

Appendix A

H_∞ - Controller Design Methodology

A.1 Introduction to the Design Method

The H_∞ controller design method discussed in the following sections of this chapter includes steps that can be summarized as follows [30].

- i) Start with a plant model and envisaged specifications for the controlled plant.
- ii) Accommodate specifications in weighting functions.
- iii) Augment the plant with these weights.
- iv) Carry out the H_∞ synthesis on the augmented plant.
- v) Measure performance of the design against specifications.
- vi) Repeat the procedure from step ii) if design requirements have not been satisfied.

A.2 The H_∞ -Methodology

A.2.1 The Plant Model

The plant model must be FDLTI. The method tolerates non-square , non-minimum phase, and unstable plants given that an augmented plant fulfills certain conditions. It will be discussed later in this chapter what the augmented plant and the conditions are. An upper bound for unstructured uncertainties (generic errors which are associated with all design models[28]) should be known as it serves as a basis for robustness specifications.

A.2.2 Specifications

The control system requirement for nominal stability can be met by H_∞ controllers. Other typical singular value specifications are the following.

- i) Robustness specification: 20 dB/decade roll-off and at least -20 dB at e.g. 100 rad/sec, which then ensures a certain bandwidth.
- ii) Performance specification: Minimize the sensitivity function as much as possible.

The robustness specification for multivariable systems can also be given more classically as multivariable gain and phase margins. For each loop in a multivariable system these margins exist independently at the same time and can be calculated as,

$$\begin{aligned}
 \text{downward gain margin} \quad & GM \downarrow \leq \frac{k}{k+1}, \\
 \text{upward gain margin} \quad & GM \uparrow \geq \frac{k}{k-1}, \tag{A.1}
 \end{aligned}$$

$$\text{phase margin} \quad |PM| \geq 2 \sin^{-1}(1/2k)$$

$$\text{where} \quad k = \|S(s)\|_\infty$$

$$\text{and} \quad S(s) = [I + G(s)K(s)]^{-1}.$$

$S(s)$ is the sensitivity function for the control system in Fig.A.1 with $K(s)$ and $G(s)$ representing the controller and the plant respectively. The robustness specification expressed in terms of gain and phase margins give an indication of the extent to which gain and phase may change in each loop before the system becomes unstable. A small k , i.e. a minimized sensitivity, gives more advantageous gain and phase margins according to Eq.A.1.

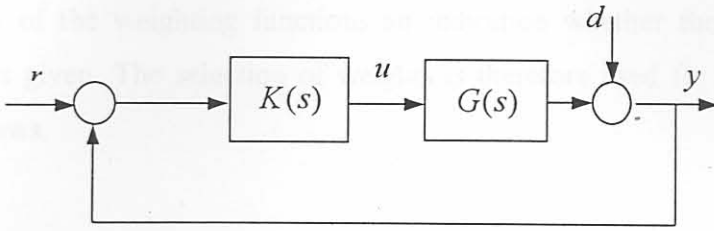


Figure A.1: Standard feedback configuration.

A well known relationship to be considered together with sensitivity is the complementary sensitivity,

$$\begin{aligned}
 C(s) &= [I + G(s)K(s)]^{-1} G(s)K(s) \text{ in} \\
 S(s) + C(s) &= I.
 \end{aligned}
 \tag{A.2}$$

Eq.A.2 says that either output disturbance or measurement noise rejection is possible in the same frequency range but not both. A small $S(s)$ expresses output disturbance rejection and good tracking whereas a small $T(s)$ means measurement noise rejection.

Regarding performance specifications one can build integrator characteristics into weighting functions in order to achieve zero steady state errors on the controlled output variables. The weighting functions contain parameters that can be tuned until the closed-loop system adheres to specifications. The structure of weights will be discussed in the next section.

A.2.3 The Structure of Weights

Weights are typically connected to the standard feedback configuration as shown in Fig.A.2. Their role is to serve as tuning parameters for the controller $K(s)$. $K(s)$ is designed for the plant $G(s)$ augmented by weights, i.e. for $P(s)$. This means that because $K(s)$ is designed for $P(s)$ instead of only $G(s)$, information about the weights is accommodated in $K(s)$.

The structure of weights $W_i(s) = c_i(s)I_{n \times n}$ ($i = 1, 2, 3$) is normally used. The purpose of this diagonal form is to enforce decoupled responses for the closed loop. By computing singular values of the weighting functions an indication whether the closed-loop meets specifications is given. The selection of weights is therefore used for singular value loop shaping as follows.

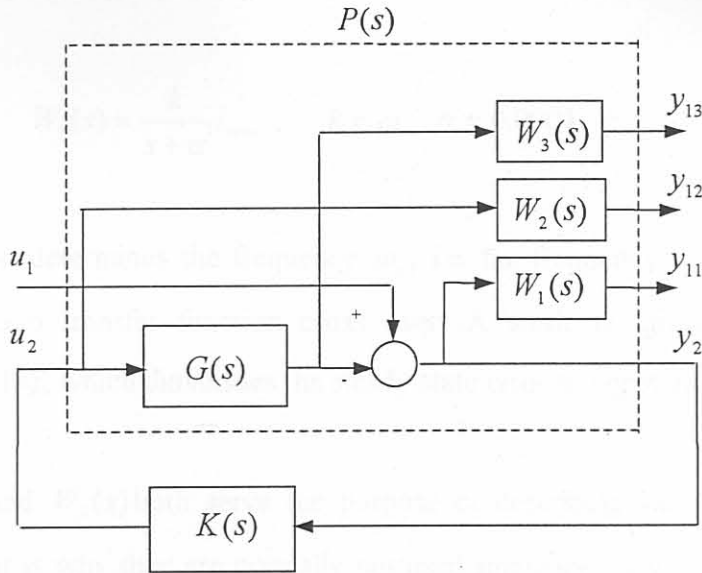


Figure A.2: Standard feedback configuration with weights.

Because $S(s)$ is the closed loop transfer function from disturbance d to output y (see Fig.A.1) singular values of $S(s)$ determine the disturbance attenuation. Therefore the disturbance attenuation performance specification is given as,

$$\lim_{\gamma \rightarrow \gamma_{\min}} \bar{\sigma}[S(j\omega)] \approx \bar{\sigma}[W_1^{-1}(j\omega)] \quad \omega < \omega_c \tag{A.3}$$

$$\lim_{\gamma \rightarrow \gamma_{\min}} \underline{\sigma}[S(j\omega)] \approx \underline{\sigma}[W_1^{-1}(j\omega)] \quad \omega < \omega_c$$

where γ_{\min} is the smallest γ still leading to a stabilizing controller for the plant in the γ -iteration. More about the γ -iteration is in section A.2.5 $\bar{\sigma}$ and $\underline{\sigma}$ denote the maximum and minimum singular values respectively.

