

Development of a Unified Mass and Heat Integration Framework for Sustainable Design: An Automated Approach

by

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ABSTRACT

Title: Development of a Unified Mass and Heat Integration Framework for Sustainable Design – An Automated Approach

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The successful industrial applications of pinch analysis techniques in energy optimisation and wastewater minimisation have resulted in the recent studies of combined mass and heat integration. Kim and Smith (2001) have demonstrated that operation of cooling water networks in series, rather than the conventional parallel arrangement improve cooling tower and cooling water network performance in new and retrofit design.

In this work, utilising a superstructure to determine the mathematical formulation that characterises a cooling water network supplied by multiple cooling water sources, which often occurs in practice, extends this methodology. It is further demonstrated that the optimum cooling water supply to a network of cooling-water-using operations supplied by multiple sources is determined by considering the entire framework of sources and cooling-water-using operations, that is, unified targeting. This optimum is better than that obtained from considering individual subsets of cooling-water-using operations and its respective source, that is, single source targeting. Relevant practical constraints were included in the formulations to enhance robustness and applicability to real life situations. Practical constraints consisted of maximum return temperatures to cooling water sources, as well as dedicated water sources and sinks of cooling-water-using operations.

This concept was applied to an illustrative example and a case study of the Sasol Synfuels (Pty) Limited cooling water system that consisted of individual networks supplied by separate water sources. For the case with maximum water reuse the single source targeting method yielded an improvement of 11.6% over the parallel target for the illustrative example. In comparison, superior results were obtained with the developed unified targeting method, which yielded an improvement of 18.4%. Likewise, for the case with the aforementioned practical constraints 6.8% and 7.6% improvements were forecasted for the single source and unified targeting methods respectively.

For the maximum reuse scenario of the case study, improvements of 37.9% and 41.0% over the parallel target were obtained using the single source and unified targeting methods, respectively. Similarly, considering practical constraints improvements of 20.3% and 31.1% were obtained. In both the illustrative example and case study the unified targeting method resulted in superior results than the single source targeting methods.

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Chapter 1. Introduction	1
1.1. Background.....	1
1.2. Thesis Structure.....	3
1.3. Objective.....	3
1.4. References.....	4
Chapter 2. Literature Review	6
2.1. Wastewater Minimisation.....	7
2.1.1. <i>Insight-based Pinch Analysis Techniques</i>	8
2.1.2. <i>Mathematical Optimisation-Based Techniques</i>	9
2.2. Cooling Water Network Synthesis.....	11
2.2.1. <i>Previous Work on Cooling Tower Analysis</i>	11
2.2.2. <i>Cooling Water System Design (Kim and Smith, 2001)</i>	11
2.2.3. <i>Automated Retrofit Design of Cooling Water Systems (Kim and Smith, 2003).</i>	15
2.3. Further Developments of Cooling Water Network Design and Mass Integration.....	16
2.4. Global Optimisation of Bilinear Process Networks with Multi-Component Flows (Quesada and Grossmann, 1995).....	18
2.5. References.....	20
Chapter 3. Methodology Development.....	23
3.1. Assumptions.....	23
3.2. Mathematical Formulation.....	24
3.2.1. <i>Nomenclature</i>	24
3.2.2. <i>Case 1 – Maximum Reuse</i>	25
3.2.2.1. <i>Linearisation</i>	28
3.2.3. <i>Case 2 – No Temperature Restriction with a Single Source and Sink</i>	29
3.2.4. <i>Case 3 – Maximum Return Temperature with Multiple Sources and Sinks</i> ..	31
3.2.4.1. <i>Reformulation and Linearisation</i>	32
3.2.5. <i>Case 4 - Maximum Return Temperature with a Single Source and Sink</i>	36
3.3. Mathematical Solver - GAMS	37

3.4.	References.....	37
Chapter 4. Application of the Mathemaical Model.....		38
4.1.	Illustrative Example – Single Source Targeting	38
4.1.1.	<i>Case 1 – Maximum Reuse.....</i>	38
4.1.2.	<i>Case 3 – Temperature Restriction with Multiple Sources and Sinks</i>	39
4.2.	Illustrative Example – Multiple Source Targeting.....	40
4.2.1.	<i>Targeting Without Cooling Water Return Temperature Limitation</i>	43
4.2.1.1.	<i>Case 1 and Case 2: Single Source Targeting</i>	43
4.2.1.2.	<i>Case 1: Unified Targeting - Maximum Reuse.....</i>	44
4.2.1.3.	<i>Case 2: Unified Targeting – Single Source and Sink.....</i>	45
4.2.2.	<i>Targeting with Cooling Water Return Temperature Limitation</i>	47
4.2.2.1.	<i>Case 3 and 4: Single Source Targeting.....</i>	47
4.2.2.2.	<i>Case 3: Unified Targeting – Multiple Sources and Sinks</i>	48
4.2.2.3.	<i>Case 4: Unified Targeting – Single Source and Sink.....</i>	49
4.3.	Case Study	51
4.3.1.	<i>Base Case Cooling Water Network.....</i>	52
4.3.2.	<i>Optimisation of the Base Case by reuse of Cooling Water</i>	53
4.3.2.1.	<i>Targeting and Cooling Water Network Design without Cooling Water Return Temperature Limitation - Case 1</i>	54
4.3.2.2.	<i>Targeting and Cooling Water Network Design with Cooling Water Return Temperature Limitation – Case 3.....</i>	57
4.4.	References.....	60
Chapter 5. Conclusion.....		61
5.1.	Referencs	62
Appendix A		63
A.1.	Illustrative Example – Single Source Targeting	63
A.1.1.	<i>Maximum reuse: Case 1 – GAMS Input File.....</i>	63
A.1.2.	<i>Temperature Restriction: Case 3 - GAMS Input File.....</i>	65
A.2.	Illustrative Example – Multiple Sources Targeting	69
A.2.1.	<i>Case 1 – GAMS Input File.....</i>	69
A.2.1.1.	<i>Case 1 – Single source targeting – Sub-problem A.....</i>	69

A.2.1.2.	<i>Case 1 – Single source targeting – Sub-problem B</i>	71
A.2.1.3.	<i>Case 1 – Single source targeting – Sub-problem C</i>	73
A.2.1.4.	<i>Case 1 – Unified Targeting</i>	75
A.2.2.	<i>Case 2 – GAMS Input File</i>	78
A.2.2.1.	<i>Case 2 – Single source targeting – Sub-problem A</i>	78
A.2.2.2.	<i>Case 2 – Single source targeting – Sub-problem B</i>	78
A.2.2.3.	<i>Case 2 – Single source targeting – Sub-problem C</i>	78
A.2.2.4.	<i>Case 2 – Unified Targeting</i>	78
A.2.3.	<i>Case 3 – GAMS Input File</i>	82
A.2.3.1.	<i>Case 3 – Single source targeting – Sub-problem A</i>	82
A.2.3.2.	<i>Case 3 – Single source targeting – Sub-problem B</i>	85
A.2.3.3.	<i>Case 3 – Single source targeting – Sub-problem C</i>	88
A.2.3.4.	<i>Case 3 – Unified Targeting</i>	91
A.2.4.	<i>Case 4 – GAMS Input File</i>	95
A.2.4.1.	<i>Case 4 – Single source targeting – Sub-problem A</i>	95
A.2.4.2.	<i>Case 4 – Single source targeting – Sub-problem B</i>	95
A.2.4.3.	<i>Case 4 – Single source targeting – Sub-problem C</i>	95
A.2.4.4.	<i>Case 4 – Unified Targeting</i>	95
A.3.	<i>Case Study</i>	100
A.3.1.	<i>Case 1 – GAMS Input File</i>	100
A.3.1.1.	<i>Case 1 – Single source targeting – Sub-problem A</i>	100
A.3.1.2.	<i>Case 1 – Single source targeting – Sub-problem B</i>	103
A.3.1.3.	<i>Case 1 – Unified Targeting</i>	105
A.3.2.	<i>Case 3 – GAMS Input File</i>	108
A.3.2.1.	<i>Case 3 – Single source targeting – Sub-problem A</i>	108
A.3.2.2.	<i>Case 3 – Single source targeting – Sub-problem B</i>	112
A.3.2.3.	<i>Case 3 – Unified Targeting</i>	115

LIST OF FIGURES

Figure 2.1: Developments leading to the present study	6
Figure 2.2: Cooling water composite and cooling water supply line	13
Figure 2.3: Cooling water mains method for cooling water network design.....	14
Figure 2.4: Cooling water network with maximum reuse	14
Figure 3.1: Superstructure of cooling-water-using operations supplied by multiple cooling water sources.....	23
Figure 3.2: Reformulation and linearisation procedure	32
Figure 4.1: Cooling water network for Case 1	39
Figure 4.2: Cooling water network for Case 3	40
Figure 4.3: Cooling water network under parallel arrangement	42
Figure 4.4: Cooling water networks for Cases 1 and 2 - single source targeting.....	43
Figure 4.5: Unified network for Case 1	44
Figure 4.6: Unified network for Case 2	46
Figure 4.7: Cooling water networks for Case 3 and 4 - single source targeting	47
Figure 4.8: Unified network for Case 3	48
Figure 4.9: Unified network for Case 4	50
Figure 4.10: Base case parallel cooling water network of case study	52
Figure 4.11: Cooling water network by single source targeting method for Case 1	55
Figure 4.12: Unified network for Case 1	56
Figure 4.13: Single source targeting for Case 3	58
Figure 4.14: Unified Targeting for Case 3	59

LIST OF TABLES

Table 2.1: Hot Process Stream Data (Example 1).....	12
Table 2.2: Limiting cooling water data (Example 1).....	12
Table 4.1: Limiting cooling water data (Example 1).....	38
Table 4.2: Cooling water supply information (Example 2)	41
Table 4.3: Hot process stream data for the cooling water network (Example 2).....	40
Table 4.4: Limiting cooling water data (Example 2).....	42
Table 4.5: Comparison of single source and unified targets for Case 1	45
Table 4.6: Model Statistics for Case 1	45
Table 4.7: Comparison of single source and unified targets for Case 2	45
Table 4.8: Model Statistics for Case 2	46
Table 4.9: Comparison of single source and unified targets for Case 3	49
Table 4.10: Model statistics for relaxed model, MR and exact model M* for Case 3 ..	49
Table 4.11: Comparison of single source and unified targets for Case 4	49
Table 4.12: Model Statistics for Case 4	50
Table 4.13: Hot process data of cooling-water-using operations for case study.....	51
Table 4.14: Limiting cooling water data for case study	53
Table 4.15: Comparison of single source and unified targets for Case 1	54
Table 4.16: Model Statistics for Case 1	57
Table 4.17: Model Statistics for relaxed model, MR and exact model, M* for Case 3	60

CHAPTER 1. INTRODUCTION

1.1. Background

There are inherently large quantities of energy and water used in chemical processes. Since the late seventies, significant efforts have been put in developing systematic procedures for energy optimisation in grassroot and retrofit design. This resulted in the development of Pinch Technology, which has been successfully applied to various industries worldwide. Pinch Technology was followed by the development of similar techniques for water minimisation (Wang and Smith, 1994; Alva-Argáez *et al.*, 1998; Savelski and Bagajewicz, 2000; Hallale, 2002) in chemical processing industries. Whilst this technology has not fully developed, various industries, globally, have started reaping the benefits of its application. The main drawback of the previous developments is that mass and heat integration problems are treated in a dichotomous manner.

Also, significant work has been done on cooling water systems relating to the reliability of cooling towers (Milford, 1986; Sudret *et al.*, 2005), the optimum sizing of cooling towers (Soylemez, 2001), energy conservation (Knoche and Bošnjaković, 1998), cooling water treatment (Martinez *et al.*, 2004) and other operational issues of cooling water systems. These developments do not consider the impact of the cooling water network on the performance of the cooling tower. Bernier (1994) investigated the impact on heat removal in cooling towers relative to cooling water return temperature and flowrate, from a cooling water network of constant duty. It was emphasized that reduction in cooling water to the cooling tower, which is concomitant with increased return temperature, results in improved cooling tower performance.

It is only recently that combined mass and heat integration have been studied. Kim and Smith (2001), following the observations by Bernier (1994), have developed a conceptual graphical technique for maximisation of cooling tower performance through minimisation of supply water to the cooling water network. The latter is supplied by only one cooling tower and includes all cooling-water-using operations including heat exchangers. This

methodology demonstrated that cooling water minimisation could be achieved by the reuse of cooling water between different cooling-water-using operations, because not all cooling-water-using operations require cooling water at the supply temperature.

Practical observations of cooling water networks in process industries are the motivation of the present study. It has been observed that it is common that there are several cooling water networks supplied by separate sources. This work is an extension of the aforementioned work, by considering multiple sources supplying a unified cooling water network with typical practical constraints.

A mathematical optimisation technique is presented that simultaneously determines the target and structure of the resulting cooling water network. Contrary to graphical counterparts, mathematical methods are readily adaptable to handling constraints of practical relevance, including various performance indices. A typical practical constraint could be that individual cooling-water-using operations are supplied by a dedicated source, implying that there is no pre-mixing of cooling water supply. Similarly, post mixing of return water to sources could be prohibited. Four cases were explored investigating practical constraints.

The benefits of the proposal is realised by a reduction in cooling water makeup costs and increased cooling tower availability for retrofit cases. In grassroots plants, the benefit is realised by reduced capital expenditure for cooling tower capacity. These benefits are achieved by considering only the mass and energy balance of the cooling tower network. Hence, the objective is to reduce the water consumption of the cooling water network. Reduced water flowrate will result in improved heat removal of the cooling tower and reduced makeup water requirements.

It is demonstrated that the approach of targeting cooling water systems within a unified framework yields more optimal results than optimisation of the individual cooling water system with its dedicated cooling water source and cooling-water-using operations.

1.2. Thesis Structure

The structure of the thesis is as follows:

- The relevant cited literature is discussed in Chapter 2
- The developed mathematical formulations for the simultaneous targeting and design of cooling water systems is explained in Chapter 3
- Chapter 4 presents illustrative examples and a case study to show the applicability of the developed scientific method
- Final conclusions and recommendations are discussed in Chapter 5

1.3. Objective

The objective of this study is to:

- create a mathematical model that results in the same target as the graphical method and results in a feasible network design
- manage cooling water systems with multiple cooling water sources
- consider practical scenarios that arise in cooling water systems

The application of the proposed mathematical formulation can be expressed with the following problem statement. Given,

- a set of cooling water sources with different supply temperatures and design capacities, and
- a set of cooling-water-using operations with limiting temperature and duty requirements,

determine the minimum cooling water requirement that fulfils the cooling duty of the entire network. Further the cooling water requirement of the individual cooling water sources is required without compromising its performance.

Bernier (1994) showed that the cooling tower performance is improved with reduced inlet flowrate, which is concomitant with increased inlet temperature to the cooling tower. Following these observations, a performance indicator for the cooling tower can be defined, as the quotient of its thermal difference and inlet flowrate. An increase in this

performance indicator will result in improved cooling tower performance. Thus, the minimum cooling water is required without decreasing the performance indicator of individual cooling towers of the base case cooling water system.

It is assumed that minimum cooling water flow requirements to the network result in minimum cooling water makeup. In this investigation the following four cases, that reflect potential practical constraints, are considered:

Case 1: Unspecified cooling water return temperature to individual cooling water sources, without a dedicated source and sink per cooling-water-using operation;

Case 2: Unspecified cooling water return temperature to individual cooling water sources, with a dedicated source and sink per cooling-water-using operation;

Case 3: Specified maximum cooling water return temperature to individual cooling water sources, without a dedicated source and sink per cooling-water-using operation;

Case 4: Specified maximum cooling water return temperature to individual cooling water sources, with a dedicated source and sink per cooling-water-using operation.

Other practical scenarios such as pressure drop constraints, capital and operating cost targets can also be incorporated into the model.

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CHAPTER 2. LITERATURE REVIEW

The key developments in the field of mass and heat integration techniques leading to the present study are shown in Figure 2.1.

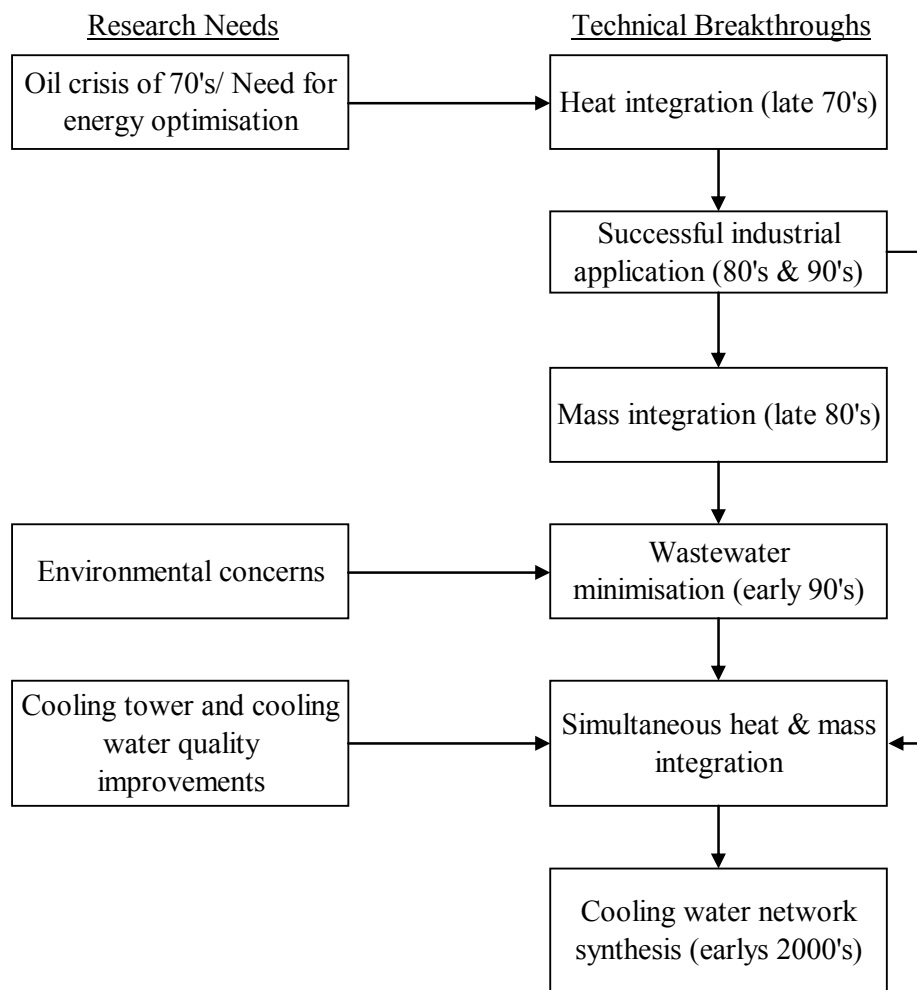


Figure 2.1: Developments leading to the present study

The well-known oil crisis of the seventies resulted in a concerted focus on improved energy utilisation in chemical plants. The primary outcome was Pinch Technology, which is a conceptually based method to reduce heating and cooling utility consumption by optimally exchanging heat between processes. This successful application together with increasing environmental concerns resulted in the development of novel wastewater

minimisation techniques in the early nineties. These developments resulted in the improved approach of simultaneous heat and mass integration to optimise cooling water networks. The seminal work in this field is presented by Kim and Smith (2001), with the use of pinch analysis technique. The present study stems from the aforementioned work. The cited literature pertaining to the present study is discussed below.

2.1. Wastewater Minimisation

Fresh water is used extensively in chemical plants in water-using operations such as desalters, stripper columns, liquid-liquid extraction, washing and sluicing operations. The spent effluent water is treated in water-treatment operations such as gravity-settlers, hydrocyclones, centrifuges, filters and membranes before disposal. In recent years, more stringent environmental restrictions have resulted in increased cost of wastewater treatment. Reduction in fresh water consumption and wastewater discharge is of paramount importance to reduce utility costs and capital expenditure of water-treatment operations and to comply with the changing environmental regulations.

In an effort to fulfil these industrial challenges, reuse, regeneration-reuse and regeneration-recycle of wastewater in water-using operations and water treatment units, is exploited to minimise wastewater (Wang and Smith, 1994). Wastewater minimisation was developed with insights from pinch analysis (Linnhoff and Hindmarsh, 1983).

There are two approaches with regards to the synthesis of water networks to minimise wastewater, that is, graphically based conceptual methods and mathematical programming techniques. Graphical methods have benefits of allowing the engineer to assess the problem during network synthesis. However, solutions are obtained significantly quicker with mathematical methods, which can also manage common practical constraints, such as dedicated cooling water sources. A further shortcoming of graphical methods over mathematical counterparts is the limitations associated with large networks. The following contributions are the key developments in this regard during the past two decades.

2.1.1. *Insight-based Pinch Analysis Techniques*

Wang and Smith (1994a) is the pioneering work in the field of graphically based conceptual methods to minimise wastewater in chemical plants. A methodology, which is based on constant pollutant load extracted in each process to optimally pick reuse options, is proposed. Maximum water inlet and outlet concentrations in individual processes, which are dictated from solubility, flowrate limitations, corrosion and fouling, characterise the methodology. Fresh water consumption is targeted and the corresponding network consisting of water-using operations is designed.

The method entails plotting the mass loads versus concentration limits for individual processes. A limiting water profile considering all processes is then plotted. The fresh water supply line is a straight line drawn from zero concentration and intersects the composite curve at one or possibly more points, referred to as pinch points. The length of the supply line satisfies the cumulative mass load. Minimising the number of intersections with the composite curve, such that the supply line just touches the composite curve at the pinch point(s), realises an optimal target. The inverse of the slope of the supply line yields the minimum fresh water consumption. The targeting procedure ensures that the outlet concentration of fresh water to the network is maximised hence ensuring that its flowrate is minimised.

A preliminary network is obtained by using a water grid diagram (Wang and Smith, 1994a). Two design techniques are proposed to achieve the target. The first maximises concentration driving forces in individual processes taking full advantage of the concentration difference between the composite curve and the limiting water supply line. The second minimises the number of water sources for individual processes, as an increased number of water sources to a mass transfer operation reduces its operability.

Later work by Savelski and Bagajewicz (1999a,b, 2000a) showed that a necessary condition for optimality for water allocation requires all processes to operate at limiting outlet concentrations. Degenerate solutions with lower outlet concentrations can occur which achieve the same target. However, these degenerate solutions have higher water

flowrates through individual processes and are not preferred. Hence, the method that minimises the number of water sources proposed by Wang and Smith (1994a) will result in degenerate solutions and the method that maximises concentration driving forces will result in true optimum networks.

Olesen and Polley (1997) used the water pinch method to obtain a target and designed the network by inspection. However, the approach is limited to a maximum of five operations.

Kuo and Smith (1998) proposed a simplified graphical procedure for network design. The method consisted of identifying pockets that can be created by successfully bending the water supply line upwards with the composite curve. Next, water ‘mains’ are created at the end of each pocket and processes are identified that are fed by these mains. Since some processes exist in different regions separated by these mains, merging between mains needs to occur. However, limitations also exist with this method. When many processes exist below the pinch that have maximum inlet concentrations larger than zero, difficulties arise in identifying which process needs to be supplied with fresh water first and reuse options.

Wang and Smith (1994a) investigated both single and multiple contaminants in the water-using operations. The approach used allows individual process constraints relating to minimum mass transfer driving forces, fouling, corrosion limitations, etc. to be easily incorporated.

2.1.2. Mathematical Optimisation-Based Techniques

Takama *et al.* (1980) was the pioneering work regarding mathematical optimisation of water allocation in chemical plants. During the seventies, studies on wastewater minimisation were exclusive to water-treating processes. A method to design an integrated network consisting of both water-using operations and wastewater treatment processes by exploiting reuse options was presented. This ensures a reduction of both fresh water consumption and wastewater generation. The objective of the developed

method was to minimise the total cost subject to constraints derived from material balances and inter-relationships amongst water-using operations and water-treatment processes.

Since the work by Takama *et al.* (1980) there were no publications on the mathematical formulation of water allocation in networks of water-using or water-treatment operations for many years. Doyle and Smith (1997) and Alva-Argáez *et al.* (1998a,b) presented mixed-integer-non-linear-programming (MINLP) and non-linear-programming (NLP) models, which exhibited suboptimal solutions with convergence problems.

