

**A STRUCTURAL GARCH MODEL:  
AN APPLICATION TO PORTFOLIO RISK MANAGEMENT**

by

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**Abstract**

The primary objective of this study is to decompose the conditional covariance matrix of a system of variables. A structural GARCH model is proposed which makes use of existing multivariate GARCH (MGARCH) models to decompose the covariance matrix. The variables analysed in the study are the All Share index (ALSI) on the Johannesburg stock exchange, the South African Rand/US Dollar exchange rate (R/\$) and the South African 90-day Treasury bill interest rate (Tbill).

The contemporaneous structural parameters in the system of endogenous variables are identified using heteroscedasticity. Although the structural parameters of the system of variables hold important and interesting information, it is not the main focus of this study. Identifying the structural parameters can be seen as a necessary condition to decompose the conditional variance covariance matrix into an endogenous and exogenous part.

The contribution of the study is twofold. The first contribution is methodological in nature, while the second is empirical. The study proposes a methodology that utilises two multivariate GARCH models to decompose the time-varying conditional covariance matrix of a system of assets, without imposing unnecessary constraints on the system. In doing so more information is obtained from decomposing the covariance matrix than what is available from existing or traditional multivariate GARCH models. The information allows the investor to analyse the structural relationships between variables in the system in both the first and the second moments. On an empirical level, the study analyses the structural relationship between financial variables in the South African economy using high-frequency data. The methodology utilised allows for consistent and efficient estimates of the structural contemporaneous relationships between these variables. The study also decomposes the volatility of each individual variable as well as the volatility between the variables. More information is gained on what drives the volatility of these variables, i.e. is volatility generated within the system, alternative to volatility generated from structural innovations or latent factors outside the system. The study finally shows how the information can be utilised in a portfolio management context.

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Praise the Lord!

**Walter Albert de Wet (10 March 2005)**

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## Chapter 1

### INTRODUCTION AND BACKGROUND

#### 1.1 INTRODUCTION

Understanding how different variables react to one another has long been at the core of economics. Variables react to one another not only through the mean, but also through the second moments. This implies that the change in one variable might result not only in a change in the level of another variable, but also affect the volatility of other variables. Depending on the purpose of the research one will be interested in the mean effect or the higher moments, or perhaps both. Many techniques have been developed to obtain a consistent, efficient and unbiased estimate for these relationships that allows for the most accurate analysis. These analyses differ in objective – it might be for forecasting purposes, or understanding the structure of the relationships for policy analysis. Whatever the objective, the best estimate (i.e. in terms of bias, efficiency and consistency) under the given circumstances is always important.

Since modern finance theory has been developed it has been generally accepted that there is a trade-off between risk and return. In efficient markets, the higher the risk, the higher the expected return. Therefore, at the heart of financial analysis is both the level of variables i.e. how they influence one another in the mean, as well as the variance of variables and the relationship between the variances. The levels of these variables represent the expected return, while the variance represents total risk of the variable. Forecasting the variance of these variables has therefore become increasingly important. Being able to forecast the variance of a variable will give some indication of what return to expect from a given investment. This variance is therefore extremely important in the pricing of financial variables. Equally important is to understand the behaviour and structural relationship between these variables. The structural relationship gives an indication of how variables will react when there is a change to other variables in the system.

When working with financial variables, the timeframe under scrutiny differs. If the investor is interested in compiling a long-run strategy, the type of econometric tools used will typically be suited for long-run analysis. The most common tool used is cointegration analysis, which focuses on identifying long-run structural relationships between variables. In most cases economic theory defines the expected direction of the relationship between the variables. The purpose of the econometric analysis is to obtain an estimate of the magnitude of the relationship. Of lesser importance in a long-run strategy (although not neglectable) is the variance of the variables.

If the aim of the analysis is to compile a short-term tactical strategy, the focus will differ from the long-run strategy. The problem that arises here is that in the short-run financial variables often behave different than what economic theory would suggest. These variables are often driven by sentiment and external shocks. The structural relationships between the variables are still important in this strategy, but most important in these short-run strategies are the second moments of the variables. How, and to what extent the volatility is generated between these variables is often at the centre of the short-run analysis.

