CHAPTER FOUR

4.0 DATA ANALYSIS AND RESULTS

4.1 Introduction

This chapter contains an analysis of the data and presents the results of the main study. Statistical procedures (outlined in Sections 3.7.3.1 and 3.7.3.2) were used to analyse the quantitative data by categorising the responses and lesson observations of the participating teachers according to the components of PCK (pedagogical content knowledge) in order to answer the research questions. The results are presented in the following order:

- Conceptual knowledge exercise
- Concept mapping exercise
- Classroom practice (lesson observation)
- Teacher interview
- Teacher questionnaire
- Teacher written report
- Classroom observations and video recordings
- Document analysis

4.2 Conceptual knowledge exercise

The main purpose of the conceptual knowledge exercise was to make a performance-based selection of teachers for the second phase of the study. The second phase consisted of a concept mapping exercise, an interview, lesson observations, questionnaires, written reports, and document analyses.

The percentage scores of the top four teachers, designated A, B, C, and D, in the conceptual knowledge exercise were 85, 90, 90, and 75 respectively.

4.3 Teacher demographic profiles

The profiles of the four selected teachers are presented below.

Table 4.2: Teacher A, B, C, and D profiles

Name of teacher	Qualification	Subject taught	Teaching experience (in years)	Grade taught
Teacher A	BEd (Mathematics Education), BA (Psychology), Diploma (Mathematics and Science)	Mathematics	21 years	11 and 12
Teacher B	BSc (Mathematics and Statistics)	Mathematics & Mathematical Literacy	10 years	11 and 12
Teacher C	BSc (Mathematics)	Mathematics & Mathematical Literacy	5 years	11 and 12
Teacher D	BEd (Mathematics Education), SED (Mathematics and Biology)	Mathematics	15 years	11 and 12

It is clear that the participants are qualified and experienced mathematics teachers and it was assumed that they have sufficient subject matter content knowledge to competently teach statistics in school mathematics.

4.4 Concept mapping

The four teachers drew a concept map (ref Section 3.5.1.2) on statistics. The results of this exercise, assessed according to the guidelines used to evaluate their responses (ref Section 3.5.1.2), showed that teachers A and C scored 100% each; Teacher B scored 92%; and Teacher D scored 80%. Teachers A and C arranged their topics according to the scheme used, so no marks were deducted. Teachers A and C had greater knowledge than teachers B and D of the school statistics curriculum content and how it should be taught logically so that one topic formed the basal knowledge for the next topic.

4.5 Classroom practice (lesson observation)

The purpose of the lesson observation was to examine interaction patterns in the classroom for each of the teachers, in other words how they used their content knowledge in teaching particular statistics topics. The instructional skills and strategies used by the teachers, the ways in which they tried to identify learners' preconceptions and learning difficulties, and what they did to rectify these misconceptions and learning difficulties, if any, were also examined. The topic in which most lessons were observed was graphing in statistics (line graphs, bar graphs, histograms, pie charts, frequency polygons, ogives, box-and-whisker plots, and scatter plots) since this topic is one of the most challenging in school statistics (DoE, 2010). Two periods of lessons were observed at a time, during site visits to each of the teachers. The observations focused on what the teacher did before (e.g. lesson planning), during (e.g. asking oral probing questions to determine learners' prior knowledge), and after the lesson (e.g. post-teaching discussions and other interventions to address identified learning difficulties).

The same format of analysis was used for all the teachers to identify the components of PCK used in teaching the lessons. The next section presents an analysis of the lesson observation of Teacher A. While observing the teachers, the focus was on how the teachers demonstrated their content knowledge, pedagogical knowledge, knowledge of learners' preconceptions and learning difficulties such as indicating how their assumed PCK manifested during classroom practice. The analysis of the lesson observation will also take into account the coding and categorisation of the themes as shown on the table.

4.5.1 Lesson observation of Teacher A

This section briefly describes Teacher A's lesson observations on teaching statistical graphs. The lesson focused on the construction, analysis, and interpretation of histograms and box-and-whiskers plot respectively. The condition of the classroom is first described, followed by the teacher's classroom practice.

Table 4.5.1 Description of classroom condition and lesson observation of Teacher A

DESCRIPTION OF LESSON	CATEGORISATION/THEMES
<u>Condition of the classroom</u> There were 15 male and 20 female learners of mixed ability. Learners were comfortably seated in six columns of single chairs and desks, with sufficient space to move between the desks. The teacher had a full view of the entire class during the lessons. The classroom walls were decorated with science wall charts. The furniture, windows and door were in good condition, with electrical wiring that permitted the use of appliances such as an overhead projector. The mathematics class was resourced with textbooks, calculators, exercise books, and graph sheets for each learner, as well as construction instruments for the teacher (ruler, protractor, and pair of dividers). The classroom had locks, and burglar bars for supervised entry	1) The classroom presented a safe learning environment for both boys and girls. 2) Learners were well resourced with textbooks and other learning materials including workbooks.
CLASSROOM PRACTICE (FIRST LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of a histogram. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: After Teacher A had greeted the class, he introduced the lesson on histograms with oral questioning, distributed evenly to different learners, and requested them to define the mode, the median, and the mean in a distribution of ungrouped data	Oral probing questioning was used as an instructional strategy (**pedagogic knowledge**) to introduce the lesson on histograms and determine learners' conceptions and definitions of basic concepts linked to the grouping of data in histogram construction (line 1)
Line 2: One of the learners defined mode as: *'….. The number that appears most often in a distribution,'* and gave an example of mode by verbally listing some numbers and locating the mode within the listed numbers. A second learner defined the median as: '*… The middle number when a distribution of numbers is arranged according to size.'* A third learner defined the mean as: '*… The average of the distribution.'* The last answer was followed by an example from the same learner, who listed some numbers, added them all together, and divided the sum by the number of numbers on the list, to determine the mean. All three learners identified or pointed out by the teacher provided correct definitions for the terms 'mode', 'median', and 'mean'.	Learners correctly defined mode, median and mean (line 2), attesting to **teacher A's content knowledge.** Using a questioning strategy, Teacher A was able to identify learners' previous knowledge about the statistics lesson topic.

Line 3: After the introduction, Teacher A gave the class an example of how to construct and interpret a histogram. He said, '*Write down this example*.' : (i) Construct a frequency table of five classes, starting from 16, (ii) calculate the mean, (iii) draw a histogram, and (iv) use the histogram to calculate the mode of the ages, correct to the nearest year, of 27 members of a netball club. The ages are as follows: 17, 21, 23, 19, 27, 38, 20, 21, 28, 31, 18, 21, 24, 30, 25, 19, 22, 27, 35, 18, 27, 22, 20, 30, 27, 21, and 23. The solution to these questions was presented as follows by Teacher A and the learners, working together:	**Teacher content knowledge** was used to work through an example of how to construct and interpret a histogram (line 3)
(1) Construction of frequency table Line 4a: Teacher A drew a frequency table with the given class intervals, as shown in table 4.5.1a. The table contained the ages of the members of a netball club, the frequencies of the age groups, the mid-values (x) of the age groups, the class boundaries and fx. The teacher did not explain the meaning of the terms. It may be assumed that the learners had come across terms such as class interval (ages), frequencies, mid-values, and the product of frequency and mid-values before because preparing a frequency table of ungrouped data is taught before grouped data according to the curriculum. Teacher A showed the learners how the class intervals (ages) are calculated, using a class of five: for example, he said, '*Beginning from 16 and with five classes, the next class is 20. Therefore, 16–20 is a class interval*.' The teacher continued, '*The next class is 21–25, the other class intervals are: 26-30, 31-35, and 36-40'* (see Table 4.5.1a). Line 4b: Teacher A listed the frequencies of the frequency (f) column on the chalkboard as learners individually counted the ages within the intervals (see Table 4.5.1a) under his instruction. For instance, he asked, '*How many persons are within the ages 16-20?'* The learners counted individually and indicated the frequencies to the teacher who wrote them in the frequency column.	**Teacher content knowledge** was used to describe and complete a frequency table from raw data (lines 4a and 4b) He engages learners by asking them to determine the frequencies within the class intervals row by row (line 4b).

Table 4.5.1a. A frequency table showing the age distribution of members of a netball club

Ages	Freq. (f)	Mid-values (x)	fx	Class boundaries
16–20	6	18	108	15.5–20.5
21–25	10	23	230	20.5–25.5
26–30	8	28	224	25.5–30.5
31–35	2	33	66	30.5–35.5
35–40	1	38	38	35.5–40.5
	27		666	

Line 5a: Teacher A showed the learners how to calculate the mid-values: e.g. he said, '*Mid-value* = $\frac{16+20}{2}$ = 18 (*for the first row*).' Teacher A continued, '*For the second row: mid-value* = $\frac{21+25}{2}$ *= 23 (for the second row) .Now continues with row 3, 4 and 5.'* The learners continued with the calculation of mid-values while the teacher wrote the acceptable values on the chalk board.

Line 5b: The next step was to calculate fx, meaning frequency multiplied by mid-values (x). Teacher A demonstrated: '*To calculate fx, you multiply the value of frequency and mid-values, i.e. fx = 6 x 18 = 108 for the first row; for the second row, fx= 10 x 23 = 230; for the third row, fx = 8 x 28 = 224; for the fourth row, fx = 2 x 33=66; and the fifth row, fx = 1x38= 38.'*

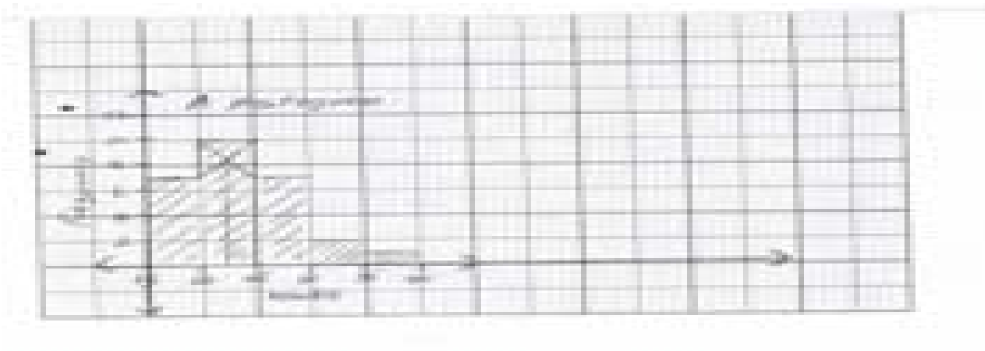
Teacher content knowledge was used to describe how to calculate mid-values and fx (lines 5a and 5b).

Learner content knowledge was used to complete mid-values (line 5b).

Line 6: Teacher A began by describing how to find the class boundaries, beginning with the first row. He then selected an example from table 4.5.1 and calculated the lower class boundary = $\frac{15+16}{2} = 15{,}5$ (for the first row). In the learners' mother tongue, he said, *'15 tlhakanya le 16 arola ka 2, e lekana le 15.5; Meaning add 15 to 16 and divide by 2, equal to* 15.5.' He continued: '*The upper class boundary* = $\frac{20+21}{2}$ *= 20,5.'* (see table 4.5.1a.). Teacher A Further requested the learners to complete the class boundaries for other rows.	**Teacher content knowledge** was used to describe how to complete the frequency table by calculating mid-values and class boundaries to construct the histogram (lines 5a and 5b). The learners' mother tongue (**instructional strategy**) was used to further reinforce a point on how to calculate class boundaries (line 6).
Line 7: The learners completed the table after the teacher had shown them how to calculate the frequencies, mid-values (x), fx, and class boundaries.	**Teacher content knowledge** was used to demonstrate how to complete the frequency table by calculating the frequencies, mid-values, class boundaries, and fx (Line 7).
Teacher A indicated that the next exercise would comprise (II) **Calculating the mean from frequency table:** to begin the demonstration on how to calculate mean from the frequency table. Line 8: Teacher A wrote on the chalkboard: '*Mean is calculated by using the formula,* $\frac{\sum fx}{\sum f}$ *, where* $\sum fx$ *means the sum of frequencies(f) multiplied by the mid-values (x) and* $\sum f$*, means summation of frequencies only, as shown in Table 4.51a*.'Using the formula, he showed the learners how to calculate the mean as follows: Mean = $\frac{\sum fx}{\sum f} = \frac{666}{27} = 24{,}67$	**Teacher content knowledge** was used to calculate mean from the frequency table (line 8) using a **procedural knowledge** approach. *Procedural knowledge approach* is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately to accomplish a given mathematical task. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). An algorithmic approach was used to calculate the mean from the frequency table (line 8).

(iii) **Constructing the histogram** Line 9: Teacher A defined a histogram as: '… *a statistical graph which is used to represent grouped data; a histogram helps to understand complex data in a simpler manner through visualisation*.' He then described how to construct a histogram without explaining what the term grouped data meant. He began by drawing the horizontal and vertical axes on the chalkboard and reinforced the terms using the learners' mother tongue. He said, '*Thala mola o o horizontal le o o vertical',* meaning, draw the horizontal and vertical axis. This was followed by stating the chosen scale. He indicated that the scale was chosen by considering the highest and lowest values of the frequencies and data values as well as the dimension of the graph paper provided, but without demonstrating it mathematically to the learners. He continued with the labelling of the axes and said: '*O be o tsenya di nomore mo meleng',* meaning, label the axes. He drew the first two bars of the histogram. He instructed the class to complete the graph and stated the chosen scale again with no mathematical explanation of how the scale was chosen. To do so would have required a more detailed conceptual explanation.	Teacher A defined a histogram and described how to construct a histogram using a **procedural** as opposed to **conceptual knowledge approach**. A *Procedural knowledge* is a formal symbolic representation system of a given mathematical task using algorithms, or rules, to complete the mathematical tasks (Star, 2002). As indicated above, the participating teachers used more of a procedural knowledge approach than a conceptual knowledge approach because the topic required a particular procedure. It is the common way in which the teachers used algorithms or rules to complete statistics task.He did not explain what was meant by grouped data. Once again the mother tongue equivalent of the technical terms was used to enhance comprehension. Topic specific graph construction skills of drawing horizontal and vertical axes, choosing a scale' and labelling the axes were used to teach the the learners histogram construction (line 9). Teacher A stated and used a chosen scale for constructing the histogram without a conceptual explanation of how it was done (line 9).
Line 10: The learners completed the histogram individually in their workbooks after the teacher had demonstrated on the chalkboard (ref Figure 4.5.1a) with the assistance of another learner how to construct a histogram from the grouped data given.	Learners completed the histogram based on the teacher's demonstration of histogram construction on the chalk board (line10)
Line 11: Some learners seemed to have understood how to construct a histogram for they completed the exercise in	Some learners experienced difficulty in selecting the

their workbooks correctly. Others had difficulties in choosing an appropriate scale so that the histogram could not be accommodated on the graph paper provided. The teacher identified those who were experiencing difficulties because these learners were erasing and correcting their mistakes. He intervened by asking one of the learners, '*Why are you erasing your work*?' The learner answered '*My work is not correct compared to the one on the chalkboard,*' Teacher A then asked, '*Do you understand why your diagram is wrong?'* The learner answered '*Yes, I have seen it on the chalkboard*' and the teacher directed the same question to the other learners who were also erasing their work. They all agreed that they had detected their mistakes from the correction on the chalkboard. The teacher had to allow the learners to write the corrections from the chalkboard into their exercise books for a few minutes before proceeding to calculate the mode from the histogram. The intention of allowing the learners to complete the diagram was to ensure that all participated in using the same diagram to calculate the mode. The next part of the lesson was on how to calculate mode from the histogram.  **Figure 4.5.1a: Histogram of the age distribution of members of a netball club (with a continuation line from the vertical axis)**	appropriate scale (line 11) for constructing a histogram. Insufficient explanation was provided by Teacher A about how to choose scale for constructing a histogram (line 11). Learners who were experiencing some difficulties corrected them with the histogram constructed by the teacher and the learners on the chalkboard (line 11). The learners grasped the rule for the construction of a histogram (line 11).
Line 12a: After the histogram was constructed by Teacher A and the learners, the teacher described another method of constructing histograms. This method allows the histogram to be constructed without a continuation line from origin of the data axis even if the data does not start from o, to reinforce the learners' understanding of histogram construction (ref Figure 4.5.1c). He used the same rule-oriented procedural approach.	**Teacher content knowledge** was used to explain another method of constructing a histogram. It involved creating a continuation line beginning from the vertical axis. The second method helped to reinforce learners'

 Figure 4.5.1b: **A Histogram showing the age distribution of members of a netball club with labelling of the data axis without a continuation line starting from the vertical axis** Line 12b: Teacher A demonstrated the construction of a histogram by beginning the labelling of the data values from the vertical axis, plotting the points, and joining the line of best fit using the same table of values and histogram that had just been constructed. Having constructed the histogram the, next step was to show how to calculate the mode from it..	knowledge of histogram construction and interpretation (line 12a). **Teacher content knowledge** was used to describe the procedure (procedural knowledge approach) of constructing a histogram (line 12b).
(iv) **Calculating the mode from the histogram** Line 13: Teacher A demonstrated how to calculate the mode (using Figure 4.5.1a as presented on the chalkboard). He first drew a diagonal line from the top right-hand corner of the highest bar of the histogram to the top right-hand corner of the bar to the left of it. The next step was to draw another diagonal from the top left-hand corner of the highest bar to the top left-hand corner of the next bar to the right of it. He then drew a line from the meeting point of the two diagonal lines to the horizontal axis and read out the mode at that point (ref Figure 4.5.1a). No explanation was given as to how the drawing of diagonal lines leads to the determination of the mode.	Teacher A used a **procedural knowledge approach** to determine the mode of a histogram (line 13) without explaining the conceptual reasoning behind the drawing of the diagonal lines. *Conceptual understanding* consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts.

	Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. It is called a conceptual knowledge approach when applied in teaching.
Line 14: A learner asked, '*Why do you have to draw a diagonal? Why don't you simply add 20 and 25 and divide by 2 to get the mode?'* This question was posed by the learners because some of them had done it in that way. Many learners nodded their heads in agreement with the question.	Some learners wondered why they should draw diagonals to locate the mode because they calculated the average of the interval of the highest bar instead of locating the mode within the interval of the highest bar identified (line 14). They might have experienced this difficulty of understanding why diagonals should be drawn before locating the mode because the teacher had not explained the term grouped data from the beginning.
Line 15: Teacher A tried unsuccessfully to explain why diagonals should be drawn from both bars on either side of the tallest bar in the histogram to calculate the mode. He said, '*Drawing the diagonals is a procedure for calculating the mode of the grouped data, and the diagonals help to locate the mode within the intervals*.' A conceptual knowledge approach of explaining the relationships among the concepts in histogram construction such as the class boundaries, class intervals, frequency and drawing the line of best fit of a histogram should have been used to answer the question, so as to provide clarity and the answer to the question the learners asked.	The teacher used **procedural knowledge** to answer the learner's question, but the question demanded a conceptual knowledge (explaining the relationship and mathematical connections among the concepts in histogram construction explanation), which the teacher did not provide at this stage (line 15).
Line 16: Teacher A continued with the learner's question on why the diagonal should be drawn and the average of 20 and 25 cannot be used to calculate the mode from the histogram (line 14) when he answered, '*You cannot find the average of 20 and 25 to give you the mode, because the intervals do not contain only the numbers 20 and 25'There are other numbers within the intervals*. 'He referred them to stem-and-leaf diagrams (drawn previously)	A **conceptual knowledge** approach was used to explain why it is not correct to add 20 and 25 in order to determine mode. Comparing the answers obtained from a stem-and-leave with the histogram (line 16) showed

to show how the mode was located and said '*Open to the stem-and-leaf you drew last time and let somebody tell us how we can locate the mode.*' One of the learners raised his hand and explained, *'23 is the most occurring number in the stem-and-leaf diagram and that is the mode.'' Now compare the answer we got from the stem-and-leaf and the one from the histogram, are they the same*?' the teacher asked to elicit an answer as to whether they could link the relationship between the two methods for calculating the mode in grouped data. The learners answered in a chorus, *'Yees sir*.'	that the teacher possesses the content knowledge required to teach histogram construction.
Line 17: Teacher A continues '*Now that you have understood the procedure I have described, write it down in your notebook'*.	Teacher A instructed learners to copy the procedure for calculating the mode on the chalkboard (line 17).
Line 18: The learners wrote the procedure for calculating a mode from a histogram in their exercise books, as provided by Teacher A (see Figure 4.5.1a and 4.5.1b) and shown on the chalkboard. Using photocopied materials, Teacher A provided examples of the useful application of histograms to everyday life situations. For example, '*They can be used to represent the age distribution of teachers in the school and the performance of groups or cohorts of learners in an examination'* he said.	Teacher A related the application of histograms to everyday life familiar situation (line 18) (**instructional strategy).** Learners copy the procedure as written on the chalkboard (line 18).
Line 19a: As the lesson progressedTeacher A asked one of the learners, '*What is the difference between a histogram and a bar graph?'* Line 19b: A learner answered, '*There are constant spaces between the bars in the bar graph, but there is no space in the histogram between the bars. Second, the bar graph is used to represent simple data and histogram is used to represent large groups of data. Because the data that histogram represent are large, they are grouped as class intervals or boundaries in the frequency table. Bar graph do not contain class interval or boundaries'* This answer was satisfactory to the teacher, who asked a second question.	A higher level of questioning (explanation, not recall) was used as an **instructional strategy** to assess how well learners had understood the lesson (line 19a). Learners showed evidence of comprehension in the answer provided about the differences between a bar graph and a histogram (line 19b).
Line 20a: Teacher A asked: *'How can you calculate the percentage of players within the age group of 26–40 in the histogram?'* (ref Figure 4.5.1a). A few learners indicated an interest in answering the question; one was asked to give an answer and she said. '*You add 7 + 2 + 1 = 10* '(from the frequency table), *'then divide 10 by 27 and*	Oral questioning based on **application of knowledge** was used to assess learners' content knowledge about histogram construction (line 20a).

<table>
<tr><td>multiply by 100; i.e. the percentage of players between 26 and 40 = $\frac{10}{27}$ X 100 = 37%. Therefore, 37% of the players are between the ages of 26 and 40.'</td><td>Learner content knowledge: an algorithmic approach was used to answer the teacher's oral question on how to calculate the percentage of players within an age group (line 20a).</td></tr>
<tr><td>Line 21: Teacher A assigned classwork in which the learners were asked a similar question on histogram construction and interpretation to the one they had already done. The classwork required learners to construct a histogram and use it to determine the mode and the percentage of learners who had completed a test. Table 4.5.1b (below) shows the mark distribution of the test. The teacher walked around the class to monitor the learners.

Table 4.5.1b: Frequency table showing learners' performance in a test

<table><tr><th>Class interval (%)</th><th>Frequency</th></tr><tr><td>40–49</td><td>2</td></tr><tr><td>50–59</td><td>6</td></tr><tr><td>60–69</td><td>12</td></tr><tr><td>70–79</td><td>8</td></tr><tr><td>80–89</td><td>4</td></tr></table>
a) Draw a histogram to illustrate learners' performance in the test.
b) From your diagram, calculate the mode.
c) If the pass mark is 60%, calculate the percentage of learners who failed the test.</td><td>Classwork was used to spontaneously assess how well learners had grasped the content of the lesson (instructional strategy to provide immediate feedback) (line 20b).

Teacher A monitored and analysed learners' responses to classwork on construction and interpretation of histograms (line 21) to ascertain how well the learners were responding to the classwork and to detect learning difficulties and misconceptions, if any.</td></tr>
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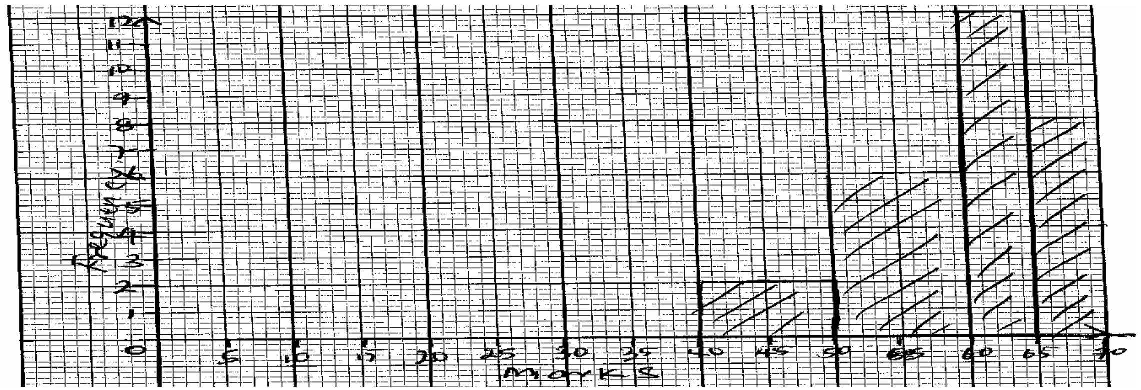
Line 22a: While most learners completed the class work efficiently, some could not finish it in class. The difficulties experienced were in (i) labelling of the axes with the types of grouped data provided (which began at 40 marks and not from 0 as was the case in the example which the teacher worked on), and ii) the construction (scaling and labelling of the axes) of the histogram. Figure 4.5.1c (below) shows an example of a graph drawn by a learner who experienced difficulties in histogram construction. The histogram could not be accommodated on the graph paper provided due to incorrect scaling.  **Figure 4.5.1c: An example of an incomplete classwork exercise on histogram construction** In this graph, the scale chosen by the learner(s) could not accommodate the histogram on the graph paper; hence part of the histogram was not represented. This made it difficult to calculate the mode and determine the percentage of learners who failed the test. Line 22b: A learner said, '*My graph is not like the one you constructed on the chalkboard.*'	Learning difficulties experienced by learners were labelling and scaling of data axes of grouped data (ref Figure 4.5.1c) (lines 22a and 22b). Lack of comprehension was evident in a learner's statement-(line 22b).
Line 23a: Teacher A analysed what the learner had drawn and said, '*You constructed a bar graph instead of a histogram. It is a wrong histogram.*' He continued, '*I shall organise an extra lesson to rectify the difficulties you are having and explain why your answer is wrong after the lesson.*' (The lesson period had expired.) After the	A learning difficulty of constructing a bar graph instead of histogram was detected by Teacher A from the classwork that the learners were doing (line 23a). A

lesson, some learners asked the teacher to explain aspects of the lesson where they lacked clarity (post-teaching discussion). Line 23b: Teacher A gave the learners homework on construction and interpretation of histograms using their textbook in mathematics, to be submitted the following day. The entire lesson was based on the learners' mathematics textbooks, photocopies of mathematics-related materials, and study guides.	post-teaching discussion took place after the lesson to help them (line 23a). Learners' learning difficulties were discovered through an analysis of classwork (**instructional strategy**) (line 21 and 23a). Homework was used as an opportunity for learners to demonstrate their understanding of histogram construction, and later to assess how well the learners had understood the lesson (**instructional strategies** for teaching) (line 23b).

CLASSROOM PRACTICE (SECOND LESSON)

Topic: Construction, analysis, and interpretation of ogives and box-and-whisker plots. Class: Grade 11

DESCRIPTION	CATEGORISATION/THEMES
Line 1: Teacher A began the lesson on box-and-whisker plots by checking and marking the homework on cumulative frequency tables and ogives (a distribution curve in which the frequencies are cumulative).	The checking and marking of homework was used to try to determine learners' conceptions (preconceptions) (line 1) (**instructional strategy**) in box-and-whisker plot construction.
Line 2: Teacher A and the learners provided the correct answers to the homework on the construction, analysis, and interpretation of a cumulative frequency table and ogive by calculating the cumulative frequencies and further explaining how it was used to construct an ogive.	The teacher and learners together consolidated the concept previously taught by providing corrections to the difficulties the latter must have experienced while doing the homework (**instructional strategy**) (line 2) on ogive construction.

Line 3a: Teacher A wrote the topic (box-and whisker plots) on the board and referred the learners to photocopied material on ogives, from which they could interpret an ogive using quartiles obtained from an ogive. They were to work in groups of 4 to 5 learners and calculate the quartiles as a way of demonstrating their knowledge of how to construct an ogive. Line 3b: Teacher A said '*Look at the photocopied paper I have given you, question 2.'* He continued and read, '*Find out the percentage of learners who obtained (i) less than the lower quartile; (ii) less than the median; and (iii) less than the upper quartile; and (iv) Minimum and maximum values of the ogive.'*	**Instructional strategies** such as group work were used to interpret ogives and to demonstrate learners' content knowledge and understanding of how to construct an ogive (line 3a). **Teacher content knowledge** and **instructional strategies** were used to design the task to be used to demonstrate box-and-whisker plot construction (line 3b).
Calculation of quartiles from an ogive after the learners had completed the exercise Line 4: The learners interpreted the ogive (it was assumed that Teacher A had provided a description of ogive construction in the previous lesson) as the question on the photocopy indicated, with the first quartile (see definition below)(Q_1) = 20, using the formula; $Q_1 = \frac{(n+1)th}{4}$ to calculate the position of Q_1. The next step was to calculate the second quartile (Q_2) = 23, using the formula, $Q_2 = \frac{(n+1)th}{2}$ to calculate Q_3; and the third quartile (Q_3) = 27, using the formula, $\frac{3(n+1)th}{4}$ (where a quartile is a division of the data distribution into four equal parts). '*The minimum is 15 and maximum is 38 (read from the ogive,*' one of the learners said in response to the questions on the photocopied question. Teacher A accepted the answers provided by the learners for Q_1, Q_2, Q_3, minimum and maximum values as correct, and said, '*Now, we are going to use these values to construct a box-and-whisker plot.*' He defined a box-and-whisker plot as *'… a graph showing the distribution of a set of data along a number line.'* With no further explanation, he went on to describe how to construct a box-and-whisker plot.	An algorithmic approach (**procedural knowledge**) was used by the learners to determine the quartiles (line 4). **Teacher content knowledge** was used to provide the definition of a box-and-whisker plot with no further explanation regarding the basic knowledge or skills to required for the construction of the graph. The teacher did not indicate or anticipate any possible difficulties or misconceptions that the learners might possibly encounter (line 4).
Construction of box-and-whisker plot	Teacher A used a **procedural knowledge** approach to determine the quartiles which were used to construct

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<tr>
<td>Line 5a: Because Teacher A was satisfied with the learners' answers on the quartiles derived from the ogive in line 4, he used the quartile values to show the learners how to construct and interpret box-and-whisker plots. He did this by first drawing a number line with a scale of 1 cm = 5 units (see below). The box was drawn above the number line using the values for Q_1, (23) Q_2 and Q_3 (27) (Fig 4.5.1d). The whisker was then represented by a line, according to the maximum (38) and the minimum value (15) as obtained from the ogive as shown below (Fig 4.5.1d).

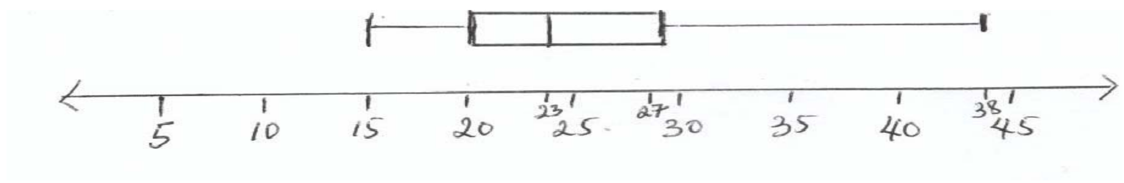
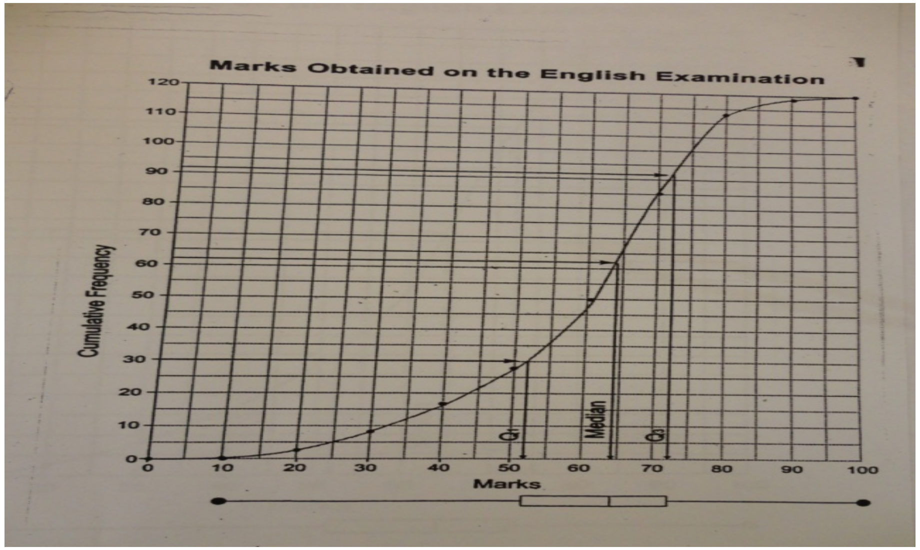


Figure 4.5.1d represents a box-and-whisker plot constructed with the values of the quartiles obtained from the ogive
Line 5b: Some learners experienced difficulties making sense of why the minimum and maximum values of Q_1, Q_2 and Q_3, had to be used for constructing a box-and-whisker plot. This was largely because the teacher did not explain the meaning of this term.</td>
<td>a box-and-whisker diagram (line 5a) (instructional strategy).

Insufficient teacher content knowledge and explanation of box-and-whisker plot resulted in learners' learning difficulties (line 5b).

Construction skills were used to construct a box-and-whisker plot with the quartiles obtained from the ogive using a procedural knowledge approach (line 5a) without explanation.</td>
</tr>
<tr>
<td>Line 6: Most of the learners requested clarity on interpreting ogives. For example: how the values of the quartiles were obtained and used to construct the box-and-whisker plot. 'Listen learners,' the teacher said, 'it appears that some of you do not understand the description I have given about the construction of box-and-whisker plot. Now let me give you another example from the textbook.'</td>
<td>Learners experienced difficulties as to how the teacher obtained the quartiles. They had no understanding of how the values of the quartiles from an ogive were obtained and used to construct a box-and-whisker plot (line 6). The teacher resorted to the use of the textbook to work through a textbook ogive example.</td>
</tr>
<tr>
<td>Line 7a: Teacher A referred the learners to their textbook, unit 8 (containing examples of what they had done). Using these textbook examples (while individual learners took note of the example from their textbook), he then tried to</td>
<td>Teacher subject matter content knowledge was supplemented by the use of textbooks to provide</td>
</tr>
</table>

explain the mathematical connection between ogives and box-and-whisker plots. He described an ogive as *'… a cumulative line graph and it is best used when you want to display information involving grouped data,'* He continued, *'To interpret an ogive, quartiles are usually used. The quartile values are used to construct the box-and whisker plot to provide more clarity about what the data tend to convey.'* ', Teacher A said. He continued *''Now I*



want you to study this example in unit 8 in your textbook for five minutes.'

Figure 4.5.1e: Ogive showing the mark distribution of learners in an English examination

This diagram of an ogive from the learners' textbook was used to provide another example of the way in which to construct ogives and box-and-whisker plots. The box-and-whisker diagram (ref Fig 4.5.1e) was constructed from information derived from the analysis and interpretation of the ogive.

definitions on the concepts of ogive such as the quartiles. There was no attempt to relate the concepts being studied to any context or examples familiar to the learners. When and how are ogives used for example? The teacher does not demonstrate any flexibility (or insufficient flexibility) in the approaches or methods used to present the topic (line 7a).

An example from the learners' mathematics textbook was used (as an **instructional strategy)** to provide some clarity on how quartiles were obtained from the ogive and used in constructing a box-and-whisker plot (line 7a)

Teacher content knowledge (Figure 4.5.1e) was used to describe the interrelationship between ogives and box-and whisker plots by reading out the quartile values from the ogive in Figure 4.5.1e and used to construct a box-and-whisker plot (line 7c).

Class work was used as an **instructional strategy** to reinforce learners' grasp of how to calculate quartiles from the ogive (line 7c).