The specification in Eq.A.3 may be used in the form of a bound as,

$$\bar{\sigma}(S(j\omega) \leq \bar{\sigma}(W_1^{-1}(j\omega)) \quad , \quad \omega < \omega_c \tag{A.4}$$

also known as the performance bound with ω_c the crossover frequency. A guideline for a good $W_1(s)$ is to set $\bar{\sigma}(W_1^{-1}(j\omega))$ equal to $\bar{\sigma}(S(j\omega))$ at frequencies below ω_c . A typical structure for $W_1(j\omega)$ [29][30] is,

$$W_1(s) = \frac{k}{s + \alpha} I_{n \times n} \quad ; \quad k = \omega_c, \alpha = 0.0001 \tag{A.5}$$

The parameter k determines the frequency ω_c , i.e. the frequency at which the singular values of the loop transfer function cross over. A small α gives an approximated integrator for $W_1(s)$, which diminishes the steady state error to constant controller outputs.

Weights $W_2(s)$ and $W_3(s)$ both serve the purpose of describing stability margins of the closed loop. That is why they are normally not used simultaneously. Stability margins are an indication of the robustness of a closed-loop system. A multiplicative stability margin is defined as the size of the smallest stable $\Delta_M(s)$ that can destabilize the system in Fig.A.3 with $\Delta_A(s) = 0$.

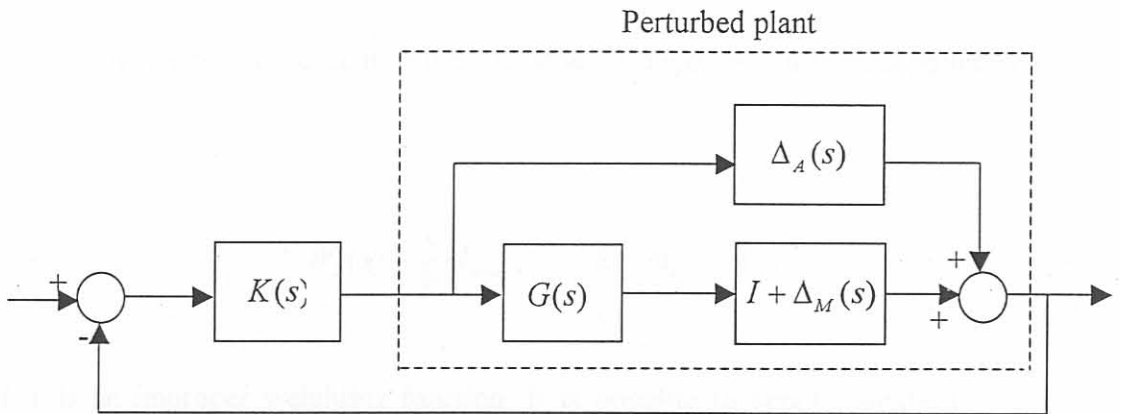


Figure A.3: System uncertainty.

An additive stability margin is defined as the size of the smallest stable $\Delta_A(s)$, which destabilizes the system with $\Delta_M(s) = 0$. These stability margins can be expressed in terms of $C(s)$ and $R(s)$ where $R(s) = K(s)[I + G(s)K(s)]^{-1}$ and because the controller $K(s)$ is

related to the weights the stability margins can be specified via the following singular value inequalities.

$$\bar{\sigma}[R(j\omega)] \leq \bar{\sigma}[W_2^{-1}(j\omega)] \quad \omega > \omega_c \quad (\text{A.6})$$

$$\underline{\sigma}[R(j\omega)] \leq \underline{\sigma}[W_2^{-1}(j\omega)] \quad \omega > \omega_c$$

$$\bar{\sigma}[C(j\omega)] \leq \bar{\sigma}[W_3^{-1}(j\omega)] \quad \omega > \omega_c \quad (\text{A.7})$$

$$\underline{\sigma}[C(j\omega)] \leq \underline{\sigma}[W_3^{-1}(j\omega)] \quad \omega > \omega_c$$

Eq.'s A.6 and A.7 are also known as robustness bounds. It is however common practice to take all plant uncertainty as represented by multiplicative perturbations such that the specifications for controller design become

$$\bar{\sigma}[S(j\omega)] \leq \bar{\sigma}[W_1^{-1}(j\omega)] \quad \omega < \omega_c \quad (\text{A.9})$$

$$\bar{\sigma}[C(j\omega)] \leq \bar{\sigma}[W_3^{-1}(j\omega)] \quad \omega > \omega_c$$

For the weight which is present in the robustness bound, W_3 , a typical structure is given by,

$$W_3(s) = \frac{s}{k} \cdot I_{n \times n} ; \quad k = \omega_c \quad (\text{A.10})$$

$W_3(s)$ is an improper weighting function. It is possible to accommodate W_3 within the proper plant, which becomes recognizable in the D and C matrices of the plant augmented by the weights in a state space description. The plant augmentation is described in the next section.

A.2.4 Plant Augmentation

The design of the controller in the H_∞ synthesis problem is based on a plant that is augmented with weights as shown in Fig.A.2. The following is an illustration of the relationships between the signals in Fig.A.2 and of the way in which the weights are incorporated to form the state space description of the augmented plant $P(s)$. For the following descriptions the plant $P(s)$ can be partitioned as a block transfer function matrix,

$$P(s) := \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}. \quad (\text{A.11})$$

The signals in Fig.A.2 are related to each other as,

$$\begin{aligned} y_1 &= P_{11}u_1 + P_{12}u_2 \\ y_2 &= P_{21}u_1 + P_{22}u_2 \end{aligned}$$

or

$$y_1 = \begin{bmatrix} y_{11} \\ y_{12} \\ y_{13} \end{bmatrix} = \begin{bmatrix} W_1 \\ 0 \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} -W_1G \\ W_2 \\ W_3G \end{bmatrix} u_2, \quad (\text{A.12})$$

$$y_2 = Iu_1 - Gu_2,$$

while a detectable and stabilizable state space description of $P(s)$ is,

$$P(s) := \left[\begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right] \quad (\text{A.13})$$

For the purpose of the synthesis in the next section let the state space form of the plant and weights be described as,

$$G(s) = \begin{bmatrix} A_p & B_p \\ C_p & D_p \end{bmatrix}$$

and

$$W_i = \begin{bmatrix} A_{wi} & B_{wi} \\ C_{wi} & D_{wi} \end{bmatrix}$$

(A.14)

with $i = 1, 2, 3$.

