3. Data and analysis

In every branch of knowledge the progress is proportional to the amount of facts on which to build, and therefore to the facility of obtaining data. James Maxwell

3.1. Introduction

This chapter contains a description of the data sources and management involved in SLR data collection (used in this research study). Firstly, the global network of SLR tracking stations is discussed mainly to highlight some of the factors affecting the accuracy of SLR data. The data collected from these tracking stations is discussed focusing on the main steps in data management (i.e. formation of SLR normal points) required to ensure that the collected data is adequate and in good format. In this chapter I also describe the general methods used throughout this project. For the sake of conciseness the analysis methods are described here and not repeated in each presented chapter. However, other chapters may contain methods that are chapter specific. Furthermore, the chapter explains some aspects (e.g., parameterizations) of the software used to analyse the SLR data.

3.2. Data

As already explained in Chapter 2, the main observable in SLR is the distance or range (round-trip, station to satellite and back). Together with this TOF, are other auxiliary (derived parameters) such as the correction information due to atmospheric effects, which are to be applied to the data. Raw SLR data are formatted at the remote station before transmission to operational data centres where the data are translated into the appropriate format. There are about six ILRS accredited analysis centres. Included are the NERC Space Geodesy Facility (NSGF), Joint Centre for Earth System Technology/Goddard Space Flight Centre (JCET/GSFC), Greenbelt, Maryland, USA, GeoForschungs Zentrum (GFZ) German Research Centre for Geosciences, Germany, Centro de Geodasia Spaziale (CGS), Italy, etc. Currently, SLR data are available in two forms: original observations (full-rate data) and compressed range observations generated from the original observations (these are called the normal points) (Noll, 2010).

Full-rate data sets include all the valid satellite returns and are often larger in volume. These data are not routinely provided by all stations in the laser tracking network. The data are useful for both engineering evaluation and scientific applications (e.g., studying the performance of retro-reflectors, discerning satellite signatures, understanding the statistical nature of satellite returns, calibration of satellite targets, validating system quality of laser station co-locations, etc.) (Noll, 2010). Full-rate data which range in size from 10 to 100 kbytes are transmitted to the Crustal Dynamics Data Information System (CDDIS) in files containing all data from a specific tracking station and satellite on a particular day. The CDDIS then combines the transmitted daily files from all tracking stations into a monthly satellite specific file. These files are then made freely available in the ILRS full-rate format in subdirectories by satellite and year.

Satellite Laser Ranging normal points constitute the primary ILRS data product (these products are archived at e.g., http://ilrs.gsfc.nasa.gov). The normal points are compressed data, i.e. the compression involves sampling over time based on certain minimum number of data points within the sampling interval. The length of this normal point interval is primarily dependent upon the satellite altitude; lower orbiting satellites have a shorter normal point interval than high orbit satellites. Normal points are computed via two steps. Firstly, the observed range with the computed reference ranges and thereafter a series of predicted residuals is generated. Suppose that d_0 represents the observed ranges and d_p are the computed reference ranges, the generated observation residuals can be described by Equation (64)

$$d_r = d_0 - d_p. \tag{64}$$

Previous studies have predicted that when the relative data density drops to a very low rate, it is plausible that the “time-isolated” measurements are highly dominated by noise or outliers (Seago, 1998). These outliers are often removed by using a suitable range or isolation window. To restrain the formation of time-isolated outliers into bad single-point normal points, ILRS analysis data centres often implement algorithms also known as a leverage point pre-filter that initially flags heavily leveraged points as noise (Seago, 1998). A datum is considered leveraged if it is the only observation within a specified time period (isolation window). The isolation window is arbitrarily chosen to be equal to either the recommended integration step size for a specific dynamic model integrator, or twice the normal point bin size. These values are passed via a satellite data file containing other satellite specific parameters. The isolation windows for various satellites as estimated by Seago (1998) are presented in Table 3.

Table 3. Isolation intervals for leverage filtering

Satellite	Normal point bin (sec)	Isolation window (sec)
GFZ-1	5	60
ERS 1/2	15	60
Starlette	30	90
Ajisai	30	90
Stella	30	90
LAGEOS 1/2	120	240
Etalon 1/2	300	600
GLONASS	300	600

In order to remove systematic trends in the observation residuals, orbital parameters are often solved by fitting a trend function, $f\left(d_r\right)$, to the residuals d_r. The fit residuals which analyses any remaining outliers can be iteratively computed as given in Equation (65),

$$f_r = d_r - f\left(d_r\right). \quad (65)$$

In the second step of formation of normal points the resulting observed trajectory is segmented into fixed intervals or bins starting from 0^h UTC. The proposed interval sizes for various satellites are listed in Table 4.

Table 4. Examples of bin sizes for specific satellites

Satellite	Bin size (seconds)
GPS, GLONASS	300
LAGEOS 1/2	120
Starlette, Stella	30
ERS 1/2	15
GRACE	5

In each bin i, the mean value $\overline{f}_{r_i}$ of all deviations is computed and added to the trend function at the centre of the interval. The normal points representing all single observations of a given interval may be computed according to Equation (66),

$$NP_i = O_i - f_{r_i} + \overline{f}_{r_i}, \quad (66)$$

where O_i is an observation located in bin i and f_{r_i} represents fit residuals in the same bin. The ILRS normal point file format exists as uncompressed ASCII files containing a header record followed by a data record. A header record contains satellite and station designators, general station configuration information and normal point calculation parameters. The data record contains laser fire times in units of 0.1 μs, system delay in picoseconds, bin RMS, meteorological data (e.g., pressure, temperature and relative humidity) and number of ranges used in the normal point formation. Normal points may be computed either at the on-side tracking stations or at ILRS data analysis centres. The ILRS operational data centres forward normal point data to the CDDIS in hourly and daily files by satellite with a typical delay of less than one day following the observations. The CDDIS updates the received files containing all normal point data on a daily basis. Daily files contain all normal point data for each satellite received at the ILRS operational data centres in the previous 24 hour period. Thus, these daily files often contain data spanning several operating days. The monthly files contain all normal point data for each satellite during the month. Daily and monthly normal point data are available from the CDDIS in subdirectories by satellite and year and can be freely accessed at http://cddis.nasa.gov/.

The normal point data analyzed in this study were selected from ILRS tracking stations (Pearlman *et al.* 2002). Tracking stations were selected in order to ensure good global distribution. As mentioned earlier, global distribution of SLR stations is dense in US, Europe, and Australia. The Southern Hemisphere suffers from a lack of SLR tracking stations. This is one major disadvantage of SLR compared to other geodetic techniques such as VLBI, GPS and DORIS. In Africa there is only one active SLR tracking station situated in South Africa at HartRAO, joining other geodetic instruments (e.g., VLBI, GPS, and DORIS). The selection of the SLR stations is based on the number of daily normal points contributed by each station. Note that the daily normal points are not generally contributed by all the selected SLR stations rather the actual normal points per day are contributed by fewer stations. Stations with the highest number of points were selected.

3.3. Satellites

The data analysed was collected from LAGEOS 1 and 2 satellite missions. LAGEOS 1 was launched in 1976 by the American Space Agency, NASA in a near circular orbit. This satellite was later joined by a sister satellite, LAGEOS 2, launched during 1992 in a joint collaboration between United States and Italy. Both satellites have a high mass-to-area ratio of 1450 kg/m and orbit the Earth at an altitude of about 6000 km above Earth's surface. LAGEOS satellites carry a total of 426 corner cube reflectors inset in the outer aluminium shell surrounding a solid cylindrical brass core. These retro-reflectors are used to reflect laser beams which are reflected back to the ground stations. Each reflector is mounted with its front face perpendicular to the radius vector at the mounting point (Otsubo *et al.*, 2004).