The focus of this study is to estimate the structural relationship between financial variables; not only through the means but also through the second moments. These estimates will provide information that can be used in the compilation of portfolios, pricing of assets and the better understanding of the structural short-run relationship between different variables. High frequency weekly data is used in determining these relationships. Because of the short-term nature of the analysis, the variances as well as the mean effects are of interest. The first aim is to estimate consistent, efficient and unbiased structural parameters or contemporaneous effects between the high frequency variables. Second, once the structural parameters are estimated, a methodology is proposed to analyse the conditional covariance matrix of these variables. This methodology allows one to decompose the conditional covariance matrix into the volatility that is generated within the system and the volatility that is generated outside the system, due to structural innovations or latent factors. Put

differently, this methodology allows one to estimate the endogenous volatility in the system as well as the volatility driven by exogenous factors (or the exogenous volatility).

When working with financial variables it is not sufficient to make use of single-equation estimates. These variables are determined contemporaneously, and therefore estimates should be solved simultaneously in a system of equations. If the estimates are not solved simultaneously, they will be biased and inconsistent. This type of analysis presents numerous econometric problems. First, when working with a system of endogenous variables, the system is not identified. It is therefore impossible to estimate the structural parameters without any additional information. Only a linear transformation of these parameters is observable, or the so-called reduced-form parameters. Therefore, the first challenge is to obtain additional information that will allow one to recover the structural parameters. This study uses the heteroscedasticity that financial variables so often exhibit, to identify the system.

The second challenge is to estimate the time-varying conditional covariance matrix of the system. The variance as well as the covariance between these variables are important in understanding how the volatility is generated inside (endogenous volatility) and from outside (exogenous volatility) the system. The literature proposes a multivariate GARCH model to analyse this problem. However, most of these models use reduced-form estimates while ignoring the contemporaneous interaction between variables. If it is possible to identify the system, the endogenous and exogenous volatility of the variables can be modelled separately.

## **1.2 OBJECTIVE AND RESEARCH METHODOLOGY**

The primary objective of this study is to decompose the conditional covariance matrix of a system of variables. Therefore, a structural GARCH model is proposed which makes use of existing multivariate GARCH (MGARCH) models to decompose the covariance matrix. This type of analysis allows for the structural analysis of the volatility generated within a

system of variables, as well as the volatility generated from factors outside the system. In most multivariate GARCH models the structural relationships between the variables are ignored, thereby leaving the investor without any idea of how the volatility is generated and what drives it. However, this type of analysis is important, for depending on the source of the innovation, the volatility of variables will differ in periods following the innovation.

In order to satisfy the objective, the contemporaneous parameters in the system of endogenous variables are identified using heteroscedasticity. Moreover, a GARCH model developed by Rigobon and Sack (2003) is employed to identify the structural parameters as well as the time-varying conditional covariance matrix of the structural innovations (exogenous innovations) that drives the variables from outside the system. Once the system is identified the variation of the variables that is explained within the system (endogenous variation) can be recovered. The endogenous variation's time-varying volatility is modelled using the standard multivariate specification proposed by Baba, Engle Kraft and Kroner in Engle and Kroner (1995). Although the structural parameters of the system of variables hold important and interesting information, it is not the main focus of this study. Identifying the structural parameters can be seen as a necessary condition to decompose the conditional variance covariance matrix into an endogenous and exogenous part.

This research analyse the structural relation (in both the first and second moments) between three financial variables of the South African economy. These variables are the All Share index (ALSI) on the Johannesburg stock exchange, the South African Rand/US Dollar exchange rate (R/\$) and the South African 90-day Treasury bill interest rate (Tbill).

### **1.3 CONTRIBUTION OF THE STUDY**

The contribution of the study is twofold. The first contribution is methodological in nature, while the second is empirical. The study proposes a methodology that utilises two multivariate GARCH models to decompose the time-varying conditional covariance matrix

of a system of assets, without imposing unnecessary constraints on the system. In doing so more information is obtained from decomposing the covariance matrix than what is available from existing or traditional multivariate GARCH models. The information allows the investor to analyse the structural relationships between variables in the system in both the first and the second moments.

On an empirical level, the study analyses the structural relationship between financial variables in the South African economy using high-frequency data. The methodology utilised allows for consistent and efficient estimates of the structural contemporaneous relationships between these variables. The study also decomposes the volatility of each individual variable as well as the volatility between the variables. More information is gained on what drives the volatility of these variables, i.e. is volatility generated within the system, alternative to volatility generated from structural innovations or latent factors outside the system. The study finally shows how the information can be utilised in a portfolio management context.

#### **1.4 OUTLINE OF THE STUDY**

The outline of the study is as follows. In chapter 2 the problems associated with the estimation of simultaneous equations are discussed. The problem of identification is explained as well as solutions proposed in the literature. This problem is very important, for identifying structural parameters can be extremely problematic. Wrong applications of solutions can result in spurious relationships.

