

Chapter 10: Appendix C - Computer Algorithm for Simulator

Time-related variables

- t*: time, in seconds after some base date/time
dt: basic time-step e.g. 2s for calculation of all actuator loops and plant models
dte: time-step e.g. 10s for calculation of main control algorithm

Indices and numbers of indices

- i*: 1 to number of plant inputs
j: 1 to number of plant outputs
ku: 1 to number of time-shifts of length *dt* for dead-time states of plant inputs
kx: 1 to number of time-shifts of length *dt* for dead-time states of plant states
ky: 1 to number of time-shifts of length *dt* for dead-time states of plant outputs
mku: maximum number of dead-time states of length *dt* provided for storing plant inputs
mky: maximum number of dead-time states of length *dt* provided for storing plant outputs

Conventions

The variables were given multi-character names (convenient for programming). Three basic variables were specified for the simulation of the plant:

- u*: Contains all input variables
x: Contains all state variables
y: Contains all output variables

Plant input variables (*u[i]*)

- u[i].PI_Setpoint*: main controller output, provided as a setpoint for a local PI controller
u[i].PI_last: value of *u[i].PI_Setpoint* at last local PI increment
u[i].local: signal to actuator from local PI controller
u[i].slack: modified *u[i].local* to provide another state incorporating actuator slack
u[i].state: 1st-order response of actuator to *u[i].slack*
u[i].smooth: noise-free input to plant, a non-linear function (valve characteristic) of *u[i].state*
u[i].noise: state for additive noise to measurement of input to plant
u[i].local_meas: local PI loop's measurement of input to plant
u[i].local_meas_last: value of *u[i].local_meas* at last local PI increment
u[i].meas: main controller's measurement of input to plant, filtered according to *dte*
u[i].disturb: state for additive disturbance to plant input
u[i].input: final actual input to plant
u[i].input_last: final actual input to plant at time *dt* ago
u_dead[i,ku]: shift vector to provide for dead time of actuator

Plant input parameters ($u[i].p$)

$u[i].p.knob$:	multiplier "knob" (0 to 100 %) for all plant-input non-ideal parameters
$u[i].p.base0$:	initial base level for u used at initialisation
$u[i].p.base$:	drifting base level for u
$u[i].p.drift_base$:	drift rate of base level for u
$u[i].p.min$:	minimum u
$u[i].p.max$:	maximum u
$u[i].p.soft0.min$:	initial minimum achievable u used at initialisation
$u[i].p.soft.min$:	drifting minimum achievable u
$u[i].p.drift.min$:	drift rate of minimum achievable u ($u[i].p.soft.min$)
$u[i].p.soft0.max$:	initial maximum achievable u used at initialisation
$u[i].p.soft.max$:	drifting maximum achievable u
$u[i].p.drift.max$:	drift rate of maximum achievable u ($u[i].p.soft.max$)
$u[i].PI.gain$:	gain of local PI controller
$u[i].PI.ti$:	integral time constant of local PI controller
$u[i].p_actuator.gain$:	actuator gain
$u[i].p_actuator.tau$:	actuator time constant
$u[i].p_actuator.tdead$:	dead-time of actuator's response
$u[i].p_slack$:	two-sided dead-band to get from $u[i].state$ to $u[i].slack$
$u[i].p_nonlinear$:	non-linearity of valve (-1 to +1, with 0 for linear) to get from $u[i].slack$ to $u[i].smooth$
$u[i].p_noise.gain$:	magnitude of noise, to update $u[i].noise$
$u[i].p_noise.tau$:	time constant for filter of noise, to update $u[i].noise$
$u[i].p_tdead$:	dead-time of plant input (excluding actuator's)
$u[i].p_disturb.gain$:	magnitude of disturbance, to update $u[i].disturb$
$u[i].p_disturb.tau$:	time constant for filter of disturbance, to update $u[i].disturb$
$u[i].p_offset$:	offset to get $u[i].input$

Calculated plant-input parameters ($u[i].c$)

$u[i].c_exp$:	0 if $u[i].p_actuator.tau = 0$, else $exp(-dt/ u[i].p_actuator.tau)$
$u[i].c_exp_noise$:	0 if $u[i].p_noise.tau = 0$, else $exp(-dt/ u[i].p_noise.tau)$
$c_exp_u_dte$	$exp(-dt/dtc)$
$u[i].c_exp_disturb$:	0 if $u[i].p_disturb.tau = 0$, else $exp(-dt/ u[i].p_disturb.tau)$
$u[i].c_ku_actuator$:	$Minimum(Integer(1+u[i].p_actuator.tdead/dt), mku)$
$u[i].c_ku_plant$:	$Minimum(Integer(1+(u[i].p_actuator.tdead+u[i].p_tdead)/dt), mku)$