However, Savelski and Bagajewicz (2000) showed that the mathematical model for the minimum water usage for the extraction of single contaminants in a network of water-using operations could be linearised. The model is set-up within specified maximum inlet and outlet water concentrations. A generic superstructure is used to characterise the network, from which material balance equations across individual processes are described. The objective function is to minimise the sum of fresh water to individual processes. This formulation is similar to the formulation developed in the present study for cooling water system design. Concentration and mass load are analogous to temperature and heat duty, respectively.

It is proven via contradiction that a network with all processes operating at maximum outlet water concentration will result in optimal water allocation. Degenerate solutions with lower outlet concentrations can occur and achieve the same target. As aforementioned, these degenerate solutions have water flowrates through individual processes that are higher and are not preferred. Setting all processes at maximum outlet concentrations renders the formulation linear, which results in a global optimum solution. The NLP formulation for the present study, shown in section 4.2.1 has a similar structure. Hence, setting all cooling-water-using operations at limiting outlet temperatures results in a linear formulation.

Savelski and Bagajewicz (2000) also showed that if a solution to the water allocation problem is optimum, then at every process, the outlet concentrations are higher than the

concentration of the combined wastewater stream coming from all precursors. This reduces the number of variables in the model.

2.2. Cooling Water Network Synthesis

2.2.1. Previous Work on Cooling Tower Analysis

A substantial amount of work has been done on cooling towers and cooling water quality, which were independent of the impacts of the cooling water network. These contributions included studies on the reliability of cooling towers (Milford, 1986; Sudret et al., 2005), the optimum sizing of cooling towers (Soylemez, 2001), energy conservation (Knoche and Bošnjaković, 1998), cooling water treatment (Martinez et al., 2004) and other operational issues of cooling water systems.

2.2.2. Cooling Water System Design (Kim and Smith, 2001)

It is only recently that combined mass and heat integration have been studied. Kim and Smith (2001) focussed on the optimum design of recirculating cooling water networks. Traditional design of such networks has a parallel arrangement that is supplied with cooling water from a cooling tower. In a parallel arrangement the fresh cooling water is used in individual cooling-water-using operations and then returned to the cooling tower. The total cooling water demand to the network is achieved by minimising the flowrate through individual cooling-water-using operations, by operating at limiting outlet temperatures for individual operations.

However, not all the cooling-water-using operations require water at the supply temperature from the cooling water source in order to achieve the desired cooling of the process stream. Thus, it is possible to reuse water between different cooling units and operate a cooling-water-using operation at a higher inlet temperature. Reuse water from other cooling-water-using operations in the network can achieve some or all of the required cooling duty of a particular cooling-water-using operation. A cooling-water-using operation will require less fresh water than in the parallel arrangement or possibly

even no fresh water if it utilises only reuse water. Kim and Smith (2001) developed a methodology based on pinch analysis techniques to determine the minimum cooling water demand that maximises cooling water reuse.

To illustrate the targeting methodology a simple problem (Example 1) presented by Kim and Smith (2001) is discussed as follows. The cooling water network in Example 1 consists of four heat exchangers using cooling water to cool the hot process streams. The supply ($T_{hot, in}$) and return ($T_{hot, out}$) temperature, heat capacity flowrate (CP) and cooling duty (Q) of the individual heat exchangers are given in Table 2.1. Note that the CP values are the product of the heat capacity and cooling water flowrate.

Table 2.1: Hot process stream data (Example 1)

Heat exchanger	$T_{hot, in}$ (°C)	$T_{hot, out}$ (°C)	CP (kW/°C)	Q (kW)
1	50	30	20	400
2	50	40	100	1000
3	85	40	40	1800
4	85	65	10	200

Table 2.2 gives the limiting inlet and outlet cooling water temperatures for this example. The maximum inlet and outlet temperatures are limited by the minimum temperature difference between the hot and cold streams, corrosion, fouling, cooling water treatment, etc. In Example 1, a minimum temperature difference of 10°C is assumed for all streams.

Table 2.2: Limiting cooling water data (Example 1)

Heat exchanger	$T_{in, lim}$ (°C)	$T_{out, lim}$ (°C)	CP (kW/°C)	Q (kW)
1	20	40	20	400
2	30	40	100	1000
3	30	75	40	1800
4	55	75	10	200

The target is achieved in the follows. A cooling water composite curve based on the individual limiting cooling water profiles is represented on the temperature-enthalpy diagram. The limiting cooling water profile represents the maximum inlet and outlet conditions of the cooling water stream for each cooling-water-using operation. The cooling water supply line is thereafter constructed, as shown in Figure 2.2. The cooling water supply line is drawn from the supply temperature of the cooling tower. Its slope is increased until it touches the cooling water composite curve, creating a pinch point. The length of the cooling water supply line must ensure that the cooling networks duty is fulfilled. The point at which the cooling water supply line ends represents the return temperature to the cooling tower.

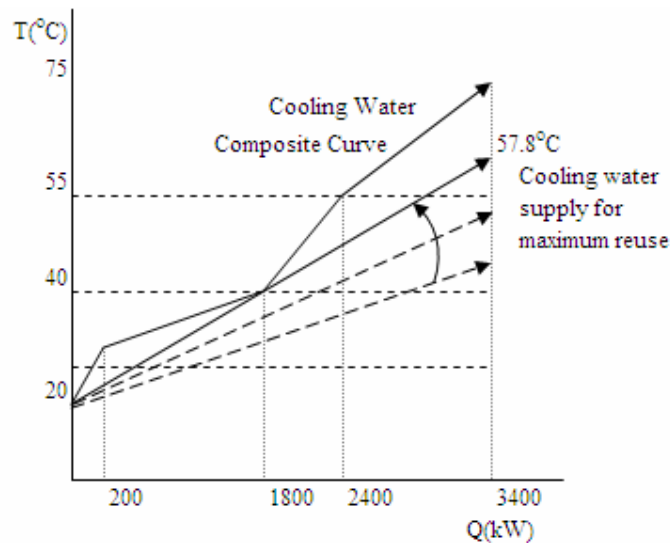


Figure 2.2: Cooling water composite and cooling water supply line

The cooling water network that satisfies the target is determined by the adaptation of the water main method of Kuo and Smith (1998a) which was designed for water reuse systems that was constrained by concentration limits as discussed in section 2.1.1. Figure 2.3a shows the applicability of this approach to cooling water networks. The composite curve is divided at the pinch point to separate the concave regions, which creates two design regions. The slope of a line drawn across each design region determines the cooling water flowrate. The cooling water mains are setup at the different temperatures, i.e., the cooling water supply temperature, the pinch temperature and exit temperature.

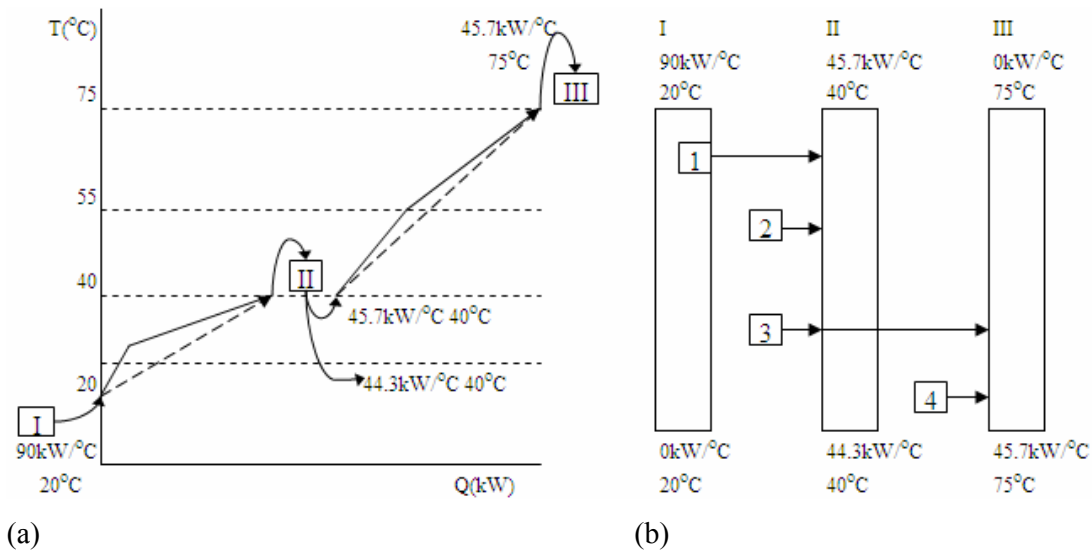


Figure 2.3: Cooling water mains method for cooling water network design (Kim and Smith, 2001)

The method to obtain the network is in four steps. Firstly, generate a grid diagram with cooling water mains and plot the cooling-water-using operations as shown in Figure 2.3b. Secondly, connect the operations with cooling water mains. The third stage is to merge operations that cross mains. Lastly, remove intermediate (pinch) cooling water mains. A suitable network that can be extracted from the cooling water mains method is shown in Figure 2.4. In principle there can be other networks that also meet the design target. Moreover, there could be more than one intermediate mains.

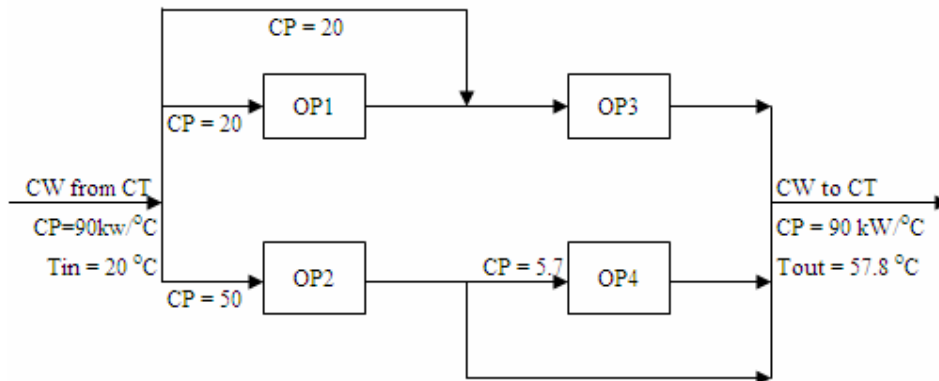


Figure 2.4: Cooling water network with maximum reuse

The procedure developed thus far is based on cooling water minimisation with maximum reuse. However it is common in industry that cooling water sources have a maximum return temperature from the network due to fouling, corrosion or problems with the cooling tower packing. The graphical method is based on the cooling water supply line creating a pinch point with the cooling water composite curve. The cooling water supply line that meets the temperature requirement will not create a pinch point and will lie below the cooling water supply line for maximum reuse and above that with no reuse. Therefore, the water mains method cannot be applied.

The supply line with temperature specification is in a feasible region, therefore, the cooling water composite curve is modified to create a pinch point with the new supply line. The cooling water composite curve is moved downwards along the temperature axis to create a new pinch point. The individual duties that partake in creating the original point need to undergo a temperature shift to shift the composite curve to the new pinch point. As modified profiles may cross over the supply line the flowrate is increased to touch the supply line. The cooling water network is thereafter developed.

It is important to note that lower limiting inlet and outlet temperatures of cooling-water-using operations will result in degenerate solutions with increased flowrates through individual units. The conceptual insights from Kim and Smith (2001) have set the foundation for the developed mathematical programming.

2.2.3. Automated Retrofit Design of Cooling Water Systems (Kim and Smith, 2003)

The series arrangement of cooling water networks for debottlenecking purposes developed by Kim and Smith (2001) results in a pressure drop increase across the network because of cooling water reuse. Graphical methods are limited to a two-dimensional plane and cannot simultaneously consider other objectives, such as, pressure drop and practical system constraints. A mathematical optimisation model that considers pressure drop constraints, complexity of networks, and efficient use of the cooling tower, is developed. The methodology focuses on the retrofit design and is aimed at exploring possibilities for

the most efficient use of existing cooling towers to prevent investment in another cooling tower or supplementary cooling capacity (such as air coolers).

The developed MINLP mathematical model minimises pressure drop across the cooling water network. Pressure drop of individual components of the network, namely the heat exchanger and piping network were characterised in terms of flowrate. A new node-superstructure model is suggested to consider the intensive property of pressure and the sequence of connections simultaneously. The cooling water flowrate and return temperature are determined by the conceptual graphical method. These are used as inputs into the mathematical model. Binary variables are used to determine the choice of fresh water and reuse water flow to individual operations.

MINLP models typically result in multiple local optima and can cause convergence failures. Observations of design profile characteristics in coolers led to the development of a simple but robust solution strategy. The formulation is linearised to a mixed-integer-linear-programming (MILP) model by using maximum outlet temperatures for individual coolers to obtain a starting point for the MINLP model.

This work highlighted the benefits of mathematical methods to deal with practical system constraints. Further, insight into the solution strategy of MINLP models, which is used in the present study, is presented.

2.3. Further Developments of Cooling Water Network Design and Mass Integration

Increased restrictions on temperature of effluent streams before discharge necessitates that streams with a higher temperature than the permitted level need to pass through a cooling system, which is typically a cooling tower. Inappropriate mixing of effluent streams with different temperatures reduces opportunities to recover heat and degrades driving forces in cooling systems.

Kim *et al.* (2001) developed a systematic method to deal with effluent temperature problems by a combination of heat recovery and effluent cooling. The duty of the cooling

tower is inherently reduced and leads to savings of capital and operating costs. This leads to distributed effluent cooling systems. The targets are set before design and an optimisation model has been developed to search for the most economic cooling system design. This paper has analysed a retrofit case study for effluent temperature reduction while considering systematically interactions between reuse, energy recovery and effluent treatment systems. The retrofit analysis procedure is, as follows:

- (i) Identification of water reuse for wastewater minimisation by firstly, distributing the reuse water from the hottest source. Next, connect the reuse water source with the nearest heat sink in terms of temperature. Lastly, introducing non-isothermal mixing points if the temperatures of the sink operations are intermediate to the temperature of the source.
- (ii) Heat integration for energy recovery
- (iii) Optimisation of the distributed cooling systems to solve the thermal pollution problem.

Combining simultaneous water and energy minimisation with distributed cooling systems leads to reduction of cooling costs and efficient use of water and energy resources. Kim and Smith, (2004) developed new design methods to reduce cooling tower makeup, which is achieved by water recovery between wastewater generating processes and cooling systems. It is proposed that wastewater before or after treatment is used as cooling tower makeup. This can cause the cooling tower to become bottlenecked because of the higher temperatures than normal makeup water. Thus cooling water reuse design (Kim and Smith, 2001) can be combined with wastewater recovery. As reuse design of cooling systems allows the cooling tower to take wastewater as makeup, substituting makeup with wastewater can yield water savings, as well as reductions in wastewater discharge without overloading the cooling tower.

A new systematic methodology has been developed (Savulescu *et al.*, 2005) for the simultaneous management of energy and water systems. A two-dimensional grid diagram is proposed to exploit different options within water systems and also enable reduced complexity of the energy and water network. Isothermal and non-isothermal stream

mixing between water streams are introduced to create systems between hot and cold water streams in the energy composite curves and provide a design basis for a better structure with fewer units for the heat exchanger network.

2.4. Global Optimisation of Bilinear Process Networks with Multi-Component Flows (Quesada and Grossmann, 1995)

In order to avoid the direct use of nonlinear models whose bilinear terms are nonconvex and concomitant with major mathematical difficulties, the reformulation and linearisation technique for bilinear programming models proposed by Sherali and Alameddine (1992), is applied. Firstly a relaxed linear solution is obtained, which provides an initial guess for the original NLP formulation.

The reformulation and linearisation technique is as follows. Given a bilinear term xy , linear bounding constraints can be derived to approximate a solution to the original equation. The linear bounding constraints are derived based on the lower and upper bounds of the continuous variables x and y , which are defined as follows.

$$x^L \leq x \leq x^U \quad (1)$$

$$y^L \leq y \leq y^U \quad (2)$$

These inequalities can be expressed as independent equations given below.

$$\gamma_1 = x - x^L \geq 0 \quad (3)$$

$$\gamma_2 = x^U - x \geq 0 \quad (4)$$

$$\gamma_3 = y - y^L \geq 0 \quad (5)$$

$$\gamma_4 = y^U - y \geq 0 \quad (6)$$

The above equations can be multiplied with each other appropriately to yield applicable constraints that contain the original term xy . For example, the non-linear constraint

$$\gamma_1 \times \gamma_3 \geq 0 \quad (7)$$

is valid when each of the constraints, γ_1 and γ_3 , are fulfilled. Substituting constraints (3) and (5) into (7) results in

$$xy \geq xy^L + x^L y - x^L y^L \quad (8).$$

Similarly, defining the non linear constraints

$$\gamma_1 \times \gamma_4 \geq 0 \quad (9)$$

$$\gamma_2 \times \gamma_3 \geq 0 \quad (10)$$

$$\gamma_2 \times \gamma_4 \geq 0 \quad (11)$$

leads to additional linear bounding constraints

$$xy \leq xy^U + x^L y - x^L y^U \quad (12)$$

$$xy \leq xy^L + x^U y - x^U y^L \quad (13)$$

$$xy \geq xy^U + x^U y - x^U y^U \quad (14)$$

for the bilinear term xy . Constraints (8), (12), (13) and (14) are the linear bounding constraints, which describe the feasible region for the term xy . A more optimal solution is obtained if the difference between the relaxed LP solution and the exact NLP solution is smaller. If these solutions are the same then a globally optimum solution is obtained.

2.5. References

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CHAPTER 3. METHODOLOGY DEVELOPMENT

A superstructure of cooling-water-using operations supplied with cooling water from many cooling water sources can be described generically, as shown in Figure 3.1. The superstructure allows for mathematical optimisation formulations to be described by writing mass and energy balances across processes. Cases 1 to 4 can be formulated using the same superstructure.

The objective function for all cases is to minimise the total cooling water flow supplied from all sources, without violating design capacities of individual cooling water sources. The solution to all cases is obtained subject to the constraints described below. The complete mathematical formulation is given in section 3.1 underneath.

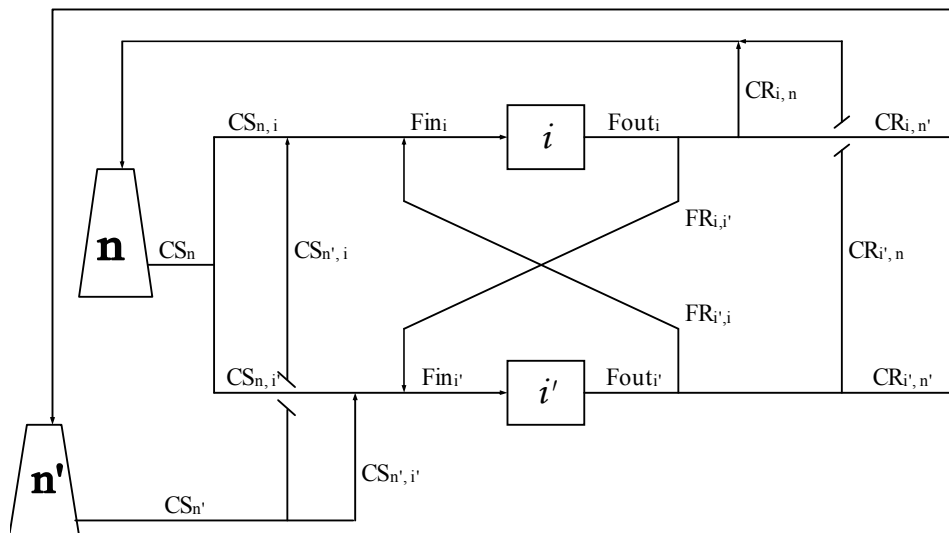


Figure 3.1: Superstructure of cooling-water-using operations supplied by multiple cooling water sources

3.1. Assumptions

- It is assumed that minimum cooling water supply to the network, and hence maximum cooling tower performance, result in minimum cooling water makeup and increased

cooling tower availability. This represents minimum operating and capital costs. The latter is not exclusive to grassroots projects only. For the retrofit case, increased cooling tower availability reduces future capital expenditure in new cooling towers. Hence, the objective function of the mathematical formulation will be to minimise cooling water demand.

- Heat capacity is assumed to be constant in the mathematical formulation. This represents negligible inaccuracies within the temperature range of the supply and return temperatures.

3.2. Mathematical Formulation

3.2.1. Nomenclature

Sets

I = Cooling-water-using operations that complies with a mass and energy balance of a counter current heat exchanger

N = Cooling water sources supplying the cooling water network

Parameters

Q_i = Duty of cooling-water-using operation i , (kW)

T_n = Cooling water supply temperature from cooling water source n , (°C)

CS_n^{\max} = Design capacity of cooling water source n , (t/h)

c_p = Specific heat capacity of water, assumed to be constant at 4187 J/kg.°C

$T_{out,i}^{\lim}$ = Limiting outlet temperature from cooling-water-using operation i , (°C)

$T_{in,i}^{\lim}$ = Limiting inlet temperature to cooling-water-using operation i , (°C)

Fin_i^{\max} = Maximum flowrate through cooling-water-using operation i , (t/h)

$Tret_n^{\max}$ = Maximum return temperature to cooling water source n , (°C)

Continuous variables

CW = Total cooling water flow supplied from all cooling water sources, (t/h)

CS_n = Total cooling water flow supplied from cooling water source n , (t/h)

$CS_{n,i}$ = Cooling water flow supplied from cooling water source n to cooling-water-using operation i , (t/h)

$CR_{i,n}$ = Return cooling water flow from cooling-water-using operation i to cooling water source n , (t/h)

$FR_{i',i}$ = Reused cooling water flow from any other cooling-water-using operation i' to cooling-water-using operation i , (t/h)

Fin_i = Total cooling water flow into cooling-water-using operation i including supply and reused water, (t/h)

$Fout_i$ = Total cooling water flow from cooling-water-using operation i including return and reused water, (t/h)

Tin_i = Inlet cooling water temperature to cooling-water-using operation i , (°C)

$Tout_i$ = Outlet cooling water temperature from cooling-water-using operation i , (°C)

Binary Variables

$$ys_{n,i} = \left\{ \begin{array}{l} 1 \leftarrow \text{if there exists a direct cooling water stream from cooling water source} \\ \quad n \text{ to operation } i \\ 0 \leftarrow \text{otherwise} \end{array} \right\}$$

$$yr_{n,i} = \left\{ \begin{array}{l} 1 \leftarrow \text{if there exists a return cooling water stream from operation } i \text{ direct to} \\ \quad \text{cooling water source } n \\ 0 \leftarrow \text{otherwise} \end{array} \right\}$$

3.2.2. Case 1 – Maximum Reuse

The total cooling water supply is the sum of the cooling water from each cooling water source n , which is shown in constraint (15).

$$CW = \sum_n CS_n \quad \forall n \in N \quad (15)$$

Constraints (16) and (17) ensure that the inlet and outlet cooling water flow of individual cooling water sources are equal.

$$CS_n = \sum_i CS_{n,i} \quad \forall i \in I, n \in N \quad (16)$$

$$CS_n = \sum_i CR_{i,n} \quad \forall i \in I, n \in N \quad (17)$$

The total water to cooling-water-using operation i is the sum of cooling water from all cooling water sources and reused water from all other operations i' , as given in constraint (18).

$$Fin_i = \sum_n CS_{n,i} + \sum_{i'} FR_{i',i} \quad \forall i, i' \in I, n \in N \quad (18)$$

The total water from cooling-water-using operation i consists of return water from that operation to each cooling water source n and reused water to all other operations i' , as given in constraint (19).

$$Fout_i = \sum_n CR_{i,n} + \sum_{i'} FR_{i,i'} \quad \forall i, i' \in I, n \in N \quad (19)$$

Constraint (20) gives the energy balance across cooling-water-using operation i .

$$Q_i = Fin_i c_p (Tout_i - Tin_i) \quad \forall i \in I \quad (20)$$

Constraint (20) is the definition of the inlet temperature into cooling-water-using operation i , assuming a constant specific heat capacity.

$$Tin_i = \frac{\sum_n CS_{n,i} T_n + \sum_{i'} FR_{i',i} Tout_{i'}}{Fin_i} \quad \forall i, i' \in I, n \in N \quad (21)$$

Since water is conserved across individual cooling-water-using operations, constraint (22) is necessary.

$$Fin_i = Fout_i \quad \forall i \in I \quad (22)$$

The following equation ensures that the maximum operating capacity of cooling water source, n is not violated.

$$CS_n \leq CS_n^{\max} \quad \forall n \in N \quad (23)$$

The inlet and outlet temperatures from a particular cooling-water-using operation cannot exceed its respective limiting inlet and outlet temperatures. Therefore, constraints (24) and (25) are required.

$$Tout_i \leq T_{out,i}^{\lim} \quad \forall i \in I \quad (24)$$

$$Tin_i \leq T_{in,i}^{\lim} \quad \forall i \in I \quad (25)$$

Given the parameters, declared in section 4.1, the objective value, CW can be minimized subject to constraints (15 – 25). The variables, which have also been declared in section 4.1, are consequently determined and provide the information to determine the structure of the cooling water network.

