CHAPTER 2

2.0 LITERATURE REVIEW

2.1 Introduction

This chapter focuses on the literature review, which tries to address the empirical and theoretical issues related to teachers' PCK development and its use in mathematics and statistics teaching. The discussion about the process of PCK is derived from a review of the NCS for mathematics and statistics teaching and the research questions guiding the study. Studies on teaching statistics in school mathematics and the models for capturing PCK are discussed. The techniques of studying PCK are highlighted and studied in order to justify the validity of the instruments used to investigate PCK. The chapter concludes with a summary of the theoretical framework that allows for the development of the research instruments, data analysis and results.

2.2 National Curriculum Statements for Mathematics and Statistics

The National Curriculum Statement (NCS) for Mathematics is based on the nature of the discipline and societal expectations of learners of mathematics (DoE, 2009). Mathematics is a subject that enables creative and logical reasoning about problems in the physical and social world, and in the context of mathematics itself (DoE, 2009:9). From this, mathematics is seen as a human activity that deals with patterns, problem solving, and logical thinking, in an attempt to understand the world and to make use of that understanding (Lebeta, 2006).

According to the views of the Department of Education (2009) and Lebeta (2006), it may be concluded that 'mathematics is part of day-to-day human experiences and relates to human activities that use features of one natural object as a tool for acting on other objects. This means that mathematics is an organic activity'. According to Davydov (1999), human activity is linked to conceptual activity. The purpose of mathematics is to demonstrate how human activity is linked to conceptual activity. Therefore, 'knowledge in mathematical science is constructed by establishing descriptive, numerical and symbolic relationships that are based on observing patterns, using rigorous logical thinking that can lead to theories of abstract relations' (DoE, 2009). By implication, mathematical knowledge can help learners to engage in problem solving to understand the world, and they can use that understanding in their daily lives. Hence, the subject statement for mathematics for Grades 10 to 12 expects learners to expand on their understanding of Learning Outcome 4 (LO4) of the NCS under the category 'Data handling and probability' (DoE, 2007:22), through appropriate teaching and learning of the topic in the classroom context. This learning outcome 'requires learners to be able to collect, organise, analyse, and interpret data, in order to establish statistical and probability models to solve related problems with a focus on human rights issues, inclusivity, and current matters involving environmental and health issues' (DoE, 2009:10). What, then, is the purpose of mathematics, according to the NCS?

According to the NCS (2009:11), the purpose of mathematics is to provide powerful tools:

- To analyse situations and arguments, make and justify critical decisions, and take transformative action, thereby empowering people to work towards the reconstruction and development of society
- To develop equal opportunities and choices
- To contribute towards the widest development of society's cultures, in a rapidly changing, technological, global context
- To derive pleasure and satisfaction through the pursuit of rigour, elegance, and the analysis of patterns and relationships
- To engage with political, organisational and socio-economic relations (DoE, 2009:11)

However, the focus of this study is on statistics, which is part of the mathematics curriculum. Research reports by Gattuso (2006) show that there is a link or relationship between mathematics and statistics. For example, linear function is used in describing the relationship between two variables in scatter plots. Using the stem-and-leaf diagram, one can distinguish between units and tens in mathematics. And in the workplace, statistics is used in representing the records of employees' weekly, monthly and yearly attendance at work on a frequency table and statistical graphs. That is why it is important that mathematics teachers understand this relationship, so that it can be addressed in the teaching and learning situation (DoE, 2009).

For many teachers, the relationships are not clear. They face difficulties in teaching statistics and addressing the relationships between mathematics and statistics in classroom practice (DoE, 2010). As early as 1988, Garfield and Ahlgren (1988) reported that although statistics is related to the learning of mathematics and other disciplines, a large proportion of learners do not understand many of the basic statistical concepts they have studied. The authors reported that 'inadequacies in prerequisite mathematics skills and abstract reasoning' are part of the difficulties encountered by the learners of statistics. Poor learner performance in statistics was also noted at the joint conference of the International Commission for Mathematics Instruction and the International Association for Statistics Educators (ICMI/IASE, 2007).

2.3 Research on teaching statistics in school mathematics

The important role of statistics in mathematics education and other disciplines has now been recognised worldwide. This was confirmed by the introduction of statistics in school mathematics in the school curricula at all levels in South Africa and elsewhere (DoE, 2009). However, recent research on teaching of statistics in school mathematics shows that learners encounter difficulties in learning the subject (Godino et al., 2011).

Baker, Corbett and Koedinger (2001) observed that learners are often confused about the construction of bar graph and histogram. According to these authors, most learners construct a histogram in the same way as a bar graph. The authors noted that in the stage of learning how to construct a histogram, learners transferred their existing knowledge about a bar graph to the construction of a histogram, instead of using knowledge specific to the target representation. And because learners were already familiar with bar graph construction, they found it easy to construct a bar graph instead of histogram (Baker et al, 2001).

Meletiou-Mavrotheris and Lee (2002) note that learners perceive histograms as two-dimensional graphs that must have two variables and thus tend to interpret a histogram as two-variable scatter plots. In addition, learners tend to perceive histograms as displays of raw data on the Y-axis with each bar standing for individual observation and with individual cases on the X-axis. These authors reported that when comparing two histograms with regard to their variability, learners used the vertical axes of the histogram instead of the horizontal axes to compare their variability or spread (Meletiou-Mavrotheris & Lee, 2002).

Baker et al. (2001) extended this research to include the construction and interpretation of statistical graphs with emphasis on scatter plots and stem-and-leaf. Their reports show that the axes of a scatter plots were drawn by the learners as if a bar graph was to be represented and plotted the points on the wrong axes. Consequently, a misinterpretation was obtained from a wrongly constructed scatter plot.

Other research studies (NCTM, 2007; Baker et al., 2001, Cazorla, 2006 and DoBE, 2012) attributed learners' learning difficulties to the way teachers taught the construction and interpretation of stem-and-leaf diagrams. The authors noted that although learners can read and represent stem-and-leaf diagrams, they were unable to interpret them because they had not been exposed to the types (varieties of ways) of stem-and-leaf representation.

