCD43 product). It has been found that even when considering only the mean and seasonal FFT components [56], reliable class separation can be achieved. A drawback of using FFT-based methods is that the underlying process is assumed to be stationary. This assumption is often invalid in the case of NDVI time-series data, especially if a land-cover change is present.

The EKF is a non-linear estimation method that can potentially be employed to estimate unobserved parameters (process model) using noisy observations of a related measurement model (Section 3.4). EKF techniques in remote sensing have been used for parameter estimation of values related to physical, biogeochemical processes or vegetation dynamics models [95, 96].

In this chapter an FFT approach will firstly be shown that separates different land-cover types based on their FFT mean and annual components [56]. A novel method is then proposed that models the NDVI time-series as a triply modulated cosine function. An EKF is used to track the parameters of the mean ($\mu$), amplitude ($\alpha$) and the phase ($\phi$) parameters of the proposed model for each time-step.
The objective is to show that using MODIS MOD43 data (Section 2.5), the $\mu$, $\alpha$, and $\phi$ parameter sequences over time are similar for same class land-cover types and dissimilar for different land-cover types representing natural vegetation and settlement land-cover types in northern South Africa.

4.2 LAND-COVER CLASS SEPARATION USING THE FFT

As discussed in section 4.1, the Fourier analysis of the NDVI time-series has proved to be insightful because the signal can be decomposed into a series of cosine waves with varying amplitude, phase and frequency. The DFT can be written in matrix form as:

$$Y = F_N y,$$
(4.1)

where $y^T = [y_0 \ y_1 \ y_2 \ \ldots \ y_{N-1}]$ is the NDVI time-series of length $N$ in vector form. $Y^T = [Y_0 \ Y_1 \ Y_2 \ \ldots \ Y_{N-1}]$ is the DFT of $y$ and $F_N$ is the DFT matrix in the form:

$$F_N(r, c) = \frac{1}{\sqrt{N}} e \left(-\frac{2\pi}{N} \right)^{(r-1)(c-1)},$$
(4.2)

where $F_N(r, c)$ is the value of row $r$ and column $c$ of the $F_N$ matrix [77]. The first 30 FFT components of a typical seven-year natural vegetation NDVI time-series is shown in Figure 4.1.
As expected, the majority of signal energy is contained in the mean and the annual component which relates to FFT component zero and seven respectively when considering a seven-year time-series. As proposed in [56], the similarity of any two arbitrary NDVI time-series can be evaluated by computing their FFT transformation respectively and then comparing the first and seasonal FFT component of each FFT series. A distance metric based on the mean \((\mu)\) and seasonal \((\alpha)\) FFT component difference for any two FFT series can then be formulated as follows:

\[
D_\mu = \| Y_0^1 - Y_0^2 \|, \tag{4.3}
\]

and

\[
D_\alpha = \| 2(Y_7^1 - Y_7^2) \|, \tag{4.4}
\]

where \(D_\mu\) and \(D_\alpha\) are the Euclidean distance between the mean and annual FFT components respectively of two NDVI time-series.

### 4.3 TRIPLY MODULATED COSINE MODEL

It can be seen from Figure 4.1 that the majority of the signal energy is contained in the mean and annual FFT component. This implies that the signal is well represented in the time-domain as a single cosine function with a specific mean offset, amplitude and phase, as shown in Figure 4.2. This single cosine model is, however, not a very good representation when the time-series is non-stationary, which is often the case because of, for example, inter annual variability or land-cover change. It is proposed that an NDVI time-series for a given pixel be modeled using a triply modulated cosine function given as

\[
y_k = \mu_k + \alpha_k \cos(\omega_k + \phi_k) + v_k, \tag{4.5}
\]

