CHAPTER 5

5.0 DISCUSSION OF RESULTS

5.1 Introduction

The results of the study are discussed in this chapter. The similarities and differences in the ways in which the participating teachers develop their pedagogical content knowledge (PCK) in teaching statistics are examined.

The discussion begins by highlighting the research questions about teaching school statistics. The following four components of PCK were used as the theoretical framework: (1) subject matter content knowledge, (2) pedagogical knowledge (instructional skills and strategies), (3) learners' conceptions (preconceptions and misconceptions), and (4) individual learning difficulties in the topics investigated. Pedagogical content knowledge in statistics teaching represents a category of knowledge that teachers need to have assimilated in order to teach the subject effectively.

These research questions were:

1 What subject matter content knowledge of statistics do mathematics teachers who are considered to be competent have and demonstrate during classroom practice?
2 What instructional skills and strategies do these teachers use in teaching statistics?
3 What knowledge of learners' preconceptions and learning difficulties, if any, do they have and demonstrate during classroom practice?
4 How do these teachers develop their PCK in statistics teaching?

Components (1) and (2) above were used to answer research questions 1 and 2. In the third component, the learners' preconceptions and learning difficulties were identified and discussed in order to understand how the teachers acquired their knowledge in teaching statistics. The fourth research question was discussed as an amalgam of the key findings for the other PCK components.

The assumed PCK profiles of the participating teachers were examined in order to determine the similarities and differences, if any, in the ways in which the teachers develop their PCK in school statistics teaching.

The chapter concludes with a detailed discussion of how the results of the study provide insight into the way in which teachers who are reputed to be competent in teaching school mathematics develop their PCK in school statistics and evaluation of the theoretical framework.

5.2 Teacher development of PCK

5.2.1 Teacher A

Teacher A was observed teaching histogram construction and box-and-whisker plots in a step-wise fashion (ref Section 4.5.1: first lesson observation, and line 9; second lesson observation, and line 5a), using the recommended mathematics textbooks and work schedule. He started the lesson by asking the learners to name orally components of measures of central tendency such as modes, medians and means of ungrouped data (ref Section 4.5.1, first lesson observation, and line 1) in an attempt to determine their prior knowledge of histogram construction. The components of measures of central tendency having been identified, the teacher and learners prepared a frequency table from the raw data (ref Section 4.5.1, first lesson observation and line 4a). Using this table, the histogram was constructed by first drawing its horizontal and vertical axes. The axes were labelled with data values on the horizontal axis, and frequencies on the vertical axis. A scale was chosen by the teacher, who stated that the highest and lowest values of the frequencies and data values, as well as the dimensions of the graph paper provided, had been considered (ref Table 4.5.1a). Next, the bars of the histogram were drawn by joining the line of best fit (ref Figure 4.5.1a). Teacher A's lesson showed that he had adopted a rule-oriented procedural approach to teaching histogram construction.

In teaching the construction of histograms, he gave further evidence of using more procedural knowledge, focusing primarily on rules and algorithms, than conceptual knowledge. The procedural approach requires simply plugging the data into the appropriate formulae, and then working out the correct values of the quartiles for the box-and-whisker plots (ref Section 4.5.1, second lesson observation, and line 4). The most challenging aspect for this teacher was knowing how to move from an algorithmic stage to a conceptually meaningful one as far as the students' learning was concerned.

However, he used a conceptual teaching approach during the lesson and demonstrated the mathematical connections and relationships between ogives and box-and-whisker plots by describing how quartiles were obtained from the ogive and used in the construction of the box-and-whisker plot (ref Section 4.5.1, second lesson observation, and line 8cii). The relationships between the ogive and box-and-whisker plot, the calculation of the first, second, and third quartiles, and the description of the number line on which the box-and-whisker was drawn, with its mathematical connections, were elucidated during his lesson. A conceptual-based instructional approach endeavours to provide the reasons that make algorithms and formulae work (Peal, 2010). The emphasis is placed on the learners' understanding of the relationships and connections between important statistical concepts such as the use of quartiles to construct the box-and whisker plots on a number line (ref Figure 4.5.1c). Overall, Teacher A implemented more of a rule-oriented procedural knowledge approach in teaching histogram and box-and-whisker plot construction than a conceptual one. What can be surmised from this is that he did use both knowledge approaches except, of course, that one was dominant.

Interestingly enough, through the non-verbal cue of nodding their heads, the learners seemed to grasp the lesson on histogram construction through the use of conceptual knowledge better than when Teacher A adopted a rule-oriented approach. This observation was illustrated by the fact the learners were able to answer questions involving recall and application of procedures posed by him in order to assess how well they had understood the lesson on histogram construction. In answering the question how do you calculate the percentage of learners in the age group of 26–40?, learners first of all calculated the number of learners, divided by 27 and multiplied the result by 100 to get the percentage of learners within that age group. (ref Section 4.5.1, first lesson observation, and line 20). In the explanation, based on his conceptual knowledge, he demonstrated his PCK in a manner that enhanced learners' comprehension of histogram and box-and-whisker plot construction.

During the lesson, a few of the learners experienced learning difficulties such as being uncertain about choosing a scale for labelling the data axis of the histogram (ref Section 4.5.1, first lesson observation, and line 11). The teacher identified such difficulties as being due to lack of comprehension on the part of the learners (ref Section 4.5.1, first lesson observation, and line 22a).

Teacher A's preference for the use of procedural knowledge in teaching histograms was confirmed in the learners' workbooks (document analysis). It was discovered that the learners had written down the teacher's rules or steps on how to construct histograms and box-and-whisker plots, as well as the diagrams of histogram and box-and-whisker plot (ref appendix 21, learner workbooks). Teacher A might have adopted the use of procedural knowledge because the construction of histograms, which demands that specific procedural rules must be followed, is consistent with a conceptual understanding of the term. In a study conducted by Flockton, Crooks and Gilmore (2004) and Leinhardt et al (1990) on graphing, they stress that the construction of graphs requires the sequence of drawing the axes, choosing the scale, labelling the axes, plotting the points, and joining the lines of best fit. The order of steps, in the case of Teacher A, demonstrated the knowledge and skills required for histogram construction.

As observed, the learners experienced learning difficulties, particularly in labelling the data axis with incorrect scale, which could mean that he possibly presented his lesson in a limited way, that is, solely procedurally, without providing the reasons underlying these procedures and clarifying the relationship between concepts (a conceptual knowledge approach) in histogram construction (ref Section 4.5.1, first lesson observation, and line 12a). The teacher omitted a detailed description of how to choose a scale of given data before labelling the data axis. He merely stated the scale and used it to demonstrate the construction of a histogram. During classwork, the learners tried to draw a histogram, which could not be accommodated on the graph paper provided because they scaled the data axis incorrectly (ref Section 4.5.1, Figure 4.5.1c).

It may be said that Teacher A's PCK in terms of subject matter content knowledge presentation did not always reveal the required variety of ways of presenting the data handling topics to his learners for ease of access. In some instances, he demonstrated the use of both procedural and conceptual knowledge in teaching histograms and box-and-whisker plots, but he predominantly used a set of algorithms to demonstrate graph construction. In the main lesson on histogram and box-and-whisker plots, he displayed factual knowledge, procedural proficiency and conceptual understanding of the data handling topics that were taught.