Line 7b: Learners studied the example for about five minutes and compared it with their previous homework box-and-whisker plot construction to try and comprehend how the values for constructing the box-and-whisker plot had been obtained. Line 7c: Teacher A described: *'The first quartile is obtained by first locating the quartile position on the frequency axis, draw a line from there to join the curve, and join the line to the horizontal axis to locate first quartile (Q_1)'* using Figure 4.5.1e. The same procedure is used for Q_2 and Q_3. Teacher A asked, '*Do you understand?*'The learners answered, '*Yes sir.*' .As a follow up the teacher gave them a task: Using the same Figure 4.5.1d, the learners were asked to (i) find the estimate of a) the lower quartile (Q_1); b) the median (Q_2); c) the upper quartile (Q_3). (ii) Find out what percentage of the learners had obtained marks that were a) less than the lower quartile, b) less than the median, and c) less than the upper quartile. The intent was to find out if the learners had understood how to obtain the quartiles from the ogive, which could then be used to construct the box-and-whisker plot. Line 7d: As the teacher monitored and analysed learners' classwork assignment, he discovered that the majority of the learners were unable to locate the position of the quartile from the ogive even after applying the correct formula.., This was either because the learners lacked the knowledge and skills of scaling and labelling of data axis, or that the teacher's oral explanation was not sufficient for them to grasp the concept.. Teacher A said '*I can see that some of you cannot locate the quartiles even after you have calculated the position of the quartiles. Now, let me do it with you.*'	Learners experienced some difficulties in locating the quartiles from the data axis due to insufficient **learner content knowledg**e about scale and labelling of the data axis (line 7d). The teacher intervened regarding the errors that the learners were making on their classwork and had to work with them using Figure 4.5.1e to clarify the learning difficulties.
Finding the quartiles (Q_1, Q_2 and Q_3) Line 8a: Using the formula for calculating the position of quartiles as in line 4 and Figure 4.5.1e, Teacher A showed the learners how to calculate quartile positions by the use of a ruler to trace the quartiles beginning from the cummulative frequency axis to the curve and down to the data axis to obtain : a) the lower quartile (Q_1) which was 52. He continued in a similar manner to obtain: b) the median (Q_2) which was 63, and c) the upper quartile (Q_3)	**Teacher content knowledge** was used to demonstrate the procedure for calculating the quartiles from the data axis (line 8a) in order to clarify learning difficulties about box-and-whisker

which was 73. **Calculating the percentage of learners that score marks less than the quartiles** Line 8b: Teacher A said, ' *Let us solve the remaining questions,'* and continued,, *'you calculate 25% of 120 as:* $\frac{25}{100}$ x $\frac{120}{1}$ = 30. *With your ruler at 30 on the cummulative frequency axis, trace it to join the curve and down to the data axis. Therefore, a) 25% of the learners obtained marks of less than 52%. In a similar manner, b) 50% of the learners obtained marks of less than 63%, and c) 75% of the learners obtained marks of less than 73%.'* The answers to the two questions (i) and (ii) are the same but the question was asked in two different ways.The teacher probably wanted to demonstrate varieties of ways of asking questions about quartiles and provide various strategies of answering the question, which illustrates the teachers' PCK. Line 8ci: A learner raised her hand and asked, '*Why is the method of calculating the median in the ogive different from the one we did last week?'* The learner referred Teacher A to her exercise book and showed him that the method was different from what she had in her book. Some learners nodded their heads in support of the question. But one of learners raised his hand up and he was recognised by by the teacher to answer the question: And he said '*In the previous example, we calculated the median of ungrouped data. But in this case we are calculating the median of grouped data'.* The methods were different, but the learners had misunderstood the ways the median is calculated in ungrouped data and in grouped data. This learning difficulty may have arisen because the teacher did not explain the difference between determining the median of ungrouped and grouped data in line 8b and in any previous ungrouped data lesson. Line 8cii: Teacher A explained, pointing at the previous example in one of the learners' exercise books and directing the whole class to the same example in their individual exercise books, '*The previous example used ungrouped data,*	plot construction. The learners' oral questions indicated that they had some learning difficulties concerning the formula for calculating the median of grouped and ungrouped data (line 8ci) which may have been due to insufficient teacher explanation of the differences between the way the median in ungrouped and group data is calculated (line 8cii). A **conceptual knowledge approach** was used to address the learners' lack of understanding of the differences between how to calculate the median of group and ungrouped data (line 8cii) by comparing the differences between the way the median is calculated from grouped data in the current lesson and ungrouped data from previous lesson.

in which you arrange the data according to size of the numbers, but the current example used grouped data in which some data were grouped together. You cannot arrange them in the same way like the ungrouped data because, the particular number within the groups are not known. Hence, the formula method was appropriate to calculate the median within the class intervals or group.'	
Line 9: Teacher A asked, as a way of concluding the lesson, '*How do you calculate the first, second and upper quartiles of an ogive? How can you use the quartiles to construct a box-and-whisker plot?'*	Oral questioning was used to assess the learners and evaluate the lesson by requesting the learners to explain how quartiles are calculated (line 9) (**instructional strategy**).
Line 10a: Several learners volunteered to answer the question; the teacher selected one who said: '*Using the formula* $\frac{n+1}{4}th$ *, you can calculate Q_1 position and locate Q_1. Using the formula* $\frac{n+1}{2}th$ *, you can calculate Q_2 position and locate Q_2. Using the formula* $\frac{3(n+1)}{4}th$ *, you can calculate Q_3.and locate Q_3.*' (rote learning regarding the use of an algorithm). Line 10 b: Teacher A called on another learner to demonstrate how the values of Q_1, Q_2 and Q_3 could be used to construct a box-and-whisker plot. Line 10c: The learner used the teacher's *example to answer the question in a procedural manner by indicating:'Using the formula (pointing on the chalkboard), you calculate Q_1 position by substituting the value of n. After that the quartile position is located on the frequency axis and by drawing a line from that position to the curve and down to the horizontal axis, you locate the first quartile (Q_1).' Q_2 and Q_3 were calculated in the same way*, the learner said.	**Learner content knowledge** mostly of a procedural or algorithmic nature was used to answer the question on the application of a formula (line 10a). The learners continued with their responses to the teacher's question to indicate that they had grasped the lesson (line 10c)
Line 11a: The learners were then referred to their textbooks for homework. This required the learners to calculate the quartiles from a constructed ogive and use the quartiles to construct a box-and-whisker plot. The assessment task tested learners' conceptual understanding of how to construct, analyse, interpret and apply the knowledge of box-	Homework was used as **instructional strategy** to assess and provide feedback on learners' conceptual understanding of the lesson on box-and-whisker plots

and-whisker plots to a familiar situation. The homework showed that the teacher complied with the assessment guidelines and learning outcomes of data handling-but provided no examples in his teaching of the application of those plots in contexts familiar to the learners. Obviously, Teacher A has displayed inadequate PCK in teaching box-and whisker plot construction at this stage.. Line 11b: A post-teaching discussion took place after the lesson. Some learners asked: '*How do you represent the fractions we got from the graphs during the interpretation of the ogive?'* (following the results from their calculations). The teacher replied: '*The fractions can be represented by rounding off to the nearest whole number.'*	(line 11a). A **post-teaching discussion** was used to address learners' questions and to clarify the method of representing fractions on the box-and-whisker plot (lines 11b).
Line 12: The teacher promised to organise extra tutoring after school for the learners who were experiencing difficulties with the construction and interpretation of ogives and box-and-whisker plots as he could not attend to everybody in the post–teaching discussion.	A **post-teaching discussion** was used to address aspects of the topic which the learners did not grasp (confusion over the use of quartile values to construct box-and-whisker plots), and additional tutoring was proposed (lines 12).

Summary of lesson observation of Teacher A

Teacher A demonstrated that he has the required content knowledge to teach statistical graphs such as histograms, ogives and box-and-whisker plots. He described, and demonstrated how to construct, a histogram and tried to elucidate the differences between ogives and box-and-whisker plots, using a mostly rule-oriented procedural approach; but less of a conceptual knowledge explanation. Using his procedural knowledge he followed a stepwise sequential approach to demonstrate the construction of a histogram and box-and-whisker plot: namely drawing of the axes, choosing a scale, labelling the axes, plotting the points, and then drawing the line of best fit. With regard to section 8cii of the second lesson observation, Teacher A also applied a conceptual approach in clarifying learners' misunderstanding of how to construct a box-and-whisker plot using the quartiles calculated from the ogive. The conceptual approach entails explaining in detail the relationship between the quartiles obtained from the ogive (e.g.$Q_{1,}$ median, and Q_3) of a box-and-whisker plot (ref Section 4.5.1, second lesson observation, and line 8cii), the mathematical connections between quartile positions and the quartiles obtained from the ogive. Teacher A used topic-specific construction skills (as earlier defined) in statistics to construct histograms and box-and-whisker plots. Instructional skills of oral questioning, checking and marking of learners' classroom and homework assignments were also used to try to identify learners' preconceptions and learning difficulties in constructing histograms and box-and-whisker plots. But the teacher identified learners' previous knowledge of histogram and box-and-whisker plot construction using the strategy oral questioning and checking and marking of learners' homework. Other instructional strategies which Teacher A applied in his teaching were the use of examples drawn from everyday familiar situations for the histogram, but for the ogive and box-and-whisker plot he applied the mother tongue to reinforce learners' comprehension. There was no evidence that he anticipated the difficulties learners were likely to have in first coming across the topics of histograms and box-and-whisker plots that he taught. For example, when he tried to identify learners' preconceptions using oral probing questioning on measures of central tendency, learners displayed evidence of having a previous knowledge of histogram construction and no preconception was identified, meaning the teacher may well not likely have knowledge of learners' preconceptions, which would have allowed him to address any anticipated learning difficulty.

From the observed lessons, it can be construed that the PCK of Teacher A consists largely of the procedural use of rules to construct histograms and box-and-whisker plots (statistical graphs) and, less frequently, of conceptual knowledge.

4.5.2 Lesson observation of Teacher B

This section briefly describes Teacher B's lessons on teaching statistical graphs. The lessons, which were observed during two periods of site visits, focused on the construction, analysis, and interpretation of the bar graph and the ogive. The condition of the classroom is described first, followed by the teacher's classroom practice in delivering the lesson.

Table 4.5.2a: Description of lesson observation and classroom conditions at School B

DESCRIPTION OF LESSONS	CATEGORISATION/THEMES
Condition of the classroom There were 16 male and 24 female learners of mixed ability. Learners were comfortably seated in the science laboratory in two columns of single chairs surrounding some big desks with sufficient space to move between the desks. The laboratory was safe and conducive to teaching and learning. The wall of the laboratory was decorated with science charts such as the human circulatory system. The learners were individually resourced with learning material such as the mathematics textbooks, exercise books and calculators. The science laboratory is sometimes used when the teacher want to use an overhead projector for demonstration.	1) Forty learners were seated in single chairs surrounding some big desks in two columns. 2) The school was safe and well protected. 3) The science laboratory is not used exclusively for science subjects. 4) The learners were resourced with learning materials
CLASSROOM PRACTICE (FIRST LESSON OBSERVATION). **Topic: construction and interpretation of bar graphs. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: The teacher arrived in the class and greeted the learners '*Good afternoon learners*?' Learners answered '*Good afternoon sir*' A frequency table was used to introduce **Teacher B's** first observed lesson. Learners were expected to prepare a frequency table of the scores of learners in a test. The data presented to the learners by **Teacher B** was based on the scores that learners had obtained in a 10-mark test, and involved arranging these scores on a frequency table: 2, 3, 4, 5, 5, 6, 4, 7, 5, 6.	Teacher B greeted the class and placed a **pre-activity** on the chalk board to gain information about learners' conceptions **(preconceptions)** of the construction and interpretation of bar graphs (line 1) (**instructional strategy**).
Line 2: The **learners** individually prepared a frequency table within five minutes (ref Figure 4.5.2a).	Learners showed that they had assimilated the knowledge of how to construct a frequency table from their previous lesson as they prepared it efficiently (line 2 and table 4.5.2a).

Table 4.5.2b: Frequency table showing the performance of learners in a test

Scores (x)	Tally	Freq. (f)	Fx
2	/	1	2
3	/	1	3
4	//	2	8
5	///	3	15
6	//	2	12
7	/	1	7
		$\sum f = 10$	$\sum fx = 47$

Construction of a bar graph

Line 3a: **Teacher B** described algorithmically how to construct a single bar graph, using the data from the frequency table (ref Figure 4.5.2a) prepared by the learners as indicated in line 2. '*Now, watch out, you begin by drawing the vertical and horizontal axes'* he said. **Teacher B** drew the horizontal and vertical axes and asked the learners to explain how to choose the scales for the axes. He asked, '*How do we choose the scale for labelling the axes?'*

Line 3b: Some learners raised their hands and the teacher pointed at one to explain.

Line 3c: Learners stated how the numbers should be written on both the horizontal and vertical axes, by indicating 1, 2, 3, 4 etc for the horizontal axis and 2, 3, 4, 5, 6, and 7 on the vertical axis, while the teacher wrote the numerals on the chalkboard and elucidated why the scale had been accepted for constructing the bar graph, for such reasons as considering the highest and lowest values on the frequency table and data and the dimension of the graph paper.

Teacher content knowledge was used to describe how a bar graph is constructed (lines 3a and 4).

Teacher B probes learners with a question to find out if they know how to choose a scale for constructing a bar graph (line 3a).

Teacher B merely indicated the scale that was chosen by the learners and why it accepted and wrote them on the chalk board with no mathematical justification of how either the learners or himself had selected the scale (line 3a)

Line 4a: **Teacher B** showed how to label and draw the bars, using the appropriate frequencies on table 4.5.2a: *'Watch and see how to draw the bars; the first score is 2, and the corresponding frequency is 1',* the teacher said. Learners watched as the teacher demonstrated how to draw one of the bars on the axes corresponding to the score (data axis) with a value of 2 and frequency is 1.

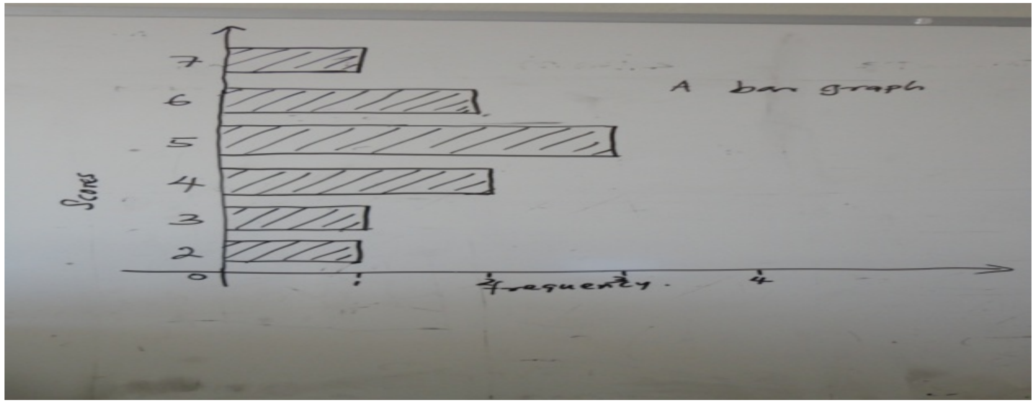


Figure 4.5.2a **Bar graph of the scores of learners in test on how to construct, analyse, and interpret a bar graph using the scores in column 1**

Line 4b: The teacher asked, *'How can I draw the second bar graph?*' The teacher nominated one of the learners, who answered, '*The second score is 3, and the frequency is 1*'. The teacher demonstrated how to draw the second bar (indicating that he was satisfied with how the first bar was constructed) and instructed the learners to copy and complete the bar graph in their exercise books while he monitored them. While monitoring, he discovered that certain learners experienced some difficulties because they had not left a constant space between the bars, which he indicated without explanation. He intervened by helping the learners to complete the bar graph and indicated that there should be constant spacing between the bars.

Teacher content knowledge was used to describe how to construct a bar graph using a **procedural approach** (**instructional strategy**) (lines 3a, 3c, 4a, 4c and 5).

Teacher B analysed learners' classwork as he monitored their work on bar graphs (line 4c).

Graph construction skills (drawing the axes, choosing scales, labelling axes, plotting the points and drawing the lines of best fit) were used by learners in drawing a bar graph (lines 3, 4a and 4b, and Figure 4.5.2a) (**instructional skill**).

Misconceptions and learning difficulties in constructing a histogram instead of a bar graph were identified by monitoring and analysing learners' responses to classwork and in the class discussion (lines 4c and 4d). Learners may have experienced such difficulties due to insufficient explanation of why there should be constant spacing between the bars of a bar graph (line 4b).

Line 4c: He asked the learners to watch while he completed the bar graph on the board. Learners who were experiencing learning difficulties (e.g. constructing a bar graph like a histogram) corrected their mistakes as he did so (see Figure 4.5.2a). Line 4d: The **learners** asked, 'Why *they had to leave spaces between the bars?*'	
Line 5: **Teacher B** referred to the graph on the chalkboard and answered: '*The bars represent different scores; the height of the bars represents the number of learners that scored a particular mark, e.g. two learners scored 4 marks, and the constant spacing differentiates one score from another, as the number of learners that score a particular mark is not the same*'	**Teacher B** answered the learners' question by demonstrating how to label the axes and explaining why it is necessary to leave constant spaces between the bars (line 5) (**teacher content knowledge**).
Line 6: **Learners** were given time to correct their misconceptions in their notebooks, as well as learning difficulties. Afterwards, the teacher explained again how to construct the frequency table and bar graph as he did in line 4a to 4c, as some of the learners continued to ask for clarity on why there should be constant spacing between the bars.	**Teacher B used the instructional strategy** of **again explaining** the preparation of a frequency table and bar graph construction to clarify learners' understanding of the need for constant spacing between the bar**s of a bar graph** (line 6).
Line 7: The **learners** asked: '*How do you know that the 10-mark test was easy or difficult?'* This question demanded that the teacher explain the relevance of frequency tables and bar graphs, which he had not done initially.	Learners asked a question that required the teacher to explain the relevance of frequency table and bar graphs (line 7).
Line 8: **Teacher B** explained: '*Other factors could be used to determine whether the test is easy or difficult, but at the moment, the pass mark is considered'*. For example, '*If the pass mark is 4 and the number of learners that scored 4 and above is 8 out of 10 learners, then the test was easy*'. **Teacher B** read out the number of persons who scored 4 and above as 8. '*This means that about 90% of the learners scored between 4 and 10. But if the number of learners that scored between 1 and 3 is 8 (Teacher B read from the graph), and the highest score was 5, the test was difficult, as 80% of the learners scored below 4 marks'*. He continued, *'Thus, with a bar graph, it is easy to show and interpret learners' performance in a test. From Figure 4.5.2a, it is evident that the test was within the level of the learners, as the learners' marks were not too low, and if the pass mark was 4 (40%), then only two of the learners failed'*.	**Teacher content knowledge** was used to explain the criteria and demonstrate how to determine whether the 10-mark test was difficult or easy (line 8) (**teacher content knowledge).**

<table>
<tr>
<td>Line 9: Teacher B gave out photocopies of classwork, in which learners were asked to construct a bar graph individually. The teacher monitored and analysed their responses as they worked. Some learners had drawn their diagrams, but had failed to consider the concept of equal spacing (maybe the learners had not understood the teacher's earlier explanation of how and why to leave constant spacing between the bars), causing them to construct bar graphs that resembled histograms. 'The spaces between the bars and the width of each bar should be the same to differentiate one item from the other, although the height of the bars will be different, because of differences in frequency', the teacher said as a way of correcting the learning difficulty during the lesson.</td>
<td>Iindividual learners did classwork on bar graphs efficiently (independent instructional strategy) (line 9).

Learning difficulties occurred from misconceptions (constructing a histogram instead of a bar graph) (line 9).