LAGEOS retro-reflectors are distributed in rings and equally spaced along lines of latitude (Fitzmaurice, 1977). The reflectors are arranged on the surface of the sphere in rows that form small circles parallel to the satellite's "equator" (circle perpendicular to the axis of rotation of the satellite) (Otsubo *et al.* 2004). These reflectors are symmetrically arranged in rows, each hemisphere (designated "N" and "S" hemispheres) having about 10 rows. The rows contain different numbers of reflectors, according to their "latitude" namely 32, 32, 31, 31, 27, 23, 18, 12, 6, 1, giving a total of 213 in each hemisphere (Otsubo *et al.*, 2004.). A total of 422 of the LAGEOS reflectors are made of fused silica glass. The remaining four are made of germanium and they are used to obtain measurements in the infrared region of the spectrum. More properties of the two satellites are summarized in Table 5.

Table 5. Mission parameters of LAGEOS 1 and 2 satellites.

Properties	LAGEOS 1	LAGEOS 2
COSPAR ID	7603901	9207002
Launch date	May 4 1976	October 22 1992
Reflectors	426 corner cubes	426 corner cubes
Orbit	Circular	Circular
Orbit inclination	109.84^{0}	52.64^{0}
Eccentricity	0.0045	0.0135
Perigee height	5860 km	5620 km
Period	225 minutes	223 minutes
Weight	406.965 kg	405.38 kg

Figure 11 portrays the retro-reflectors of LAGEOS satellites. The LAGEOS series were designed to provide an orbiting benchmark for geodynamical studies of the Earth (http://ilrs.gsfc.nasa.gov). These include studies of Earth's gravity field, determination of EOPs and investigation of various geophysical phenomena such as tectonic plates, polar motion and tides (Smith *et al.* 1990; Sengoku, 1998; Bouille *et al.*, 2000). Due to their high mass-to-area ratio and attitude the LAGEOS orbits are less sensitive to Earth's gravity field and to non-gravitational forces. Thus they provide precise measurements of the satellite's position with respect to Earth. In addition the high altitude of the two satellites causes them to be sensitive up to degree 20 of the underlying gravity field model.

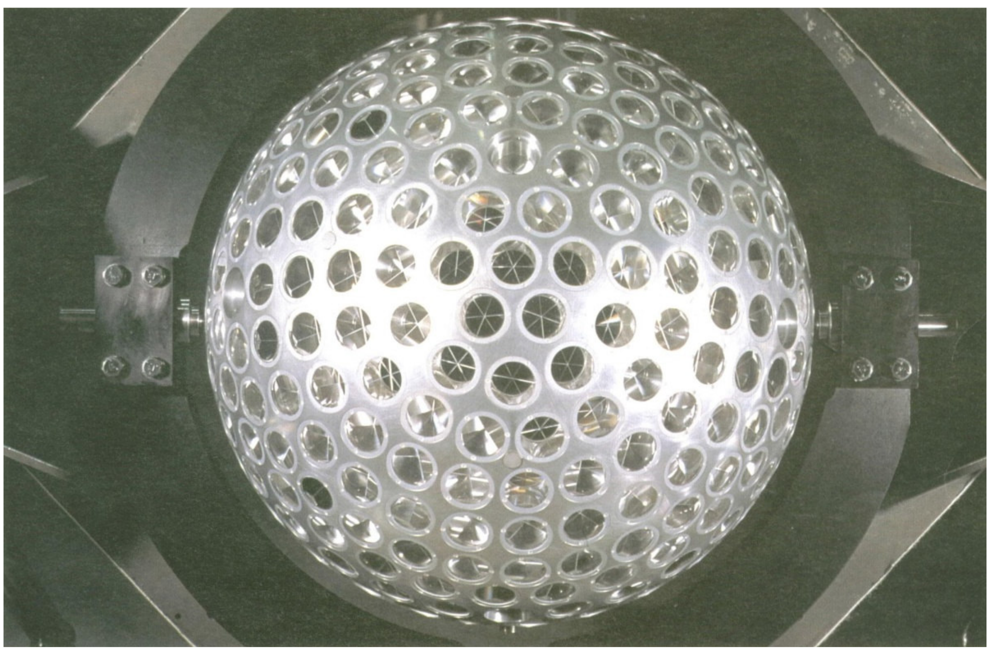

Figure 11. Retro-reflectors on LAGEOS satellite. http://ilrs.gsfc.nasa.gov.

Figure 12 illustrates the distribution of the normal point data analysed in one of the studies presented in this thesis. This time series comprises three years of SLR data, i.e. spanning December 2005 to December 2008. Figure 12 serves to illustrate a general distribution of normal points. The correct specifications on the data used are mentioned in relevant chapters. Typically there are between 200 and 400 normal points per day (for a total of ~15 stations) over a 24 hour satellite arc. Sometimes there are less, especially over international holiday periods (e.g. Christmas) and statistically there are less data available over weekend periods. This is due to the fact that not all SLR stations operate in full 24x7 mode. A percentage (typically 10%-20%) of data can be filtered out statistically as outliers, depending on analysis strategy.

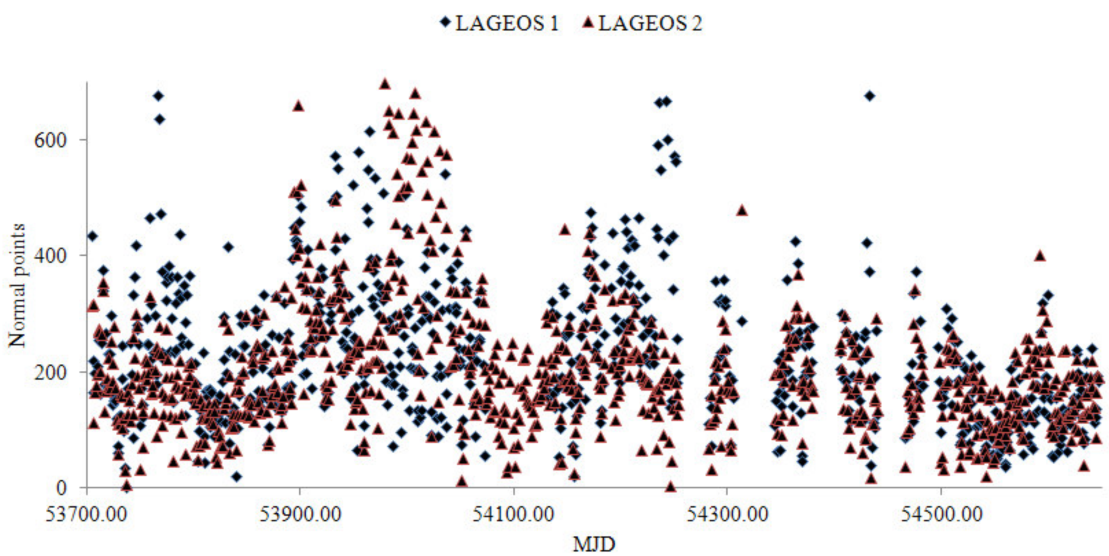

Figure 12. Distribution of normal points analysed.

3.4. SLR analysis software

The SLR data throughout this project was analysed using the SDAS package developed at HartRAO (Combrinck and Suberlak, 2007) mainly for POD and estimation of geodetic parameters. This software comprises the following main elements:

- Generation of initial setup files for the computation of SLR stations positions and their velocity solutions,
- Satellite orbit and parameter adjustment module for orbit improvement,
- Parameter estimation module which introduces constraints in the form of adjusting the outlier rejection term with a predefined weight.