Gersten and Benjamin (2012) note that the use of different strategies for teaching mathematics helps to anchor the learners behaviourally and mathematically, avoids possible learning difficulties, and achieves effective learning. This finding conforms with the suggestion being made here, based on this study's results, that teachers' flexibility or the ability to use a variety of instructional approaches (both conceptual and procedural knowledge) should make data-handling concepts (which are said to be difficult for learners to grasp) more meaningful and accessible to more learners. Teacher A can thus be said to have possessed and demonstrated the required knowledge of histogram and box-and-whisker plot construction.

Grouping method was also used as an instructional strategy for teaching the construction of ogive by teacher A in order to provide interactive engagement , collaborative learning and to ensure sustainability of interest in learning statistics among the learners. Learners work in groups of four to five to calculate the quartiles of an ogive for constructing box-and-whisker plots. The use of grouping method to sustain learners' interest in learners was given an empirical support by Adodo and Agbayewa (2011) who report that effective classroom lesson is achieved using grouping method for teaching. Adodo and Agbayewa (2011) further noted that grouping method allows the teacher to better tailor the pace and content of instruction to learners' ability level and needs and easy management of the classroom is achieve especially in the homogeneous grouping which teacher A adopted.

With regard to his pedagogy, Teacher A often used examples that are familiar to learners for teaching data handling. Using the mark distribution of learners' performances in an English examination, he described in a step-by-step fashion how ogives are constructed, and how quartiles are obtained and used to construct the box-and-whisker plot (ref Section 4.5.1: second lesson observation, and line 8a). The use of familiar examples and contexts by Teacher A is consistent with the approaches used by other workers to make the topic more meaningful and accessible (Ball & Bass, 2000; Meletiou-Mavrotheris & Stylianou, 2002). For example, Meletiou-Mavrotheris and Stylianou (2002) used familiar situations as examples in the context of teaching statistics in order to improve learner access and comprehension. According to these researchers, the teaching of rules alone (algorithmic teaching) does not always convey meaningful relationships between the mathematics knowledge taught in class and daily life situations (Meletiou-Mavrotheris & Stylianou, 2002).

So this disconnects with and is seen to obscure the relevance of statistics teaching and mathematics education in general.

Teacher A's knowledge of learners' preconceptions of the statistics lessons observed was derived largely from what transpired in the classrooms, notably through his analysis of learners' responses to teacher classroom questions (oral probing questioning and pre-activities) and classwork or assignments. During the lessons, Teacher A was able to identify some of these difficulties or inaccurate conceptions – such as the learners' inability to select appropriate scales for labelling the data axis of the histogram correctly through monitoring learner activity and questioning (ref Section 4.5.1, first lesson observation, and line 21; Figure 4.5.1c).

In the lessons observed, for instance, the teacher did not display evidence of anticipating learners' potential difficulties with any of the topics. The teacher went into the lessons without necessarily having prior knowledge or expectations of the type and nature of learning difficulties that his learners were likely to have in teaching histogram construction. For example, at the beginning of the lesson on histogram construction, Teacher A requested learners to define mode, median and mean. The learners did so efficiently, based on knowledge that they had been taught (ref Section 4.5.1, first lesson observation, and line 2). Thus the teacher detected learners' previous knowledge instead of preconceptions. Since the teacher could not identify their preconceptions of histogram construction, learners were likely to experience misconceptions and learning difficulties such as constructing a bar graph instead of a histogram because of their poor background in scaling. Teacher A can therefore be said to have displayed insufficient PCK in terms of the knowledge of learners' preconceptions of histogram and box-and-whisker plot construction.

Teacher A could have addressed possible learning difficulties before or during the lesson if he had had sufficient knowledge of learners' preconceptions of histogram construction. When asked in the questionnaire about his expectations of learners' difficulties, he said merely that there were no major problems, but he would deal with these when the learners asked him (ref Appendix xx, item 10). The insufficiency or inadequacy of his PCK in terms of his insight into learners' preconceptions was a knowledge deficit that was common to all the four teachers that were studied. The finding justifies further investigation into the reasons that teachers, in spite of many years of teaching experience, do not seem to give much thought to possible misconceptions or alternative frameworks their learners are likely to bring with them when they first come across new topics.

Penso (2002) noted that learners' thinking about and prior knowledge of a topic is an important aspect that should be taken seriously into consideration during teaching as it helps to avoid possible learning difficulties that learners may encounter during the lesson. Penso (2002) suggested that during their lesson planning, practising teachers should be encouraged to explore varieties of instructional strategies that could elicit learners' thinking and prior knowledge of the concept being taught in order to be able to deal with their learning difficulties effectively. Hill *et al* (2008) note that the sequence of teaching and learning may be distorted if learners' preconceptions are not identified in order to address learning difficulties that learners are likely to encounter during teaching.

Teacher A addressed the learning difficulties through individual after-lesson or post-teaching discussions, including additional exercises that were given as homework (ref Section 4.5.1, first lesson observation, lines 23a and 23b). In his interview and written reports (ref Sections 4.7.2) the teacher confirmed the use of oral questioning, classwork and homework assignments as strategies that he purposefully uses to evaluate how well learners have understood the lesson and to gain insight into their pre-existing knowledge of histogram and box-and-whisker plot constructions.

In sum, Teacher A used several instructional strategies of oral questioning, group work, using contexts and examples familiar to learners to introduce a topic, checking and marking learners' classroom and homework assignments, as well as using content-specific rule-oriented graphing skills (drawing axes, choosing scale, labelling axes, plotting points and joining line of best fit) for constructing histograms. By identifying learners' learning difficulties, using diagnostic questioning and monitoring techniques (already indicated), Teacher A can be said to have used effective pedagogical strategies to elicit learners' difficulties. But these monitoring strategies were not usually followed up with probing questions to determine the sources of difficulty or of incorrect preconceptions.

From the discussion so far, the question is how Teacher A developed his PCK. Specifically Teacher A's PCK on the construction of histogram and box-and-whisker plots could be said to have been developed over time through a series of teaching and learning experiences. It would be useful to identify and briefly discuss the sources of such experiences. First, in terms of his formal education, Teacher A received further training in the teaching of mathematics after his initial teacher training programme. He holds a BEd degree, majoring in mathematics education, and has an Advanced Certificate in Education, specialising in teaching mathematics and science. His qualifications may be part of the reason that his content knowledge of the subject matter can be considered adequate. In his teaching he demonstrated a good grasp of the various topics of histograms and box-and-whisker plots related to school statistics.

Teacher A has 21 years' mathematics teaching experience. Over the years his pedagogy or instructional strategies in teaching statistics would have involved lesson planning based on the recommended work schedule and textbooks in school statistics, delivery of lessons based on his teaching philosophy, learned skills and feedback from his learners. Other sources of development would have included reviews of his teaching portfolios and learners' workbooks. All of these activities would have contributed to the development of topic-specific PCK in statistics teaching.