Initialisation of plant-input variables (at $t=0$)

$u[i].p.base$	$= u[i].p.base0$
$u[i].PI_setpoint$	$= u[i].p.base0$
$u[i].PI_last$	$= u[i].PI_setpoint$
$u[i].local$	$= u[i].PI_setpoint$
$u[i].slack$	$= u[i].PI_setpoint$
$u[i].state$	$= u[i].PI_setpoint$
$u[i].smooth$	$= u[i].PI_setpoint$

```

u[i].noise           = 0
u_dead(i,ku)         = u[i].PI_setpoint, for all ku
u[i].local_meas      = u[i].PI_setpoint
u[i].local_meas_last = u[i].PI_setpoint
u[i].meas            = u[i].PI_setpoint
u[i].disturb         = 0
u[i].input           = u[i].PI_setpoint

```

Plant-input dynamics (to be executed every dt seconds)

```

//Calculate PI control for future over dt
if (u->PI.gain == 0)
    u->local += (u->PI_setpoint-u->PI_last);
else {
    if (u->PI.ti == 0)
        u->local += u->PI.gain*((u->PI_setpoint-u->PI_last)-(u->local_meas-u->local_meas_last));
    else
        u->local += u->PI.gain*((u->PI_setpoint-u->PI_last)-(u->local_meas-u->local_meas_last)+
            (u->PI_setpoint-u->local_meas)*sim_param.dt/u->PI.ti);
}

u->local_meas_last = u->local_meas;
u->PI_last = u->PI_setpoint;

//If not generating the model for the MPC-controller
if (!model){

    //Update soft maximum and restrict to hard limits
    u->p.soft.max += (u->p.drift.max * sim_param.dt) * u->p.knob;
    if (u->p.soft.max > u->p.max) u->p.soft.max = u->p.max;
    if (u->p.soft.max < u->p.min) u->p.soft.max = u->p.min;

    //Update soft minimum and restrict to between hard minimum and soft maximum
    u->p.soft.min += (u->p.drift.min * sim_param.dt) * u->p.knob;
    if (u->p.soft.min < u->p.min) u->p.soft.min = u->p.min;
    if (u->p.soft.min > u->p.soft.max) u->p.soft.min = u->p.soft.max;

    //Update base level drift and restrict to soft limits
    u->p.base += (u->p.drift_base * sim_param.dt) * u->p.knob;
    if (u->p.base < u->p.soft.min) u->p.base = u->p.soft.min;
    if (u->p.base > u->p.soft.max) u->p.base = u->p.soft.max;

    //Limit actuator, allowing slack on both sides
    if (u->local > u->p.max + u->p_slack) u->local = u->p.max + u->p_slack;
    if (u->local < u->p.min - u->p_slack) u->local = u->p.min - u->p_slack;

    //Calculate slack state (Unchanged if within the slack band)
    if (u->local > u->slack + u->p_slack) u->slack = u->local - u->p_slack;
    if (u->local < u->slack - u->p_slack) u->slack = u->local + u->p_slack;

}
else
    u->slack = u->local;

//Update state
u->state = u->state * u->c_exp + (u->slack*u->p_actuator.gain)*(1 - u->c_exp);

```



```

//Calculate nonlinearity and limit it to soft limits
//Function: y = (1+a)x + ax^2, a between -1 and 1
if (u->p.soft.max == u->p.soft.min)
    temp = 1;
else
    temp = (u->state - u->p.soft.min) / (u->p.soft.max - u->p.soft.min); //Scale variable between 0 and 1;
u->smooth = (1.0 + u->p_nonlinear) * temp - u->p_nonlinear * temp * temp;

//Scale back to normal values
u->smooth = u->smooth * (u->p.soft.max - u->p.soft.min) + u->p.soft.min;

//Store in shift vector (dead time)
for (long ku=(u->c_ku_plant-1); ku>0; ku--)
    u_dead[ku] = u_dead[ku-1];
u_dead[0] = u->smooth;

//Measurement noise
if (!model)
    u->noise = u->noise*u->c_exp_noise + ((double(rand())/RAND_MAX)*2 - 1)*u->p_noise.gain*
    (1 - u->c_exp_noise);

//Extract local measurement (with dead time) and add noise
u->local_meas = u_dead[u->c_ku_actuator-1] + u->noise;

//Update disturbance
if (!model)
    u->disturb = u->disturb + ((double(rand())/RAND_MAX)*2 - 1)*u->p_disturb.gain*(1 - u-
    >c_exp_disturb);

//Extract plant input (with dead time) and add disturbance
u->input_last = u->input;
u->input = u_dead[u->c_ku_plant-1] + u->disturb + u->p_offset;

//Update measurement for main controller
u->meas = u->meas*u->c_exp_dtc + u->local_meas*(1 - u->c_exp_dtc);