The output solution includes the following:

- O-C RMS residuals, the mean and the standard deviation (SD) of the O-C residuals,
- Components of the stations' positions and their velocities,
- Empirical coefficients for atmospheric drag, solar radiation pressure, Earth's elasticity, Earth's albedo, once-per-cycle per revolution empirical parameters (9 coefficients) and coefficients of un-modelled components,

- Time and range bias values,
- Gravitational spherical harmonic coefficients (solve-for parameters) such as J_2 to J_5, S_{21} and C_{21}.

3.4.1. Software parameterization

The SDAS package utilises the station and satellite coordinates provided by the IERS and ILRS in the ITRF. These satellites coordinate a-priori values can be selected from a menu to be at a specific epoch, such as ITRF2000 and SLRF2005. The satellite coordinates incorporated in ITRF2000 were integrated by using primary core stations observed by VLBI, LLR, SLR, GPS and DORIS and were also densified by regional GPS networks in Alaska, Antarctica, Asia, Europe, North and South America and the Pacific. On the other hand, coordinates in ITRF2005 were constructed by using long-term input data in the form of time series of station positions, velocities and EOPs. These input solutions are provided as a weekly sampling by the IAG International Services of satellite techniques: the International GNSS Service (IGS) (Dow *et al.*, 2005), the ILRS (Pearlman *et al.*, 2002) and the International DORIS Service (IDS), (Tavernier *et al.* 2006), and in a daily basis by the International VLBI Service (IVS) (Schlueter *et al.*, 2002).

The SLRF2005 reference frame is a dedicated reference frame derived from a combination of ITRF2000, rescaled ITRF2005 and a global SLR solution based on data spanning 1993 to 2007 with new SLR stations included (for further details on combination strategy used to derive SLRF2005, see for example, http://ilrs.gsfc.nasa.gov/working_groups/awg/SLRF2005.html and Luceri and Bianco, 2008). The satellite a-priori coordinates are provided by the ILRS in the consolidated prediction format (CPF). During data processing, both satellite and SLR station position vectors are transformed to a non-rotating (inertial) frame, the International Celestial Reference Frame (ICRF). The ICRF is a geocentric inertial coordinate system, defined by the precise J2000.0 equatorial coordinates of extragalactic radio sources determined from VLBI measurements (Johnston and de Vegt, 1999). The J2000 standard reference epoch is given by 01-Jan-2000 12:00:00 ephemeris time. This is the beginning of the Julian year 2000, and corresponds to a Julian date of 2451545.0. The fundamental inertial frame definition uses the Earth as the reference body, its mean equator as the reference plane, the vernal equinox of its mean orbit as the reference direction, and J2000 as the reference epoch. Hence, this frame is called the Earth Mean Equator and Equinox of Epoch J2000 (also known as EME2000) (Lyons and Vaughn, 1999).

The Jet Propulsion Laboratory (JPL) DE-405 planetary ephemeris (Standish, 1998), which is based on the ICRF inertial coordinate system has been utilised to determine exact vectors and distances to solar system objects and to account for the gravitational perturbations on the satellite orbit by the Sun, Moon and planets. These coordinates have been converted from barycentric inertial to geocentric inertial. In ICRF inertial coordinate system algorithms are designed to maintain three directions of orthogonal axes:

- The Z axis (Z_{J2000}) is the unit normal to the Earth's mean equator of epoch J2000
- The X axis (X_{J2000}) is chosen to be the vernal equinox, the node with the Earth's mean orbit plane where the orbit ascends through the equator plane for the J2000.
- The Y axis (Y_{J2000}) is chosen to complete the right-handed orthogonal coordinate system

The vector axes are shown in Figure 13 and are used as the basis for expressing the positions and velocities of satellites in space.

The SLR tracking station coordinates which are normally expressed in the Earth-fixed, geocentric, rotating systems are transformed to the ICRF reference frame by taking into account precession and nutation of the Earth, its polar motion and the UT1 transformation. The relation between the ICRF and the ITRF may be described by Equation (67),

$$\vec{X}_{ICRF} = PNTXY.\vec{X}_{ITRF}. \tag{67}$$

In this equation P is the precession matrix, N is the nutation matrix, T is a matrix expressing the rotation by true sidereal time S and XY are the transformation matrixes from the terrestrial frame to the frame connected to the instantaneous ephemeris pole, $\vec{X}_{ICRF}$ and $\vec{X}_{ITRF}$ are the vectors relating to ICRF and ITRF axes respectively. The data obtained from the IERS and Bulletin B (actually file eopc04_62.now, consistent with ITRF2005) were utilised, with all values interpolated via polynomial fits to the epoch of SLR measurement. Bulletin B of the IERS provides current values of the EOPs in the IERS Reference System. While Bulletin A gives an advanced solution of EOPs as well as predictions updated on a weekly basis, the standard solution is given on weekly basis in Bulletin B. Details of file eopc04_62.now can be obtained at http://hpiers.obspm.fr/iers/eop/eopc04_05/ in the document C04_05.guide. The EOP values provide an exact link between the ICRF and the ITRF.

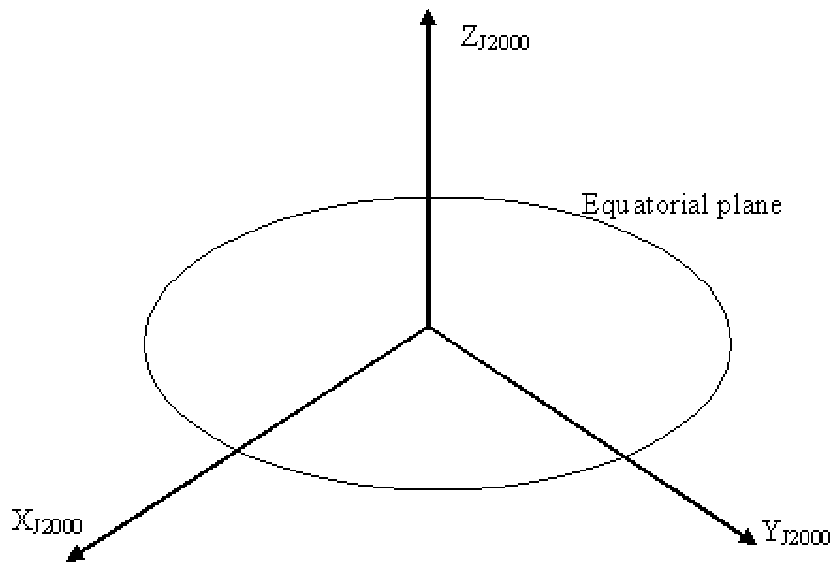

Figure 13. The J2000 inertial reference frame.

During data processing the software conforms to the IERS96 conventions as far as precession and nutation of the Earth's polar motion is concerned. For example, the model of the Earth's precession and nutation specifies the 1976 International Astronomical Union (IAU) precession (Lieske *et al.*, 1977; Lieske, 1979) and the 1980 IAU nutation formula (Seidelmann, 1982). The software utilises the UT1-UTC values as provided by IERS Bulletin B database (http://hpiers.obspm.fr/eoppc/bul/bulb_new) or eopc04_62.now.

