

CHAPTER ONE

OVERVIEW

1.1 Introduction

How does one mathematically determine whether the gradient of a straight line is positive or negative? I asked this of a mathematics student teacher I was observing and was surprised that he could provide no mathematical explanation. Instead he explained that a positive gradient could be recognised by the fact that if you were walking along the line, it would be like walking up a mountain so you would feel really positive. On the other hand the negative gradient or slope is like coming down a mountain and one usually feels negative coming down a mountain. He confessed that he relied mainly on memorisation to explain mathematical concepts.

This is one of many similar examples where mathematics is endorsed as a process of rote memorisation rather than a discipline requiring understanding. In my role as a mathematics educator (or specialisation lecturer), I became increasingly concerned about the low level of content knowledge as well as teaching and learning strategies being demonstrated by pre-service mathematics students during practical teaching periods. Despite the global reform being initiated in mathematics education, the students continued to demonstrate a traditional and rote learning approach to teaching mathematics with only superficial motions towards a more constructivist paradigm. With their own experiences of mathematics teaching at school most likely being limited to a traditional approach, and the lack of deep change occurring in most schools where they would teach, I began to wonder how we could most effectively achieve the change in pedagogy we are aiming towards.

Along with many other countries, South Africa has experienced radical curriculum reform during the past ten years. Our latest curriculum, based on a philosophy of Outcomes-Based Education ([OBE], see for example Jansen, 1998, 1999; Muller, 1998), demands a range of teaching strategies and roles on the part of the teacher as outlined in the Norms and Standards for Educators (Department of Education [DoE], 2000). These include being mediators of learning, interpreters and designers of Learning Programmes

and materials, leaders, administrators and managers, scholars, researchers and lifelong learners, community members, citizens and pastors, assessors and Learning Area or Phase specialists. The curriculum statement also makes the point that setting and achieving outcomes encourages a learner-centred and activity-based approach to education.

This reform in the type of teacher envisioned has also brought about changes in pre-service teacher training programmes. Much of the research focusing on teacher training makes an attempt to find out how training should be tailored in order to optimally prepare students and teachers for the changing role of teaching they have to fulfil (e.g. Shulman, 1986, 1987; Ball, 1990; Ma, 1999; Peressini, Borko, Romagnano, Knuth & Willis, 2004; Adler, 2005; Adler, Davis & Kazima, 2005). The aim of this research project is to contribute to the existing body of research in this regard, by *investigating the relationship between the **mathematics profiles**¹ of secondary school pre-service mathematics teachers, and the **instructional behaviour** they develop relating to the teaching and learning of mathematics.*

I hold the position of lecturer at a university in South Africa. I graduated from this same university in 1993 with a Bachelor of Arts (majoring in Psychology and Northern Sotho²) and a Higher Education Diploma, specialising in teaching Northern Sotho, Mathematics and French. In 1994 I began to teach mathematics at an urban girls school where I remained for eight years. The headmistress of the school during that time was a mathematics educator herself and provided me with the freedom to try new approaches in the teaching and learning of mathematics. This largely formed the basis for the social constructivist approach I assume within my role as a mathematics educator. During this time I also spent some time teaching in the United Kingdom and writing a series of

¹ The term “mathematics profile” I introduce in this study is further elaborated on in the following section. It refers to the combination of each participant’s subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and beliefs about the teaching and learning of mathematics. The term profile is more commonly used in the field of Psychology, for example, a personality or a brain profile.

² This is one of the 11 official languages of South Africa. It is an indigenous language spoken predominantly in the Limpopo and Gauteng provinces.

mathematics textbooks with an experienced panel of authors. These opportunities kept challenging me with regard to the traditional approach to teaching mathematics that I had experienced as a learner at school and how these practices could be reformed in order to equip learners to be stronger mathematical thinkers rather than rote learners. In 2002 I joined the university as a lecturer in mathematics education. While making that shift from a teacher orientation to a researcher, I found literature that supported what I had been experiencing during my prior years of teaching with regard to the traditional versus reform tension. I embraced social constructivism and this gave me a framework within which I could develop my instructional behaviour as a mathematics educator. In this position, however, I also became increasingly frustrated at the apparent lack of change evident in the mathematics pre-service training as well as in mathematics classrooms I visited when assessing my students. This frustration pre-empted a curiosity about what either enables or constrains pre-service teachers in reforming their approach to the teaching and learning of mathematics. This curiosity eventually led to this study. This is further outlined in Section 1.2.1.

One of my responsibilities at the university is teaching the mathematics specialisation module for the one-year Post Graduate Certificate in Education (PGCE)³ programme. I therefore elected to use data obtained from these students⁴, specifically those enrolled for a Further Education and Training (FET)⁵ qualification. As part of their end-of-year summative evaluation for the PGCE, students are required to prepare portfolios representing their professional development as mathematics facilitators throughout the year. These include personal profiles such as brain profile tests, personality assessments, their daily reflections, their lesson plans, video-recordings from their school-based practice, assessment records from their specialisation lecturer, their school-based mentor

³ This was previously known as the Higher Education Diploma. Students complete an undergraduate degree, such as a Bachelor of Arts and then enrol fulltime for this one-year diploma that certifies them as teachers. The other academic option that students in South Africa have to qualify as teachers is to enrol for a four-year Bachelor of Education degree.

⁴ I use the term student and pre-service teacher interchangeably in this study with regard to tertiary education. The term “learner” refers to school education.

⁵ The 12 years of compulsory schooling in South Africa comprises four phases of education, namely, the Early Childhood Education (Grade R – 3), the Intermediate phase (Grades 4 – 6), the Senior phase (Grades 7 – 9) and the Further Education and Training phase (Grades 10 – 12).

(a teacher at the school), peers (fellow PGCE students) and self-evaluations. Their final presentation of their portfolio as well as their verbal defence thereof is also video-recorded by the university. I chose to do the study "in arrears" (post-hoc) and at the end of 2008 obtained permission from the 2006, 2007 and 2008 FET students in mathematics to use their portfolios and any other relevant documentation/material from their PGCE year as my data set.

Using this data set I embarked on three data reduction processes. The first was to select data from the participants' portfolios to compile the participants' reflections, which are written in their own voice. The second was to use these reflections as well as the mathematics specialisation lecturers'⁶ assessment reports and my own experience of working with each participant to write a researcher reflection. The researcher reflection is divided into two parts: one part reflects on the mathematics profile and the other on the instructional behaviour of each participant. The third reduction involved using participant and researcher reflections are to construct a visual representation of each participant's mathematics profile and instructional behaviour profile. These visual presentations facilitated the in-case and cross-case comparisons to establish the possible influence or links between their mathematics profiles and the instructional behaviour/approach the pre-service teachers display during their school-based practice teaching periods.

The main aim of the study was to explore the influence of the mathematics profiles of pre-service teachers on their instructional behaviour. The mathematics profiles were constructed from four components foregrounded in the literature, namely subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and beliefs relating to the teaching and learning of mathematics. The instructional behaviour of each participant portrays their approach to teaching and learning mathematics. This is depicted specifically with regard to traditional versus reform teaching practices and democratic versus authoritarian learning experiences offered to learners. The seven participants came

⁶ During 2006 I was the only specialisation lecturer in mathematics responsible for the students. In 2007 and 2008 I had study leave for one of the semesters each year during which another lecturer took responsibility for the module and visited the students during their school-based experiences. On occasion we also both visited a student as part of the training process of the relief lecturer and to ensure consistency.

from the same university in South Africa and all enrolled for a one-year Post Graduate Certificate in Education between 2006 and 2008.

1.2 Background to the research

The intellectual puzzle I engage with in this research project has emerged from a variety of experiences I have had over the last few years in my position as a lecturer training pre-service mathematics teachers. The experiences involved workshops, lectures, interviews, observations and general interactions with pre-service as well as current teachers. The problem is encapsulated in the limited conceptual understanding of mathematics demonstrated by teachers and students of mathematics and the poor performance of learners in South Africa in mathematics. My assumption is that improving the mathematical understanding of mathematics teachers will result in stronger mathematics learners. In the sections that follow, further insight into the background to the problem is provided.

1.2.1 Training of pre-service mathematics students

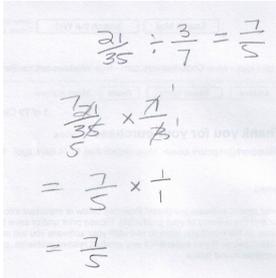
Ma (1999) conducted a study investigating and comparing the mathematical understanding of a cohort of teachers in the United States and China. She concluded that the Chinese teachers demonstrated a deeper conceptual understanding of division in fractions than teachers in the United States. Using her research I adopted some of the questions she posed to the participants as a departure point for discussions in my methodology classes. Students in a third year methods class were asked if the calculation in Figure 1.1 could be performed by dividing the numerators and then dividing the denominators.

$$\frac{21}{35} \div \frac{3}{7} = \frac{7}{5}$$

Figure 1.1 *Division of fractions calculation*

The immediate response of most of the class was a resounding "no." After doing the calculation their own way (see Figure 1.2), most of the students then noted that the

solution presented in the calculation in Figure 1.1 was in fact correct. At least half the class were still adamant, however, that the calculation could not be done using the thinking process suggested above, even though the answer was correct. When asked to write down why they thought it could not be done that way, the general response was that "we were not taught to do it that way."



Handwritten student solution for $\frac{21}{35} \div \frac{3}{7} = \frac{7}{5}$. The student shows the original problem, then crosses out the 21 and 35 in the numerator and denominator of the first fraction, leaving $\frac{7}{5}$. They then multiply this by the reciprocal of the second fraction, $\frac{7}{3}$. The final result is $\frac{7}{5}$.

Figure 1.2 Example of a solution provided by a student

Students were then further requested to indicate how they would approach teaching the topic of division of fractions to a class. All the students focused their approach on teaching learners to multiply by the reciprocal. Without exception, none of the students could produce a mathematically correct reason why the method they were proposing to teach learners is acceptable and why it worked. The most common reason they gave was that division and multiplication are inverse operations and that the second fraction should therefore be inverted. When confronted with the counter example of applying their conjecture to the addition and subtraction of fractions, although aware of the incorrectness in their thinking, students were unable to find a suitable mathematical reason why we multiply by the reciprocal instead of dividing fractions.

This is one of many available vignettes providing anecdotal evidence of how students demonstrated their lack of conceptual understanding and their limited, instrumentalist view of mathematics. Ernest (1988, p.10) explains an instrumentalist view of mathematics as:

...the view that mathematics, like a bag of tools, is made up of an accumulation of facts, rules and skills to be used by the trained artisan skilfully in the pursuance of some external end. Thus mathematics is a set of unrelated utilitarian rules and facts.

I became increasingly concerned about students who may continue to hold this view of mathematics as they enter the teaching profession. How would this view of mathematics

enable them to be effective "mediators of learning" and "Learning Area specialists" as required by the norms and standards set out for educators⁷ (DoE, 2000, p. 3)? Would this view and lack of insight perhaps confine them to a more traditional approach to teaching mathematics in their pedagogy?

1.2.2 South African learners' performance in mathematics

South Africa took part in the Third International Mathematics and Science Study (TIMSS – now referred to as Trends in Mathematics and Science Study) in 1995, 1999 and 2003, of which the latter two were conducted on Grade 8 learners. On all three occasions, South Africa was placed in the last position (in 2003 out of approximately 50 countries), being outperformed by other African countries such as Botswana, Tunisia, Egypt and Morocco (Howie, 2002; Reddy, 2006). TIMSS made use of Item Response Theory to calculate the achievement scores, with a scale of 800 points and a standard deviation of 100 points. In the 2003 results, the average scale score for Grade 8 South African learners was 264 (SE = 5.5) which was significantly lower than the international average scale score of 467 (SE = 0.5). The average scale score of South Africa in the 2003 TIMSS study compared to the average scale scores of other African countries that took part is depicted in Table 1-1 below (Mullis, Martin, Gonzalez & Chrostowski, 2004).

Table 1-1 Comparison of South African average mathematics scale score in TIMSS 2003 with other African countries

Country	Average age of learner	Average scale score	Standard Error
South Africa	15.1	264	5.5
Botswana	15.1	366	2.6
Tunisia	14.8	410	2.2

⁷ This document outlines the norms and standards required of teachers entering the profession and acts as a guideline for teaching training programmes.

Country	Average age of learner	Average scale score	Standard Error
Ghana	15.5	276	4.7
Egypt	14.4	406	3.5
Morocco	15.2	387	2.5

Table 1-2 is a breakdown of the mathematics enrolment and performance at Senior Certificate level from 2003 - 2007, the national performance in terms of the mathematics achievement of South African learners at school-leaving level is also of concern.

Table 1-2: National Senior Certificate Examination results (2003 - 2007)

	2003	2004	2005	2006	2007
Total no. candidates passing	322 492	330 717	347 184	351 217	368 217
Percent passing mathematics	40%	40%	40%	39%	41%
Pass on SG	33%	33%	32%	32%	34%
Pass on HG	7%	7%	8%	7%	7%

SG: Standard Grade

HG: Higher Grade

In 2008, there was no longer a distinction between Higher and Standard Grade. All learners in Grade 12 in 2008 had to write either mathematics or mathematical literacy⁸. There were 298 821 learners who wrote mathematics of which 46 % of them passed.

⁸ Prior to 1996 learners in Grade 10 were able to select mathematics as one of their six subjects for the FET phase. They then had the option to take mathematics on the higher (more difficult) grade or on the standard grade. From 1996 the policy was amended to ensure that all learners take some form of mathematics throughout their FET phase. Higher grade and standard grade options were removed and all learners have to now either select mathematics or mathematical literacy as one of their six subjects for the FET phase.

There were 263 464 learners who wrote mathematical literacy with 79 % of these learners passing.

According to the above results, learners in South Africa are underperforming in mathematics both nationally and internationally. Studies, where factors that contribute to mathematics performance have been analysed (Howie, 2002; Reddy, 2006), have been done to explore why this is the case. Howie (2002), analysing data collected from teachers in the TIMSS 1999 study found that in South African classrooms significantly more time (21%) was spent on re-teaching and clarification of content or procedures than in other countries on average (13%). South African teachers also spent more time on administrative tasks (13% compared to average 5% in other countries) and reviewing homework (26%) compared to the average of other countries (12%). The same study found that with respect to pedagogical practices, teachers of 16% of South African learners placed a high emphasis on mathematics reasoning and problem solving, which was comparable with the international average. However, while the pattern internationally appeared to be that learners of teachers who claimed to have this approach would achieve a correspondingly higher achievement, this was not the case with South African learners. In fact, the opposite was true. South African learners whose teachers reported placing a high emphasis on reasoning and problem solving achieved lower results (260 points) compared to learners whose teachers placed a lower emphasis on this approach (303 points).