Let the states of the augmented plant $P(s)$ be described by the state vector x_{ap} as,

$$x_{ap} = [x_p \quad x_{w1} \quad x_{w2} \quad x_{w3}]^T$$

where the state vectors of the plant $G(s)$ and the weights $W_i(s)$ are represented by x_p and x_{wi} respectively. The matrices of the augmented plant can be written in terms of the state space matrices of the plant and the weights as follows.

$$A = \begin{bmatrix} A_p & 0 & 0 & 0 \\ -B_{w1}C_p & A_{w1} & 0 & 0 \\ 0 & 0 & A_{w2} & 0 \\ B_{w3}C_p & 0 & 0 & A_{w3} \end{bmatrix}$$

$$[B_1 \mid B_2] = \begin{bmatrix} 0 & B_p \\ B_{w1} & -B_{w1}D_p \\ 0 & B_{w2} \\ 0 & B_{w3}D_p \end{bmatrix}$$

(A.15)

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -D_{w1}C_p & C_{w1} & 0 & 0 \\ 0 & 0 & C_{w2} & 0 \\ D_{w3}C_p & 0 & 0 & C_{w3} \\ \hline -C_p & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} = \begin{bmatrix} D_{w1} & -D_{w1}D_p \\ 0 & D_{w2} \\ 0 & D_{w3}D_p \\ \hline I & -D_p \end{bmatrix}$$

It is possible to write the augmented plant for configurations of plants and weights other than shown in Fig.A.2 in a similar way as shown above because Eq.A.12 is a general description for all plants that can be solved as H_∞ problems.

A.2.5 H_∞ Synthesis and γ -Iteration

Fig.4.6 is the diagram on which the H_∞ feedback problem is based. The control objective is to find a controller $K(s)$ for the FDLTI augmented plant such that the closed loop is nominally stable and the H_∞ norm of the closed loop transfer function from $u_1(t)$ to $y_1(t)$, $H_{y_1 u_1}(s)$, is minimized.

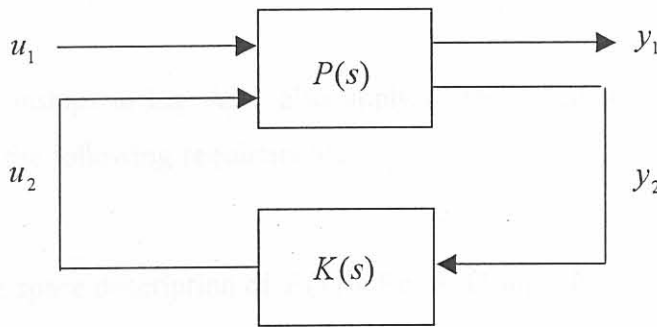


Figure A.4: Synthesis block diagram

In terms of the signals in Fig.A.4 the control problem is to find a controller $K(s)$ which uses the information in the measured output, y_2 , to generate a control signal, u_2 , which counteracts the influence of the exogenous inputs (weighted), u_1 , on the exogenous outputs (weighted), y_1 , thereby minimizing the closed loop ∞ - norm from u_1 to y_1 [31]. When using weights $W_1(s)$ and $W_3(s)$ to find the H_∞ -norm of $H_{y_1 u_1}(s)$, the applicable relationship is,

$$\|H_{y_1 u_1}(j\omega)\|_\infty = \max_\omega \bar{\sigma} \begin{bmatrix} W_1(j\omega)S(j\omega) \\ W_3(j\omega)C(j\omega) \end{bmatrix}$$

The controller is designed by way of the γ - iteration. The result of such an iteration would be a sub-optimal controller.

γ - Iteration:

The γ -iteration is a procedure that originates from Doyle et al [32] with the objective to solve the H_∞ output feedback problem. Part of this procedure is solving two modified Riccati equations in order to find a stabilizing controller $K(s)$ such that,

$$\|H_{y_1 u_1}\|_\infty \leq \gamma \quad (\text{A.16})$$

with

$$\min_{K(s)} \|H_{y_1 u_1}\|_\infty = \gamma_{optimal}$$

$$\gamma_{optimal} < \gamma$$

The signal relationships in Eq. A.11 also apply to the signals in Fig.A.4. The augmented plant must meet the following requirements:

- i) The state space description of $P(s)$ in Eq. A.13 must be stabilizable and detectable.
- ii) The transfer functions $P_{11}(s)$ and $P_{22}(s)$ must be strictly proper.
- iii) The transfer functions $P_{12}(s)$ and $P_{21}(s)$ must be proper but not strictly proper.

The second condition is satisfied if D_{11} and D_{22} are zero. By adding fast filters to the appropriate weights this condition can be met. Otherwise for D_{11} to be zero weight $W_1(s)$ needs to be strictly proper. D_{22} will also be zero if the plant $G(s)$ is strictly proper.

A standard feedback configuration such as shown in Fig. A.2 guarantees $P_{21}(s)$ to be proper. $P_{12}(s)$ is proper if $G(s)$ and either $W_2(s)$ or $W_3(s)$ or both are used. If the plant is strictly proper ($D_p = 0$), and only one of $W_2(s)$ and $W_3(s)$ is used, the weight, $W_2(s)$, will have to be present. If the use of an improper weight, $W_3(s)$, is preferred to the use of weight, $W_2(s)$, $W_3(s)$ can be used by accommodating it into a strictly proper plant.

Further requirements of the γ -iteration are on D_{12} and D_{21} . D_{12} must have full column rank while D_{21} must have full row rank.

The steps of the γ -iteration are given as follows [30]:

- i) Guess γ which indicates the achievable performance.
- ii) u_1 and y_1 have to be scaled such that γ in Eq. A.16 is unity, i.e. $\|\tilde{H}_{y_1 u_1}\|_{\infty} \leq 1$, with $\tilde{H}_{y_1 u_1}$ being scaled appropriately. Fig. A.5 shows what the scaling in this step entails.