where \(y_k\) denotes the observed value of the NDVI time-series at time \(k\) and \(v_k\) is the noise sample at time \(k\). The noise is additive but with an unknown distribution. The cosine function is based on a number of parameters (that are not directly observable), namely the frequency \(\omega\), the nonzero mean \(\mu\), the amplitude \(\alpha\) and the phase \(\phi\). The frequency can be explicitly computed as \(\omega = 2\pi f\) where \(f\) is based on the annual vegetation growth cycle. Given the 8 daily composite MCD43 MODIS data, \(f\) was calculated to be 8/365. The values of \(\mu_k\), \(\alpha_k\) and \(\phi_k\) are functions of time and must be estimated given \(y_k\) for \(k \in 1, \ldots, N\), where \(N\) is the total number of observations. The estimation of these parameters is non-trivial and require a non-linear estimator. The estimator that was used in this thesis is an EKF.
4.4 NEW CLASS SIMILARITY METRIC

As shown in [56], substantial separability can be achieved when comparing mean and annual FFT components of NDVI time-series of different land-cover types. The underlying idea in [56] is that a similarity index can be calculated by considering the mean and annual FFT components of two pixel’s time-series. It is proposed that instead of taking a single FFT of the entire seven-year time-series and considering a scalar mean and amplitude value, the estimated values for $\mu_k$, $\alpha_k$ and $\phi_k$ as presented in (4.5) be estimated for each value of $k$ using an EKF. The state vector is defined as:

$$ x_k = [\mu_k \alpha_k \phi_k]^T. $$

and the process and observation models are formulated as:

$$ x_k = x_{k-1} + w_{k-1}, $$

and

$$ y_k = x_{k,1} + x_{k,2} \cos(2\pi f k + x_{k,3}) + n_k. $$

The process model assumes that the state vector remains constant from one time-step to the next.
with an additive process noise component. The observation model is based on (4.5) with the mean, amplitude and phase parameter being replaced by the state parameters representing each of these variables. Having an estimate of each of the state parameters for each time-step \( k \) effectively results in a time-series for each of the three parameters, the advantage of which will become apparent when considering the following example.

Consider the seven-year natural vegetation and settlement NDVI time-series shown in Figure 4.3. As expected, the NDVI for the natural vegetation time-series shows a much higher peak in the summer time due to the increased biomass compared to a typical settlement time-series. The low peak in the NDVI for the vegetation time-series in the summer of 2004 is attributed to a very dry season leading to a reduction in biomass. Using the methodology shown in section 4.2, a similarity metric can be computed by comparing the FFT components of each of these NDVI time-series. In particular, the mean and annual FFT components are compared (Section 4.2). The annual FFT components of both the natural vegetation and settlement NDVI time-series are shown in Figure 4.4. When considering the natural vegetation annual FFT component, it can be seen that the non-stationarity due to the dry year reduces the natural vegetation annual FFT component which effectively reduces the separability between the two time-series.

On the other hand, when tracking the vegetation amplitude using an EKF, it can be seen that an accurate estimate of the amplitude is produced for each time-step (Figure 4.3). The separability between the
natural vegetation and settlement amplitude (tracked using the EKF framework) remains at a maximum when considering the years that are unaffected by the dry year. It is thus proposed that a distance metric describing the similarity between the two time-series be formulated by taking the maximum distance between the EKF derived parameter streams of the time-series given as

\[ D_\mu = \max \{ \mu_{k,1} - \mu_{k,2} \}, \quad 1 \leq k \leq N, \]  

(4.9)

and

\[ D_\alpha = \max \{ \alpha_{k,1} - \alpha_{k,2} \}, \quad 1 \leq k \leq N. \]  

(4.10)

\( D_\mu \) is the maximum distance between the first \( (\mu_1) \) and second \( (\mu_2) \) parameter sequence over time \( k \). \( D_\alpha \) is calculated in a similar manner finding the maximum distance between the amplitude parameter sequences.