Once the problem of identification has been discussed, the methodology of identification through heteroscedasticity is explained in chapter 3. This methodology utilises the heteroscedasticity in data (i.e. the volatility of variables differ across time) to obtain additional information for identification of the structural parameters. Some empirical applications of this methodology are briefly highlighted in order to put its application into perspective.

Chapter 4 discusses existing multivariate GARCH models available in the literature. Understanding how these GARCH models are structured is important to identify their uses and shortcomings. The GARCH models used in this study are also discussed in relation to other standard multivariate GARCH models.

Chapter 5 presents a detailed exposition of the proposed methodology to decompose the time-varying methodology of a system of variables. The two GARCH models under consideration are explained as well as the underlying derivation of the models.

A brief empirical review on the relationship between the three variables of interest (i.e. the ALSI, the R/\$ and the Tbill) is given in chapter 6. There are numerous international studies in the literature that estimate the relationship between these variables. A thorough understanding of the relationships between the variables is relevant to conceptualise the importance of the results of this study.

Once the methodology has been explained, in chapter 7 follows the empirical application of the methodology using South African data. The results are analysed and discussed in detail. The structural parameters are identified and the covariance is decomposed into an “endogenous” covariance matrix and an “exogenous” covariance matrix. That is, the covariance is divided into the endogenous volatility inside the system and the exogenous volatility outside the system.

Chapter 8 applies impulse responses to the empirical results obtained in chapter 7. The results are essential in understanding the importance of utilising the additional information contained in decomposing the time-varying covariance matrix of a system of assets. The research is concluded with an application to portfolio risk management to highlight the importance of the proposed methodology.

Finally, in chapter 9, a summary of the research is given and some concluding comments are made.

Chapter 2

THE PROBLEM OF IDENTIFICATION

2.1 INTRODUCTION

In single-equation estimation there exists a one-way or unidirectional effect from the explanatory variables to the dependent variable. In a system of equations, the endogenous variables are random variables determined within the system. These variables are determined by not only other variables in the system, but also by disturbance terms specific to the variables. That implies that the change in one variable will change all the other variables in the system since they are determined simultaneously. When estimating a system one cannot determine the parameters in the system without taking into account the information provided by the other variables. Examples in the literature are in abundance. Perhaps most notable are the demand and supply case analysed by Working (1927) and Klein’s model at the Wharton School (Klein, 1974).

When endogenous variables also serve as explanatory variables, one of the assumptions of the classical linear regression model (CLRM) is violated. This is the assumption that the endogenous variables are assumed fixed in repeated samples. In a system of equations the endogenous variables used as regressors are not distributed independently of the disturbance terms in the equation. When a disturbance term to a specific variable changes, that endogenous variable changes directly. Since the variables are determined contemporaneously within the system, the change in one variable will result in a change of other variables in the system. Equation 2.1 shows such a system.

$$y_i \quad \Gamma + x_i \quad \mathbf{B} = \varepsilon_i \quad \text{for } i = 1, 2, \dots, n. \tag{2.1}$$

$\begin{matrix} 1 \times g & g \times g & 1 \times k & k \times g & 1 \times g \end{matrix}$

Here  $y_i$  is a vector of  $g$  endogenous variables,  $x_i$  is a vector of  $k$  predetermined variables and  $\varepsilon_i$  is a vector of  $g$  stochastic disturbance terms. The covariance matrix of the error terms is assumed to be the same for each observation. Without loss of generalisation, the system in equation 2.1 can be simplified to the bivariate case with only endogenous variables in equation 2.2

$$\begin{aligned} y_{it} &= \delta y_{jt} + \eta_{it} \\ y_{jt} &= \beta y_{it} + \varepsilon_{jt} \end{aligned} \quad \text{or} \quad \begin{matrix} \mathbf{A} & \mathbf{Y} & = & \boldsymbol{\mu} \\ 2 \times 2 & 2 \times 1 & & 2 \times 1 \end{matrix} \quad (2.2)$$

Equation 2.2 represents the structural-form of the system. The standard assumption is that the covariance matrix of the system will be constant at each observation. The covariance matrix of the structural-form is given by equation 2.3

$$\boldsymbol{\Omega} = \begin{bmatrix} \sigma_{\eta\eta} & \sigma_{\eta\varepsilon} \\ \sigma_{\varepsilon\eta} & \sigma_{\varepsilon\varepsilon} \end{bmatrix} \quad (2.3)$$

From equation 2.2 it is clear that given the bivariate case, and  $\delta$  and  $\beta$  non-zero, a change in the disturbance term of one variable will not only result in a change in that variable but will also result in a change in the other variable. Because the endogenous variables are not distributed independent of their disturbance terms, estimators will be biased and inconsistent, even asymptotically (Green, 2000).