The SLR tracking stations' positions are affected by fluctuations in the position of the axis of the Earth's crust. Such fluctuations are due to horizontal and vertical displacements resulting from the solid Earth tidal perturbations as well as from large scale motions of Earth's lithosphere (tectonic plate velocity). Displacements of the tectonic plate motions can be accounted for by calculating the plate velocity using ITRF station velocities and adjusting the stations' positions in the ITRF to the epoch of the SLR measurements (see for example Combrinck and Suberlak, 2007). Tidal forces (solid and ocean) arise from changes to the Earth's geo-potential induced by variations in the mass distribution of Earth. Contributions to the solid Earth tide force arise from the gravitational effects of the Sun and Moon, which deform the shape of the Earth. Other effects result from ocean loading on the crust and wobbles of the mantle and core region. Station displacements due to solid Earth tides were accounted for according to the model reported in Petrov (2005). Ocean loading is modelled by the ocean tides, whence our analysis utilised a model derived by Scherneck (1991). Atmospheric and pole tides were accounted for in accordance with IERS conventions 2003 (McCarthy and Petit, 2003). In the SLR analysis reported in this thesis, the transformation of the COM corrections of satellites was not considered although it is a selectable option in the software. In general a total of 48 parameters were adjusted during SLR data processing. The main standard parameters are the position and velocity of the satellite, solar pressure coefficient (set at 1.13), satellite drag coefficient (set at 4.9) and Earth albedo coefficient (set at 0.34). All the implemented models were aimed at achieving optimal solutions thereby minimising the O-C residuals (this is the main parameter in POD). In summary, the parameters considered during data processing are listed in Table 6.

Table 6. Constants, reference frames and empirical models used in the SLR data processing.

N_{max}	20×20
Inertial reference frame	J2000
Pole-tide correction (station position)	IERS 2003
Relativity (space-time curvature)	IERS 2003
Earth–tide correction (station position)	Petrov 2005
Earth-tide acceleration of satellite	(Rizos and Stolz, 1985)
Ocean loading correction (station position)	Scherneck, 1991
Atmospheric loading	Special Buro for Loading, IERS
Tectonic plate model	ITRF2000 velocity field
Earth orientation	a-priori Earth orientation parameters and UTC-UT1 values as per IERS extrapolated to observation epoch
O-C outlier rejection	Selectable
Satellite centre-of-mass offset (LAGEOS)	251 mm, ILRS standard value (Otsubo and Appleby, 2003)

3.5. Data analysis

A schematic representation of data analysis followed throughout this project is given in Figure 14. The SDAS package utilises a dynamical data analysis procedure (this is the dynamic orbit determination discussed in Chapter 2) where the satellite's equations of motion i.e. gravitational and non-gravitational forces are taken into account. Using this method is advantageous since it does not only determine the satellite orbits, but also improves or estimates the force models such as the Earth's gravity field model. Also given that the force model can accurately describe the movement of an orbiting satellite, the least-squares solution method can be used to reduce the orbit error caused by measurement noise or errors. However, in this case the orbit accuracy is highly dependent on force models used for dynamic orbit determination. The analysis procedure involves numerical integration of the LAGEOS equations of motion from nominal initial conditions within a given force field and reference frames as listed in Table 6. A linear system of normal equations is set up and its solution is computed. The software computes derivatives of the observations with respect to the "solve-for parameters" of interest by integrating the equation of motion and solving for the unknown parameters using a least-squares adjustment. The procedure is iterated (e.g. 20 times) until convergence is reached, presumably on the last selected iteration number. During the data processing, the orbital arc integration length[8] was fixed at 24 hours. Although short arcs are mostly affected by various discontinuities, in this study it was chosen considering the density of the analysed data as well as the possibility of reducing some of the discontinuities through smoothing procedures. In addition, a short arc length was selected in order to prevent the increase of residual errors in non-gravitational accelerations.

[8] An orbital arc integration length may be defined as the interval of time from the initial point to some chosen final point of specific repeated period of satellite tracked data.

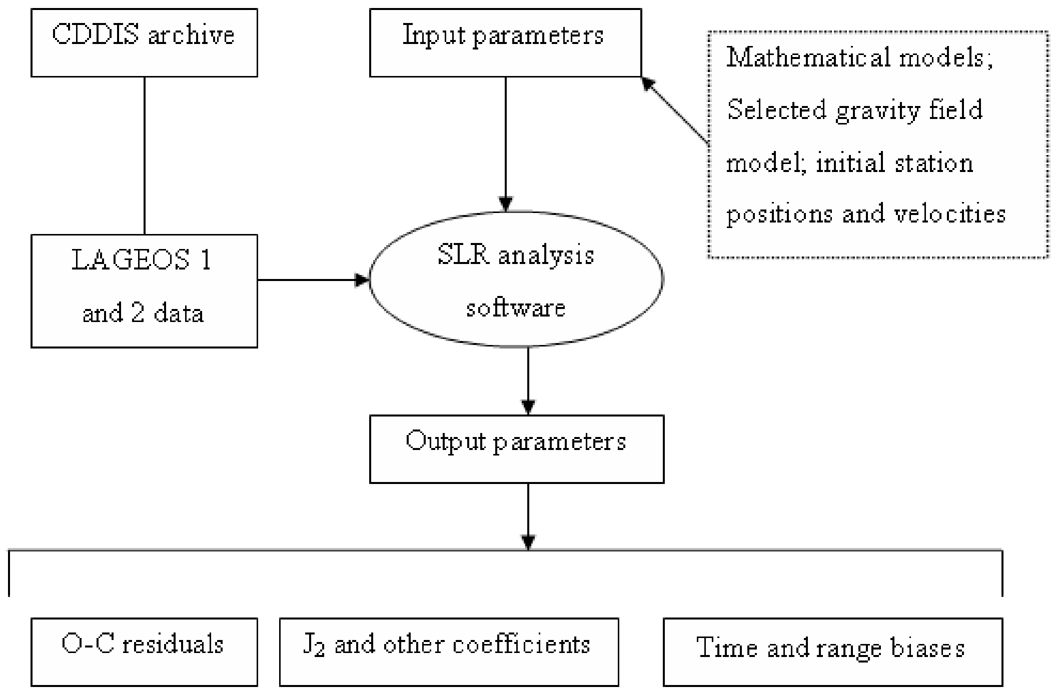

Figure 14. Schematic representation of data processing

3.6. Concluding remarks

SLR tracking data particularly from LAGEOS satellites have over the years allowed precise determination of satellite orbits as well as the investigation of orbital perturbations and their possible origins. About 30 SLR tracking stations coordinated by the ILRS are mostly distributed in European, Australian and Asian countries, this concentration of stations in the Northern Hemisphere has an impact on results as the network geometry is weakened and often data are not captured if one or more of the few Southern Hemisphere stations are not operational. Currently the accuracy of SLR data range between 1 cm and 3 cm for good tracking stations. However the accuracy decreases to 5 cm or more in some cases. In this study LAGEOS 1 and 2 data collected from selected tracking stations were analysed using the SDAS package developed at HartRAO. The analysis of SLR data requires adequate modelling of the orbit via an orbit integrator that includes modelling of gravitational and non-gravitational forces perturbing the orbit of the satellite. In order to adjust the range as determined by the SLR system, corrections due to physical effects such as those caused by the solid Earth tide and tidal deformations on the static gravity field need to be made. The SDAS package at HartRAO takes into account all the mathematical models in order to achieve a suitable solution. In the following chapters results obtained from LAGEOS 1 and 2 data analyses are discussed.

4. Investigating the accuracy of gravity field models using satellite laser ranging data

"...the various models are not as good as they are said to be. If they were, the differences between them should not be as great as they are..."