Learning difficulties were identified through analysis of their classwork (line 9).</td>
</tr>
<tr>
<td>Line 10: Teacher B attempted to correct the misconceptions by explaining again how to construct the bar graph on the chalkboard, while the learners watched. The teacher again demonstrated how the axes were drawn, followed by choosing the scale, labelling the axes and drawing the bars. The problem arose because the teacher had not explained the reasons for the spaces between bars at the beginning.</td>
<td>Instructional strategy of again demonstrating how to construct a bar graph was used to correct learners' misconceptions (line10). The difficulties that the learners experienced could be traceable to insufficient explanation of how to construct a graph using a procedural knowledge approach (line10)</td>
</tr>
<tr>
<td>Line 11: Teacher B provided additional problem-solving activities based on familiar situations (ref table 4.5.2c). For example, learners were provided with a table containing the amount spent on groceries purchased from a supermarket, and were asked to draw a bar graph and to determine what percentage, of the total amount spent, the most expensive item constituted.

Table 4.5.2c: Frequency table showing the distribution of the amount spent on buying some groceries from a supermarket

<table>
<tr><td>Item</td><td>Tomatoes</td><td>Rice</td><td>Chicken</td><td>Maize meal</td><td>Onions</td></tr>
<tr><td>Amount</td><td>R10</td><td>R70</td><td>R35</td><td>R42</td><td>R3</td></tr>
</table>
</td>
<td>Problems related to a familiar situation were used by Teacher B to try to address the learning difficulty of drawing a histogram instead of a bar graph (line 11).</td>
</tr>
</table>

Table 4.5.2b contains items bought in a supermarket and the amount spent on each. For example, R10 was spent on buying tomatoes, R35 on buying chicken, etc.	
Line 12: Using table 4.5.2b, some of the learners tried to construct the bar graph quickly and efficiently, beginning with the labelling of the axes, choosing the scale for drawing the bar graph, labelling the vertical and horizontal axes, plotting points and drawing the bars, but a few still experienced certain difficulties, as they continued to ask why each bar should be separated from the other. This might indicate that either the learners lack the ability to understand or that the teacher's explanation was not sufficient to elicit an understanding of what had been explained.	**Learners** showed evidence of having understood the lesson on the construction of bar graph (line 12) (**construction skills** of drawing the axes, labelling axes, choosing scale, plotting the points and drawing the line of best fit). Some learners continued to experience difficulties despite the teacher's further explanation of bar graph construction (line 12). The teachers' explanation may not have sufficiently helped the learners to grasp what he had taught or the learners lacked the ability to understand the explanation (line 12).
Line 13: The lesson concluded with oral questioning. For example, the teacher asked, '*Why do we separate one bar from the other with a space?'* Homework on the construction and interpretation of bar graphs from their textbook was also given to the learners to reinforce their understanding of the construction of bar graphs. Teacher B promised to use extra tutoring to help learners who were still experiencing difficulties.	Teacher B asked oral questions and gave homework to learners on construction and interpretation of bar graphs to reinforce their understanding (line 13).
Line 14: A post-teaching discussion took place after the lesson in which some of the learners sought clarity on how to calculate the percentage of the most expensive items bought in the supermarket, which was one of the questions that had not been answered from the classwork. The teacher had to explain orally and asked the learners to complete it at home.	**Post-teaching discussion** was used to address learners' questions (line 14).
CLASSROOM PRACTICE (SECOND LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of ogives. Class: Grade 11**	

Line 1: Teacher B, standing in front of the class, introduces the lesson *'Today's lesson is about the construction and the interpretation of ogives'* Oral questions were directed at individual learners as in line 2.	The lesson was introduced by **oral probing questioning (instructional strategy)** (lines 2 and 4) to **identify learners' conceptions (preconception)**
Line 2: The teacher pointed to individual learners and asked them to mention ways in which data may be represented.	Teacher B identified learners who would answer the question (line 2). **(instructional strategy).**
Line 3: The learners referred to the frequency table, the bar graph, the pie chart, the histogram, the line graph, etc.	The learners' responses to the oral probing showed that they had insight into how to represent data (line 3) (**content knowledge).**
Line 4: **Learners** were referred to page 199 of their textbooks, activity 8.11, question 3, which contains the mark distribution of learners' performance in an English examination. The teacher requested the learners to:' *(a) prepare a cumulative frequency table of the learners' performance; (b) construct an ogive; (c) interpret the ogive by calculating the five-number summary (minimum, first quartile (Q_1), median (Q_2), third quartile (Q_3) and maximum value'*. Although question was set for the learners, but the teacher has to use it as an example to demonstrate how to construct and interpret ogive.	**Instructional strategy** of assessing how to construct and interpret an ogive was set for the learners and to be used to demonstrate how to do so (line 4).
a) Preparation of cumulative frequency table **Line 5a: Teacher B** demonstrated how a cumulative frequency table is constructed (see table 4.5.2c), using the first three rows of the table, and said '*add the frequency of the first and second rows to give the cumulative frequency of the second row (0 + 2 = 2 of second row). The cumulative frequency of the second row is added to the frequency of the third row to give the cumulative frequency of third row (2 + 6 = 8 of the third row), and so on'* (table 4.5.2c). He added, '*In groups of eight, complete the table by calculating the cumulative frequencies of the remaining intervals within 10 minutes.*'	**Teacher content knowledge** was used to prepare a cumulative frequency table (line 5). **Instructional strategy** to assess learners' understanding of a cumulative frequency table took the form **of group work activities** in class (**interactive instruction**) (line 4), (line 6b). Teacher's **procedural knowledge** was used to demonstrate how to prepare a frequency table (line 5a).

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Table 4.5.2d: Mark distribution of learners in an English examination

Marks	Freq (f)	Cumulative frequency
1–10	0	0
11–20	2	0 + 2 = 2
21–30	6	2 + 6 = 8
31–40	7	8 + 7 = 15
41–50	14	15 + 14 = 29
51–60	20	29 + 20 = 49
61–70	35	49 + 35 = 84
71–80	29	84 + 29 = 113
81–90	6	113 + 6 = 119
91–100	1	119 + 1 = 120

Line 5b: The **learners** completed the cumulative frequency table.

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a) Construction of ogive

Line 6a: **Teacher B** explained procedurally '*An ogive is constructed by drawing and labelling the axes with data on the horizontal axis and the cumulative frequencies on the vertical axis. The cumulative frequencies will help in the construction of the ogive' he said.*

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Instructional skill mostly used in constructing an ogive was a **topic-specific construction skill** (lines 6a and 6bi).

A procedural knowledge approach was used (**content knowledge and instructional strategy**) to demonstrate how to construct an ogive (line 6a and 6bi).

Teacher content knowledge was utilised to provide

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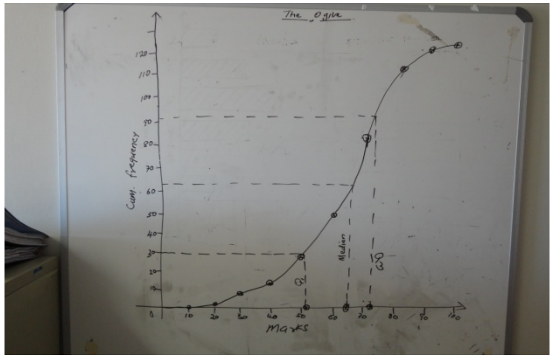

Figure 4.5.2b: Ogive representing learners' performance in an English examination

a) Line 6bi: The teacher demonstrated how to construct an ogive by plotting the cumulative frequency against the marks (e.g. 10, 0; 20, 2; and 30, 8) as indicated on the frequency table and joining the line of best fit. Afterwards the analysis and interpretation were performed using the formula for calculating the quartile positions and the **quartiles.**

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descriptions on how to plot the points from the frequency table on the axes of the ogive (teacher **content and instructional strategy)** (lines 6a and 6bi).

Interpretation of ogive by calculating the five-number-summary (minimum value, Q_1, Q_2, Q_3 and maximum values) (line 6bii) was carried out by Teacher B.

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Line 6bii: The teacher showed the learners how to calculate the position of the quartiles and said '*Using the formula* $\frac{n+1}{4}th$, *you can calculate* Q_1 *position and locate* Q_1. *Using the formula* $\frac{n+1}{2}th$, *you can calculate* Q_2 *position and locate* Q_2. *Using the formula* $\frac{3(n+1)}{4}th$, *you can calculate* Q_3.*and locate* Q_3.' All answers were obtained by using the position of the quartile calculated to read out the values of the five-number summary, such as minimum value = 10, Q1 = 52, Q_2 = 63, Q_3 = 73 and maximum value is 120, from the ogive.

Line 6c: A learner asked, '*Must the cumulative frequency always be on the vertical axis? Why don't you put it on the horizontal axis?'* This question showed that the learner did not understand how to label the axes of the ogive because

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Teacher B provided insufficient explanation **(PCK).** He focused on procedure at the expense of conceptual understanding. Hence learners were obliged to request further clarification about the position of the cumulative frequency on the ogive, which the teacher had not previously explained (line 6c) **(teacher content knowledge and instructional strategies).**

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the teacher had not explained this from the beginning, depicting the fact that the teacher displayed insufficient content and pedagogical knowledge to demonstrate how to label the axes of an ogive. Line 6d: **Teacher B** responded, *'You can label it on the horizontal axis, but it is more convenient to label it on the vertical axis, as you are expected to plot the cumulative frequency against the marks'* (see Figure 4.5.2b).	
Line 7a: **Teacher B** referred the learners to a photocopied exercise for classwork with a similar question in which learners were requested to prepare a frequency table, construct an ogive with the table prepared and calculate the five-number- summary (min, Q_1, Q_2, Q_3 and maximum values) from the ogive, but with class intervals starting from 20. He monitored them while they were doing their classwork. Line 7b: Most learners misunderstood the concept of labelling class intervals 0–10, 11–20, 21–30, and 31–40, etc. Instead, they labelled the class intervals on the horizontal axis 20–30, 30–40, 40–50, and 50–60, etc, instead of 10, 20, 30, etc. This approach does not allow the learners to plot the points on the data axis.	**Teacher content knowledge** was used to set classwork on ogive construction (line 7a) to ascertain how well learners have understood the lesson. **Learners' misconceptions** (line 7b) involving how to label the horizontal axis were identified through **analysis of their classwork.** The labelling could result in drawing a histogram instead of an ogive. Lack of understanding stemmed from insufficient elucidation, focusing on the procedural knowledge approach at the expense of conceptual knowledge (line 7b) (**Learning difficulty)**
Line 8a: The lack of understanding of how to label the axes was addressed by **Teacher B** in a class question and answer session (see line 8b). He also explained again how to construct an ogive, as in line 6a, and interpret the ogive, as in line 5a. Line 8b: **Teacher B** referred the learners to the diagram on the chalkboard (Figure 4.5.2b). He again explained how the ogive was interpreted by means of quartiles by using the formula $Q_1 = \frac{1}{4}(n+1)th$ for the first quartile, Q_2 =	**A rule-oriented procedural approach** was used to re-explain ogive construction (line 8b). **Teacher B** explained once more how to construct ogive and position of quartiles to reinforce learners' understanding of ogive construction and interpretation (line 8a)

$(\frac{n+1}{2})th$ for the second quartile, and Q_3 = ¾(n + 1) for the third quartile, to calculate the first, second, and third quartile position and the first, second and third quartile. These quartiles were used to interpret the ogive by deciding the percentage of learners who passed or failed the examination by gaining a given pass mark such as the median.	
Line 9: The other strategy used to address the learning difficulties was the provision of extra-class activities in their textbooks for the learners to solve after normal school hours. Its focus was on drill and practice, using the exercises from their textbooks, in order to make the lesson more accessible and comprehensible to the learners.	**Extra-class activities** on ogive construction were given to the learners from their textbook **(instructional strategy**) (line 9) to deepen their understanding of ogive construction and address learning difficulties.
Line 10a: **Teacher B** concluded the lesson with oral questioning. For instance, **Teacher B** asked, '*What does 'n' represent in the formula for calculating the quartiles? Where can I locate the quartiles using the formula?*' **Teacher B** nominated learners to answer the questions after many of them raised their hands. Line 10b: A **learner** answered the first question by saying '*n = 120 (meaning the sum of the frequencies as in the diagram on the chalkboard)'*. A second learner indicated the answer on the vertical axis but got it wrong. A third learner explained, '*you have to trace it through the vertical axis to meet the curve, and then go down to the horizontal axis, where you have to read off the value for the quartiles, e.g. Q1 = 52'* The teacher and learners accepted the answer.	**Oral questioning** was used in addition to monitoring classwork and homework to assess how well learners had achieved the learning outcomes of the lesson (line 10a). The intention of continuous learner assessment is to ascertain how well learners have understood the teacher's elucidation of ogive construction during the lesson. (**Teacher topic-specific content knowledge and pedagogical knowledge were used to determine learners progress**) (line 10a).
Line 11: **Teacher B** gave the learners homework by referring them to the same exercise in their textbook as mentioned in line 4, as well as to photocopies of past question papers containing questions related to the construction, analysis, and interpretation of ogives.	**Homework** was used as an **instructional strategy** to assess how well learners understood the lesson on ogives and consolidate the lesson (line 11).

Line 12: After the lesson, some learners asked him how to label the horizontal axis if the class boundaries did not start from zero. The teacher explained once more to the learners one by one using the example that had previously been given in class.	**Post-teaching discussion** took place between the teacher and the learners immediately after the lesson to address the learning difficulty (line 12) (**teacher content knowledge and instructional strategy).**

Summary of lesson observation of Teacher B

From the two lessons observed, it is evident that Teacher B demonstrated his knowledge of the content of school statistics which may have been developed through formal education and teaching with the recommended textbooks and work schedule. Teacher B used appropriate topic-specific instructional skills and strategies, such as the use of examples drawn from familiar situations and a formal procedural approach in teaching the construction of the bar graph and ogive. In statistical graph construction and interpretation, measures of central tendency, knowledge of graphing involving drawing axes, choosing scale, etc, are regarded as prior knowledge. In order to identify learners' preconceptions in bar graph and ogive construction, he applied diagnostic techniques of pre-activity that focused on the preparation of a frequency table of ungrouped data and oral questioning on different ways of representing data. The learners displayed evidence of possessing previous knowledge of bar graphs and ogive constructions but with no preconception identified, depicting the fact the teacher may not have had sufficient knowledge of the learners' likely preconceptions of bar graphs and ogives.

The learners' misconceptions in drawing a histogram instead a bar graph, and the learning difficulties that emanated from these, were identified through analysis of their classwork while monitoring, checking and marking their responses to the tasks. Further explanations, extra-class activities and post-teaching discussion were provided to correct their misconceptions and learning difficulties. Teacher B's PCK is largely procedural, focusing on rules and algorithms, and is not always responsive to the needs of the learners, especially when these involve clarification of the construction of grouped data (the ogive). The frequent use of procedural knowledge may stem from the nature of the topic, which requires learners to collect, organise, construct, analyse, interpret statistical and probability model to solve related problems (DoBE, 2010) and demonstrate how graphs should be constructed (Leinhardt et al, 1990). This approach did not appear to accommodate the needs of the learners, because most of them still experienced difficulties with labelling the data axes of graphs of grouped data. Teacher B can be said to have displayed insufficient ability to elucidate concepts of ogive construction **(PCK),** focusing on procedural, at the expense of conceptual understanding.

4.5.3 School C: Lesson observation of Teacher C

In this section, the teacher's classroom practice on teaching the construction of ogives and scatter plots is described. The condition of the classroom is described first, followed by his classroom practice in the construction, analysis, and interpretation of ogives and scatter plots.

Table 4.5.3a: Description of lesson observation and classroom conditions in Teacher C's mathematics lesson

DESCRIPTION OF LESSONS	CATEGORISATION/THEMES
<u>**Condition of the classroom**</u> Teacher C's classroom was safe and protected. The teacher had a full view of the entire class during lessons. The classroom walls were decorated with science wall charts; the furniture, windows and door were in good condition, with electrical wiring that permitted the use of appliances such as an overhead projector. The individual learners were resourced with textbooks, calculators, exercise books, and graph sheets for each learner, as well as construction instruments for the teacher (ruler, protractor, and pair of dividers). There were 45 learners, consisting of 26 females and 19 males, seated comfortably in twos in four columns of double chairs and desks.	1) The classroom was conducive for learning, safe and well protected. 2) There were 45 learners in the class, who were seated in double chairs in four columns. .3) The individual learners have all the necessary materials for learning statistical graphs.
CLASSROOM PRACTICE (FIRST LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of ogives. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: A histogram had been taught in the previous lesson, and learners had been given homework.	The ogive was taught (line 1). **(Teacher content knowledge).**
Line 2a: **Teacher C** and the learners marked the homework on the construction, analysis, and interpretation of the histogram. Line 2bi: To determine learners' prior knowledge of ogives, **Teacher C** asked, '*What is the difference between a class interval and a class boundary?'* Line 2bii: One of the **learners** voluntarily answered, '*A class interval and a class boundary are the same thing,*	**Oral probing questioning** to identify learners' conceptions **(preconceptions)** (lines 2bi and 2c) was used to introduce the lesson (**Instructional strategy**). **Analysis of homework** (checking if answers were right or wrong) (line 2a) was used to try to identify learners'

because both of them contain a group of numbers between them.' The question was not answered correctly, but none of the other learners volunteered to answer. Other learners, Teacher C indicated, could not provide the answer. Therefore, the **teacher** explained, using an example, '*0–10, 11–20, 21–30, etc., are class intervals. But 0–10, 10–20, 20–30, etc' are class boundaries of a prepared ogive on a photocopied exercise*.' Line 2c: **Teacher C** requested. '*indicate to me how data can be represented based on your experience'* Line 2d: **Learners** referred to the bar chart, the pie chart, scatter plots, the line graph, ogive, etc. This response indicated that learners held some conceptions about ogives, which included data representation, since they had been taught previously.	conceptions in ogive construction (**instructional strategy).** **Teacher content knowledge** was used to explain the differences between class boundaries and intervals (line 2bii). The learners displayed evidence of having previous knowledge about data representation in statistics (line 2c)
Line 3a: **Teacher C** explained the construction of the ogive procedurally, using a frequency table on a photocopied exercise containing the ages of cars, in years, in a sample of 100 car owners. Learners were also asked to interpret the ogive in terms of the five-number-summary. A five-number-summary consists of the minimum value, $Q_{1,}$ $Q_{2,}$ $Q_{3,}$ and Maximum value of the given data. Line 3b: A cumulative frequency distribution table was individually constructed by the learners, based on the teacher's instruction (ref Table 4.5.3b). For instance, Teacher C explained: '*The frequency of the first row (25) should be written under the column for cumulative frequency. The cumulative frequency of the first row is then added to the frequency of the second row (25 + 32 = 57), to get the cumulative frequency of the second row, etc'*.	**Teacher content knowledge** was used to set the example to demonstrate histogram construction (line 3a). **Learners' content knowledge** was used to complete the cumulative frequency table in a procedural manner (**instructional strategy)** following certain algorithms (lines 3a and 3b). **Teacher content knowledge in statistics (**data collection) was used to prepare a frequency table (lines 3b and 3c).