Case 1 is a NLP formulation, due to the bilinear terms in constraints (20) and (21), which are naturally non-convex. Hence, a global optimum solution cannot be guaranteed, as local minimum optima could also be obtained. The formulation can be linearized to cast it as a linear programming (LP) problem to determine a global optimum objective.

3.2.2.1. Linearisation

Savelski and Bagajewicz (2000) have examined a problem of a similar structure to that exhibited by constraints (15) – (25), albeit in water utilisation systems for wastewater minimisation. This is discussed in Section 2.1.2. It was demonstrated that a water network in which individual operations operate at maximum outlet concentration will always require minimum water and postulated this as the necessary condition for optimality. Similarly, it can be demonstrated in this case that a cooling water network in which individual operations have maximum outlet temperature will require minimum water flowrate from the supply.

Using the necessary conditions of optimality as aforementioned allows constraints (20) and (21) to be linearized to yield an overall linear problem, which ensures global optimality. This is achieved by substituting constraint (21) and (22) into (20) to eliminate $T_{in,i}$ and replacing the continuous variable $T_{out,i}$ with the parameter $T_{out,i}^{lim}$. Thus, constraints (20) and (21) are replaced by constraint (26). The elimination of the variable $T_{in,i}$ reduces the number of continuous variables and hence the computation time.

$$\frac{Q_i}{c_p} + \sum_n CS_{n,i} T_n + \sum_{i'} FR_{i',i} T_{out,i'}^{lim} = F_{out,i} T_{out,i}^{lim} \quad \forall i, i' \in I, n \in N \quad (26)$$

Subsequently, constraint (24) is not required. As the above constraints are independent of $T_{in,i}$ constraint (25) can be expressed in terms of the water flowrate to cooling-water-using operation i , $F_{in,i}$.

$$F_{in,i} \leq F_{in,i}^{max} \quad \forall i \in I \quad (27)$$

where,

$$F_{in,i}^{max} = \frac{Q_i}{c_p (T_{out,i}^{lim} - T_{in,i}^{lim})} \quad \forall i \in I \quad (28)$$

The total cooling water flow to the network is minimized subject to constraints (15 – 19, 22, 23 and 26 – 28), which forms an overall LP model. This formulation ensures the sequential use of all available supply water from the source with the greatest thermal driving force to that with the lowest. This formulation can readily be extended to practically constrained situations characterized by cases 2, 3 and 4.

3.2.3. *Case 2 – No Temperature Restriction with a Single Source and Sink*

The traditional arrangement in chemical plants is that individual cooling-water-using operations have one dedicated source and sink, that is, pre-mixing of cooling water supply from different cooling water sources and post splitting of cooling water return to different cooling water sources is prohibited. Typical process reasons for such operation are: differences in cooling water supply quality, minimisation of piping required, more stable start-up operations, and plant geographic constraints, e.g. a particular cooling-water-using operation could be much closer to the cooling tower.

This scenario explores such a case, in which individual cooling-water-using operations have at most one dedicated source and sink. It is possible that a cooling-water-using operation is supplied by only reused water or all its water is reused in another operation. If a cooling-water-using operation receives and delivers cooling water to a cooling tower, then a common source and sink is required.

As with Case 1, there is no limit on cooling water return temperatures to individual cooling water sources. The objective function remains the same, that is, the total cooling water supply from all sources is to be minimized. In addition to the constraints used in Case 1, constraints to prevent pre-mixing of cooling water supply, prevent post splitting of cooling water return and ensure common sources and sinks, are required.

To prevent pre-mixing of cooling water supply from many cooling water sources a binary variable $y_{s_{n,i}}$ is introduced, which operates as a decision variable. If there is cooling water supply from cooling water source n to cooling-water-using operation i then the variable $y_{s_{n,i}}$ is one, else it is zero.

This specification is achieved twofold with constraints (29) and (30). Firstly, the cooling water supply from a source to a specific cooling-water-using operation should be less than the product of $ys_{i,n}$ and a suitable upper bound for that streams flow. As the flow to a cooling-water-using operation from a source cannot exceed that source's design capacity, the design capacity was used as an upper bound.

Secondly, the sum of all the binary variables of a particular cooling-water-using operation for each cooling water source should be zero or one. This ensures that a cooling-water-using operation can only receive water from one cooling water source, but it is not a requirement that a cooling-water-using operation uses fresh water. This enables an operation to be designed with only reused water, if required.

$$CS_{n,i} \leq CS_n^u ys_{n,i} \quad \forall i \in I, n \in N \quad (29)$$

$$\sum_n ys_{n,i} \leq 1 \quad \forall i \in I, n \in N \quad (30)$$

Similarly, to prevent post splitting of cooling water return to more than one source constraints (31) and (32) are required, with the use of the binary variable $yr_{i,n}$. Constraints (33) and (34) ensure that the same source and sink is used, if a cooling-water-using operation receives and delivers cooling water to a cooling tower. This is achieved by setting up constraints (33) and (34) such that they are valid for all conditions of possible supply and return from a particular cooling tower.

$$CR_{i,n} \leq CS_n^u yr_{i,n} \quad \forall i \in I, n \in N \quad (31)$$

$$\sum_n yr_{i,n} \leq 1 \quad \forall i \in I, n \in N \quad (32)$$

$$yr_{i,n} \leq ys_{n,i} + (2 - ys_{n,i} - yr_{i,n}) \quad \forall i \in I, n \in N \quad (33)$$

$$yr_{i,n} \geq ys_{n,i} - (2 - ys_{n,i} - yr_{i,n}) \quad \forall i \in I, n \in N \quad (34)$$

The total cooling water flow to the network is minimized subject to constraints (15 – 19, 22, 23 and 26 – 34). This formulation results in a MILP model, for which global optimality can be ensured.

3.2.4. Case 3 – Maximum Return Temperature with Multiple Sources and Sinks

Maximum return temperature limitations and undedicated sources and sinks characterise Case 3. The objective function remains as in the previous cases, that is, the total cooling water supply from all the cooling water sources is to be minimized. In addition to the constraints required in Case 1, constraint (35) is required to specify the maximum return temperature limitation to a cooling water source.

$$\frac{\sum_i CR_{i,n} Tout_i}{CS_n} \leq Tret_n^{\max} \quad \forall i \in I, n \in N \quad (35)$$

The flowrate through individual sources is allowed to vary subject to the above specification. Therefore, the outlet temperature of a cooling-water-using operation is no longer a parameter, as in the LP formulation of Case 1, but a variable. Subsequently, the linearisation of constraint (26) is no longer valid and is replaced by,

$$\frac{Q_i}{c_p} + \sum_n CS_{n,i} T_n + \sum_{i'} FR_{i',i} Tout_{i'} = Fout_i Tout_i \quad \forall i, i' \in I, n \in N \quad (36)$$

Further, the upper bound for the outlet temperature from cooling-water-using operation i , once again needs to be included as an inequality in the formulation to ensure a feasible solution. Hence constraint (24) used in the NLP formulation of Case 1 is required.

$$Tout_i \leq T_{out,i}^{\lim} \quad \forall i \in I \quad (24)$$

The minimum cooling water flow for Case 3 can be determined subject to constraints (15 – 19, 22 – 24, 27, 28, 35, 36), which is referred to as model M^* . This formulation exhibits a nonconvex NLP structure due to the bilinear terms $CR_{i,n}Tout_i$, $FR_{i,i}Tout_i$ and $Fout_iTout_i$ contained in constraints (35) and (36) respectively. This NLP model can converge to local optima, hence global optimality cannot be guaranteed. This structure is concomitant with major mathematical difficulties largely due to the possibility of an infeasible starting point.

3.2.4.1. Reformulation and Linearisation

To ensure convergence of the mathematical solver and that a more optimal solution is obtained, the formulation should be solved in two steps. Firstly a relaxed linear solution is obtained (Figure 3.2), which provides an initial guess for the original NLP formulation, M^* . A more optimal solution is obtained if the difference between the relaxed LP solution and the exact NLP solution is smaller. If these solutions are the same then a globally optimum solution is obtained. The relaxed LP solution is obtained by defining linear bounding constraints for the bilinear terms of constraints (35) and (36) using the method described in Quesada and Grossmann (1995), which is shown in some in Section 2.4.

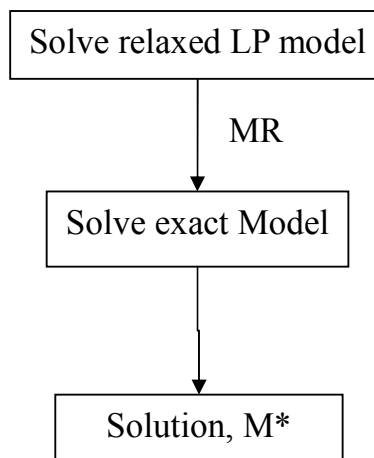


Figure 3.2: Reformulation and linearisation procedure

Constraints (35) and (36) become,

$$\frac{\sum_i \gamma_{1,i,n}}{CS_n} \leq Trel_n^{\max} \quad \forall i \in I, n \in N \quad (37)$$

$$\frac{Q_i}{c_p} + \sum_n CS_{n,i} T_n + \sum_{i'} \gamma_{2,i',i} = \gamma_{3,i} \quad \forall i, i' \in I, n \in N \quad (38)$$

where,

$$\gamma_{1,i,n} = CR_{i,n} Tout_i \quad \forall i \in I, n \in N \quad (37a)$$

$$\gamma_{2,i',i} = FR_{i',i} Tout_{i'} \quad \forall i, i' \in I \quad (38a)$$

$$\gamma_{3,i} = Fin_i Tout_i \quad \forall i \in I \quad (38b)$$

The linear bounding constraints for the bilinear terms represented in equation (37a, 38a, and 38b) are:

$$\gamma_{1,i,n} \geq CR_{i,n}^L Tout_i + CR_{i,n} Tout_i^L - CR_{i,n}^L Tout_i^L \quad \forall i \in I, n \in N \quad (39)$$

$$\gamma_{1,i,n} \geq CR_{i,n}^U Tout_i + CR_{i,n} Tout_i^U - CR_{i,n}^U Tout_i^U \quad \forall i \in I, n \in N \quad (40)$$

$$\gamma_{1,i,n} \leq CR_{i,n}^U Tout_i + CR_{i,n} Tout_i^L - CR_{i,n}^U Tout_i^L \quad \forall i \in I, n \in N \quad (41)$$

$$\gamma_{1,i,n} \leq CR_{i,n}^L Tout_i + CR_{i,n} Tout_i^U - CR_{i,n}^L Tout_i^U \quad \forall i \in I, n \in N \quad (42)$$

$$\gamma_{2,i',i} \geq FR_{i',i}^L Tout_{i'} + FR_{i',i} Tout_{i'}^L - FR_{i',i}^L Tout_{i'}^L \quad \forall i, i' \in I \quad (43)$$

$$\gamma_{2,i',i} \geq FR_{i',i}^U Tout_i + FR_{i',i} Tout_{i'}^U - FR_{i',i}^U Tout_{i'}^U \quad \forall i, i' \in I \quad (44)$$

$$\gamma_{2,i',i} \leq FR_{i',i}^U Tout_i + FR_{i',i} Tout_{i'}^L - FR_{i',i}^U Tout_{i'}^L \quad \forall i, i' \in I \quad (45)$$

$$\gamma_{2,i',i} \leq FR_{i',i}^L Tout_i + FR_{i',i} Tout_{i'}^U - FR_{i',i}^L Tout_{i'}^U \quad \forall i, i' \in I \quad (46)$$

$$\gamma_{3,i} \geq Fin_i^L Tout_i + Fin_i Tout_i^L - Fin_i^L Tout_i^L \quad \forall i \in I \quad (47)$$

$$\gamma_{3,i} \geq Fin_i^U Tout_i + Fin_i Tout_i^U - Fin_i^U Tout_i^U \quad \forall i \in I \quad (48)$$

$$\gamma_{3,i} \leq Fin_i^U Tout_i + Fin_i Tout_i^L - Fin_i^U Tout_i^L \quad \forall i \in I \quad (49)$$

$$\gamma_{3,i} \leq Fin_i^L Tout_i + Fin_i Tout_i^U - Fin_i^L Tout_i^U \quad \forall i \in I \quad (50)$$

These linear bounding constraints are simplified by substituting appropriate values for the upper and lower bounds. The upper bound for $Tout_i$ which appears in all the bilinear terms of constraints (37) and (38) has been defined in constraint (24). The lower bound for this variable is zero. The bounds for $CR_{i,n}$ which appears in the bilinear term of constraint (37) are,

$$0 \leq CR_{i,n} \leq Fin_i^{\max} \quad \forall i \in I, n \in N \quad (51)$$

The bounds for $FR_{i',i}$ which appears in the first bilinear term of constraint (38) are,

$$0 \leq FR_{i',i} \leq Fin_i^{\max} \quad \forall i, i' \in I \quad (52)$$

The upper bound for Fin_i which appears in the second bilinear term of constraint (38), is given in constraint (27). The lower bound is zero.

After the substitution of these bounds, the linear bounding constraints defined in constraints (39 – 50) become,

$$\gamma_{1,i,n} \geq Fin_i^{\max} Tout_i + CR_{i,n} Tout_i^{\lim} - Fin_i^{\max} Tout_i^{\lim} \quad \forall i \in I, n \in N \quad (53)$$

$$\gamma_{1,i,n} \leq Fin_i^{\max} Tout_i \quad \forall i \in I, n \in N \quad (54)$$

$$\gamma_{1,i,n} \leq CR_{i,n} Tout_i^{\lim} \quad \forall i \in I, n \in N \quad (55)$$

$$\gamma_{2,i,i'} \geq Fin_i^{\max} Tout_{i'} + FR_{i',i} Tout_{i'}^{\lim} - Fin_i^{\max} Tout_{i'}^{\lim} \quad \forall i, i' \in I \quad (56)$$

$$\gamma_{2,i,i'} \leq Fin_i^{\max} Tout_{i'} \quad \forall i, i' \in I \quad (57)$$

$$\gamma_{2,i,i'} \leq FR_{i',i} Tout_{i'}^{\lim} \quad \forall i, i' \in I \quad (58)$$

$$\gamma_{3,i} \geq Fin_i^{\max} Tout_i + Fin_i Tout_i^{\lim} - Fin_i^{\max} Tout_i^{\lim} \quad \forall i' \in I \quad (59)$$

$$\gamma_{3,i} \leq Fin_i^{\max} Tout_i \quad \forall i \in I \quad (60)$$

$$\gamma_{3,i} \leq Fin_i Tout_i^{\lim} \quad \forall i \in I \quad (61)$$

The reformulation technique requires two models to be solved, which supplement each other. The relaxed LP model, referred to as model MR consists of constraints (15 – 19, 22, 23, 28, 37, 38, and 53 – 61) and is a linear relaxation of the original model M*, as it does not contain any bilinear terms. The bilinear terms are replaced by linearisation variables

γ_1 , γ_2 and γ_3 , and suitable linear bounding constraints for the linearisation variables. The linear bounding constraints for these bilinear terms are valid as shown by the method described in section 2.5. Constraints (24) and (27) are included in the bounds of the linear bounding constraints and are therefore not required.

The LP solution of model MR supplies an initial guess to the original NLP model M*, which consists of constraints (15 – 19, 22 - 24, 27, 28, 35, 36). This initial guess assists the convergence of the NLP formulation. The solution obtained by the NLP model M* is not a global optimum solution. A global optimum solution is obtained only if the solution of model MR and M* are the same.

3.2.5. Case 4 - Maximum Return Temperature with a Single Source and Sink

In this case, the return temperature to individual cooling water sources is specified, with a dedicated source and sink per cooling-water-using operation. The objective function remains as in the previous cases, that is, the total cooling water supply from all the cooling water sources is minimized. In addition to the constraints used in Case 3, constraints to prevent pre-mixing of cooling water supply and post splitting of cooling water return, that were described in the formulation of Case 2, are required.

Thus, the objective of Case 4 is obtained subject to constraints (15 – 19, 22 – 24 and 27 – 36), which is called model M*. This formulation results in a mixed-integer-non-linear programming problem (MINLP), which is concomitant with major mathematical difficulties, largely due to the possibility of an infeasible starting point and convergence to local optima. The objective is determined by the reformulation linearisation technique, which requires two models to be solved. The relaxed linear model MR, which is a mixed-integer-linear-programming (MILP) problem, consists of constraints (15 – 19, 22, 23, 28 – 34, 37, 38, and 53 – 61). The solution of model MR supplies an initial guess to the exact model M*. Constraints (35) and (36) is reformulated and linearized by the method described in the formulation of Case 3.

3.3. Mathematical Solver - GAMS

In this work, the optimisation software General Algebraic Mathematical Software (GAMS) was used to determine the target and variables that constitute the network. All calculations were performed in a 3.0GHz Pentium 4 processor with 1GB RAM. GAMS/CPLEX solver was used for LP and MILP models. For NLP and MINLP models, GAMS/CONOPT and GAMS/DICOPT solvers were used respectively. In using DICOPT, CPLEX and CONOPT solvers were respectively used for MILP and NLP sub-problems.

3.4. References

- Majozi, T. and A. Moodley, 2007, Simultaneous targeting and design for cooling water systems with multiple cooling water supplies. *Computers and Chemical Engineering*, (In press)
- Quesada, I. and I.E. Grossmann, 1995, Global optimisation of bilinear process networks with multicomponent flows. *Computers and Chemical Engineering*, 19: 1219 – 1242

CHAPTER 4. APPLICATION OF THE MATHEMATICAL MODEL

4.1. Illustrative Example – Single Source Targeting

The developed mathematical formulation, for the simultaneous targeting and design of cooling water networks, is accurately validated with the example presented by Kim and Smith (2001), which used the graphical water mains method. The results of the graphical approach are discussed in Section 2.2.2. Table 4.1 shows the limiting cooling water data for this example. The cooling water network is supplied with cooling water at 20°C from a single cooling water source. Hence, in this example only case 1 and 3 are applicable.

Table 4.1: Limiting cooling water data (Example 1)

Operation	$T_{in,i}^{lim}$ (°C)	$T_{out,i}^{lim}$ (°C)	CP(kW/°C)	Q_i (kW)
1	20	40	20	400
2	30	40	100	1000
3	30	75	40	1800
4	55	75	10	200

4.1.1. Case 1 – Maximum Reuse

The cooling water network with maximum permissible water reuse (Case 1), which was obtained with the mathematical formulation, is shown in Figure 4.1. The formulation that was developed for single source targeting without return temperature restrictions as described in Case 1, Chapter 3, was used to obtain the target and network. This method yielded the same target (90 kW/°C or 77.4 t/hr) and cooling water return temperature (57.78°C), as that obtained by the graphical technique presented by Kim and Smith (2001). The target was obtained in 0.125 CPU seconds. The LP mathematical model, for which global optimality is guaranteed, consists of 25 linear constraints and 33 continuous variables.

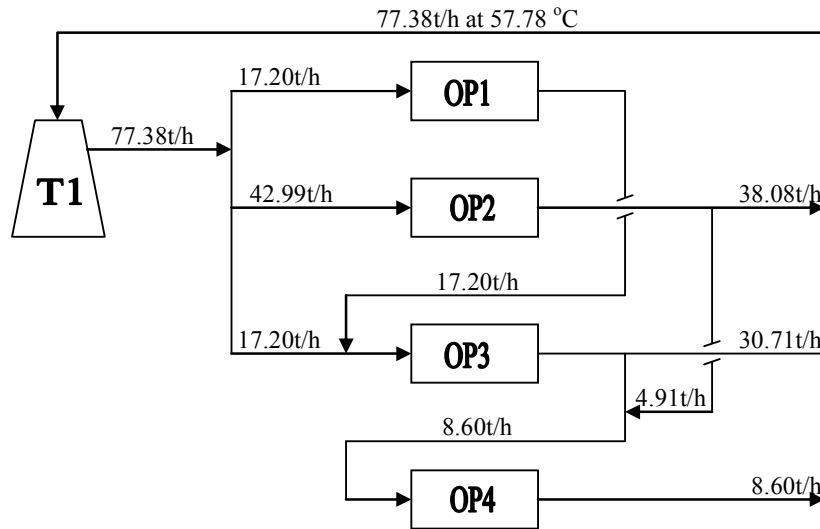


Figure 4.1: Cooling water network for Case 1

4.1.2. Case 3 – Temperature Restriction with Multiple Sources and Sinks

The algebraic formulation with a return temperature specification for single source targeting (Case 3) was also validated with the work presented by Kim and Smith (2001). The algebraic method was successfully applied to obtain a cooling water network with a return temperature restriction of 55 °C. As the supply and return temperature of the cooling tower and the duty of the network are specified, the target is subsequently pre-defined, as shown below.

$$CW = \frac{\sum Q_i}{c_p (T_{ret}^{max} - T_n)} \quad \forall i \in I \quad (62)$$

Equation (62) is valid provided that the maximum return temperature is less than the return temperature for Case 1 and yields a target of 97.14 kW/°C or 83.5 t/hr. This target is also obtained with developed mathematical formulation in 0.110 CPU seconds. A suitable cooling network design is obtained from the formulation, which is presented in Figure 4.2. The NLP mathematical model, for which global optimality is not guaranteed, consists of 30 constraints and 37 continuous variables. The relaxed LP solution, which is used as a starting point for the exact model is 75.38 kW/°C or 64.8 t/h.

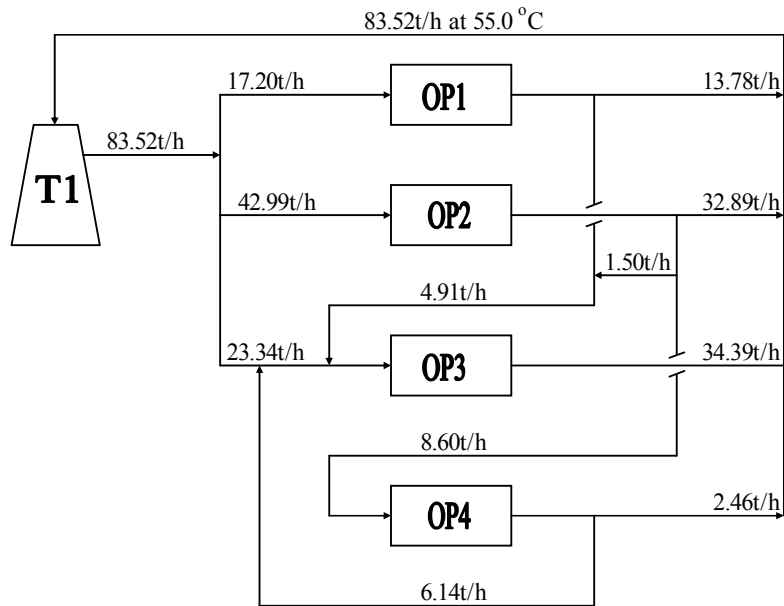


Figure 4.2: Cooling water network for Case 3

4.2. Illustrative Example – Multiple Source Targeting

A simple problem (Example 2) will be used to illustrate the conceptual design techniques for cooling water systems consisting of multiple sources. It will be shown that unified targeting of cooling water networks supplied by multiple sources yields better results than consideration of individual sources supplying their subset of cooling-water-using operations. Table 4.2 gives the cooling water supply information. Source T1 supplies operations OP1 and OP2, source T2 supplies OP3 and OP4, and source T3 supplies OP5 and OP6. The hot process stream data for the cooling-water-using operations is given in Table 4.3, containing the duty, flowrate and inlet and outlet temperatures.