Reddy (2006) compared the results of the 1999 and 2003 TIMSS data to find that on average the scores had decreased, although the difference was not statistically significant. She makes the following comment in her report (p. 52):

Since 1998 (with the introduction of C2005), there have been many professional development courses and programmes for teachers. In addition, numerous interventions by government, private sector, business and non-governmental organisations have been made in schools, especially the African schools, with the objective of improving the state of mathematics and science education. However, it seems that despite these programmes there has been a decrease in mathematics performance in many schools.

Perhaps it is time to start asking ourselves why our professional development courses (both in-service and pre-service) are not having the desired improved effect on the

mathematics performance of our learners. Are we perhaps expecting teachers to change their pedagogical beliefs and practices when in fact their subject matter knowledge is a limiting factor in enabling them to effectively do this? Is there a specific type of mathematics profile that is more likely to end up breaking out of a more traditional approach? Perhaps teachers' views of teaching and learning mathematics are the factor we need to be foregrounding in professional development? These are the corner pieces of the puzzle I hope to unravel and understand more of within the context of this investigation on pre-service teachers.

1.2.3 Contract research project

During the course of 2004 I managed an independent evaluation for a non-governmental organisation (NGO) in the form of contract research (Barnes, 2008). The task was to evaluate the impact/effectiveness of an intervention⁹ they were funding. The particular intervention was aimed at additional training and support for Intermediate Phase mathematics educators, mostly in rural areas. The evaluation was carried out in a randomly selected sample of 12 schools in one South African province. The evaluation sought to examine the impact and effect of the intervention on firstly the educators (at which the intervention was primarily aimed) and as a more distant outcome, the learner performance.

The evaluation collected data from approximately 1 104 learners and an average of 17 educators from the 12 schools. Three instruments were used in collecting data from educators. These included semi-structured interviews (that were recorded and transcribed), observation schedules (completed by fieldworkers sitting in on lessons) and educator questionnaires that the educators involved completed. Learner performance was measured through the administration of pre- and post-tests, which were identical. Once completed, the tests were manually coded (marked) by fieldworkers and the data captured

⁹ According to the NGO, the intervention focused on the following key aspects (outcomes): content knowledge of teachers, curriculum management, assessment and teaching practices. The intervention was designed with a view to improving teachers' skills with regard to the four aspects mentioned, in order to have a positive effect on learners' performance.

by data typists. A team from the university consisting of a statistician and two researchers specialising in mathematics education analysed and interpreted the data.

During the semi-structured interview, educators were asked by a fieldworker to offer their definition and views of mathematics as a subject. This was done in order to ascertain how the educators valued the place of the subject in the curriculum and how confident they were in teaching it. An educator's view of mathematics is often an indicator of the way they are likely to teach it. To quote Dossey (1992):

The conception of mathematics held by the educator may have a great deal to do with the way in which mathematics is characterized in classroom teaching. (p. 42)

Hersh (1986) makes the same point:

One's conception of what mathematics is, affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it... (p. 13)

Most of the educators that were interviewed held positive views of mathematics and claimed that it was a very necessary part of the curriculum. Quotes from the educators¹⁰, such as those included below substantiate this:

I would say it's a very lovely subject, what is important is we are doing maths everyday of our lives. You go to the bus you pay, that's maths, you look at the watch, you go to the shop you buy that's maths. We are doing maths unconsciously, so maths is the subject to be taught everywhere.

I would say it's the mother of all the subjects because even if you didn't go to school but maths is always there, even if you can't read or write but some other people are able to calculate their money, they are able to say I want 1kg bag of rice or I want 10kg, that is maths, it's the most important subject, whether you like it or not but you are doing it anyway, unconsciously.

However, some also admitted they find it difficult and challenging to teach, but that they are "trying to rub all those stereotypes" that learners and educators often attach to the subject. Some of the educators felt they were succeeding in this since they had learnt

¹⁰ Grammatical corrections to respondent comments were only made when meaning was adversely affected. Respondents were not first language English speakers.

ways to make more use of practical resources in their teaching, through the intervention. Others voiced their continued fear and concerns about the subject.

... but although we are not good on it but we love it.

Because even our learners they are so difficult to grasp, so you don't know whether it's language or what.

You can say that maths is an interesting subject, but we, including our kids we are afraid of it.

The definitions of mathematics provided by the educators pertained mainly to the use of the subject as it relates to figures and the four basic operations (addition, subtraction, multiplication and division) as used in our daily lives.

But if I can define it [mathematics], with the knowledge I've got - I can say mathematics is measurement, because everything you measure is mathematics included. It can be information because you can get information from the radio, bearing in mind that it's four o'clock now, it's use, so now I'm using a watch through mathematics. There are so many things that I can say about defining, it can be measurement, I can say the distance, the counting ..learners can count, they can count change, when they get into the bus they must know the bus here from to town it's R10, it's R9.50, so they must know if I gave them R10 they must know that there's 50 cents change. So that is how mathematics works to me.

Maths is a subject dealing with numbers and measurements. It is used daily in our lives e.g. when buying groceries.

Only one of the educators alluded to it in the sense of "problem-solving" and another to the benefits of mathematics in improving the thinking of learners.

... maths to me as a whole it, is dealing with problem solving. It's true, the main concept of maths is to solve the problem.

... just in short I can say - I would say mathematics creates fast thinking in our pupils, they think very fast. So they will think very fast.

Data collected from the interviews were supported by observations from the fieldworkers who observed the educators teaching lessons. Out of the 25 classes observed, most of the educators explained the work by means of showing the learners examples. In 16 of these lessons, the educators used examples relating to real life situations, while in 15 of the classes fieldworkers also observed learners solving contextual problems relating to their

lives. However, only seven of the classes showed learners having the opportunity to negotiate meaning through discussing their understanding of concepts and strategies for solving problems with each other and the educator. In addition to this, learners posing problems to their educator and to each other was only observed in six of the lessons.

What can be concluded from the analysis done in relation to the educators' views and definitions of mathematics is that although the educators believe it is an important and worthwhile subject, they are not all very comfortable or confident teaching it. This could be due to a limited level of content knowledge as depicted in many of the definitions offered by the educators of what mathematics is. An emerging trend though is that educators are making an effort to teach the subject in a practical manner and to make it as relevant as possible to the daily lives of learners.

While it was encouraging to see educators moving towards a more practical approach to teaching mathematics, it concerned me because this encompasses only a small part of the scope of mathematics as a subject. In the National Curriculum Statement (NCS) for the FET phase the Department of Education provides the following definition for mathematics (DoE, 2003, p.9):

Mathematics is a human activity that involves observing, representing and investigating patterns and quantitative relationships in physical and social phenomena and between mathematical objects themselves. Through this process, new mathematical ideas and insights are developed. Mathematics uses its own specialised language that involves symbols and notations for describing numerical, geometric and graphical relationships. Mathematical ideas and concepts build on one another to create a coherent structure. Mathematics is a product of investigation by different cultures – a purposeful activity in the context of social, political and economic goals and constraints.

It is my understanding that this definition, as well as the purpose, unique features and scope of mathematics as provided in the NCS (DoE, 2003) is calling for more than a greater emphasis on a practical approach to teaching mathematics. The definition and purpose require educators to apply a range of teaching and learning strategies so that learners can gain the full benefit of mathematics. I therefore began to question what it is that either enables or limits educators from being more flexible in the range of teaching and learning strategies they apply in their classrooms. Reflecting on the data obtained

from the evaluation outlined above, I noticed that this particular sample of educators did not seem resistant to making changes in their teaching strategies. They also felt that the resources and training provided during the course of the intervention had enabled them to be more practical in their teaching. I decided to further analyse the definitions teachers provided for mathematics to gain further insight into their conceptions and knowledge of mathematics. As the literature suggests (Ball, 1990; Ma, 1999), teachers' conceptions and understanding of what mathematics is could be a factor limiting the optimisation of a broader range of teaching and learning strategies within their classrooms. Many of the educators interviewed emphasized the practical day-to-day uses of mathematics when stating their definitions for the learning area. Classroom observations provided evidence of a greater emphasis on this practical aspect in their teaching. I began to see an articulation between educators' knowledge of mathematics, how they acquire this knowledge and how this manifests in relation to the range of teaching and learning strategies they employ in their classrooms. This awareness became foundational to the conceptualisation of this study.

In summary, there are three main parts that constitute the background to this study. Firstly, mathematics pre-service teachers I was training demonstrated limited depth of mathematical understanding that appeared to constrain them in the traditional approach to the teaching and learning of mathematics. Secondly, the international, regional and national performance of South African learners does not demonstrate a trend of strong mathematics learners. Finally, teachers in an in-service programme began adopting a more practical approach to the teaching of mathematics. However, their conceptions of mathematics appeared to limit broader and deeper changes in their practices aligned with the definition of mathematics as defined by the new curriculum in South Africa. This background led me to adopt two assumptions underpinning this study. Improving the mathematics performance of learners in South Africa requires a focus on the training of mathematics teachers. This training should consider the complexity of the mathematics "make-up" of the teacher, including their content knowledge and conceptions of mathematics and their beliefs about the teaching and learning thereof. The challenge of this complexity led me to the literature on content knowledge for mathematics teachers, which is expanded in the following section.

1.2.4 Consulting the literature

To begin the process of searching for relevant literature on the content knowledge of mathematics teachers, I used a combination of the following keywords (content knowledge; mathematics; education; pre-service; student teachers) and initiated a search on various internet search engines and academic databases. This led me to a paper entitled "Developing measures of teachers' mathematics knowledge for teaching" by Hill, Schilling and Ball (2004). The article contains an overview of literature on content knowledge for teaching which was most helpful in setting me off on a literature "trail".

The literature trail led to me to more work, mostly by Ball (1988a; 1988b; 1990; 1991) on mathematics knowledge for teaching. Ball and her colleagues draw on Shulman's contribution (1986) of pedagogical content knowledge as well as the well-known work of Ma (1999). Other authors, such as Grossman, Wilson & Shulman (1990) and Leinhardt and Smith (1985) are also regarded as experts on the research in this regard.

Through the literature trail it became evident that the term "content knowledge" is generally accepted as being more loaded than one's knowledge of mathematical content. Shulman (1986), for example, distinguishes three categories of content knowledge: subject matter content knowledge, pedagogical content knowledge and curricular knowledge. Ball (1990) differentiates between the execution of a mathematical operation and the teacher's ability to represent that operation accurately for learners. She therefore coined the terms "knowledge of mathematics" and "knowledge for mathematics". Her later work, supported by other researchers such as Hill and Bass, attempts to identify, measure and address the mathematics knowledge necessary for teaching.

Leinhardt and Smith (1985) suggest that the most important two distinctions one should make regarding content knowledge of teachers relates firstly to their lesson structure knowledge and secondly to their subject matter knowledge. Grossman et al. (1990), on the other hand, extended the number of categories to four. They suggest subject matter knowledge, general pedagogical content knowledge, pedagogical content knowledge and knowledge of the context. Ma (1999) did not define categories. She studied the profound understanding of fundamental mathematics in order to compare the subject matter school knowledge of elementary mathematics teachers in the United States and China.

In a current research project in South Africa, known as the Quantum Project (e.g. Adler, Davis, Kazima, Parker & Webb, 2005; Adler & Davis, 2006a; Adler & Davis, 2006b; Adler & Pillay, 2007), Adler and her colleagues investigate and describe mathematics for teaching within an in-service training context. Their project is mostly focused on middle and senior school mathematics teachers, foregrounding what mathematics they need to know and their knowledge of how to use this mathematics in order to teach mathematics effectively in diverse classroom contexts. Adler and Pillay (2007) summarise mathematics for teaching as “the mathematical ‘problems’ a teacher confronts, the knowledge resources he [the teacher] draws on to solve these problems and the teacher’s explanations of why he does what he does” (p. 16).

Reflecting on my puzzle through the lens I had now constructed from the literature, I first decided that the term “subject matter knowledge” was most appropriate for the particular input that I wanted to investigate. It is central to all the findings emerging from the “content knowledge” literature and depicts the specific construct I planned to examine more closely. My aim was therefore initially to study the classroom practice of pre-service secondary school mathematics teachers in order to ascertain how their subject matter knowledge manifests in their classroom practice.

However, this term did not fully embrace my experience that not only subject matter knowledge but also students' conceptions of mathematics play an influential role in the teaching practice they adopt in the teaching and learning of mathematics. This also ties in with literature where the relationship between single components such as subject matter knowledge (Ball, 1990) or conceptions (Thompson, 1984; 1992) and instructional behaviour is not a simple one. I was also concerned about the limitation of only looking at students' classroom practice instead of also investigating how they think about teaching mathematics and what theories and beliefs they subscribe to in this regard. Reviewing the literature again, I identified a pattern in the research of four main components that researchers have investigated in relation to the instructional behaviour or classroom practice of mathematics teachers. These are the subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and beliefs about the teaching and

learning of mathematics. Each of these components is discussed in more detail in chapter 2.

I therefore broadened the focus of my study to look at the mathematics “make-up” or profiles of the students in relation to their instructional behaviour that they develop as student teachers. The Oxford Dictionary (1994, p. 637) defines the word "profile" in its noun form as:

- A drawing or other representation of this;
- A side view, especially of the human face;
- A short account of a person's character or career.

This definition encouraged me to construct the term "mathematics profile". As I view it, the pre-service teachers all present the "faces" of their professional development through their final portfolios. Looking back on the data, I am taking on a side rather than front view. The term "mathematics" indicates my intention to focus this profile on data from their PGCE year that are possible indicators of their mathematical knowledge, understanding, beliefs, experiences and performance. The mathematics profile of each participant is depicted by a narrative and visual representation of their subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and their beliefs relating to the teaching and learning of mathematics.