Teacher A attended workshops arranged by his educational district office. Most of these workshops dealt with aspects of how to teach various mathematics topics that are considered difficult to learn, such as data handling, analytical geometry and trigonometry. It would appear, however, that the workshops barely considered facets of teacher knowledge of learners' preconceptions and sources of learning difficulties in data handling. But if they did, the teacher did not demonstrate their potential usefulness in planning his lessons. Teacher A appears to have limited knowledge of learners' preconceptions that could have been used in teaching on learners' behalf.

In summary, Teacher A may have developed his pedagogical content knowledge from the formal initial teacher education programme that he received; the further training obtained at the completion of his tertiary education; attendance at in-service training workshop programmes; periodic reviews of his own lessons and learner workbooks; and feedback over his many years of mathematics teaching.

5.2.2 Teacher B

Teacher B planned and taught his statistics lessons on bar graphs and ogives from the recommended mathematics textbooks and work schedule (ref Section 4.5.2, second lesson observation and line 9). He used a predominantly rule-driven formal procedural approach to statistical graphs (ref Section 4.5.2: second lesson observation, lines 6a and 6b; and section 4.5.2, first lesson observation, lines 3a, 3b, 3c and 4a). As observed, in starting his lessons he tried to identify learners' prior knowledge of the new topic. For instance, he introduced bar graph construction and interpretation with a pre-activity (ref Section 4.5.2: first lesson observation, and line 1) that assessed learners' understanding of the way in which to prepare a frequency table. His use of pre-activities as diagnostic strategies to identify learners' pre-existing knowledge was also attested to in his responses to the teacher questionnaire and written reports (ref Sections 4.7.3).

Teacher B taught graphical constructions of bar graphs and ogives according to the learning outcomes of data handling as stated in the mathematics curriculum (DoBE, 2010) (ref Section 2.2). These outcomes require that learners should be able to use appropriate measures of central tendency and spread to collect, organise, analyse, and interpret data, in order to establish statistical and probability models for solving related problems (DoE, 2007). Teacher B followed precisely the order in which the learning outcomes were stated in teaching his learners how to construct bar graphs and ogives. In practice, this meant, as observed in his lesson, drawing the axes, choosing the scale, labelling the axes, plotting the points, and joining the line of best fit, in that order (ref Section 4.5.2, first lesson observation, lines 3a, 3c, 4a, 4c and 5). Teacher B demonstrated his PCK for drawing bar graphs in line with the sequence described. Flockton *et al* (2004) confirm that for a person to understand a graph, he or she should be able to use the construction skills of drawing the axes, labelling the axes, plotting the points, and joining the line of best fit to construct a graph.

Teacher B's assumed PCK on bar graphs and ogive constructions could be characterised as procedural in terms of his lesson planning and teaching approach. Teacher B's predominant use of a formal procedural approach was also triangulated in the analysis of his learners' workbooks (document analysis). The learners drew the bar graph and wrote down the teacher's steps on how to construct bar graphs and ogives (ref Appendix xxi; learners' workbooks). Teacher B might have been influenced to adopt a formal procedural approach because of the learning outcomes of data handling as laid down in the Curriculum and Assessment Policy Statement (CAPS) (DoBE, 2012). Besides, the construction of bar graphs and ogives demands specific procedural rules (Flockton *et al*, 2004 and Leinhardt *et al,* 1990).

Having said that, when the teacher merely taught them the rules for constructing bar graphs, some learners experienced certain misconceptions, confusing bar graphs with histograms, and histograms with ogives (ref Section 4.5.2: first lesson observation, and line 9; second lesson observation, and line 7a). A histogram is usually used to display continuous data. The horizontal axis shows class intervals, and there are no gaps between the bars. The area of each bar shows the frequency for the class interval. Teacher B can be said to have presented his lesson in a limited way with insufficient explanations of how to choose the scales of grouped data (consisting of histogram, frequency polygon, ogive, scatter plot) that are used to analyse and interpret large data. Further, Teacher B seems not to have the flexibility to present the topics to the learners in different ways because his lessons were presented solely according to the procedural knowledge approach.

A detailed description of the construction of bar graphs and ogives using a conceptual knowledge approach would have been ideal in presenting the lesson and would have avoided possible misconceptions and learning difficulties that the learners might have encountered in the lesson. Conceptual knowledge involves understanding mathematical ideas and procedures and includes basic arithmetic facts (Engelbrecht, Harding & Potgieter, 2005). It is rich in relationships among important mathematical concepts such as calculating the quartile positions and locating the quartile itself on the ogive, class intervals and boundaries, frequencies and cumulative frequencies of an ogive. But Teacher B's teaching of bar graphs and ogives was dominated by a procedural knowledge approach, which involves following a rule or procedure without a detailed explanation of the relationships and mathematical connections between the concepts being learned, such as calculating a quartile position and locating it in an ogive. Thus, the teacher is probably unable to present his lesson in a variety of ways to ensure better comprehension and understanding. A detailed description of the concepts and their relationships, and the mathematical connections between these concepts and even existing ideas, may help to avoid possible misconceptions and learning difficulties that learners are likely to encounter during and after the lessons.

Baker *et al* (2001) and Bornstein (2011) note that a teacher who is unable to present mathematics content to learners in a variety of ways tends to expose them to learning difficulties, such as constructing a histogram instead of an ogive because of the use of an incorrect scale for labelling the data axis. A combination of procedural and conceptual knowledge approach would have helped to deepen learners' understanding and would have avoided misconceptions and learning difficulties that learners might develop during the lesson, as suggested by Engelbrecht, Harding & Potgieter (2005).

Teacher B often used familiar situations as examples for teaching data handling (ref Section 4.5.2: first lesson observation, lines 1 and 11). For instance, he described how a bar graph is constructed using a frequency table prepared by the learners from the raw scores obtained by learners in a mathematics test (ref Section 4.5.2, first lesson observation, and line 1). In his lesson on bar graphs (as explained earlier) he demonstrated the construction of bar graphs using a procedural knowledge approach. The use of familiar contexts is consistent with the recommendations of Meletiou-Mavrotheris and Stylianou (2002), who employ everyday situations as examples in order to make the topic accessible and meaningful to more learners. Although Engelbrecht *et al* (2005) suggest that a procedural knowledge approach could help learners to understand important demanding rule-oriented concepts, they affirmed that the use of both procedural and conceptual knowledge would be more effective and would create greater opportunities for improving learners' conceptual understanding of mathematics during the lesson (Engelbrecht *et al*, 2005; and Star, 2002).

During the lesson, Teacher B identified the learners' inability to label the data axis of the histogram correctly (ref Section 4.5.2: second lesson observation, and line 7b) by monitoring and analysing their responses to classwork. In one example, the learners chose the scale of grouped data and labelled the axes for data values incorrectly (ref Section 4.5.2: second lesson observation, line 7b). Teacher B addressed such learning difficulties through extra class activities in the form of drills and practice, as well as individual post-teaching discussions after formal classes (ref Section 4.5.2: second lesson observation, lines 9 and 12). The use of classwork and homework to evaluate how well learners had understood the lesson was confirmed in the teacher's responses to the questionnaire and written reports (ref Sections 4.7.3).