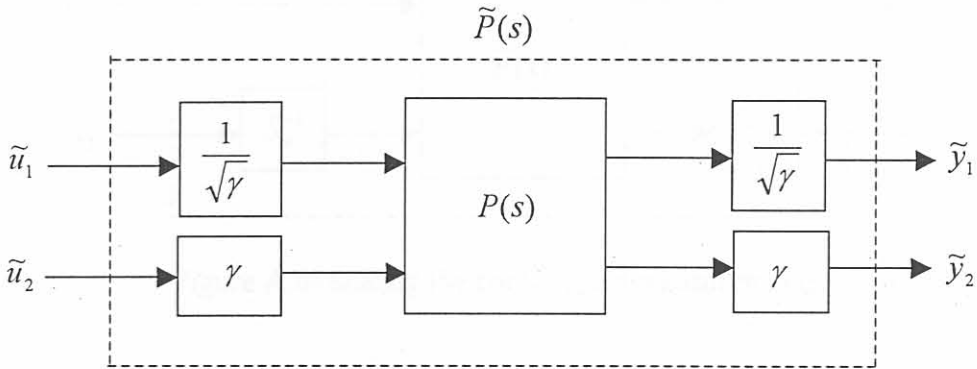


Figure A.5: Scaling the augmented plant.

The signal relationship for the scaled augmented plant $\tilde{P}(s)$ is then as follows:

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{\gamma}} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & \gamma P_{22}(s) \end{bmatrix} \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix} = \tilde{P}(s) \begin{bmatrix} \tilde{u}_1 \\ \tilde{u}_2 \end{bmatrix}$$

After scaling, the state space description for $\tilde{P}(s)$ becomes,

$$\tilde{P}(s) = \left[\begin{array}{cc|cc} A & & \frac{1}{\sqrt{\gamma}} B_1 & \sqrt{\gamma} B_2 \\ \hline \frac{1}{\sqrt{\gamma}} C_1 & & \frac{1}{\gamma} D_{11} & D_{12} \\ \sqrt{\gamma} C_2 & & D_{21} & \gamma D_{22} \end{array} \right]$$

iii) Scaling of controls u_2 and measurements y_2 is done with matrices S_u and S_y such that,

$$S_u^T S_u = D_{12}^T D_{12} = I$$

$$S_y^{-1} (S_y^{-1})^T = D_{21} D_{21}^T = I$$

where S_u and S_y are two square, nonsingular matrices. They can be obtained by a Cholesky decomposition of the identity matrix.

Scaling of u_2 and y_2 is done as shown in Fig. A.6.

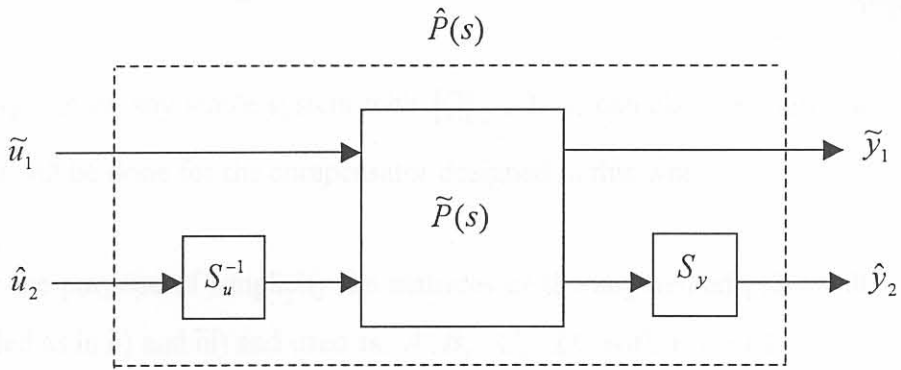


Figure A.6: Scaling the controls and measurements.

S_u and S_y is also used to scale the state space matrices of the plant as follows:

$$\hat{B}_2 = \tilde{B}_2 S_u^{-1}$$

$$\hat{C}_2 = S_y \tilde{C}_2$$

$$\hat{D}_{12} = \tilde{D}_{12} S_u^{-1}$$

$$\hat{D}_{21} = S_y \tilde{D}_{21}$$

$$\hat{D}_{22} = S_y \tilde{D}_{22} S_u^{-1}$$

iv) Fig. A.7 shows the H_∞ controller structure.

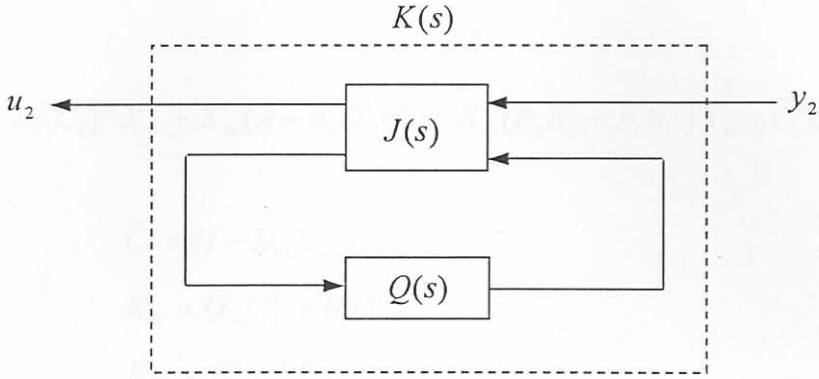


Figure A.7: H_∞ controller structure.

$Q(s)$ can be any stable system with $\|Q\|_\infty \leq 1$. It can also be chosen as, $Q(s) = 0$, as it will be done for the compensator designed in this work.

For the purpose of simplicity the matrices of the augmented plant will be assumed scaled as in ii) and iii) and used as, A, B_j, C_i, D_{ij} with $i, j = 1, 2$.

For the controller $K(s)$ in Fig. A.4 the state space description is given as,

$$J(s) = \left[\begin{array}{c|c} A_J & B_J \\ \hline C_J & D_J \end{array} \right]$$

where

$$A_J = A - K_F C_2 - B_2 K_C + Y_\infty C_1^T (C_1 - D_{12} K_C),$$

$$B_J = [K_F \quad K_{F1}],$$

$$C_J = \begin{bmatrix} -K_C \\ K_{C1} \end{bmatrix},$$

$$D_J = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix},$$

$$K_C = (B_2^T X_\infty + D_{12}^T C_1)(I - Y_\infty X_\infty)^{-1},$$

$$K_{C1} = (D_{12} B_1^T - C_2)(I - Y_\infty X_\infty)^{-1},$$

with X_∞ the unique, real, symmetric solution of the Algebraic Riccati equation (ARE),

$$(A - B_2 D_{12}^T C_1)^T X_\infty + X_\infty (A - B_2 D_{12}^T C_1) - X_\infty (B_2 B_2^T - B_1 B_1^T) X_\infty + \tilde{C}_1^T \tilde{C}_1 = 0$$

with

$$\tilde{C}_1 = (I - D_{12} D_{12}^T) C_1,$$

$$K_F = (Y_\infty C_2^T + B_1 D_{12}^T),$$

$$K_{F1} = (Y_\infty C_1^T D_{12} + B_2),$$

and Y_∞ is the unique, real, symmetric solution of the ARE,

$$(A - B_1 D_{21}^T C_2) Y_\infty + Y_\infty (A - B_1 D_{21}^T C_2)^T - Y_\infty (C_2 C_2^T - C_1 C_1^T) Y_\infty + \tilde{B}_1^T \tilde{B}_1 = 0,$$

with

$$\tilde{B}_1 = B_1 (I - D_{21}^T D_{21}).$$

If the following three conditions hold, then a solution can be obtained and γ in i) does not have to be increased any further.

