

**HOW COMPETENT MATHEMATICS TEACHERS DEVELOP  
PEDAGOGICAL CONTENT KNOWLEDGE IN STATISTICS  
TEACHING**

**BY**

**SUNDAY BOMBOI IJEH**

Submitted in partial fulfilment of the requirement for the degree

**PhD (MATHEMATICS EDUCATION)**

Department of Science, Mathematics and Technology Education

Faculty of Education

University of Pretoria

Pretoria

**Supervisor: Professor G.O.M. Onwu**

**AUGUST, 2012**

## CERTIFICATION

**This thesis has been examined and approved as meeting the required standard of scholarship for the fulfilment of the Degree of Doctor of Philosophy in Mathematics Education.**

**Prof. G. O. M. Onwu**

.....

**Date.....**

**SUPERVISOR**

**UNIVERSITY OF PRETORIA**

**DECLARATION OF ORIGINALITY**

Full names of student .....SUNDAY BOMBOI IJEH

Students number .....27488111

**Declaration**

1. I understand what plagiarism is and am aware of the university's policy in this regard.
2. I declare that this THESIS is my own original work. Where other peoples' work has been used, this has been properly acknowledged and referenced in accordance with departmental requirements.
3. I have not used work previously produced by another student or any other person to hand in as my own.
4. I have not allowed, and will not allow, anyone to copy my work with the intention of passing it off as his or her own work.

Signature of student.....

Signature of Supervisor.....

## ACKNOWLEDGEMENTS

I would like to express my heartfelt gratitude to the following people for their contribution towards the successful completion of this project.

- The Almighty God for giving me the strength, wisdom and courage to complete this project. Without Him, I would not have made it.
- My supervisor: Professor Gilbert Oke Onwu for his patience, invaluable guidance and encouragement throughout this research project. It was indeed a privilege to have worked with him. His constructive criticism and quick responses led to the successful completion of this project.
- To Professor Irma Eloff, Professor W. M. Braun, Professor K. Dvzimbo, Professor L. D. M. Lebeloane and Dr E. C. Ochonogor for their moral and courageous support.
- To Professor L.C. Jita, Professor D. Mogari, Professor L. Nyaumwe, Dr M. L. Mokhele, Dr A. Motlhabane and Mr M. Phoshoko for their professional advice and mentorship throughout the study.
- To Mr James Matshego for helping to lay the foundation for this study.
- The Gauteng Department of Education, for permission to conduct the research in public schools.
- To my wife and family for encouragement, patience and support.

## TABLE OF CONTENTS

### CHAPTER 1

#### Introduction

1.1	Background to the study	1
1.2	The research problem	5
1.3	Aims of the Study	6
1.4	Statement of the problem	6
1.5	Research questions	6
1.6	Significance of the study	7
1.7	Theoretical framework	8
1.7.1	Subject matter content knowledge	11
1.7.2	Pedagogical knowledge	12
1.7.3	Knowledge of learner's conception in statistics teaching in school mathematics	13
1.7.4	Knowledge of Learners' learning difficulties	14
1.8	Scope of the study	15
1.9	Criteria for selecting the topic	15
1.10	Definition of terms	16
1.11	The chapter structure of the thesis	18
1.12	Summary of the chapter	20

## CHAPTER 2

### 2.0 LITERATURE REVIEW

2.1	Introduction	21
2.2	National Curriculum Statement for mathematics and statistics	21
2.3	Research on teaching of statistics in school mathematics	23
2.4	Assessing Teachers' PCK	25
2.4.1	Description of PCK	25
2.4.2	Teacher knowledge and PCK	29
2.4.3	Pedagogical content knowledge and subject matter for teaching	31
2.4.4	PCK and pedagogical knowledge (Instructional skills and strategies)	38
2.4.5	PCK and knowledge of learners' preconceptions and learning difficulties	42
2.6	Summary of the chapter	44

## CHAPTER 3

### 3.0 RESEARCH METHODOLOGY AND PROCEDURE

3.1	Introduction	46
3.2	Assumption of PCK development during classroom practice	46
3.3	Research design and methods used in this study	46
3.3.1	Research design	46
3.3.2	Research methods	46
3.4	Population and sample description	47
3.4.1	Study population	47
3.4.2	Study sample	48