In terms of his knowledge of learners' learning difficulties, Teacher B was able to detect the misconception and learning difficulty of drawing a histogram instead of an ogive (ref Section 4.5.2: second lesson observation and line 7b). This misunderstanding could have been because of insufficient explanation of the construction of bar graphs and ogives via the procedural knowledge approach. As explained earlier, these learning difficulties were discovered while monitoring and analysing the learners' responses to classwork on bar graph and ogive construction (ref Section 4.5.2: first lesson observation, and line 9; second lesson observation, and line 10a). These problems were addressed by re-demonstrating the construction of bar graphs and by using extra class activities in the case of the ogive (ref Section 4.5.2: first lesson observation and line 10; second lesson observation, and line 9). The teacher interview, questionnaire, written reports and teacher's portfolios confirmed that learners had difficulties with the construction of graphs of grouped data such as the ogive (ref Appendix xvii, item 14; Appendix xx, item 10; items 1 and 2; and Appendix xxi, teacher portfolios).

In terms of his knowledge of learners' conceptions (preconceptions and misconceptions) in statistics teaching, Teacher B tried to identify them from pre-activities and oral probing questioning. Learners demonstrated previous knowledge of frequency tables and how data is represented by preparing the frequency table efficiently and explaining the way in which data is represented, but the strategy that was adopted failed to elicit learners' preconceptions of bar graph construction. In other words, the teacher therefore displayed insufficient knowledge of the learners' preconceptions of bar graphs and ogives. Learners are likely to experience misconceptions and learning difficulties, such as an inability to label the data axis due to incorrect scaling during the construction of ogive (ref Section 4.5.2, second lesson observation, line 7b) when the procedural knowledge approach was adopted to teach ogive construction. Teacher B would have been able to tackle this learning difficulty had the learners' preconceptions had been detected at the beginning of the lesson. When asked what learning difficulties did the learners experience during the lesson? (ref Appendix xx, item 6), he indicated that learners could not choose a scale of grouped data, revealing that their learning difficulties may have emanated from the teacher's insufficient knowledge of learners' preconceptions.

Inadequacy of knowledge of learners' preconceptions appeared to be common to all four teachers observed during the case study period. This finding points to a further investigation into the reasons that teachers with so many years of experience do not possess the knowledge of learners' possible preconceptions that may be necessary for effective teaching in the topics. Hill *et al* (2008) note that the sequence of teaching and learning may not lead to easy understanding of a concept and may not permit effective teaching if learners' preconceptions are not detected at the beginning of the lesson. Penso (2002) opines that teachers should consider several opportunities to detect learners' prior knowledge of a topic in their planning so that the anticipated learning difficulties can easily be addressed during lesson planning and presentation. This is an important agenda for inclusion in mathematics teachers' education programmes to ensure continuous improvement of PCK in statistics teaching.

How then does Teacher B develop his PCK in statistics teaching? In terms of his formal education, Teacher B received further training in the teaching of mathematics and statistics. He holds a BSc degree, majoring in mathematics and statistics. His qualifications may have contributed to his content knowledge of the subject matter which can be considered adequate. In his teaching he did not demonstrated sufficiently a good grasp of the various topics of bar graph and ogive construction related to school statistics because his teaching was dominated with a procedural knowledge approach that resulted to more questions from the learners during and after the lesson seeking for clarity of the misconceptions and learning difficulties they have encountered.

Teacher B has 10 years' mathematics teaching experience. Within these years of teaching, his pedagogy or instructional strategies in teaching statistics would have involved lesson planning based on the recommended work schedule and textbooks in school statistics, delivery of lessons based on his teaching ideology, learned skills and learners' responses to class activities in statistics. The review of his teaching portfolios and learners' workbooks were other sources for PCK development. All of these activities would have contributed to the development of topic-specific PCK in statistics teaching.

Teacher B attended workshops organised by his educational district office. As in the case of Teacher A, most of these workshops dealt with aspects of how to teach various mathematics topics that are considered difficult to learn, such as data handling, analytical geometry and trigonometry. The workshops sometimes appeared not to consider different aspects of teacher knowledge of learners' preconceptions and sources of learning difficulties in statistics teaching. But if they did, the teacher did not demonstrate their potential usefulness of the workshop in planning his lessons as the participating teachers were unable to demonstrate their knowledge of learners' anticipated learning difficulties of statistical graphs. Teacher B appears to have limited knowledge of learners' preconceptions that could have been used in teaching bar and ogive construction.

In summary, Teacher B may have developed his pedagogical content knowledge from the formal initial teacher education programme that he received, attendance at in-service training workshop programmes, periodic reviews of his own lessons and learner workbooks; and learners' responses to class activities in bar graphs and ogives construction.

5.2.3 Teacher C

During classroom practice, Teacher C taught his planned lessons on ogives and scatter plots as laid out in the work schedule (DoBE, 2010). He used the recommended and supplementary mathematics and statistics-related textbooks as sources of information for planning and teaching his lessons on data handling (statistics) (ref Section 4.5.3, first lesson observation, and line 16). Teacher C also displayed evidence of a procedural rather than a conceptual knowledge approach to teaching the construction of ogives and scatter plots (ref Section 4.5.3, first lesson observation, lines 3a, 3c and 4). Teachers need to possess good understanding of both conceptual knowledge and procedural knowledge of mathematics to be able to provide learners with clear explanations (Engelbrecht *et al*, 2005 and Star, 2002). Schneider and Stern (2010) view conceptual knowledge as mastery of the core concepts and principles and their interrelations in the mathematics domain. It is knowledge that is rich in relationships. On the other hand, procedural knowledge can be viewed as consisting of rules and procedures for solving mathematics problems. Procedural knowledge in mathematics allows learners to solve problems quickly and efficiently because to some extent it is automated through drill work and practice.

Teacher C demonstrated the requisite knowledge of and skills for constructing ogives in a step-by-step manner (ref Section 4.5.3, first lesson observation, and line 4) and scatter plots (see Section 4.5.3, second lesson observation, and line 4di). In his teaching, he moved from the algorithmic to the conceptually meaningful stage. He began his lesson on ogives and scatter plots by identifying the learners' prior knowledge of the concept of ogives through oral questioning, and the accuracy of the homework on histograms that had previously been taught (ref Section 4.5.3, first lesson observation, line 2bi; second lesson observation, and line 1). Subsequently, using a cumulative frequency table prepared by the learners, an ogive was constructed by first drawing its horizontal and vertical axes (ref Section 4.5.3, first lesson observation, and line 4). The data values were labelled on the horizontal axis (the upper class boundaries), and the cumulative frequencies on the vertical axis. A scale was chosen by the teacher, who indicated that he had chosen it by considering the highest and lowest values of the frequency and data values. The points were plotted and the line of best fit was joined to produce the ogive (ref Section 4.5.3, first lesson observation, line 9b).