Nicholson and Darnton (2005) researched the challenges for the classroom teacher in teaching statistics. In an analysis of questions used in statutory national tests, learners' scripts were used to collect data on their reasoning processes and learning difficulties. The results of an analysis of the questions and scripts at the early stage in the primary school were compared with the difficulties seen at the later stage of secondary statistics. The findings of this study show that pupils at the early stage struggle to articulate their reasoning processes explicitly. Furthermore, teaching and learning at the later stage of their secondary examination were based on computational accuracy and procedural competence in statistics, and less time was spent on interpretational skills. The implication of these findings is that mathematics teachers who are not familiar with the common difficulties and misconceptions may not be able to help learners to overcome their learning difficulties in statistics and achieve a deeper understanding of core concepts (Nicholson & Darnton, 2005).

Mavrotheris and Stylianou (2003) observed that one of the sources of learning difficulties in a statistics classroom is that most mathematics teachers are too formalistic in their approach to the subject. The authors noted that statistics lessons are presented in rigidly established bodies of mathematical knowledge without any reference to the real-world context (Mavrotheris & Stylianou, 2003). Formalist ways of teaching have led to educators failing to convey to the learners the relationship between knowledge they acquire in the statistics classroom and its uses in everyday life (Mavrotheris & Stylianou, 2003). For example, learners were taught first to build a cumulative frequency table, and construct an ogive by drawing the axes, labelling the axes, plotting the points and joining the line of best fit. During interpretation and analysis, values were read off from the vertical and horizontal axes without being linked to the learners' real world (Libman, 2010). Hence, learners had difficulties in understanding what the teacher had taught using the formalistic approach.

Watson, Callingham and Donne (2008) carried out research on establishing PCK for teaching statistics from Grades 1 to 12. The PCK of 42 teachers selected as part of a professional learning programme in statistics was examined. The results of the Rasch analysis to obtain a measure of teacher ability levels in relation to PCK indicate that teachers who did not respond appropriately to the survey items often missed or left out those items that required a response to a specific student misunderstanding (Watson, Callingham & Donne 2008). The inability of the teachers to respond to specific student misunderstanding could mean either that they were not able to move students towards a higher level of statistics understanding or to design instructional interventions to address students' learning difficulties. This study represents an initial attempt to establish the nature of teachers' demonstrated PCK in teaching school statistics.

The intention of the researcher through this study is to determine whether the participating teachers are aware of their learners' difficulties with statistical graphs and the means used by them to elicit these difficulties. PCK is seen as a relevant construct for this study as teachers' topic-specific content knowledge influences what is taught in the classroom context. It therefore becomes necessary to explore the PCK of a mathematics teacher who demonstrates good content-specific knowledge (Godino, Batanero & Font, 2011) to see how this teacher's PCK is enacted while teaching these difficult topics.

2.4 Assessing teachers' PCK

2.4.1 Description of PCK

The concept of pedagogical content knowledge (PCK) was introduced by Shulman (1986) in a paper in which he argued that research on teaching and teacher education ignored questions dealing with the contents of lessons, the questions asked, and the explanations offered. As indicated in the theoretical framework of this study, PCK goes beyond knowledge of the subject per se to encompass the dimension of subject matter knowledge for teaching. It refers to how the teacher interprets the subject matter knowledge in the context of facilitating learning.

Shulman (1986), while categorising a knowledge base for teaching, noted that the way in which the subject matter is presented and formulated is a key element in the conceptualisation of PCK. According to him, this knowledge could originate from research or teaching practice. Other elements in Shulman's categorisation of the knowledge base for teaching are awareness of strategies that may be fruitful in reorganising the understanding of learners, and learners' preconceptions and misconceptions about a particular topic.

In the two decades since Shulman introduced the concept of PCK there have been a number of studies on the subject. Various scholars across the discipline have elaborated on Shulman's work and proposed different conceptualisations of PCK (Grossman, 1990; Marks, 1990; Cochram et al., 1993; Van Driel et al., 1998; Magnusson, Krajcik & Borko, 1999; Gess-Newsome and Lederman, 2001; Barnett & Hodson, 2001; Jong, 2003; Halim & Meerah, 2002). This amplification is in terms of what they include or do not include in their conceptualisations of PCK.

Grossman (1988) developed and expanded the definition of PCK. Her definition is based on four central components: knowledge of learners' understanding; the curriculum; instructional strategies; and the purpose of teaching. Knowledge of learners' understanding refers to how the learners comprehend what is taught. In other words, how do learners understand the subject matter being presented to them? The curriculum pertains to the content of the subject matter, as contained in it. Knowledge of instructional strategies constitutes understanding of the stratagems employed in teaching the subject. The purpose of teaching is to achieve the learning outcomes, as outlined in the curriculum. Using these components, Grossman (1988) examined the influence of teacher education on knowledge growth. The findings regarding the impact of teacher education on knowledge growth demonstrate that teacher education can influence knowledge growth by teachers.

Teacher education involves the disciplinary tutoring through which the subject matter knowledge and pedagogical knowledge can be acquired. This education can provide an opportunity to acquire more knowledge and growth if the teacher continues to practise in the particular discipline (Grossman, 1988). The influence of teacher education on knowledge growth is related to this study in the sense that one can speculate that the disciplinary education acquired by teachers could influence the way in which their PCK is developed and used for teaching statistics in school mathematics, hence the need to examine and assess the level of teachers' subject matter content knowledge, as already indicated. However, in the context of delivering a particular curriculum (DoE, 2007), the model fails to indicate any specific programme and how it influences the teachers' knowledge and its uses during classroom practice (Ibeawuchi, 2010).