Lambeck and Coleman (1983).

The following chapter is based on a paper by Botai and Combrinck (2011).

4.1. Introduction

In this chapter, improvements in gravity field modelling over a period of 18 years are studied based on SLR data analysis using the SDAS package. In particular, this analysis is concerned with investigating the accuracy of GGMs focusing on spherical harmonic coefficients up to degree and order 20. In addition, since SDAS is still under development the present analysis is also to investigate whether SDAS output (e.g., O-C residuals and J_2) compares with outputs from other existing SLR analysis software packages such as GEODYN. The O-C residuals computed during SLR data analysis utilizing various gravity field models are used as a proxy for the accuracy of the satellite orbits and thus a measure of improvement in gravity field modelling. In this study three different analyses were conducted:

a) Seven months of SLR data collected from LAGEOS 1 and 2 were analysed by considering 12 gravity field models. Here, the main goal was to review the general improvement in gravity field modelling.
b) Three years of LAGEOS 1 and 2 data were analysed in order to investigate the trend in the improvement in the range bias associated with gravity field models derived from 1996 to 2008.
c) Lastly, the analysis of SLR tracking data sets for LAGEOS 1 and 2 (here 26 months of data) also focused on investigating improvements in the SDAS package based on two recent gravity field models.

Based on our analysis, there has been a factor of ~2 improvement in the SLR range bias computed from LAGEOS 1 and 2 SLR data analysis based on gravity field models developed from 1990 to 2008. However from the analysis of the O-C range residuals, the majority of the gravity field models released from 1999 exhibit negligible improvement. Models developed between 1999 and 2008 depict subtle differences of O-C residuals across the analysed models suggesting stability in the accuracy of gravity field modelling according to the SDAS package. Furthermore, using the most recent version of SDAS, gravity field models have shown significant improvements (the current value of O-C residuals is ~1.5 cm to 2.0 cm) compared to earlier versions of the analysis software.

4.2. Background

Continuous tracking of geodetic satellite orbits using the SLR data have provided an unprecedented opportunity in the history of gravity field modelling. The GGMs derived from such observations allow researchers to probe the long- to medium-wavelength components (half-wavelengths longer than 200 km, or spherical harmonic degrees 2 to 100) of the Earth's gravitational field. Numerous gravity field models have been derived from the mid-1960s. Furthermore, gravity field modelling is still in progress with new models being derived and old models being modified continuously. The accuracy of most of the latest models in terms of precise orbit determination is currently at cm level.

Improvements in gravity field modelling in terms of accuracy and spatial resolution is necessary in order to understand the physics of the interior of the Earth, the dynamics of the ocean and the interaction of continents, ice and ocean in sea-level studies, as well as for a better determination of satellite orbits and height systems in science and engineering (Rummel *et al.*, 2002). Such improvements are warranted owing to the availability of SLR tracking data, especially from the low Earth orbiting satellites. Satellite missions such as CHAMP, GRACE and GOCE launched in 2000, 2002 and 2009 respectively are believed to have improved the spatial resolution, sensitivity and accuracy of the newly developed GGMs. These satellites are designed to resolve the long-wavelength part of the gravity field and hence provide unprecedented accuracy (Featherstone, 2003). In contrast to the sporadic tracking by the SLR station network of the ILRS, the three satellite missions (CHAMP, GRACE and GOCE) carry GPS receivers on board that allow continuous orbit tracking. Furthermore, these satellites are equipped with accelerometers which provide direct measurements of the non-conservative forces (e.g. air-drag). In the case of GOCE, six accelerometers are installed in a gradiometer arrangement which additionally allows for direct measurement of the Earth's gravity gradients which gives an improvement in the medium wavelength part of the gravity. The three satellite missions also provide a homogeneous and near complete global coverage of gravity field information.

Research focusing on gravity field modelling has led to the unprecedented improvement in the resolution of various gravity field models i.e., to higher degree and order spherical harmonics. Such improvements can be measured by studying the inherent characteristics (e.g., the statistics) of the GGMs based on several factors. For example, the behaviour of GGMs can be analyzed by performing orbit adjustment tests on artificial satellites, GPS/leveling tests, comparing spectral behaviour of the models or ocean geoid (Foerste *et al*., 2008). While earlier geopotential models derived up to degree and order 70 could resolve spatial features (geoid computation) at a half-wavelength of about 290 km, models (particularly the most recent) computed up to degree and order 360 can resolve spatial features down to 55 km (Moore *et al*. 2006). Now-a-days gravity field modelling has reached a new era where new gravity field models are being derived reaching even higher degree/order (1000 or more) providing even further unprecedented accuracies, see for example Pavlis *et al*. (2008).

Early evaluations of gravity field models by Zhang and Featherstone (1995) reported that the OSU91A geopotential model provided the best fit to the gravity field over the Australian region compared to prior released models. In contributions by Pearse and KearsIey (1996) and Kirby *et al*. (1998) the accuracy of the OSU91A gravity model was inferior to the EGM96 gravity model where the latter was reported to give better solutions for the computation of geoid heights. Evaluations of GGMs released between 1996 and 2002 by Amos and Featherstone (2003) based on comparisons of gravity anomalies, free-air gravity anomalies, geoid heights and GPS/levelling tests found that EIGEN-1S was the best satellite-only GGM when applied in the Australian and New Zealand region while the best combined GGM over the same region was reported to be PGM2000A (Pavlis *et al*., 2000). The quality of the GGM01 model was assessed by Ellmann (2004) based on a comparison with the combined gravity field model EGM96. It was reported that the GGM01 model gives better solutions of gravity anomalies and geoidal heights over Fennoscandia (e.g., Finland, Germany, Norway, and Sweden) and the Baltic Sea region.

As reported in Foerste *et al*. (2009), a comparison study of ten geopotential models (EGM96, GGM02C, GGM03S, ITG-GRACE03, JEM01-RL03B, EIGEN-GL04C, EIGEN-5C/5S and EGM2008) using geoid heights and GPS/leveling data points revealed that the

EGM2008 model provided the best solution compared with the other models at degree 360. A much improved solution was also reported for EGM2008 when its coefficients were increased to degree 2190. A similar study by Yilmaz *et al*. (2010) evaluating GGMs EGM96, EIGEN-5C and EGM2008 based on the comparison of geoid heights with the GPS/levelling over Afyonkarahisar in Western Turkey also confirmed the improvements of EGM2008 model in the computation of geoid heights.

Improvement in the Earth gravity field modelling is anticipated as new and qualitative SLR tracking data and new algorithms of processing the data become available in the future. This expectation therefore motivates for assessment and validation of the accuracy and precision of existing gravity field models. Orbit tests are considered as tools for testing the long wavelength components of the gravity field model. In particular, the quality of orbits (and indirectly the quality of gravity field models) can be obtained by computing orbits to a variety of low and high artificial satellites with different orbit parameters. This can be done via a dynamic approach as well as by analysing the statistics of the satellites orbital residuals (also known as the difference between the observed orbital elements and the computed ones, e.g., O-C range residuals) for available tracking data to such satellites. In this study, we evaluate the accuracy of gravity field models in terms of POD by analyzing different data sets from LAGEOS 1 and 2 SLR data. In addition, improvements in the SDAS package are also investigated by analyzing 26 months of LAGEOS 1 and 2 SLR data considering two recent satellite-only and combined gravity field models.

