## **APPENDIX**

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## **Appendix A1: Technical efficiency in panel frontier models**

Two types of panel data production frontier models can be adopted in measuring technical efficiency. In the first, technical efficiency is allowed to vary across industries, but is assumed constant through time for each industry. However, the assumption of time invariance of technical efficiency may be weak in long panels. The second type of panel data production frontier models allows technical efficiency to vary across industries and through time for each industry. Kumbhakar and Lovell (2000:97) provide a detailed discussion regarding panel frontiers.

### **A1.1 Time-invariant technical efficiency**

Since observations exist on *I* industries indexed  $i = 1, \ldots, I$  by through *T* time periods, indexed by  $t = 1,...,T$ . A Cobb-Douglas production frontier with time invariant technical efficienc y can be written as:

$$
\ln Y_{it} = \alpha_0 + \sum_{n} \alpha_n \ln X_{nit} + v_{it} - \mu_i,
$$
 A.1

Where  $v_{it}$  represents random statistical noise and  $\mu_i \geq 0$  represents technical inefficiency. The structure of the production function is assumed to be constant through time and no allowance is made for technical change. In essence, this model is similar to the cross-section production frontier except for the addition of time subscripts to output, inputs and to the statisti cal noise. The parameters of the model and technical efficiency can be estimated in a number of ways<sup>54</sup>.

 $54$  It should be pointed out that maximum likelihood estimation of panel data estimation of a stochastic production frontier panel data with time invariant technical efficiency is structurally similar to the procedure followed in cross-sectional data.

#### **(a). Fixed effects model**

This is the simplest panel model to estimate. In order to adapt this model to the efficiency measurement the requirement that  $\mu_i \geq 0$  has to be met. It is further assumed that  $v_i$  are  $iid(0, \sigma_v^2)$  and are uncorrelated with the regressors. No distributional assumption is required for the  $\mu_i$ , it can be correlated with the regressors or with  $v_{it}$ . Since  $\mu_i$  are treated as fixed or non-random, they become industry specific intercept parameters to be estimated along with the  $\alpha_n$  s. This model can be estimated by OLS in the form:

$$
\ln Y_{it} = \alpha_{0i} + \sum_{n} \alpha_n \ln X_{nit} + v_{it},
$$

Where the  $\alpha_{0i} = (\alpha_0 - u_i)$  are the industry-specific intercepts. Estimation is accomplished either by suppressing  $\alpha_0$  and estimating *I* industry specific intercepts or by retaining  $\alpha_0$  and estimating  $(I-1)$  industry-specific intercepts or by applying a within transformation, in which all data are expressed in terms of deviations from industry means and the *I* intercepts are recovered as means of industry residuals<sup>55</sup>. A normalisation is employed after estimation in which:  $\hat{\alpha}_0 = \max_i {\hat{\alpha}_{0i}}$  A.3

and the resultant  $\mu_i$  is derived from:

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$$
\hat{\mu}_i = \hat{\alpha}_0 - \hat{\alpha}_{0i} \tag{A.4}
$$

this ensures that all  $\hat{\mu}_i \geq 0$  and the industry-specific estimates of technical efficiency are then given by:

$$
TE_i = \exp\{-\hat{\mu}_i\}
$$
 A.5

In a fixed-effects model one industry will be 100 per cent technically efficient, and the technical efficiencies of others are computed relative to the technically

<sup>55</sup> Each of these variants is referred to as the least squares dummy variables (LSDV).

efficient industry. The fixed effects model is simple and has nice consistency properties for the industry-specific technical efficiency. The drawback is that while the fixed effects  $(\mu_i)$  are intended to capture variation across industries in time-invariant technical efficiency, they also capture the effects of all phenomena that vary across industries but are time invariant for each industry.

### **(b). Random effects model**

In this model the  $\mu_i$  are randomly distributed with constant mean and variance, are assumed to be uncorrelated with the regressors and with the  $v_{it}$ . No distributional assumptions are made regarding  $\mu_i$  only that it should be non negative. The  $v_{it}$  are required as before to have zero expectation and constant variance. This modification allows for the inclusion of time-invariant regressors in the model and the model can be written as:

$$
\ln Y_{it} = [\alpha_0 - E(\mu_i)] + \sum_{n} \alpha_n \ln X_{nit} + v_{it} - [\mu_i - E(\mu_i)]
$$
  