Based on an explicit constructivist view of teaching, Cochram et al. (1993), in their research on PCK as an integrative model for teacher preparation, renamed PCK 'pedagogical content knowing' (PCKg), to acknowledge the dynamic nature of knowledge development. In their model, PCKg is conceptualised far more broadly than in Shulman's view. They define PCKg as 'a teacher's integrated understanding of four components of pedagogy, subject matter content knowledge, learner characteristics and the environmental context of teaching' (Cochram et al., 1993). According to these authors, PCKg is generated as a synthesis of the simultaneous development of these four aspects in the context of the integrative model of teaching. Following this argument, it means that the components of PCK, as highlighted above, do not exist independently of one another. In this study, however, the components of PCK were captured individually during classroom practice. Even though the elements of PCK do not exist independently of one another as conceptualised, it is still seen as an amalgam of these components during classroom practice. PCK is individualistic, tacit, and ever changing with time and experience (Miller, 2007).

But according to Lee and Luft (2008), there are two models of PCK, integrative and transformative. In the integrative model, the PCK components exist separately, and at the beginning of teachers' careers they enable teachers to rely on only one of the PCK components to cope with teaching (subject matter content) (Lee & Luft, 2008). Transformative PCK is held by experienced teachers who combine all the components of PCK and convert it into classroom practice. Lee and Luft (2008) claimed that during teaching it is difficult to distinguish subject matter knowledge or general pedagogical knowledge from PCK, which means the components do not exist independently of one another. In this study, based on notion of amalgam, the components of PCK can exist independent of one another or together. The ways the teachers used them were established by attempting to describe the PCK profiles of the participating teachers as evidence in their practice.

Van Driel, Verloop and De Vos (1998) conducted research on developing science teachers' PCK, using classroom observation and interviews. According to them, the idea of integration of knowledge components is also central to the way PCK is conceptualised by Fernandez-Balboa and Steel (1995). These authors identify five knowledge components of PCK: subject matter, the learners, instructional strategies, the teaching context, and the teaching purpose.

Magnusson et al. (1999) presented a model of the relationship between the constituent domains of PCK. According to them, subject matter knowledge (e.g. substantive knowledge, and syntactic knowledge), pedagogical knowledge, knowledge of educational aims, knowledge of the classroom, and context knowledge (e.g. knowledge of specific learners and school characteristics) could be used to interpret PCK. In the teaching process, these domains could be combined (Rollnick et al., 2008) to provide effective teaching and promote learners' understanding of the lesson.

Barnett and Hodson (2001), in their research on how to understand what science teachers know, considered PCK a constituent of pedagogical context knowledge, together with other components. These other components were academic knowledge, classroom knowledge, and professional knowledge. But the components of PCK are not always clear and consistent; rather they look blurry; and the development of a teacher's PCK is not linear, but advances from different angles (Loughran et al., 2004).

Although different researchers have varying opinions about the conceptualisation of PCK, Jong (2003) and Van Driel et al. (1998) stated that these elements seem to be germane to any conceptualisation of PCK with respect to a chosen content area

- Knowledge of learners' learning difficulties, conceptions, and misconceptions concerning the topic
- Knowledge of how to represent specific topics

Several scholars have researched PCK development, and their studies are concerned with how a teacher uses his/her knowledge of the content that the learners are expected to learn and the best approaches to employ to access that content; hence it is called the knowledge base for teaching. A teacher's PCK is therefore unique (Bucat, 2004) as it depends on how he or she interprets learners' preconceptions and learning difficulties and what the learners need in order to understand the content being taught (Mitchell & Mueller, 2006). The development of PCK is mutual and hence the development of one component influences the development of another (Henze, Van Driel & Verloop, 2008). Hill et al. (2008) argued that the impact of teachers' PCK on learners' learning was still to be proven, since there seemed to be a relationship between the teacher's PCK and what the teacher does in the classroom. So far, these authors have agreed that the development of a teacher's PCK is rooted in the classroom and this could contribute to effective teaching and learning of statistics in school mathematics.

The first component of PCK, namely knowledge of learners' understanding and their conceptions of a specific topic, helps teachers to interpret learners' actions and ideas, as well as plan effective instruction (Loughran, Mulhall & Berry, 2004; Halim & Meerah, 2002). These authors argued that ignorance of learners' misconceptions may be due to teachers' lack of content knowledge. The second component, knowledge of how to teach a particular topic, refers to awareness of specific areas that are useful in helping learners understand specific concepts. This involves knowledge of ways of representing specific concepts, in order to facilitate learning (Halim & Meerah, 2002). This component of PCK, which aims to develop learners' conceptual understanding, seems necessarily dependent on having subject matter knowledge relative to the concept being taught. Furthermore, 'the PCK for representing specific topics is a product of previous planning, teaching and reflecting' (Halim & Meerah, 2002).

2.4.2 *Teacher knowledge and PCK*

According to Gess-Newsome (in Jong, 2003), all the various views of PCK can be categorised as integrative or transformative. Where PCK is categorised as integrative, knowledge of teaching is merely the integration of forms of teacher knowledge, such as subject matter content knowledge, knowledge of learners' learning difficulties, and knowledge of learners' preconceptions concerning a topic. In this integrative view, PCK is seen as a mixture. In other words, 'PCK does not really exist in its own domain, and teaching is seen as an act of integrating knowledge of subjects, pedagogy and context' (Gess-Newsome & Lederman, 2001). In classroom practice, knowledge of all these domains is integrated by the teacher to create effective teaching and learning opportunities. Most teacher education programmes that are organised in separate courses of subject matter, pedagogy, and practice follow this model of teacher knowledge (Ibeawuchi, 2010).

In the transformative view (Jong, 2003), forms of teacher knowledge, such as subject matter knowledge, pedagogical knowledge, and contextual knowledge are transformed into a new form of knowledge such as understanding of a concept. In this view, PCK is seen as a compound. This model supports teacher education programmes that contain integrated courses and allow prospective teachers to quickly develop the required skills and knowledge. The integrative view and the transformative view can be considered opposite ends of the PCK spectrum (Jong, 2003). In this study, it is assumed that the transformative view was used by the participating teachers during classroom practice for teaching statistical graphs because the teachers uses the conceptual knowledge approach to describe the concept of histogram, ogive and bar graph which the learners appear to have understood.