4.2. Analysis of gravity field models

4.2.1. Improvements in gravity field modelling

In this section of the study, seven months of SLR data collected from LAGEOS 1 and 2 and spanning December 2005 to June 2006 were analysed for the purpose of assessing general improvements in gravity field modelling. In particular, twelve (12) gravity field models comprising of satellite-only and the combined (satellite and terrestrial data) categories which were developed and released to the geodetic community between 1990 and 2008 were considered during the SLR tracking data analysis. A brief description of each of these models is presented in Chapter 2 and a summary is also given in Table 7.

Table 7. GGMs evaluated in this study. Data: S = Satellite tracking data, G = Terrestrial gravity data, A = Altimetry data.

Model	Year	Degree/order	Data	Reference
AIUB-GRACE01S	2008	120	S	Jaeggi *et al.* (2008)
EIGEN-5C	2008	360	S,G,A	Foerste *et al.* (2008)
EIGEN-5S	2008	150	S	Foerste *et al.* (2008)
GGM03C	2007	360	S,G,A	Tapley *et al.* (2007)
EIGEN-GL04S1	2006	150	S	Foerste *et al.* (2006)
EIGEN-CG03C	2005	360	S,G,A	Foerste *et al.* (2005)
EIGEN1	2002	119	S	Reigber *et al.* (2003)
GRIM5C1	1999	120	S,G,A	Gruber *et al.* (2000)
EGM96	1996	360	EGM96S,G,A	Lemoine *et al.* (1998)
JGM3	1994	70	S,G,A	Tapley *et al.* (1996)
OSU91A	1991	360	GEMT2,A,G	Rapp *et al.* (1991)
GRIM4C1	1990	50	S,G,A	Schwintzer *et al.* (1991)

The data processing technique is discussed in Chapter 3. The stations selected for data analysis and their global performance during the period between 2006 and 2008 are listed Table 8. As featured in Table 8, ILRS tracking stations (column 1) have different total passes per year (i.e., different data volumes which is generally determined by ILRS scheduling program) and the annual averaged data quality (which is influenced by the local atmospheric conditions at the SLR site). Stations which were not able to provide any data or provided insufficient data were not evaluated.

In general, Table 8 illustrates that there has been an improvement in SLR tracking data over the years. It is important however to note that the individual station data are distributed heterogeneously with respect to the length of the time span and to the available number of normal points. For example while the best tracking stations observed about 2000 passes for LAGEOS (e.g., at Yarragadee) more than half of the 19 SLR stations selected delivered less than 15% of that data amount (e.g., Katzively and Lviv tracking stations). Nowadays the accuracy of data collected from ILRS tracking stations ranges from the 1 cm-level (for stations which perfom well) up to 3 cm-level for those stations that generally underperform. It is however important to also underline that the local atmospheric conditions such as the fraction of cloud cover, as well as turbulence degrade the quality of the data recorded at the SLR sites.

Table 8. Performance parameters of global SLR tracking stations recovered from the ILRS website i.e., http://ilrs.gsfc.nasa.gov/stations/site_info/. The stations are listed based on data volume contributed from 2006 to 2008, a map showing the distribution of these stations is given in Figure 5.

Station Name	Station No.	LAGEOS data volume			LAGEOS data quality [mm]		
		2006	2007	2008	2006	2007	2008
Yarragadee	7090	2038	1799	2078	9.1	9.5	9.6
Zimmerwald	7810	1147	1192	74	12.2	17.6	11.0
Graz	7839	858	825	653	7.7	8.0	5.2
Wettzell	8834	978	1041	1011	15.6	18.5	18.5
Monument Peak	7110	894	484	363	14.5	15.9	16.0
Herstmonceux	7840	929	932	426	16.3	12.9	13.5
Changchun	7237	423	772	605	14.3	17.5	12.4
Matera_MLRO	7941	872	753	799	6.5	5.9	4.9
Hartebeesthoek	7501	720	304	254	8.9	10.4	10.5
Potsdam_3	7841	307	304	313	20.1	19.2	17.4
Greenbelt	7105	269	321	511	9.1	9.5	9.9
San_Fernando	7824	260	523	440	14.7	15.1	14.1
Concepcion_847	7405	590	1078	816	14.5	12.0	19.2
McDonald	7080	369	412	335	11.8	12.5	12.5
Beijing	7249	178	339	311	19.4	16.6	16.3
Riga	1884	98	111	57	13.0	12.0	12.4
Katzively	1893	80	287	310	8.3	40.2	42.5
Tokyo-(CRL)	7308	63	248	472	17.4	15.5	15.0
Arequipa	7403	37	218	130	7.0	6.9	5.6

In this study we have computed orbit residuals for LAGEOS 1 and 2 using the SDAS package and considering the 12 selected gravity field models (see Table 8). The orbit residuals are derived from the SLR data analysis which utilizes dynamical modelling (e.g., gravity fields) during precise orbit determination (Yunck, 1997; Lemoine *et al.*, 1998). In particular, orbit residuals (which also represent the differences between the satellite position as calculated from SLR observations and the satellite position (orbit) computed from dynamical models) are commonly referred to as the O-C range residuals which are dependent on the type of gravity field model under consideration (different gravity field models are associated with different O-C range residuals). The computed O-C residuals are used in this study as a measure of accuracy in the gravity field modelling.

Table 9 contains the mean SD values of the O-C residuals based on the 12 considered gravity field models. The results presented in Table 9 indicate that the oldest gravity field models, GRIM4C1 and OSU91A (released in 1990 and 1991 respectively) are linked to a mean residual (~10 cm and ~8 cm for LAGEOS 1 and 2 respectively) that is approximately twice the O-C range residuals computed from SLR analysis using some of the more recent models. Higher SD values associated with gravity field models (GRIM4C1 and OSU91A) may be due to the systematic errors or range bias in the ephemeris and/or as a result of inappropriate calibration of the models (Milani *et al*., 1995). The gravity field models released from 1999 onwards seem to remain at approximately the same level, although there are many specific differences (e.g. type of data used, degree and order of coefficients) amongst these later models.

Table 9. Statistical comparative accuracies of the evaluated gravity field model in terms of O-C residuals.

Model	Year	Mean SD [cm] LAGEOS 1	Mean SD [cm] LAGEOS 2
AIUB-GRACE01S	2008	3.79	3.63
EIGEN-5C	2008	3.89	3.73
EIGEN-5S	2008	3.85	3.32
GGM03C	2007	3.88	4.86
EIGEN-GL04S1	2006	3.89	3.72
EIGEN-CG03C	2005	3.81	3.69
EIGEN1	2002	6.09	7.52
GRIM5C1	1999	3.82	3.70
EGM96	1996	4.14	4.41
JGM3	1994	4.49	5.57
OSU91A	1991	10.17	8.10
GRIM4C1	1990	10.36	9.94

The average difference among the gravity field models released between 1999 and 2008 is at mm level with maximum difference being less than 2%. This may imply that gravity field models released from 1999 to 2008 have less or no influence on the current cm accuracy level of the precise orbit determination. However, since the addition of CHAMP and GRACE data, an improvement in gravity field modelling is expected, though systematic errors might be dominant in the analysis set up. In the case where modelling errors dominate, the inaccuracies caused by the modelling of other perturbing forces are greater than the contribution from gravity field models and thus obscure the improvement in the gravity modelling. Nevertheless, with the inclusion of long term series of data there still could be room for further improvements in the models.

Figure 15 depicts the mean SD values of the O-C residuals computed from LAGEOS 1 and 2 SLR tracking data based on varying the 12 selected gravity field models. In general, gravity field modelling progressively improved between 1990 and 2008. Noticeable improvements occurred with the development of gravity field models that were released between 1990 and 1996. For example, while the GRIM4C1 and OSU91A models exhibit SD mean values of ≥8 cm the most recent models show an SD of ~3 cm. Based on the SLR analysis of LAGEOS 1 and 2 SLR data, the mean SD values corresponding to the 12 gravity field models indicate that gravity field modelling significantly improved over the 18 years period.