In response to this problem researchers have turned to estimating a linear transformation of the structural-form, the reduced-form of the system of equations. In the reduced-form, every endogenous variable is expressed as a function of all exogenous variables in the system. No endogenous variable in the reduced-form is expressed as an explanatory variable. This transformation takes care of the contemporaneous feedback that makes regressors and disturbance terms dependent. Using this transformation, estimators will be unbiased,

consistent and efficient. Equation 2.4 displays the linear transformation (i.e. the reduced-form) of equation 2.2

$$\begin{aligned}
 y_{it} &= \frac{1}{1-\delta\beta} [\delta\varepsilon_{jt} + \eta_{it}] \\
 y_{jt} &= \frac{1}{1-\delta\beta} [\beta\beta_{it} + \varepsilon_{jt}]
 \end{aligned}
 \quad \text{or} \quad
 \begin{matrix}
 \mathbf{Y} &= & \mathbf{A}^{-1} & \boldsymbol{\mu} \\
 2 \times 1 & & 2 \times 2 & 2 \times 1
 \end{matrix}
 \quad (2.4)$$

The reduced-form covariance matrix is given by equation 2.5

$$\Sigma = \begin{matrix} \mathbf{A}^{-1'} & \boldsymbol{\Omega} & \mathbf{A}^{-1} \\ 2 \times 2 & 2 \times 2 & 2 \times 2 \end{matrix} \quad (2.5)$$

One observation from equation 2.4 is important. The structural parameters of equation 2.4 are not directly observable. Only a combination of them is observable. This combination is the reduced-form parameters. If one is interested in predicting the movement of an endogenous variable in the system, the reduced-form parameters are sufficient. However, if one is interested in the structural parameters it is necessary to recover them from the reduced-form parameters. This is not always straightforward, and in some instances, impossible. The problem of identifying structural parameters from the reduced-form is referred to as the problem of identification.