Recently the statistics education community's attention has been drawn to the statistical knowledge for teaching (SKT) measures by scholars such as Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball (2008). According to these authors, statistical knowledge for teaching included statistical information that is common to individuals working in diverse professions and the subject matter knowledge that supports such teaching, for example why and how a statistical procedure works, how best to define a statistical term for a particular grade level, and the particular content (Hill et al., 2008). To these authors, the impact of teachers' PCK on learners' learning had yet to be proven, but there seemed to be a relationship between a teacher's PCK and what the teacher did during classroom practice. Following these arguments, the development of PCK is explored in the classroom, and this can contribute to effective teaching and learning.

Toerien (2011) conducted preliminary research on the development of PCK of in-service science teachers and conceptualised PCK as including subject matter content knowledge, the context of the school, knowledge of the curriculum, and teachers' pedagogical knowledge. Using semi-structured interviews and lesson observation, Toerien (2011) noted that these four components could be used to investigate the development of PCK of in-service science teachers in the classroom context.

In looking at how various researchers have conceptualised PCK, it appears that investigating PCK may not always be a straightforward matter, because of its unarticulated and tacit nature. Jong et al. (2005) argued that investigating PCK development is a complex process, because PCK is determined, among other things, by the nature of the topic, the context in which the topic is taught, and the way in which a teacher reflects on the teaching experience (Park & Oliver, 2008). This is because different topics may require different teaching approaches, depending on the learning outcomes. This study sought to determine how PCK is developed by investigating participating teachers through the use of multiple sources for data triangulation.

In summary, Figure 2.1 describes the components of PCK that are likely to be used for teaching statistics in school mathematics. They include subject matter content knowledge, pedagogical knowledge, knowledge of learners' conceptions and knowledge of learners' learning difficulties. In the context of this study, the pedagogical knowledge in statistics teaching will be assessed using multi-evaluation comprising of the lesson observation, written reports, and questionnaire and documents analysis.

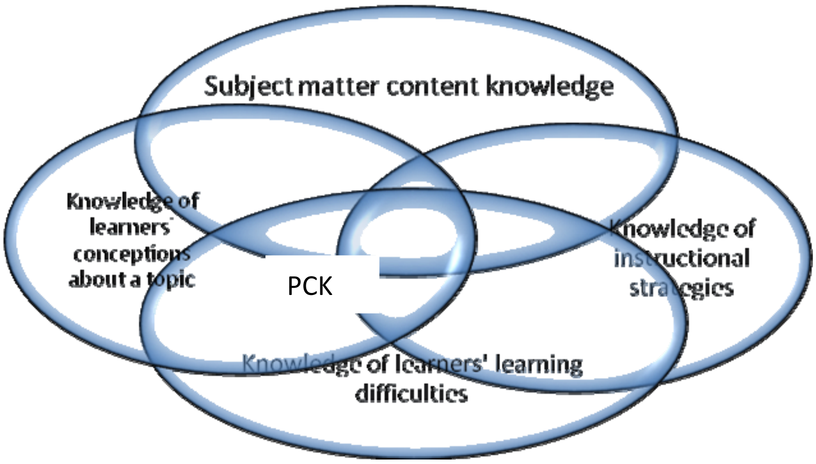

Figure 2.1: Components of PCK used in this study

2.4.3 Pedagogical content knowledge and subject matter for teaching

Several researchers have used the terms 'subject matter knowledge' and 'subject matter content' to describe the kind of knowledge that teachers need for teaching (Shulman, 1986; Ma, 1999; Vistro-Yu, 2003; Jong, 2003; Jong et al., 2005; Halim et al., 2002; Rollnick et al., 2008). In terms of mathematics teaching, Plotz (2007) referred to subject matter content knowledge as 'mathematical content knowledge'. With regard to PCK development in statistics teaching it is necessary to define what each of the concepts means, so that they can be used to define the construct of PCK that was used in statistics teaching. Plotz (2007) argued that mathematical content knowledge is acquired mostly by studying mathematics in school, and this may be described as 'in-school acquired knowledge'. Van Driel et al. (1998), Jong (2003) and Jong et al. (2005) described subject matter knowledge as the knowledge obtained through formal training at universities and colleges, which may be regarded as disciplinary education. From these assertions, it would seem that subject matter knowledge is acquired through formal training in a subject area.

Ball and Bass (2000) researched the interweaving of content and pedagogy in the teaching and learning of mathematics. The findings of their study indicated that the subject matter knowledge needed by teachers is found not only in the list of topics of the subject matter to be learned, but in the practice of teaching itself (Ball and Bass, 2000; Plotz, 2007). In other words, knowing the content of a subject is not enough to justify the capacity of a teacher to teach; what makes a teacher capable of teaching is also how well the teacher facilitates the learning. According to these authors, little is known about the way in which 'knowing' a topic from a list of topics affects teachers' capabilities. And if one depends on analysing the curriculum to identify the subject matter content knowledge needed for teaching the topics without focusing on practice as well, not much will be gained (Ball and Bass, 2000; Plotz, 2007). Plotz's (2007) study also reveals that mathematical content knowledge and pedagogical knowledge are both needed for effective teaching and can motivate the development of the PCK used for teaching. He stressed that teachers' prior knowledge needs to be exposed for effective content knowledge transformation and understanding as the prior knowledge aided the teachers in the written problem-solving activities to design to assess their mathematical content knowledge state.

Capraro, Capraro, Parker, Kulm and Raulerson (2005) researched the role of mathematics content knowledge in developing pre-service teachers' PCK, using performance in a previous mathematics course, a pre- and post-test assessment instrument, success in the state-level teacher certification examination, and journals. Their study outlined the connection between mathematics content knowledge and pedagogical knowledge in developing PCK, in order to address the increasing expectations of what learners should know and be able to do, and knowledge that the teachers must have in order to meet the educational goals during instruction and learning. A total of 193 undergraduate students who enrolled in integrated method block courses prior to the teaching practice programme were involved in the research project on teaching practice in mathematics. The findings of Capraro et al. (2005) indicated that the teachers' previous mathematics abilities are valuable predictors of students' success in teacher certificate examinations. Secondly, the mathematically competent pre-service teachers exhibited progressively more PCK, as they had been exposed to mathematical pedagogy comprising subject matter content and teaching practice during their mathematics method course. Therefore, for one to have pedagogically powerful representations of a topic, one should first have a comprehensive understanding of the topic.