Large mean SD values of O-C range residuals observed in earlier GRACE gravity field models, EIGEN1, may be explained by inherent systematic errors in the SLR observations, uncertainties in the conceptual gravity field model, model error, as well as outliers related to weak station geometry and lack of data on some days. Overall, an improvement by a factor of 2 in the O-C range residuals based on the analysis of the LAGEOS 1 and 2 data sets considering the various gravity field models is observed since 1990. In particular, the satellite-only gravity field models, AIUB-GRACE01S and EIGEN-5S, yield the lowest O-C results therefore they seem to be the most accurate in terms of our evaluation.

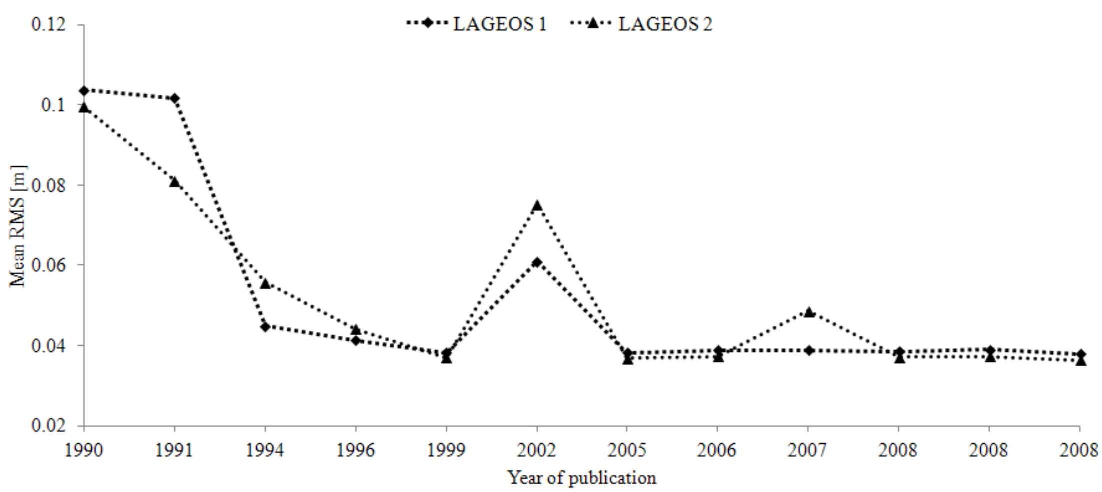

Figure 15. Time series of the mean SD values for the 12 evaluated GGMs.

Handling outliers in the O-C residuals based on selected gravity field models

In this part of work, the inherent outliers in the O-C residuals across the 12 selected gravity models have been assessed and corrected based on the 3σ-rule. This rule uses the fact that 99.73% of all values of a normally distributed parameter fall within three standard deviations of the average value. Suppose that we have a sample of O-C residuals given by $X = \{x_1, x_2, x_3, ..., x_n\}$, outliers in the data can be identified by iteratively applying the outlier tests given by Equation (68),

$$t_i = \frac{|\bar{x} - x_i|}{s}. \tag{68}$$

Here $\bar{x}$ is the mean, *s* is the standard deviation for the entire data set, x_i is the suspected single outlier, i.e., the value furthest away from the mean. Normally, a 3σ-rule considers any observations with $|t_i| > 3$ as possible outliers and discards such observations or adjusts them to one of the values $\bar{x} \pm 3s$, whichever is nearer. Another way of detecting outliers in the data is by fitting a linear regression on the data. Suppose the relationship between two variables *x* and *y:* (x_i, y_i), with *i* = 1, …, n is given by a straight line regression model,

$$y_i = \alpha + \beta x_i + u_i, \qquad i = 1, ..., n. \tag{69}$$

Here x_i and y_i are the predictor and response variable values respectively, and u_i are random errors. Possible outliers can be detected by estimating the parameters α and β with the least-squares estimates given by Equation (70),

$$\vec{\beta} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$
$$\alpha = \bar{y} - \bar{x}\vec{\beta}. \tag{70}$$

The 3σ-rule given by Equation (68) was used to investigate possible outliers in the O-C residuals computed from LAGEOS 1 and 2 SLR data based on the 12 gravity field models listed in Table 7.

Table 10 lists the new mean SD values of the O-C residuals computed by using LAGEOS 1 data after applying the 3σ-rule. A significant improvement in the mean SD of O-C residuals is noticed in all 12 gravity field models. In particular, the positive influence of the gravity field models on the O-C range residuals is more noticeable in all the models in the LAGEOS 1 data analysis. For example, the mean SD values computed from LAGEOS 1 based on GRIM4C1 and OSU91A ranges between ~6 cm and ~8 cm respectively. Application of the 3σ-rule on the O-C residuals derived from LAGEOS 1 indicates that ~6% and ~5% of the residuals were rejected as outliers. The combined gravity field model, EIGEN1 exhibits a high mean SD with data rejection of only 2% as compared to the other more recent models (models derived from CHAMP and GRACE data).

The mean SD values for the rest of the models lie at ~3 cm with range residual rejection ranging from 8% for JGM3 to ~27% for EIGEN-5S, EIGEN-5C and AIUB-GRACE01S. The AIUB-GRACE01S, EIGEN-5C, EIGEN-5S, EIGEN-CG03C and GRIM5C1 seem to be the best considering their reduced SD values, though they also reject a high quantity of data. In particular, the GRACE satellite-only model, AIUB-GRACE01S, is found to be the best with the least mean SD of 3.07 cm. It is however important to point out that, the percentage of data rejection is a function of the number of data points, as opposed to the variance of the O-C range residuals (here, older gravity field models have less data points compared with the most recent gravity field models).

Table 10. Mean SD values of the O-C residuals computed from LAGEOS 1 based on the 12 gravity field models after the application of the 3σ-rule.

Model	Mean SD [cm] LAGEOS 1	Residuals rejection %
AIUB-GRACE01S	3.07	26.8
EIGEN-5C	3.15	26.7
EIGEN-5S	3.18	27.4
GGM03C	3.29	15.4
EIGEN-GL04S1	3.25	20.7
EIGEN-CG03C	3.12	25.3
EIGEN1	5.16	2.1
GRIM5C1	3.13	25.1
EGM96	3.86	1.0
JGM3	3.81	8.3
OSU91A	8.38	4.9
GRIM4C1	6.87	5.9

Table 11 presents results for the 3σ-rule applied on O-C residuals computed from LAGEOS 2 based on the 12 selected gravity field models. In Table 11 the GRIM4C, OSU91A and EIGEN1 contain the highest mean SD values of ~8 cm and ~6cm with ~3%, ~4% and ~5% residual rejection. In particular, the EIGEN1 model exhibits a higher mean SD than any other recent models computed from CHAMP and GRACE data. The combined models, JGM3 and GGM03C are very close with a mean SD of ~4.7 cm and ~4.2 cm and residual rejection of ~3% and ~2% respectively. The rest of the gravity field models show an average SD of ~3 cm with residual rejection ranging from ~0.5% to ~1%. In particular, the EIGEN-5S, EIGEN-5C, EIGEN-GL04S1 and EIGEN-CG03C models exhibit equal average SD values of 3.30 cm with residual rejection of 0.48%. Since EIGEN-5S solution was incorporated in the computation of EIGEN-5C it may imply that the EIGEN-5C model and perhaps EIGEN-CG03C are dominated by the satellite-only information up to a certain degree/order. Based on this study the best model for computing LAGEOS 2 orbits is found to be AIUB-GRACE01S (similar to LAGEOS 1) considering the improvements in O-C residuals.