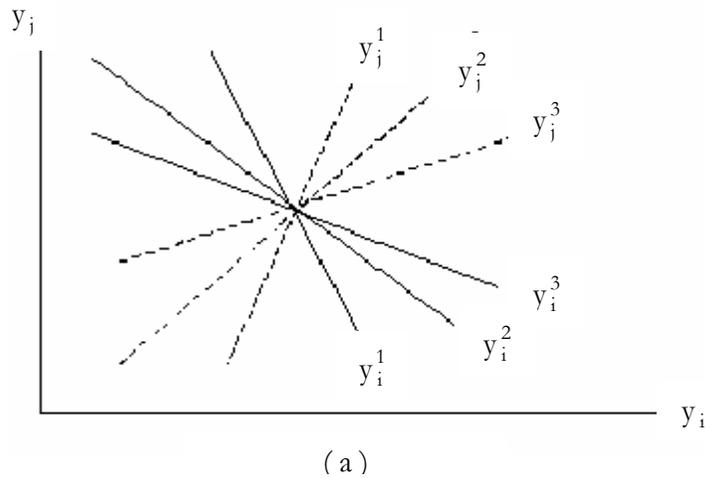
## 2.2 IDENTIFICATION

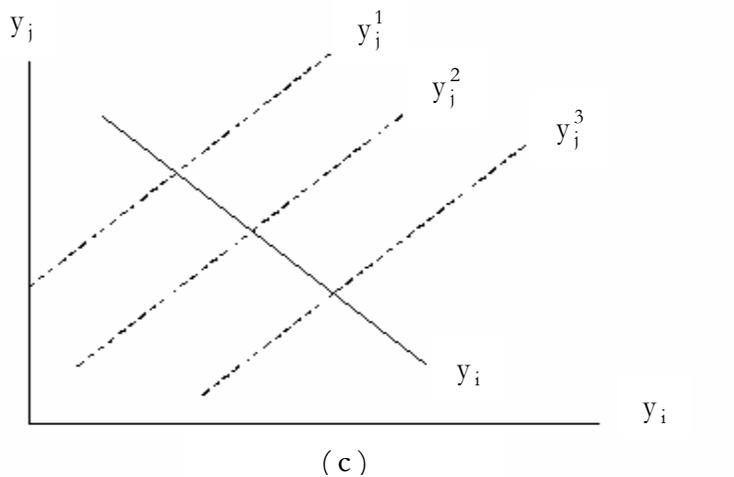
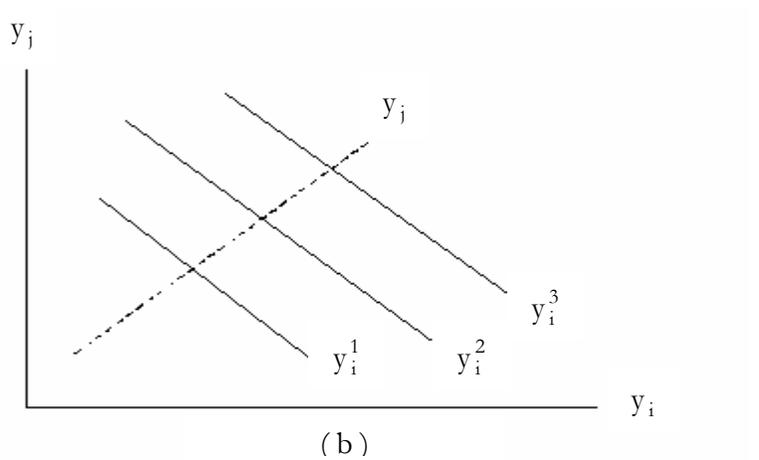
When estimating the behavioural parameters (structural parameters) of a system, one has to solve the reduced-form equations. The reduced-form expresses the endogenous variables simply as a function of the predetermined variables and the stochastic disturbance terms. From these parameters (as expressed in equation 2.4 and equation 2.5) the structural parameters have to be recovered. The reduced-form allows for the application of standard

estimation techniques, since the endogenous variables are expressed as a function of only exogenous (predetermined) variables and disturbance terms, which are assumed independent.

When attempting to retrieve structural parameters from reduced-form parameters, there are three possible cases. The first case is where it is possible to extract unique structural parameters from the reduced-form. In this case the equations are exactly identified, or just identified. The second case, when the system of equations is over-identified, is where it is possible to retrieve the structural parameters, but more than one solution exist for every structural parameter in the system. The third and more problematic case is where it is impossible to retrieve any structural parameters from the reduced-form parameters without any additional information. In this case the system is said to be under-identified or unidentified.

**Figure 2.1: Hypothetical functions for variables  $y_j$  and  $y_i$  and the identification problem**





The problem of identification exists because different sets of structural parameters may be compatible with the same set of data. Figure 2.1 explains the problem of identification. Given variables  $y_i$  and  $y_j$  in equation 2.4 and no other information, there is no way the researcher can be sure that he or she is estimating the true and exact function for  $y_j$  or the function for  $y_i$ . That is, a single observation of  $y_j$  and  $y_i$  represents simply the point of intersection of the appropriate two functions. This is indicated in figure 2.1a. With no additional information, it is not possible to obtain unique estimates for the structural parameters. Given a specific point of intersection, there exist many possible functions for  $y_j$  and  $y_i$  that go through that point. Some additional information on the nature of the two variables is necessary to identify unique functions for them. If some additional information exists on say, variable  $y_j$ , it is possible to identify the function for  $y_j$ . Of course, the reverse

also holds. Figure 2.1b and figure 2.1c indicate these cases. In these two cases the equations are said to be identified since it is possible to obtain unique estimates for each equation.

A more formal method to establish if an equation in a system is under-, exactly- or over-identified is the order and rank conditions for identification (Intriligator et al., 1996). The order and rank conditions deal with the number of endogenous and predetermined variables in a system of equations. For expositional reasons the following notation is introduced to explain the order and rank conditions:

- M = number of endogenous variables in the system
- m = number of endogenous variables in the equation
- K = number of predetermined (exogenous) variables in the system including the intercept
- k = number of predetermined variables in a given equation
- P = partitioned matrix of the reduced-form coefficients containing the coefficients of the predetermined variables

Given the M endogenous variables in the system, there should be M equations. The order conditions for identification are as follows:

*Order condition for Identification of Equation j:*

$$K_j^* \geq M_j.$$

*The number of exogenous variables excluded from equation j ( $K_j^*$ ) must be at least as large as the number of endogenous variables included in equation j ( $M_j$ ).*

The order condition is only a counting rule. It is a necessary but not sufficient condition for identification. It ensures that each structural coefficient has at least one solution, but does not

ensure that it has only one solution. The sufficient condition for uniqueness is the rank condition. The rank condition requires the researcher to partition the matrices and impose restrictions.