I was also not satisfied with the term “classroom practice”. This is of course a broad term to define and in general terms could be understood to be what happens in a classroom. The European literature (see for example, Brosseau, 1997; Goffree, Oliveira, Serrazina & Szendrei, 1999) often describes classroom practice by the components of the so called didactic contract or didactical triangle between the learner, the teacher and the subject matter and the interaction between these components. This practice includes classroom management, administration, instructional practices, discipline, assessment practices, questioning techniques, communication between teachers and learners, time on task, planning, learning environment and media, to mention a few.

However, this study aims to determine a relationship between the mathematics “make-up” of pre-service teachers and the approach to teaching and learning they adopt during

their school-based practices. I therefore needed a term that would put the focus specifically on the teacher and their instruction, rather than on what was generally happening in the classroom. I have therefore selected the term *instructional behaviour*¹¹ to denote actions, decisions and interpretations the participant makes in the classroom. This particular construct is limited to observable behaviour and, where necessary, I also consider reference to applicable reflections by the participant being observed. The two main components of the instructional behaviour construct are: the type of teaching stance that the pre-service teachers adopt and the approach to learning that they encourage from their learners.

1.3 Problem Statement

This study seeks to investigate the relationship between the mathematics profiles of secondary school pre-service mathematics teachers and the instructional behaviour they develop in the teaching and learning of mathematics.

1.3.1 Rationale

In South Africa, the last ten years have been full of an educational reform initiative that was conceived after the demise of apartheid. This educational reform has been driven by two imperatives: firstly the need to overcome the damage done by apartheid, and provide a system of education that builds democracy, human dignity, equality and social justice and secondly to establish a system of lifelong learning (DoE, 2002).

In order to do this, one of the key policies created to facilitate this process in South Africa was Curriculum 2005 (C2005), which:

... envisaged for general education a move away from a racist, apartheid, rote learning model of learning and teaching to a liberating, nation-building and learner centred outcomes-based one. In line with training strategies, the re-formulation is intended to allow greater mobility between different levels and institutional sites, and the integration of knowledge and skills through "learning pathways." (DoE, 2002, p. 9)

¹¹ I borrowed this term from Thompson (1984) who conducted a similar study, but focused on the relationship between the mathematics conceptions of teachers and their instructional behaviour.

In addition to this, C2005 also defined a set of critical and developmental outcomes that are intended to overarch all programme development. All learning programmes and assessment standards in curriculum design are required to express these critical outcomes in the various defined fields of learning, of which mathematics is one. The critical outcomes include skills such as problem-solving, critical thinking, working in teams, communicating and using science and technology (DoE, 2002). The principles of Outcomes-based education have been employed in defining these outcomes in the curriculum and underpin the design and intended implementation of the new curriculum.

This implementation has been fraught with challenges, one of which has been and continues to be the training of teachers. Teachers are ultimately responsible for defining and delivering the curriculum at classroom level (Hargreaves, 1989) and a grasp of the relationship between teachers and the curriculum and to curriculum reform is therefore vital. Teachers' beliefs and knowledge of a subject may have a direct impact on their decisions, which in turn could affect the classroom instruction they embark on (Ernest, 1991).

Research projects that have been carried out in South Africa since the introduction of OBE and the new curriculum (see for example Howie, 2002; Howie, Barnes, Cronje, Herman, Mapile & Hattingh, 2003; Barnes, 2004; Venter, Barnes, Howie, & Janse van Vuuren, 2004; Aldous-Mycock, 2008) indicate that the type of classroom instruction dominant in many mathematics classrooms in South Africa does not resemble the intended curriculum or philosophy as outlined in our reform policy documents. We know from existing literature that a strong relationship between teachers' content knowledge and how they teach has certainly been empirically established in research done in the United States, predominantly in Elementary and Primary schools. In South Africa, the work being done by Jill Adler and her colleagues (see Section 1.2.4) focuses on middle and senior school mathematics with in-service training. The empirical gap I have therefore identified in the research is one that focuses on pre-service teachers in the secondary (high school) phase. The conceptual gap I am researching focuses on the relationship between not just one component (such as subject matter knowledge) of pre-service mathematics teachers and their instructional behaviour. Rather I am trying to

understand the relationship between the complexity of their mathematical make-up (or profiles) and their instructional behaviour.

Beyond a personal interest, I believe this research can have an impact on the way we train our pre-service mathematics teachers for the FET phase. It could also inform the continued support we could provide to beginning teachers during their first few years of teaching. My intention is that this study should produce rich data that will help us further understand the influence of pre-service teachers' subject matter knowledge, pedagogical content knowledge and their conceptions and beliefs about mathematics as a whole, on their resulting instructional behaviour. This in turn will hopefully shed more light on furthering our progress in solving the quest for optimum pre-service training of mathematics teachers. If South Africa can produce more effective mathematics teachers, the opportunities to improve learner achievement are much greater. The results of the TIMSS studies conducted in 1995, 1999 and 2003 (see for example Howie, 2002) revealed that this is a domain within education which remains a great challenge to our education system. With the introduction of Mathematical Literacy as a compulsory subject for all FET learners from 2006, the need for effective mathematics teachers is even more foregrounded.

In summary, the rationale for the study is embedded in a personal interest and experience of working with and training mathematics teachers, an empirical gap in the research literature on teachers in the secondary phase, and an intention to add value to the pre-service training programmes of secondary school mathematics teachers at tertiary institutions. The research questions that guide the inquiry follow.

1.3.2 Research Questions

The research questions configured to direct the study consist of a main research question that has been divided into subsidiary questions that will help to operationalise the inquiry.

The main research question is as follows:

*How does the **mathematical profile** of a pre-service teacher of mathematics influence **instructional behaviour**?*

To address this main question, the following subsidiary questions guide the inquiry:

- a) How are the mathematics profiles of PGCE pre-service mathematics teachers reflected in their instructional behaviour?
- b) What similarities or incongruities are there between the pre-service teachers' instructional behaviour and the mathematics profiles they portray?
- c) Are differences among the pre-service teachers in their instructional behaviour related to differences in their mathematics profiles?

1.4 Conceptual framework

The theoretical underpinning of my own instructional behaviour is premised on a social constructivist framework. This therefore implicitly influenced my initial search for and selection of literature. However, as the study progressed and my literature base gained depth and breadth, I began to examine other theories such as the theory of educational change (e.g. Fullan, 1982; 1995), sociocultural theory (e.g. Lave, 1988), symbolic interactionism (e.g. Blumer, 1969), constructivism (e.g. Piaget, 1970) and radical constructivism (e.g. von Glaserfeld, 1984; Steffe & Kieren, 1994) as possible lenses for doing the analysis and discussing the results.

The two main constructs of this study focus on the individual (namely the pre-service teacher) and therefore one could argue that constructivism would have been an appropriate underpinning theory for the study. However, in this study the participants exist and develop within a number of social contexts: the main contexts being the PGCE course and the classrooms within which the pre-service teachers conduct their school-based experiences. This adds the additional dynamic interplay of lecturers, fellow students, mentor teachers, learners and the subject of mathematics. I therefore considered sociocultural theory as an alternative option. Sociocultural theory makes it possible to characterise mathematics as a complex human activity by foregrounding meaning through an emphasis on taken-as-shared meanings, instead of on socially accepted ways of behaving. However, historically this stance assumes that the developed disciplines of mathematics, teaching and learning exist independently of the pre-service teachers and the learners. In this study, practice is viewed as an emergent phenomenon as opposed to

an existing manner of reasoning and communicating (Cobb, Stephan, McClain & Gravemeijer, 2001). Social constructivism solved this dualism for me in its acknowledgement of the development of the participants and their interaction with mathematics and the social contexts of the classroom and their PGCE course.

The literature review not only confirmed my choice of social constructivism as the overarching theory being applied, but also assisted me in developing a more focused conceptual framework to apply in dealing with the complexity of the constructs being studied within the cases. The conceptual framework draws extensively on the work of Ernest (1988, 1991, 1998) in analysing the two main constructs of mathematics profiles and instructional behaviour. However, where there was not sufficient literature in Ernest's work, the conceptual framework was supplemented by other authors such as Ball (1988a, 1988b, 1990, 1991), Thompson (1984, 1992), Shulman (1986), Mason (1989), Veal and MaKinster (2001) and Davis (1997). The literature reviewed and the resulting conceptual framework are presented in chapter 2.

There are a number of studies in the literature (as cited in chapter 2) that explore the relationship between one of the components, for example, *conceptions of mathematics and the teaching thereof* and *instructional behaviour* (Thompson, 1984) or the level of *subject matter knowledge* and the quality of *classroom practice* (Leinhardt & Smith, 1985) or *pedagogical content knowledge* and *classroom practice* (Leinhardt, 1989). While much light has been shed on these relationships and the complexities thereof, my aim is to try to present and study the components as a “package” and to explore the relationship between the package and the instructional behaviour. I do not intend to go into the depth on each component as the afore-mentioned studies have done, but to rather explore the complexity of the four components that comprise the mathematics profile. This study therefore investigates the relationship between the participants' mathematics profiles (the “package”) and how this profile relates to the instructional behaviour they exhibit towards the teaching and learning of mathematics.

1.5 Teacher training in South Africa

Qualifying as a teacher in South Africa currently requires one of two possible routes: a Bachelor of Education (four year degree) or an appropriate undergraduate degree (e.g. Bachelor of Science, Bachelor of Arts, Bachelor of Commerce) followed by a Post Graduate Certificate in Education. This pre-service phase is known as the Initial Professional Education of Teachers (IPET), with in-service training being referred to as Continuing Professional Teacher Development (CPTD) in the latest policy documents (DoE, 2006).

Tertiary education in South Africa is outlined in the National Qualifications Framework (NQF) using credits and levels 1 – 8 (South African Qualifications Authority [SAQA], 2000; see APPENDIX A). The PGCE is a 120 NQF credit, Level 6 qualification. According to the *Norms and Standards for Educators* (DoE, 2000) the PGCE is defined as:

...a generalist educator's qualification that 'caps' an undergraduate qualification. As an access requirement candidates are required to have appropriate prior learning which leads to general foundational and reflexive competence. The qualification focuses mainly on developing practical competence reflexively grounded in educational theory (p. 29).

The Council on Higher Education (CHE) conducted a national review of PGCE courses in South Africa during 2006 and 2007 through their Higher Education Quality Committee (HEQC)¹². The document released by the HEQC (2006) on the criteria and minimum standards for PGCE courses stated that a one year full-time or two year part-time PGCE programme should:

- Consolidate subject knowledge and develop appropriate pedagogical content knowledge.

¹² The Council on Higher Education (CHE) is an independent statutory body responsible for advising the Minister of Education in South Africa on all matters related to higher education policy issues, and for quality assurance in higher education and training. The Higher Education Quality Committee is the only permanent committee of the CHE and is responsible for carrying out the quality assurance.

- Cultivate a practical understanding of teaching and learning in a diverse range of South African schools, in relation to educational theory, phase and/or subject specialisation, practice and policy.
- Foster self-reflexivity and self-understanding among prospective teachers.
- Nurture commitment to the ideals of the teaching profession and an understanding of teaching as a profession.
- Develop the professional dispositions and self-identity of students as teachers.
- Develop students as active citizens and enable them to develop the dispositions of citizenship in their learners.
- Promote and develop the dispositions and competences to organise learning among a diverse range of learners in diverse contexts (HEQC, 2006, p. 1).

Students achieving these exit level outcomes should be competent novice teachers who over time, through experience and with the appropriate support will develop as fully-fledged extended professionals. Such professionals (teachers) are required to be specialists in: their particular learning area, subject or phase, teaching and learning, assessment and curriculum development. Each one is also expected to be a leader, administrator and manager, a lifelong learner and a professional who plays a community, citizenship and pastoral role (DoE, 2000).

These are all very general guidelines that are offered by the current policies in guiding tertiary institutions with the training of teachers. Institutions are left to develop their own conceptual frameworks and content for their PGCE programmes. The outline of the PGCE programme that forms the context for this study is presented in chapter 3, Section 3.4.1.

1.6 Research design and methods

As already outlined, the purpose of this study is to investigate how the mathematics profiles of pre-service mathematics teachers influences the instructional behaviour they develop and exhibit during their school-based practice. This implies, firstly, a detailed understanding of their mathematics profiles as well as insight into their instructional behaviour.

When this study was first conceptualised, I had intended to measure the subject matter knowledge of the teachers using only a quantitative instrument. Subsequent readings in the literature led me to broaden the study to the use of mathematics profiles instead, and to the conclusion that the design of the study would benefit from a qualitative nature. Grossman et al. (1990) report on how the earliest research on teacher subject matter knowledge tried to identify statistical relationships between the knowledge of teachers and the achievement of their learners. The subject matter knowledge of teachers was represented either as the number of classes that a teacher had taken in the subject, their grades obtained in the subject or their score on a standardised achievement/performance test. The majority of studies, however, showed no significant relationship and it was suggested that perhaps teacher subject matter knowledge had not been adequately conceptualised (Byrne, 1983 as cited in Grossman et al., 1990) and that it is a complex phenomena that encompasses more than can be measured on a test or by the level or grades of a teacher's qualification. To operationalise my main research question, therefore, I chose a qualitative design for this study within a social constructivist epistemology as outlined in the following section.

1.6.1 Paradigm

Social constructivism is discussed in chapter 2 as a philosophy of mathematics education (Ernest, 1991) as well as a paradigm or worldview which is “a basic set of beliefs that guide action” (Creswell, 2007). This epistemology informs my approach to the teaching and learning of mathematics as well as to research. It was interesting during the study to reflect on the development of the inquiry and the writing up of this study in relation to how I usually approach my instruction of mathematics. Both foreground my ontological assumption that individuals may not share the same “reality” (Creswell, 2007) and therefore multiple perspectives need to be presented. I favour transparent thinking and presenting the challenge, while facilitating the learner (in the case of this study, the reader) through the process of understanding my thinking while also constructing their own autonomous understanding. As previously indicated, the lens of social constructivism guided my literature review, but also later became my chosen theoretical underpinning for the analyses. I suspect this was largely motivated by the fact that social

constructivism also guides my interpretation and approach to the teaching and learning of mathematics.

This type of paradigm views the world as an emergent social process (Burrell & Morgan, 1979) and aims to characterise how people experience the world, ways in which they interact together, and the settings in which these interactions take place (Packer, 2007). It seeks to explain behaviour from the individual's point and understand the subjectively created world "as it is" (Burrell & Morgan, 1979).