Table 4.5.3b: Table showing the ages of cars in a sample of 100 cars

Age (years)	Freq. (f)	Mid-values (x)	fx	Cum. freq.
0 < x < 2	25	1	25	25
2 < x < 4	32	3	96	57
4 < x < 6	20	5	100	77
6 < x < 10	12	8	96	89
10 < x <15	7	12.5	87.5	96
15 < x < 20	4	17.5	70	100
	$\sum f = 100$		$\sum fx = 474.$	

Line 3c: '*Continue in the same way to calculate the remaining cumulative frequencies,'* **Teacher C** said. The learners, as shown in table 4.5.3b completed the table.

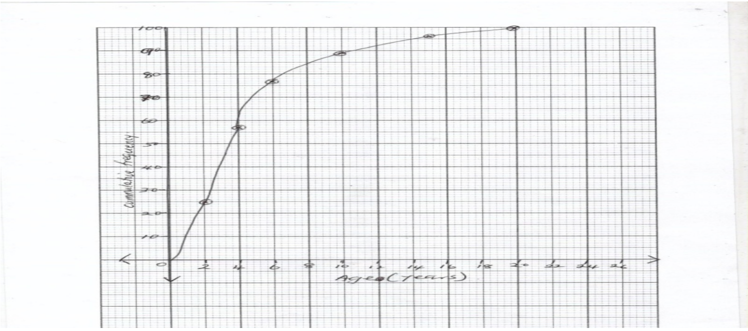
a) Construction of ogive

Line 3d: **Teacher C** used Table 4.5.3b to explain how to construct the ogive, as shown in Figure 4.5.3a. He requested, 'draw the vertical and horizontal axes, choose a scale by considering the highest and lowest value on the cumulative frequency and class boundaries'. The teacher used topic-specific algorithmic knowledge of ogive construction in his demonstration.

Instructional skills such as topic-specific construction skills (drawing of axis, choosing of scale, labelling of axes, plotting the points and joining the line of best fit) (line 3d) were used to construct the ogive.

Topic-specific teacher content knowledge and instructional strategy were used to demonstrate how to construct an ogive (line 3d) using an algorithmic approach. Thus the teacher has content and pedagogical knowledge of histogram construction.

A procedural knowledge approach was used to explain how to construct an ogive (lines 3c and 3d) (**instructional skill and strategy**)

 **Figure 4.5.3a: Ogive of age distribution of sample of 100 cars owners park in a car park**	
Line 4: **Teacher C** explained the ogive construction, using a rule-oriented approach, plotted two points and asked the learners to complete the plotting and join the lines of best fit for the ogive as part of their classwork.	**Teacher C's** use of an algorithmic approach to explain how to construct an ogive (**Teacher content knowledge and instructional strategy**) (line 4)
Line 5: **Learners** completed the ogive by plotting (6; 77), (10; 89), (15; 96) and 20; 100) and joining the line of best fit. (ref Figure 4.5.3a). But some learners were uncertain about the labelling of the data axis.	**Learner content knowledge** was used to complete the ogive (line 5) but some of the learners appeared not to have understood how the ogive was completed especially the labelling of the data axis with data from the frequency table.
Line 6a: **Teacher C** monitored the learners and offered a further explanation of the preparation of the cumulative frequency table to those who were experiencing difficulties, such as being uncertain how to label the data axis with the class boundaries provided on the table of values. He indicated, '*The cumulative frequency was used to label the cumulative frequency axis (vertical axis) and data axes on the horizontal axis*'.	Teacher C **monitored and guided** learners while they were doing their classwork **(instructional skills and strategies)** (line 6a). Insufficient explanation was provided because a procedural approach was used where a conceptual

Line 6b: A **learner** asked, *"Why do we need to add these numbers (frequencies) together?'* Line 6c: **Teacher C** answered, '*Adding the frequencies together to give the next frequency on the cumulative frequency column makes it a cumulative frequency that you are required to calculate for constructing the ogive. Cumulative means adding more numbers each time to get the next number.'* Through non-verbal cues of nodding their heads up and down, learners showed that they had understood the explanation, indicating that the conceptual knowledge approach was sufficient to enable them to comprehend how a cumulative frequency table is prepared. Teacher C demonstrated the required content knowledge of preparing a cumulative frequency table in his explanation regarding the construction of an ogive to the learners.	explanation was more appropriate **(**line 6b). **Teacher content knowledge** was used to explain how the cumulative frequencies were obtained (conceptual knowledge approach) (line 6c) (**instructional strategy**).
Line 7a: **Teacher C** observed a misconception, which resulted in drawing a histogram instead of an ogive with the given data, while he monitored and analysed the learners' responses to classwork. . Line 7b: **Teacher C** told a learner who was experiencing this misconception, '*Look, you were asked to complete the ogive we were plotting on the chalkboard and not to draw something else. Clean it off and continue with the diagram on the chalkboard by plotting the points and joining the line of best fit. For example, when cumulative frequency is 57, age is 4; when cumulative frequency is 77, and age is 6; etc*,' the teacher said.	**Misconception** of drawing a histogram instead of an ogive was identified during monitoring of classwork (line 7a). **Learning difficulties** resulting from this misconception were identified through analysis of learners' responses to classwork (line 7a). Teacher C addressed the misconception through reviewing the learners' work and instructing them to continue with plotting the points and joining the line of best fit (line 7b). (Teacher C displayed **knowledge of the topic content, instructional strategy and learning difficulty.)**

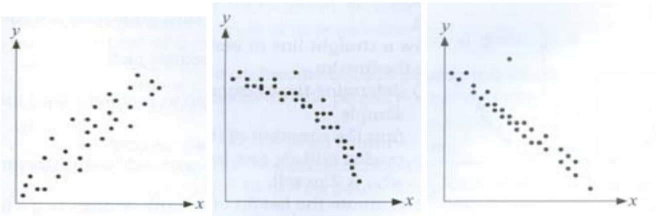
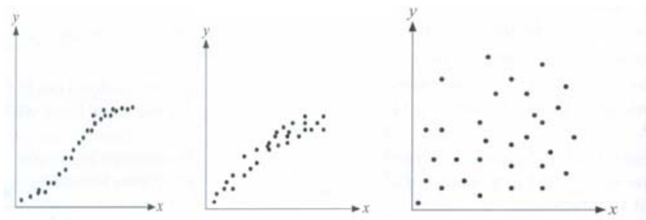
Line 8a: Referring to how the horizontal axis was labelled, a **learner** asked, '*Why do you indicate the numbers that were not on the table?*' The learner displayed a lack of knowledge of selecting a scale of given grouped data, which may not have been addressed through the procedural approach adopted by the teacher.	Teacher C identified lack of knowledge or his insufficient explanation (learning difficulty) of how to choose a scale for constructing an ogive through **oral questioning** from the learners (line 8a).
Line 9a: **Teacher C** re-explained the construction of an ogive by analysing the table of values of the cars and how they were used to construct the ogive, as in line 5. He explained, '*The numbers were not omitted , but grouped together as: 0 < x < 2; 2 < x < 4; 4 < x < 6; etc. And 6 < x < 10 contains 6 < x < 8; 8 < x < 10, In addition, 10 < x < 12, 12 < x < 14, 14 < x < 16, 16 < x < 18, 18 < x < 20 is within 10 < x < 15 and 15 < x < 20, as indicated in the diagram. Indicating those numbers that were not on the table ensured sequential numbering of the data axis that could help in the construction and interpretation of the ogive'*. Line 9b: After plotting the points, **Teacher C** demonstrated how to join the line of best fit, which gave an S shape. He instructed learners to copy the description from the chalkboard.	**Teacher content-specific knowledge of the construction of an ogive** was used to explain how to label the horizontal axis (line 9a). **A conceptual knowledge approach** based on **teacher's content specific knowledge** of how to label graphs of grouped data was used to explain the construction of an ogive (lines 9a and 9b) (**instructional strategy**).
Line 10: **Learners** listened, and copied notes from the board. One asked, '*Does it mean that the graph of the ogive must be in the form of an S?'*	This question showed lack of understanding of the nature of an ogive. It required further clarification from the teacher from his **content knowledge** of ogive construction using a conceptual knowledge approach (line 10).
Line 11: **Teacher C** answered, '*Yes.*' He explained, *'ogive graphs are typically in an S shaped. If the constructed graph does not display this shape, then it is not an ogive or is constructed wrongly'*.	Teacher C answered learners' oral questions and provided greater clarification to reinforce comprehension of the nature of an ogive **(teacher content knowledge)** (line 11).
b) Interpretation of ogive (calculating the quartiles from an ogive)	The teacher asked how the median is calculated from grouped data as a way of determining learners'

Line 12: **Teacher C** posed this question to the learners, '*How would you calculate the median from the ogive, according to the question?'*	conception in ogive interpretation (line 12).
Line 13a: A **learner** (pointed out by Teacher C) answered, **'***you have to arrange the data in ascending order and locate the middle number. But if they are more than one number at the middle, the average of the two middle numbers is considered as the median.'* The learner quoted the wrong formula for finding the median of ungrouped data, instead of quoting the formula for finding the second quartile of a grouped data showing a lack of understanding of how to calculate median of grouped data. Line 13b: Teacher C explained the formula for calculating all the quartiles and focused on the formula for calculating the median position by indicating, '*Median (second quartile)(* $Q_{2)} = (\frac{n+1}{2})th$ *). The position of the median calculated (second quartile) was used to locate the median on the ogive*. '*Median age = 3 years'*, the teacher said. Line 13c: **Teacher C** and the learners calculated the first and third quartile from the ogive using the formulae ($Q_1 = \frac{1}{4}(n+1)th$ and $Q_3 = \frac{3(n+1)th}{4}$ to locate Q_1 and Q_3. The five number summary was i) minimum age = 1year; Q1 = 2 years; Q_2 = 3 years; Q_3 = 8 years and the maximum age = 20 years. These were all calculated and listed. But some learners appeared to be confused because they regarded the quartile position as the quartile itself. For example, the first quartile position was calculated as 25.5th position. Rather than using this position to find the value of first quartile from the data, the learners simply wrote Q1= 25,5th instead of Q1= 2. Some learners displayed a lack of understanding of how to calculate quartiles from the ogive due to the teachers' procedural knowledge description of how to calculate quartiles.	The learners showed lack of comprehension of how to calculate the median from a graph of grouped data (line 13) **An algorithmic approach** was used, in that the quartiles were calculated according to a particular procedure or formula, without explanation of the use of that algorithm **(insufficient knowledge of learners' conceptions and learning difficulties)** in calculating the median of grouped data, and the difference between calculating the medians of grouped and ungrouped data) (line 13a). **Procedural knowledge** was used to explain how to calculate the quartile's position and locate the quartile itself from the ogive **(lines 13b and 13c).** **Learners experience some difficulties** of using the quartile position to represent the quartile itself (line 13c) which may be linked to the procedural knowledge description adopted by Teacher C during the lesson on ogive construction (line 13c).

Line 14: The teacher provided the following detailed explanation of the mathematical connections between the quartile positions and how they were used to calculate the quartiles from the ogive. The teacher first explained, '*the meaning of 'n' is the number of cars in the park. The value of 'n' was obtained from the table by calculating the frequencies, and substituting the value of 'n' into the formula (in line 13b and c), you can determine the first quartile position (*Q_1*)'*. His next step was to show the mathematical connection between the quartiles position and the value of the quartile from the ogive by using the quartile positions to locate the values of the quartiles from the ogive as indicated in line 13c. Following his explanation in which he substituted 'n' into the formulae as indicated in line 13b, the quartile positions were calculated and used to locate the values Q1 = 2 years; Q_2 = 3 years; Q_3 = 8 years, from the ogive. The learners were able to use the same formula and procedure to calculate the quartile positions and the quartiles in their classwork based on the teachers' conceptual explanation.	**Teacher content knowledge** was used to show the mathematical connections between the quartile position, the quartiles and how they are utilised in interpreting the ogive (line 14) employing a **conceptual knowledge approach.** More learners understood the explanation given via a **conceptual knowledge approach** (line 14).
Line 15: Individualised teaching took the form of post-teaching discussion, so that each learner presented the areas in which he or she was still experiencing problems. The difficulties included labelling data axes and determining the median value of an ogive. The teacher provided more activities applicable to familiar situations using their mathematics textbook as a way of reinforcing learners' competency in ogive construction.	The instructional strategy of using more activities applicable to familiar situations from their mathematics textbook was used to address **learners' learning difficulties** in labelling the data axes of grouped data and determining the median of an ogive (line 15) (**knowledge of learners' learning difficulties and instructional strategy).**
Line 16: The mathematics textbook, as well as examination aids and publications of *Study mate* containing past questions in statistics and mathematics, were used by **Teacher C** to prepare and teach the construction and interpretation of the ogive, as well as to assign homework.	**Textbook and other materials** were used as sources of information for teaching ogive construction **(development of teacher's PCK in respect of content knowledge and instructional strategies) (lines** 4 and 16).

CLASSROOM PRACTICE (SECOND LESSON OBSERVATION) **Topic: Construction and interpretation of scatter plots. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: Marking and checking homework on the construction and interpretation of scatter plots was used to start the lesson and to identify learners' knowledge or conceptions about scatter plot construction after Teacher C had greeted the class. After the marking and checking were concluded, Teacher C gave the correct answers, while the learners wrote down the corrections in their notebooks.	Teacher C used the **instructional strategy** of checking learners' homework on scatter plot construction and interpretation to try to identify their knowledge and preconceptions of scatter plot construction (line 1).
Line 2: **Teacher C** wrote the topic, '*Construction and interpretation of scatter plots*' on the chalkboard and presented a photocopied exercise containing different types of scatter diagrams to the learners.	**Teacher content knowledge** of scatter plots was utilised to indicate the topic of the lesson and set activities to ascertain learners' knowledge of scatter plot constructions (line 2)
Line 3a: The learners were asked to work in groups and to determine (by analysis and interpretation of the scatter plots) which of the scatter diagrams had a positive correlation, a negative correlation, or no correlation. They had previously been taught how to construct a scatter plot. Line 3b: Learners worked in groups to analyse the scatter plots, to determine the nature of the points plotted and the lines of best fit.	Learners worked in groups (**instructional strategy**) to analyse and interpret scatter plots as a way of identifying how well they had grasped how to construct a scatter plot from their previous lesson (lines 3a and 3b).
Line 4a: After the analysis, learners (in groups) were asked to interpret the graph by indicating their conclusions: whether the diagrams showed a positive correlation, a negative correlation, or no correlation. Line 4bi: Learners through their spokespersons for each group indicated, '*The first diagram displays a positive linear relationship.'* Another group concluded, *'the second diagram displays a graph of negative relationship, but not linear.*' Some of the groups did not seem to be satisfied with the answers presented for two of the graphs B and C.	Learner activity on data handling and interpretation by responding to class activities was undertaken in groups **(Line 4bi).** **Teacher instructional strategy** of giving and monitoring classwork on scatter graph interpretation was used to identify learner knowledge and conceptions

Line 4bii: Teacher C monitored the way in which learners were analysing and interpreting the scatter plots in groups. '*In terms of analysis, you were expected to know the values of Y and the corresponding value of X as used in constructing the scatter plots,*' he said. He continued, '*Based on the relationship between X and Y values, one can say whether there is positive correlation, negative correlation, or no correlation* (**interpretation)** *as previously explained*.' Recognising that some learners appeared to be experiencing difficulties in interpreting a negatively correlated scatter plot as having no correlation in interpreting the diagrams, which could indicate that they lack an understanding or the teachers' previous lesson explanation on scatter plot construction was not sufficient to enable them to grasp what he had taught them on the topic, Teacher C further handed out another photocopied exercise showing a table of values reflecting the age and mass distribution of players in a rugby game. He asked one of the learners (who appeared to have interpreted the diagram more efficiently), '*Plot the numbers of players against the masses to construct a scatter plot. Can I see you do that on the chalkboard*?' The learners constructed the scatter plot efficiently. But Teacher C decided again to assess learners' conceptions in scatter plot construction (using extra-class activity) which would have aided them in interpreting the scatter plot if they had known how to construct these efficiently. Teacher C used his topic-specific content and pedagogical knowledge to assess the learners' understanding of scatter plots using more activities on their construction in order to improve their grasp of the latter. In this activity, Teacher C plotted some points using the frequency table that he has provided on the activity on the scatter plot and requested learners to complete the remaining points. He said, '*let someone complete the scatter plot?'*	of scatter plots (line 4a). Learners misinterpreted a scatter plot owing to insufficient comprehension of scatter plot construction as a result of inadequate teacher explanations regarding how to determine the relationship between X and Y in a scatter plot (**learning difficulty**) (Line 4bii). A negatively correlated scatter plot was interpreted as having no correlation due to an outlier. **Teacher content knowledge** was used to explain (**instructional strategy**) the construction and interpretation of a scatter plot (lines 4di and 4dii).
Line 4c: More learners volunteered and they were requested individually to plot other points on the graph using the table provided by the teacher on the chalkboard, while the other learners watched.	**A procedural approach** of drawing the axes, choosing scale, labelling axes, plotting the points and drawing the line of best fit was used to describe and complete the scatter plot (line 4di).
Line 4di: Teacher C completed the graph that the learners had been plotting, and explained algorithmically how to construct a scatter plot. He then analysed it by reading the value on the vertical axis and the corresponding value on the horizontal or data axis. '*From this analysis, the meaning of what the graph intended to convey about the*	**Graph construction skills (**drawing axes, choosing scale, labelling axes, plotting the points and joining the

relationship between the number of players and their masses (correlation or no correlation) was determined', the teacher said. Line 4dii: Some of the learners seemed dissatisfied, as they shook their heads. More explanations were offered by **Teacher C,** who utilised **a conceptual approach** to again demonstrate scatter plot construction and interpretation using the classwork. For instance, Teacher C explained; '*The characteristics (nature of points and shape of line of best fit) of a linear positive correlation with its line of best fit moves from right to left through the origin, and related it to diagrams A and E of Figure 4.5.3b. In a linear negative correlation the line of best fit drops down from the vertical axis to the horizontal axis, as in diagrams B and C, Figure 4.5.3b. And a scatter plot with no correlation has all the points spread through the vertical to the horizontal axis as in diagram F, Figure 4.5.3b*'. '*Diagram D shows a positive correlation, but it is not linear because the points spread through the origin from right to left, but not in a straight line,'* the teacher concluded  Diagram A Diagram B Diagram C  Diagram D Diagram E Diagram F **Figure 4.5.3b: Scatter diagrams showing different kinds of correlation between X and Y**	line of best fit) were used to create a scatter plot (line 4di). Teacher C provided further explanation (**using conceptual knowledge**) to address learners' difficulties, showing that he has insight into learners' learning difficulties,; hence the strategy he adopted to provide clarification and reinforce understanding (line 4dii)