$$X_\infty \geq 0$$

$$Y_\infty \geq 0$$

$$\bar{\lambda}(Y_\infty X_\infty) \leq 1$$

The iteration has to be repeated from step ii) with an increased γ if the conditions above are not satisfied.

v) u_2 and y_2 on the designed compensator $\hat{K}(s)$ (see Fig. A.8) have to be scaled to,

vi)

$$B_K = \sqrt{\gamma} \hat{B}_K S_y,$$

$$C_K = \sqrt{\gamma} S_u^{-1} \hat{C}_K,$$

$$D_K = \gamma S_u^{-1} \hat{D}_K S_y$$

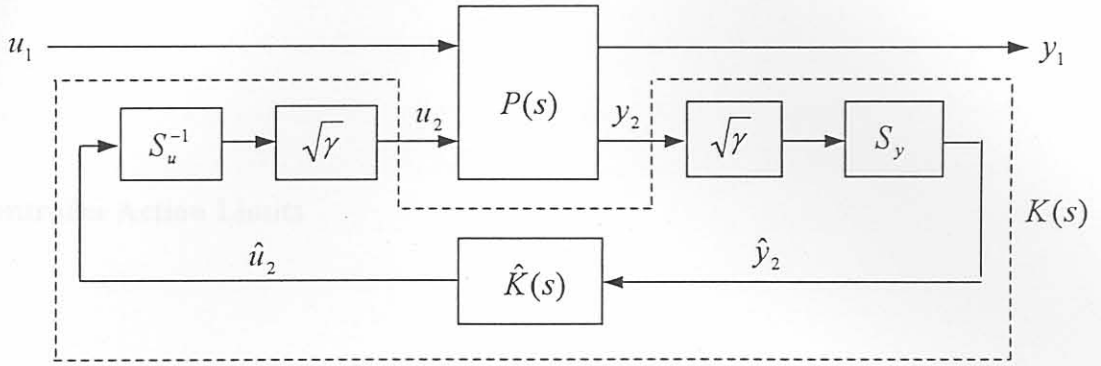


Figure A.8: The designed controller scaled.

The state space matrices of the compensator $\hat{K}(s)$ with $Q(s) = 0$ give the state space description as,

$$\hat{K}(s) = \left[\begin{array}{c|c} A - K_F C_2 - B_2 K_C + Y_\infty C_1^T (C_1 - D_{12} K_C) & K_F \\ \hline & K_C \end{array} \right] \begin{array}{c} \\ 0 \end{array}$$

Appendix B

Controller Action Limits

B.1 Hydraulic Actuator Limits

Hydraulic actuator stroke, x_s , mill stretch, $y_{stretch}$, and displacement of the upper roll pack, y_1 , are in [4] related by

$$x_s = y_{stretch} - y_1 \quad (B.1)$$

At the maximum rolling force $P_{max} = 40 \text{ MN}$, the mill stretch limit is

$$y_{stretchmax} = 6.5 \cdot 10^{-3} \text{ m} \quad [4].$$

The worst case for the mill is that a rolling force is exerted by the hydraulic actuators while the strip in the roll gap does not compress, i.e. $y_1(t, z) = 0$. Then the maximum actuator stroke is determined by the limit to which the mill can stretch:

$$x_{smax} = y_{stretchmax} = 6.5 \cdot 10^{-3} \text{ m.} \quad (B.2)$$

B.2 Coiler Motor Speed Change Limits

The most severe control action of the coiler motors will be demanded when a fully loaded coiler has to change its speed such that tensions increase from almost zero to their set up values . Therefore this case is considered here in order to determine coiler motor speed change limits.

Based on the graphs in chapter 5 in which the control actions of the back and front coilers can be seen, it is assumed that the back and front coiler speeds change linearly by

$$v_{bc} = a_{bc}t \tag{B.3}$$

$$v_{fc} = a_{fc}t \tag{B.4}$$

such as the peripheral speed of the work rolls which changes as

$$v_{main} = a_{main} \cdot t \approx 0.79 \cdot t , \tag{B.5}$$

which is obtained from the speed up ramp of the main drive shown in Fig. 3.2. In the relationships given above the accelerations represent the following.

- a_{bc} : peripheral acceleration of back coiler,
- a_{fc} : peripheral acceleration of front coiler and
- a_{main} : acceleration of mill main drive.

From relationship 2.5 it can be shown that a difference in a_{bc} and a_1 , the acceleration of strip material at the roll gap entrance, for T_1 to increase up to its setup value. This difference is denoted by

$$\Delta a_1 = a_1 - a_{bc} . \tag{B.6}$$

Equivalently on the mill delivery side the difference between a_{fc} and a_2 , the acceleration of strip material at the roll gap exit, for T_2 to increase up to its set up value, is denoted by

$$\Delta a_2 = a_{fc} - a_2 . \tag{B.7}$$

a_1 and a_2 can be computed from a_{main} as follows. First it is assumed that the neutral point is at $\phi = \phi_h$ between the roll gap entrance and exit before the tensions are established where

ϕ has the meaning as in Fig. 3.17. ϕ_h indicates the position halfway along the horizontal distance, L_x , between the roll gap entrance and exit. The throughput of material through the roll gap can be expressed as

$$h(\phi_h)v(\phi_h) = h_1v_1 = h_2v_2. \quad (\text{B.8})$$

From this throughput the entrance and exit acceleration of the strip material can then be computed as

$$a_1 = \frac{h(\phi_h)}{h_1} \cdot a_{main}, \quad (\text{B.9})$$

$$a_2 = \frac{h(\phi_h)}{h_2} \cdot a_{main}. \quad (\text{B.10})$$

With ϕ_m being the bite angle it can be determined from

$$\phi_m \approx \cos^{-1}\left(\frac{R - \delta/2}{R}\right) = 5.45^\circ. \quad (\text{B.11})$$