Table 4.2: Cooling water supply information (Example 2)

Source	T_n (°C)	CS_n^{\max} (t/h)	$Tret_n^{\max}$ (°C)
T1	20.0	30.0	52.0
T2	22.0	40.0	52.0
T3	25.0	40.0	50.0

Table 4.3: Hot process stream data for the cooling water network (Example 2)

Heat Exchanger	Q_i (kW)	$T_{in, hot}$ ($^{\circ}$ C)	$T_{out, hot}$ ($^{\circ}$ C)	CP (kW/ $^{\circ}$ C)
OP1	610	40	55	40.7
OP2	420	50	70	21.0
OP3	800	35	60	32.0
OP4	555	55	63	69.4
OP5	345	50	65	23.0
OP6	700	40	55	46.7

Traditional targeting procedures design cooling water systems in parallel arrangement, in which the return temperature to the cooling water source is not maximised, but the thermal difference across individual cooling-water-using operations is maximised. This is achieved by minimising the flow of cooling water through individual cooling-water-using operations by designing at or near to the limiting cooling water outlet temperature, without exceeding the maximum return temperature to the cooling water source.

Under parallel configuration source T1 has a load of 30.0 t/h with a return temperature of 49.5 $^{\circ}$ C, source T2 has a load of 40.0 t/h with a return temperature of 51.2 $^{\circ}$ C and source T3 has a load of 40.0 t/h with a return temperature of 47.5 $^{\circ}$ C. The cooling water networks under parallel arrangement are used as the base case for evaluation (Figure 4.3). The performance indicators, which are defined in section 1.3, of the cooling water sources are 0.98 $^{\circ}$ C/t/hr, 0.73 $^{\circ}$ C/t/hr and 0.56 $^{\circ}$ C/t/hr for source T1, T2 and T3, respectively.

The graphical technique developed by Kim and Smith (2001) has only been applied to cooling water networks supplied by a single source. Hence, by using the graphical technique the minimum cooling water demand of the whole network can be calculated as the sum of the minimum cooling water demand of individual sources supplying its subset of cooling-water-using operations. The limiting cooling water inlet and outlet temperatures (Table 4.4) are required to obtain the minimum cooling water flowrate that exploits cooling water reuse between cooling-water-using operations. In practice, the limiting data is determined by the operational conditions of individual cooling-water-using operations. In this example a ΔT_{min} of 10 $^{\circ}$ C was assumed.

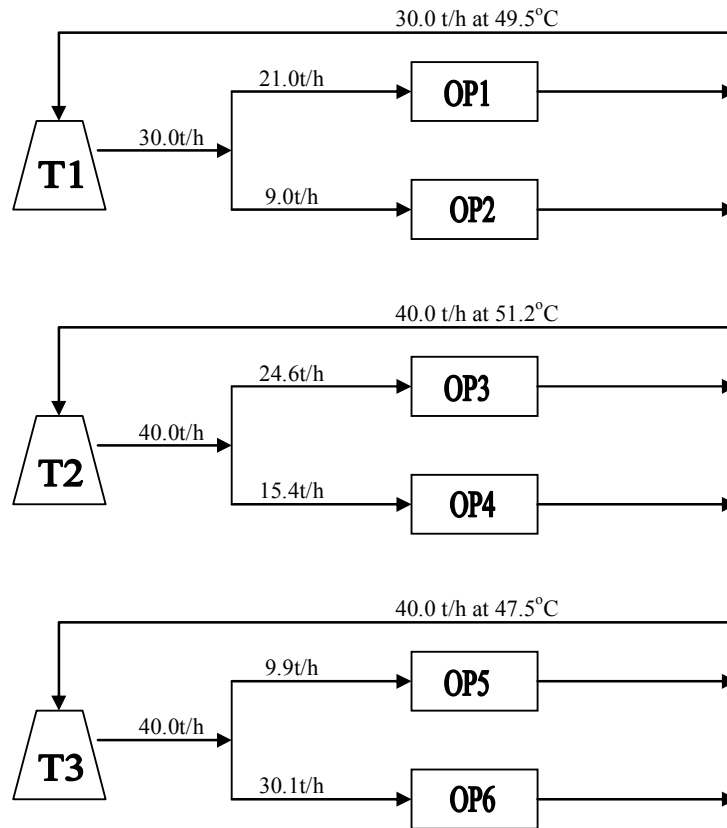


Figure 4.3: Cooling water network under parallel arrangement

Table 4.4: Limiting cooling water data (Example 2)

Heat Exchanger	$T_{in,i}^{lim}$ (°C)	$T_{out,i}^{lim}$ (°C)	CP (kW/°C)	Q_i (kW)
OP1	30	45	40.7	610
OP2	40	60	21.0	420
OP3	25	50	32.0	800
OP4	45	53	69.4	555
OP5	40	55	23.0	345
OP6	30	45	46.7	700

The optimal cooling water demand and resulting network were obtained by the developed mathematical formulation, by considering the entire system as a whole, that is, unified targeting. The unified target was obtained for Cases 1 - 4 and compared with the target

that would have been obtained if the graphical technique was applied, that is, single source targeting. The single source target was also obtained by the developed mathematical formulation, as it results in the same target as using the graphical method (as shown in section 4.1). The results for each case are presented graphically and discussed below.

4.2.1. Targeting Without Cooling Water Return Temperature Limitation

4.2.1.1. Case 1 and Case 2: Single Source Targeting

The resultant cooling water networks using the single source targeting procedure (Figure 4.4) has a combined target of 97.2 t/h.

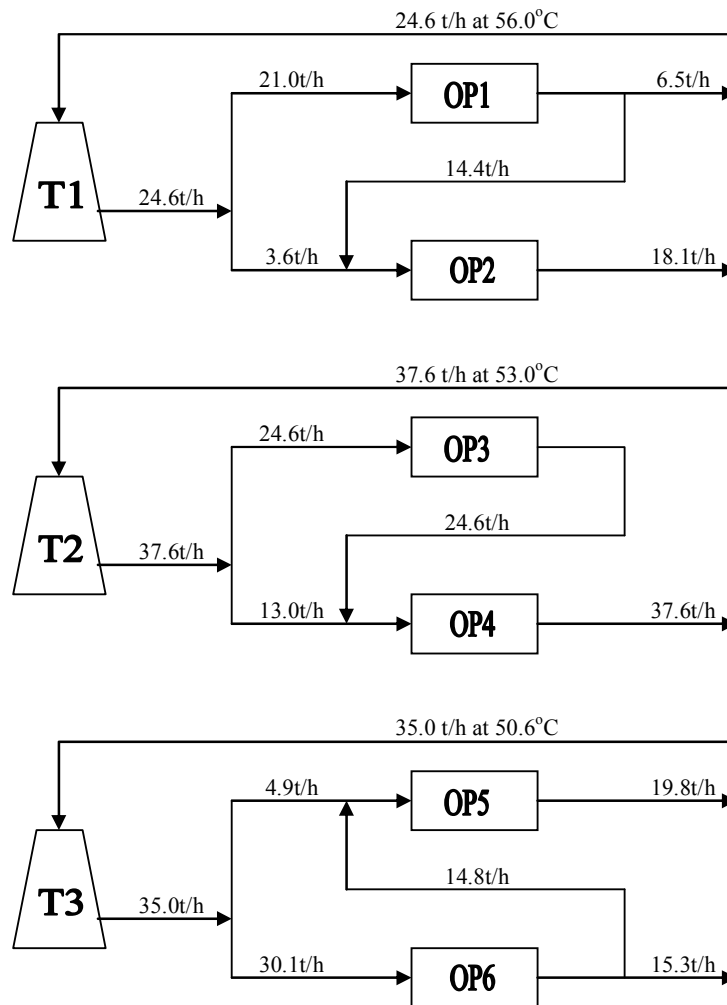


Figure 4.4: Cooling water networks for Cases 1 and 2 - single source targeting

4.2.1.2. *Case 1: Unified Targeting - Maximum Reuse*

Case 1 is characterised by no restrictions on return temperature to the cooling water source and no supply and return flow restrictions from sources. The unified targeting approach for Case 1 yields a target of 89.8 t/h, which is 7.7% less than the single source target, as shown in Figure 4.5 and Table 4.5. Further, the target is 18.4% less than the target required using a parallel arrangement of the network (110 t/h). The single source and unified targeting methods for Case 1 are both LP problems. Both methods indicate an improvement of the performance indicators of individual cooling water sources compared to that of the parallel network. The model statistics displaying the number of continuous variables, number of constraints and CPU time are shown in Table 4.6.

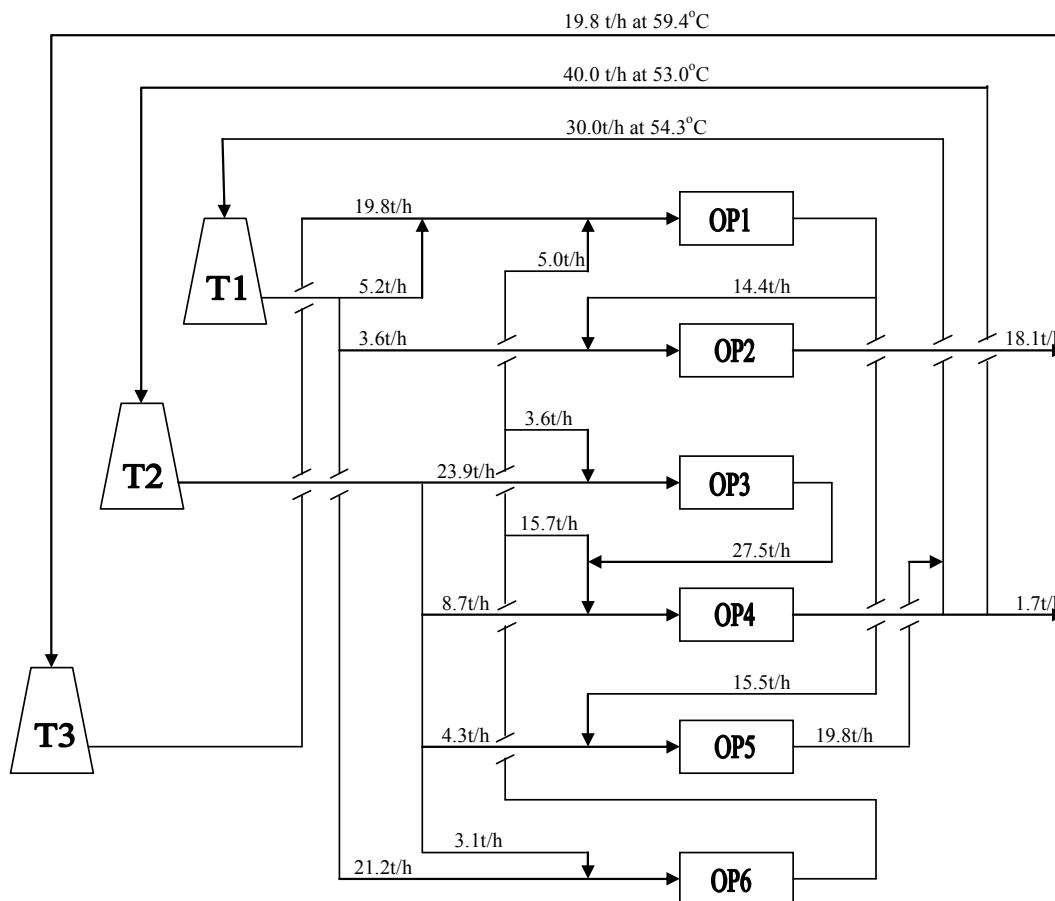


Figure 4.5: Unified network for Case 1

Table 4.5: Comparison of single source and unified targets for Case 1

	Single Source Targeting				Unified Targeting			
	T1	T2	T3	Total	T1	T2	T3	Total
Flowrate (t/h)	24.6	37.6	35.0	97.2	30.0	40.0	19.8	89.8
Ret. Temp (°C)	56.0	53.0	50.6		54.3	53.0	59.4	
Perf. Ind (°C/t/h)	1.46	0.88	0.87		1.14	0.78	1.74	

Table 4.6: Model Statistics for Case 1

Model	Single Source Targeting			Unified Targeting
	T1	T2	T3	
Continuous Variables	13	13	13	88
Constraints	14	14	14	46
CPU Time (s)	0.015	0.032	0.046	0.141

4.2.1.3. Case 2: Unified Targeting – Single Source and Sink

The unified target for Case 2 is 93.0 t/h, which is 4.3% less than the single source target (Table 4.7). The single source target is the same as that for Case 1. In this case, however, individual cooling-water-using operations have dedicated sources and sinks, as indicated in Figure 4.6. There are no restrictions for the cooling water return temperatures to sources. There is a 15.4% reduction of the target obtained using a parallel configuration. The model statistics displaying the number of continuous variables, discrete variables, constraints and CPU time are shown in Table 4.8.

Table 4.7: Comparison of single source and unified targets for Case 2

	Single Source Targeting				Unified Targeting			
	T1	T2	T3	Total	T1	T2	T3	Total
Flowrate (t/h)	24.6	37.6	35.0	97.2	29.0	37.8	26.2	93.0
Ret. Temp (°C)	56.0	53.0	50.6		53.0	57.4	50.0	
Perf. Ind (°C/t/h)	1.46	0.88	0.87		1.14	0.94	0.95	

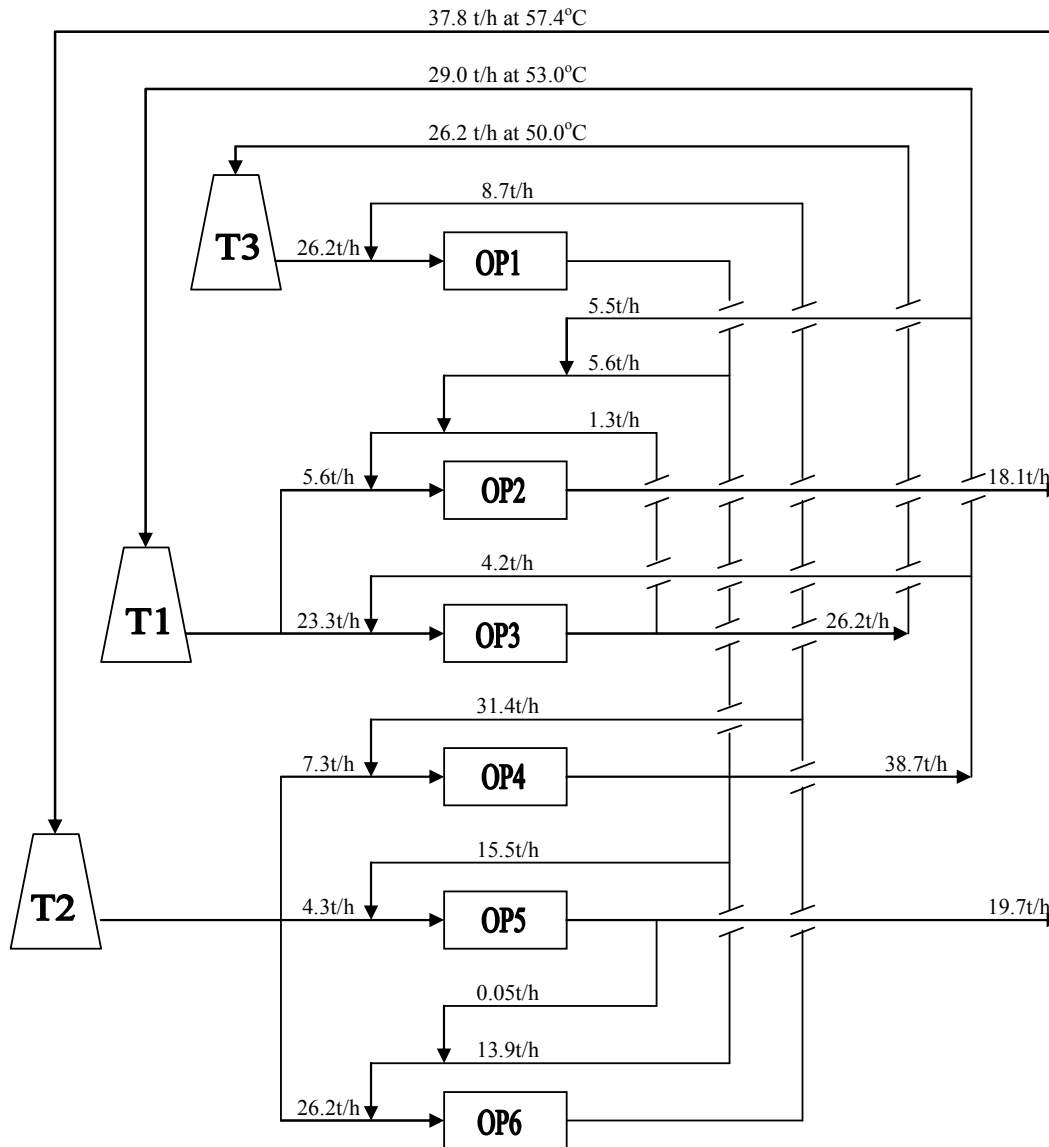


Figure 4.6: Unified network for Case 2

Table 4.8: Model Statistics for Case 2

Model	Single Source Targeting			Unified Targeting
	T1	T2	T3	
Continuous Variables	13	13	13	124
Binary Variables				36
Constraints	14	14	14	94
CPU Time (s)	0.047	0.032	0.046	0.047

4.2.2. Targeting with Cooling Water Return Temperature Limitation

4.2.2.1. Case 3 and 4: Single Source Targeting

Case 3 is characterized by the allowance of pre-mixing of cooling water supply and the post-splitting of return water, but the return temperatures to cooling towers T1, T2 and T3 have been limited to 52°C, 52°C and 50°C, respectively. Dedicated sources and sinks differentiates Case 4 from Case 3. Return temperature constraints are common in practice, as the hot return may cause problems such as fouling, corrosion and sub-optimal performance of packing. Thus, the single source cooling water networks (Figure 4.7), with limits on the return temperature, are different than that used in Cases 1 and 2.

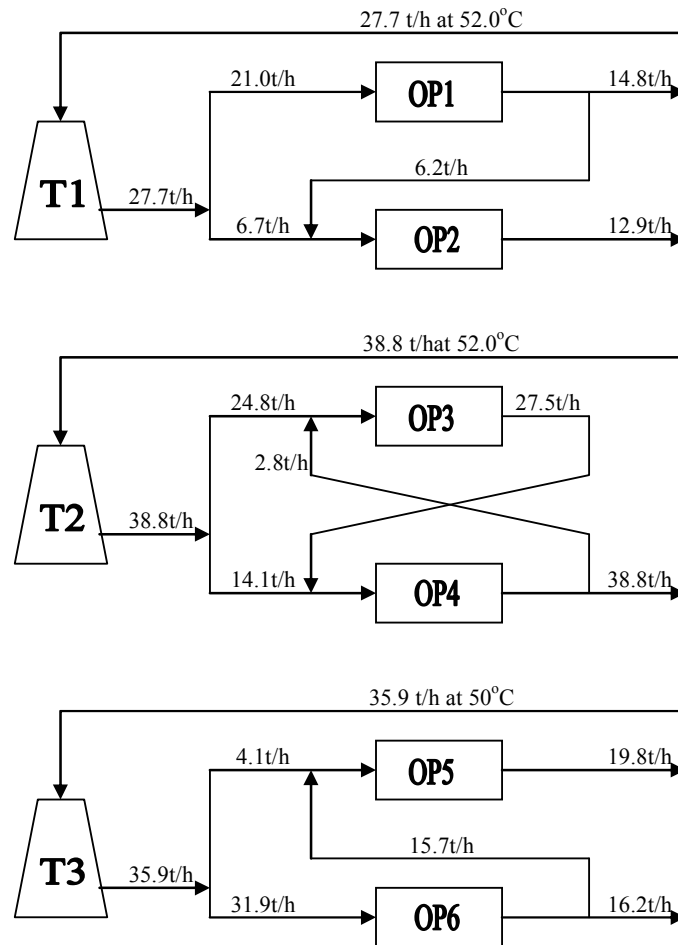


Figure 4.7: Cooling water networks for Cases 3 and 4 - single source targeting

4.2.2.2. Case 3: Unified Targeting – Multiple Sources and Sinks

The unified approach target shows an improvement of 0.9%, as shown in Table 4.9, than the combined single source targets of 102.4 t/h. A cooling water network that meets the target for Case 3 is shown in Figure 4.8. The model statistics displaying the number of continuous variables, number of constraints and CPU time are shown in Table 4.10. The formulation exhibits a nonconvex NLP structure that is concomitant with major mathematical difficulties. A reformulation linearisation technique is used to obtain a starting point for the exact NLP model.

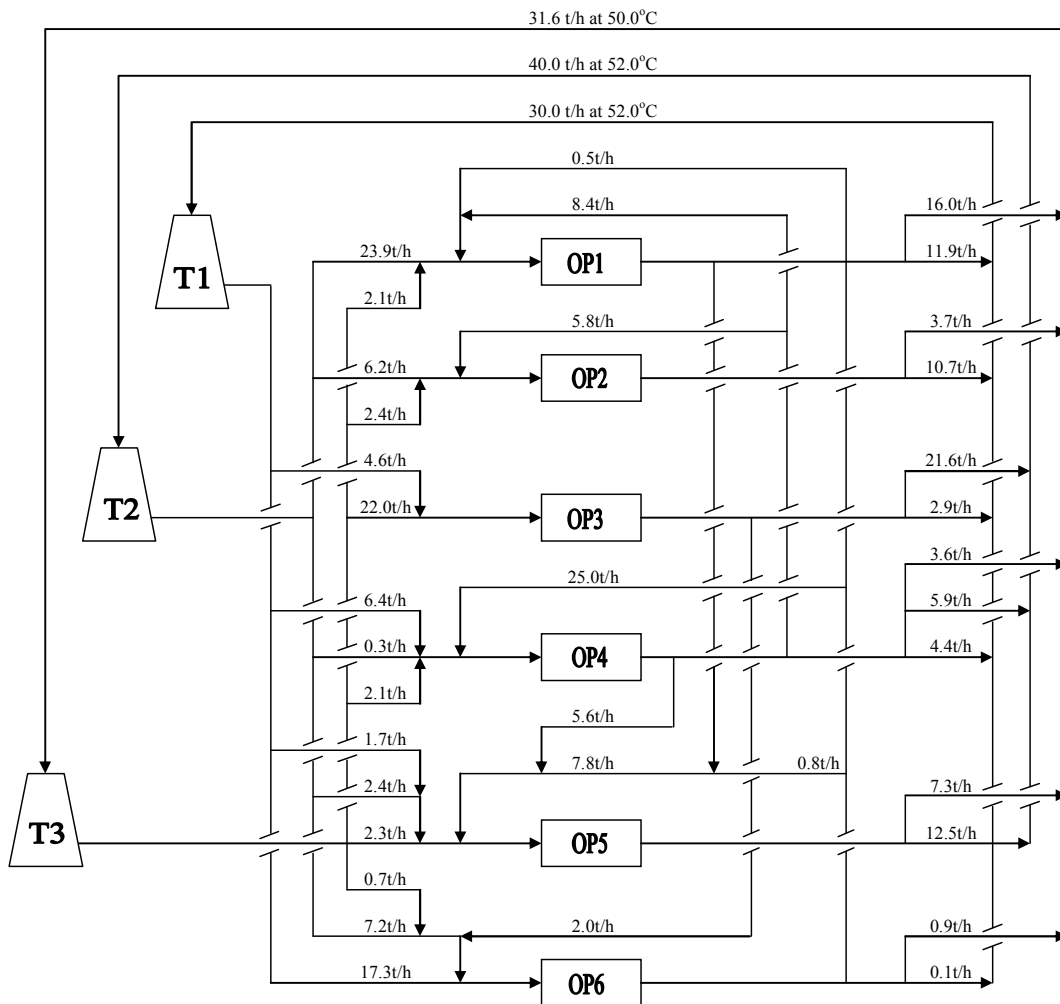


Figure 4.8: Unified network for Case 3

Table 4.9: Comparison of single source and unified targets for Case 3

	Single Source Targeting				Unified Targeting			
	T1	T2	T3	Total	T1	T2	T3	Total
Flowrate (t/h)	27.7	38.8	35.9	102.4	30.0	40.0	31.6	101.6
Ret. Temp (°C)	52.0	52.0	50.0		52.0	52.0	50.0	
Perf. Ind (°C/t/h)	1.16	0.77	0.70		1.07	0.75	0.79	

Table 4.10: Model Statistics for relaxed model, MR, and exact model, M*, for Case 3

	Single Source Targeting						Unified targeting	
	T1-MR	T1-M*	T2-MR	T2-M*	T3-MR	T3-M*	MR	M*
Cont. var.	23	15	23	15	23	15	154	94
Constraints	36	16	36	16	36	16	223	55
CPU time (s)	0.36	0.31	0.06	0.27	0.03	0.17	0.3	0.21

4.2.2.3. Case 4: Unified Targeting – Single Source and Sink

In this case individual cooling-water-using operations have dedicated sources and sinks, as indicated in Figure 4.9. The results, as shown in Table 4.11, for the unified target for Case 4 indicate a minor improvement of 0.9% than the combined single source targets of 102.4 t/h. The model statistics displaying the number of continuous variables, discrete variables, constraints and CPU time are shown in Table 4.12. Due to the presence of binary variables and bilinear terms, the exact model exhibits a MINLP structure. To provide a starting point for the solution of the exact model, the reformulation linearisation technique used in case 3 is also applied.