*Rank condition for Identification:*

$$\text{rank}[P] = M - 1$$

*The rank of the partitioned matrix of the reduced-form coefficients containing the coefficients on the predetermined variables is equal to the number of endogenous variables less one.*

This condition imposes a restriction on the partitioned matrix of the reduced-form coefficient matrix. In practice it is easy to check both conditions for a small model. For large models, frequently only the order condition is verified (Green, 2000). Given the order and rank conditions one can distinguish between four cases:

1. If  $K-k > m-1$  and the rank of the partitioned matrix  $P$  is  $M-1$ , the equation is over-identified.
2. If  $K-k = m-1$  and the rank of the partitioned matrix  $P$  is  $M-1$ , the equation is exactly identified.
3. If  $K-k \geq m-1$  and the rank of the partitioned matrix  $P$  is less than  $M-1$ , the equation is under-identified.
4. If  $K-k < m-1$  the structural equation is unidentified. The rank of the partitioned matrix  $P$  is bound to be less than  $M-1$ .

From the above discussion it can be determined that the equations in the system such as presented in equation 2.4 are unidentified. As mentioned before, if an equation is identified

(exactly or over) it does not present much of a statistical problem. If the system is under-identified, the only way to obtain the structural parameters from the reduced-form parameters is through imposing some restrictions on the equations. Such restrictions, of course, can only be imposed if their validity can be verified. The additional information for the restrictions is obtained from several sources:

1. Normalisation. In each equation one variable has the coefficient of one. It is similar to putting one variable on the left hand side of the equation. Normalisation directly scales down the number of coefficients to estimate in each equation.
2. Identities. Variable definitions or equilibrium conditions imply that all the coefficients in a particular equation are known. This implies that there are less parameters to estimate which adds additional information to the system.
3. Exclusions. The omission of variables from an equation places zeros on certain coefficients to be estimated.
4. Linear restrictions. Restrictions on the structural parameters may serve to rule out false structures. One example is the restriction of the coefficients in a production function to add up to unity.
5. Restrictions on the disturbance covariance matrix. In the identification of a system, this is similar to restrictions on the slope parameters. For example, one may assume that the structural disturbance terms are uncorrelated.
6. Nonlinearities. In some systems the variables, the parameters or both enter non-linear. This will usually complicate the analysis, but may aid in identification.

### **2.3 REDUCED-FORM VS. STRUCTURAL PARAMETERS**

The question of when it is necessary to use reduced-form parameters and when it is necessary to use structural-form parameters depends on the purpose of the estimation. If the purpose of estimation is to forecast variables, to describe various characteristics of the data, or to search for hypotheses of interest to test a theory, the reduced-form parameters are sufficient. However, using the reduced-form of a system is not sufficient if the aim is to evaluate structural innovation and economic policy. Also, related impulse response functions are less useful if not done using structural equations (Cooley and LeRoy, 1985). Since the aim of this paper is to analyse, amongst others, the effects of structural innovations on a portfolio of assets, the reduced-form is not sufficient for using in the research.

### **2.4 OTHER METHODS OF ESTIMATING CONSISTENT PARAMETERS IN A SYSTEM OF EQUATIONS**

Apart from restricting the parameters that need to be estimated the literature also proposes other methods to estimate consistent and efficient structural parameters.

#### **2.4.1 Instrumental Variables (Two-stage least squares)**

Instrumental variable technique (IV) is a general estimation procedure in situations where the independent variable is correlated with the disturbance terms. If an instrument can be found for each endogenous variable that appears as regressor in the system, the structural parameters can be estimated consistently. However the instrument must be highly correlated with the exogenous regressors and uncorrelated with the disturbance terms.

Two-stage least squares (2SLS) are a special case of IV and as the name suggests contains two steps. Step 1 estimates the reduced-form parameters by regressing each endogenous variable acting as a regressor on all the exogenous variables in the system of simultaneous

equations. Step 2 then uses these estimated values as instrumental variables for these endogenous variables in estimating the parameters using OLS. 2SLS gives consistent estimators for the parameters in the system of equations.

#### **2.4.2 Three-Stage Least Squares**

Three-Stage Least Squares (3SLS) is the system counterpart of 2SLS. The 3SLS estimator is consistent and in general is more efficient than 2SLS. The first step in 3SLS calculates the 2SLS estimates as defined above. Step 2 the use the 2SLS estimates to estimate the individual structural equations' disturbance terms and use them to calculate the variance –covariance matrix of the errors. The last step then applies generalized least squares and the variance-covariance matrix to estimate the system of equations once again.

In general, the superiority of 3SLS over 2SLS is slight if the computational intensity of 3SLS is taken into account. For this reason 3SLS has not been very popular in empirical studies in the past.