1.6.2 Research approach

To provide depth in investigating the research questions, I selected a case study approach and used a convenience sample of seven participants. The participants were selected through convenience sampling based on their willingness to be part of the study, as well as the fact that they were all enrolled for the mathematics specialisation course in obtaining a Post Graduate Certificate in Education (PGCE) at the same university in South Africa in 2006, 2007 or 2008. They completed this one-year post-graduate qualification on completion of their undergraduate degrees, in order to qualify as teachers. I was the mathematics specialisation lecturer for all of the participants. The education backgrounds of the participants were different, but they all followed a similar route to qualify in becoming teachers.

This setting simulates that of a *case study* as defined in the literature on research designs (e.g. Adelman, Jenkins & Kemmis, 1980; Guba & Lincoln, 1981; Merriam, 1988; Cohen, Manion & Morrison, 2000; Yin, 2003). Merriam (1988) cites definitions from various authors who support this, such as a case study being defined as "the examination of an instance in action" (MacDonald & Walker, 1977, p. 181) and a process "which tries to describe and analyse some entity in qualitative, complex and comprehensive terms not infrequently as it unfolds over a period of time" (Wilson, 1979, p. 448). The context of this inquiry is also dynamic and provides a unique example of real teachers in a real classroom situation (Cohen et al., 2000).

More specifically, Bennet and George (1997) refer to the type of case study research I used as the “method of structured, focused comparison” (p. 2). They make the point that:

Comparative case studies can use within-case analysis of individual cases as well as case comparisons to assess and refine existing theories, and more generally, to develop empirical theory. The method of doing is “structured” in that the same general questions are asked of each case in order to guide the data collection, thereby making possible systematic comparison and cumulation of the findings of the cases. The method is “focused” in that it deals with only certain aspects of the cases; that is, a selective theoretical focus guides the analysis of the cases.

The theoretical focus that guides these case studies is to establish the existence, nature and extent of any relationship between the mathematics profiles and instructional behaviour of the participants. The point of departure was to first examine that relationship within the individual cases before comparing the different cases, namely the seven students. Bennet and George (1997) identify this type of theory-building objective as having “heuristic purposes” (p. 5). This includes searching for new variables, hypotheses and causal mechanisms and paths, through an inductive process. They propose that the structure and focus of such studies are more easily attained when a single investigator plans and carries out all of the case studies. The data collection and analysis are further outlined in the sections that follow.

1.6.3 Data collection strategies

This study has been placed within a social constructivist worldview thereby drawing on qualitative data collection and analysis methods. The primary source of data comes from the final portfolios that the pre-service teachers hand in as part of their final summative mark for the PGCE. As indicated, these portfolios contain a selection of personal information such as a storyline, brain profiles, personality tests, daily reflections during their school-based period, learning task designs, video-recordings from their school-based periods, their vision and mission statements on education and any other information they deem important to demonstrate their professional development throughout the year. In addition to this, I also had documents available from a baseline assessment (see Appendix D) on mathematics content that students complete on entering the course as well as assessment reports from lessons I had observed the students presenting. More details on the data set are provided in chapter 3.

1.6.4 Data analysis

Since the inception of this study, the ideas on data analysis evolved as I worked through more literature on other empirical studies conducted in this domain. The dynamic nature of interpretive, qualitative studies allows and encourages the iterative process but my reading and experience were fundamental to the conceptualisation and design of this study.

I analysed the data using a deductive but, to a lesser extent, also an inductive approach. The deductive approach facilitated the indicators and categories already identified in the literature. The inductive approach allowed for the formulation of new themes that came out of the data (see Section 3.6). This means that the scheme for analysing the themes associated with the content become apparent during the analysis itself and are not predetermined as is the case with the deductive approach. This type of inductive analysis (Miles & Huberman, 1994; Creswell, 2003; Gay & Airasian, 2003) allowed me to construct patterns that emerge from the data in order to make sense of them. In such an analysis one usually starts with a large set of issues and, through an iterative process, progressively narrows them down into small important groups of key data. From this data variables are then identified through further examination and analysis that can be interpreted and discussed. This therefore creates a multistage process of organising, categorising, synthesising, interpreting and reporting on the available data (Gay & Airasian, 2003).

1.6.5 Methodological norms

The data collected for this study took the form of video data and documents. My own reflections, thoughts, observations and uncertainties during the course of the study were recorded in a journal to provide an audit trail and assist me in identifying and acknowledging possible personal biases and preferences that affected the data analyses (Gay & Airasian, 2003). Due to the post-hoc nature of the research approach, member checking was not employed with the participants. However, I did use member checking in consultation with two other colleagues with regard to the participant reflections, mathematics profiles and instructional behaviour profiles. For the visual representation of the mathematics profiles I consulted an architect who assisted me to conceptualise and design the symbolic drawings and interrogate their meaning and consistency.

A further source used to increase the trustworthiness of the qualitative data was to draw on literature discussing (where possible) similar and conflicting findings to the outcome of this study (Eisenhardt, 2002). It is also envisioned that the theory building process (Bennet & George, 1997; Eisenhardt, 2002) strengthened the study in its credibility and transferability by the high number of case studies (7 in total) being depicted and compared.

1.6.6 Ethical considerations

As I was the lecturer of the participants, I wanted to ensure that they did not feel coerced or compelled to be part of the study. I also did not want to engage with the power-play element that is present when a lecturer chooses to use their students as participants. I therefore waited until after their final portfolios had been handed in and defended in order to request their permission (see Appendix E for participant consent form) to be part of the study. In approaching the participants, the following steps were followed (Gay & Airasian, 2003):

- The purpose and an outline of the study were provided to them and they were asked if they would consider availing their portfolios and other relevant documentation from their PGCE year as data for this study;
- It was emphasised that their participation was entirely voluntary.
- They were promised full confidentiality and anonymity on events that took place during the study, but were given the option to give full release on the video data for use in public domains such as training and presentations or to limit the use of video data display to this report. Six of the participants signed full release of their video data.

I did not obtain ethical clearance from any of the learners present in the classes that were video-recorded. The reason for this is that they were not the focus of the study. Any data used as evidence here or in presentations arising from the study have been suitably “doctored” or edited to ensure anonymity of the learners. This was mainly done through an editing technique known as blurring.

1.7 Limitations of the study

I was confronted with two initial main limitations of this study. Firstly the fact that I am both a lecturer of the participants and the researcher has an impact on the investigation. Although I did the data analyses on a post-hoc basis, I still had a relationship with each of the students as their lecturer and therefore also formed opinions of them during their PGCE year. The advantage of this situation is that it affords me even further insight into the participants outside of the data being collected. I envision this contributing to the overall depth and richness of the case studies.

The second limitation pertains to the lack of generalisability of case studies. While this study is restricted to one tertiary institution, the participants have gained their undergraduate degrees at a variety of different institutions and represent a variety of gender, cultures and languages. It has not been my intention to generalise these results of individual cases but to add to the body of knowledge on the influence of pre-service teachers' mathematics profiles on their instructional behaviour.

1.8 Outline of the study

This dissertation is divided into seven chapters, each serving an individual purpose, but overlapping and intertwining nonetheless. The first chapter serves as an introduction to the study and its origins. Chapter 2 reports on the literature review, during which the theoretical and analytical frameworks of the study are also foregrounded. Chapter 3 serves as the research design chapter. It firstly establishes the epistemological paradigm of the study before discussing the methodology (case study) and elaborating on the methods to collect and analyse data. The context and sample of the study are also further introduced in chapter 3 and ethical issues as well as issues of quality control are dealt with. The fourth chapter of this report depicts the first data reduction in the form of the participant reflections. In chapter 5 the researcher reflections are included as the second data reduction, followed in chapter 6 by the third data reduction, the visual representation of the profiles. In chapter 6 the cross-case comparison is also discussed. The final chapter reflects on the study and its research process as a whole before making final conclusions and recommendations.

CHAPTER TWO

LITERARY FRAMEWORK

2.1 Introduction

This chapter presents the literary framework. This is the interaction between my epistemological underpinning of social constructivism with the literature reviewed, resulting in a conceptual framework. The conceptual framework is defined by the two main constructs within this study (mathematics profiles and instructional behaviour) and the complexity of their relationship. Within the conceptual framework the various components of each of the main constructs were identified through the literature review and draw mainly on the work of Ernest (1988, 1991, 1998) supplemented by the work of other researchers.

The literature review informed the proposed conceptual framework for the study but was also initially informed by social constructivism as the lens through which I regard my own teaching and research. An iterative process of reviewing developments relating to research within mathematical content knowledge¹³ was first studied, followed by a synthesis of recent empirical studies that are relevant to this domain and the broader range of components that may influence the classroom practice of teachers. Further literature on theories and approaches to the teaching and learning of mathematics was also then explored. Subsequently the theory of social constructivism was chosen as the preferred overarching theory. The literature was then again reviewed with regard to this epistemological underpinning. It presents both the position I take on the teaching and learning of mathematics as well as offering the necessary interpretive framework for this research. From this above-mentioned iterative process, the conceptual framework for analysing the data was constructed.

Consequently this chapter firstly discusses social constructivism as the overarching epistemological underpinning for this study. A synopsis of the literature review is then

¹³ Although the focus of this study is on mathematics profiles, I use the phrase content knowledge purposely here to denote the more comprehensive domain of pedagogical content knowledge and subject matter knowledge.

provided, leading into the conceptual framework that was constructed to guide the presentation and analysis of the data.

2.2 Social constructivism

Ernest (1991, 1998) suggests social constructivism as a philosophy of mathematics and also discusses it as a philosophy of mathematics education. Through this lens mathematics is viewed as a social construction and knowledge is a result of a process of coming to know including processes leading to the justification of mathematical knowledge. Ernest (1991) mentions three philosophical perspectives as a basis for his “unified philosophy of mathematics” (p. 85) of social constructivism namely quasi-empiricism, conventionalism and radical constructivism. From quasi-empiricism social constructivism takes the fallibilist epistemology, including the view that mathematical knowledge and concepts develop and change. From conventionalism, it draws on the notion that human language, rules and agreement play a key role in establishing and justifying the truths of mathematics. However, the most central claim of social constructivism is that “no certain knowledge is possible, and in particular no certain knowledge of mathematics is possible” (Ernest, 1991, p. 89), which has its origins in radical constructivism. These latter two tenets of conventionalism and radical constructivism may seem to contradict each other and Ernest reconciles this contradiction with the following explanation:

Thus although the primacy of focus of each of conventionalism and radical constructivism is sacrificed in social constructivism, their conjunction in it serves to compensate for the individual weaknesses, yet this conjunction raises the question as to their mutual consistency. In answer it can be said that they treat different domains, and both involve social negotiation at their boundaries. Thus inconsistency seems unlikely, for it could only come about from their straying over the interface of social interaction, into each other's domains (p.86).

Ernest (1991) also foregrounds the relationship between objective and subjective knowledge (see Figure 2.1) as part of his theory of social constructivism. This view places subjective and objective knowledge in mutually supportive and dependent positions. I offer a summarised overview of the distinction Ernest (1991) makes between subjective and objective knowledge here. For a more detailed explanation see Ernest (1991, 1998).

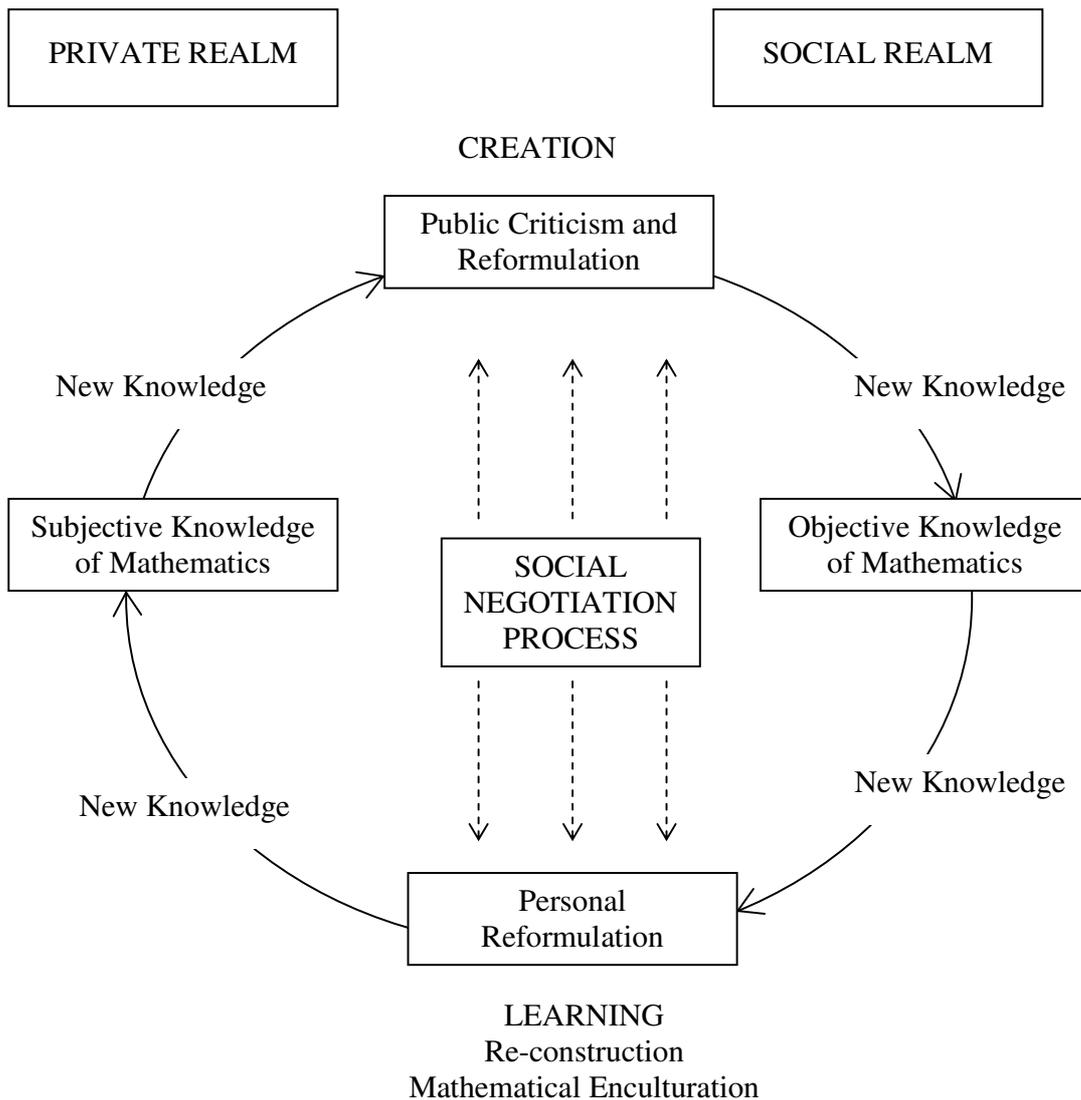


Figure 2.1 *The relationship between subjective and objective knowledge (Ernest, 1991)*

Ernest (1991) describes *subjective thought* as the mathematical thought of an individual (both the process and its product, mathematical knowledge). This is mostly learned or reconstructed objective knowledge, but it is subject to certain powerful constraints in that the process of re-creation results in unique subjective representations of mathematical knowledge. Individuals then use this knowledge to construct their own, unique mathematical productions which leads to the creation of new subjective mathematical knowledge.