Table 11. Mean SD values of the O-C residuals computed from LAGEOS 2 based on the 12 gravity field models after the application of the 3σ-rule.

Model	Mean SD [cm] LAGEOS 2	Residuals rejection %
AIUB-GRACE01S	3.23	0.49
EIGEN-5C	3.30	0.49
EIGEN-5S	3.30	0.48
GGM03C	4.16	2.00
EIGEN-GL04S1	3.30	0.48
EIGEN-CG03C	3.30	0.48
EIGEN1	6.21	5.10
GRIM5C1	3.29	0.49
EGM96	3.74	1.50
JGM3	4.74	3.00
OSU91A	6.46	4.70
GRIM4C1	8.12	3.20

4.2.2. Trends in O-C residuals based on developments in gravity field modelling

In this study three (3) years of LAGEOS 1 and 2 tracking data spanning December 2005 to December 2008 were used to investigate the trend in the improvement of O-C residuals based on a set of four gravity field models (i.e., EGM96, GRIM5C1, GGM03C and AIUB-GRACE01S) released between 1996 and 2008. The motivation for considering the selected models arises from the stable pattern (there was no noticeable improvement) in the O-C range residuals observed (see Figure 15) during the analysis of SLR data while using the models. In Table 12 the average SD values were calculated from the original O-C residuals (before filtering the outliers) of LAGEOS 1 and 2 considering the four gravity field models while, Table 13 presents slightly improved mean SD values of the O-C residuals after applying the 3σ-rule. Direct comparison of the average SD values computed from the four different models depicts that the GRIM5C1 and AIUB-GRACE01S have comparable accuracy (e.g., 3.35 and 3.36 based on LAGEOS 1 SLR and 3.35 and 3.34 cm based on LAGEOS 2 SLR data).

Table 12. Mean SD values calculated from the O-C residuals based on LAGEOS 1 and 2 data using EGM96, GRIM5C1, GGM03C and AIUB-GRACE01S models.

Model	Year	Mean SD [cm] LAGEOS 1	Mean SD [cm] LAGEOS 2
EGM96	1998	4.32	4.22
GRIM5C1	1999	3.94	3.84
GGM03C	2006	4.18	4.32
AIUB-GRACE01S	2008	3.92	3.82

Table 13. Mean SD values of the four models after 3σ-rule filtration.

LAGEOS 1		
Model	Mean SD [cm]	Residuals rejection %
EGM96	3.66	12.0
GRIM5C1	3.35	16.2
GGM03C	3.52	12.6
AIUB-GRACE01S	3.36	15.9
LAGEOS 2		
EGM96	3.60	10.0
GRIM5C1	3.35	9.5
GGM03C	3.84	11.4
AIUB-GRACE01S	3.34	9.3

As tabulated in Table 13 the O-C range residuals derived from LAGEOS 1 SLR data exhibit high residual rejection (~14% overall) during filtering, compared with ~10% rejected from O-C range residuals derived from the analysis of LAGEOS 2 SLR data sets. The EGM96 and GGM03C exhibits slightly higher O-C range residuals (3.7cm and 3.8cm respectively) based on the analysis of LAGEOS 1 and 2 SLR data respectively. The high average SD based on the SLR analysis while considering the EGM96 and GGM03C gravity field models could be attributed to possible inherent biases in the O-C range residuals due to weak station geometry, systematic errors, poor tracking on certain days, especially during raining seasons and poor distribution of tracking data (due to network asymmetries).

4.3. Investigating possible improvements in the SDAS package

The SDAS package is still under development hence the estimates of O-C range residuals are expected to be optimized as more features are introduced into the software. The SDAS package has undergone considerable upgrades since the SLR data processing started back in 2008. This includes implementation of different IERS models to correct for the effects of tidal deformations due to solid Earth and pole tides. Hence the main focus in this section is to investigate the possible improvements in the estimation of O-C range residuals as realized by the general upgrade of SDAS. For this purpose we have analysed twenty-six months of LAGEOS 1 and 2 data spanning May 2008 to April 2010 while considering two recent gravity field models, EGM2008 (partly because this model has the highest degree/order 2159, though SDAS is only configured to process up to degree/order 20) and AIUB-GRACE01S (partly because this gravity field model exhibited the lowest O-C range residuals in the previous SLR data analysis).

The results for this analysis are presented in Table 14 (from the original O-C residuals) and Table 15 (after 3σ-rule filtration). The mean SD obtained in this study using a new version of the SDAS package shows an improvement by more than a half compared with the older version of the software. This suggests that the added features in the software have increased its capability to compute satellite orbits with unprecedented accuracy. In addition, the results reported in this study are comparable to those published in the literature. Generally, the SD values of other LAGEOS orbit computations, based on the most recent gravity field models, are found to be ≤ 1.5 cm, see for example Cheng *et al*. (2009). In this study we find the mean SD values for EGM2008 and AIUB-GRACE01S to be 1.8 cm based on LAGEOS 1 and 1.6 cm based on LAGEOS 2 data. This gives a difference of about 3 mm and 2 mm between our results and those reported by Cheng *et al*. (2009).

Table 14. Mean SD values calculated from the O-C residuals based on LAGEOS 1 and 2 data using EGM2008 and AIUB-GRACE01S models.

Model	Mean SD [cm] LAGEOS 1	Mean SD [cm] LAGEOS 2
EGM2008	2.01	1.77
AIUB-GRACE01S	2.00	1.80

Table 15. Mean SD values of the O-C residuals for LAGEOS 1 and 2 data based on EGM2008 and AIUB-GRACE01S models after 3σ-rule filtration.

LAGEOS 1		
Model	Mean SD [cm]	Residuals rejection %
EGM2008	1.81	3.90
AIUB-GRACE01S	1.84	3.10
LAGEOS 2		
EGM2008	1.64	3.30
AIUB-GRACE01S	1.62	3.20

4.4. Concluding remarks

Analysis of the accuracy of satellite orbits calculated from SLR measurements partly entails assessment of the influence of various gravity field models on the O-C range residuals. As a result, a more accurate gravity field model would manifest in the form of an improvement of the O-C range residuals calculated from the analysis of SLR data while considering the gravity model in question. In this study, the accuracy of twelve gravity field models released between 1990 and 2008 were analysed in terms of precise orbit determination by comparing their O-C range residuals. The results from a seven month data period indicated that there has been an improvement in the development of gravity field models over the period of evaluation. The evaluated models show an improvement by a factor of at least 2 since 1990 in terms of O-C range residuals. Furthermore, our analysis indicated that gravity field models released from 1999 onward are likely to be accurate at approximately the same level, at least to the sensitivity of our O-C tests, although there are many specific differences amongst these later models. A further analysis (for a period of three years) of a set of four gravity field models released between 1999 and 2008 demonstrates subtle differences in their O-C range residuals which could be associated with data quality. Overall, in the SLR data analysis (this includes the seven months and ~3 years of LAGEOS 1 and 2 SLR data) undertaken in this study, it was found that the satellite-only derived gravity field model AIUB-GRACE01S could be the most accurate due to the low average SD of the corresponding O-C range residuals. The SDAS package has undergone numerous upgrades with promising results; current level of accuracy of the O-C range residuals is comparable to those published in the literature.