Line 5a: A learner asked, '*Do we need to draw the line to show how the two variables X and Y are correlated?*' This question demanded a conceptual explanation which was provided in line 4dii, but the learner may have developed certain misconceptions about drawing the line of best fit in a scatter plot from the earlier procedural explanation which led to a lack of understanding of why such a line has to be drawn based on the nature of the points plotted, to determine the relationship between X and Y'. Another misconception was, '*There were no lines of best fit in Figure 4.5.3b which they had worked on earlier*', the learner indicated. The learner had posed a legitimate question seeking clarification because the teacher simply did not provide a conceptual explanation for the different scatter plots as indicated in the introductory exercise for the lesson and in line 4di. Line 5b: Teacher C answered, '*Yes*' and repeated what he had said in line 4dii by explaining the characteristics of scatter plots, how their correlation can be determine and how they relate to each other as in the diagrams in Figure 4.5.3b.	This learner's question displayed a lack of understanding of how to construct and interpret scatter plots─precisely because of inadequate explanation, using learned rules to explain. The question is how does the teacher makes the leap from the algorithmic to the conceptually meaningful explanation (line 5a).
Line 6: Teacher C observed that in the graphs the learners analysed in groups, they misinterpreted diagram C (Figure 4.5.3b. For example, diagram C was interpreted as a graph with no correlation between X and Y, owing to outliers (the point or points that are farthest from the line of best fit). '*Using one point alone to indicate that diagram C had no correlation may not be adequate as there are other clustered points that would display the correlation between X and Y*,' the teacher explained. This was a misconception of using the nature and shape of a scatter plot with no correlation to interpret a graph of negative linear correlation. In addition, some learners indicated in their exercise book that the line of best fit meant a change in X caused by a change in Y, as in a line graph, which means if Y increases, then X increases by the same percentage. '*Yes, when X increases, Y also increases, which means X and Y are related,*' one of the learners indicated. In a scatter diagram, *'The line of best fit only indicates the association or connection between X and Y, as indicated in diagrams A and B,'* the teacher explained. He continued, '*And depending on how clustered the points are close to the line of best fit, one can say that it is strong, moderate of weak correlation.*' As indicated earlier, *'You were expected to analyse and interpret the scatter plots to determine the relationship between X and Y,'* he emphasised.	**Teacher content knowledge** was used to address learners' misinterpretation of scatter plot (line 6) by explaining why diagram C could not be adjudged to have a negative correlation. A more conceptual explanation was provided of how to describe the relationship between X and Y in a scatter plot and indicate the kind of correlation that the scatter plot is showing (line 6).
Line 7: Teacher C corrected the misconception of using the characteristics of a scatter plot with no correlation to	The **topic-specific content and instructional strategy**

<table>
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<td>
interpret a scatter plot with a negative linear correlation, as well as interpreting a linear scatter plot as if it were a line graph, as in lines 5 and 6, and diagram C of Figure 4.5.3b. He provided more activities on scatter plots and photocopied activities on their construction and interpretation of scatter plots. For example, he said, ‘In this exercise, you were required to construct a scatter plot and indicate the relationship between test 1 and test 2 (see Table 4.5.3c below). The data in the frequency table give the marks (out of 20) that 12 learners attained in the two tests’.

Line 8: Teacher C gave out the classwork as shown below.

Table 4.5.3c: Frequency table showing the distribution of learners’ performance in two tests

<table>
<tr><td>learner</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td><td>L</td></tr>
<tr><td>Test 1</td><td>10</td><td>18</td><td>13</td><td>7</td><td>6</td><td>8</td><td>5</td><td>12</td><td>15</td><td>15</td><td>10</td><td>20</td></tr>
<tr><td>Test 2</td><td>12</td><td>20</td><td>11</td><td>18</td><td>9</td><td>6</td><td>6</td><td>12</td><td>13</td><td>17</td><td>10</td><td>19</td></tr>
</table>
a) Draw a scatter plot and describe by means of two examples whether there is a positive or a negative correlation in the learners’ performance in the tests.
b) How do you account for the outliers, if any?
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<td>
of providing more examples was used to address the learners’ misconceptions concerning outliers and interpreting a linear correlated scatter plot as if it were a line graph (line 7). Topic-specific content and pedagogical knowledge was utilised to address learners’ misconceptions.
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<td>
Line 9: As he monitored the learners’ doing the first classwork, he discovered that some of them did the classwork efficiently. He gave a second classwork activity involving a frequency table of the age distribution of persons infected with HIV/Aids in two towns. They were to work on their own individually to construct a scatter plot showing the relationship between the age distributions of persons infected with HIV/AIDS in the two towns. The objective of using several activities on scatter plots constructions was to identify and correct any difficulties or errors related to the construction and interpretation of scatter plots and reinforce learners’ grasp of scatter plot construction.
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<td>
Instructional strategy of using real-life context based examples to assess learners’ conceptual understanding of the construction and interpretation of scatter plots and address their learning difficulties (line 9). Several class activities were used to reinforce learners’ grasp of how to construct and interpret scatter plot (line 9)
</td>
</tr>
</table>

Line 10: Learners carried out the exercise individually. A few still experienced difficulties in drawing the line of best fit and determining the type of correlation.	An individualised or independent learning strategy/approach was used to evaluate how well learners had learned the construction of a scatter plot (line 10).
Line 11: After the classwork, oral questioning, and homework (as in line 8), were made use of by Teacher C to further assess learning. For instance, he asked a learner, '*What is an outlier?*' '*An outlier is a data value or point that lies apart from the rest of the data'*, the learner replied. Teacher C adjudged the learner to be correct and instructed the learners to answer other questions on the photocopied exercise as homework.	**Oral questioning** and the homework assignment comprised the instructional strategy used to assess how well learners had grasped the concept of constructing scatter plots (line 11).
Line 12: At the end of the lesson, some learners asked more questions about the work that they did, especially the misinterpretation of a negative linear scatter plot and interpreting the line of best fit in scatter plot as if it were a linear algebraic graph. Teacher C held individual discussions with a few learners about diagram C, and asked the others to see him after school the following day.	**Teacher content knowledge and instructional strategy was used to** clarify the misinterpretation of a negative linear scatter plot and interpreting the line of best fit as if it is an algebraic linear graph **in a post-teaching discussion (**responding to learners' oral and written questions after lesson) and various examples (line 12).

Summary of lesson observation of Teacher C

The way in which Teacher C taught his lessons on the ogive and scatter plot showed that he possessed the subject matter content knowledge of school statistics. He utilised recommended statistics and statistics-related textbooks and materials (mathematics study guides) to teach statistical graphs such as the ogive and scatter plot. He demonstrated his subject matter content knowledge by describing how the ogive and scatter plot should be constructed, by adopting an approach that emphasised procedural knowledge and application of formulae, rather than conceptual knowledge. For example, the teacher made greater use of algorithms by slotting values into equations for calculating quartiles without eliciting clear comprehension of the relationships of concepts in the equations. At times he did not provide adequate explanation, and merely repeated the procedures for arriving at an answer when the learner experienced misconceptions and learning difficulties in interpreting an ogive using the calculated quartile positions. Having said that, the teacher used his conceptual knowledge, for instance on how to teach ogive and scatter plot construction, especially when learners encountered misconceptions and learning difficulties such as drawing a histogram instead of an ogive, being unable to label the data axis because of incorrect scaling, and not knowing the distinction between quartile position and quartile value to teach ogive and scatter plots. While the teacher used his procedural knowledge to explain in a step-by-step manner how ogive and scatter plots are constructed, he employed his conceptual knowledge to demonstrate the mathematical connections between quartile positions and to utilise the calculated quartile position to work out the quartile value from the ogive in order to provide the meaning or information that the ogive conveys (interpretation). For example, while the quartile position for Q_1 was calculated to be 25.5^{th}, Q_1 value from ogive was found to be, $Q_1 = 2$.

Concerning the instructional knowledge component of his assumed PCK in data handling, Teacher C used appropriate topic-specific scatter plot construction skills of drawing the axes, choosing the scale, labelling of axes, plotting the points and joining the lines of best fit to make data-handling lessons on ogives and scatter plots accessible to more learners. Post-activity and post-teaching discussions were among the instructional strategies he used to address errors and construction difficulties, etc, in ogives and scatter plots. He applied the required diagnostic techniques of oral probing / questioning, checking and marking of homework at the beginning of the lesson to try to identify learners' prior knowledge about ogive and scatter plot construction. Teacher C identified learners' previous knowledge instead of preconceptions which could indicate that the teacher may not have possessed sufficient knowledge of learners' preconceptions in ogives and scatter plots constructions. The lack of sufficient knowledge of learners' preconception which could have been used to address any anticipated learning difficulties during lesson planning and implementation may have further created room for learners to develop some misconceptions and such learning difficulties as an inability to label data axis, constructing a histogram instead of an ogive and misinterpreting a negative correlated scatter plot as having no correlation. These misconceptions in using content knowledge about algebraic line graph construction to interpret the line of best fit of a scatter plot and learners' inability to label the data axis were identified through analysis of their responses to classwork and homework, and pre- and post-teaching discussions. Teacher C provided additional class activities and individualised teaching, post-teaching discussion on the classwork, and further elucidation on scatter plots immediately after the lesson in order to correct any remaining misconceptions and learning difficulties.

From the analysis of the lesson observations of Teacher C, it appears that his PCK was more frequently a procedural approach to teaching, and less often a conceptual approach. The frequent use of procedural knowledge may be a result of the nature of the topic, which requires learners to be able to collect, organise, construct, analyse, and interpret statistical and probability models to solve related problems (DoBE, 2010) and to demonstrate the construction skills of graphs in statistics (Leinhardt et al, 1990). Following this sequence, the teacher may have decided to use his procedural knowledge to teach the construction and interpretation of ogive and scatter plots. On the other hand, the teacher adapted his conceptual knowledge to explain the construction and interpretation of ogives, especially when learners experienced misconceptions and learning difficulties. For example, when some of them misinterpreted a negative linear scatter plot as having no correlation because of an outlier, the teacher explained the meaning and nature of the scatter plot and its line of best fit, which can be used to determine the extent of the correlation (strong, moderate, weak or no correlation) (ref Second lesson observation, line 6). The mathematical connection between calculating the quartile position and using the calculated position to locate the quartile in an ogive was explained conceptually to the learners when they could not distinguish between them during his lesson on ogive construction that involved a procedural approach (line 14).

While the teacher can be said to comprehend learners' learning difficulties by identifying problem areas through the analysis of learners' classwork, homework and from pre-and post-teaching discussion, as well as addressing the difficulties using familiar context-based examples, his knowledge of learners' conceptions may have been developed through the use of oral questioning, checking and marking of learners' homework to assess learners' conceptions in ogive and scatter plot construction.

4.5.4 School D: Lesson observation of Teacher D Grade 11

This section describes briefly the teacher's classroom practice on the teaching of the construction of bar graph and histogram. The condition of the classroom is described first. It is followed by a description of the teachers' classroom practice in the implementation of the planned lesson on the construction and interpretation of bar graph and histogram.

Topic: Construction, analysis, and interpretation of bar graphs

Table 4.5.4a: Description of lesson observation and classroom conditions in School D

<table>
<tr><th>DESCRIPTION OF LESSONS</th><th>CATEGORISATION/THEMES</th></tr>
<tr><td><u>Condition of the classroom</u>

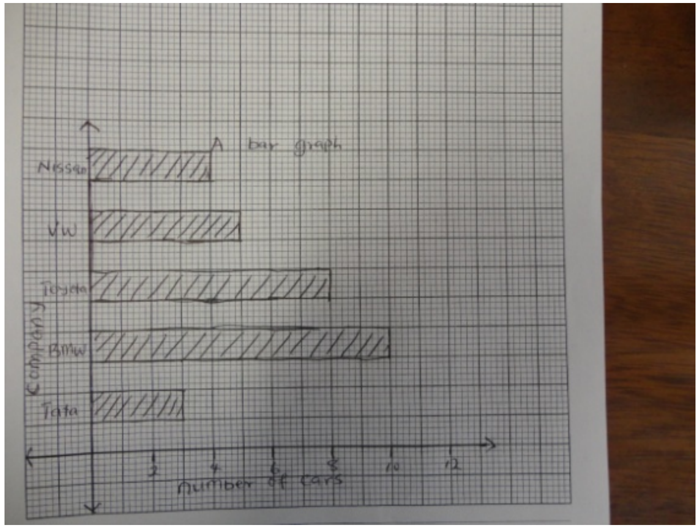
There are 17 male and 23 female learners of mixed ability. Forty learners are seated comfortably in twos in four columns of double chairs and desks. The teacher had a full view of the entire class during lessons. The classroom walls were decorated with science wall charts; the furniture, windows and door were in good condition, with electrical wiring that permitted the use of appliances such as an overhead projector. The individual learners were resourced with textbooks, calculators, exercise books, and graph sheets for each learner, as well as construction instruments for the teacher (ruler, protractor, and pair of dividers). The classroom presented a conducive learning environment, with locks, keys, and burglar bars for supervised entry</td><td>1) The classroom is safe and conducive to teaching and learning.
2) The individual learners were resourced with learning materials.
3) There were forty learners in the class.</td></tr>
<tr><td>CLASSROOM PRACTICE (FIRST LESSON OBSERVATION)

Topic: Construction and interpretation of bar graphs. Class: Grade 11</td><td>CATEGORISATION/THEMES</td></tr>
<tr><td>Line 1: Teacher D introduced the lesson on bar graphs after greeting the class with a pre-activity exercise in which learners were asked to individually prepare a frequency table (shown below) of raw data about the number of cars in a car park manufactured by different companies.

Table 4.5.4bi: Table showing the number of makes of cars in a car park

<table><tr><th>Company</th><th>Nissan</th><th>VW</th><th>Toyota</th><th>BMW</th><th>Tata</th></tr><tr><td>Number of cars</td><td>4</td><td>5</td><td>8</td><td>10</td><td>3</td></tr></table></td><td>Teacher D utilised a learner pre-activity exercise of frequency table preparation, which he regards as important for successful bar graph construction, to try to identify learners’ prior knowledge or conceptions (preconception) about bar graphs(line 1) (teacher content specific knowledge and instructional strategy)</td></tr>
</table>

a) Definition of bar graph Line 2a: The learners prepared a frequency table as displayed in table 4.5.4bi. Line 2b: **Teacher D** defined and described a bar graph orally and wrote it on the chalkboard: '*It is a statistical graph used in representing data in the form of a bar. A bar graph is used for representing discrete data. When a bar graph is used to represent information, you can easily see the information physically and understand how one discrete piece of data is different from another. A bar graph can be represented vertically or horizontally.*' The next step was for the teacher to demonstrate how a bar graph is constructed.. **Construction of bar graph** Line 2c: **Teacher D described this on the chalkboard as follows: '***You draw vertical and horizontal axes and labelled them (the horizontal axis represents the frequencies, and the vertica axis represents the companies). The scale of the horizontal axis were determine by considering the lowest and the highest value of the number of cars and appropriately labelling the horizontal axis with names of the companies*. In learners' mother tongue, he said, ***labella ga ke go bontsha***, meaning '*Watch me as I demonstrate it'*. Teacher D continued**, '** *For the first bar Tata, the frequency is 3; For the second bar, the frequency is 10; for the third bar, the frequency is 8 etc.'*	**Learners** showed evidence of knowing how to prepare a frequency table as they had been taught it previously **(line 2a).** **Teacher content knowledge** was used to define and explain bar graph construction and its uses (line 2b). **Instructional skills** such as **construction skill** involving the drawing of the axes, choosing of scale, labelling of axes, plotting of points, and joining the line of best fit were utilised in constructing a bar graph (line 2c). Teacher D taught a bar graph using a procedural knowledge approach (line 2c) (**content knowledge and instructional strategy).** **Graph construction skills of** drawing the axes, choosing scale, labelling axes, plotting points, and joining the line of best fit were used to construct a bar graph (line 2c). **The learner's mother tongue** was used to direct the learners' attention to the lesson and reinforce their comprehension of the material (line 4b) (**instructional strategy**) (line 2c).

 **Figure 4.5.4a: Bar graph showing the numbers of makes of cars in a car park**	
a) Interpretation of bar graph Line 3: **Teacher D** drew the bar graph, as in Figure 4.5.4a, and interpreted it by indicating that Tata was the least frequent make of car in the car park, while BMW was the most frequent. The second most frequent was Toyota.	**Teacher content knowledge** was made use of to interpret the bar graph (line 3).
Line 4a: **Teacher D** asked, '*Why do you think the most frequent make of car in the car park was BMW?'* Learners answered one by one and gave the following answers: *'BMW produce the most popular cars.' 'BMW produce prestigious cars,' 'BMW produce cars of high quality,' etc.* Line 4b: **Teacher D** further answered the question, *'BMW produced the highest number of cars in the car park.'* In their mother tongue he said, '*ke mang a sahlaloganyeng,* meaning *'who does not understand the explanation?'*	**Oral questioning (instructional strategy)** was used to probe learners' views about the most frequent make of car (line 4a). **Open-ended questions that called for reasoning and analytical skills** (line 4a). Reasoning skills were employed to arouse interest and focus the learners' minds on the construction and interpretation of the bar graph.

<table>
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<td>
b) Classwork
Line 5a: Teacher D set the learners an activity to solve individually. It involved a table of values of the distribution of marks obtained by 50 learners in a class test. Learners were asked to construct the bar graph and calculate the percentage of learners who failed the test if the pass mark was 5 out of 10 or 50%.