In order for an individual's subjective mathematical knowledge production to become objective, it must first enter the public domain through publication. This allows it to become scrutinised and criticised by others which may result in its reformulation and acceptance as *objective* (socially accepted) *knowledge* of mathematics, although this objective knowledge still always remains open to challenge. During the "genesis of mathematical knowledge" (p. 84), objective criteria are used in the critical scrutiny of mathematical knowledge. These include shared ideas of basic inference and other basic methodological assumptions. These criteria rest ultimately on the common knowledge of language (linguistic conventions) which are also socially acceptable. Ernest (1991) therefore sums objective knowledge up as both "published mathematical knowledge and the linguistic conventions on which its justifications rest..." (p. 84).

Ernest (1991) uses Popper's (1979) definition of three distinct worlds, and the associated types of knowledge to clarify his distinction between objective and subjective knowledge. According to Popper (1979, p. 74, as cited in Ernest, 1991):

We can call the physical world 'world 1', the world of our conscious experiences 'world 2', and the world of the logical contents of books, libraries, computer memories, and suchlike 'world 3'.

Ernest (1991) places subjective knowledge as a world 2 knowledge and objective knowledge as a world 3 knowledge, which includes products of the human mind, such as published theories, discussions of such theories, related problems and proofs. All of these are human-made and changing and in mathematical terms include theories, axioms, conjectures and formal and informal proofs. Ernest (1991) then also adopts the social theory of objectivity as offered by Bloor (1984) to extend objective knowledge to also include shared (but possibly implicit) conventions and rules of language usage. According to Bloor (1984, p. 229 as cited in Ernest, 1991):

Here is the theory: it is that objectivity is social. What I mean by saying that objectivity is social is that the impersonal and stable character that attaches to some of our beliefs, and the sense of reality that attaches to their reference, derives from these beliefs being social institutions.

I am taking it that a belief that is objective is one that does not belong to any individual. It does not fluctuate like a subjective state or personal preference. It is not mine or yours, it can be shared. It has an external thing-like aspect to it.

This places objective knowledge and its rules outside of individuals (in the community) where, like culture, it develops autonomously in keeping with tacitly accepted rules rather than the arbitrary dictates of individuals.

Creswell (2007) depicts social constructivism as a worldview in which individuals

... seek understanding of the world in which they live and work. They develop subjective meanings of their experiences – meanings directed toward certain objects or things. These meanings are varied and multiple, leading the researcher to look for the complexity of views rather than narrow the meanings into a few categories or ideas (p. 20).

Research in this worldview relies on the participants' views of the situation. These subjective meanings of individuals are formed through interaction with others and through historical and cultural norms that operate in individuals' lives. In order to understand these historical and cultural settings, constructivist researchers focus on the specific contexts in which people live and work. It is therefore also important for the researcher to recognize and acknowledge how their interpretation flows from their own personal, cultural and historical experiences. My intent in this study was to make sense of the meanings participants have relating to the main constructs, but this interpretation was shaped by my own background and experiences. These are further outlined in chapter 3.

2.3 Literature review

In the initial design of the study, I focused the literature review on subject matter knowledge as one of the two main constructs, the other construct being classroom practice. However, as the study proceeded and the two main constructs evolved into *mathematics profiles* and *instructional behaviour*, the literature review had to be broadened. Not discarding the literature I had already synthesized on subject matter knowledge, I went back to the literature and started to look for additional studies on pre-service mathematics teachers as well as other studies researching the components of the mathematics profile.

The components of the mathematics profile construct as I define it, appear in the literature within studies focusing on one or two components, for example, subject matter knowledge (for example Ball, 1988a, 1988b, 1990, 1991, 2002), beliefs and conceptions

of mathematics (Thompson, 1984, 1992), pedagogical content knowledge (Shulman, 1986) or classroom practice (Cobb et al., 2001). I could not find a similar study where the complexity of mathematics profiles of pre-service teachers had been constructed in order to determine the influence thereof on the instructional practices they develop as student teachers. However, a number of researchers acknowledge (for example, Ball, 1988a; Fenemma & Franke, 1992; Nespor, 1987) that the interaction between quality of teaching and learning and aspects of the teacher, such as subject matter knowledge, beliefs, etc. is a complex one. It is my aim to try and embrace some of that complexity within this study. However, as I examined a number of components to make up the mathematics profiles of the participants, it is not possible to get an in-depth view and analysis of each. I have opted rather to go for a broader (and therefore possibly less accurate) description of each in order to foreground the mathematics profile as a whole rather than the individual components.

The most closely related empirical study I identified was an ongoing study conducted by Rowland and his colleagues from Cambridge as well as Thompson's (1984, 1992) work on conceptions. Rowland, Martyn, Barber and Heal (2001) looked at how the subject matter knowledge of pre-service primary teachers manifests in their classroom practice. Thompson (1984) studied the relationship of teachers' conceptions of mathematics and the teaching thereof to instructional practice. I report on their work later on in this section. This gave me a good starting point from which to build my review.

From there I sought other scholarly work (mainly within the domain of mathematics education) pertaining to the various components that make up the two main constructs. I drew largely on the work of Ernest (1988, 1991, 1998) in developing the conceptual framework in this regard. For the mathematics profile construct, I discuss the following components identified in the literature and define them for the purpose of this study: subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and beliefs regarding teaching and learning of mathematics. The instructional behaviour construct draws on literature relating to classroom practice and the components contained therein are: teacher's ideology (teaching approach) and learners' mathematics experiences (learning approach).

2.3.1 Subject matter knowledge

Leinhardt and Smith (1985) offer a basic definition of subject matter knowledge that is still quoted by more recent researchers in this domain (Hill, Schilling & Ball, 2004). They define it as including “concepts, algorithmic operations, the connections among different algorithmic procedures, the subset of the number system being drawn upon, the understanding of classes of student errors, and curriculum presentation” (p. 247). Both Shulman (1986) and Grossman et al. (1990) expanded this definition to include the syntactic and substantive structures of a subject. Drawing on the work of Schwab (1978), they identified substantive structures as the different ways in which the fundamental principles and concepts of a discipline are organised (Shulman, 1986), that guide inquiry in the field and enable one to make sense of the data (Grossman et al., 1990).

The syntactic structure relates to the set of rules that assists one in determining what is true or false, valid or invalid within a discipline (Shulman, 1986). New knowledge or claims can be deemed legitimate or unwarranted through these rules. Syntactic structures also consist of the tools of inquiry within a discipline (Grossman et al., 1990). Grossman et al. (1990) then also included an additional dimension into their view of subject matter knowledge which relates to teachers’ beliefs about and orientation towards the subject matter. In her work Ball (1988b) makes a differentiation between knowledge *of* mathematics (knowledge of concepts and ideas, and how they work) and knowledge *about* mathematics (for example how one decides that a solution is correct). Grossman et al. (1990) refer to these two collectively as content knowledge for teaching.

Dewey (1983) claimed that “every study of subject thus has two aspects: one for the scientist as a scientist; the other for the teacher as teacher” (p. 273). Teachers do not just teach, they teach a specific subject. Their knowledge therefore needs to extend beyond just the tacit knowledge of that subject to a more explicit knowledge (Ball, 1991) that enables them to make the subject accessible to their learners. It is not uncommon to find pre-service teachers who hold a high qualification in mathematics, who appear to get answers right when they do mathematics and yet do not show advanced proficiency in connecting underlying concepts, principles and meanings (Ball, 1988a). It is therefore important to not only look at the knowledge pre-service teachers have about mathematics but the conceptual depth of this knowledge and how it is organised. As Grossman et al.

(1990) concluded, when teachers demonstrated a deeper knowledge, this resulted in more emphasis on conceptual explanations in their teaching. Concurring with Leinhardt and Smith (1985), they agreed that teachers who displayed a better organisation of subject matter knowledge tended to be more effective in their teaching. In this study, I have therefore sought to evaluate the subject matter of the participants in terms of the depth and organisation thereof, rather than how much mathematics they know.

Ball's work (later assisted by other colleagues) has made a good theoretical contribution to literature on subject matter knowledge in mathematics during the last two decades. In her initial work, Ball (1988a) challenged three existing myths on the preparation of prospective mathematics teachers by studying 19 teacher education students' knowledge of mathematics relating to the topic of division. She analysed their substantive knowledge along three qualitative dimensions, namely the value of truth in their knowledge, the legitimacy of their knowledge and the connectedness thereof. She firstly challenged the myth that "traditional school mathematics is simple" (p. 32) by showing that even students majoring in mathematics struggled when required to work below the surface of simple maths. While these students could perform procedures, they seemed to lack the warranted understanding of the content. They would for example know how to "invert and multiply" when required to do division by fractions but not be able to provide any mathematical explanation for why this procedure is valid (see Section 1.2.1 where I experienced a similar phenomenon with my students). The second assumption she contested was that "elementary and secondary school math classes can serve as subject matter preparation for teaching mathematics" (p. 33). She found that when teacher candidates tried to respond to tasks and questions drawing on what they had learnt in school, they typically exhibited loose fragments in their knowledge and understanding. Most of them did not display meaningful understanding. The third myth she opposed was that "majoring in mathematics ensures subject matter knowledge" (p. 33). Some of the students in her study were mathematics majors and had obviously done more maths than some of the other students. Although these students appeared to know more (in that they got more of the answers right), the additional studies did not seem to afford them any significant advantage in explaining and connecting underlying concepts, principles and meanings. This work of hers is important in my study in that a departure point of this investigation is one that stands on the falsehood of these very myths.

Ball (1988b, 1990) then went on to develop a framework for understanding what prospective mathematics teachers know and believe when they enter teacher education. She used interviews and structured tasks to explore the students':

- knowledge of and about mathematics
- ideas about the teaching and learning of mathematics
- feelings about themselves in relation to mathematics

She then presented the thesis that “teachers’ subject matter knowledge interacts with their assumptions and explicit beliefs about teaching and learning, about students, and about context to shape the ways in which they teach mathematics to students” (Ball, 1991, p. 1). She developed this argument in three parts. Firstly she analysed past investigations of the role of teachers’ subject matter knowledge in teaching mathematics. Secondly by unpacking the concept of subject matter knowledge for teaching mathematics and what is entailed in finding out what teachers know, and finally by presenting three case analyses of teachers’ understanding of mathematics as displayed in their teaching of multiplication.

Her work has since gone on to focus on the subject matter preparation of teachers (Ball & Cohen, 1999; Ball & McDiarmid, 1990), intertwining pedagogy with knowledge (Ball, 2002; Ball & Bass, 2000) and how to go about measuring teachers’ mathematics knowledge specifically for teaching (Hill et al., 2004). Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball (2008) examined the relationship between five teachers’ knowledge for teaching and the mathematical quality of their instruction. Their study illuminated claims that teachers’ mathematical knowledge plays an important role in their teaching of the subject.

Finally the work of Skemp (1971, 1989) on understanding also plays an important part in evaluating the mathematics subject matter knowledge of students in this study. Skemp differentiates between *relational* and *instrumental* understanding. Instrumental understanding on the one hand, he suggests is "rules without reasons" in that learners may possess the necessary rules and ability to use them, without actually comprehending why or how that rule works. Often learners will need to memorise more and more of these rules in order to avoid errors and this type of understanding therefore encompasses a

"multiplicity of rules rather than fewer principles of more general application" (1989, p. 5). Relational understanding, on the other hand, involves integrating new ideas into existing schemata and understanding both "what to do and why". This building up of a schema (or conceptual structure) becomes an intrinsically satisfying goal in itself and the result is, once learnt, more lasting. Skemp (1989) uses an analogy of a stranger in a town to differentiate between the two types of understanding. One could have a limited number of fixed plans that take one from particular starting locations to particular goal locations in the town. He provides this as an example of instrumental understanding. On the other hand one could have a mental map (schema) of the town, from which one can produce, when needed, an almost infinite number of plans to guide one from a starting point to a finishing point, provided only that both can be imagined on the mental map (relational understanding).

Other research I also found useful in the domain of subject matter knowledge in mathematics is the work of Tim Rowland and his colleagues in the United Kingdom¹⁴. Although they are working with primary school teachers, their study supported the rationale for this study. Their research provided statistical evidence that sound knowledge of mathematics topics¹⁵ is associated with more competent teaching of mathematics in the case of pre-service primary school teachers (e.g. Rowland, Martyn, Barber & Heal, 2001). Similarly they were also able to relate weak subject matter knowledge with less competent teaching of the subject. When a similar study was carried out in Ireland though (using the same instruments), they were not able to establish any significant association between a quantitative measure of the subject matter knowledge of pre-service primary teachers and their teaching performance (Corcoran, 2005).

2.3.2 Pedagogical content knowledge

This phrase was coined by Shulman (1986, 1987) when he started asking questions about how subject matter is transformed from the knowledge of the teacher into the content of instruction. In order to investigate this, he worked with colleagues on a research

¹⁴ Their project is known as SKIMA (subject matter knowledge in mathematics) and is ongoing collaborative work between researchers at the universities of Cambridge, London, Durham and York.