Following this argument, subject matter knowledge in the context of PCK development becomes a product of the interaction between mathematical competence and concern for the instruction and learning of mathematics (Plotz, 2007). In other words, the concern for instruction and learning shown by a competent mathematics teacher must demonstrate that he or she has adequate knowledge of the subject matter, and this may be necessary for PCK development. In this study, it is assumed that during their university preparation programmes, the participating teachers acquired the subject matter knowledge of mathematics and the pedagogical knowledge necessary for PCK development in statistics.

However, the South African mathematics (Grades 10–12) teaching force is made up mainly of practitioners who have three-year teaching diplomas obtained from the old (pre-1994) colleges of education (Rollnick et al., 2008). Less than 40% of these teachers hold a junior degree on the subject they teach. The mathematics content measures only up to that of first year at a university (Rollnick et al., 2008). In this study, the key question is, given that the teachers show competence or understanding of these concepts in mathematics, irrespective of their training, how does this influence their teaching and therefore their PCK for teaching statistics in school mathematics?

Vistro-Yu (2003) conducted a study on how secondary school mathematics teachers faced the challenges of teaching mathematics (in terms of the pedagogical knowledge requirements of PCK in mathematics) in a new mathematics class in college algebra. Thirty-three secondary school mathematics teachers were initially involved in the research project. They were made to write a standardised test in high-school mathematics to determine the level of their subject matter content knowledge. Based on this performance, six teachers were selected for the research project. These six teachers were asked to prepare and teach an assigned topic in a college algebra module, while the researcher conducted classroom observations of the lessons presented by them. They were interviewed before teaching commenced, and after the lessons, the six teachers were given a questionnaire to complete by reflecting on their teaching performance. The findings of the study showed that the teachers were limited in the ways they prepared their lessons. According to Vistro-Yu (2003), they were not able to teach in an organised manner and lacked in-depth subject matter knowledge. The results of the interview showed that some of the participants were dissatisfied with their teacher education preparatory programmes because they lacked thorough content knowledge of the subject matter. In this study, the methods adopted by Vistro-Yu (2003), namely teachers' content knowledge exercise, lesson observation, and interviews, were used to determine the subject matter content knowledge and pedagogical content knowledge of these mathematics teachers (the participants in the study).

Jong et al. (2005) conducted a study of the PCK of pre-service teachers using particle models to teach chemistry at secondary-school level. Responses to written assignments, transcripts of workshop discussions, and reflective reports by the participants were used to collect data. The findings of this study indicated that the pre-service teachers were able to understand and describe the learning difficulties of their learners during teaching with particle models. In addition, they developed PCK using particle models, although development varied among the participants (Jong et al, 2005).

The research methods of Ball and Bass (2000), Vistro-Yu (2003), Capraro et al. (2005) and Jong et al. (2005) provided the rationale for the assessment of subject matter content, pedagogical knowledge, knowledge of learners' conceptions and learning difficulties as constituent elements needed to develop PCK for teaching. However, there were deficiencies in their studies. One of these was that their research was conducted within a relatively short time (Vistro-Yu, 2003; Capraro et al., 2005). For instance, using one, two or four lesson periods to conduct an investigation on the challenges in the instruction and learning of mathematics (Vistro-Yu, 2003; Capraro et al., 2005) may not be adequate, since most topics in mathematics take more than one period to teach.

Second, some of the researchers (Capraro et al., 2005; Ball & Bass, 2000) used grades obtained in their university courses to justify the competency of a teacher in instructing a subject. This may not be adequate, as the number of mathematics courses that a teacher has studied at university or college does not necessarily ensure effective or quality teaching in a classroom situation (Plotz, 2007; Capraro et al., 2005; Geddis, 1993). Rather, what makes him or her an effective teacher is how well he or she understands what learners have to learn,

and the way he or she presents the subject matter content (Muijs & Reynolds, 2000; Graffin et al., 1996). Therefore, more precise measures are needed to specify in greater detail the relationships between the various components of PCK and how they are developed in order to improve learner performance in mathematics (DoE, 2008). A third deficiency is lack of lesson observation in conducting some of the investigations (Capraro et al., 2005; Ball & Bass, 2000). The use of lesson observation would have afforded the researchers the opportunity to determine how mathematics teachers use their PCK, for example preparation and presentation of the lesson based on adequate knowledge of the subject matter; and identification of learners' preconceptions and learning difficulties, conceptions and misconceptions concerning the topic (Jong, 2003).
In order to avoid these deficiencies, the study was carried out with the following features:

1) The PCK of teachers were investigated over a relatively long period (between four and six weeks).
2) The study was carried out with experienced secondary school mathematics teachers.
3) Lesson observation was undertaken to determine how the teachers demonstrated their PCK and subject matter knowledge during the teaching process and how they identified learners' preconceptions and misconceptions of the topic.
4) Teachers' and learners' portfolios and workbooks were examined to determine what had made the instruction and learning of the topic easy or difficult.
5) These features were adapted to investigate the way competent mathematics teachers developed their PCK for teaching statistics in school mathematics, in the hope of discovering a further directive for the continuous improvement of the mathematics teachers' PCK in statistics teaching as well as of educational programmes for in-service and pre-service teachers of statistics.

In terms of measuring teachers' subject matter content knowledge in a topic, several techniques and methods have been used by several researchers in the field of mathematics and science education. For instance, Gess-Newsome and Lederman (2001) and Jong (2003) reported that a teachers' subject matter content knowledge can be measured using concept mapping, card sorting and pictorial representation. In this study, the subject matter content knowledge of the participating teachers was assessed with the conceptual knowledge exercise, concept mapping, interview and lesson observation.