ϕ_h and $h(\phi_h)$ are then computed from

$$\phi_h \approx \sin^{-1}\left(\frac{L_x}{2R}\right) = 2.72^\circ \quad (\text{B.12})$$

where

$$L_x \approx R \sin \phi_m = 36 \text{ mm} \quad (\text{B.13})$$

then

$$h(\phi_h) = h_2 + 2(R - R \cos \phi_h) = 10.7 \text{ mm}. \quad (\text{B.14})$$

Since the operating point of the controllers will be at $v_{main} = 3.5 \text{ m/s}$ the peripheral speeds of the coilers will be

$$v_{bc} = \frac{h(\phi_h)}{h_1} \cdot v_{main} = 2.8 \text{ m/s} \quad (\text{B.15})$$

and

$$v_{fc} = \frac{h(\phi_h)}{h_2} \cdot v_{main} = 3.78 \text{ m/s}. \quad (\text{B.16})$$

For the radius of a fully loaded coiler, $r_{c\ full} = 1.25\text{ m}$, and of a coiler mandrel, $r_{c\ mandrel} = 0.675\text{ m}$, the rotating speeds of the coilers will be $\omega_{bc} = 2.24\text{ rad/s}$ and $\omega_{fc} = 5.6\text{ rad/s}$. With a gear box ratio of $1/4.875$ these rotating speeds lie in the speed range of 0 to 720 rpm for typical coiler motors [38].

The computation of the torques, $T_{m\ bc}$ and $T_{m\ fc}$, available on the back and front coiler mandrels respectively is based on coiler motor power, $P_c = 1500\text{ kW}$ [1] and on the coiler rotating speeds:

$$T_{m\ bc} = \frac{P_c}{\omega_{bc}} = 0.67\text{ MNm} \quad (\text{B.17})$$

$$T_{m\ fc} = \frac{P_c}{\omega_{fc}} = 0.268\text{ MNm} . \quad (\text{B.18})$$

For a step change of $T_{i\ setup}$, $i \in (1,2)$, the possible angular acceleration of a fully loaded coiler, α_{full} , and the coiler mandrel, $\alpha_{mandrel}$, can be determined as

$$\alpha_{full} = T_{m\ bc} / I_{c\ full} \quad (\text{B.19})$$

$$\alpha_{mandrel} = T_{m\ fc} / I_{c\ mandrel} , \quad (\text{B.20})$$

where $I_{c\ full} \approx 40310\text{ kgm}^2$ and $I_{c\ mandrel} \approx 1502\text{ kgm}^2$ is obtained from dimensions of a typical Steckel mill [38] as the axial moments of inertia of a fully loaded coiler and a coiler mandrel respectively.

The back and front coiler accelerations that are then possible are

$$a_{bc} = \alpha_{full} \cdot r_{c\ full} = 20.8\text{ m/s}^2 \quad (\text{B.21})$$

$$a_{fc} = \alpha_{mandrel} \cdot r_{c\ mandrel} = 120.4\text{ m/s}^2 . \quad (\text{B.22})$$

The possible acceleration changes Δa_1 and Δa_2 can then be determined from Eq. B.6 and B.7 as $\Delta a_1 = -21\text{ m/s}^2$ and $\Delta a_2 = 120\text{ m/s}^2$.

Appendix C

H_∞ Controller State Space Matrices

The continuous state space matrices of the controllers are given in the following. In their state space form the controllers are given by

$$\dot{z}(t) = Fz(t) + Ge(t) \quad (C.1)$$

$$u(t) = Hz(t) + Ne(t) \quad (C.2)$$

where

- $e(t)$: the control error vector,
- $z(t)$: the controller state vector,
- $u(t)$: the controller action vector,
- F : the controller state matrix,
- G : the controller input matrix,
- H : the controller output matrix and
- N : the controller feed-through matrix.

C.1 H_∞ Controller for the Second Linear Model

Matrix F:

Columns 1 through 6

$$\begin{pmatrix} -1.3492e+005 & -3.4164e+005 & 4.4832e+006 & 1.6519e+006 & -2.8014e+006 & 5.5820e+005 \\ 3.8654e-011 & -6.5782e+001 & -6.1100e-005 & 8.3823e+001 & -1.2418e+003 & -3.9187e+003 \\ 1.6591e-012 & -3.0468e-011 & -6.5781e+001 & 8.5290e+000 & 8.5183e+001 & -3.5105e+002 \\ -7.5346e-012 & 1.1937e-011 & -1.0402e-010 & -5.2462e+000 & -1.6127e-001 & 7.2129e-003 \\ 2.4570e-011 & 3.2620e-010 & -1.2354e-009 & 1.0392e-009 & -9.9389e-007 & 1.3478e-008 \\ 3.5988e-011 & 9.2143e-011 & -1.3832e-009 & -1.1961e-009 & 8.7299e-010 & -1.0015e-006 \\ -3.6507e-013 & -1.0703e-012 & 1.5298e-011 & 1.6314e-011 & -1.1590e-011 & 1.9835e-011 \end{pmatrix}$$

Column 7

$$\begin{pmatrix} 1.4051e+004 \\ 1.5770e+002 \\ 1.1620e+001 \\ -4.0151e+000 \\ 7.0147e-010 \\ -6.3876e-010 \\ -9.9999e-007 \end{pmatrix}$$

Matrix G:

$$\begin{pmatrix} 8.2441e+004 & -2.0559e+007 & -2.2805e+006 \\ -1.4344e+001 & -2.0401e+003 & -7.8316e+002 \\ -1.4631e+000 & -1.4419e+002 & -2.2255e+002 \\ -1.0345e+001 & 1.0493e-003 & 4.9010e-002 \\ -2.1480e-008 & -1.9791e-007 & 1.0175e-006 \\ -9.5162e-008 & -4.5064e-007 & -5.2139e-007 \\ -2.4050e-006 & 1.9722e-008 & 6.7007e-00 \end{pmatrix}$$

Matrix H:

Columns 1 through 6

$$\begin{bmatrix} 1.6305e-003 & 4.2899e-003 & -5.7355e-002 & 4.1393e+000 & 7.6829e+003 & 1.0539e+004 \\ 4.9458e-005 & -3.5084e-001 & 1.2342e+000 & 3.0441e-001 & 5.5834e+008 & 1.0870e+009 \\ 4.7142e-006 & -9.4548e-002 & 1.3482e+000 & 2.0210e-002 & 1.9912e+008 & -9.1235e+007 \end{bmatrix}$$

Column 7

$$\begin{bmatrix} 3.2787e+005 \\ -4.7954e+007 \\ 1.4909e+006 \end{bmatrix}$$

Matrix N:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

C.2 H_∞ Controller for the Third Linear Model

Matrix F:

Columns 1 through 6

-7.5938e+001	8.5808e+001	4.6065e+003	1.0655e+004	5.1611e+003	1.1942e+004
4.0000e+000	0	0	0	0	0
5.1963e-001	1.0964e+001	2.3278e+002	1.3073e+003	6.4514e+002	1.4927e+003
0	0	1.6000e+001	0	0	0
5.1963e-001	1.0964e+001	5.7581e+002	1.3319e+003	3.0208e+002	1.4676e+003
0	0	0	0	1.6000e+001	0
-4.4761e-004	-9.4442e-003	-5.0560e-001	-1.1715e+000	-1.2745e-002	-2.9498e-002
-5.7295e-002	-1.2089e+000	-6.4716e+001	-1.4995e+002	-1.6313e+000	-3.7757e+000
0	0	0	0	0	0
-5.5952e-005	-1.1805e-003	-6.3200e-002	-1.4643e-001	-1.5931e-003	-3.6872e-003
1.3918e-003	2.9365e-002	-2.9568e-002	-6.8236e-002	1.6413e+000	3.8040e+000
0	0	0	0	0	0
1.7398e-004	3.6706e-003	-3.6960e-003	-8.5296e-003	2.0517e-001	4.7550e-001
0	0	0	0	0	0
2.2269e-002	4.6984e-001	-4.7309e-001	-1.0918e+000	2.6261e+001	6.0864e+001
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 7 through 12

-4.6036e+000	1.3753e+007	4.4164e+005	-5.6263e+002	-5.9274e+001	4.6276e+001
0	0	0	0	0	0
-5.7545e-001	1.7191e+006	5.5206e+004	-7.0329e+001	-7.4092e+000	5.7845e+000
0	0	0	0	0	0
-5.7545e-001	1.7191e+006	5.5206e+004	-7.0329e+001	-7.4092e+000	5.7845e+000
0	0	0	0	0	0
-9.5043e-003	-1.4413e+003	-2.9218e+001	1.3898e-003	6.3830e-003	-4.9829e-003
6.3450e-002	-1.9359e+005	-6.2160e+003	1.7789e-001	8.1702e-001	-6.3781e-001
0	2.0480e+003	0	0	0	0
6.1963e-005	-1.8017e+002	-3.6522e+000	-9.8263e-003	7.9787e-004	-6.2286e-004
-1.5413e-003	-8.8333e+001	-2.8422e+000	-1.7923e-001	-5.6025e+002	-1.3851e+000
0	0	0	0	4.0000e+000	0
-1.9266e-004	-1.1042e+001	-3.5527e-001	-2.2404e-002	-2.4809e-003	1.9367e-003
0	0	0	0	0	0
-2.4661e-002	-1.4133e+003	-4.5474e+001	-2.8677e+000	-3.1755e-001	2.4789e-001
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0

Columns 13 through 18

6.1582e+002	1.0788e+004	-3.9644e+007	-1.2732e+006	1.7142e+004	1.7494e+006
0	0	0	0	0	0
7.6978e+001	1.3485e+003	-4.9555e+006	-1.5914e+005	2.1427e+003	2.1868e+005
0	0	0	0	0	0
7.6978e+001	1.3485e+003	-4.9555e+006	-1.5914e+005	2.1427e+003	2.1868e+005
0	0	0	0	0	0
-6.7515e-002	-1.1862e+000	9.7890e+001	3.1429e+000	-1.8458e+000	-1.9232e+002
-8.6420e+000	-1.5183e+002	1.2530e+004	4.0229e+002	-2.3626e+002	-2.4617e+004
0	0	0	0	0	0
-8.4394e-003	-1.4827e-001	1.2236e+001	3.9286e-001	-2.3072e-001	-2.4040e+001
-3.9537e-003	-6.9081e-002	-1.2120e+004	-2.3740e+002	5.7392e+000	-1.1212e+001
0	0	0	0	0	0
-5.6023e+002	-2.8098e+000	-1.5150e+003	-2.9675e+001	7.1740e-001	-1.4015e+000
2.0000e+000	0	0	0	0	0
-6.3260e-002	-1.1053e+000	-2.0210e+005	-6.4892e+003	9.1827e+001	-1.7940e+002
0	0	2.0480e+003	0	0	0
0	0	0	0	-1.0000e-006	0
0	0	0	0	0	-1.0000e-006
0	0	0	0	0	0

Column 19

1.8910e+006
0
2.3638e+005
0
2.3638e+005
0
-4.6706e+000
-5.9783e+002
0
-5.8382e-001
6.0231e+002
0
7.5288e+001
0
9.6369e+003
0
0
0
-1.0000e-006

Matrix G:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 3.9063e-003 & 0 & 0 \\ 0 & 1.0000e+000 & 0 \\ 0 & 0 & 1.0000e+000 \end{pmatrix}$$

Matrix H:

Columns 1 through 6

$$\begin{pmatrix} 2.0785e+000 & 4.3854e+001 & 2.3032e+003 & 5.3275e+003 & 2.5805e+003 & 5.9709e+003 \\ -3.5809e-003 & -7.5553e-002 & -4.0448e+000 & -9.3718e+000 & -1.0196e-001 & -2.3598e-001 \\ 1.3918e-003 & 2.9365e-002 & -2.9568e-002 & -6.8236e-002 & 1.6413e+000 & 3.8040e+000 \end{pmatrix}$$

Columns 7 through 12

$$\begin{pmatrix} -2.3018e+000 & 6.8763e+006 & 2.2082e+005 & -2.8131e+002 & -2.9637e+001 & 2.3138e+001 \\ 3.9656e-003 & -1.1531e+004 & -2.3374e+002 & 1.1118e-002 & 5.1064e-002 & -3.9863e-002 \\ -1.5413e-003 & -8.8333e+001 & -2.8422e+000 & -1.7923e-001 & -1.9847e-002 & 1.5493e-002 \end{pmatrix}$$

Columns 13 through 18

$$\begin{pmatrix} 3.0791e+002 & 5.3940e+003 & -1.9822e+007 & -6.3658e+005 & 8.5710e+003 & 8.7470e+005 \\ -5.4012e-001 & -9.4893e+000 & 7.8312e+002 & 2.5143e+001 & -1.4766e+001 & -1.5386e+003 \\ -3.9537e-003 & -6.9081e-002 & -1.2120e+004 & -2.3740e+002 & 5.7392e+000 & -1.1212e+001 \end{pmatrix}$$

Column 19

$$\begin{pmatrix} 9.4551e+005 \\ -3.7364e+001 \\ 6.0231e+002 \end{pmatrix}$$

Matrix N:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Appendix D

Signals for Controller Design and Implementation

The controllers as discussed in chapter 5 were designed for models with deviational inputs and outputs such that the signals at the inputs and outputs of both the controller and plant in Fig. D.1 had the values as in Tab. D.1.