This process of constructing an ogive from grouped data depicted a rule-oriented procedural approach. His procedural knowledge in teaching ogives (which was understandable to his learners) is believed to have been developed as a result of his five years' mathematics teaching experience, using the recommended lesson plan and work schedule of the Department of Education (DoE, 2010). The same procedural approach was used to teach scatter plots (ref Section 4.5.3, second lesson observation, and line 4di). To demonstrate the construction of a scatter plot, the teacher followed an algorithmic approach with progressively less conceptual knowledge. That is, the teacher's lesson was dominated to a large extent by a procedural knowledge approach rather than by conceptual knowledge. Some of the factors that may have contributed to Teacher C teaching scatter plots in a step-wise manner, following a particular order or sequence, could be attributed to the way in which the learning outcome of data handling is stated in the mathematics curriculum (DoBE, 2010). The document indicates that competency in graphing requires that the learner is able to construct, analyse, interpret statistical and probability models to solve related problem. The construction of graphs, as stated, entails scaling, drawing axes, labelling the axes, plotting points, and joining the line of best fit (Flockton *et al*, 2004; Leinhardt et al, 1990**)**. Teacher C followed this sequence for teaching scatter plots (ref Section 4.5.3, second lesson observation). In the lessons observed, the teacher gave a full explanation of how to construct a scatter plot before demonstrating how to analyse and interpret it. The learners did their classwork in groups. They were presented with exercises on scatter plots, and were requested to analyse and interpret the plots to determine whether there was a correlation between the variables X and Y (ref Section 4.5.3, second lesson observation, lines 3a and 4a).

Teacher C's preferred procedural approach to teaching the topic was confirmed in the learners' workbooks, portfolios and teacher's written reports (see appendices xx and xx1). Owing to the limited use of the conceptual knowledge approach rather than the procedural one – namely knowledge of the core concepts and principles and their interrelations in teaching ogive and scatter plots – it did not come as a surprise that some learners displayed certain misconceptions and learning difficulties in their analysis and interpretation of scatter plots (ref Section 4.5.3, second lesson observation, and line 6). For example, a negatively correlated linear scatter plot was interpreted by the learners as having no correlation because of an outlier that lay far from the line of best fit (ref Section 4.5.3, second lesson observation, lines 6 and 7). This misconception could be attributed to the rule-oriented approach that had been adopted to describe the construction of scatter plots (ref Section 4.5.3, second lesson observation, and line 4di), which did not allow for sufficient explanation of the interrelationships among the data values, frequencies, lines of best fit and outliers. The learning difficulty of interpreting a negatively correlated scatter plot as having no correlation owing to outliers may further indicate that in teaching the construction of scatter plots the teacher did not explain an outlier, line of best fit, type and nature of correlation, and how the presence of an outlier affects the correlation of the X and Y variables of the scatter plot.
What can be gleaned from the discussion so far is that teachers need to possess deep conceptual understanding of the mathematics concept that they are teaching and must be able to illustrate why mathematical algorithms work and how these algorithms could be used to solve problems in real-life situations (Nicholson & Darnton, 2003). The learning difficulties experienced by the learners were subsequently addressed by Teacher C during post-activity discussions (instructional strategy). This strategy was frequently used by Teacher C (ref Section 4.5.3, second lesson observation, line 12) during his lessons on ogives and scatter plots.

An important task of any teacher is to attempt to transform the content to be taught in such a way as to make it comprehensible to the learners (Mohr & Townsend, 2002). Teacher C also displayed evidence of a conceptual approach by providing the reasons that make the algorithm and formula work, and by explaining the relationships between important statistical concepts, as well as the mathematical connection between them during the lessons on ogives (ref Section 4.5.3, first lesson observation, lines 13b and 14). It was significant that more learners seemed to possess a better grasp of the topic in that they were able to construct and interpret ogives by means of this approach rather than the procedural approach (ref Section 4.5.3, first lesson observation, and line 14). In the particular lessons observed, Teacher C explained the mathematical connections and relationships between quartile positions and the quartiles and how quartiles can be used to interpret ogives (ref Section 4.5.3, first lesson observation, lines 13b and 14). In doing so, Teacher C could be regarded as having displayed progressively adequate PCK.

In his pedagogy, Teacher C used activities from everyday-life situations as examples (ref Appendix xxi, learner workbook). For example, he demonstrated how to construct a scatter plot using the frequency distribution table of the ages of persons infected with HIV/AIDS in two towns (ref Section 4.5.3, second lesson observation, line 9). This use of examples drawn from everyday life situation to illustrate scatter plot construction is in accordance with the view held by Shulman (1987) and Krebber (2004) that transformation of the subject matter by the teacher into a form that is more easily understood by the learners involves explanation with examples and instructional selection of teaching methods that are adaptable to the general characteristics of the learners. Teacher C may have decided to use examples drawn from everyday-life situations because the topic is new in the curriculum and may be looking for a more manageable way of presenting it to the learners in order to reinforce their understanding.

Teacher C gained knowledge of learners' preconceptions and learning difficulties mostly during classroom practice. The results of this study show that he had limited knowledge of learners' preconceptions. As observed, learners revealed previous knowledge of ogives and scatter plots from their responses to homework on these topics. For instance, at the beginning of the lesson on scatter plot construction, he checked and marked learners' homework on scatter plots based on their previous knowledge of what they had been taught and corrected some of their errors. While he was doing the corrections, he did not display any indication of having knowledge of other anticipated learning difficulties. Instead, he presented the correction procedurally, with no emphasis on the way in which previous errors that learners had committed could be avoided during the lesson or subsequently. Learners' learning difficulties led to Teacher C having to provide corrections to the homework. This leads one to the conclusion that he may not have considered identifying learners' preconceptions in scatter plot construction. This information should have been used in planning the current lesson and avoiding probable learning difficulties. The learning difficulties (constructing a histogram instead of an ogive and misinterpreting negatively correlated scatter plots as having no correlation due to an outlier) that were identified through monitoring and analysing the learners' responses to classwork (ref Section 4.5.3, second lesson observation, and line 4bii) would have been taking into consideration during lesson planning on scatter plot construction. Their inability to label the data axis due to incorrect scaling was identified by oral questioning from the learners (ref Section 4.5.3, first lesson observation, and line 8a).

Penso (2002) opines that practising teachers should be encouraged to consider learners' thinking and prior knowledge in lesson planning to avoid possible learning difficulties that learners may experience during lesson. Hill *et al* (2008) also reported that the sequence of teaching and learning may be altered if learners' prior knowledge is not considered during lesson planning and presentation. Teacher C addressed the learning difficulties by using a conceptual knowledge approach, and reviewing the learners' homework to reinforce their understanding. He also conducted post-teaching discussions during and after ogive and scatter plot construction lessons (ref Section 4.5.3 second lesson observation, and line 12).
The difficulties in terms of labelling the data axis of grouped data graph incorrectly were confirmed through analysis of the learners' workbooks (ref Appendix xxi, learners' workbooks), as well as the teacher's responses to the questionnaire and written reports (ref Section 4.7.3) in which he indicated that he identified learners' learning difficulties on graphs of grouped data through analysis of their classwork, homework and assignments. The learners, however, still followed the teacher after the lesson on scatter plot construction, demanding clarification about misinterpretation of a negative linear scatter plot that he had re-explained during the lesson. The teacher had evidently not addressed their learning difficulties sufficiently, which means that in teaching the construction of scatter plots his PCK was not comprehensive enough to cater for the learners' learning difficulties (Westwood, 2004). At this stage Teacher C did not exhibit enough PCK because his teaching could not cater for all the learners' learning difficulties in ogive and scatter plot construction. He subsequently addressed the learning difficulties experienced by the learners (such as misinterpreting a scatter plot because of outliers) in post-activity discussions, a strategy that he used frequently in his lessons (ref Section 4.5.3, second lesson observation, line 13). Capraro *et al* (2005) note that a competent mathematics teacher should be able to exhibit progressively more PCK in his or her lessons since he or she has acquired more experience

from formal education programmes and should plan his or her lessons in a way that is designed to avoid any learning difficulty that learners are likely to encounter.