Table 4.11: Comparison of single source and unified targets for Case 4

	Single Source Targeting				Unified Targeting			
	T1	T2	T3	Total	T1	T2	T3	Total
Flowrate (t/h)	27.7	38.8	35.9	102.4	30.0	40.0	31.6	101.6
Ret. Temp (°C)	52.0	52.0	50.0		52.0	52.0	50.0	
Perf. Ind (°C/t/h)	1.16	0.77	0.70		1.07	0.75	0.79	

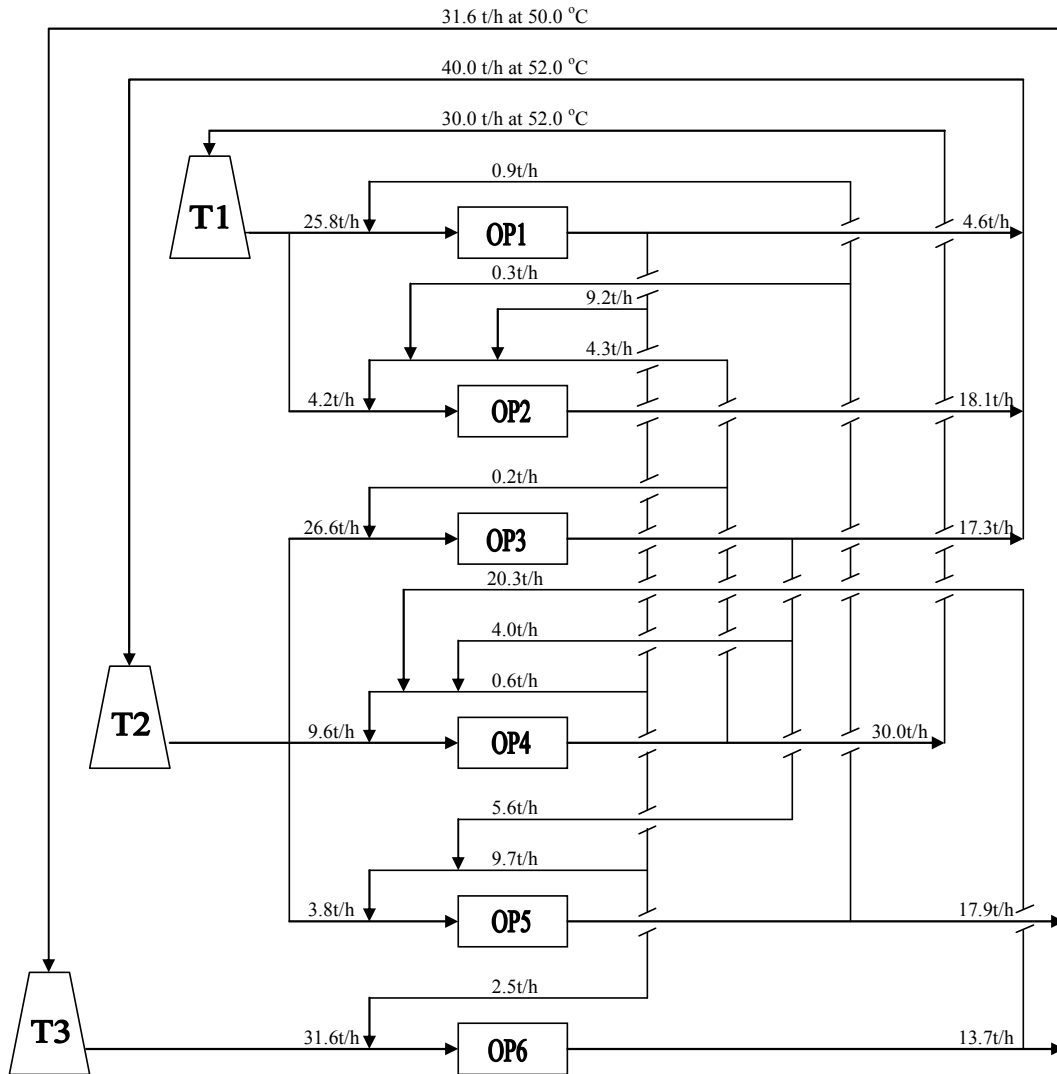


Figure 4.9: Unified network for Case 4

Table 4.12: Model Statistics for relaxed model, MR, and exact model, M*, for Case 4

	Single Source Targeting for cooling tower T1 and T2						Unified targeting	
	T1-MR	T1-M*	T2-MR	T2-M*	T3-MR	T3-M*	MR	M*
Cont. var.	23	15	23	15	23	15	190	130
Binary var.							36	36
Constraints	36	16	36	16	36	16	271	103
CPU time (s)	0.37	0.31	0.06	0.26	0.03	0.17	0.33	0.39

4.3. Case Study

To further illustrate the concept of the developed mathematical targeting approach of cooling water networks supplied by multiple cooling water sources, a real life case study of the Sasol Synfuels (Pty) Ltd petrochemical plant in Secunda, South Africa was investigated. Due to the vast size of the facility, lack of availability of process data and lack of integration options between the various units because of start-up and shutdown procedures, the case study was limited to the cooling-water-using operations listed in Table 4.13. Individual cooling-water-using operations represent the process data for a set of identical heat exchangers on all the trains supplied by a cooling water header. For example, OP1 represents process data for 6 identical heat exchangers supplied by one cooling water header.

Table 4.13: Hot process data of cooling-water-using operations for case study

Heat Exchanger	Q_i (MW)	$T_{in,i}^{hot}$ ($^{\circ}$ C)	$T_{out,i}^{hot}$ ($^{\circ}$ C)	CS_i (t/h)
OP1	100.815	146.5	40.0	7866
OP2	90.583	135.5	40.0	7866
OP3	19.590	145.2	40.0	2785
OP4	27.180	184.2	40.0	2785
OP5	19.459	142.5	40.0	2785
OP6	952.402	45.8	45.8	63600
OP7	100.668	93.9	47.0	4130
OP8	16.119	47.0	37.0	1737
OP9	66.775	94.7	54.0	2130
OP10	56.489	72.9	38.0	4870

The utility producing units that support the processing units are supplied by cooling water (termed utility cooling water) at a temperature of 24 $^{\circ}$ C. The heat exchanger operations OP1 – OP6 are supplied by utility cooling water from cooling tower T1. The processing units, chosen for optimisation, contain heat exchangers OP7 – OP10 and supplied with cooling water (termed process cooling water) from cooling tower T2 at 29 $^{\circ}$ C.

4.3.1. Base Case Cooling Water Network

The current operation (Figure 4.10) utilises 24.36 t/s (87,686 t/h) of utility cooling water, and 3.57t/s (12,866 t/h) of process cooling water to cool the hot process streams under parallel configuration.

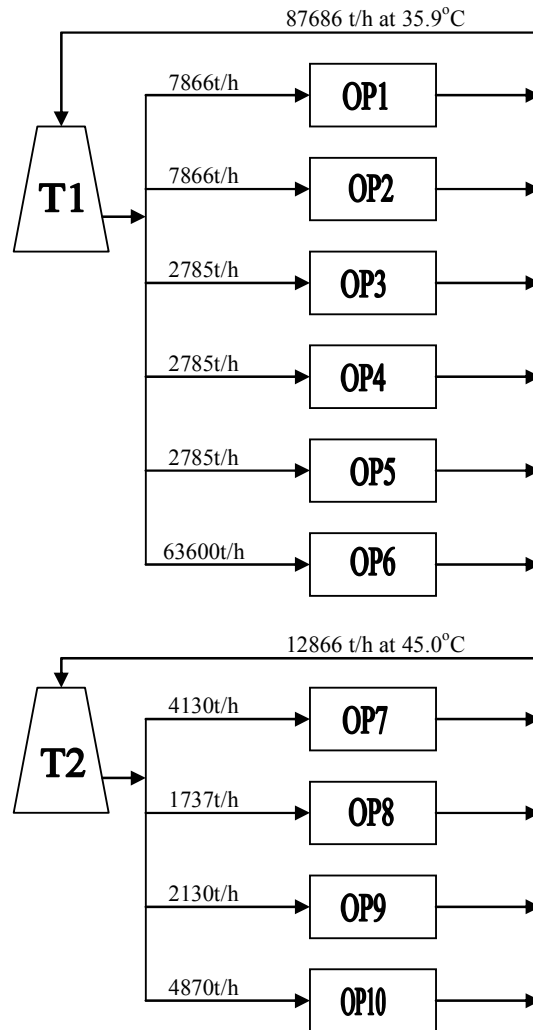


Figure 4.10: Base case parallel cooling water network of case study

The large demand of utility cooling water necessitates two identical cooling towers operating in parallel to service the utility cooling water network, which is represented as source T1. In the current parallel configuration, utility cooling water is returned to the

utility-cooling tower at 35.9°C, which has a performance indicator of 0.49 °C/t/s. Similarly the process cooling water is returned to the process-cooling tower T2 at 45°C, which has a performance indicator of 4.48 °C/t/s.

4.3.2. Optimisation of the Base Case by Reuse of Cooling Water

The cooling water flowrate of individual sources cannot exceed the current operation, of 87686 t/h and 12866 t/h for cooling tower T1 and T2, respectively. The limiting cooling water data (Table 4.14) was extracted from data sheets of the corresponding heat exchangers. Further, the current cooling water flowrate through individual operations was not exceeded in selecting the limiting inlet cooling water temperature.

Table 4.14: Limiting cooling water data for case study

Heat Exchanger	$T_{in,i}^{lim}$ (°C)	$T_{out,i}^{lim}$ (°C)	c_p (kW/°C)	Q_i (kW)
OP1	30	60	3,361	100.815
OP2	30	60	3,019	90.583
OP3	30	60	653	19.590
OP4	30	60	906	27.180
OP5	30	60	649	19.459
OP6	27	40	73,262	952.402
OP7	37	60	4,377	100.668
OP8	29	37	2,015	16.119
OP9	33	60	2,473	66.775
OP10	29	60	1,822	56.489

The unified target was determined for Cases 1 and 2. As it will be shown in Figures 4.12 and 4.13 for Cases 1 and 3, respectively, no process cooling water is required from source T2. Thus, Cases 2 and 4, in which cooling-water-using operations have dedicated sources and sinks, are not required. The unified targets for Cases 1 and 3 were compared with the corresponding single source targets. These results for Cases 1 and 3 for the case study of Sasol Synfuels (Pty) Ltd are discussed below.

4.3.2.1. *Targeting and Cooling Water Network Design without Cooling Water Return Temperature Limitation - Case 1*

The single source targeting method yielded networks displayed in Figure 4.11. The unified target allows supply and return water from and to different sources. The results for the unified targeting approach of Case 1, as shown in Table 4.15, indicate a required target of 16.49 t/s (59,374 t/h), which is 4.9% less than the single source target of 17.35 t/s (62,453 t/h).

Further, the target is 41.0% less than the current demand of 27.93 t/s (100,552 t/h) using the current parallel configuration of the network. By using the unified targeting approach no process cooling water from cooling tower T2 is required. The cooling water network obtained from the unified approach is shown in Figure 4.12.

Table 4.15: Comparison of single source and unified targets for Case 1

	Single Source Targeting			Unified Targeting		
	T1	T2	Total	T1	T2	Total
Load (t/s)	15.50	1.85	17.35	16.49	0.0	16.49
Ret. Temp (°C)	42.6	60.0		45.0	-	
Perf Ind (°C/t/s)	1.2	16.8		1.27	-	

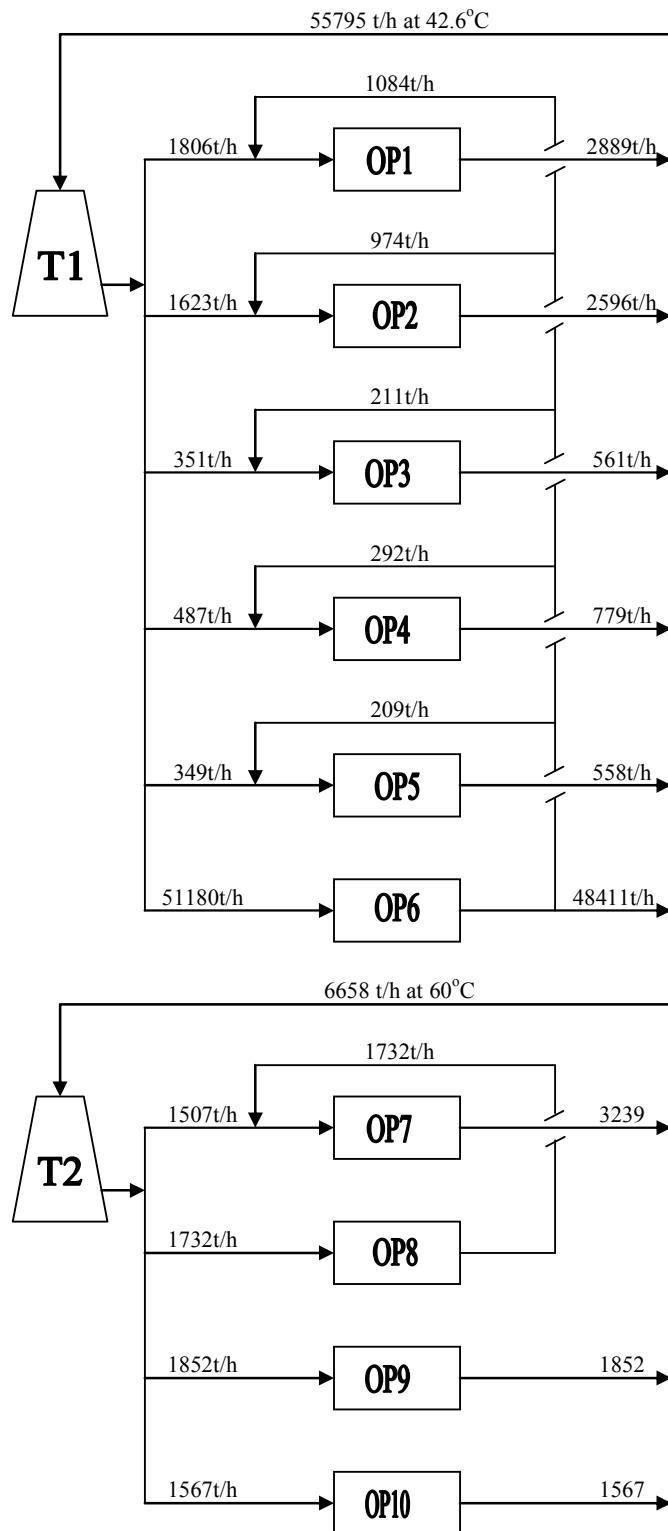


Figure 4.11: Cooling water network by single source targeting method for Case 1

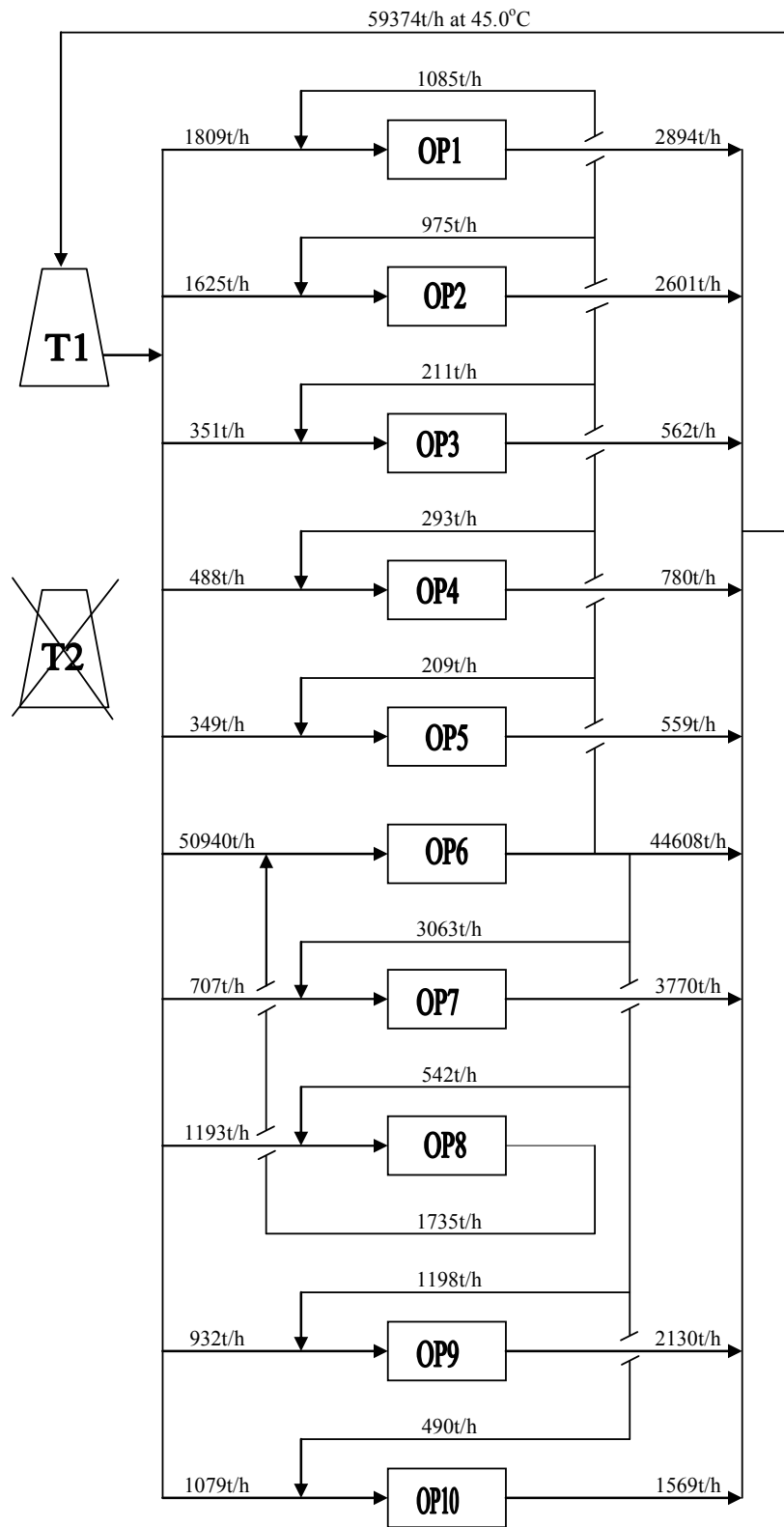


Figure 4.12: Unified network for Case 1

The model statistics displaying the number of continuous variables, number of constraints and CPU time are shown in Table 4.16. The single source and unified targeting are both LP problems.

Table 4.16: Model Statistics for Case 1

Model	Single Source Targeting		Unified Targeting
	T1	T2	
Continuous Variables	61	33	163
Constraints	37	25	67
CPU Time (s)	0.05	0.087	0.12

4.3.2.2. Targeting and Cooling Water Network Design with Cooling Water Return Temperature Limitation – Case 3

The results obtained with the unified targeting procedure of Case 1 predict a return temperature to the utility cooling tower, T1, of 45.0°C. No process cooling water is required with this targeting method. However, the maximum design temperature to the utility cooling tower is 42°C; hence the unified and single source target was calculated with the new specification on temperature. The cooling water network obtained by unified targeting (Figure 4.13) results in a cooling water requirement of 19.24 t/s (or 69,224 t/h).

The single source target of the utility cooling water network, with the return temperature of 42°C, is 16.06 t/s (or 57,799 t/h). The return temperature to the process cooling tower, T2, is also limited to 42°C. However, due to increases in the plant throughput over the years, this temperature under parallel configuration is 45.0°C. Hence reducing the cooling water requirement and maintaining a temperature of 42°C to the process cooling tower, using the single source method is not possible. Therefore, the current parallel target of 3.57 t/s (or 12,866 t/h) is used as the single source target for the process cooling water network. The utility and process cooling water networks using single source targeting is shown in Figure 4.14.