#### **2.4.3 Full Information Likelihood Estimation**

In this technique estimates of all the reduced-form parameters are found by maximising the likelihood function of the reduced-form disturbances, subject to zero restrictions on all the structural parameters in the system of equations. The usual assumption made is that the structural disturbances, and thus the reduced-form disturbances is distributed multivariate normally. The variance-covariance matrix under this assumption is as efficient as the variance-covariance matrix in 3SLS.

## 2.5 CONCLUSION

When estimating a system of equations it is not possible to directly estimate the structural parameters of the system. Rather, in order to obtain unbiased and consistent estimates, a linear transformation of the structural-form (i.e. the reduced-form) is estimated. In order to obtain the structural-form parameters from the reduced-form estimates, the system has to be identified (either exactly or over-identified). When the system is not identified it is not possible to recover the structural parameters without additional information. The literature proposes a solution to this problem by placing restrictions on the equations. These restrictions are difficult to defend when working with high-frequency data (e.g. daily asset return data). If the researcher wants to recover structural parameters when working with this type of unidentified system, alternative sources of restrictions have to be imposed. Alternative methods for estimating systems have been proposed. Most notable of these methods is 2SLS. The main problem with 2SLS is finding suitable instrumental variables that are highly correlated with the regressors but uncorrelated with the disturbance terms. If no suitable instruments can be found, 2SLS will still give inconsistent estimators. However, recently the heteroscedasticity that prevails in data has been successfully used to identify equations where traditional long-run constraints are not applicable. It is this identification methodology that will be used to identify and decompose the system of equations.

## Chapter 3

### IDENTIFICATION THROUGH HETEROSCEDASTICITY

#### 3.1 INTRODUCTION

When modelling economic variables it is often useful to distinguish between long-run and short-run relationships between variables. The long-run relationship represents the equilibrium between variables while the short-run relationship represents the adjustment of the variables towards the long-run equilibrium. To illustrate the idea, Johansen and Juselius (2000) use an analogy from physics and think of the economy as a system of balls connected by springs. When left alone the system will be in equilibrium but pushing any ball will bring the system away from equilibrium. Through the connection of the balls the movement or shock will influence the whole system of balls. When there is no shock present the economy is in a “steady-state” moving along at some controlled speed. The long-run relationships between economic variables represent the steady-state in the economy. The magnitude of the parameters will dictate at what speed the balls move in the controlled state. However, as soon as one of the balls is shocked the effect is transmitted to all the balls. At some stage the springs are stretched to its limit and the balls move back towards the steady-state observed before the shock. This adjustment away and towards the steady state represents the dynamic short run relationship between variables. Parameters of this nature measure the short run dynamics between economic variables. The nature of the parameters differs and their use in economic research depends on the problem at hand. Identifying the parameters are essential in economics as it gives a picture of the transmission of shocks through the economy and how the economy adjust to shocks back towards its steady-state. The parameters will also give an indication of the speed at which the economy is moving in its steady-state. The parameters in the system need to be estimated simultaneously. Depending on the frequency of the data either the long-run or short-run dynamic parameters will be estimated. High frequency data (e.g. daily data) on certain economic variables will typically measure average short-run relationships as “noise” in the system is likely to affect the steady-state of the

system. This “noise” consists of other variables that may drive relationships in the short-run as what is suggested by economic theory (Harasty and Roulet, 2000). Over lower frequency data (e.g. quarterly data) noise in the system is likely to average out and the parameters is more likely to represent long-run steady state relationships. However, the econometric problems in obtaining parameters stay the same in both cases. The solution to the problem will differ depending on the type of system at hand.

If a system of equations is unidentified, structural parameters cannot be recovered from the reduced-form estimation. The literature presents a solution by constraining the number of parameters to be estimated and thereby indirectly increasing the number of equations (Fisher, 1976, Haavelmo, 1947, Koopmans et al., 1950). These restrictions differ in nature. Zero restrictions (the coefficients of variables in an equation are assumed to have zero values) are the most commonly used restriction (Gujarati, 2003). This type of restriction is often found in cointegration analysis where long-run relationships are analysed. Other restrictions take on a variety of forms such as the use of extraneous estimates of parameters, knowledge of relationships that exist between parameters, knowledge of the variances of the disturbance terms, normalisation, sign restrictions and covariance constraints (Kennedy, 2003).