In summary, the PCK profile of Teacher C may be construed as an amalgam of the various components of PCK, as defined earlier. His presumed PCK in teaching data-handling topics lies in his ability to use oral questioning and homework to identify the learners' preconceptions, as well as his use of construction skills and recommended mathematics and statistics-related textbooks, and past Senior Certificate Examination question papers in statistics to plan how to teach the construction of ogives and scatter plots. A combination of procedural and conceptual approaches, as well as the use of everyday situations and examples in teaching the statistics topics, constituted the instructional strategies that Teacher C employed to teach ogives and scatter plots. By identifying learners' learning difficulties through monitoring and analysing learners' responses to classwork, Teacher C can be said to have knowledge of learners' learning difficulties. But these difficulties were not always followed up in terms of taking them into consideration when planning the next lesson in order to identify learners' preconceptions of the new topic.

The question that one would want to ask at this stage is how, then, do the teachers develop their PCK in statistics teaching? Precisely, Teacher C's PCK on the construction of ogive could be said to have been developed through classroom practice and learning experiences over time. In terms of his formal education, Teacher C received further training on the teaching of mathematics. He holds a BSc degree, majoring in mathematics. His qualifications may have informed the reason that his content knowledge of the subject matter can be considered adequate.

Teacher C has five years of mathematics teaching experience. His instructional strategies in teaching statistics would have involved lesson planning, using the recommended work schedule and textbooks in school statistics, delivering lessons, and checking and marking learners' responses to homework. Other sources of PCK included reviews of his teaching portfolios and learners' workbooks. These activities may have contributed to the development of topic-specific PCK in statistics teaching

Teacher C attended workshops arranged by the District office of the Department of Basic Education. Most of these workshops focused on the new topic of data handling and particularly on how to teach it.

5.2.4 Teacher D

In Teacher D's observed lessons, it was noted that he had planned and taught his lessons on bar graphs and histograms using the Department of Basic Education's mathematics work schedule, and the recommended textbooks as sources of information (ref Section 4.5.4 first lesson observation, and line 11). During his teaching of bar graph and histogram construction (ref Section 4.5.4, first lesson observation, and line 2c), he gave more evidence of a procedural approach to teaching bar graphs and histograms than a conceptual one. For example, Teacher D taught the lesson on bar graphs in a step-by-step manner, beginning with pre-activities to identify learners' prior knowledge of bar graph construction, followed by the preparation of a frequency table compiled by the learners using a familiar daily life example (ref Section 4.5.4, first lesson observation, lines 1 and 2c). In this case, a frequency table was prepared of the number of cars in a car park according to their make (ref Section 4.5.4, first lesson observation, line 1). Next, with the help of the frequency table, a bar graph was constructed by first drawing its horizontal and vertical axes and labelling them appropriately. A scale was chosen by the teacher with the explanation that this was done by considering the highest and lowest values of the frequencies and the companies that manufactured the cars. Next, the points were plotted and the line of best fit was joined to produce the bar graph (ref Section 4.5.4, first lesson observation, lines 2c and 3). The teacher's specific strategy for teaching bar graph construction followed a rule-oriented procedural approach using procedural knowledge.

Engelbrecht *et al* (2005) describe the procedural knowledge approach as "following a rule or procedures flexibly, accurately, efficiently and appropriately in completing a given task". For example, in constructing a statistical graph, procedural knowledge approach requires a series of actions such as drawing the axes, choosing the scale, labelling the axes, plotting the points and joining the line of best fit. But what may be sometimes challenging is knowing how to move from the procedural stage to a conceptual meaningful one in terms of the students' learning.

As with the other teachers, Teacher D's procedural knowledge may have been developed over his 15 years of teaching mathematics in high school, using the recommended lesson plan and work schedule for statistics (DoBE, 2010). It could be suggested that although Teacher D possesses adequate ways of presenting bar graph construction to his learners, his PCK may be limited in the sense that he presented his lesson procedurally, an approach that was not always responsive to the learners' needs. Consequently, some of the learners constructed the classwork task without leaving spaces between the bars of the graph. The inability to consider the consistency of spaces between the bars of a graph during lesson presentation resulted in the learning difficulties that the learners experienced during classroom practice.

According to Shulman (1987), representation involves the teacher thinking through the key ideas and identifying alternative ways of presenting them to the learners. It is a stage in which suitable examples, demonstrations and explanations are used to build a bridge between the teacher's comprehension of the subject matter and what is required for the learners (Ibeawuchi, 2010). Multiple forms of representations are highly desirable if one is to be successful in the teaching process (Rollnick *et al*, 2008). Teacher D, however, in certain graphing topics, did display evidence of an alternative conceptual knowledge approach in teaching histograms (ref Section 4.5.4, second lesson observation, lines 11). Engelbrecht *et al* (2005) describe the conceptual knowledge approach as "involving an understanding of mathematical ideas and procedures consisting of the knowledge of basic arithmetic facts". Therefore it is knowledge that is rich in relationships and understanding of important statistical concepts in bar graph and histogram constructions. In the lesson observed, Teacher D explained in detail the meaning of a histogram. According to Teacher D, "a histogram is a graphical representation, showing a visual impression of the distribution of grouped data. It consists of tabular frequencies shown as adjacent rectangular bars, erected over discrete intervals, with an area equal to the frequency of the observations in the interval. Unlike the bar graph, a histogram is used to represent a large set of data (e.g. a population census) visually, but with no spaces between the bars" (ref Section 4.5.4, second lesson observation, lines 5b). His conceptual approach (presumably PCK) to teaching the construction of a histogram enhanced conceptual understanding of the topic as the learners seemed to be satisfied with Teacher D's conceptual explanation (ref Section 4.5.4, second lesson observation, and line 11) of how to construct a histogram after the learners had experienced misconceptions and learning difficulties in labelling the data axis (ref Section 4.5.4,second lesson observation, and line 10). They displayed the non-verbal cue of nodding their heads in agreement with the teacher's explanation (ref Section 4,5,4, second lesson observation, and line 12).

From the lessons observed with Teacher D, he used a procedural knowledge approach more rather than a conceptual knowledge approach. His preferred use of this approach was confirmed in the document analysis conducted in Teacher D's learner workbooks. The learners had completed the diagrams on bar graphs and histograms efficiently, with indications of the procedures that had been adopted in constructing these statistical graphs. Star (2002) argues that it is important for practising teachers to possess both kinds of knowledge in order to impart teaching to the learners in a meaningful way. The use of a rule-oriented procedural approach and a conceptual knowledge approach reveals that teachers are looking for ways of making the teaching of bar graphs and histogram comprehensible and accessible to their learners. Moreover, the construction of graphs demands that a particular order of actions should be followed, consistent with conceptual understanding. Teacher D can therefore be said to possess and demonstrate the required knowledge of bar graph and histogram construction.