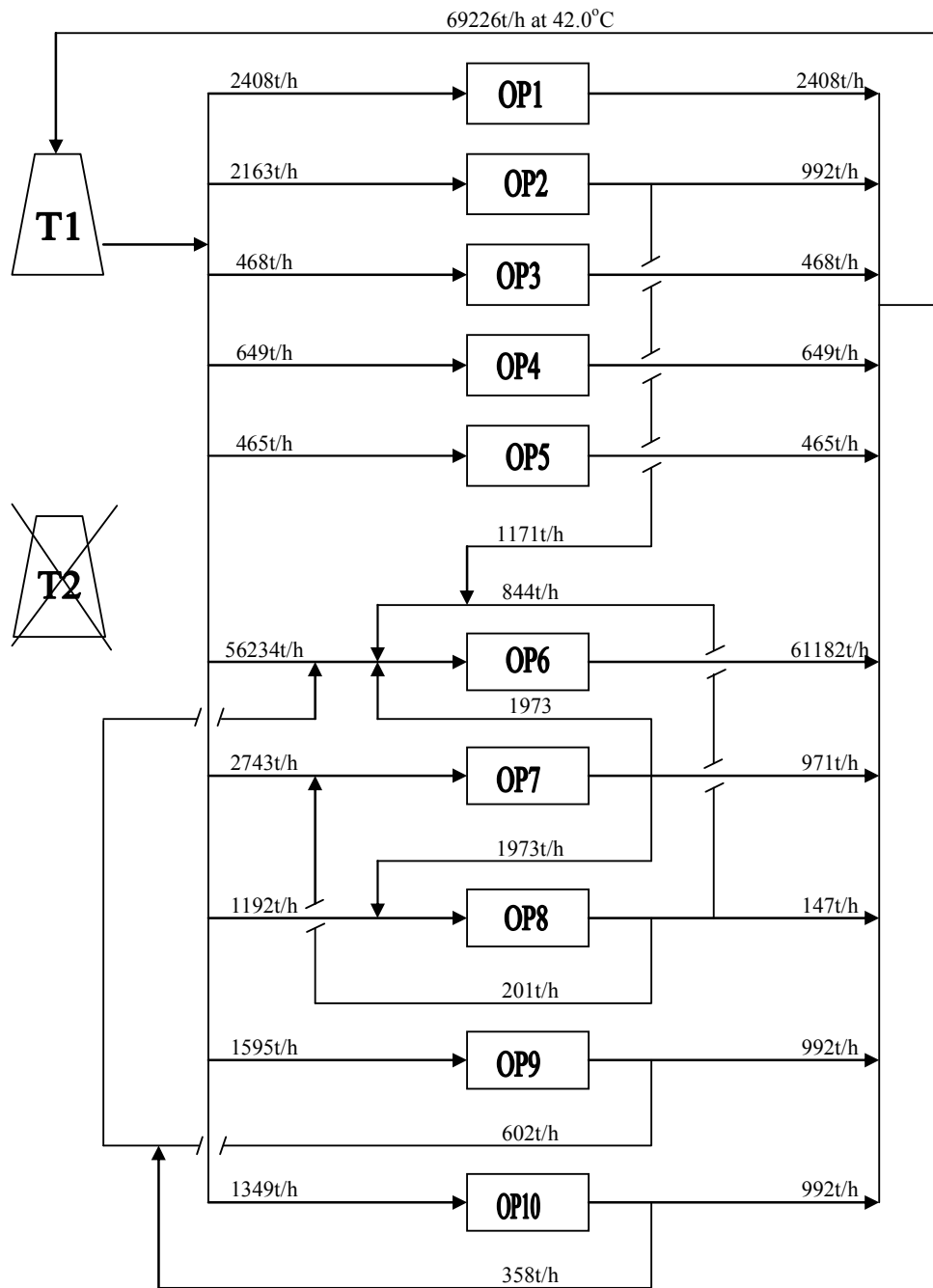


Figure 4.13: Unified Targeting for Case 3

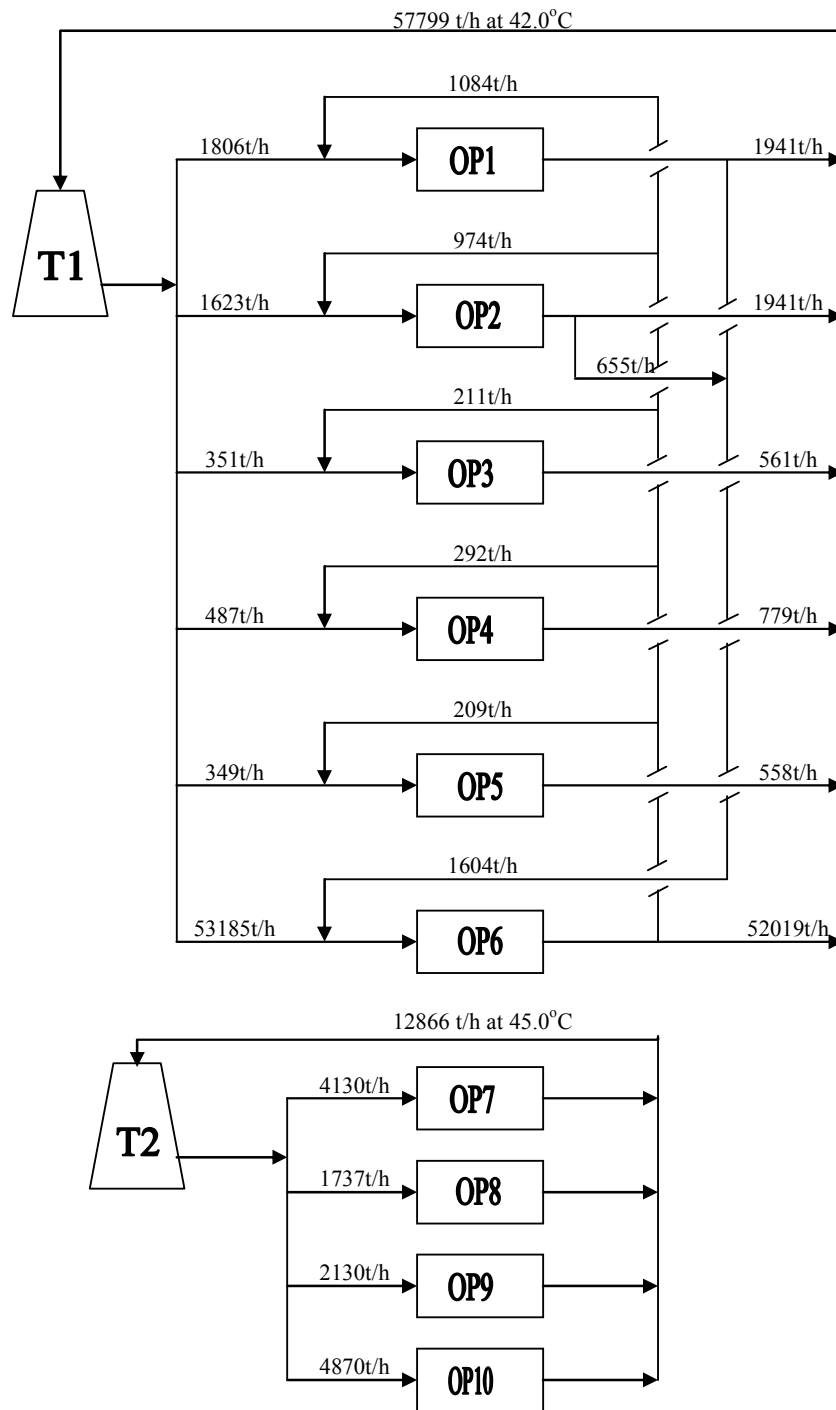


Figure 4.14: Single source targeting for Case 3

Therefore the unified target is compared with the current parallel cooling water requirement. The unified target with temperature specification is 16.6% greater than the unified target without temperature specification, but is still 19.8% better than the current parallel cooling water requirements. Hence there is significant scope for improvement on the Sasol Synfuels cooling water systems, to operate at the design return conditions of 42°C.

The model statistics displaying the number of continuous variables, number of equations and resource usage are shown in Table 4.17. The single source and unified targeting are both linear programming problems.

Table 4.17: Model Statistics relaxed model, MR, and exact model, M*, for Case 3

	Single Source Targeting for T1 and T2				Unified targeting	
	T1-MR	T1-M*	T2-MR	T2-M*	MR	M*
Continuous var.	115	67	61	37	303	173
Constraints	176	44	118	30	449	79
CPU time (s)	0.031	0.109	0.016	0.047	0.063	0.031

4.4. References

Kim, J.K. and R. Smith, 2001, Cooling water system design. *Chemical Engineering Science*, 56: 3641 – 3658.

CHAPTER 5. CONCLUSION

Both the simple illustrative example and the case study of the Sasol Synfuels (Pty) Ltd cooling water system show significant improvements on cooling water consumption and cooling tower performance. The illustrative example demonstrates that the unified targeting approach relative to the single source targeting approach results in a 7.7% and 0.9% reduction of cooling water requirements, for the case of maximum water reuse and that with flow and temperature constraints, respectively. Similarly, the case study shows a 4.9% and 13.5% improvement for Case 1 and 3, respectively.

These savings can be justified in terms of pumping cost, make-up water cost and additional cooling capacity to prevent investment in a new cooling tower. In the final design these savings will be a trade-off with the piping cost and pipe pressure drop.

Kim and Smith (2001) demonstrated using a novel graphical methodology how the conditions necessary for maximum cooling tower performance, that is, minimum cooling water supply flowrate and maximum return temperature, could be achieved for a given set of cooling-water-using operations. The minimum cooling water supply flowrate is achieved by exploiting reuse opportunities in a manner similar to that proposed by Wang and Smith (1994a). For a constant duty, this results in increased return temperature to the cooling water source. The basis of the mathematical formulation adheres to these requirements.

All previous work in cooling water system design has not considered multiple sources in targeting. In industry, it is often found that multiple sources are available for cooling purposes. It has been successfully demonstrated that the optimum cooling water target is achieved when all the sources and networks are considered as a whole. The developed mathematical formulation allows targeting for cooling water requirements of cooling water systems with single and multiple water sources. The structure of the model for the cases with no return temperature limitation ensures that a global optimum solution is achieved. For the cases with return temperature limitations, the NLP and MINLP models

encounter mathematical difficulties. This is overcome by using the reformulation and linearisation technique. However, the disadvantage is that a global optimum cannot be guaranteed. This is a resultant limitation of the mathematical formulation with temperature constraints.

The mathematical method can be easily adapted to include other performance indices, e.g., an economic objective function or further practical restrictions such as restricted flow. This can be considered in future work.

5.1. References

- Kim, J.K. and R. Smith, 2001, Cooling water system design. *Chemical Engineering Science*, 56: 3641 – 3658.
- Wang, Y.P. and R. Smith, 1994a, Wastewater minimisation. *Chemical Engineering Science*, 49: 981 – 1006.

APPENDIX A

A.1. Illustrative Example – Single Source Targeting

A.1.1. Maximum reuse: Case 1 – GAMS Input File

Sets

```
i processes / OP1, OP2, OP3,OP4 /;
```

```
alias(i,j);
```

Parameters

```
Q(i) Heatload (kW)  
  / OP1 400  
    OP2 1000  
    OP3 1800  
    OP4 200 /
```

```
Tinlim(i) Limiting inlet temperature (deg C)  
  / OP1 20  
    OP2 30  
    OP3 30  
    OP4 55 /
```

```
Toutlim(i) Limiting outlet temperature (deg C)  
  / OP1 40  
    OP2 40  
    OP3 75  
    OP4 75 /
```

```
T Temperature of fresh cooling water supplied from cooling  
tower n (deg C)  
  / 20 /
```

```
Tret      Return Temperature
```

```
cp        Heat Capacity (kJ per kg per deg C)
```

```
Finmax(i) Maximum inlet flow (kg per hour);
```

```
cp = 4187;
```


$$\text{Finmax}(i) = Q(i) * 3600 / (\text{cp} * (\text{Toutlim}(i) - \text{Tinlim}(i)));$$

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg per hour)

CR(i) Return water flow from operation i to Cooling Tower n (kg per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

CPtot Total fresh cooling water from all sources (kg per hour);

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq10(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq11(i) Maximum inlet Flow;

$$\text{Eq1}.. \text{CW} = \sum(i, \text{CS}(i));$$

$$\text{Eq4}(i).. \text{CS}(i) + \sum(j, \text{FR}(j,i)) = \text{Fin}(i);$$

$$\text{Eq5}(i).. \text{CR}(i) + \sum(j, \text{FR}(i,j)) = \text{Fout}(i);$$

$$\text{IntRec}(i,j) \$ (\text{ord}(i) = \text{ord}(j)).. \text{FR}(i,j) = 0;$$

$$\text{Eq10}(i).. Q(i) * 3600 / \text{cp} + \text{CS}(i) * T + \sum(j, \text{FR}(j,i) * \text{Toutlim}(j)) = \text{Fout}(i) * \text{Toutlim}(i);$$

$$\text{Eq8}(i).. \text{Fin}(i) = \text{Fout}(i);$$

$$\text{Eq11}(i).. \text{Fin}(i) = \text{Finmax}(i);$$

Model Fresh water /all/;

Solve Fresh water using LP minimising CW;

$$\text{Tret} = \sum(i, \text{CR}.1(i) * \text{Toutlim}(i)) / \text{CW}.1;$$

Display CS.1, CR.1, Tret, Finmax;

A.1.2. Temperature Restriction: Case 3 - GAMS Input File

Sets

```
i processes / OP1, OP2, OP3, OP4 /;
```

```
alias(i,j);
```

Parameters

```
Q(i) Heatload (kW)
```

```
/ OP1 400
  OP2 1000
  OP3 1800
  OP4 200 /
```

```
Tinlim(i) Limiting inlet temperature (deg C)
```

```
/ OP1 20
  OP2 30
  OP3 30
  OP4 55 /
```

```
Toutlim(i) Limiting outlet temperature (deg C)
```

```
/ OP1 40
  OP2 40
  OP3 75
  OP4 75 /
```

```
T          Temperature of fresh cooling water supplied
from cooling tower n (deg C)
```

```
/ 20 /
```

```
Tretmax    Return Temperature to cooling tower
```

```
/ 55 /
```

```
cp          Heat Capacity (kJ per kg per deg C)
```

```
Finmax(i)  Maximum inlet flow (kg per hour);
```

```
cp = 4187;
```

```
Finmax(i) = Q(i)*3600/(cp*(Toutlim(i)-Tinlim(i)));
```

Positive Variables

```
CTS          Fresh cooling water from Cooling Tower n (kg
per hour) (NB. called CS(n) in formulation)
```

CS(i) Fresh cooling water flow to process i from Cooling Tower n (kg per hour)
 Fin(i) Total water flow to process i (kg per hour)
 FR(i,j) Recycle flow to process j from process i (kg per hour)
 CR(i) Return water flow from operation i to Cooling Tower n (kg per hour)
 Fout(i) Total water flow exiting process i (deg C)
 Tout(i) Outlet temperature from process i (deg C)
 G1 Linearisation variable 1 for term CPR(j.i)*Tout(j)
 G2 Linearisation variable 2 for term CPin(i)*Tout(i)
 G3 Linearization variable 2 for term CPin(i)*Tout(i)
 Treturn Return temperature;

Variable

CW Total fresh water usage (kg per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq10(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq11(i) Maximum inlet Flow

Eq18(i) Duty definition of process i

Eq19(i) Limiting outlet temperature specification for process i

Eq20 Return temperature to cooling tower2

Eq21(i) Duty definition of process i

Eq30(j,i) G1 equality 1

Eq31(j,i) G1 equality 2

Eq32(j,i) G1 equality 3

Eq33(j,i) G1 equality 4

Eq40(i) G2 equality 1

Eq41(i) G2 equality 2

Eq42(i) G2 equality 3

Eq43(i) G2 equality 4

```

Eq44      Return temperature to cooling tower
Eq51(i)   G3 equality 1
Eq52(i)   G3 equality 2
Eq53(i)   G3 equality 3
Eq54(i)   G3 equality 4;

Eq1..     CW =e= sum(i, CS(i));
Eq4(i)..  CS(i) + sum(j, FR(j,i)) =e= Fin(i);
Eq5(i)..  CR(i) + sum(j, FR(i,j)) =e= Fout(i);
IntRec(i,j)$ (ord(i)=ord(j)).. FR(i,j) =e= 0;
Eq8(i)..  Fin(i) =e= Fout(i);
Eq11(i).. Fin(i) =l= Finmax(i);

Eq21(i).. Q(i)*3600/cp + CS(i)*T + sum(j,G1(j,i)) =e=
G2(i);
Eq30(j,i).. G1(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq31(j,i).. G1(j,i) =g= FR(j,i)*30;
Eq32(j,i).. G1(j,i) =l= Finmax(i)*Tout(j) + FR(j,i)*30 -
Finmax(i)*30;
Eq33(j,i).. G1(j,i) =l= FR(j,i)*Toutlim(j);
Eq40(i)..  G2(i) =g= Fin(i)*30;
Eq41(i)..  G2(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq42(i)..  G2(i) =l= Finmax(i)*Tout(i) + Fin(i)*30 -
Finmax(i)*30;
Eq43(i)..  G2(i) =l= Fin(i)*Toutlim(i);

Eq44..     Tretmax*CW =e= sum(i, G3(i));
Eq51(i)..  G3(i) =g= CR(i)*30;
Eq52(i)..  G3(i) =g= Finmax(i)*Tout(i) + CR(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq53(i)..  G3(i) =l= Finmax(i)*Tout(i) + CR(i)*25 -
Finmax(i)*30;
Eq54(i)..  G3(i) =l= CR(i)*Toutlim(i);

Eq18(i)..  Q(i)*3600/cp + CS(i)*T + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);
Eq19(i)..  Tout(i) =l= Toutlim(i);
Eq20..     Tretmax*CW =e= sum(i, CR(i)*Tout(i));

Model Fresh water1
/Eq1,Eq4,Eq5,IntRec,Eq8,Eq11,Eq19,Eq21,Eq30,Eq31,

```

Eq32,Eq33,Eq40,Eq41,Eq42,Eq43,Eq44,Eq51,Eq52,Eq53,Eq54/;

Model Fresh water2

/Eq1,Eq4,Eq5,IntRec,Eq8,Eq11,Eq18,Eq19,Eq20/;

Solve Fresh water1 using LP minimising CW;

Solve Fresh water2 using NLP minimising CW;

A.2. Illustrative Example – Multiple Sources Targeting

A.2.1. Case 1 – GAMS Input File

A.2.1.1. Case 1 – Single source targeting – Sub-problem A

Sets

```
i processes / OP1, OP2 /;
```

```
alias(i,j);
```

Parameters

```
Q(i)          Heatload (kW)
/ OP1 610
  OP2 420 /
```

```
Tinlim(i)    limiting inlet temperature (deg C)
/ OP1 30
  OP2 40 /
```

```
Toutlim(i)   limiting outlet temperature (deg C)
/ OP1 45
  OP2 60 /
```

```
T            Temperature of fresh cooling water supplied from cooling
tower n (deg C)
/ 20 /
```

```
cp           Heat Capacity (kJ per kg per deg C)
/ 4187 /
```

```
Tret        Return Temperature
```

```
Finmax(i)   Maximum inlet flow (kg per hour);
```

```
Finmax(i) = Q(i)*3600/(cp*(Toutlim(i)-Tinlim(i)));
```

Positive Variables

```
CS(i)       Fresh cooling water flow to process i from Cooling
Tower n (kg per hour)
```

```
Fin(i)      Total water flow to process i (kg per hour)
```

```
FR(i,j)     Recycle flow to process j from process i (kg per
hour)
```

```
CR(i)       Return water flow from operation i to Cooling
Tower n (kg per hour)
```

Fout(i) Total water flow exiting process i (deg C)
 Tout(i) Outlet temperature from process i (deg C)
 CPtot Total fresh cooling water from all sources (kg per hour);

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq12(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq13(i) Maximum inlet Flow;

Eq1.. CW =e= sum(i, CS(i));

Eq4(i).. Fin(i) =e= CS(i) + sum(j, FR(j,i));

Eq5(i).. Fout(i) =e= CR(i) + sum(j, FR(i,j));

IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

Eq12(i).. $Q(i) \cdot 3600 / cp + CS(i) \cdot T + \sum(j, FR(j,i) \cdot Toutlim(j)) =e= Fout(i) \cdot Toutlim(i)$;

Eq8(i).. Fin(i) =e= Fout(i);

Eq13(i).. Fin(i) =l= Finmax(i);

Model Fresh water /all/;

Solve Fresh water using LP minimising CW;

Tret = sum(i, CR.l(i) * Toutlim(i)) / CW.l;

Display CS.l, CR.l, Tret;

A.2.1.2. Case 1 – Single source targeting – Sub-problem B

Sets

i processes / OP3, OP4 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)
/ OP3 800
OP4 555 /

Tinlim(i) limiting inlet temperature (deg C)
/ OP3 25
OP4 45 /

Toutlim(i) limiting outlet temperature (deg C)
/ OP3 50
OP4 53 /

T Temperature of fresh cooling water supplied from cooling tower n (deg C)
/ 22 /

cp Heat Capacity (kJ per kg per deg C)
/ 4187 /

Tret Return Temperature

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg per hour)

CR(i) Return water flow from operation i to Cooling Tower n (kg per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

CPtrot Total fresh cooling water from all sources (kg per hour);

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq12(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq13(i) Maximum inlet Flow;

Eq1.. CW =e= sum(i, CS(i));

Eq4(i).. CS(i) + sum(j, FR(j,i)) =e= Fin(i);

Eq5(i).. CR(i) + sum(j, FR(i,j)) =e= Fout(i);

IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

Eq12(i).. Q(i)*3600/cp + CS(i)*T + sum(j, FR(j,i)*Toutlim(j)) =e= Fout(i)*Toutlim(i);

Eq8(i).. Fin(i) =e= Fout(i);

Eq13(i).. Fin(i) =l= Finmax(i);

Model Fresh water /all/;

Solve Fresh water using LP minimising CW;

Tret = sum(i, CR.l(i)*Toutlim(i))/CW.l;

Display CS.l, CR.l, Tret;

A.2.1.3. Case 1 – Single source targeting – Sub-problem C

Sets

i processes / OP5, OP6 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)

/ OP5 345
OP6 700 /

Tinlim(i) limiting inlet temperature (deg C)

/ OP5 40
OP6 30 /

Toutlim(i) limiting outlet temperature (deg C)

/ OP5 55
OP6 45 /

T Temperature of fresh cooling water supplied from cooling tower n (deg C)

/ 25 /

cp Heat Capacity (kJ per kg per deg C)

/ 4187 /

Tret Return Temperature

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg per hour)

CR(i) Return water flow from operation i to Cooling Tower n (kg per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

CPTot Total fresh cooling water from all sources (kg per hour);

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq12(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq13(i) Maximum inlet Flow;

Eq1.. CW =e= sum(i, CS(i));

Eq4(i).. CS(i) + sum(j, FR(j,i)) =e= Fin(i);

Eq5(i).. CR(i) + sum(j, FR(i,j)) =e= Fout(i);

IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

Eq12(i).. Q(i)*3600/cp + CS(i)*T + sum(j, FR(j,i)*Toutlim(j)) =e= Fout(i)*Toutlim(i);

Eq8(i).. Fin(i) =e= Fout(i);

Eq13(i).. Fin(i) =l= Finmax(i);

Model Fresh water /all/;

Solve Fresh water using LP minimising CW;

Tret = sum(i, CR.l(i)*Toutlim(i))/CW.l;

Display CS.l, CR.l, Tret;

A.2.1.4. Case 1 – Unified Targeting

Sets

i processes / OP1, OP2, OP3, OP4, OP5, OP6 /
n Towers / T1, T2, T3 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)
/ OP1 610
OP2 420
OP3 800
OP4 555
OP5 345
OP6 700 /

Tinlim(i) Limiting inlet temperature (deg C)
/ OP1 30
OP2 40
OP3 25
OP4 45
OP5 40
OP6 30 /

Toutlim(i) Limiting outlet temperature (deg C)
/ OP1 45
OP2 60
OP3 50
OP4 53
OP5 55
OP6 45 /

T(n) Temperature of fresh cooling water supplied
from cooling tower n (deg C)
/ T1 20
T2 22
T3 25 /

CSmax(n) Maximum capacity of cooling tower n (kg per
hour)
/ T1 30
T2 40
T3 40 /

cp Heat Capacity (kJ per ton per deg C)
/ 4187 /

Tret(n) Return Temperature

Finmax(i) Maximum inlet flow (kg per hour);

Finmax(i) = $Q(i) * 3600 / (cp * (T_{outlim}(i) - T_{inlim}(i)))$;

Positive Variables

CTS(n) Fresh cooling water from Cooling Tower n (ton per hour) (NB. called CS(n) in formulation)

CS(n,i) Fresh cooling water flow to process i from Cooling Tower n (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton per hour)

CR(i,n) Return water flow from operation i to Cooling Tower n (ton per hour)

Fout(i) Total water flow exiting process i (deg C);

Variable

CW Total fresh water usage (kg per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq2(n) Total fresh water material balance supplied from cooling tower n

Eq3(n) Total cooling water material balance returned to cooling tower n

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal recycle of process i

Eq12(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq9(n) Maximum capacity of cooling tower n

Eq13(i) Maximum inlet Flow;

Eq1.. $CW = \sum(n, CTS(n))$;

Eq2(n).. $CTS(n) = \sum(i, CS(n,i))$;

```

Eq3(n)..   CTS(n) =e= sum(i, CR(i,n));
Eq4(i)..   Fin(i) =e=sum(n, CS(n,i)) + sum(j, FR(j,i));
Eq5(i)..   Fout(i) =e= sum(n, CR(i,n)) + sum(j, FR(i,j));
IntRec(i,j)$ (ord(i)=ord(j))..   FR(i,j) =e= 0;
Eq12(i)..  Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) + sum(j,
FR(j,i)*Toutlim(j))
           =e= Fout(i)*Toutlim(i);
Eq8(i)..   Fin(i) =e= Fout(i);
Eq9(n)..   CTS(n) =l= CSmax(n);
Eq13(i)..  Fin(i) =l= Finmax(i);

Model Fresh water /all/;
Solve Fresh water using LP minimising CW;
   Tret(n) = sum(i, CR.l(i,n)*Toutlim(i))/sum(i, CR.l(i,n));

Display CTS.l, CS.l, CR.l, Tret, Fin.l, Finmax;

```

A.2.2. *Case 2 – GAMS Input File*

A.2.2.1. *Case 2 – Single source targeting – Sub-problem A*

As Case 1

A.2.2.2. *Case 2 – Single source targeting – Sub-problem B*

As Case 1

A.2.2.3. *Case 2 – Single source targeting – Sub-problem C*

As Case 1

A.2.2.4. *Case 2 – Unified Targeting*

Sets

```
i processes / OP1, OP2, OP3, OP4, OP5, OP6 /
n Towers    / T1, T2, T3 /;
```

```
alias(i,j);
```

Parameters

```
Q(i)          Heatload (kW)
/ OP1 610
  OP2 420
  OP3 800
  OP4 555
  OP5 345
  OP6 700 /
```

```
Tinlim(i)    Limiting inlet temperature (deg C)
/ OP1 30
  OP2 40
  OP3 25
  OP4 45
  OP5 40
  OP6 30 /
```

```
Toutlim(i)   Limiting outlet temperature (deg C)
/ OP1 45
  OP2 60
  OP3 50
  OP4 53
  OP5 55
```

OP6 45 /

T(n) Temperature of fresh cooling water supplied
from cooling tower n (deg C)

/ T1 20

T2 22

T3 25 /

CSmax(n) Maximum capacity of cooling tower n (kg per
hour)

/ T1 30

T2 40

T3 40 /

cp Heat Capacity (kJ per ton per deg C)