The conceptual knowledge exercise in statistics was designed in multiple-choice formats. The multiple-choice questions in statistics consist of a series of question, each with five possible options from which the participating teachers have to choose the best to answer the questions. Critics say that the multiple-choice format may not accurately depict the respondent's personal views about teaching because there is no provision for the reasons for the selection of a particular option. But researchers continue to use multiple-choice questions with success, because the many advantages of this type of question offset their demerits (Gess-Newsome and Lederman, 2001; Kazeni, 2006). For example, multiple-choice questions can be set at different cognitive levels. They are versatile if designed and used appropriately (Miller, 2006). Multiple-choice question assessments can be completed in a short time, and they ensure better coverage of content. In this study, multiple-choice questions were used to assess the changes in statistics content knowledge of the participants (since they have been teaching the topic) as they may have covered enough content area of statistics and to select them for the second phase of the qualitative research.

Considering the role of concept mapping in teaching and learning, Ochonogor and Awaji (2005) and Novak and Cannas (2006) described concept mapping as a learning strategy that aids understanding of complex ideas and clarifies ambiguous relationships between ideas. According to these authors, concept maps may be seen as graphical tools for representing topics, by depicting key concepts and organising knowledge clearly. Following this argument, organising and representing the knowledge of a particular topic can take the form of connecting the concepts by means of arrows, boxes, words or phrases in order to elicit the meaning of the relationships between the concepts. In this connection, concept maps are seen as a special form of web diagram for exploring knowledge and gathering and sharing information visually (Novak & Cannas, 2006). Concept maps can depict how we think, which influences how and what we teach (Miller, 2006). Hence, concept maps can provide opportunities to see relationships between types of knowledge.

Novak and Gowin (1994:96) argued 'that concept maps provide visual representations of knowledge'. According to these authors, concept maps allow researchers to create concrete representations of knowledge that can be used to determine knowledge changes in a teacher. Since concept maps create physical representation of knowledge, changes in this representation are assumed to provide evidence of teacher knowledge change (Miller, 2006:96).

Miller (2006) used concept maps to analyse the construction of pre-service teachers' PCK during a science method course. The participants of the study were asked to construct a concept map of important concepts in a specific chemistry unit that focuses on numerous teaching activities. The findings of this study show that the changes in the structure of the concept map were related to the changes in the personal knowledge of the learner.

Ferry, Hedberg and Harper (1997) investigated how pre-service teachers used a concept map to organise curriculum content knowledge. Participants of the study were asked to use a concept map to plan science-based instruction that could be delivered to an elementary science class. The results of the study showed that pre-service teachers had different perceptions of the connections between the basic statistical concepts, which enhanced their conceptual understanding of the concepts and aided the sequential planning of the sequence of the concepts for teaching (Ferry, Hedberg & Harper, 1997).

Concept mapping may lack reliability in terms of representing all that an individual knows about the content knowledge being assessed (Miller, 2006). Furthermore, if a teacher does not continue with classroom practice, the changes in knowledge of the topic may be short lived.

However, concept maps have been credited with many advantages. For instance, a concept map allows teachers to organise their knowledge of teaching their primary content area much better with high cognitive demand. In this study, a concept mapping exercise was used to indirectly assess teachers' content knowledge of statistics in school mathematics by arranging statistics topics in logical sequence according to the way in which the teachers would present them in their classroom practice.

The interview was used to triangulate the data gathered with the concept mapping. The interview consists of open-ended questions that the interviewer asked the interviewees to respond to. The interview allows the respondent the opportunities to create options for responding and to voice their experiences unconstrained by any perspective of the researcher or past research that may not directly be observed in the respondent action (Cresswell, 2008:225). Some researchers argued that an interview is deceptive and provides the perspective the interviewees want the interviewer to hear, which renders the information inarticulate, perceptive and unclear (Cresswell, 2008). Several researchers (Vistro-Yu, 2003;

Loughran et al, 2004; Hill, 2008) have used the interview to assess teachers' educational background that must have assisted them to develop their topic-specific content knowledge and PCK. In this study, an interview schedule was used to gather data to assess the teachers' educational background that had enabled them to develop their topic-specific content and PCK in statistics teaching. The use of lesson observations in assessing teachers' content and pedagogical knowledge will be discussed in Section 2.4.4.

The research procedures used by researchers such as Jong et al. (2005), Capraro et al. (2005), Vistro-Yu (2003), Jong (2003) and Van Driel et al. (1998) share the same research procedure as this study in terms of the use of these instruments: a conceptual knowledge exercise, interview schedules, concept mapping, to assess subject matter content knowledge and PCK.

2.4.4 PCK and pedagogical knowledge (instructional skills and strategies)

Pedagogical knowledge is believed to be the kind of information that a teacher needs and uses to perform everyday teaching tasks. It involves teaching styles and strategies, classroom management and teaching and learning processes relating to learners in the classroom (Cochram et al., 1993; Vistro-Yu, 2003). Pedagogical knowledge includes knowing and understanding the content to be taught and the specific demands of that content, such as instructional skill and strategies (Kreber, 2004; Loughran et al., 2004; Ball, Thames & Phelps, 2008). Instructional knowledge entails knowing how to sequence the learning outcomes, prepare the lessons, facilitate discussion and group work, construct tests and evaluate learners' understanding through the use of examinations, among others (Kreber, 2004).

In general, different kinds of instructional strategies, representations and activities are used in teaching mathematics. Knowledge of instructional strategies entails understanding ways of representing specific concepts, in order to facilitate student learning. Representations include illustrations, examples, models, and analogies. Each representation has a conceptual advantage and disadvantage over other representations (Ibeawuchi, 2010). PCK in this area includes awareness of the relative strengths and weaknesses of a particular representation. Activities can be used to help learners understand specific concepts or relationships, for example demonstrations, simulations, investigations and even experimentations. PCK of this type incorporates teachers' knowledge of the conceptual power of a particular activity

(Magnusson et al., 1999). For a representation to be powerful or comprehensible, the teacher must know the learners' conceptions about a particular topic, and the possible difficulties they will experience during the teaching and learning of the topic. Representations during teaching must be clearly linked, and the relationships between concepts must be comprehensible (Ibeawuchi, 2010). However, most mathematics teachers are not able to identify learner misconceptions and to teach for conceptual change since most of them have not yet dealt with their own alternative conceptions, and are working with very limited resources, time, and necessary skills (Van Driel, 1998).