```

Plant-state module

Plant states ($x[i,j]$)

$x[i,j].base$: base level for state x
 $x[i,j].xx$: plant state associated with input i and output j
 $x[i,j].disturb$: state for additive noise to measurement of state x
 $x_dead(i,j,kx)$: shift vector to provide for dead time of state x

Plant-state parameters ($x[i,j].p$)

$x[i,j].knob$: multiplier "knob" (0 to 100 %) for all plant-state non-ideal parameters
 $x[i,j].p.base0$: initial base level for x used at initialisation
 $x[i,j].p.base$: drifting base level for x
 $x[i,j].p.drift_base$: drift rate of base level for x
 $x[i,j].p.min$: minimum x
 $x[i,j].p.max$: maximum x
 $x[i,j].p.soft0.min$: initial minimum achievable x used at initialisation
 $x[i,j].p.soft.min$: drifting minimum achievable x

$x[i,j].p.drift.min$: drift rate of minimum achievable x ($x[i,j].p.soft.min$)
 $x[i,j].p.soft0.max$: initial maximum achievable x used at initialisation
 $x[i,j].p.soft.max$: drifting maximum achievable x
 $x[i,j].p.drift.max$: drift rate of maximum achievable x ($x[i,j].p.soft.max$)
 $x[i,j].p_tf.a$: steady-state gain for state x
 $x[i,j].p_tf.b$: immediate-response gain for state x
 $x[i,j].p_tf.tau$: 1st-order time constant for state x
 $x[i,j].p_tf.ti$: integrator time constant for state x (0 for no integrator)
 $x[i,j].p_tf.tdead$: dead-time for state x
 $x[i,j].p_disturb.gain$: magnitude of disturbance, to update state x
 $x[i,j].p_disturb.tau$: time constant for filter of disturbance, to update state x

Calculated plant-state parameters ($x[i,j].c_$)

$x[i,j].c_exp$: 0 if $x[i,j].p_tf.tau = 0$, else $exp(-dt / x[i,j].p_tf.tau)$
 $x[i,j].c_exp_disturb$: 0 if $x[i,j].p_disturb.tau = 0$, else $exp(-dt / x[i,j].p_disturb.tau)$
 $x[i,j].c_kx$: $Minimum(Integer(1 + (x[i,j].p_tf.tdead)/dt), mkx)$

Initialisation of plant-state variables (at $t=0$)

$x[i,j].p.base = x[i,j].p.base0$
 $x[i,j].base = x[i,j].p.base$
 $x[i,j].xx = x[i,j].p.base$
 $x_dead(i,j,kx) = x[i,j].p.base$, for all kx
 $x[I,J].DISTURB = 0$

Plant-state dynamics (to be executed every dt seconds)

//If not generating the model for the MPC-controller
 if (!model) {

//Update soft maximum and restrict to hard limits
 $x[0].p.soft.max += (x[0].p.drift.max * sim_param.dt) * x[0].p.knob$;
 if ($x[0].p.soft.max > x[0].p.max$) $x[0].p.soft.max = x[0].p.max$;
 if ($x[0].p.soft.max < x[0].p.min$) $x[0].p.soft.max = x[0].p.min$;

//Update soft minimum and restrict to between hard minimum and soft maximum
 $x[0].p.soft.min += (x[0].p.drift.min * sim_param.dt) * x[0].p.knob$;
 if ($x[0].p.soft.min < x[0].p.min$) $x[0].p.soft.min = x[0].p.min$;
 if ($x[0].p.soft.min > x[0].p.soft.max$) $x[0].p.soft.min = x[0].p.soft.max$;

}

for ($j=0$; $j<tfdim$; $j++$)

{

if (!model) {

//Update base level drift and restrict to soft limits
 $x[j].p.base += (x[j].p.drift_base * sim_param.dt) * x[j].p.knob$;
 if ($x[j].p.base < x[j].p.soft.min$) $x[j].p.base = x[j].p.soft.min$;
 if ($x[j].p.base > x[j].p.soft.max$) $x[j].p.base = x[j].p.soft.max$;