3.5	Research instruments used for collecting data	49
3.5.1	Development of instrument	49
3.5.1.1	Teacher conceptual knowledge exercise in statistics	49
3.5.1.2	Concept mapping for teachers	52
3.5.1.3	Interview schedule for teachers	53
3.5.1.4	Lesson observation schedule	55
3.5.1.5	The teacher questionnaire	55
3.5.1.6	Teacher written reports	57
3.5.1.7	Document analysis	58
3.5.1.8	Video recording	58
3.6	Validation of the research instrument	59
3.6.1	Validity and reliability of the concept map	59
3.6.2	Validity and reliability of the interview schedule	60
3.7	The Pilot study	61
3.7.1	Subjects used in the pilot study	61
3.7.2	Administration of pilot study	61
3.7.3	Result of the pilot study	62
3.7.3.1	The conceptual knowledge exercise	62
3.7.3.2	The concept mapping	68
3.7.3.3	The lesson observation schedule	68
3.7.3.4	The interview schedule	69
3.7.3.5	The questionnaire for teachers	69
3.7.3.6	The written report guide	70
3.8	The Main study	70
3.8.1	Subjects used in the Main study	70
3.8.2	Administration of the Main study	70

3.9	Data analysis and results of the Main study	71
3.9.1	Quantitative data analysis	71
3.9.2	Qualitative data analysis	71
3.10	Ethical issues	72
3.11	Summary of the chapter	72

## CHAPTER 4

### 4.0 DATA ANALYSIS AND DISCUSSION

4.1	Introduction	73
4.2	Conceptual knowledge exercise	73
4.3	Teacher demographic profile	73
4.4	Concept mapping	74
4.5	Classroom practice (Lesson observation)	74
4.5.1	Lesson observation of teacher A	75
4.5.2	Lesson observation of teacher B	96
4.5.3	Lesson observation of teacher C	110
4.5.4	Lesson observation of teacher D	127
4.6	Video Recording of the lesson observation of the four teachers	141
4.7	Teacher development of PCK	141
4.7.1	Teacher development of subjects matter content knowledge	141
4.7.2	Teacher development of pedagogical knowledge (instructional skill and strategies)	143
4.7.3	Teacher development of knowledge of learner's preconception	146



	and learning difficulties	
4.7.4	Teacher development of PCK in statistics teaching	149
4.8	Summary of the chapter	149

## **CHAPTER 5**

### **5.0 Discussion of results**

5.1	Introduction	151
5.2	Teacher development of PCK	152
5.2.1	Teacher A	152
5.2.2	Teacher B	159
5.2.3	Teacher C	165
5.2.4	Teacher D	170
5.3	Evaluation of theoretical framework	175
5.4	Summary of chapter	179

## **CHAPTER 6**

### **6.0 Summary and recommendations of the study**

6.1	Introduction	180
6.2.	Focus of the study	180
6.3	Summary of the results according to the theoretical framework	182
6.3.1	Subject matter content knowledge	182
6.3.2	Pedagogical knowledge (instructional skills and strategies)	182
6.3.3	Knowledge of learners' preconceptions and learning difficulties	184

6.4	Concluding remarks	185
6.5	Educational implication of the study	186
6.6	Suggestions for further study	187
6.7	Limitations of the Study	188
6.8	The role of the researcher in the non-participatory lesson observation.	189
6.9	Summary of the chapter	190
	<b>BIBLIOGRAPHY</b>	192
	<b>APPENDICES</b>	206

## ABSTRACT

This study is concerned with how competent mathematics teachers develop pedagogical content knowledge (PCK) in statistics teaching. Pedagogical content knowledge was used as the theoretical framework that guided the research and data collection.