¹⁵ This includes topics that extend beyond those found in the primary curriculum

programme aimed at addressing issues such as knowledge of teaching, how teachers decide what to teach, the questions they ask and the explanations and content they provide in their lessons. In his study, he acknowledged along with other researchers in this domain (e.g. Leinhardt & Smith, 1985; Grossman et al., 1990) the fallibility and inaccuracy of administering achievement tests as the index of teacher knowledge. Instead they followed participants (secondary teachers in English, biology, mathematics and social studies) through their post-graduate teacher-education year as well as into their first year of teaching where possible. The theoretical framework that emerged from their inquiry into how content knowledge grows in the minds of teachers distinguished between three categories of content knowledge, namely, subject matter knowledge, pedagogical content knowledge and curricular knowledge. In this section I will foreground their discussion of pedagogical content knowledge as a means to defining this construct for the purpose of this particular study.

While subject matter content knowledge focuses on the facts, concepts, connections, structures and syntax of a subject, pedagogical content knowledge also includes the subject matter knowledge for teaching. As Shulman (1986) puts it:

Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations – in a word, the ways of representing and formulating the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. (p. 9)

Also included in his explanation of pedagogical content knowledge is an understanding of what makes the learning of specific topics easy or difficult for learners of different ages. This encompasses knowledge of common misconceptions and the errors learners typically make (Hill et al., 2004). This means teachers need to have strategies to call on in order to assist learners in re-organising their understanding, depending on the conceptions and preconceptions brought into the subject by learners (Shulman, 1986).

Leinhardt and her colleagues (Leinhardt, 1989; Leinhardt, Putnam, Stein & Baxter, 1991) analysed teachers' pedagogical content knowledge and reasoning using constructs of "script", "agenda" and "explanation". The "script" acts as an organising structure that

underpins the planning of the lessons. It consists of the goals, tasks and actions for a particular curricular topic and incorporates sequences of action and argumentation, relevant representations and explanations and markers for anticipated learner problems. The lesson “agenda” fits into the script and is a mental plan that guides lesson outcomes, how to achieve these and the order thereof and important decision points in the lesson. Within the script is an “explanation” of each new idea and these include the teachers’ systematic organization of learners’ experiences designed to help them construct a meaningful understanding of the concept or procedure. This may include actions of the teacher as well as managing contributions from learners.

In the PGCE course, students are required to provide similar documentation as part of their planning for their school-based practices. In the mathematics specialisation module, students develop a Learning Task Design (LTD) for each topic or section of work. This corresponds to the “script” used by Leinhardt. The LTD’s are broken up into individual planning for each lesson which parallels this concept of “agendas”. Within the individual lesson plans, students are required to outline their role as well as that of the learners, which has aspects of the “explanations” described above.

Mason (1989) suggests six levels of mathematical process that provide a basis for designing mathematics assessment and a technique for helping learners make sense of a topic for themselves through forming and verifying their own meanings. A picture of these levels is presented below in Figure 2.2.

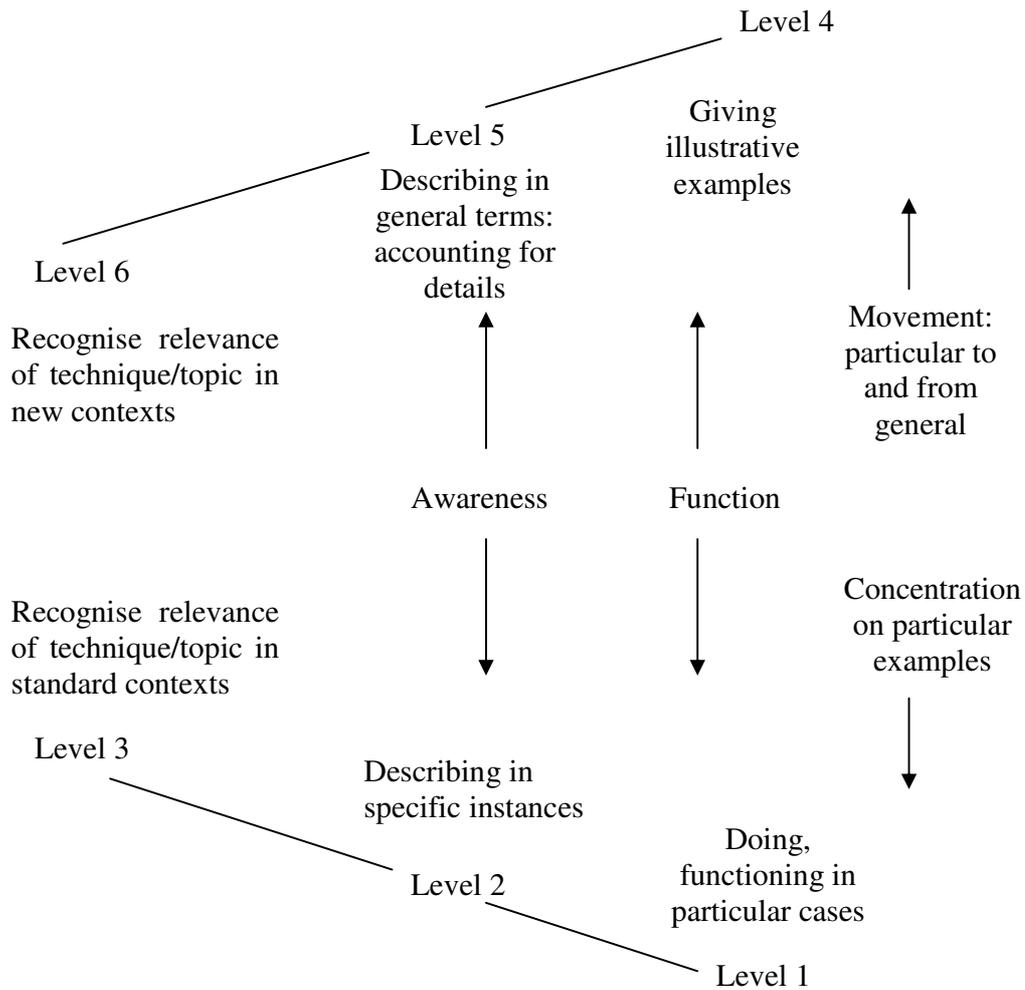


Figure 2.2 Six levels of mathematical processes

This figure can be read from right to left as a flow from the functional to the perceptive, from left to right as an unfolding of the essence into the functional, or as levels developing clockwise from bottom right round to top right. Levels 1 to 3 relate to describing while levels 4 to 6 are more about explaining. A short synopsis of each level is presented in Table 2-1 below:

Table 2-1 Mason’s levels of mathematical process

<i>Level</i>	<i>Summary</i>	<i>Examples</i>
1	Doing specific calculations, Functioning with practical apparatus	Add fractions of a particular type Make measurements

<i>Level</i>	<i>Summary</i>	<i>Examples</i>
	Recalling specific aspects of a topic and specific technical terms	Fractions can be added, multiplied, compared
2	Giving an account of how a technique is carried out on an example in own words and describing several contexts in which it is relevant	You multiply these together and add those Fractions arise as parts or shares of a whole
	Giving a coherent account of the main points of a topic in relation to a specific example	Fractions can be compared by subtracting or by dividing
	Giving a coherent account of what a group did, in specific terms	We tried this and this and noticed this
3	Recognising relevance of technique or topic/idea in standard contexts	If two thirds of a team have flu
4	Giving illustrative examples (standard and own) of generalisations drawn from a topic, or of relationships between relevant ideas	The simplest denominator is not always the product – give an example
	Identifying what particular examples have in common and how they illustrate aspects of the technique or topic	What does $5/6 + 3/8 = 29/24$ illustrate about adding fractions?
5	Describing in general terms how a technique is carried out to account for anomalies, special cases, particular aspects of the technique	To add two fractions you
6	Recognising relevance of technique or topic in new contexts	
	Connecting topic coherently with other mathematical topics	Fractions are one way to get hold of certain kinds of numbers

I decided these levels would be useful in analysing the tasks participants designed for their learners in terms of what Leinhardt (1989) describes as scripts, agendas and explanations. Investigating the lessons that participants prepare and present to their learners and analysing these according to Mason's levels can act as an indicator of the participants' pedagogical content knowledge. Participants with a stronger pedagogical content knowledge should be able to design and implement lessons and tasks for learners that cover a range of Mason's levels, including the higher levels of 4, 5 and 6.

Even (1990, 1993) investigated pre-service teachers' subject matter knowledge and its interrelations with pedagogical content knowledge, in the context of the concept of functions. She concluded that better subject matter preparation for teachers needs to focus on constructing mathematics courses for these teachers differently. The courses need to be presented in line with the constructivist views on teaching and learning and include "environments that foster powerful constructions of mathematical concepts" (p. 113).

Even (1993) also suggests that results of her study concurred with similar findings (Ball & McDiarmid, 1990) that teachers tend to follow their own teachers' footsteps unless they have developed a different repertoire of teaching skills. Developing this repertoire forms part of pedagogical reasoning which is the process of transforming subject matter knowledge into forms that are pedagogically powerful (Shulman, 1987). Hence she reinforces the notion that while subject matter knowledge has a strong influence on the quality of pedagogical content knowledge, it is not sufficient to focus on one without considering the development of the other.

In the domain of science education, Veal and MaKinster (2001) suggest two taxonomies of pedagogical content knowledge (PCK); a general taxonomy and the taxonomy of pedagogical content knowledge attributes. In their general taxonomy, they differentiate between *general*, *domain specific* and *topic specific* pedagogical content knowledge. The *general* PCK refers to the discipline being taught, in the case of this study, mathematics. The *domain specific* PCK focuses on specific subject matter within the discipline, for example, algebra. The *topic specific* PCK is the various sections within the domain that each have their own set of concepts and terms (some of which overlap), for example, the topic of functions within the domain of algebra. Topics may be introduced differently in

different domains. For example, the concept of gradient in mathematics is taught in both the algebra and analytical geometry domains of mathematics, but it is approached differently depending on which domain it is being taught in. In my understanding of the literature, a teacher demonstrating a high level of pedagogical content knowledge will be able to create learning environments for learners that will enable them to see the different use of the topic in the two domains but still recognize and understand that the topic or concept remains the same.

In Veal and MaKinster's (2001) taxonomy of PCK attributes, they identify content knowledge as the basis, with knowledge of learners building on that, and PCK with its components of context, assessment, environment, nature of discipline, pedagogy, curriculum, socio-culturalism and classroom management hierarchically on top of the knowledge of learners (see Figure 2.3). Given that the aim of this study is not focusing solely on PCK, it is not possible to report on the participants' knowledge of their learners except where direct reflections, statements or observations are offered from the data from their portfolios. Also for the purpose of this report, the PCK components or attributes reported on are limited to assessment, pedagogy, curriculum, context and classroom management.

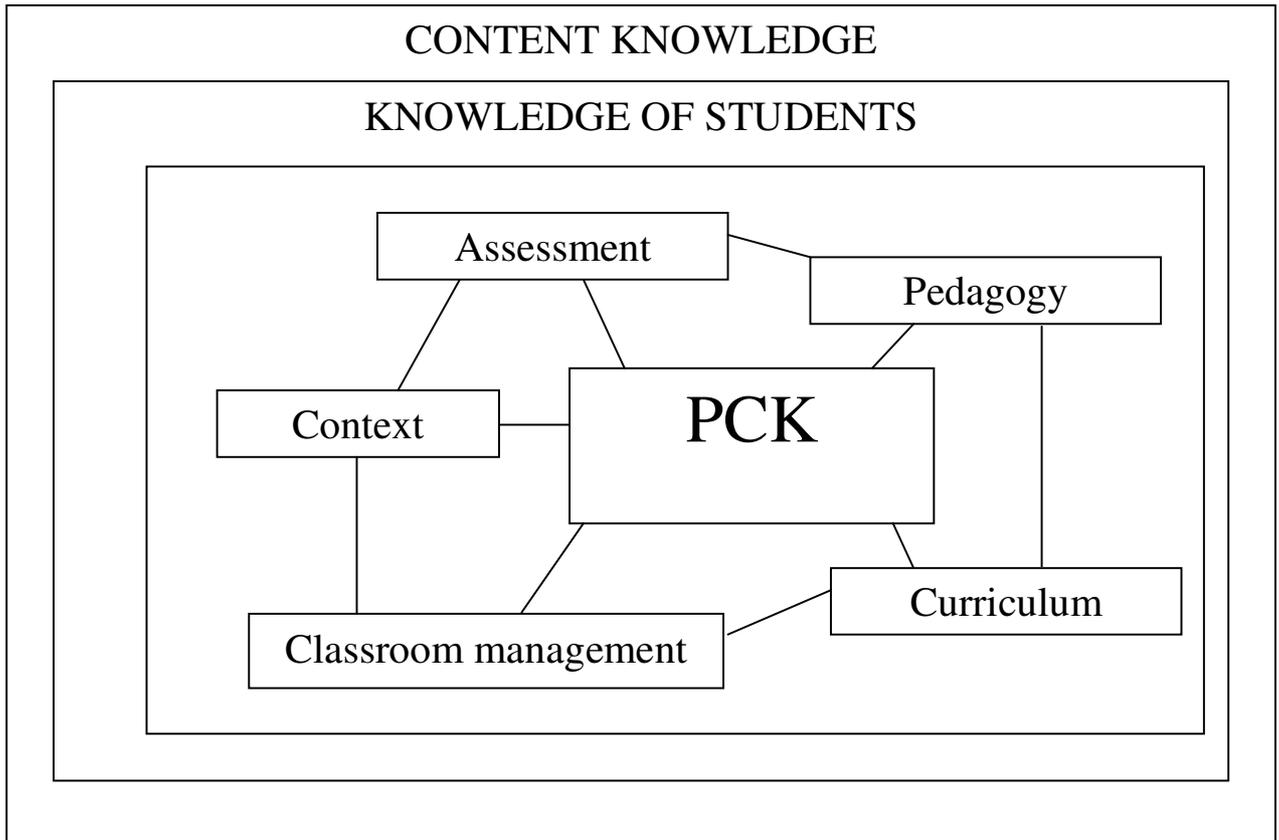


Figure 2.3 Taxonomy of attributes of pedagogical content knowledge (Adapted from Veal & MaKinster, 2001)

The major distinction I make in this study between subject matter knowledge and pedagogical content knowledge relates to the interface between the participant (as the teacher), the mathematics and the learners. In evaluating the participants' subject matter knowledge, I investigate their interaction with the mathematics through their lesson preparation and presentation. For pedagogical content knowledge, the communication between the participants, the mathematics and the learners is the focus. Subject matter knowledge focuses on the pre-service teachers' knowledge of mathematics in general, the domains contained therein (e.g. algebra) and their knowledge and understanding of the various topics (e.g. functions) within that domain and how they relate to other topics and domains within the subject (Veal & MaKinster, 2001). Pedagogical content knowledge, however, foregrounds the pre-service teachers' knowledge and understanding of the learners they will be teaching within the context of the subject and how to translate subject matter to a diverse group of learners (Veal & MaKinster, 2001). This includes the conceptual and procedural knowledge learners bring to the learning of the topic, the

stages of understanding learners are likely to pass through in mastering the content as well as possible errors, misconceptions or alternative conceptions learners may have or develop with regard to the topic (Carpenter, Fennema, Peterson & Carey, 1988). It also includes the pre-service teachers' knowledge of assessment, instructional techniques (pedagogy), context, curriculum and classroom management.