Table 4.5.4bii: Frequency table showing the mark distribution of learners in a class test

<table>
<tr><td>Marks</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Frequency</td><td>3</td><td>1</td><td>2</td><td>7</td><td>10</td><td>12</td><td>9</td><td>3</td><td>2</td><td>1</td></tr>
</table>
Line 5b: Learners solved the question individually by constructing the bar graph and determining the percentage of learners that had failed the test.
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<td>
Instructional strategy was to set an activity on bar graph construction which learners had to solve individually (line 5a)

Learners solve activity of bar graph individually as a way of assessing how well they have understood what the teacher taught them (line 5b).
</td>
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<td>
Line 6: While the teacher monitored how the learners were progressing in their classwork, he offered additional explanations for labelling the axes and drawing the bars. For instance, one of the learners asked, ‘Why do you leave equal spaces between the bars when the companies produce a different number of cars in the car park?’
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<td>
Learners asked the teacher to explain why there should be constant spaces between the bars, meaning that they did not understand this from the earlier explanation that the teacher had provided using procedural knowledge (line 6). Learning difficulty of their lack of understanding of the construction of bar graph was discovered by Teacher D.
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<td>
Line 7a: Teacher D explained: ‘All the companies manufacture cars only, but of different makes, hence they have to be separated by equal spacing by choosing appropriate scale, which differentiates one make of car from another. The difference in height of the bars is because of the difference in the number of cars produced. In terms of your classwork, the differences in the height of the bars are as a result of the number of students which correspond to the marks they scored,’ the teacher said. A conceptual knowledge was used to explain the frequencies, the cars manufactured and while there should be constant spacing between the bars.
</td>
<td>
Teacher content knowledge was used to explain why there should be constant spacing between the bars (line 7a).

Teacher used conceptual knowledge requiring the drawing of the bars with constant spacing based on the company and the scale that was chosen for constructing the graph and the differences in height resulting from the varying frequencies of
</td>
</tr>
</table>

Line 7b: Learners showed evidence of a grasp of the lesson as they constructed the bar graph more efficiently, especially after the teacher demonstrated how to construct the bar graph using a conceptual knowledge approach.	the cars manufactured by each company (line 7a). Learners demonstrated evidence of a grasp of the lesson as they constructed the graph more efficiently (line 7b).
Line 8: **Teacher D** identified learners' difficulty in constructing bar graphs during the monitoring of the learners while they are doing classwork, such as unequal spacing between the bars (as most learners think this merely indicates the space and bars without considering the sizes),. For example, while most **learners** used the space between the first bar and the horizontal axis to determine the spaces between the other bars, some did not consider the consistency of the spacing between the bars, irrespective of the size of the space between the first bar and the horizontal axis, as in Figure 4.5.4a. Some **learners** drew the bar graph with different spacing between the bars, and others drew histograms instead of bar graphs.	Teacher D identified **misconceptions,** involving drawing a histogram instead of a bar graph through not considering the spacing between the bars, during the examination of their work on bar graph construction (line 8). Another misconception concerns the inconsistency in spacing and sizes of the bars (line 8).
Line 9: These misconceptions (as stated above) in which learners drew a histogram instead of a bar graph and drew the bars without considering the size of the latter were addressed by **Teacher D** through extra explanations to individual learners-as well as by compulsory additional activities from the textbook which the learners did in class individually.	Extra elucidation on how to construct and interpret a bar graph, especially with respect to the drawing of the histogram instead of a bar graph and inconsistency of spacing between the bars, was offered on a one-on-one basis to correct the misconceptions and learning difficulties (line 9). Extra class activities was given to the learners' to deepen their understanding of bar graph construction (line 9).
Line 10a: During the lesson, **Teacher D** repeated what he said in line 2b and 2c and provided further explanations on the meaning of a bar graph, construction of bar graph with emphasis on the space between the bars drawn according to scale, the size of the bars and the consistency of the space between the bars individually to some learners who were experiencing difficulties. For example, one of the **learners** whose classwork had been marked wrong, because she had constructed a histogram instead of a bar graph, requested clarity as to why her answer was wrong. Line 10b: **Teacher D** stated that the learner had not left spaces between the bars, as explained in the example on the	**Conceptual knowledge** was used to explain the meaning of a bar graph and how it can be constructed by considering the frequency and drawing the bars with appropriate scale. How the scaling affected the consistency of the spaces between the bars and sizes of the bars, and the learners' misconceptions and learning difficulties (inconsistency of spaces between the bars and sizes of the bars) (Line 10b) were addressed. **Teacher used**

chalkboard. The spaces between the bars in a bar graph help to differentiate between categories of data (companies) and must be equal because we are dealing with cars, though of different makes (categories). '*In a bar graph, there should be a constant spacing between the bars and the sizes of the bars must be the same,'* he said. Line 10c: The learner nodded her head in agreement with the teacher's explanation, as explained in line 10b. Teacher D corrected the classwork, and the learners wrote down the corrections in their class workbook.	**content knowledge and instructional strategy** to explain conceptually the construction and interpretation of bar graph (line 10b) to the learners. **Teacher content knowledge** was used to address learners' misconceptions and learning difficulties using teacher's conceptual knowledge (line 10b)
Line 11: At the end of the lesson, the **learners** were given homework from the school supplementary textbook,	**A supplementary mathematics textbook** was used as a source of information for teaching bar graph and assigning homework (line 11).
CLASSROOM PRACTICE (SECOND LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of histograms. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: After greeting the class, **Teacher D** began the lesson on histogram construction by checking and marking homework on the construction and interpretation of stem-and-leaf diagrams. The teacher and learners provided corrections to the homework so that learners who experienced difficulties could correct their mistakes. While providing the corrections, **Teacher D** explained once more how a stem-and-leaf diagram is constructed by arranging the leaves in the right-hand column and the stem in the left-hand column. '*Just as the stem-and-leaf diagram is used to represent group data, the histogram we are about to study now is also used to represent grouped data,'* he added.	Knowledge of stem and leaf diagrams is regarded by the teacher as an important part of learner's prior knowledge before the histogram can be successfully taught to learners, Checking learners' homework on the construction and interpretation of stem-and-leaf diagrams was used as an **instructional strategy** to introduce the lesson and to determine **learners' background knowledge or conceptions** in histogram construction (line 1) (**teacher's PCK**).

Line 2: **Learners** did corrections, which were written on one side of the chalkboard by Teacher D, while he wrote the new topic on the other side of the chalkboard.	Teacher C wrote the new topic while learners corrected their mistakes in their homework (line 2).
Line 3: **Teacher D** presented a photocopy of an activity on the construction of a histogram representing the mass of each player in a 2003 South African rugby squad. The masses of the 30 players were: 115, 122, 110, 110, 105, 112, 80, 98, 90, 93, 85, 87, 99, 84, 112, 76, 96, 128, 110, 108, 118, 105, 108, 118, 90, 89, 90, 88, 103, and 85 kg. The activity requests the learners to: a) prepare a frequency table of the data presented with a class of 10; b) use the frequency table to construct a histogram; and c) determine from the histogram (i) the mean; (ii) interval that has the highest frequency; (iii) percentage of players whose weight fell between 110 and 120 kg and (iv) the mode.	**Instructional strategy** of using photocopied material to provide a source of information for lesson activity was used to set exemplar questions to demonstrate the construction and interpretation of histogram (line 3).
a) Preparation of frequency table Line 4: **Learners** were instructed to prepare a frequency table by calculating the frequencies of each interval. The class boundaries, mid-values, and fx were later calculated to help in answering question (b) and (c), as normally done if the need arises, or based on the questions in the learners' activities (see table 4.5.4c), and to calculate the measures of central tendency (the mean, and the mode) that best describe the masses of the players. The instruction presupposed that learners knew how to prepare a frequency table; hence class boundaries, mid-value and fx were not explained.	**Teacher D** instructed learners to prepare a frequency table from the raw data presented (line 4) (**Instructional strategy**).
Line 5a: The frequency table was constructed by the teacher and the learners. While Teacher D wrote down the frequencies, learners counted the masses within each interval. The mid-values were calculated by finding the average of the upper and lower class of each class interval while fx was calculated by finding the product of mid-value (x) and frequencies (f) of the individual classes, row by row.	**Teacher content knowledge** on the preparation of frequency tables was used to create a frequency table, and to explain how to prepare the frequency table of grouped data by grouping the data according to class; also to determine the frequency as well the class boundaries, mid-values and fx and calculating measures of central tendency, as indicated in questions (b) and (c) (line 5a).

Table 4.5.4c: Frequency table showing the masses of players in the 2003 South African rugby squad

Class intervals	Class boundaries	Freq. (f)	Mid-values (x)	Fx
70-79	70-80	1	75	75
80-89	80-90	6	85	510
90-99	90-100	7	95	665
100-109	100-110	5	105	525
110-119	110-120	9	115	1035
120-129	120-130	2	125	250
		$\sum f = 30$		$\sum fx = 3060$

Line 5b: **Teacher D** defined and described a histogram orally and wrote it down on the chalkboard as indicated in the textbook: '*A histogram is a graphical representation, showing a visual impression of the distribution of grouped data. It consists of tabular* <u>*frequencies*</u> *shown as adjacent rectangular bars, erected over discrete intervals, with an area equal to the frequency of the observations in the interval. Unlike the bar graph, a histogram is used to represent a large set of data (e.g. a population census) visually, but with no spaces between the bars,*' the teacher said. After the explanation, he referred to the frequency table and indicated the usefulness of the table in the construction of the histogram beginning with the class boundaries, followed by the frequencies. He thereafter began to demonstrate how to construct the histogram.

Teacher procedural knowledge was used in preparing the frequency table with learners (line 5a).

Procedural knowledge was utilised to describe how a histogram should be constructed, an approach that the teacher felt would make the histogram more accessible to the learners (teacher topic-specific **content knowledge and instructional strategy)** (line 5c).

Teacher content knowledge was used to explain the usefulness of the frequency table in constructing a histogram, beginning with the class boundaries, and followed by the frequencies (line 5b).

Topic-specific **construction skills** of drawing the axes, choosing scale, labelling axes, plotting the points and joining the line of best fit were used to construct a histogram (**instructional skill)** (line 5c).

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a) Construction of a histogram

Line 5c: **Teacher D** illustrated the histogram construction visually using procedural knowledge by drawing the vertical and horizontal axes and labelling them using a scale chosen by the teacher by considering the lowest and highest values of the frequencies, with the vertical axis representing the frequencies, and the horizontal axis representing the masses on the chalkboard. He drew two bars of the histogram and instructed learners to complete it according to the class boundaries and frequencies.

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Line 6a: **Learners** completed the histogram individually in their workbooks while Teacher D monitored and examined their responses. Most of the learners who had correctly completed the table drew a histogram, as shown in Figure 4.5.4b. Other learners who had not drawn their histogram correctly because of incorrect scaling and labelling of the horizontal axis, among other errors, and also because of lack of comprehension, corrected their mistakes by copying the correct diagram presented on the chalkboard. Some learners drew bar graphs instead of histograms by leaving spaces between the bars. The difficulties experienced in scaling could have arisen because at the beginning of the activity the teacher did not describe and explain how to choose a scale for constructing a graph of grouped data.

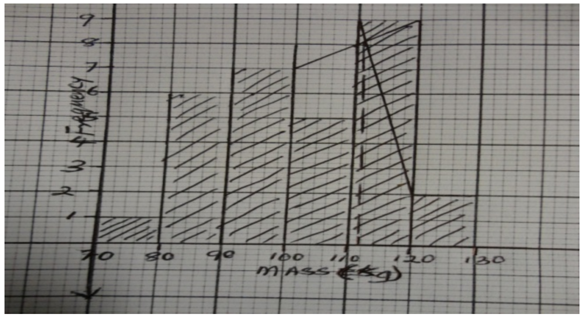

Figure 4.5.4b: Histogram showing the distribution of the masses of players in a 2003 South African rugby squad

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Learning difficulties of drawing a bar graph instead of a histogram were identified through analysis of learners' responses to classwork (line 6a).

Insufficient teacher explanation (pedagogical knowledge) of choosing the scale for constructing a histogram with a procedural approach led to learners constructing a bar graph instead of histogram (line 6a).

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Line 6b: After completing the histogram, **Teacher D** answered follow-up questions (see line 3) such as question (c) which requested the learners to determine (i) the mean; (ii) the interval that had the highest frequency; (iii) the percentage of players whose weight fell between 110 kg and 120 kg and (iv) the mode from the histogram. **c) (i) Calculating the mean** The mean was calculated: '*Mean* = $\frac{\sum fx}{\sum f} = \frac{3060}{30}$ = *102 kg*'; the teacher said	Learners drew a bar graph instead of a histogram (line 6a) **(misconception).** **Teacher's procedural knowledge was** used in calculating the mean (line 6b). This was done by substituting the values in the equation: $\frac{\sum fx}{\sum f} = \frac{3060}{30}$ **= 102kg**
ii) **Identifying the interval with highest frequency** Line 7a: **Teacher D** analysed the histogram (in which learners determined which interval (110–119) kg had the highest frequency, and which intervals had the next highest frequencies (90–100) kg). He then wrote the answer, 'The class with the highest frequency is 110–119 kg'. '*Any question about how we determine the class that has the highest frequency*?' he asked. As there was no question from the learners, he answered the next question about the percentage of learners that fail the test. iii) The % of players that fall within (110–120) kg = $\frac{9}{30}$ x $\frac{100}{1}$ = 30% From Figure 4.5.4b, it was determined that the individual mass of most of the players (9 out of 30) in the squad fell between 110 kg and 120 kg, which formed 30% of the players in the squad.	**Teacher content knowledge** was used to analyse and interpret the histogram (line 7a), demonstrating the application of **analytical and interpretational skills** by calculating the class with the highest frequency. **Procedural knowledge** was made use of to demonstrate how to determine mode from a histogram (line7b) (**instructional strategy**).

iv) Calculating mode from a histogram Line 7b**: Teacher D** then determined the mode using procedural knowledge by drawing a diagonal line from the top-right corner of the highest bar to the top-right corner of the bar next to it on the left-hand side, and drawing a diagonal line from the top-left corner of the highest bar to the top-left corner of the bar next to it on the right-hand side (as in case A). A line was drawn from the meeting point of the two diagonals down to the horizontal axis to locate the mode. '*After the identification of the interval where the mode will be located, the diagonal lines help to locate the mode within the class interval*,' the teacher said. '*By drawing a line from the point of intersection of the diagonals, the mode was located as 113kg (see Figure 4.5.4b),*' he added.	**Teacher content knowledge and instructional knowledge** were employed to demonstrate how to determine the mode from the histogram by drawing intersecting diagonals and using the point of intersection to locate the mode (line 7b).
Line 8: After rule-oriented procedural knowledge was used to demonstrate how to calculate the mode from the histogram, learners were given time to write the explanation of how the mode was calculated from the histogram that Teacher D had written on the chalkboard into their workbooks. "*Now you can write down the explanation I have given on the chalkboard into your workbooks*," the teacher said.	**Learners** wrote down in their notebooks what the teacher had explained as he instructed them. Teacher's **instructional knowledge** was used to provide time for the learners to write down the explanation given by him on how to calculate the mode.
Line 9: Classwork based on construction and interpretation of bar graphwas then given to the learners to solve individually from their supplementary textbook. Learners had to complete their classwork in their workbooks at home, as they were not able to complete it by the end of the lesson period.	**A supplementary recommended mathematics textbook** was employed as a source of information for teaching histograms (line 9). Using a classwork (line 9) assignment for feedback was part of the teacher's instructional **strategy** during the lesson.
Line 10: When the lesson was about to end and learners were still busy doing the classwork; a **learner** enquired (referring to Figure 4.5.4b), *'why it was necessary to label the horizontal axis from 70, and not from 0, as was done on the vertical axis?'* This question demanded a conceptual knowledge approach, which was provided in line 11.	A **misconception** was identified through oral questioning from the learners on the labelling of the data axis (line 10).
Line 11: **Teacher D** replied that, '*One labels the horizontal axis from 70, because 70 is the lowest value on the table. In addition, a scale of 1cm = 10 units was used to label the data axis. Therefore, if you begin from 0, all the values as indicated on the table of values will not be accommodated on the graph paper provided*,' he added. Alternatively, '*One*	**Teacher's conceptual knowledge** was used to clarify the reason that it was necessary to start labelling the horizontal axes

can label from 0 and make a continuation line between the 0 and 70. The continuation line indicates that the intervals below 70 have been omitted so that the graph can be contained on the graph paper,' the teacher said. A related example was drawn from the same supplementary mathematics textbook.	from 70 (line 11). **Teacher content knowledge and instructional strategy** were applied to explain conceptually why it was not necessary to start labelling from zero as a result of the scale of 1cm = 10 units, which was chosen because of the dimensions of the graph paper (line 11).
Line 12: More **learners** seemed to be satisfied with the teacher's explanation by using a conceptual knowledge approach as explained in line 11. They nodded their heads, while a few others were still experiencing difficulties and shook their heads which may be as a result of lack of understanding due to inadequate explanation regarding why the labelling of the data axis has to start with 70 and not 0 .	While some learners indicated that they were satisfied with the teachers' explanation, others felt that the teacher had not cleared up the difficulty (line 12). Insufficient **teacher content knowledge** was made use of to address learners' difficulties in labelling the data axis correctly (line 12).
Line 13: **Teacher D** gave them homework and promised to organise extra tutoring after normal school hours, where he would try to explain once more how to construct, analyse, and interpret a histogram using activities related to everyday life.	**The instructional strategy** of employing homework (line 13) to assess how well learners understood the lesson was adopted during the lesson. **Extra tutoring** was also proposed for helping learners with difficulties.

Summary of lesson observation of Teacher D

Teacher D demonstrated aspects of procedural knowledge of the topics of bar graph and histogram construction. He combined appropriate pedagogical knowledge of teaching bar graphs and histograms with a rule-oriented procedural and conceptual knowledge approach. The content knowledge of bar graph and histogram construction used for teaching the observed lessons was both procedural and conceptual, but mostly procedural. For example, Teacher D demonstrated procedurally how bar graphs and histograms are constructed using the construction skills of drawing the axes, and choosing a scale by considering the lowest and highest values of the data and frequencies as well as the dimension of the graph paper provided. The next step was to plot the points and draw the line of best fit (ref Section 5.5.4, first lesson observation and line 2c; second lesson observation, and line 5c). In terms of his conceptual knowledge, he explained how histograms should be constructed with a scale, even when data values do not start from zero, so that the values can be accommodated on the graph paper provided (refSection 4.5.4, second lesson observation, and line 11) when he discovered that the learners were experiencing some difficulties.