Several studies have highlighted certain instructional strategies as a component of PCK. Hashweh (1987) for example emphasises that incorrect and misleading representations, such as analogies and examples that depict the teachers' misconceptions, could result from teaching outside one's own field of expertise. Tobin, Tippins and Gallard (1994) also state that when teachers teach outside their areas of specialisation, they give explanations and analogies that reinforce the misconceptions that learners already have.

Magnusson et al. (1999) argue that pedagogical knowledge as a component of PCK is dependent on teachers' subject matter knowledge about a particular concept. This may not always be true, as subject matter knowledge does not guarantee that PCK will be transformed into representations that will help learners understand targeted concepts, or that teachers will be able to decide when it is most appropriate pedagogically to use a particular representation. Anderson and Mitchener (1994), in their research on science education, support this view and are of the opinion that teachers' knowledge of science teaching may be limited, even if the teachers have knowledge of the subject matter. In a particular topic, pedagogical knowledge, or the way concepts are represented as a component of PCK, seems to depend on previous planning, teaching, and reflection (Halim & Meerah, 2002).

Vistro-Yu (2003) researched pedagogical knowledge in mathematics and focused his study on how the mathematics teacher faces the challenge of teaching algebra in a new class. As explained earlier, pedagogical knowledge is knowledge used for teaching, particularly awareness of instructional techniques, psychological principles, classroom management, and the teaching and learning process. Similar PCK-related studies by Jong et al. (2005) and Rollnick et al. (2008) show that science teachers with adequate pedagogical knowledge should be able to design good teaching and learning strategies that allow them to teach the concepts and manage the classroom and other instruction and learning processes. Hence, the instructional strategies used by the participants in the study for teaching school statistics were investigated in classroom practice. The question that one would ask at this stage is how do we measure the knowledge of instructional skills and strategies demonstrated by the teachers in their statistics lesson.

Current researches on PCK have suggested that the multi-method approach may be appropriate in exploring knowledge of the relevant instructional strategies (Jong, 2003; Miller, 2006; Rollnick et al., 2008; Ibeawuchi, 2010; Toerien, 2011) during classroom practice. Multi-method evaluation involves collecting multiple sources of data. Multi-method analysis tends to create increasing impact on changing knowledge, with each data source adding more dimensions to the findings from another source, thereby biasing the findings of the study (Gess-Newsome & Lederman, 2001). Nevertheless, researchers are using this method with increasing success. Multi-method evaluation is useful for triangulation of data and improving the validity of the data (Gess-Newsome & Lederman, 2001). In this study, multiple sources were used to collect data to assess the instructional skills and strategies that the participating teachers used in teaching statistics.

One of the multiple sources is the lesson observation of the participating teachers. Lesson observation is a process of gathering open-ended, firsthand information by observing the participant physically and gathering the information as it occurs at the research site (Cresswell, 2008:221). Lesson observation has the advantage of studying the actual behaviour of the participants and the difficulties they may have in demonstrating their ideas during research activities. The disadvantages of using lesson observation for data collection are that the researcher will be limited to the site and situations of the research and may have difficulty in establishing rapport with individuals. But despite the disadvantages, researchers continue to use lesson observation with success because of the firsthand information and recording the actual behaviour of the participants at the research site. The lesson observation was also used to triangulate data gathered with the concept mapping exercise (ref Section 2.4.3).

In this study, the teachers' written reports were triangulated with learners' lesson observations which form part of the multiple sources for evaluating teachers pedagogical knowledge in statistics teaching. Several researchers, including Gess-Newsome & Lederman (2001), Penso

(2002) and Jong (2003), Capraro et al (2005), have used the teacher' written report to evaluate teachers' PCK during classroom practices in science and mathematics. It has the advantage of making teachers reflect on their teaching, thereby providing opportunities for the teachers to evaluate it. In this study, the teachers' written reports were used to assess the teachers' pedagogical and triangulate the data collected with lesson observation in terms of reflecting on what transpired during the lesson.

Researchers such as Gess-Newsome and Lederman (2001) and Vistro-Yu (2003) have used questionnaire to determine teachers' pedagogical knowledge in the context of PCK development. According to them, they were able to capture what the teachers did while teaching a specific topic in science and mathematics. In this study, part of the teacher questionnaire responses was used to assess what the teachers did while teaching the assigned topic in statistics. Free-response questionnaire allows the researcher to obtain the teachers' feelings about their actions during the lesson, which they might not have displayed or expressed during the lesson and interview.

The documents analysis and video records were also used to triangulate the data from the lesson observation. Capraro et al (2005), Jong et al (2005) and Ogbonnaya (2011) have used document analysis such as journal and certification to gather data to assess the teachers' content and pedagogical knowledge in mathematics and they were successful in gathering data related to the teachers' content and pedagogical knowledge. In this study, the documents analyse included the teacher portfolios, learners' workbook and portfolios, textbooks as well as school policy guidelines for teaching and learning. They have the advantage of being readily available for reading, analysis and interpretation to the researcher.
Based on these advantages, the documents (learners class workbooks and portfolios, teacher portfolios, lesson plans, and NCS subject assessment guidelines) were considered as a source for gathering data to assess the teachers' pedagogical knowledge in terms of what has made the lesson easy or difficult

Jong (2003:375) explained that teachers are able to explain their cognition in detail while they look at a video record of a lesson that has been taught. Because of the distracting effect of a video recording being made in the classroom, an interview can be considered a replacement for it. The video recording is used as a tool for teachers to remember what they taught during the lesson, and they can experience how the lesson was delivered, unlike the interview, which only allows the respondents to verbalise their actions during the lesson. Jong (2003) noted that the stimulated-recall interview (video records) might be more appropriate in explaining teachers' actions during classroom practice. In this study, the video recorder was used to record the lessons in which the participating teachers demonstrated their pedagogical knowledge in statistics teaching and to triangulate the lesson observations in statistical graphs.

