Optimal Dimensional Synthesis of Planar Parallel Manipulators with respect to Workspaces

by

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Summary

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Parallel manipulators have attracted increasing interest from researchers over the past couple of decades. These manipulators consist of a moving platform, connected to a fixed base by means of a number of separate kinematic chains, placed in parallel. Due to this particular architecture, parallel manipulators possess a number of advantages over traditional serial manipulators. Some of the disadvantages of parallel manipulators, however, are their limited workspaces and nonlinear behavior throughout their workspaces. As a result, development of design methodologies for such manipulators is an important issue in order to ensure performance to their full potential. The methodologies developed in this study are based on the use of numerical optimization techniques.

The development of appropriate design methodologies in this study required three separate issues to be addressed. The first of these was the development, testing and selection of *numerical optimization* algorithms suitable for the solution of the practical optimization problems encountered. Two optimization algorithms, the spherical quadratic steepest descent (SQSD) algorithm[1] for unconstrained problems, and the Dynamic-Q algorithm[2] for constrained problems were developed and tested. These methods compare well with conjugate gradient, and sequential quadratic programming methods respectively, exhibiting robustness and efficiency when applied to a number of test problems.

The second topic addressed is the important issue of the determination of manipulator *workspaces*. The existing chord method for workspace determination is refined, and applied for the first time to the determination of new types of manipulator workspaces for a planar three-degree-of-freedom (3-dof) manipulator. The chord method is also modified for the determination of planar *tendon-driven* parallel manipulator workspaces. A new and efficient method for determining tension distributions in over-constrained tendon-driven manipulators is proposed. The chord method is easily applied to the determination of manipulator workspaces, and determines them accurately and efficiently.

The final issue addressed is that of *dimensional synthesis* of manipulators for prescribed and desired workspaces. Various specific methodologies are investigated and applied to a 2-dof parallel manipulator[3]. The most promising

[1]J.A. Snyman and A.M. Hay, The spherical quadratic steepest descent method for unconstrained minimization with no explicit line searches. *Computers and Mathematics with Applications*, 42:169–178, 2001.

[2]J.A. Snyman and A.M. Hay, The Dynamic-Q optimization method: An alternative to SQP? *Computers and Mathematics with Applications*, 44:1589–1598, 2002.

[3]A.M. Hay and J.A. Snyman, Methodologies for the optimal design of parallel manipulators. Accepted for publication in the *International Journal for Numerical Methods in Engineering*, 2003 (in press).

methodology is then used to optimize a 3-dof planar parallel manipulator[4,5], using various strategies for dealing with the extra angular orientational degree of freedom of the moving platform. An alternative approach is used in optimizing a planar tendon-driven parallel manipulator[6]. The numerical optimization algorithm used in all cases is the Dynamic-Q method, which performs efficiently and robustly in determining optimal designs, even when numerical noise is present in the problem. It is believed that the new methodologies presented provide efficient, practical and easily generalizable numerical alternatives to existing methods for the dimensional synthesis of parallel manipulators.

Keyterms: optimization algorithm, parallel manipulator, optimal design, workspace analysis, mechanism synthesis.

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[4] A.M. Hay and J.A. Snyman, The optimal synthesis of parallel manipulators for desired workspaces. In J. Lenarčič and F. Thomas, editors,*Advances in Robot Kinematics*, 337–346, Caldes de Malavella, Spain, June 2002. Kluwer Academic Publishers.

[5] A.M. Hay and J.A. Snyman, The synthesis of parallel manipulators for optimal desired workspaces with respect to the condition number. CD-ROM Proceedings of *ASME 2002 Design Engineering Technical Conferences*, Paper number DETC2002/MECH-34306, Montreal, Canada, October 2002.

[6] A.M. Hay and J.A. Snyman, Analysis and optimization tools for a reconfigurable tendon-driven manipulator. CD-ROM Proceedings of *CIRP 2nd International Conference on Reconfigurable Manufacturing*, Ann Arbor, MI, August 2003.

Samevatting

OPTIMALE DIMENSIONELE SINTESE VAN IN-VLAK PARALLEL-MANIPULEERDERS MET BETREKKING TOT WERKRUIMTES

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Gedurende die afgelope paar dekades is toenemende belangstelling deur navorsers in parallel-manipuleerders getoon. Hierdie manipuleerders bestaan uit 'n bewegende platform, gekoppel aan 'n vaste basis deur middel van 'n aantal afsonderlike kinematiese kettings, wat in parallel met mekaar geplaas is. As gevolg van hul besondere argitektuur, het parallel-manipuleerders 'n aantal voordele bo tradisionele serie-manipuleerders. Sekere nadele van parallel-manipuleerders is egter, hul beperkte werkruimtes en nie-lineêere gedrag binne die werkruimtes. Gevolglik, is die ontwikkeling van ontwerp-metodologië vir sulke manipuleerders van uiters belang, om te verseker dat hul tot volle potensiaal funksioneer. Die metodologië wat in hierdie studie ontwikkel is, is gebaseer op die gebruik van numeriese optimeringstegnieke.