/ 4187 /

Tret Return Temperature

Finmax(i) Maximum inlet flow (kg per hour);

Finmax(i) = $Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CTS(n) Fresh cooling water from Cooling Tower n (kg
per hour) (NB. called CS(n) in formulation)

CS(n,i) Fresh cooling water flow to process i from
Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg
per hour)

CR(i,n) Return water flow from operation i to Cooling
Tower n (kg per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

CPTot Total fresh cooling water from all sources (kg
per hour);

Binary Variable

ys(n,i) Binary variable for CS

yr(n,i) Binary variable for CR;

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied
from all cooling towers

Eq2(n) Total fresh water material balance supplied
from cooling tower n

Eq3(n) Total cooling water material balance returned to
cooling tower n

Eq4(i) Total inlet water material balance into cooling
water using operation i

Eq5(i) Total outlet water material balance from
cooling water using operation i

IntRec Internal Recycle of process i

Eq12(i) Duty definition of process i

Eq8(i) Water balance across unit

Eq9(n) Maximum capacity of cooling tower n

Eq13(i) Maximum inlet Flow

Eq15(i,n) Upper bound of CS

Eq16(i) Sum of binary variables of CS

Eq17(i,n) Upper bound of CR

Eq18(i) Sum of binary variables of CR;

Eq1.. CW =e= sum(n, CTS(n));

Eq2(n).. CTS(n) =e= sum(i, CS(n,i));

Eq3(n).. CTS(n) =e= sum(i, CR(i,n));

Eq4(i).. Fin(i) =e= sum(n, CS(n,i)) + sum(j, FR(j,i));

Eq5(i).. Fout(i) =e=sum(n, CR(i,n)) + sum(j, FR(i,j));

IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

Eq12(i).. Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) + sum(j,
FR(j,i)*Toutlim(j))
 =e= Fout(i)*Toutlim(i);

Eq8(i).. Fin(i) =e= Fout(i);

Eq9(n).. CTS(n) =l= CSmax(n);

Eq13(i).. Fin(i) =l= Finmax(i);

Eq15(i,n).. CS(n,i) =l= CSmax(n)*ys(n,i);

Eq16(i).. sum(n, ys(n,i)) =l= 1;

Eq17(i,n).. CR(i,n) =l= CSmax(n)*yr(n,i);

Eq18(i).. sum(n, yr(n,i)) =l= 1;

Model Fresh water /all/;
Fresh water.optcr = 0.0001;

Solve Fresh water using MIP minimising CW;

```
Tret(n) = sum(i, (CR.l(i,n)*Toutlim(i)))/sum(i,  
CR.l(i,n));
```

```
Display CTS.l, CS.l, CR.l, Tret;
```

A.2.3. Case 3 – GAMS Input File

A.2.3.1. Case 3 – Single source targeting – Sub-problem A

Sets

```
i processes / OP1, OP2 /;
```

```
alias(i,j);
```

Parameters

```
Q(i)          Heatload (kW)
  / OP1 610
    OP2 420 /
```

```
Tinlim(i)    Limiting inlet temperature (deg C)
  / OP1 30
    OP2 40 /
```

```
Toutlim(i)   Limiting outlet temperature (deg C)
  / OP1 45
    OP2 60 /
```

```
T            Temperature of fresh cooling water supplied
from cooling tower n (deg C)
  / 20 /
```

```
Tretmax      Maximum return Temperature to cooling tower
  / 52 /
```

```
cp           Heat Capacity (kJ per kg per deg C)
  / 4187 /
```

```
Tret         Return Temperature to cooling tower
```

```
Finmax(i)   Maximum inlet flow (kg per hour);
```

```
Finmax(i) = Q(i)*3600/(cp*(Toutlim(i)-Tinlim(i)));
```

Positive Variables

```
CS(i)       Fresh cooling water flow to process i from
Cooling Tower n (ton per hour)
```

```
Fin(i)      Total water flow to process i (ton per hour)
```

```
FR(i,j)     Recycle flow to process j from process i (ton
per hour)
```

CR(i) Return water flow from operation i to Cooling Tower n (ton per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

G1 Linearization variable 1 for term CR(i.n)*Tout(j)

G2 Linearization variable 2 for term FR(j.i)*Tout(j)

G3 Linearization variable 3 for term Fin(i)*Tout(i);

Variable

CW Total fresh water usage (ton per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq8(i) Water balance across unit

Eq10(i) Limiting outlet temperature specification for process i

Eq13(i) Maximum inlet Flow

Eq19 Return temperature to cooling tower

Eq20(i) Duty definition of process i

Eq21 Linearised equation for the return temperature to cooling tower

Eq22(i) Linearised equation for the duty definition of process i

Eq37(i) G1 equality 2

Eq38(i) G1 equality 3

Eq39(i) G1 equality 4

Eq40(j,i) G2 equality 1

Eq41(j,i) G2 equality 3

Eq42(j,i) G2 equality 4

Eq43(i) G3 equality 2

Eq44(i) G3 equality 3

Eq45(i) G3 equality 4;

Eq1.. CW =e= sum(i, CS(i));

Eq4(i).. CS(i) + sum(j, FR(j,i)) =e= Fin(i);

```

Eq5(i)..      CR(i) + sum(j, FR(i,j)) =e= Fout(i);
IntRec(i,j)$ (ord(i)=ord(j))..  FR(i,j) =e= 0;
Eq8(i)..      Fin(i) =e= Fout(i);
Eq10(i)..     Tout(i) =l= Toutlim(i);
Eq13(i)..     Fin(i) =l= Finmax(i);

Eq19..        sum(i, CR(i)*Tout(i)) =l= Tretmax*CW;
Eq20(i)..     Q(i)*3600/cp + CS(i)*T + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);
Eq21..        sum(i, G1(i)) =l= Tretmax*CW;
Eq22(i)..     Q(i)*3600/cp + CS(i)*T + sum(j,G2(j,i)) =e=
G3(i);

Eq37(i)..     G1(i) =g= Finmax(i)*Tout(i) + CR(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq38(i)..     G1(i) =l= Finmax(i)*Tout(i);
Eq39(i)..     G1(i) =l= CR(i)*Toutlim(i);
Eq40(j,i)..   G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i)..   G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i)..   G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i)..     G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i)..     G3(i) =l= Finmax(i)*Tout(i);
Eq45(i)..     G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1 / Eq1,Eq4,Eq5,IntRec,Eq8,Eq21,Eq22,Eq37,
Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45 /;

Model Fresh water2 /
Eq1,Eq4,Eq5,IntRec,Eq8,Eq10,Eq13,Eq19,Eq20 /;

Solve Fresh water1 using LP minimising CW;
Solve Fresh water2 using NLP minimising CW;
      Tret = sum(i, CR.l(i)*Tout.l(i))/CW.l

Display CS.l, CR.l, Tret;

```

A.2.3.2. Case 3 – Single source targeting – Sub-problem B

Sets

i processes / OP3, OP4 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)
/ OP3 800
OP4 555 /

Tinlim(i) Limiting inlet temperature (deg C)
/ OP3 25
OP4 45 /

Toutlim(i) Limiting outlet temperature (deg C)
/ OP3 50
OP4 53 /

T Temperature of fresh cooling water supplied
from cooling tower n (deg C)
/ 22 /

Tretmax Maximum return Temperature to cooling tower
/ 52 /

cp Heat Capacity (kJ per kg per deg C)
/ 4187 /

Tret Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CS(i) Fresh cooling water flow to process i from
Cooling Tower n (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton
per hour)

CR(i) Return water flow from operation i to Cooling
Tower n (ton per hour)

$F_{out}(i)$ Total water flow exiting process i (deg C)
 $T_{out}(i)$ Outlet temperature from process i (deg C)
 $G1$ Linearization variable 1 for term
 $CR(i,n) * T_{out}(j)$
 $G2$ Linearization variable 2 for term
 $FR(j,i) * T_{out}(j)$
 $G3$ Linearization variable 3 for term
 $Fin(i) * T_{out}(i);$

Variable

CW Total fresh water usage (ton per hour);

Equations

$Eq1$ Total fresh water material balance supplied from all cooling towers
 $Eq4(i)$ Total inlet water material balance into cooling water using operation i
 $Eq5(i)$ Total outlet water material balance from cooling water using operation i
 $IntRec$ Internal Recycle of process i
 $Eq8(i)$ Water balance across unit
 $Eq10(i)$ Limiting outlet temperature specification for process i
 $Eq13(i)$ Maximum inlet Flow
 $Eq19$ Return temperature to cooling tower
 $Eq20(i)$ Duty definition of process i
 $Eq21$ Linearised equation for the return temperature to cooling tower
 $Eq22(i)$ Linearised equation for the duty definition of process i
 $Eq37(i)$ $G1$ equality 2
 $Eq38(i)$ $G1$ equality 3
 $Eq39(i)$ $G1$ equality 4
 $Eq40(j,i)$ $G2$ equality 1
 $Eq41(j,i)$ $G2$ equality 3
 $Eq42(j,i)$ $G2$ equality 4
 $Eq43(i)$ $G3$ equality 2
 $Eq44(i)$ $G3$ equality 3
 $Eq45(i)$ $G3$ equality 4;

$Eq1..$ $CW = e = \sum(i, CS(i));$
 $Eq4(i)..$ $CS(i) + \sum(j, FR(j,i)) = e = Fin(i);$
 $Eq5(i)..$ $CR(i) + \sum(j, FR(i,j)) = e = F_{out}(i);$
 $IntRec(i,j) \$(ord(i)=ord(j))..$ $FR(i,j) = e = 0;$

```

Eq8(i)..      Fin(i) =e= Fout(i);
Eq10(i)..     Tout(i) =l= Toutlim(i);
Eq13(i)..     Fin(i) =l= Finmax(i);

Eq19..        sum(i, CR(i)*Tout(i)) =l= Tretmax*CW;
Eq20(i)..     Q(i)*3600/cp + CS(i)*T + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);
Eq21..        sum(i, G1(i)) =l= Tretmax*CW;
Eq22(i)..     Q(i)*3600/cp + CS(i)*T + sum(j,G2(j,i)) =e=
G3(i);

Eq37(i)..     G1(i) =g= Finmax(i)*Tout(i) + CR(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq38(i)..     G1(i) =l= Finmax(i)*Tout(i);
Eq39(i)..     G1(i) =l= CR(i)*Toutlim(i);
Eq40(j,i)..   G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i)..   G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i)..   G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i)..     G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i)..     G3(i) =l= Finmax(i)*Tout(i);
Eq45(i)..     G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1 / Eq1,Eq4,Eq5,IntRec,Eq8,Eq21,Eq22,Eq37,
Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45 /;

Model Fresh water2 /
Eq1,Eq4,Eq5,IntRec,Eq8,Eq10,Eq13,Eq19,Eq20 /;

Solve Fresh water1 using LP minimising CW;
Solve Fresh water2 using NLP minimising CW;
      Tret = sum(i, CR.l(i)*Tout.l(i))/CW.l

Display CS.l, CR.l, Tret;

```


A.2.3.3. Case 3 – Single source targeting – Sub-problem C

Sets

i processes / OP5, OP6 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)
/ OP5 345
OP6 700 /

Tinlim(i) Limiting inlet temperature (deg C)
/ OP5 40
OP6 30 /

Toutlim(i) Limiting outlet temperature (deg C)
/ OP5 55
OP6 45 /

T Temperature of fresh cooling water supplied
from cooling tower n (deg C)
/ 25 /

Tretmax Maximum return Temperature to cooling tower
/ 50 /

cp Heat Capacity (kJ per kg per deg C)
/ 4187 /

Tret Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CS(i) Fresh cooling water flow to process i from
Cooling Tower n (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton
per hour)

CR(i) Return water flow from operation i to Cooling
Tower n (ton per hour)

```

Fout(i)      Total water flow exiting process i (deg C)
Tout(i)      Outlet temperatruue from process i (deg C)
G1           Linearization variable 1 for term
CR(i.n)*Tout(j)
G2           Linearization variable 2 for term
FR(j.i)*Tout(j)
G3           Linearization variable 3 for term
Fin(i)*Tout(i);

Variable
  CW         Total fresh water usage (ton per hour);

Equations
  Eq1        Total fresh water material balance supplied
from all cooling towers
  Eq4(i)     Total inlet water material balance into cooling
water using operation i
  Eq5(i)     Total outlet water material balance from
cooling water using operation i
  IntRec     Internal Recycle of process i
  Eq8(i)     Water balance across unit
  Eq10(i)    Limiting outlet temperature specification for
process i
  Eq13(i)    Maximum inlet Flow
  Eq19       Return temperature to cooling tower
  Eq20(i)    Duty definition of process i
  Eq21       Linearised equation for the return temperature
to cooling tower
  Eq22(i)    Linearised equation for the duty definition of
process i
  Eq37(i)    G1 equality 2
  Eq38(i)    G1 equality 3
  Eq39(i)    G1 equality 4
  Eq40(j,i)  G2 equality 1
  Eq41(j,i)  G2 equality 3
  Eq42(j,i)  G2 equality 4
  Eq43(i)    G3 equality 2
  Eq44(i)    G3 equality 3
  Eq45(i)    G3 equality 4;

Eq1..       CW =e= sum(i, CS(i));
Eq4(i)..    CS(i) + sum(j, FR(j,i)) =e= Fin(i);
Eq5(i)..    CR(i) + sum(j, FR(i,j)) =e= Fout(i);
IntRec(i,j)$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

```

```

Eq8(i)..      Fin(i) =e= Fout(i);
Eq10(i)..     Tout(i) =l= Toutlim(i);
Eq13(i)..     Fin(i) =l= Finmax(i);

Eq19..        sum(i, CR(i)*Tout(i)) =l= Tretmax*CW;
Eq20(i)..     Q(i)*3600/cp + CS(i)*T + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);
Eq21..        sum(i, G1(i)) =l= Tretmax*CW;
Eq22(i)..     Q(i)*3600/cp + CS(i)*T + sum(j,G2(j,i)) =e=
G3(i);

Eq37(i)..     G1(i) =g= Finmax(i)*Tout(i) + CR(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq38(i)..     G1(i) =l= Finmax(i)*Tout(i);
Eq39(i)..     G1(i) =l= CR(i)*Toutlim(i);
Eq40(j,i)..   G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i)..   G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i)..   G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i)..     G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i)..     G3(i) =l= Finmax(i)*Tout(i);
Eq45(i)..     G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1 / Eq1,Eq4,Eq5,IntRec,Eq8,Eq21,Eq22,Eq37,
Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45 /;

Model Fresh water2 /
Eq1,Eq4,Eq5,IntRec,Eq8,Eq10,Eq13,Eq19,Eq20 /;

Solve Fresh water1 using LP minimising CW;
Solve Fresh water2 using NLP minimising CW;
      Tret = sum(i, CR.l(i)*Tout.l(i))/CW.l

Display CS.l, CR.l, Tret;

```

A.2.3.4. Case 3 – Unified Targeting

Sets

i processes / OP1, OP2, OP3, OP4, OP5, OP6 /
n Towers / T1, T2, T3 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)
/ OP1 610
OP2 420
OP3 800
OP4 555
OP5 345
OP6 700 /

Tinlim(i) Limiting inlet temperature (deg C)
/ OP1 30
OP2 40
OP3 25
OP4 45
OP5 40
OP6 30 /

Toutlim(i) Limiting outlet temperature (deg C)
/ OP1 45
OP2 60
OP3 50
OP4 53
OP5 55
OP6 45 /

T(n) Temperature of fresh cooling water supplied
from cooling tower n (deg C)
/ T1 20
T2 22
T3 25 /

CSmax(n) Maximum capacity of cooling tower n (ton per
hour)
/ T1 30
T2 40
T3 40 /

Tretmax(n) Maximum Return Temperature to cooling tower
 / T1 52
 T2 52
 T3 50 /

cp Heat Capacity (kJ per kg per deg C)
 / 4187 /

Tret(n) Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CTS(n) Fresh cooling water from Cooling Tower n (ton per hour)

CS(n,i) Fresh cooling water flow from Cooling Tower n to process i (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton per hour)

CR(i,n) Return water flow from operation i to Cooling Tower n (ton per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

G1 Linearization variable 1 for term $CR(i,n) * Tout(j)$

G2 Linearization variable 2 for term

$FR(j,i) * Tout(j)$

G3 Linearization variable 3 for term

$Fin(i) * Tout(i)$;

Variable

CW Total fresh water usage (ton per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq2(n) Total fresh water material balance supplied from cooling tower n

Eq3(n) Total cooling water material balance returned to cooling tower n

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq8(i) Water balance across unit

Eq9(n) Maximum capacity of cooling tower n

Eq10(i) Limiting outlet temperature specification for process i

Eq13(i) Maximum inlet Flow

Eq19(n) Return temperature to cooling tower

Eq20(i) Duty definition of process i

Eq21(n) Linearised equation for the return temperature to cooling tower

Eq22(i) Linearised equation for the duty definition of process i

Eq37(i,n) G1 equality 2

Eq38(i,n) G1 equality 3

Eq39(i,n) G1 equality 4

Eq40(j,i) G2 equality 1

Eq41(j,i) G2 equality 3

Eq42(j,i) G2 equality 4

Eq43(i) G3 equality 2

Eq44(i) G3 equality 3

Eq45(i) G3 equality 4;

Eq1.. $CW = e = \sum(n, CTS(n));$

Eq2(n).. $CTS(n) = e = \sum(i, CS(n,i));$

Eq3(n).. $CTS(n) = e = \sum(i, CR(i,n));$

Eq4(i).. $Fin(i) = e = \sum(n, CS(n,i)) + \sum(j, FR(j,i));$

Eq5(i).. $Fout(i) = e = \sum(n, CR(i,n)) + \sum(j, FR(i,j));$

IntRec(i,j)\$(ord(i)=ord(j)).. $FR(i,j) = e = 0;$

Eq8(i).. $Fin(i) = e = Fout(i);$

Eq9(n).. $CTS(n) = l = CSmax(n);$

Eq10(i).. $Tout(i) = l = Toutlim(i);$

Eq13(i).. $Fin(i) = l = Finmax(i);$

Eq19(n).. $\sum(i, CR(i,n)*Tout(i)) = l = Tretmax(n)*CTS(n);$

Eq20(i).. $Q(i)*3600/cp + \sum(n, CS(n,i)*T(n)) + \sum(j, FR(j,i)*Tout(j)) = e = Fout(i)*Tout(i);$

Eq21(n).. $\sum(i, G1(i,n)) = l = Tretmax(n)*CTS(n);$

Eq22(i).. $Q(i)*3600/cp + \sum(n, CS(n,i)*T(n)) + \sum(j, G2(j,i)) = e = G3(i);$

```
Eq37(i,n).. G1(i,n) =g= Finmax(i)*Tout(i) +
CR(i,n)*Toutlim(i) - Finmax(i)*Toutlim(i);
Eq38(i,n).. G1(i,n) =l= Finmax(i)*Tout(i);
Eq39(i,n).. G1(i,n) =l= CR(i,n)*Toutlim(i);
Eq40(j,i).. G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i).. G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i).. G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i).. G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i).. G3(i) =l= Finmax(i)*Tout(i);
Eq45(i).. G3(i) =l= Fin(i)*Toutlim(i);
```

Model Fresh water1

```
/Eq1,Eq2,Eq3,Eq4,Eq5,Eq8,Eq9,IntRec,Eq21,Eq22,
```

```
Eq37,Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45/;
```

Model Fresh water2

```
/Eq1,Eq2,Eq3,Eq4,Eq5,Eq8,Eq9,IntRec,Eq10,Eq13,Eq19,Eq20/;
```

Solve Fresh water1 using LP minimising CW;

Solve Fresh water2 using NLP minimising CW;

```
Tret(n) = sum(i, CR.l(i,n)*Tout.l(i))/CTS.l(n)
```

```
Display CTS.l, CS.l, CR.l, Fin.l, Finmax, Tout.l, Toutlim,
Tret;
```

A.2.4. *Case 4 – GAMS Input File*

A.2.4.1. *Case 4 – Single source targeting – Sub-problem A*

As Case 3

A.2.4.2. *Case 4 – Single source targeting – Sub-problem B*

As Case 3

A.2.4.3. *Case 4 – Single source targeting – Sub-problem C*

As Case 3

A.2.4.4. *Case 4 – Unified Targeting*

Sets

```
i processes / OP1, OP2, OP3, OP4, OP5, OP6 /
n Towers    / T1, T2, T3 /;
```

```
alias(i,j);
```

Parameters

```
Q(i)          Heatload (kW)
/ OP1 610
  OP2 420
  OP3 800
  OP4 555
  OP5 345
  OP6 700 /
```

```
Tinlim(i)    Limiting inlet temperature (deg C)
/ OP1 30
  OP2 40
  OP3 25
  OP4 45
  OP5 40
  OP6 30 /
```

```
Toutlim(i)   Limiting outlet temperature (deg C)
/ OP1 45
  OP2 60
  OP3 50
  OP4 53
  OP5 55
```


OP6 45 /

T(n) Temperature of fresh cooling water supplied
from cooling tower n (deg C)

/ T1 20
T2 22
T3 25 /

CSmax(n) Maximum capacity of cooling tower n (kg per
hour)

/ T1 30
T2 40
T3 40 /

Tretmax(n) Maximum Return Temperature to cooling tower

/ T1 52
T2 52
T3 50 /

cp Heat Capacity (kJ per kg per deg C)

/ 4187 /

Tret(n) Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CTS(n) Fresh cooling water from Cooling Tower n (kg
per hour) (NB. called CS(n) in formulation)

CS(n,i) Fresh cooling water flow to process i from
Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg
per hour)

CR(i,n) Return water flow from operation i to Cooling
Tower n (kg per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

G1 Linearization variable 1 for term
CPR(j,i)*Tout(j)

G2 Linearization variable 2 for term

CPin(i)*Tout(i)

G3 Linearization variable 2 for term
CPin(i)*Tout(i);

Binary Variable

ys(n,i) Binary variable for CS
yr(n,i) Binary variable for CR;

Variable

CW Total fresh water usage (kg per hour);

Equations

Eq1 Total fresh water material balance supplied
from all cooling towers
Eq2(n) Total fresh water material balance supplied
from cooling tower n
Eq3(n) Total cooling water material balance returned to
cooling tower n
Eq4(i) Total inlet water material balance into cooling
water using operation i
Eq5(i) Total outlet water material balance from
cooling water using operation i
IntRec Internal Recycle of process i
Eq8(i) Water balance across unit
Eq9(n) Maximum capacity of cooling tower n
Eq10(i) Limiting outlet temperature specification for
process i
Eq13(i) Maximum inlet Flow
Eq15(i,n) Upper bound of CS
Eq16(i) Sum of binary variables of CS
Eq17(i,n) Upper bound of CR
Eq18(i) Sum of binary variables of CR
Eq19(n) Return temperature to cooling tower
Eq20(i) Duty definition of process i
Eq21(n) Linearised equation for the return temperature
to cooling tower
Eq22(i) Linearised equation for the duty definition of
process i
Eq37(i,n) G1 equality 2
Eq38(i,n) G1 equality 3
Eq39(i,n) G1 equality 4
Eq40(j,i) G2 equality 1
Eq41(j,i) G2 equality 3
Eq42(j,i) G2 equality 4
Eq43(i) G3 equality 2

Eq44(i) G3 equality 3
Eq45(i) G3 equality 4;

Eq1.. CW =e= sum(n, CTS(n));
Eq2(n).. CTS(n) =e= sum(i, CS(n,i));
Eq3(n).. CTS(n) =e= sum(i, CR(i,n));
Eq4(i).. Fin(i) =e= sum(n, CS(n,i)) + sum(j, FR(j,i));
Eq5(i).. Fout(i) =e= sum(n, CR(i,n)) + sum(j, FR(i,j));
IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;
Eq8(i).. Fin(i) =e= Fout(i);
Eq9(n).. CTS(n) =l= CSmax(n);
Eq10(i).. Tout(i) =l= Toutlim(i);
Eq13(i).. Fin(i) =l= Finmax(i);

Eq15(i,n).. CS(n,i) =l= CSmax(n)*ys(n,i);
Eq16(i).. sum(n, ys(n,i)) =l= 1;
Eq17(i,n).. CR(i,n) =l= CSmax(n)*yr(n,i);
Eq18(i).. sum(n, yr(n,i)) =l= 1;