At the beginning of the lesson, Teacher D used his pedagogical knowledge of instructional skills and strategies to try to identify learners' preconceptions by giving them a pre-activity on the preparation of a frequency table, and by checking and marking their homework on stem-and-leaf diagrams. Through the pre-activity, learners demonstrated that they had mastered the concept of preparing a frequency table of ungrouped data and of constructing bar graphs because they had been taught these in the past (ref Section 4.5.4, first lesson observation line 1). But checking and marking learners' homework on stem-and-leaf diagrams revealed that some learners had experienced difficulties that could have been the results of inadequate explanation or of lack of comprehension by the learners (ref Section 4.5.4, second lesson observation, and line 1). These difficulties were corrected before the new lesson began. In the lesson observed, Teacher D knows that stem-and-leaf diagrams are necessary for histogram construction. There is no evidence in his lessons that he knows of the misconceptions his students are likely to have of bar graph and histogram construction. Hence, he can be said to have provided poor and inadequate explanations that resulted in certain learning difficulties. This is possibly understandable because the topic of data handling is a new one. Learners' misconceptions and learning difficulties were identified through marking and analysing the learners' classwork, as well as through oral questioning, where learners could request clarification of what they did not understand about determining the mode from a histogram. These misconceptions and learning difficulties were not adequately addressed through individual problem-solving class activities and further explanations on the construction and interpretation of bar graphs and histograms, because some learners continued to experience difficulties. For example, when the lesson was about to end and learners were doing the classwork, a learner enquired (referring to Figure 4.5.4b), *'Why is it necessary to label the horizontal axis from 70, and not from 0, as was done on the vertical axis?'* (ref Section 4.5.4, second lesson observation, and line 10). **Teacher D** replied that, '*One labels the horizontal axis from 70, because 70 is the lowest value on the table. In addition, a scale of 1cm = 10 units was used to label the data axis. Therefore, if you begin from 0, all the values as indicated on the table of values will not be accommodated on the graph paper provided*,' he added. Alternatively, '*One can label from 0 and make a continuation line between the 0 and 70. The continuation line indicates that the intervals below 70 have been omitted so that the graph can be contained on the graph paper*,' the teacher said. A few learners shook their heads to indicate that they had not understood the explanation. Teacher D probably does not command sufficient content and pedagogical knowledge to address learners' misconceptions and learning difficulties effectively in this respect.

4.6 Video recordings of lesson observation of the four teachers

The video recordings of the four participating teachers confirmed the teaching of the construction and interpretation of bar graphs, histograms, ogives, box-and-whisker plots, and scatter plots during lesson observations (see Section 4.5.1–4.5.4). The video recordings were also used to triangulate the written notes taken during classroom observations.

4.7 Teacher development of PCK

4.7.1 Teacher development of subject matter content knowledge

In the interviews, the teachers claimed that they had studied mathematics and general method courses at university, which helped them to adapt the way they taught school statistics (ref Appendix XVII, items 1, 2 and 3) by employing appropriate instructional skills and strategies to teach statistical graphs. For instance, when they were asked, "If one of the courses you studied at university is mathematics methodology, how did it help you to prepare for your lessons for teaching?" Teacher A indicated that the method course he had studied helped him to vary his instructional strategies (ref Appendix XVII, item 5a). Teacher B asserted, "The mathematics method courses help me to vary formulae and strategies for teaching statistics." Teacher C averred that the courses had helped him to prepare his lessons in line with the objectives of the lessons. And Teacher D said the courses helped him to plan his lessons in line with the work schedules, assessment and evaluation of his lessons.

The participating teachers were further requested to indicate how they knew that their teaching in statistics was effective, as a way of establishing whether the contents of statistics lessons are adequately delivered by teachers with content knowledge of statistics. Teacher A claimed that through analysis of the learners' responses to classwork, homework, and assignments, he knew that his lessons were effective (ref Appendix XVII, item 8). Teachers B, C and D said virtually the same thing, which means that the teachers may have demonstrated the content knowledge of school statistics which they possess during their lessons.

To further ascertain how the participating teachers gained their content knowledge for teaching, they were asked, "Have you attended a mathematics workshop or teacher development programme?" and also, "as a mathematics teacher, did you benefit from the workshop?" Teachers A, B and C responded that they had attended workshops on data handling (the new topic in the curriculum) and learnt how to teach challenging topics in this respect. Teacher D responded: "Yes, I attended many workshops on teacher development in content knowledge especially in data handling. I did not benefit much because I was taught what I already know in mathematics", which could mean that Teacher D became more aware that he already possessed the required content knowledge for the subject he was teaching.

From the above analysis, the teachers can be said to have developed their content knowledge in statistics teaching through formal education, which gave them the opportunity to study mathematics and the methodology of teaching and enabled them to design instructional strategies for carrying out effective teaching. Through classroom practice, lesson planning and preparation, and content knowledge workshops, they gained further content knowledge.
The teacher portfolios and concept mapping exercise confirmed that the teachers possess the content knowledge of school statistics as they listed the subject matter content of school

statistics to be taught in a sequential and logical manner (ref Appendix XXI, teachers' portfolios; Section 4.4).

In addition to listing the content of school statistics, the participating teachers taught statistical graphs using both procedural and conceptual knowledge approaches following the learning outcome of data handling as stipulated in curriculum (DoBE, 2012) and how graphing concepts should be taught (Flockton et al, 2004; Leinhardt et al, 1990) in their lessons on statistical graphs. Using topic-specific content knowledge and instructional skill (construction skill) of drawing the axes, choosing of scale, labelling of the axes, plotting the points and joining of the line of best fit, Teacher A for instance, demonstrated procedurally how to construct a histogram (ref Section 4.5.1, first lesson observation, and line 9). While some learners displayed evidence of grasp of their lesson, a few experienced some learning difficulties (ref Section 4.5.1, first lesson observation, and line 11) which resulted in the teacher adopting a conceptual knowledge approach to assist learners who are experiencing some difficulties (ref Section 4.5.1, first lesson observation, line 16). Thus, the participating teachers can be said to have mastered the content of school statistics which they developed through formal education and classroom practice, and demonstrated it by teaching with procedural and conceptual knowledge approaches, using recommended textbooks, a work schedule and by attending content-driven knowledge workshops.

4.7.2 Teacher development of pedagogical knowledge (instructional skills and strategies)

The focus of this section was to determine the instructional skills and strategies that the participating teachers utilised in teaching school statistics. The teacher questionnaire, lesson observation, written reports and documents analysis were used to collect data to ascertain the teachers' pedagogical knowledge in statistics teaching. The purpose of the questionnaire was to establish what the teachers actually did while teaching assigned topics in school statistics and to determine the pedagogical knowledge (instructional skills and strategies) they possess and use in teaching school statistics.

In their responses to the questionnaire (ref Appendix XVIII), the teachers claimed they had achieved the objectives of their lessons, in which learners are expected to construct, analyse and interpret statistical graphs, and apply the knowledge to everyday real life situations according to the learning outcomes of data handling (DoBE, 2010). This means that the teachers applied content and pedagogical knowledge that was adequate to elicit understanding of school statistics. For example, they were asked, "Do you think that the learners achieved the objective of the lesson and if not, what do you do to improve their understanding?" to establish what strategies they adopted and how good these strategies were (ref Appendix XVIII, item 7). All four teachers claimed they knew that the objectives of their lessons had been achieved through active participation of learners in their lessons, and responses to classwork, homework, assignments, tests, and examinations in statistics (ref Appendix XVIII, item 7). Teacher A tried to engage the learners in extensive class discussions to improve their understanding of statistical graphs, while Teacher B used teaching aids such as statistical charts and an overhead projector to display statistical diagrams. Teacher C indicated that he made use of extra class activities related to real life to improve learners' understanding of the lessons, whereas Teacher D claimed that he used additional examples and past questions in tests to improve learners' understanding of statistical graphs.

From the responses of the four teachers to the questionnaire, it can be understood that they gained their pedagogical knowledge through classroom practice, which involved planning and presentation of lessons, as well as using classwork, homework, exams and assignments, to assess how well learners understood the lessons on statistical graphs. The participating teachers taught statistical graphs with instructional strategies which they felt could help learners to understand the topics and learners responded positively to classwork, homework and assignments. They also claimed to have used class activities related to familiar real life and problem solving on past test questions in statistics to help learners improve their understanding of statistical graphs. The lesson observation, teacher written reports, and document analysis confirmed that the teachers used class activities related to familiar real life situations, problem solving in the form of drill and practice, as well as employing classwork, homework and assignments to assess how well learners had understood the lessons on statistical graphs. For example, during the lesson observation on scatter plot construction, Teacher C made use of the age distribution of persons infected with HIV/AIDS in two towns (familiar real life situation) as classwork to assess how well the learners understood his lesson on the construction and interpretation of scatter plot (ref Section 4.5.3, second lesson observations, and line 9). The teachers also utilised both procedural and conceptual knowledge approaches in teaching statistical graphs (ref Section 4.5.4, first lesson observation, line 2c and 7a). In the teacher's written report, Teacher D indicated that he tackled learners' learning difficulties by adopting different teaching approaches and providing additional class activities related to real life (ref Appendix XX, item 6).

In the learners' notebooks (ref Appendix XXI, learner workbooks) there are examples of statistical graphs, calculations and exercises related to the concepts they were taught according to the procedures for constructing statistical graphs, indicating also that the teachers may have used a procedural knowledge approach. For example, the workbooks of learners in Teacher A's class displayed diagrams of histograms constructed as examples by the teacher and others done as classwork by drawing the axes, labelling the axes based on a given scale, plotting points, and drawing lines of best fit (ref Appendix XXI, learner workbooks). Teachers B, C and D's learner workbooks (ref Appendix XXI, learner workbooks) contained similar records of examples in which a procedural knowledge approach may have been used for teaching statistical graphs. The conceptual knowledge was used less frequently to assist learners that were experiencing some learning difficulties (ref Section 4.5.3, second lesson observation, and line 4dii). All four teachers made use of classwork, homework and assignments as well as the SBA to assess how well learners understood the lessons on statistical graphs. The assessment tasks appeared to be similar because the four participating teachers used the same assessment guidelines, work schedules and textbooks as recommended by the Department of Basic Education (ref Appendix XXI, teacher and learners' portfolios) for teaching Grade 11 mathematics. Learners' recorded examples from extra lessons (ref Appendix XXI, learner workbooks) indicating that the teachers must have individually conducted extra tutoring to help learners who experience learning difficulties (inability to choose scale of grouped data) in order to deepen their understanding of data handling.

From the above discussion, it is evident that the participating teachers used predominantly a procedural knowledge approach and to some extent a conceptual knowledge approach, construction skills, extra tutoring, examples drawn from familiar real life situation, additional class exercises in the form of drill and practice in the teaching of statistical graphs. By doing so, the teachers may have developed more knowledge of the instructional skills and strategies for teaching school statistics.

4.7.3 Teacher development of knowledge of learners' preconception and learning difficulties

A teacher questionnaire, lesson observation, written reports and documents analysis were used to investigate whether the teachers had knowledge of learners' preconceptions and misconceptions, if any, as well as of learning difficulties about statistical graphs such as bar graphs, histograms, ogives, and scatter plots. The investigation revealed that despite many years of teaching experience held by the participating teachers, they possessed no knowledge of learners' preconceptions in statistical graphs. For instance, in the questionnaire, they were asked, "What prior knowledge does your lesson require?" Teachers A and D claimed that learners need measures of central tendency as prior knowledge for bar graphs, histograms and ogives construction (ref Appendix XIX, item 4). Teacher B said that learners need simple addition and subtraction skills, as well as measures of central tendency as prior knowledge for bar graph and ogive construction. Teacher C asserted that learners need to understand measures of central tendency and know how to interpret information from straight-line graphs as prior knowledge for scatter plot and ogive construction. All their responses indicated that they had acquired previous knowledge about the topics they were teaching. But what was needed was the knowledge the learners had before they were taught the concept of statistical graph (preconception). It means that the instructional strategies adopted by the teachers could not elicit learners' preconceptions of the various topics they taught depicting the fact that the teachers have no knowledge of learners' preconceptions in statistics teaching.

The teachers were also asked, "How did you identify the prior knowledge (preconceptions) about statistical graphs with which the learners came to the class?" Teachers A and C claimed that they used probing questioning to establish if learners had gained prior knowledge of measures of central tendency linked to histograms, ogives and scatter plot construction (ref Appendix XIX, item 4–6). This was confirmed in the lesson observation of Teacher A (ref Section 4.5.1, of the first lesson observation, and line 1) in which learners mentioned mode, median and mean when the teacher attempted to probe their preconceptions of histogram construction. Teacher B claimed that he determined their prior knowledge in statistical graphs constructions while correcting their responses to homework and using pre-activities related to the topic he was going to teach (ref Appendix XIX, item 6). This was confirmed in the observation of a bar graph construction lesson given by Teacher B (ref Section 4.5.2), of the first lesson observation, and line 1) in which learners used knowledge of simple addition to prepare a frequency table in a pre-activity in ungrouped data. Learners also mentioned different ways of representing data as prior knowledge for ogive construction. Teacher D claimed that he made use of pre-activities and probing questions to determine prior knowledge in statistical graph constructions such as bar graphs and histograms (ref Appendix XIX, item 6).

This employment of pre-activities and oral probing questions was confirmed in the lesson observation of Teacher D (ref Section 4.5.4, of the first lesson observation, and line 1) who used pre-activities, and checking and marking learners' homework, to attempt to identify learners' preconceptions of bar graphs and histogram construction.

From the responses of the participating teachers to the questionnaire, it appears that they have used topic-specific instructional strategies such as asking oral probing questions, checking and marking learners' homework, and utilising pre-activities at the beginning of the lessons to try to identify learners' prior knowledge in the topics taught in statistical graphs. By employing these strategies, all four teachers could have been adjudged to have demonstrated that they knew about the learners' possible preconceptions and were therefore able to decide which instructional strategy was best to elicit the prior knowledge that was essential for the learning of the new concepts. But the strategies only elicited learners' previous knowledge and not the preconceptions, which means the teachers possess no knowledge of the learners' preconceptions. The teachers' written reports and documents analysis confirmed that the participating teachers tried to identify learners' prior knowledge in statistical graphs using diagnostics techniques such as oral probing questioning, pre-activities as well as checking and marking of learners' homework (ref Appendix XIX, items 8 and 9; Appendix XXI, teacher portfolios).

Regarding the learners' misconceptions and learning difficulties, all the participating teachers adopted monitoring and analysis of learners' responses to classwork to identify any misconception and learning difficulty that the latter may experience during their lessons on statistical graphs. As noted in their responses to the interview (ref Appendix XX, item 14), the learners' learning difficulties range from basic computations of mode, median and mean of grouped data (as in the case of teacher A), to choosing of the scale for constructing graphs of grouped data (for Teachers B and C), and determining the mid-points of graphs of grouped data. From the teachers' responses to the questionnaire, while Teachers A and C addressed the difficulties by giving learners additional exercises in graphs of grouped data, Teacher B did so by specifically teaching the learners how to choose different scales for different data for the sake of uniformity in graph construction. Teacher B tackled the learners' difficulties in graphs of grouped data by giving them additional examples and possibly repeating the lesson in order to reinforce learners' understanding of statistical graphs. The teachers were further asked, "What is it about statistics that makes it easy or difficult?" Teachers A and B said that measures of central tendency represent an easy concept to learn. Teacher C commented that relating statistics to real life makes it lively, interesting, and easy to learn. Teacher D said that statistics is easy to learn if someone who is knowledgeable presents the topic. Therefore, teacher content knowledge of a topic should be adequate in order to make the teaching of statistics comprehensible and accessible to the learners.

In the document analysis, misconceptions such as drawing a histogram instead of a bar graph, as in the case of Teacher B, and drawing a bar graph instead of histogram, as in the cases of Teacher A, C and D, were addressed individually through extra tutoring, extra class activities and post-teaching discussions in statistical graphs (ref Appendix XX1, teacher portfolios) during and after school hours.

The lesson observations and the teacher written reports confirmed that the teachers identified learners' misconceptions and learning difficulties by monitoring and analysis of learners' responses to classwork, homework and assignments in statistical graphs and addressing the misconceptions and learning difficulties by extra tutoring, teaching learners how to choose scale, re-demonstrating or repeating the lessons, extra class activities and post-teaching discussions in statistical graphs. For example, the learners' misconception of drawing a histogram instead of an ogive (ref Section 4.5.2, second lesson observation, and line 7a) and the learning difficulty emanating from the misconceptions of interpreting a negatively correlated scatter plot as having no correlation due to an outlier (ref Section 4.5.3, second lesson observation, and line 4bii) were identified during the monitoring and analysis of learners' responses to classwork by Teachers B and C on ogive and scatter plots respectively (ref Appendix XX, items 1 and 2). The misconceptions and learning difficulties were addressed by post-teaching discussion (ref Section 4.5.3, second lesson observation, and line 12) and extra class activities in the form of drill and practice (ref 4.5.2, first lesson observation, and line 15).

From the above analysis, it can be concluded that the individual participating teachers developed their knowledge of learning difficulties through analysing and monitoring learners' responses to classwork, homework and assignments to identify learners' learning difficulties in statistical graphs. The teachers also extended their knowledge of these difficulties by addressing the difficulties using additional tutoring, extra class activities, post-teaching discussions, re-teaching, and further explanation of the lessons they taught, individually to learners during and after the lessons.

4.7.4 Teacher development of PCK in statistics teaching

By summing the ways through which the participating teachers developed the subject matter content knowledge, pedagogical knowledge and knowledge of learners' preconceptions and learning difficulties, one would be able to determine how the participating teachers developed their PCK in statistics teaching. In section 4.7.1, it was deduced that the participating teachers possess the content of school statistics which they acquired through formal education, and demonstrated it by employing procedural and conceptual knowledge approaches, using recommended textbooks, devising a work schedule and by attending content-driven knowledge workshops. In section 4.7.2, it was discovered that the participating teachers utilised both procedural and conceptual knowledge approaches, construction skills, extra tutoring, examples drawn from familiar real life situation, and additional class exercises in the form of drill and practice in the teaching of statistical graphs. By employing these instructional skills and strategies for teaching statistical graphs, the teachers may have developed more knowledge of the instructional skills and strategies for teaching school statistics. And in section 4.7.3, the individual participating teachers developed their knowledge of learning difficulties through analysing and monitoring learners' responses to classwork, homework and assignments to identify such difficulties in statistical graphs. The teachers may have also developed further knowledge of these difficulties by tackling these using additional tutoring, extra class activities, post-teaching discussions, re-teaching, and further explanation of the lessons they taught, individually to learners during and after the lessons.

4.8 Summary of chapter

In this chapter, the data collected with the instruments mentioned in section 4.1 were presented and analyse in order to determine how the participating teachers developed their assumed PCK in statistics teaching. The results of the qualitative data collected with the conceptual knowledge exercise and concept mapping were analyse in order to select the participants for the second phase of the research and determine the teachers' content knowledge of the statistics curriculum respectively. The lesson observations of the four participating teachers were analysed and discussed in detail in order to tease out how they demonstrate the PCK they have during classroom practice. The video records were used to triangulate the data collected during the lesson observations. The teacher interview, questionnaire, written reports and documents analyses were analysed by categorising the responses of the participating teachers according to the theme of the study. The chapter concluded with a highlight of how the teachers developed their assumed PCK were determined with a summation of their subject matter content knowledge, pedagogical knowledge and knowledge of learners' preconceptions and learning difficulties.