2.3.3 Conceptions of mathematics

I specifically distinguish between the use of the terms “beliefs” and “conceptions” in these next two sections. In my view, conceptions are a more general construct: the set of positions a teacher has about something (in this case mathematics) that are probably mostly subconscious and elusive (Ponte, 1999). I see beliefs as being more overt in both the individuals' thinking as well as their actions, with the individual having more of a conscious awareness of them than of conceptions.

The design of the PGCE course that forms the context for this study puts a lot of emphasis on the pre-service teachers engaging with and reflecting on their instructional practice. They are therefore continually encouraged and required to explicitly discuss and reflect on their beliefs about teaching and learning. However, this is not the case with regard to the nature of mathematics. While I touch on this aspect within the mathematics specialisation module of the course, the pre-service teachers do not engage or reflect extensively on how they view mathematics as such (beyond whether or not they enjoy it). Therefore, it remains a more subconscious and elusive construct than their beliefs on teaching and learning. Hence I use the word “conceptions” in relation to their views on mathematics.

Thompson (1984) uses the term conceptions as an umbrella term for the teachers' beliefs, views and preferences about mathematics and its teaching. Cooney (1994) and Thompson, Philipp, Thompson and Boyd (1994) also refer to conceptions as “orientations” towards mathematics. Ernest (1988) summarises the teachers' conception of the nature of mathematics as “his or her belief system concerning the nature of mathematics as a whole” (p.1). These need not be consciously held views but may rather be implicitly embedded philosophies. Ponte (1992) views conceptions as a conceptual

substratum that has a key role in thinking and action, providing ways of seeing the world and organising concepts. For the purpose of this study, the term conception of mathematics is taken to mean the way that the participant views the nature of mathematics as a whole. This may be either implicitly embedded or explicitly apparent and pertains specifically to the participant's definition and views of mathematics as a subject.

Ernest (1988) presents three possible views of mathematics. The *instrumentalist view* of mathematics assumes the stance that mathematics is an accumulation of facts, rules and skills that need to be used as a means to an end, without there necessarily being any relation between these components. The *Platonist view* of mathematics sees the subject as a static but unified body of certain knowledge, in which mathematics is discovered rather than created. The *problem solving view* of mathematics is a dynamic, continually expanding and evolving field of human creation and invention that is in itself a cultural product. Thus mathematics is viewed as a process of enquiry, not a finished product. The results remain constantly open to revision. Ernest (1988) suggests that a hierarchal order exists within these three views, placing the instrumentalist view at the lowest level and the problem solving view at the highest.

Thompson et al. (1994) discuss two main orientations towards mathematics that emerged from their research on how different teachers approached the teaching of the same task. They also allude to a third orientation which is also discussed here. A teacher with a *computational orientation* regards mathematics as a composition of computational procedures. Such teachers subscribe to “doing mathematics as computing in the absence of any reason for the computation aside from the context of having been asked to do so” (p.86). Teachers who hold a *calculational orientation* are driven by an image of mathematics as the “application of calculations and procedures for deriving numerical results” (p. 86). While not only focused on computations, this view does remain intent on procedures in order to get the answer. Typical “symptoms” of such an orientation include:

- the answer being the most important element of problem solving;
- speaking exclusively in numbers and numerical operations;

- remediating learners' difficulties with calculational procedures, not taking into account the context within which the difficulties arise;
- an emphasis on identifying and performing procedures.

Conceptually orientated teachers on the other hand strive for conceptual coherence within the subject, focusing learners' attention on the rich conception of situations, ideas and relationships rather than on the thoughtless application of procedures. Their activities are mainly motivated by:

- the expectation that learners intellectually engage in tasks and activities;
- an image of a system of ideas and ways of thinking that the learners should develop;
- an image or plan of how to develop these ideas and ways of thinking.

The latter two appear to correspond respectively to what Thompson (1984) refers to as a *content-orientated approach* and a *process-oriented approach*. Her research showed that teachers' beliefs, views and preferences about mathematics and its teaching played a significant role in shaping their instructional behaviour. The two participants in her study who conceived of mathematics as a "rather static body of knowledge (p. 119) both presented the content in their instructional practices as a finished product (content-orientated approach). One participant used a more conceptual approach though while the other portrayed mathematics as a collection of rules and procedures for finding answers to specific questions, which Thompson classified as a computational approach. The third participant, however, held a more dynamic view of mathematics, believing that engaging in creative and generative purposes is the best way for students to learn. Her practice in turn was more process-orientated.

In a more recent and slightly different study, Agudelo-Valderrama, Clarke and Bishop (2007) examined the relationship between Columbian mathematics teachers' conceptions of beginning algebra and their conception of their own teaching practice. They concluded that "teachers' conceptions of the nature of beginning algebra underpinned their conceptions of the crucial determinants of their teaching practices" (p. 86). From this they were able to establish two basic groups: teachers for whom algebra knowledge is produced externally and those for whom it is produced internally. For the "external"

group the crucial determinants of their teaching related to learners' behaviour and the knowledge was passed on from books to learners. For the "internal" group the knowledge and dispositions of the teacher were regarded as crucial determinants by the teacher for the teaching being enacted. For teachers in this group, the learners needed to create meaning in their algebra work through suitable classroom situations and activities. This again highlights the complex but important relationship between how teachers' conceptions of mathematics affects their conception of how it should be presented (Hersh, 1986).

Another way of classifying conceptions is on a continuum from *absolutist* to *constructivist views* of mathematics (Ernest, 1991). On the *absolutist end* of the continuum, teachers with this conception view mathematics as a collection of fixed and infallible skills and concepts (Romberg, 1992) and as a subject that contains absolute truths and is value-free, culture-free and has universal validity (Ernest, 1991). On the other end of the continuum, the *constructivist view* challenges the basic assumption that mathematical knowledge is infallible. This view emphasizes the reconstruction of mathematical knowledge within the practice of mathematics, using the learners' knowledge and experience as a starting point. Teachers working in this paradigm see mathematics as continually growing and being revised (Ernest, 1991) and prefer to act as facilitators rather than teachers in the teaching and learning process.

The following figure summarises the information presented above. This figure aligns the instrumentalist view with the computational orientation, the Platonist view with the calculational orientation and the problem solving view with the conceptually orientated approach. The content-orientated approach mentioned by Thompson (1984) spans across the computational and calculational (more conceptual) categories specified by Thompson et al. (1994) while the process-orientated approach corresponds with the conceptual and problem-solving views. These can be placed on a spread on the absolutist-constructivist continuum as I understand them from the literature.

Instrumentalist	Platonist	Problem solving
Content-orientated approach		Process-orientated approach
Computational	Calculational	Conceptual
Absolutist		Constructivist

Figure 2.4 Summary of conceptions of mathematics

2.3.4 Beliefs regarding the teaching and learning of mathematics

The influence of teachers' beliefs about mathematics and the teaching thereof on what they do in the classroom has been well established in the mathematics education literature (e.g., Thompson, 1984, 1992; Cooney, 1985; Confrey, 1990; Wilson & Goldenberg, 1998; Agudelo-Valderrama et al., 2007). This is therefore an integral component of the mathematics profile.

Malara and Zan (2002) see beliefs and knowledge as impossible to separate; that tacit knowledge embeds teachers' deep beliefs that influence practice. They therefore suggest studying individual teachers in depth and providing detailed analyses of their cognitive processes as a means to measuring changes in teachers' beliefs. In this study, the subject matter knowledge and beliefs of teachers are depicted as two separate components, they are still viewed as an inseparable part of the mathematics profile as a whole. Malara and Zan (2002) also highlight the importance of getting teachers to study their own practice through self-awareness and reflection. Both these suggestions are worked into this study as part of the research design as well as the design of the PGCE course through which the students qualified as teachers.

As noted by Ponte (1999) the word "belief" is often used with different meanings and regarded as a "messy" construct to define. Beliefs may be seen as dispositions to action and major determinants of behaviour (Brown & Cooney, 1982 as cited in Ponte, 1999) that are context specific (Lerman, 1994). They can also be viewed as "inconvertible personal truths, that are idiosyncratic, have strong affective and evaluative components, and reside in the episodic memory" (Nespor, 1987, p. 320). They can be implicit or explicit, espoused or enacted (Ernest, 1988) and often there can be a mismatch between

the espoused beliefs and the beliefs that are enacted in practice (Thompson, 1984; Hoyles, 1992).

Ernest (1988) identifies three models that depict the teacher’s role and intended outcome of instruction. This first is the role of *instructor* where the intended outcome is skills mastery with correct performance. The second role is as *explainer* where the intended outcome is conceptual understanding with unified knowledge. And the third role is that of *facilitator* where confident problem posing and solving are the intended outcome.

With regard to a teacher’s beliefs of the learning of mathematics, he includes “the teacher’s view of the process of learning mathematics, what behaviours and mental activities are involved on the part of the learner, and what constitutes appropriate and prototypical learning activities” (p. 2). The two key constructs in these models are *active construction* of understanding versus *passive reception* of knowledge and *developing autonomy* in the child versus the learner as *submissive and compliant*.

For the purpose of this study, beliefs regarding the teaching and learning of mathematics are therefore regarded as espoused and enacted, verbal and non-verbal indications of how the participants view teaching (their role in the instruction and what they hope to achieve with it) as well as their view on the role of the learner (mental and prototypical activities they engage in) in the teaching and learning process. These are now portrayed on a continuum.

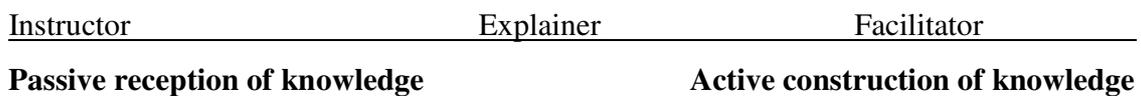


Figure 2.5 Summary of beliefs regarding the teaching and learning of mathematics

2.3.5 Teacher’s ideology (approach to teaching)

Goldin (2002) presents two “camps” as an overview of mathematics education ideologies. He calls these traditional and reform ideologies and I agree with his acknowledgement that establishing these “risks great oversimplification” (p. 199) of the picture. However, throughout the literature, it is evident that researchers are

acknowledging this divide albeit with different terms (for example, Jaworski, 1989; Rogers, 1992). While I see these as two opposing ideological stances, a continuum prevents oversimplification of the representation of these stances. Through the use of the continuum, the teaching approach of participants can be plotted according to the extent of the dominant ideology they demonstrate rather than merely labelling their instructional behaviours as one of the two opposing extremes. My assumption is that traditional practices cannot be totally abandoned in favour of only reform practices. However, in order to produce more independent and mathematical thinkers, the reform ideology needs to be embraced by teachers as the dominant approach within their instructional behaviour.

On the one end of the continuum, the *traditional ideology* values content, the correctness of learners' responses and the mathematical validity of their methods. Teaching methods include a lot of individual drill and practice. This is to ensure the correct use of efficient mathematical rules and algorithms and learners' mastering the application thereof in order to successfully move on to more complex ideas. Mathematical skills at each level are developed step-by-step and then generalised in higher level mathematics. Class groupings are dominantly homogeneous by ability and expository teaching is valued (Goldin, 2002). Rogers (1992) refers to the teaching approach that embraces this ideology as "academic mathematics" and describes it as "learning by the feet of the master" (p. 154). As Polanyi (1964 as cited in Rogers, 1992) so eloquently puts it:

To learn by example is to submit to authority. You follow your master because you trust his manner of doing things even when you cannot analyse and account in detail for its effectiveness. By watching the master and emulating his efforts in the presence of his example, the apprentice unconsciously picks up the rules of the art including those which are not explicitly known by the master himself (p. 53).

This type of teaching is also often referred to as "a transmission process where mathematical knowledge exists and may be conveyed by the teacher to the learner" (Jaworski, 1989, p. 171). The assumption underlying this approach is that if the teacher gives a clear exposition of the mathematical knowledge, the learners who have heard it should then be able to provide evidence of understanding it through exercises designed for this purpose. Boaler (1997, 2002, 2004) conducted research on different approaches to teaching mathematics and their impact on learning. In Boaler (2004) she depicts a classroom where a conventional (or traditional) approach to the teaching of algebra was applied. She calls this teaching mathematics through "demonstration and practice" (p. 1).

She explains how in such a classroom learners sat individually, the teachers presented new mathematical methods through lectures and the learners worked through short, closed problems. The vast majority of the questions teachers asked were procedural.

On the other end of the continuum, the *reform ideology* places more value on learners finding patterns, making connections, communicating mathematically and problem-solving from the earliest grades. This problem-solving usually takes the form of open-ended, real-life, contextualised problems. Alternative and authentic assessment is often used. There is a reduced emphasis on routine arithmetic computation, with hands-on, guided discovery methods, exploration and modelling being preferred approaches. High-level mathematical reasoning processes are central to this ideology which encourages learners to invent, compare and discuss mathematics techniques. Learners are also required to construct their own viable mathematics meanings and in this it is acknowledged that learners have different learning styles. Where co-operative groups are used, learners are usually grouped heterogeneously to allow interaction with these varying learning styles and other characteristics (Goldin, 2002). This is more in line with what Rogers (1992) labels as “interpreted mathematics” which he describes as “the context-bound use of mathematics as a tool, a means to an end, to solve problems in the ‘real’ world” (p. 155). Jaworski (1989) refers to this as an “investigative approach to teaching and learning” (p. 172) where opportunities are provided that impel the learners to express and explore ideas for themselves. Discussion is encouraged so that the teacher can find out what learners are thinking and so that learners can ask questions.