Over and above this, Teacher D was able to identify learning difficulties experienced by the learners during the lesson and alternative conceptions from the various graphing exercises that were carried out by the learners. One such learning difficulty was their inability to choose the correct scale for labelling the data axis. This meant that they constructed a bar graph instead of a histogram (ref Section4.5.4, second lesson observation, and line 6a). In this case, Teacher D may be said to have presented his lesson in a limited way. His lessons were dominated by procedural knowledge teaching without providing the reasons underlying such procedures. He may not have accommodated the possibility of anticipating learning difficulties during the lessons on bar graph and histogram construction in his lesson planning and presentation and resolving them. For instance, he indicated the scale for constructing a bar graph and how it was obtained without explaining his reasons for choosing it, which shows that he may have presented his lesson in a limited way. When the learners adopted the same procedure to construct the bar graph during classwork, they did not consider the consistency of spacing in a bar graph, which resulted in a histogram instead of a bar graph. In terms of subject matter content knowledge, Teacher D may not have demonstrated the required variety of ways of presenting bar graphs and histogram construction for easy comprehension by the learners. Gersten and Benjamin (2012) note that the use of several strategies for teaching mathematics helps to deepen learners' understanding behaviourally and mathematically, avoids possible alternative conceptions and learning difficulties, and achieves effective learning. This means that teachers should be flexible (able to use a variety of instructional strategies to make content more accessible to more learners) in the representation of bar graph and histogram construction.

Regarding knowledge of instructional skills and strategies, Teacher D used analysis of learners' responses to classwork on bar graphs and histograms to identify their alternative conceptions and learning difficulties (ref Sections 4.5.4, second lesson observation, and line 6a). The teacher questionnaire, written report and learner workbooks confirmed this use of monitoring and analysing learners' responses to classwork to identify their alternative conceptions and learning difficulties. He addressed these difficulties through the instructional strategies of additional explanations, extra class activities, and examples related to familiar situations. These methods are consistent with the findings of Penso (2002), Westwood (2004), Bucat (2004), Mitchel and Mueller (2006) and Cazorla (2006), who adopted the same strategies for dealing with learners' misconceptions and learning difficulties. In practice, teachers are expected to design good teaching and learning instructions that take into consideration ways of identifying and addressing learners' learning difficulties (Westwood, 2004; Jong *et al*, 2005; and Rollnick *et al*, 2008). The other instructional skills that Teacher D used in teaching bar graphs and histograms were the construction skills involving drawing the axes, choosing the scale, labelling the axes, plotting points and joining line of best fit, which require a procedural knowledge approach.

The greater part of Teacher D's knowledge of learners' preconceptions and learning difficulties was gathered while teaching the assigned topic in statistical graphs. As observed earlier, during classwork the learners' inability to choose an appropriate scale for histogram and bar graph construction was identified through monitoring and analysing their responses (ref Section 4.5.4, second lesson observation, and line 6a). But the teacher did not display any evidence of having anticipated the learners' learning difficulties with bar graph and histogram construction, revealing that he may have gone into class without necessarily having knowledge of learners' possible learning difficulties in these constructions. To this end, Teacher D can therefore be said to have displayed insufficient PCK in terms of awareness of learners' preconceptions of bar graph and histogram constructions. From instance, at the start of the lesson, Teacher D requested the learners to prepare a frequency table of the makes of cars in a car park. The learners prepared the frequency table efficiently using previous knowledge. Thus, the teacher realised that the learners had previous knowledge that could be linked to bar graph construction, and no preconception was identified. When asked about the learning difficulties that learners might have had or were likely to experience during his lesson, he indicated that although that the learners had problems in determining the mid points and constructing graphs of group data, he would deal with difficulties that might arise (ref Appendix xx, item 10).

Inadequacy in teachers' knowledge of learners' preconceptions was common to the entire group of teachers involved in this study. This suggests the need for further investigation into the reasons that such teachers with many years of experience should have such a knowledge deficit in an area that is essential for effective classroom practice. As indicated in the section on Teacher A, Penso (2002) suggested that in their lesson planning, practising teachers should explore a variety of instructional strategies that would elicit learners' thinking and prior knowledge of the concept being taught in order to deal with their learning difficulties effectively. Hill *et al* (2008) note that the sequence of teaching and learning may by interfered with and possibly create opportunities for learning difficulties to occur if learners' preconceptions are not considered when planning and presenting a lesson.

Teacher D tried to address difficulties through extra explanations and homework assignments (ref Section 4.5.4, second lesson observation, and line 13). The teacher questionnaire, written reports and document analysis confirm the use of pre-activities, extra explanations and class activities in the form of classwork and homework to evaluate how well learners have understood the lessons and to gain insight into learners' pre-existing knowledge of bar graph and histogram construction.

It can be gleaned from the above discussion that Teacher D displayed a combination of the components of PCK that were identified earlier (Hill *et al*, 2008). Teacher D's presumed PCK is evidenced in his lesson planning and preparation, and in the use of textbooks in school statistics and mathematics, as well as other learning materials, such as past Senior Certificate Examination question papers. Pre-activities and correction of homework assignments were the instructional strategies he used to identify preconceptions about bar graphs and histograms. A combination of procedural and conceptual approaches to the teaching of statistics, as well as the use of exemplars drawn from familiar situations, was another instructional strategy that Teacher D used to teach the construction of bar graphs and histograms (ref Section 4.5.4, second lesson observation, lines 5c). Teacher D at this stage of using several instructional skills and strategies can be considered to have displayed knowledge of instructional skills and strategies for teaching bar graphs and histograms. Misconceptions such as drawing a histogram instead of a bar graph (ref Section 4.5.4, first lesson observation, and line 8) were addressed through post-teaching discussions, additional explanations during the lessons and homework (ref Section 4.5.4, first lesson observation, lines 9 and 11).

In sum, the sources for the development of Teacher D's PCK can partly be linked to the formal education that he acquired from a teacher training programme. He holds a BEd and SED, majoring in mathematics education. These qualifications and his 15 years of experience may have provided Teacher D with the opportunities to develop his content and pedagogical knowledge in statistics teaching. His instructional strategies over the years would have involved the use of lesson planning in line with the recommended work schedule, textbooks in school statistics and presentation of his lessons based on learning skills and reviewing of learners' classwork, homework and assignments. Other sources of development of his topic-specific PCK would have included reviews of his portfolios and learner workbooks. Teacher D attended content knowledge workshops organised by the Department of Basic Education (DoBE). Most of these workshops dealt with new aspects of the mathematics curriculum, especially the issues around teaching topics such as data handling.

5.3 Evaluation of theoretical framework

To evaluate the theoretical framework of this study is to determine to what extent the theoretical framework has enabled the researcher to answer the research questions.