For controller implementation on the nonlinear plant simulator the signals in Fig. D.1 had the values as in Tab. D.2.

Fig. D.1 and Tab.'s D.1 and D.2 show that the inputs and outputs of controller and plant in the design stage are deviational variables, i.e.

$$e_i = \Delta e_i = \Delta r_i - \Delta y_i \quad (D.1)$$

$$u_i = \Delta u_i \quad (D.2)$$

$$y_i = \Delta y_i \quad (D.3)$$

and during the implementation stage the plant demand and output become respectively,

$$u_i = \Delta u_i + u_{ir} \quad (D.4)$$

and

$$y_i = \Delta y_i + y_{iss}(t) \quad (D.5)$$

Since the controller was designed for deviational variables as input to compute a deviational variable as controller output, it's input has to be a deviational variable during implementation, i.e.

Table D.1: Signal values for controller design

Signal	Value
u_1	Δu_1
u_2	Δu_2
u_3	Δu_3
u_{1r}	0
u_{2r}	0
u_{3r}	0
y_{11}	Δy_{11}
y_{12}	Δy_{12}
y_{13}	Δy_{13}
y_{21}	Δy_{21}
y_{22}	Δy_{22}
y_{23}	Δy_{23}
y_{31}	Δy_{31}
y_{32}	Δy_{32}
y_{33}	Δy_{33}
y_1	Δy_1
y_2	Δy_2
y_3	Δy_3
r_1	Δr_1
r_2	Δr_2
r_3	Δr_3

$$\Delta e_i = \Delta r_i - \Delta y_i = (\Delta r_i + y_{iss}(t)) - (\Delta y_i + y_{iss}(t)) = r_i - y_i \quad (D.6)$$

Therefore the reference, r_i , has to be,

$$r_i = \Delta r_i + y_{iss}(t) , \quad (D.7)$$

Table D.2: Signal values for controller implementation

Signal	Value
u_1	$\Delta u_1 + u_{1ss}$
u_2	$\Delta u_2 + 0$
u_3	$\Delta u_3 + 0$
u_{1r}	u_{1ss}
u_{2r}	0
u_{3r}	0
y_{11}	-
y_{12}	-
y_{13}	-
y_{21}	-
y_{22}	-
y_{23}	-
y_{31}	-
y_{32}	-
y_{33}	-
y_1	$\Delta y_1 + y_{1ss}(t)$
y_2	$\Delta y_2 + y_{2ss}(t)$
y_3	$\Delta y_3 + y_{3ss}(t)$
r_1	$\Delta r_1 + y_{1ss}(t)$
r_2	$\Delta r_2 + y_{2ss}(t)$
r_3	$\Delta r_3 + y_{3ss}(t)$

i.e. a steady state value, y_{iss} , has to be added to the deviational reference Δr_i . Because of the steady state evolution for all three of the nonlinear plant simulator outputs this steady state value is time dependent for all three outputs y_i .

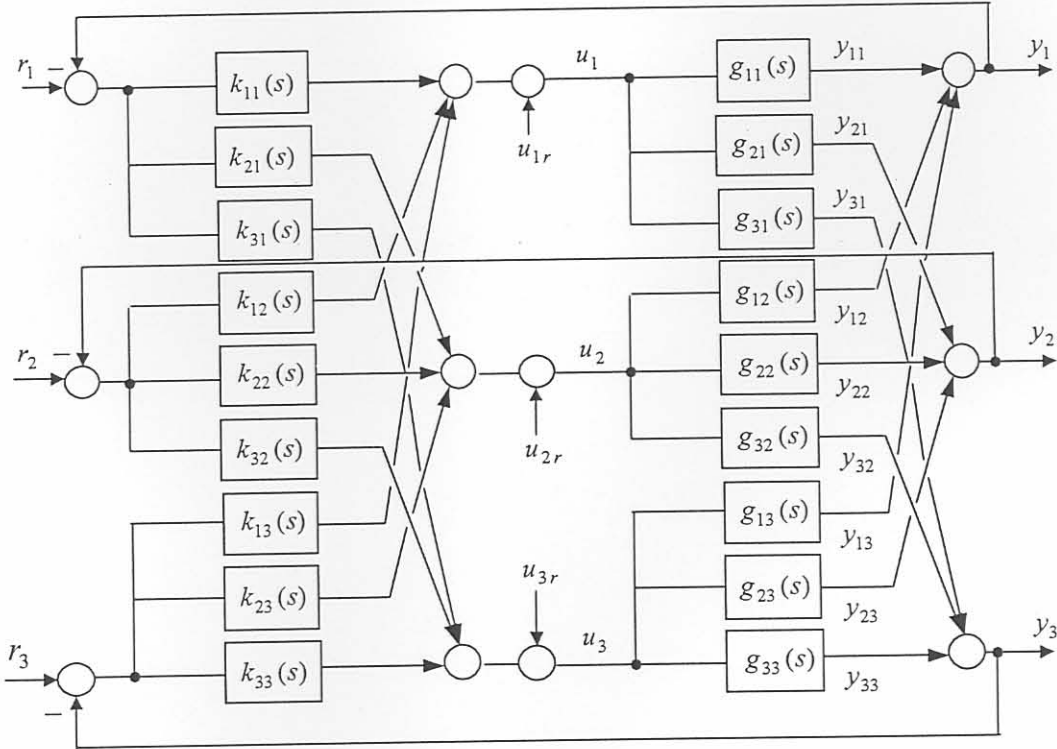


Figure D.1: Blockdiagram of MIMO control system (signs of signals at summation points are positive except where indicated else).