=  $\alpha_0^* + \sum_{n} \alpha_n \ln X_{nit} + v_{it} - \mu_i^*$  A.6

where the underlying assumption that the  $\mu_i$  are random rather than fixed permits some of the  $X_{\text{nit}}$  to be time invariant. This random effects model fits into the one way error components model in panel data literature and can be estimated by the standard two step generalised least squares GLS method. Once  $\alpha_0^*$  and  $\alpha_n s$  have been estimated using feasible GLS, the  $\mu_i^*$  can be generated from the residuals by means of equation A.7 below:

$$
\hat{\mu}_i^* = \frac{1}{T} \sum_t \left( \ln Y_{it} - \hat{\alpha}_0^* - \sum_n \hat{\alpha}_n \ln X_{nit} \right)
$$

Estimates of  $\mu_i$  are again obtained by means of normalisation such that:

$$
\hat{\mu}_i = \max_i \{ \hat{\mu}_i^* \} - \hat{\mu}_i^* \tag{A.8}
$$

The estimates will be consistent as both *T* and *I* tend to infinity. Estimates of industry-specific technical efficiency are then obtained by substituting $\hat{u}_i$  into equation A.5. In tune with the fixed effects model, estimators for the random effects model also require that at least one industry be 100 per cent technically efficiency so that the technical efficiencies of the remaining industries are measured relative to the technically efficient industries.

### **A1.1 Time-varying technical efficiency**

in operating environments that are comp etitive. Technical inefficiency cannot remain constant for long time periods. While it is desirable to relax this assumption, the relaxation however, happens only at the cost of additional second by maximum likelihood approach or method of moments<sup>56</sup>. The assumption that technical efficiency is constant through time is very strong parameters to be estimated. Two approaches have been followed in the estimation of the time-varying technical efficiency model. The first approach has time-varying technical efficiency modelled using fixed or random effects and the

## **(a) Fixed effects and random effects models**

Cornwell, Schmidt, and Sickles (1990) and Kumbhakar (1990) proposed a stochastic production panel data model with time-varying technical efficiency. The model is specified as:

$$
\ln Y_{it} = \alpha_{0t} + \sum_{n} \alpha_n \ln X_{nit} + v_{it} - \mu_{it}
$$

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<sup>56</sup> If the independence and distributional assumptions are tenable, then it is possible to use maximum likelihood estimation. It is also possible to estimate parameters in equation A.9 using method of moments.

$$
= \alpha_{it} + \sum_{n} \alpha_{n} \ln X_{nit} + v_{it}
$$

where  $\alpha_{0t}$  is the production frontier intercept common to all industries in period *t*,  $\alpha_{it} = \alpha_{0t} - \mu_{it}$  is the intercept for industry *i* in period *t*, the remainder of the variables are as previously defined. With an *I* ×*T* panel it is not possible to obtain estimates of all  $I \cdot T$  intercepts  $\alpha_{i}$ , the *N* slope parameters  $\alpha_{n}$  and  $\sigma_{v}^{2}$ . Cornwell, Schmidt, and Sickles (1990) addressed this problem by specifying:

$$
\alpha_{it} = \Omega_{i1} + \Omega_{i2}t + \Omega_{i3}t^2 \tag{A.10}
$$

While this reduces the number of intercepts to be estimated, Lee and Schmidt (1993) proposed an alternative specification in which the  $\mu_{it}$  in equation A.9 are specified as:

$$
\mu_{it} = \alpha(t) \bullet \mu_i \tag{A.11}
$$

Where the function  $\alpha(t)$  is specified as a set of time dummy variables  $\alpha_t$ . This model is appropriate for short panels. Once the  $\alpha_t s$  and  $\mu_i$  are estimated, then

$$
\mu_{ii} = \sum_{i} {\hat{\alpha}_{i} \hat{\mu}_{i}} - (\hat{\alpha}_{i} \hat{\mu}_{i}) \text{ and } TE = \exp\{-\hat{\mu}_{ii}\}\
$$

# **Appendix A2: The Battese and Coelli (1992) specification**

Maximum likelihood estimates of stochastic frontier production functions for panel data with time-varying or invariant efficiencies in the spirit of Battese and Coelli (1992) can be estimated. In particular, Battese and Coelli (1992) propose a stochastic frontier production function fo r panel data which has firm effects that are assumed to be distributed as trun cated normal random variables, and are permitted to vary systematically with time. The model may be expressed as:

$$
Y_{it} = x_{it}\alpha + (\nu_{it} - \mu_{it}) \quad i = 1, \dots, N, t = 1, \dots, T,
$$

where  $Y_{it}$  is the logarithm of the output of the  $i$ -th industry in the  $t$ -th time period;  $x_{i}$  is a  $k \times 1$  vector of inputs of the *i*-th industry in the *t*-th time period;  $\alpha$  is a vector of unknown parametes; the  $v_i$  are random variables which are assumed to be *iid*  $N(0, \sigma v^2)$ , and independent of the  $\mu_i = (\mu_i \exp(-\eta(t-T))),$ where the  $\mu_i$  are non-negative random variables that are assumed to account for technical inefficiency in production and are assumed to be *iid* as truncations at zero of the  $N(\mu, \sigma\mu^2)$  distribution;  $\eta$  is a parameter to be estimated; and the panel of data need not be complete.