Boaler (2004) refers to this type of approach to teaching as “project-based” (p. 1) where learners are taught mathematics in mixed-ability groups through open-ended projects. In her research the teachers in such a classroom posed longer, conceptual problems and combined learner presentations with teacher questioning. Teachers were seldom observed lecturing the learners who were taught in heterogeneous groups. The teachers asked more varied questions than the teachers in traditional classes, including less procedural and more conceptual questions.

2.3.6 Learners' mathematics experiences (learning approach)

This component represents values communicated to learners through their mathematics learning experiences (see Figure 2.6 as an illustration). Ernest (1989) differentiates between authoritarian and democratic experiences of learning. Learners' mathematics experiences are termed *authoritarian* when what the teacher dictates must be followed and taken in without question. Learners submit to the teacher and depend (in the extreme) on the teacher for every aspect of their mathematics learning (Ernest, 1989).

On the other hand, learners have *democratic* experiences of learning mathematics when they are respected and respect each other. The classroom atmosphere can be described as one of relative freedom, and learners are free to navigate and discuss many aspects of the curriculum. Learners therefore become increasingly independent of the teacher.

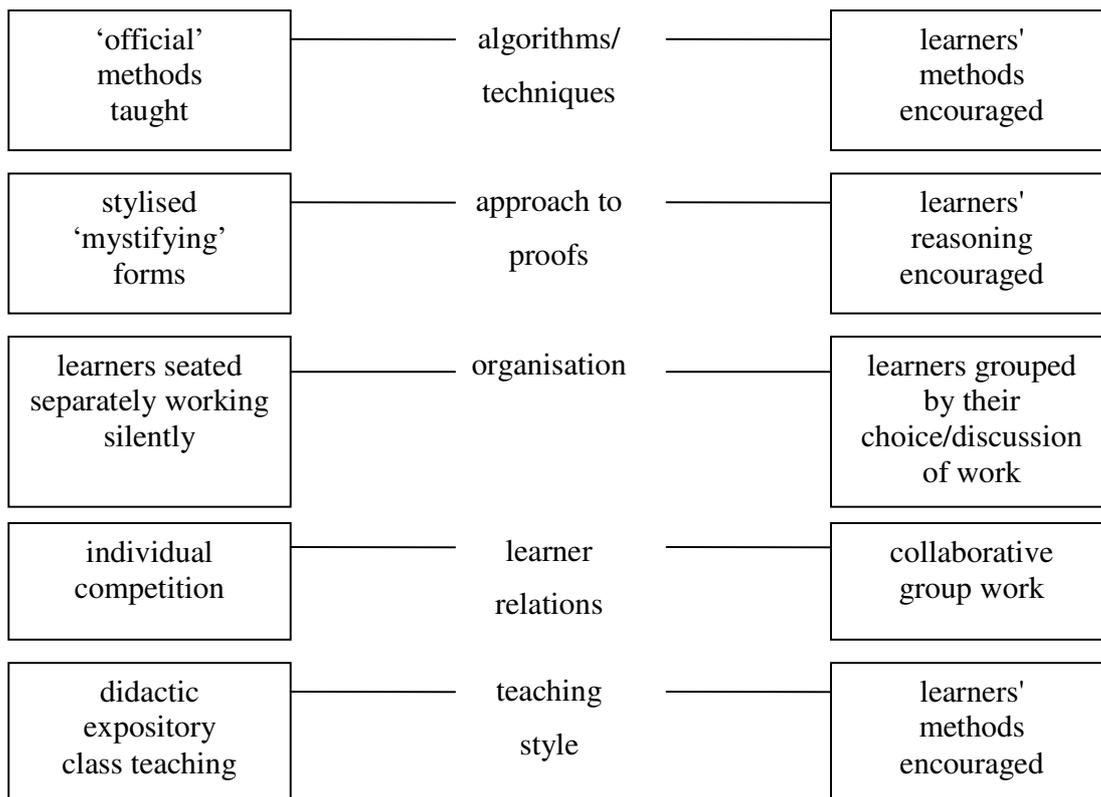


Figure 2.6 Illustration of authoritarian versus democracy continuum (Ernest, 1989)

Various aspects of school mathematics can be included in constructing the authority versus democracy continuum (Ernest, 1989):

- The ways the subject is presented (status of definitions, approach to proof, attitude to techniques and algorithms);
- The ways a learner's work is dealt with (the forms of assessment used, how errors are handled, answers checked);
- Classroom management (seating, access to resources, the way learners' tasks are selected, the sort of questions a teacher asks);
- Relationships which are permitted, encouraged or discouraged (between learners, between learners and teacher);
- The curriculum (how it is chosen, the way different parts are approached, its orientation – whether it is directed towards the learners' experience or interests).

Davis (1997) adds another aspect to these described above by using the manner in which the teacher listens to the learners as a metaphoric lens through which to interpret practice. He suggests three forms of listening: evaluative listening, interpretive listening and hermeneutic listening.

He explains the primary reason for *evaluative listening* as rather limited and limiting, as the teacher is most often listening for something (i.e. a “mathematical” explanation) rather than listening to the speaker. The motivation of such listening lies in evaluating the correctness of learner's contribution by judging it against a preconceived standard. Questions posed in this type of listening already have a “correct” answer in mind. Davis suggests that the teacher whose listening is merely evaluative “would strive for unambiguous explanations and well-structured lessons” (p. 360). He goes on to suggest that this manner of teaching (through evaluative listening) is associated with a conception of mathematics primarily as a system of already established, formal truths where mathematics teaching is a process through which one strives to avoid ambiguity.

Interpretive listening encompasses more of an attempt by the teacher to listen to the learner and to make sense of the explanations they are offering. The sorts of questions asked require more elaborate answers and may also entail a demonstration or explanation. However, although learner articulations and subject sense-making are more foregrounded

here, they might not affect the trajectory of the lesson. Passive taking in or absorption of what learners are saying in evaluative listening is replaced here by “an awareness that an active interpretation – a sort of reaching out rather than taking in” (p. 364) is involved. Communication is therefore understood to be more of a “negotiatory” process and listening becomes as vital as telling or explaining in this manner of teaching.

Davis (1997) makes the point that in both of the above manners of listening (which he likens to manners of teaching), the authority in the classroom remains with the teacher. For example, learners’ explanations are modelled on the teacher’s explanations and the teacher is the authority in deciding which answers are adequate and which require elaboration. In the third mode, *hermeneutic listening*, a collective authority is established. Such listening “demands the willingness to interrogate the taken for granted and the prejudices that frame our perceptions and actions” (p. 370). The teacher now becomes a participant in the exploration of the mathematics where class members are jointly exploring a mathematical issue rather than attempting to master already formulated bits of knowledge. This proposes that the teacher does not subscribe to the belief that teaching is a matter of causing or making learners acquire, master or construct particular understandings through some planned instructional sequence. Rather learning is viewed as a social process where the teacher participates, interprets, transforms and interrogates – in short, listens (Davis, 1997).

2.4 Conceptual Framework

The conceptual framework emerged from the background explained in chapter 1 as well as the literature review (see Figure 2.7). Two main constructs in the framework are the *mathematics profiles* of the pre-service teachers and their *instructional behaviour*. The components of the *mathematics profile* construct are subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and beliefs relating to the teaching and learning of mathematics as identified in the literature. The components of the *instructional behaviour* construct are teacher’s mathematics education ideology (teaching approach) and learners’ mathematical experiences (learning approach).

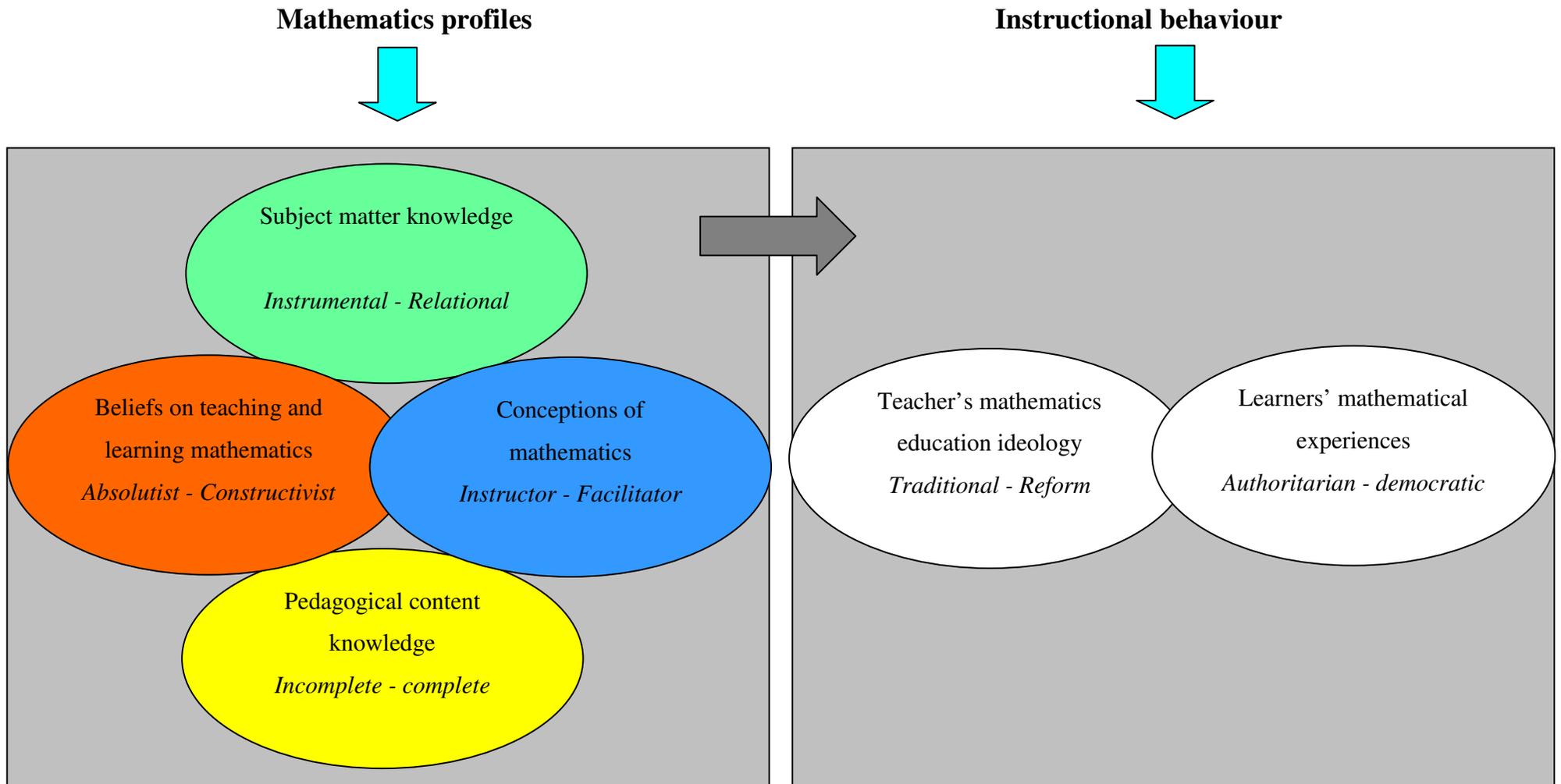


Figure 2.7 Conceptual framework

Ernest's (1991, 1998) theory of social constructivism underpins and holds together the conceptual framework represented visually above. This is an exploratory study and thus, according to Ernest's definitions of subjective and objective knowledge, the results of this study initially emerge as subjective knowledge. As the study becomes subjected to public examination and further criticism, with various reformulations, the results may then start to become more objective knowledge.

It was also Ernest's work on conceptions of mathematics (1988) and beliefs about the teaching thereof (1991) that inspired my thinking of placing participants in categories for the visual profiles. Ernest used three categories in both cases, but my data suggested that an additional category would be more explicative of the participants' profiles. I therefore added an added a fourth category to each of Ernest's three categories and for consistency conceptualised the other two components (subject matter knowledge and pedagogical content knowledge) of the mathematics profile with four categories. For the instructional behaviour profile, I drew largely on Ernest's work (1991) relating to authoritarian or more democratic learning experiences afforded to learners by the teacher. For the other components of both the mathematics as well as the instructional behaviour profiles, I drew on the ideas and research of other researchers in mainly mathematics education, but also in science and general education domains.

The component of subject matter knowledge in the mathematics profile was mostly informed by the ideas of Ball (1988a, 1988b, 1990, 2002) and Skemp (1971, 1989). The component of pedagogical content knowledge draws on the work of Shulman (1986), Mason (1989) and Veal and MaKinster (2001). The other two components in the mathematics profile (beliefs and conceptions) were developed from the work of Ernest (1988, 1991) supplemented by research from Thompson (1984) and Thompson et al. (1994). For the instructional behaviour construct, Goldin's work (2002) formed the basis for the teacher's mathematics education ideology (traditional versus reform teaching approach). Ernest's work (1989) informed the learners' mathematical experiences (authoritarian versus democratic learning approach) component, with additions from Davis (1997). The components in the mathematics profile are linked indicating my assumption that these by nature overlap each other. This is also the case for components within the instructional behaviour construct. The blue arrows indicate the

literature review process to develop the two main constructs and the grey arrow shows the focus of this study in examining the influence of the mathematics profiles on pre-service teachers' instructional behaviour. How each of these components was applied in the data analyses is discussed in chapter 3.

2.5 Conclusion

This chapter has discussed the epistemological underpinning, social constructivism, as a philosophy of mathematics as well as a worldview. Literature relevant to the scope of the study has been presented and a conceptual framework was developed from the interaction between my own background in mathematics education, social constructivism and the synthesis of the literature. The literature review covered the main aspects of the mathematics profile construct (subject matter knowledge, pedagogical content knowledge, conceptions of mathematics and beliefs on the teaching and learning of mathematics) and the instructional behaviour construct. A working definition for each for this study was espoused and a discussion of research in each domain presented. From this literature review the conceptual framework was compiled. Chapter 3 now outlines the research approach and the intricacies thereof for this study.