}

```

//Update state
if (x[j].p_tf.ti == 0)
    x[j].xx = x[j].xx*x[j].c_exp + (x[j].p.base + x[j].p_tf.a * (u->input - u->p.base)) *
    (1 - x[j].c_exp) + x[j].p_tf.b * x[j].c_exp * (u->input - u->input_last);
else
    x[j].xx = x[j].xx + x[j].p_tf.a*(u->input - u->p.base)*sim_param.dt;

//Update disturbance
if (!model)
    x[j].disturb = x[j].disturb * x[j].c_exp_disturb + ((double(rand())/RAND_MAX)*2 - 1) *
    x[j].p_disturb.gain*(1 - x[j].c_exp_disturb);

x[j].xx += x[j].disturb;

//Store in shift vector (dead time)
for (kx=(x[j].c_kx-1); kx>0; kx--)
    x_dead[j*sim_param.max_kx+kx] = x_dead[j*sim_param.max_kx+kx-1];
x_dead[j*sim_param.max_kx+0] = x[j].xx;
}

```

Plant-output module

Plant output variables (*y[j]*)

y[j].y_output: plant output
y[j].noise: state for additive measurement noise to output of plant
y[j].yy: plant output as measured
y[j].setpoint: setpoint of output for main controller
y_dead(j,ky): shift vector to provide for dead time of plant measurement

Plant-output parameters (*y[j].p*)

y[j].knob: multiplier "knob" (0 to 100 %) for all plant-output non-ideal parameters
y[j].p.base0: initial base level for *y* used at initialisation
y[j].p.base: drifting base level for *y*
y[j].p.drift_base: drift rate of base level for *y*
y[j].p.min: minimum *y*
y[j].p.max: maximum *y*
y[j].p.soft0.min: initial minimum achievable *y* used at initialisation
y[j].p.soft.min: drifting minimum achievable *y*
y[j].p.drift.min: drift rate of minimum achievable *y* (*y[j].p.soft.min*)
y[j].p.soft0.max: initial maximum achievable *y* used at initialisation
y[j].p.soft.max: drifting maximum achievable *y*
y[j].p.drift.max: drift rate of maximum achievable *y* (*y[j].p.soft.max*)
y[j].p_tf.tau: steady-state gain for output *y*
y[j].p_tf.tau: 1st-order time constant for output *y*
y[j].p_tf.tdead: dead-time for output *y*
y[j].p_noise.gain: magnitude of noise, to update output *y*
y[j].p_noise.tau: time constant for filter of noise, to update output *y*
y[j].p_parabola: 0 for linear, otherwise $y(j)=p_y_base(j) - \Sigma(x(l,j)-p_x_base(i,j))^2$

Calculated plant-output parameters (*y[i].c_*)


```

y[j].c_exp:      0 if y[j].p_tf.tau = 0, else exp(-dt / y[j].p_tf.tau)
y[j].c_exp_noise: 0 if y[j].p_tau_noise = 0, else exp(-dt / y[j].p_tau_noise)
y[j].c_ky:      Minimum(Integer(1 + y[j].p_tf.tdead / dt), mky)

```

Initialisation of plant-output variables (at $t=0$)

```

y[j].p.base = y[j].p.base0
y[j].y_output = y[j].p.base
y_dead(j,ky) = y[j].p.base, for all ky
y[j].noise = 0
y[j].yy = y[j].p.base