The study's methodology consisted of two phases. In the first phase, the six identified mathematics teachers undertook a conceptual knowledge written exercise. The result of this exercise was used to select the best four performing teachers for the second phase of the study. The second phase consisted mainly of lesson observations, interviews, written documents in the form of completed questionnaires, written diaries or reports, document analysis designed to produce rich detailed descriptions of participating teachers' PCK in the context of teaching statistics concepts at school level. The concept mapping exercise was used to indirectly assess participating teachers' content knowledge and their conceptions of the nature of school statistics and how it is to be taught. The qualitative data obtained were analysed to try to determine individual teachers' content knowledge of school statistics, related pedagogical knowledge, knowledge of learners' conceptions in statistics teaching, knowledge of learners' learning difficulties as well as how they developed their PCK in statistics teaching. The analysis was done based on iterative coding and categorisation of responses and observations made to identify themes, patterns, and gaps, in school statistics teaching. Commonalities and differences if any, in the PCK profiles of the four participating teachers were also analysed and determined.

The results of the study showed that overall, individual teachers develop their PCK in school statistics teaching by:

- (a) formally developing their knowledge of the subject matter in a formal undergraduate educational programme, as well as subject matter content knowledge during classroom practice;
- (b) using varied topic-specific instructional skills such as graphical construction skills in teaching statistical graphs;

- (c) using diagnostic techniques (oral questioning and pre-activity, class discussions and questioning) and a review of previous lessons to introduce lessons, and to determine learners' preconceptions in statistics teaching ;
- (d) Using teaching strategies that can help to identify learners' learning difficulties as well as intervention to address the difficulties;
- (e) continually updating their knowledge of school statistics by attending content knowledge workshops and other teacher development programmes designed to improve content knowledge and practice.

**Keywords:** pedagogical content knowledge (PCK), subject matter content knowledge, pedagogical knowledge, instructional strategies, conceptions, learning difficulties, competent teachers, data handling, procedural knowledge, conceptual knowledge.

## LIST OF TABLES

Table 1.1	Learners' performance in mathematics from 1999 to 2004 in the South African Senior Certificate examination.	3
Table 3.1	Schools and teachers that participated in the main study	48
Table 3.2	Mathematics assessment taxonomy and marks allocation	49
Table 3.3	Showing competency and skills and marks allocated	50
Table 3.4	A table showing the list of statistics taught in grades 10, 11 and 12 (if any).	52
Table 3.5	Item specification table for the interview	54
Table 3.6	Item specification table for the questionnaire	56
Table 3.7	Item specification table for the written reports	57
Table 3.8	Item response pattern of the conceptual knowledge exercise from the pilot study test items	64
Table 3.9	Summary of discrimination indices of the test items	66
Table 3.10	Summary of difficulty indices of the test items	67
Table 3.11	Summary of test characteristics	68
Table 4.2	Teacher A, B, C, and D profiles	74
Table 4.5.1	Description of classroom condition and lesson observation of teacher A	76
Table 4.5.1a	A frequency table of members of netball club	79
Table 4.5.1b	A frequency table showing learners' performance in a test	85
Table 4.5.2a	Description of lesson observation and classroom condition at school	97

<b>B</b>		
Table 4.5.2b:	A frequency table showing the performance of learners in a test	98
Table 4.5.2c	A frequency table showing the distribution of the amount spent on buying some groceries from a supermarket	101
Table 4.5.2d	Mark distribution of learners in an English examination	104
Table 4.5.3a	Description of lesson observation and classroom condition in Teacher C's mathematics lesson	111
Table 4.5.3b:	A table showing the ages of cars in a sample of 100 car owners	113
Table 4.5.3c	A frequency table showing the distribution of learners' performance in two tests	123
Table 4.5.4a	Description of lesson observation and classroom condition at school	128
<b>D</b>		
Table 4.5.4bi	A table showing the number of different makes of cars in a car park	128
Table 4.5.4bii	A frequency table showing the mark distribution of learners in a class test	131
Table 4.5.4c	A frequency table showing the masses of players in a 2003 South African rugby squad	135
Table 4.7.1	Teachers' responses to interview about teachers' subject matter content knowledge in statistics teaching	242
Table 4.7.2	Participants' responses to the interview, questionnaire and written reports about teachers' knowledge of instructional skills and strategies for teaching statistics.	245
Table 4.7.3	Participants responses to the questionnaire and written reports on teachers' knowledge of learners' preconceptions and misconceptions in statistics teaching	250
Table 4.7.4	Participants responses to the teachers' interview, questionnaire and written reports about teachers' knowledge of learners' learning difficulties	252
Table 4.9	A comparison of the documents used by participants in statistics teaching	255