2.4.5 PCK and knowledge of learners' preconceptions and learning difficulties

Instructional strategies, learning difficulties and misconceptions are some of the components of pedagogical content knowledge that are used in teaching a particular topic in a specific subject area (Penso, 2002). Penso (2002) conducted a study on the PCK of pre-service biology teachers, with the emphasis on how student teachers identify and describe learners' learning difficulties. The teacher used classroom observation and learners' diaries to collect data from the participants. Penso's (2002) findings showed that learning difficulties could be identified and described during teaching and by observing lessons. Penso (2002) claimed that these difficulties might originate from the way the lessons were taught, which involves the content of the lesson, lesson preparation and implementation, and the learning atmosphere. Other factors include the misconceptions that the learners and the teachers have about the topic, and the cognitive and affective characteristics of the learners.

According to Penso (2002), learners regard their learning difficulties as being caused by conditions prior to the process of teaching and to those existing in the course of teaching. While the aspect of lesson content relates to the level of difficulty and abstraction of the topic, the teaching, lesson preparation and implementation aspects are concerned with the structure and presentation of the lesson (Cazorla, 2006). Negative lesson structure conditions include overloading content and unsatisfactory sequences in the lesson. Negative lesson presentation conditions include inappropriate instructional strategies for presentation, and not contributing to the process of learning. Negative cognitive and affective characteristics entail lack of prior knowledge about a topic that would enable learners to cope with the lesson in a meaningful way, preconceptions developed by the learners because of previous experiences, partial and inconsistent thinking, and lack of motivation and concentration. These negative cognitive and affective characteristics may result in learning difficulties in a teaching and learning situation if the teacher does not have adequate prior content knowledge of the topic.

Cazorla (2006) researched the ways in which mathematics teachers teach statistics in elementary and secondary schools and teacher training colleges, and reported that mathematics teachers seemed to encounter teaching and learning difficulties during teaching. According to this author, misconceptions and the ways in which mathematics lessons are taught are among the factors that contribute to learners' learning difficulties in statistics teaching. In addition, most statistics teachers do not have adequate knowledge of the curriculum and the necessary approaches to the teaching and learning of statistics. This leads to poor content delivery in the classroom, and consequently affects learners' performance.

Jong (2003), in his research on exploring science teachers' pedagogical content knowledge, used a teacher's log, concept mapping, interviews, and convergent and inferential investigation techniques and notes in order to identify and resolve misconceptions and learning difficulties. Convergent and inferential techniques may be used by the teachers during classroom practice. These refer to data collection techniques in which questions are developed in short-answer and multiple-choice formats to probe the preconceptions and misconceptions of learners in a topic (Jong, 2003). The gap in this study is that lesson observation could have been used to determine how teachers use their PCK to identify learning difficulties during the lesson.

It is thus conclusive that inadequate subject matter knowledge and inappropriate instructional strategies employed in classroom practice can bring about misconceptions and learning difficulties among learners in statistics teaching. However, learning difficulties can be resolved if practising teachers have developed adequate PCK to solve them, which, in turn, can lead to improved learner achievement. In this study, the teachers' knowledge of learners' learning difficulties was assessed through lesson observation, questionnaires, teachers' written reports and document analysis.

In the literature review, the studies by Penso (2002) and researchers such as Jong et al. (2005), Jong (2003), Van Driel et al. (1998), Capraro et al. (2005) and Cazorla (2006) justify the need for this study to investigate how competent secondary school mathematics teachers develop PCK in statistics teaching.

Research reports by Jong (2003) and Gess-Newsome and Lederman (2001) indicated that convergent and inferential techniques may be appropriate in measuring teachers' knowledge of learners' preconceptions and learning difficulties in science. The convergent and inferential technique involves the use of predetermined verbal descriptions of teacher knowledge comprising multiple choices and short-answer questionnaire. A multiple-choice item test is a series of questions with several possible answers, from which a person has to choose the correct one. The multiple-choice format can be used to rate individual performance and ability in a test, as well as to compare the performance between participants (as in this study) (Bontis, Hardie & Serenko, 2009; Kehoe, 1995).

In this study, the teachers' knowledge of learners' preconceptions and learning difficulties were assessed using the lesson observation, as part of the interview schedule, and in the questionnaire, written reports and documents analysis. Based on the way various researchers used these instruments in assessing teachers' content and pedagogical knowledge, and the many advantages of using them to capture teachers' PCK (ref Sections 2.4.3 and 2.4.4), the lesson observation was adapted to assess the teachers' knowledge of learners' preconceptions and learning difficulties in statistics teaching in order to attest how this knowledge manifests in the teacher during classroom practice. The data gathered with the interview, questionnaire, written reports and documents analysis were used to triangulate the lesson observation and to ascertain how the teachers' knowledge of learners' preconception and learning difficulties manifests during the lesson on statistical graph.

2.5 Summary of the chapter

In this chapter, various categories of relevant literature on PCK were presented. It began with a description of the NCS for Mathematics and Statistics, and explained how these subjects relate to each other. Although the studies of Penso (2001), Gess-Newsome and Lederman (2001), Rollnick et al (2008) and Jong (2003) were in the area of the sciences, their framework for describing the PCK in science teaching seemed relevant to describing how the participating teachers developed their PCK in statistics teaching. The researches on teaching and learning statistics, mathematics and sciences provide the benchmarks and suggestions about the process that the study has to consider in describing how the participating teachers develop PCK in statistics teaching. PCK is an appropriate theoretical framework for the study as it addresses the key issues: subject matter content knowledge, pedagogical knowledge, knowledge of learners' conceptions and knowledge of learners' learning difficulties, and bridging the gap in PCK development in statistics teaching. The chapter concluded with a detailed description of how the components of PCK used for this study were assessed to determine the individual topic-specific PCK in statistics teaching.