The conceptual knowledge exercise, concept mapping exercise, teacher interviews, lesson observations and document analysis were the instruments used to examine the subject matter content knowledge of the participating teachers in school statistics in this study. The intention of the researcher in using these instruments for data collection was to determine the subject matter content knowledge that the participating teachers demonstrated in classroom practice. What can be gleaned from the results is that the instruments allowed the researcher to capture the teachers' PCK in terms of the subject matter content knowledge in statistics teaching. The concept map exercise was used as a proxy, but was not sufficient to determine how knowledgeable the teachers were about the contents of the curriculum (ref Section 4.4). The teachers should have been requested to write an examination in order to determine their content knowledge of the topic. But because it might be difficult to get the teachers to write an examination, a concept mapping exercise was considered a good proxy for assessing their content knowledge. Another way in which the teachers' content knowledge could have been examined was through certification. That is by reviewing the certificate obtained from colleges and universities. Considering a certificate in mathematics education without observing how a teacher demonstrates his or her content knowledge in the classroom may not be sufficient to determine whether that teacher possesses content knowledge of a topic. Hence, lesson observations were used to assess the teachers' subject matter content knowledge and how well they demonstrated this knowledge in statistics teaching. Although Mahvunga and Rollnick (2011) suggest that a quantitative research study may be sufficient to assess teachers' content knowledge, their study failed to indicate how to assess the quality of teachers' content knowledge, which can be determined only during classroom practice. This assertion is given wide empirical support by researchers such as Toerien (2011), Ball *et al* (2008), Capraro *et al* (2005), Jong *et al* (2005), Lee and Luft (2008), Jong (2003) and Gess-Newsome and Lederman (2001), who all note that PCK is rooted in classroom practice. Any research into teachers' PCK that does not consider the use of lesson observation may fail to fully convey the required information about how teachers develop topic-specific PCK. Through lesson observation, it was possible to determine how the teachers demonstrated their content knowledge of certain topics. Lesson observation provided opportunities to experience the details, nuances and dimensions that the teachers used in their classroom practice in order to determine the adequacy of their subject matter content knowledge (ref Sections 4.5.1–4.5.4). Through the teacher interviews, it appears that the teachers' educational backgrounds that may have enabled them to develop topic-specific content knowledge in statistics were determined (ref Section 4.7.1).

While researchers such as Shulman (1986), Van Driel *et al* (1998), and Magnusson (1999) use subject matter knowledge consisting of syntactic and substantive knowledge acquired in formal education, in this study the subject matter content knowledge focused on the content to be taught and learned by the students. The use of subject matter content knowledge as the theoretical framework for this study proved useful in determining the procedural and conceptual knowledge (component of the PCK) that a teacher demonstrates in teaching statistical graphs. Other PCK studies (Plotz, 2007; Lee & Luft, 2008; Adela, 2009; Ibeawuchi, 2010; Ogbonnaya, 2011; and Toerien, 2011) share the same view of using subject matter content knowledge as a theoretical framework for examining teachers' PCK development in mathematics. These authors also assess the subject matter by making the teachers write a test on the content of the topic under investigation. The instruments developed with the framework were therefore considered adequate to determine teachers' subject matter content knowledge in statistics teaching and the theoretical framework can be considered adequate and valid.

The teacher questionnaire, which focused on what the teachers did while teaching the assigned topic, and the written reports used to triangulating the data collected with lesson observations were used to determine the pedagogical knowledge (instructional skills and strategies) that the teachers used in teaching school statistics. Other instruments used to assess the teachers' pedagogical knowledge were lesson observation and document analyses.
The questionnaire revealed many aspects of the teachers' PCK, such as knowledge of instructional skills and strategies for teaching statistical graphs. These strategies included oral probing questioning, checking and marking learners' homework and pre-activities to determine learners' pre-existing knowledge (ref Section 4.8). The lesson observations, teacher written reports and document analyses confirmed the use of these instructional strategies. These activities were crucial in determining learners' conceptions about statistical graphs, as suggested by Krebber (2004), Westwood (2004) and Ball *et al* (2008), but did not elicit learners' preconceptions in statistical graphs. From the lesson observations, it was not possible to determine learners' preconceptions because the strategies the teachers adopted to do so did not elicit them. Instead, the learners displayed previous knowledge linked to learning the new topic. In fact, the teachers did not have knowledge of the instructional skills and strategies that might have been necessary to determine the learners' preconceptions in statistical graphs.

As Krebber (2004) and Westwood (2004) suggest, the use of the instructional strategies of oral probing questioning, pre-activities, checking and marking learners' responses to classwork, homework and examining learners' understanding, as well as identifying their misconceptions and learning difficulties in statistical graphs, is critical in learning and could motivate the development of teachers' pedagogical knowledge. Loughran *et al* (2004), Ball *et al* (2008), and Vistro-Yu (2003) regard teachers' pedagogical knowledge as crucial to PCK development. Having ascertained the instructional skills and strategies demonstrated by the teachers through the teacher questionnaire, written reports, document analyses and lesson observation, the researcher believes that the teachers' pedagogical knowledge can be considered a valid theoretical framework for determining the PCK required for teaching school statistics.

However, the framework provided an opportunity to reveal that the teachers had some knowledge of learners' misconceptions, as individually they were able to identify misconceptions through analysis of learners' responses to classwork, homework and assignments in statistical graphs. The activities of identifying and addressing learners' misconceptions are critical aspects of teaching and learning. Penso (2002), Carzola (2006) and Westwood (2004) note that a teacher who lacks the ability to identify and address learners' misconceptions may experience poor content delivery in classroom practice. Practising mathematics teachers are encouraged to learn about the possible instructional skills and strategies for identifying and addressing learners' alternative conceptions in statistical graphs.

Penso (2002), Westwood (2004) and Carzolia (2006) also posit that if learners' alternative conceptions and difficulties are not identified and addressed in the preparation and presentation of lessons, negative lesson presentations can occur. The lesson observations (ref Section 4.5.1–4.5.4), teachers' written reports, and learners' and teachers' portfolios confirmed that the participating teachers know about learners' learning difficulties in statistics (ref Sections 4.7–4.10). Therefore, knowledge of learners' learning difficulties can be considered adequate as a theoretical framework for capturing teachers' PCK in statistics teaching.

5.4 Summary of the chapter

This chapter opened with a brief recapitulation of the research questions and PCK components as a theoretical framework for this study.

The teachers demonstrated that they possess content knowledge of school statistics. However, the predominant approach to putting across statistical ideas to their learners about data handling, particularly the construction of statistical graphs, was procedural. A conceptual approach was used less to some extent and not as the same degree as procedural approach. The individual teachers are presumed to have developed their PCK in statistics teaching by extending their knowledge of the subject matter content through formal education programmes and the use of topic-specific mathematics and statistics textbooks and other publications as sources for lesson planning and teaching.

The instructional skills and strategies used by the participating teachers for teaching specific statistics topics consisted largely of oral questioning, pre-activities, and post-teaching discussions to determine preconceptions. By using these instructional skills and strategies to teaching statistical graphs, the participating teachers may have developed their PCK in statistics teaching. An analysis of the learners' classwork, homework, assignments, and post-teaching discussions was used to determine where the learners' misconceptions and learning difficulties lay. All four teachers, although at different times, used extra tutoring, problem-solving activities involving familiar daily-life contexts, individualised teaching, post-teaching discussions, and repetition of the lessons to address learners' difficulties and misconceptions (ref Section 5.2.1-5.2.4).

This chapter concludes with an evaluation of how the theoretical framework was used to ascertain whether it provided adequate opportunities to develop instruments for collecting data to answer the research questions.