## LIST OF FIGURES

Figure 2.1	The components of PCK used in this study	31
Figure 4.5.1a	A histogram showing the age distribution of members of a netball club	81
Figure 4.5.1b	A histogram showing the age distribution of members of a netball club	82
Figure 4.5.1c	An example of an incomplete classwork exercise on a histogram, showing the mark distribution of learners' performance in a test	86
Figure 4.5.1d	A box-and-whisker plot showing the marks obtained in an examination.	89
Figure 4.5.1e	An ogive showing the mark distribution of learners in an English examination	90
Figure 4.5.2a	A bar graph of the scores of learners in a test used to explain how to construct, analyse, and interpret a bar graph using the scores in line	99
Figure 4.5.2b	An ogive representing learner performance in an English examination	105
Figure 3.5.3a	Ogive of age distribution of sample of 100 cars owners park in a car park	114
Figure 4.5.3b	Scatter diagrams showing the relationships between X and Y	121
Figure 4.5.4a	A bar graph showing the number of different makes of cars in a car park	130
Figure 4.5.4b	A histogram showing the distribution of the masses of players in a 2003 South African rugby squad	136

## LIST OF APPENDICES

Appendix I	Consent letter for participating teachers	206
Appendix II	Request for permission to allow your child to participate in a research programme in mathematics	208
Appendix IIIA	Request for permission to conduct research on how competent mathematics teachers develop pedagogical content knowledge in statistics teaching in your schools	209
Appendix IIIB	Approval letter to conduct research in high schools in Gauteng Province	211
Appendix IV	Criteria for validating interview schedule for teacher on how they develop PCK in statistics teaching.	213
Appendix V	Transcription of video records of first lesson observation of teacher A	216
Appendix VI	Transcription of video records of second lesson observation of teacher A	219
Appendix VII	Transcription of video records of first lesson observation of teacher B	221
Appendix VIII	Transcription of video records of second lesson observation of teacher B	223
Appendix IX	Transcription of video records of first lesson observation of teacher C	225
Appendix X	Transcription of video records of second lesson observation of teacher C	228
Appendix XI	Transcription of video records of first lesson observation of teacher D	230
Appendix XII	Transcription of video records of second lesson observation of teacher D	232



APPENDIX XIII	Criteria for validating questionnaire schedule for teacher on how they develop PCK in statistics teaching.	234
Appendix XIV	Criteria for validating written reports schedule for teacher on how they develop PCK in statistics teaching.	236
Appendix XV	Criteria for validating document analysis schedule for teachers on how they develop PCK in statistics teaching.	237
Appendix XVI	Criteria for validating the lesson plan and observation schedule.	240
Appendix XVII	Analysis of participants' responses to interview, questionnaire and teachers' written report	242
Appendix XVIII	Participants' responses to the interview, questionnaire and written reports about teachers' knowledge of instructional skills and strategies for teaching statistics.	245
Appendix XIX	Teachers' knowledge of learners' preconceptions and misconceptions.	250
Appendix XX	Knowledge of learners' learning difficulties	252
Appendix XXI	Comparison of the texts used by participants in statistics teaching	255
Appendix XXII	An exercise in statistics for mathematics teachers	258
Appendix XXIII	Memo for final conceptual knowledge exercise, march 2010	267
Appendix XXIV	Examining the content knowledge of mathematics teachers in statistics teaching	268
Appendix XXV	Rubric for concept mapping exercise	270
Appendix XXVI	The interview schedule for mathematics teachers	273
Appendix XXVII	Report on the teaching of statistics	278
Appendix XXVIII	The questionnaire for mathematics teachers	280
Appendix XXIX	Instrument validation form	284
Appendix XXX	Ethical clearance certificate	286

Appendix XXXI	A sample of teachers' response to concept mapping exercise	286
Appendix XXXII	Lesson observation sheet	288