Die ontwikkeling van gepaste ontwerp-metodologieë in hierdie studie, vereis dat drie verskillende sake aangespreek word. Die eerste van hierdie is die ontwikkeling, toetsing en seleksie van numeriese optimerings-algoritmes wat geskik is vir die oplos van die praktiese optimeringsprobleme wat in dié studie voorkom. Twee optimerings-algoritmes, die sferiese kwadratiese steilste daling (SQSD) algoritme[1] vir onbegrensde probleme, en die Dynamic-Q algoritme[2] vir begrensde probleme, is ontwikkel en getoets. Hierdie metodes vergelyk onderskeidelik goed met die toegevoegde gradiënt en agtereenvolgende kwadratiese programmerings (SQP) metodes.

Die tweede belangrike onderwerp wat aangespreek word is die bepaling van manipuleerder werkruimtes. Die bestaande koord-metode vir werkruimte-bepaling is verfyn, en vir die eerste keer toegepas in die bepaling van nuwe tipes werkruimtes van 'n in-vlak manipuleerder met 3-vryheidsgrade. Die koord-metode is ook aangepas vir die bepaling van die werkruimtes van 'n in-vlak tendon-aangedrewe parallel-manipuleerder. Die toepassing van die koord-metode lei met gemak tot die doeltreffende en akkurate bepaling van hierdie werkruimtes.

Die finale saak wat bestudeer word is die dimensionele sintese van manipuleerders vir voorgeskrewe en verlangde werkruimtes. Verskeie spesifieke metodologieë word ondersoek en toegepas op 'n 2-vryheidsgrade parallel-manipuleerder[3]. Vervolgens is die mees belowende metodologie gebruik in die op-

[1] J.A. Snyman and A.M. Hay, The spherical quadratic steepest descent method for unconstrained minimization with no explicit line searches. *Computers and Mathematics with Applications*, 42:169–178, 2001.

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timering van 'n 3-vryheidsgrade in-vlak parallel-manipuleerder[4,5], waar verskeie strategieë gebruik is om die addisionele hoek-orientasie-vryheidsgraad te hanteer. 'n Alternatiewe benadering is gevolg in die optimering van die tendon-aangedrewe in-vlak parallel-manipuleerder[6]. In al die gevalle is die Dynamic-Q numeriese optimerings-algoritme gebruik. Dié metode is doeltreffend en betroubaar, selfs wanneer numeriese geraas in die probleem teenwoordig is. Die vertroue is dat die nuwe metodologië wat hier aangebied word, doeltreffende, praktiese en maklik veralgemeende numeriese alternatiewe tot bestaande metodes vir dimensionele sintese van parallel-manipuleerders, verteenwoordig.

Sleutelterme: optimerings-algoritme, parallel-manipuleerder, optimale ontwerp, werkruimte-analise, meganisme-sintese

[4]A.M. Hay and J.A. Snyman, The optimal synthesis of parallel manipulators for desired workspaces. In J. Lenarčič and F. Thomas, editors, *Advances in Robot Kinematics*, 337–346, Caldes de Malavella, Spain, June 2002. Kluwer Academic Publishers.

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[6]A.M. Hay and J.A. Snyman, Analysis and optimization tools for a reconfigurable tendon-driven manipulator. CD-ROM Proceedings of *CIRP 2nd International Conference on Reconfigurable Manufacturing*, Ann Arbor, MI, August 2003.

For the Sun and the Rain...

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Nomenclature

In this work vectors will be denoted in bold font. The following notation will be used:

$\Re^n$	n-dimensional Euclidean (real) space
x, y, z	Cartesian reference frame axes
$\mathbf{q}$	Manipulator generalized coordinates
$\mathbf{u}$	Manipulator output coordinates
$\mathbf{v}$	Manipulator input coordinates
$\mathbf{w}$	Manipulator intermediate coordinates
ϕ_P	Moving platform orientation
$W^C[\phi^{\text{fix}}]$	Constant [angular] orientation workspace
W^M	Maximal workspace
$W^D[\phi^{\text{min}}, \phi^{\text{max}}]$	Dextrous workspace
$W^O[\mathbf{u}^{\text{fix}}]$	Orientation workspace
∂W	Workspace boundary
$\mathbf{x}$	Vector of design variables
f	Objective function
$\mathbf{g}$	Vector of inequality constraints
$\mathbf{h}$	Vector of equality constraints
ε_x	Convergence tolerance on step size
ε_g	Convergence tolerance on gradient value
ε_f	Convergence tolerance on function value

ρ	Move limit for SQSD and Dynamic-Q algorithms
Γ	Finite difference interval
d	Chord length for chord workspace determination method
$\mathbf{d}$	Vector of manipulator design variables
$\mathbf{b}^i$	Point on workspace boundary
$\mathbf{B}^j$	Bifurcation point on workspace boundary
W_p	Prescribed workspace
W_c	Calculated workspace
δW_p	The part of workspace W_p not intersecting W_c
δW_c	The part of workspace W_c not intersecting W_p
$\mathbf{O}'$	Position of local coordinate system $x' - y'$
(β_p, r_p)	Polar coordinate description of prescribed workspace
(β_c, r_c)	Polar coordinate description of calculated workspace
l_i	Length of manipulator actuator (leg) i
$\mathbf{J}$	Manipulator Jacobian matrix
κ	Condition number of the Jacobian matrix
t_i	Tension in tendon i
$\mathbf{S}$	Manipulator structure matrix
$\mathbf{T}$	Transformation matrix from local to global coordinate systems
$\mathbf{f}^{Ci}$	Force transferred to the moving platform by tendon i
$\boldsymbol{\ell}^i$	Displacement vector along tendon i
$\mathbf{f}^P$	External load applied to the moving platform
τ^P	External torque applied to the moving platform