```

Plant-output dynamics (to be executed every dt seconds)

```

//If not generating the model for the MPC-controller
if (!model) {
    //Update soft maximum and restrict to hard limits
    y->p.soft.max += (y->p.drift.max * sim_param->dt) * y->p.knob;
    if (y->p.soft.max > y->p.max) y->p.soft.max = y->p.max;
    if (y->p.soft.max < y->p.min) y->p.soft.max = y->p.min;

    //Update soft minimum and restrict to between hard minimum and soft maximum
    y->p.soft.min += (y->p.drift.min * sim_param->dt) * y->p.knob;
    if (y->p.soft.min < y->p.min) y->p.soft.min = y->p.min;
    if (y->p.soft.min > y->p.soft.max) y->p.soft.min = y->p.soft.max;

    //Update base level drift and restrict to soft limits
    y->p.base += (y->p.drift_base * sim_param->dt) * y->p.knob;
    if (y->p.base < y->p.soft.min) y->p.base = y->p.soft.min;
    if (y->p.base > y->p.soft.max) y->p.base = y->p.soft.max;
}

//Extract plant outputs
if (y->p_parabola == 0) { //If not quadratic function
    temp = y->p.base;
    for (long i=0; i<tfdim; i++)
        temp += (x_dead[i*sim_param->max_kx*tfdim+x[i*tfdim].c_kx-1] - x[i*tfdim].p.base);
}
else { //If quadratic function
    temp = 0;
    for (long i=0; i<tfdim; i++)
        temp += (x_dead[i*sim_param->max_kx*tfdim+x[i*tfdim].c_kx-1]-x[i*tfdim].p.base);
    temp = y->p.base - pow(temp, 2);
}
y->y_output = y->y_output * y->c_exp + temp * (1 - y->c_exp);

//Store in shift vector (dead time)
for (long ky=(y->c_ky-1); ky>0; ky--) y_dead[ky] = y_dead[ky-1];
y_dead[0] = y->y_output;

//Update measurement noise
if (!model)
    y->noise = y->noise * y->c_exp_noise + ((double(rand())/RAND_MAX)*2 - 1) *
    y->p_noise.gain*(1 - y->c_exp_noise);

//Produce plant measurement with dead-time and noise

```

```
y->yy = y_dead[y->c_ky-1] + y->noise;
```

```
//Calculates performance index
```

```
if (y->use_mpc) {
    if (mpc->perform == 1)
        mpc->p_index += (pow( (y->yy - y->setpoint), 2) * y->tune_w);
    if (mpc->perform == 2)
        mpc->p_index += (fabs(y->yy - y->setpoint) * y->tune_w);
    if (mpc->perform == 3)
        mpc->p_index += (fabs(y->yy - y->setpoint) * (time*sim_param->dt) * y->tune_w);
}
```

Simulation Parameters (*sim_param*)

<i>sim_param.simtime</i> :	Time of simulation
<i>sim_param.dt</i> :	Time interval of simulation of plant models
<i>sim_param.steptime[i]</i> :	Time when step change is applied
<i>sim_param.finalSP[i]</i> :	Size of step change
<i>sim_param.initialSP[i]</i> :	Initial setpoint
<i>sim_param.recordstep</i> :	Size of interval (time) for recording the output
<i>sim_param.max_ku</i> :	Maximum input dead time steps
<i>sim_param.max_kx</i> :	Maximum state dead time steps
<i>sim_param.max_ky</i> :	Maximum output dead time steps
<i>sim_param.no_inputs</i> :	Number of inputs
<i>sim_param.no_outputs</i> :	Number of outputs
<i>sim_param.accuracy</i> :	Accuracy of simulation for steady state
<i>sim_param.dt_power</i> :	Time interval for power controller
<i>sim_param.load_change</i> :	Load change for power controller at each control interval

MPC Parameters (*mpc*)

<i>mpc.TT[j]</i> :	Model horizon(T) for each output
<i>mpc.UU[i]</i> :	Control Horizon(U) for each input
<i>mpc.VV[j]</i> :	Prediction horizon(V) for each output
<i>mpc.delt</i> :	Time interval of MPC controller
<i>mpc.f1[j]</i> :	Weighting factors for controlled variables (multiple of identity matrix)
<i>mpc.f2[i]</i> :	Weighting for manipulated variables (multiple of identity matrix)
<i>mpc.udim</i> :	Dimension for inputs of MPC controller
<i>mpc.ydim</i> :	Dimension for outputs of MPC controller
<i>mpc.steptime[j]</i> :	Time when step change is applied
<i>mpc.finalSP[j]</i> :	Final value of setpoint
<i>mpc.initialSP[j]</i> :	Initial value of setpoint

<i>mpc.qpdim</i> :	Number of controlled variables to be constrained when QP is used
<i>mpc.qp[j].constraint</i> :	Specifies if output should be constrained (when not controlled)
<i>mpc.qp[j].T</i> :	Model horizon for output that should be constrained
<i>mpc.qp[j].V</i> :	Control horizon for output that should be constrained
<i>mpc.qp[j].Vb</i> :	Beginning of constraint window
<i>mpc.qp[j].Ve</i> :	End of constraint window

PI-Controller Parameters (*PI*)

PI.gain: Controller gain
PI.ti: Integral action (ti) of PI-controller
PI.delt: Time interval for PI-control action
PI.output: Output number to be controlled with current input

Chapter 11: Appendix D - Transfer Function Matrix

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