### 4.5 SLIDING WINDOW FFT APPROACH

From figure 4.4 it is clear that the EKF produces a time-series for the mean and amplitude parameter where the comparative mean and annual component of the FFT method produces a single frequency domain estimate. A more “fair” comparison would be to use a sliding window FFT to estimate the mean and annual component for a sliding-window by keeping the window-size constant and incrementing the start and endpoints of the window by one time-step. The underlying idea is that the FFT is calculated
Figure 4.5: Two instances of a one year sliding window together with their corresponding FFT.

over each time-series window and that that mean and annual component is then recorded for each increment. As the window slides over the time-series, the mean and annual component can each be expressed as a time-series. The first FFT component is related to the mean of the time-series, the FFT component that corresponds to the annual components can be calculated as

\[ C_a = \frac{L}{P} + 1. \]  

(4.11)

Where \( L \) is the window size and \( P \) is the number of time-series samples in one year. For this study, it is assumed that \( L \) will always be a multiple of \( P \). Figure 4.5 illustrates the concept of a sliding window. In this illustration, a one year sliding window was used. Two instances of the sliding window are shown together with their corresponding FFT components.

The sliding window DFT formulation of 4.1 can be written as

\[ Y_i = F_N y_i, \quad i \in \{1, N - L + 1\}, \]  

(4.12)

where \( y_i = [y_i; y_{i+1}; y_{i+2}; \ldots; y_{i+L-1}] \), \( N \) is the length of the time-series and \( L \) is the window size. The
mean and annual time-series can then be expressed as

\[ \mu = \{ Y(1)_1, Y(1)_2, Y(1)_3, \ldots, Y(1)_{N-L+1} \} \]  \hspace{1cm} (4.13)

\[ \alpha = \{ 2|Y(C_a)_1|, 2|Y(C_a)_2|, 2|Y(C_a)_3|, \ldots, 2|Y(C_a)_{N-L+1}| \} \].  \hspace{1cm} (4.14)

Where \( \mu \) is the mean time-series and \( \alpha \) is the amplitude time-series. \( Y(z)_i \) is the \( zth \) value of vector \( Y_i \) and \( C_a \) is the location of the annual FFT component, determined by the window size (see equation 4.11). Having \( \mu^{SWF} \) and \( \alpha^{SWF} \), a distance metric \( D_\mu \) and \( D_\alpha \) can be calculated similar to (4.9) and (4.9) as:

\[ D_\mu = \max \{ \mu_{k,1} - \mu_{k,2} \}, \quad 1 \leq k \leq N - L + 1 \]  \hspace{1cm} (4.16)

and

\[ D_\alpha = \max \{ \alpha_{k,1} - \alpha_{k,2} \}, \quad 1 \leq k \leq N - L + 1 \].  \hspace{1cm} (4.17)

### 4.6 SUMMARY

Previous research has found that class separability is achievable by considering the difference in FFT components related to the NDVI time-series of two different classes [56]. In particular, the mean and annual FFT components are considered as they tend to carry the majority of signal energy [56, 57]. It is proposed that this concept be extended by modeling the NDVI time-series as a triply modulated cosine function with varying mean, amplitude and phase and estimating these parameters for each time-step using an EKF. In short, by using this time domain approach, the mean, amplitude and phase is estimated for each time-step as opposed to the frequency domain approach that assumes a stationary time-series and consequently only gives a single estimate of the mean and annual frequency component. Having iterative estimates of these components allows one to exploit the fact that the mean and annual frequency dissimilarity is more prevalent during certain parts of the seasonal cycle than others, an effect that is merely averaged out using the traditional FFT over the entire NDVI time-series. A sliding window FFT approach was also introduced for comparison, unlike the traditional FFT method [56], the sliding window FFT also produces a time-series of the mean and amplitude and using the same methodology as with the EKF method, a change metric can be derived by computing the maximum deviation in the mean and amplitude time-series respectively. A comparison between the three methods is given in chapter 6.
A further application of the EKF method is towards land-cover change detection. By following the changes of the cosine parameters through time and comparing them with neighboring pixels, a change detection method can be formulated. This possibility is further explored in the following chapter.