Coelli (1996) utilises the parameterization of Battese and Corra (1977) to replace  $\sigma v^2$  and  $\sigma \mu^2$  with  $\sigma^2 = \sigma v^2 + \sigma^2 \mu^2$  and  $\gamma = \frac{\sigma \mu^2}{(\sigma v^2 + \sigma \mu^2)}$ σν $^-$  + σμ  $\sigma^2 = \sigma v^2 + \sigma^2 \mu^2$  and  $\gamma = \frac{\sigma \mu}{(\sigma v^2 + \sigma \mu^2)}$  in the context ofe maximum likelihood estimation. The parameter,  $\gamma$ , lies between 0 and 1 and this maximization process. The log-likelihood function of this model is presented in Battese and Coelli (1992). range is searched to provide a good starting value for use in an iterative

The imposition of restrictions upon model A.13 can provide a number of the special cases of this particular model which have appeared in the literature such as Battese, Coelli and Colby (1989), Battese and Coelli (1988), Pitt and Lee (1981), Aigner, Lovell and Schmidt (1977) as well as Stevenson (1980. Predictions of individual industry technical efficiencies from stochastic production frontiers can be derived in which the measures of technical efficiency relative to the production frontier (A13) are defined as:

$$
EFF_i = E(Y_i^* | \mu_i, X_i / E(Y_i^* | \mu_i = 0, X_i))
$$

where  $Y_i^*$  is the production of the *i*-th industry, which will be equal to  $exp(Y_i)$ when the dependent variable is in logs. The  $EFF<sub>i</sub>$  will take a value between zero and one. The results of implementing the Battese and Coelli (1992) specification produces estimates in Table A2.1 and A2.2 below and efficiency estimates are found in Table A2.3 and Table A2.4. These results employ output rather than value added for the Cobb-Douglas and translog functions respectively.

Stochastic frontier model: Dependent Variable ln(y)				
Variable	Parameter	Coefficient	Standard error	t-ratio
ln(K)	$\alpha_{\scriptscriptstyle{k}}$	0.1466	0.0355	4.12
ln(N)	$\alpha_{n}$	0.2162	0.0280	7.71
ln(M)	$\alpha_{\rm m}$	0.4507	0.0225	20.04
	$\alpha_{\scriptscriptstyle t}$	0.0120	0.0020	6.17
Constant	$\alpha_0$	1.9121	0.1409	13.57
Sigma-squared	$\sigma^2$	0.2873	0.0464	6.19
Gamma	γ	0.9501	0.0093	102.60
mu	$\mu$	$-0.1026$	0.1889	$-0.54$
eta	η	$-0.1643$	0.0035	$-4.68$
Log likelihood function	366.1463			
LR test of one sided error	1076.8774			
Maximum no. of iterations	100			
No. of cross sections	28			
No. of time periods	23			
Total no. of observations	644			

**Table A2.1: Maximum Likelihood Estimates: Cobb-Douglas production function** 

Note: Error components specification using output rather than value added.

Source: FRONTIER 41 Regression output from data obtained from www.tips.org.za, www.statssa.gov.za and www.resbank.co.za



**Table A2.2: Maximum Likelihood Estimates: Translog production function** 

N ote: Error components specification using output instead of value added.

Source: FRONTIER 41 Regression output from data obtained from www.tips.org.za, www.statssa.gov.za and www.resbank.co.za





Note: Error components specification using output instead of value added.

Source: FRONTIER 41 R egression ou tput from data obtained from <u>www.tips.org.za, www.statssa.gov.za</u> and www.resbank.co.za



## **Table A2.4 : Translog Production function technical efficiency estimates**

Note: Error components specification using output instead of value added.

Source: FRONTIER 41 Regression output from data obtained from www.tips.org.za, www.statssa.gov.za and www.resbank.co.za

# **Appendix A3: Variable definitions**





Notes: Where necessary nominal variables are deflated with an appropriate price index to obtain real series.

Source:http://ts.easydata.co.za;http://www.tips.org.za;http://www.resbank.co.za; http://www.statssa.gov.za