Eq19(n).. sum(i, CR(i,n)*Tout(i)) =l= Tretmax(n)*CTS(n);
Eq20(i).. Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);

Eq21(n).. sum(i, G1(i,n)) =l= Tretmax(n)*CTS(n);
Eq22(i).. Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) +
sum(j,G2(j,i)) =e= G3(i);

Eq37(i,n).. G1(i,n) =g= Finmax(i)*Tout(i) +
CR(i,n)*Toutlim(i) - Finmax(i)*Toutlim(i);
Eq38(i,n).. G1(i,n) =l= Finmax(i)*Tout(i);
Eq39(i,n).. G1(i,n) =l= CR(i,n)*Toutlim(i);
Eq40(j,i).. G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i).. G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i).. G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i).. G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i).. G3(i) =l= Finmax(i)*Tout(i);
Eq45(i).. G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1

/Eq1,Eq2,Eq3,Eq4,Eq5,Eq8,Eq9,IntRec,Eq15,Eq16,Eq17,Eq18,Eq21

,

```
Eq22,Eq37,Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45/;
```

```
Model Fresh water2
```

```
/Eq1,Eq2,Eq3,Eq4,Eq5,Eq8,Eq9,IntRec,Eq10,Eq13,  
Eq15,Eq16,Eq17,Eq18,Eq19,Eq20/;
```

```
Solve Fresh water1 using MIP minimising CW;
```

```
Solve Fresh water2 using MINLP minimising CW;
```

```
Tret(n) = sum(i, CR.l(i,n)*Tout.l(i))/CTS.l(n)
```

```
Display CTS.l, CS.l, CR.l, Fin.l, Finmax, Tout.l,  
Toutlim,Tret;
```

A.3. Case Study

A.3.1. Case 1 – GAMS Input File

A.3.1.1. Case 1 – Single source targeting – Sub-problem A

Sets

```
i processes / OP1, OP2, OP3, OP4, OP5, OP6 /;
```

```
alias(i,j);
```

Parameters

```
Q(i)          Heatload (kW)
/  OP1 100815
   OP2 90583
   OP3 19590
   OP4 27180
   OP5 19459
   OP6 952402 /
```

```
Tinlim(i)    limiting inlet temperature (deg C)
```

```
/  OP1 30
   OP2 30
   OP3 30
   OP4 30
   OP5 30
   OP6 27 /
```

```
Toutlim(i)   limiting outlet temperature (deg C)
```

```
/  OP1 60
   OP2 60
   OP3 60
   OP4 60
   OP5 60
   OP6 40 /
```

```
T          Temperature of fresh cooling water supplied from cooling
tower n (deg C)
```

```
/ 24 /
```

```
cp          Heat Capacity (kJ per kg per deg C)
```

```
/ 4187 /
```

```
Tret       Return Temperature
```

Finmax(i) Maximum inlet flow (kg per hour);

Finmax(i) = Q(i)*3600/(cp*(Toutlim(i)-Tinlim(i)));

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg per hour)

CR(i) Return water flow from operation i to Cooling Tower n (kg per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

CPtot Total fresh cooling water from all sources (kg per hour);

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

Eq8(i) Water balance across unit

IntRec Internal Recycle of process i

Eq12(i) Duty definition of process i

Eq13(i) Maximum inlet Flow;

Eq1.. CW =e= sum(i, CS(i));

Eq4(i).. CS(i) + sum(j, FR(j,i)) =e= Fin(i);

Eq5(i).. CR(i) + sum(j, FR(i,j)) =e= Fout(i);

IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

Eq8(i).. Fin(i) =e= Fout(i);

Eq12(i).. Q(i)*3600/cp + CS(i)*T + sum(j, FR(j,i)*Toutlim(j)) =e= Fout(i)*Toutlim(i);

Eq13(i).. Fin(i) =l= Finmax(i);

Model Fresh water /all/;

Solve Fresh water using LP minimising CW;

```
Tret = sum(i, CR.l(i)*Toutlim(i))/CW.l;  
Display CS.l, CR.l, Tret;
```

A.3.1.2. Case 1 – Single source targeting – Sub-problem B

Sets

i processes / OP7, OP8 , OP9, OP10 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)

/ OP7 100668

OP8 16119

OP9 66775

OP10 56489 /

Tinlim(i) limiting inlet temperature (deg C)

/ OP7 37

OP8 29

OP9 33

OP10 29 /

Toutlim(i) limiting outlet temperature (deg C)

/ OP7 60

OP8 37

OP9 60

OP10 60 /

T Temperature of fresh cooling water supplied from cooling tower n (deg C)

/ 29 /

cp Heat Capacity (kJ per kg per deg C)

/ 4187 /

Tret Return Temperature

Finmax(i) Maximum inlet flow (kg per hour);

Finmax(i) = Q(i)*3600/(cp*(Toutlim(i)-Tinlim(i)));

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (kg per hour)

Fin(i) Total water flow to process i (kg per hour)

FR(i,j) Recycle flow to process j from process i (kg per hour)
 CR(i) Return water flow from operation i to Cooling Tower n (kg per hour)
 Fout(i) Total water flow exiting process i (deg C)
 Tout(i) Outlet temperature from process i (deg C)
 CPtot Total fresh cooling water from all sources (kg per hour);

Variable

CW Total fresh water usage;

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq8(i) Water balance across unit

Eq12(i) Duty definition of process i

Eq13(i) Maximum inlet Flow;

Eq1.. CW =e= sum(i, CS(i));

Eq4(i).. CS(i) + sum(j, FR(j,i)) =e= Fin(i);

Eq5(i).. CR(i) + sum(j, FR(i,j)) =e= Fout(i);

IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;

Eq8(i).. Fin(i) =e= Fout(i);

Eq12(i).. Q(i)*3600/cp + CS(i)*T + sum(j, FR(j,i)*Toutlim(j)) =e= Fout(i)*Toutlim(i);

Eq13(i).. Fin(i) =l= Finmax(i);

Model Fresh water /all/;

Solve Fresh water using LP minimising CW;

Tret = sum(i, CR.l(i)*Toutlim(i))/CW.l;

Display CS.l, CR.l, Tret;

A.3.1.3. Case 1 – Unified Targeting

Sets

```
i processes / OP1, OP2, OP3, OP4, OP5, OP6, OP7, OP8, OP9,  
OP10 /  
n Towers / T1, T2 /;
```

```
alias(i,j);
```

Parameters

```
Q(i) Heatload (kW)  
/ OP1 100815  
OP2 90583  
OP3 19590  
OP4 27180  
OP5 19459  
OP6 952402  
OP7 100668  
OP8 16119  
OP9 66775  
OP10 56489 /
```

```
Tinlim(i) Limiting inlet temperature (deg C)  
/ OP1 30  
OP2 30  
OP3 30  
OP4 30  
OP5 30  
OP6 27  
OP7 37  
OP8 29  
OP9 33  
OP10 29 /
```

```
Toutlim(i) Limiting outlet temperature (deg C)  
/ OP1 60  
OP2 60  
OP3 60  
OP4 60  
OP5 60  
OP6 40  
OP7 60  
OP8 37  
OP9 60
```

OP10 60 /

T(n) Temperature of fresh cooling water supplied
from cooling tower n (deg C)

/ T1 24

T2 29 /

CSmax(n) Maximum capacity of cooling tower n (kg per
hour)

/ T1 87686

T2 12866 /

cp Heat Capacity (kJ per ton per deg C)

/ 4187 /

Tret Return Temperature

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CTS(n) Fresh cooling water from Cooling Tower n (ton
per hour) (NB. called CS(n) in formulation)

CS(n,i) Fresh cooling water flow to process i from
Cooling Tower n (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton
per hour)

CR(i,n) Return water flow from operation i to Cooling
Tower n (ton per hour)

Fout(i) Total water flow exiting process i (deg C);

Variable

CW Total fresh water usage (kg per hour);

Equations

Eq1 Total fresh water material balance supplied
from all cooling towers

Eq2(n) Total fresh water material balance supplied
from cooling tower n

Eq3(n) Total cooling water material balance returned to
cooling tower n

```

Eq4(i)      Total inlet water material balance into cooling
water using operation i
Eq5(i)      Total outlet water material balance from
cooling water using operation i
IntRec      Internal Recycle of process i
Eq8(i)      Water balance across unit
Eq9(n)      Maximum capacity of cooling tower n
Eq12(i)     Duty definition of process i
Eq13(i)     Maximum inlet Flow;

Eq1..       CW =e= sum(n, CTS(n));
Eq2(n)..    CTS(n) =e= sum(i, CS(n,i));
Eq3(n)..    CTS(n) =e= sum(i, CR(i,n));
Eq4(i)..    Fin(i) =e= sum(n, CS(n,i)) + sum(j, FR(j,i));
Eq5(i)..    Fout(i) =e= sum(n, CR(i,n)) + sum(j, FR(i,j));
IntRec(i,j)$ (ord(i)=ord(j)).. FR(i,j) =e= 0;
Eq8(i)..    Fin(i) =e= Fout(i);
Eq9(n)..    CTS(n) =l= CSmax(n);
Eq12(i)..   Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) + sum(j,
FR(j,i)*Toutlim(j))
            =e= Fout(i)*Toutlim(i);
Eq13(i)..   Fin(i) =l= Finmax(i);

Model Fresh water /all/;
Solve Fresh water using LP minimising CW;
Tret = sum(i, (sum(n, CR.l(i,n))*Toutlim(i))/CW.l);

Display CTS.l, CS.l, CR.l, Tret, Fin.l, Finmax;

```

A.3.2. Case 3 – GAMS Input File

A.3.2.1. Case 3 – Single source targeting – Sub-problem A

Sets

```
i processes / OP1, OP2, OP3, OP4, OP5, OP6 /;
```

```
alias(i,j);
```

Parameters

```
Q(i)          Heatload (MW)
  / OP1 100815
    OP2 90583
    OP3 19590
    OP4 27180
    OP5 19459
    OP6 952402 /
```

```
Tinlim(i)    Limiting inlet temperature (deg C)
  / OP1 30
    OP2 30
    OP3 30
    OP4 30
    OP5 30
    OP6 27 /
```

```
Toutlim(i)   Limiting outlet temperature (deg C)
  / OP1 60
    OP2 60
    OP3 60
    OP4 60
    OP5 60
    OP6 40 /
```

```
T           Temperature of fresh cooling water supplied
from cooling tower n (deg C)
  / 24 /
```

```
Tretmax     Return Temperature to cooling tower
  / 42 /
```

```
cp          Heat Capacity (kJ per kg per deg C)
  / 4187 /
```

Tret Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

Finmax(i) = $Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton per hour)

CR(i) Return water flow from operation i to Cooling Tower n (ton per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

G1 Linearization variable 1 for term CR(i.n)*Tout(j)

G2 Linearization variable 2 for term

FR(j.i)*Tout(j)

G3 Linearization variable 3 for term

Fin(i)*Tout(i);

Variable

CW Total fresh water usage (ton per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq8(i) Water balance across unit

Eq10(i) Limiting outlet temperature specification for process i

Eq13(i) Maximum inlet Flow

Eq19 Return temperature to cooling tower

Eq20(i) Duty definition of process i

Eq21 Linearised equation for the return temperature to cooling tower

Eq22(i) Linearised equation for the duty definition of process i

Eq37(i) G1 equality 2
 Eq38(i) G1 equality 3
 Eq39(i) G1 equality 4
 Eq40(j,i) G2 equality 1
 Eq41(j,i) G2 equality 3
 Eq42(j,i) G2 equality 4
 Eq43(i) G3 equality 2
 Eq44(i) G3 equality 3
 Eq45(i) G3 equality 4;

Eq1.. CW =e= sum(i, CS(i));
 Eq4(i).. CS(i) + sum(j, FR(j,i)) =e= Fin(i);
 Eq5(i).. CR(i) + sum(j, FR(i,j)) =e= Fout(i);
 IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;
 Eq8(i).. Fin(i) =e= Fout(i);
 Eq10(i).. Tout(i) =l= Toutlim(i);
 Eq13(i).. Fin(i) =l= Finmax(i);

Eq19.. sum(i, CR(i)*Tout(i)) =l= Tretmax*CW;
 Eq20(i).. Q(i)*3600/cp + CS(i)*T + sum(j,
 FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);
 Eq21.. sum(i, G1(i)) =l= Tretmax*CW;
 Eq22(i).. Q(i)*3600/cp + CS(i)*T + sum(j,G2(j,i)) =e=
 G3(i);

Eq37(i).. G1(i) =g= Finmax(i)*Tout(i) + CR(i)*Toutlim(i)
 - Finmax(i)*Toutlim(i);
 Eq38(i).. G1(i) =l= Finmax(i)*Tout(i);
 Eq39(i).. G1(i) =l= CR(i)*Toutlim(i);
 Eq40(j,i).. G2(j,i) =g= Finmax(i)*Tout(j) +
 FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
 Eq41(j,i).. G2(j,i) =l= Finmax(i)*Tout(j);
 Eq42(j,i).. G2(j,i) =l= FR(j,i)*Toutlim(j);
 Eq43(i).. G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
 - Finmax(i)*Toutlim(i);
 Eq44(i).. G3(i) =l= Finmax(i)*Tout(i);
 Eq45(i).. G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1 / Eq1,Eq4,Eq5,IntRec,Eq8,Eq21,Eq22,Eq37,

Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45 /;

Model Fresh water2 /

Eq1,Eq4,Eq5,IntRec,Eq8,Eq10,Eq13,Eq19,Eq20 /;

```
Solve Fresh water1 using LP minimising CW;  
Solve Fresh water2 using NLP minimising CW;  
    Tret = sum(i, CR.l(i)*Tout.l(i))/CW.l  
  
Display CS.l, CR.l, Tret;
```


A.3.2.2. Case 3 – Single source targeting – Sub-problem B

Sets

i processes / OP7, OP8, OP9, OP10 /;

alias(i,j);

Parameters

Q(i) Heatload (kW)
/ OP7 100668
OP8 16119
OP9 66775
OP10 56489 /

Tinlim(i) Limiting inlet temperature (deg C)
/ OP7 37
OP8 29
OP9 33
OP10 29 /

Toutlim(i) Limiting outlet temperature (deg C)
/ OP7 60
OP8 37
OP9 60
OP10 60 /

T Temperature of fresh cooling water supplied
from cooling tower n (deg C)
/ 29 /

Tretmax Return Temperature to cooling tower
/ 42 /

cp Heat Capacity (kJ per kg per deg C)
/ 4187 /

Tret Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

Finmax(i) = Q(i)*3600/(cp*(Toutlim(i)-Tinlim(i)));

Positive Variables

CS(i) Fresh cooling water flow to process i from Cooling Tower n (ton per hour)
 Fin(i) Total water flow to process i (ton per hour)
 FR(i,j) Recycle flow to process j from process i (ton per hour)
 CR(i) Return water flow from operation i to Cooling Tower n (ton per hour)
 Fout(i) Total water flow exiting process i (deg C)
 Tout(i) Outlet temperature from process i (deg C)
 G1 Linearization variable 1 for term CR(i.n)*Tout(j)
 G2 Linearization variable 2 for term FR(j.i)*Tout(j)
 G3 Linearization variable 3 for term Fin(i)*Tout(i);

Variable

CW Total fresh water usage (ton per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers

Eq4(i) Total inlet water material balance into cooling water using operation i

Eq5(i) Total outlet water material balance from cooling water using operation i

IntRec Internal Recycle of process i

Eq8(i) Water balance across unit

Eq10(i) Limiting outlet temperature specification for process i

Eq13(i) Maximum inlet Flow

Eq19 Return temperature to cooling tower

Eq20(i) Duty definition of process i

Eq21 Linearised equation for the return temperature to cooling tower

Eq22(i) Linearised equation for the duty definition of process i

Eq37(i) G1 equality 2

Eq38(i) G1 equality 3

Eq39(i) G1 equality 4

Eq40(j,i) G2 equality 1

Eq41(j,i) G2 equality 3

Eq42(j,i) G2 equality 4

Eq43(i) G3 equality 2

```

Eq44(i)      G3 equality 3
Eq45(i)      G3 equality 4;

Eq1..        CW =e= sum(i, CS(i));
Eq4(i)..     CS(i) + sum(j, FR(j,i)) =e= Fin(i);
Eq5(i)..     CR(i) + sum(j, FR(i,j)) =e= Fout(i);
IntRec(i,j)$ (ord(i)=ord(j)).. FR(i,j) =e= 0;
Eq8(i)..     Fin(i) =e= Fout(i);
Eq10(i)..    Tout(i) =l= Toutlim(i);
Eq13(i)..    Fin(i) =l= Finmax(i);

Eq19..       sum(i, CR(i)*Tout(i)) =l= Tretmax*CW;
Eq20(i)..    Q(i)*3600/cp + CS(i)*T + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);
Eq21..       sum(i, G1(i)) =l= Tretmax*CW;
Eq22(i)..    Q(i)*3600/cp + CS(i)*T + sum(j,G2(j,i)) =e=
G3(i);

Eq37(i)..    G1(i) =g= Finmax(i)*Tout(i) + CR(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq38(i)..    G1(i) =l= Finmax(i)*Tout(i);
Eq39(i)..    G1(i) =l= CR(i)*Toutlim(i);
Eq40(j,i)..  G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i)..  G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i)..  G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i)..    G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i)..    G3(i) =l= Finmax(i)*Tout(i);
Eq45(i)..    G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1 / Eq1,Eq4,Eq5,IntRec,Eq8,Eq21,Eq22,Eq37,
Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45 /;

Model Fresh water2 /
Eq1,Eq4,Eq5,IntRec,Eq8,Eq10,Eq13,Eq19,Eq20 /;

Solve Fresh water1 using LP minimising CW;
Solve Fresh water2 using NLP minimising CW;
      Tret = sum(i, CR.l(i)*Tout.l(i))/CW.l

Display CS.l, CR.l, Tret;

```

A.3.2.3. Case 3 – Unified Targeting

Sets

```
i processes / OP1, OP2, OP3, OP4, OP5, OP6, OP7, OP8, OP9,  
OP10 /  
n Towers / T1, T2 /;
```

```
alias(i,j);
```

Parameters

```
Q(i) Heatload (kW)  
/ OP1 100815  
OP2 90583  
OP3 19590  
OP4 27180  
OP5 19459  
OP6 952402  
OP7 100668  
OP8 16119  
OP9 66775  
OP10 56489 /
```

```
Tinlim(i) Limiting inlet temperature (deg C)  
/ OP1 30  
OP2 30  
OP3 30  
OP4 30  
OP5 30  
OP6 27  
OP7 37  
OP8 29  
OP9 33  
OP10 29 /
```

```
Toutlim(i) Limiting outlet temperature (deg C)  
/ OP1 60  
OP2 60  
OP3 60  
OP4 60  
OP5 60  
OP6 40  
OP7 60  
OP8 37  
OP9 60
```

OP10 60 /

T(n) Temperature of fresh cooling water supplied
from cooling tower n (deg C)

/ T1 24

T2 29 /

CSmax(n) Maximum capacity of cooling tower n (kg per
hour)

/ T1 101813

T2 14939 /

Tretmax(n) Return Temperature to cooling tower

/ T1 42

T2 42 /

cp Heat Capacity (kJ per kg per deg C)

/ 4187 /

Tret(n) Return Temperature to cooling tower

Finmax(i) Maximum inlet flow (kg per hour);

$Finmax(i) = Q(i) * 3600 / (cp * (Toutlim(i) - Tinlim(i)))$;

Positive Variables

CTS(n) Fresh cooling water from Cooling Tower n (ton
per hour)

CS(n,i) Fresh cooling water flow from Cooling Tower n
to process i (ton per hour)

Fin(i) Total water flow to process i (ton per hour)

FR(i,j) Recycle flow to process j from process i (ton
per hour)

CR(i,n) Return water flow from operation i to Cooling
Tower n (ton per hour)

Fout(i) Total water flow exiting process i (deg C)

Tout(i) Outlet temperature from process i (deg C)

G1 Linearization variable 1 for term

$CR(i,n) * Tout(j)$

G2 Linearization variable 2 for term

$FR(j,i) * Tout(j)$

G3 Linearization variable 3 for term

$Fin(i) * Tout(i)$;

Variable

CW Total fresh water usage (ton per hour);

Equations

Eq1 Total fresh water material balance supplied from all cooling towers
 Eq2(n) Total fresh water material balance supplied from cooling tower n
 Eq3(n) Total cooling water material balance returned to cooling tower n
 Eq4(i) Total inlet water material balance into cooling water using operation i
 Eq5(i) Total outlet water material balance from cooling water using operation i
 IntRec Internal Recycle of process i
 Eq8(i) Water balance across unit
 Eq9(n) Maximum capacity of cooling tower n
 Eq10(i) Limiting outlet temperature specification for process i
 Eq13(i) Maximum inlet Flow
 Eq19(n) Return temperature to cooling tower
 Eq20(i) Duty definition of process i
 Eq21(n) Linearised equation for the return temperature to cooling tower
 Eq22(i) Linearised equation for the duty definition of process i
 Eq37(i,n) G1 equality 2
 Eq38(i,n) G1 equality 3
 Eq39(i,n) G1 equality 4
 Eq40(j,i) G2 equality 1
 Eq41(j,i) G2 equality 3
 Eq42(j,i) G2 equality 4
 Eq43(i) G3 equality 2
 Eq44(i) G3 equality 3
 Eq45(i) G3 equality 4;

Eq1.. CW =e= sum(n, CTS(n));
 Eq2(n).. CTS(n) =e= sum(i, CS(n,i));
 Eq3(n).. CTS(n) =e= sum(i, CR(i,n));
 Eq4(i).. Fin(i) =e= sum(n, CS(n,i)) + sum(j, FR(j,i));
 Eq5(i).. Fout(i) =e= sum(n, CR(i,n)) + sum(j, FR(i,j));
 IntRec(i,j)\$ (ord(i)=ord(j)).. FR(i,j) =e= 0;
 Eq8(i).. Fin(i) =e= Fout(i);
 Eq9(n).. CTS(n) =l= CSmax(n);

```

Eq10(i)..      Tout(i) =l= Toutlim(i);
Eq13(i)..      Fin(i) =l= Finmax(i);

Eq19(n)..      sum(i, CR(i,n)*Tout(i)) =l= Tretmax(n)*CTS(n);
Eq20(i)..      Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) + sum(j,
FR(j,i)*Tout(j)) =e= Fout(i)*Tout(i);

Eq21(n)..      sum(i, G1(i,n)) =l= Tretmax(n)*CTS(n);
Eq22(i)..      Q(i)*3600/cp + sum(n, CS(n,i)*T(n)) +
sum(j,G2(j,i)) =e= G3(i);

Eq37(i,n)..    G1(i,n) =g= Finmax(i)*Tout(i) +
CR(i,n)*Toutlim(i) - Finmax(i)*Toutlim(i);
Eq38(i,n)..    G1(i,n) =l= Finmax(i)*Tout(i);
Eq39(i,n)..    G1(i,n) =l= CR(i,n)*Toutlim(i);
Eq40(j,i)..    G2(j,i) =g= Finmax(i)*Tout(j) +
FR(j,i)*Toutlim(j) - Finmax(i)*Toutlim(j);
Eq41(j,i)..    G2(j,i) =l= Finmax(i)*Tout(j);
Eq42(j,i)..    G2(j,i) =l= FR(j,i)*Toutlim(j);
Eq43(i)..      G3(i) =g= Finmax(i)*Tout(i) + Fin(i)*Toutlim(i)
- Finmax(i)*Toutlim(i);
Eq44(i)..      G3(i) =l= Finmax(i)*Tout(i);
Eq45(i)..      G3(i) =l= Fin(i)*Toutlim(i);

Model Fresh water1
/Eq1,Eq2,Eq3,Eq4,Eq5,Eq8,Eq9,IntRec,Eq21,Eq22,
Eq37,Eq38,Eq39,Eq40,Eq41,Eq42,Eq43,Eq44,Eq45/;

Model Fresh water2
/Eq1,Eq2,Eq3,Eq4,Eq5,Eq8,Eq9,IntRec,Eq10,Eq13,Eq19,Eq20/;

Solve Fresh water1 using LP minimising CW;
Solve Fresh water2 using NLP minimising CW;
      Tret(n) =      sum(i, CR.l(i,n)*Tout.l(i))/CTS.l(n)

Display CTS.l, CS.l, CR.l, Fin.l, Finmax, Tout.l,
Toutlim,Tret;

```