However, when working with high-frequency financial data, most of these restrictions, based on long-run relationships, are difficult to defend. Financial assets tend to influence one another in a very different way in the short run than what economic and finance theory suggest should hold in the longer run. In the short run financial assets are influenced by investor sentiment rather than fundamentals. Therefore, if the researcher is working with an unidentified system of equations containing high-frequency data (e.g. daily financial asset returns), obtaining any structural parameter from the reduced-form estimation is extremely difficult, and in many cases, subject to (invalid) ad hoc constraints. Rigobon (2003) presents a methodology based upon the heteroscedasticity in the data that solves the identification problem in the case of an unidentified system of equations. When working with high-frequency data, it is often the case that none of the standard identification assumptions can be defended. However, high-frequency financial data often exhibits heteroscedasticity.

### 3.2 IDENTIFICATION THROUGH HETEROSCEDASTICITY

Wright (1928) and Wright (1921) first introduced the use of second moments as a source of identification<sup>1</sup>. Rigobon (2003) extended this literature by developing the methodology whereby heteroscedasticity is used as an instrument to solve the identification problem. Reconsider the general case of two assets,  $y_i$  and  $y_{jt}$ :

$$\begin{aligned} y_{it} &= \delta y_{jt} + \eta_{it} \\ y_{jt} &= \beta y_{it} + \varepsilon_{jt} \end{aligned} \quad (3.1)$$

The system in equation 3.1 includes only endogenous variables and asset specific disturbance terms. The parameters of interest are  $\beta$  and  $\delta$ , and the variances of the innovations are  $\sigma_\eta^2$  and  $\sigma_\varepsilon^2$ . As explained in chapter 2, if  $\beta$  and  $\delta$  are non-zero, the parameters in equation 3.1 cannot be estimated unbiased and consistently without any further information. It is only possible to estimate the covariance matrix of the reduced-form of the system given by

$$\hat{\Sigma} = \begin{bmatrix} \frac{\delta^2 \sigma_\varepsilon^2 + \sigma_\eta^2}{(1 - \delta\beta)^2} & \frac{\delta \sigma_\varepsilon^2 + \beta \sigma_\eta^2}{(1 - \delta\beta)^2} \\ \cdot & \frac{\sigma_\varepsilon^2 + \alpha^2 \sigma_\eta^2}{(1 - \delta\beta)^2} \end{bmatrix}. \quad (3.2)$$

Given the estimated covariance for the reduced-form in equation 3.2, the problem of identification is that the covariance matrix only provides three moments while there are four unknowns to recover. Many constraints have been used to solve this problem. These constraints have proven very helpful in many economic problems, but are not practical in all

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<sup>1</sup> Wright (1928) and Wright (1923) showed that when heteroscedasticity is present in an equation, it reduces the bias in simultaneous equations in the OLS estimation. The bias is reduced because the heteroscedasticity present in the data serves as instrument to identify the structural parameters.

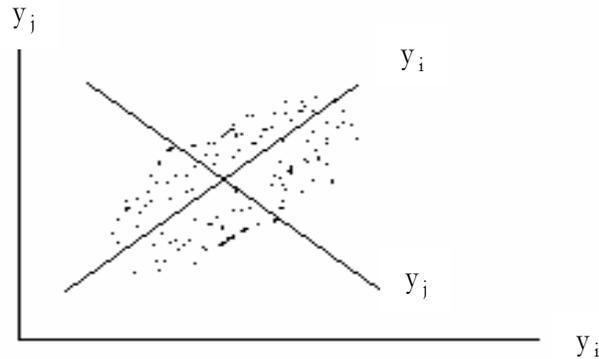
instances. Therefore, in cases where traditional constraints cannot be justified, identification based on heteroscedasticity may be helpful.

The method of identification through heteroscedasticity is intuitively appealing. Consider the case where there are two regimes in the variances of the structural disturbance terms. One regime exhibits high volatility in the disturbance terms while the other regime exhibits low volatility in the disturbance terms. Also assume the structural parameters of interest remain constant across both regimes. Under these two assumptions the reduced-form covariance matrices in both regimes have the same structure:

$$\hat{\Sigma}_s \equiv \begin{bmatrix} \varpi_{11,s} & \varpi_{12,s} \\ \cdot & \varpi_{22,s} \end{bmatrix} = \begin{bmatrix} \frac{\delta^2 \sigma_{\varepsilon,s}^2 + \sigma_{\eta,s}^2}{(1-\delta\beta)^2} & \frac{\delta\sigma_{\varepsilon,s}^2 + \beta\sigma_{\eta,s}^2}{(1-\delta\beta)^2} \\ \cdot & \frac{\sigma_{\varepsilon,s}^2 + \alpha^2\sigma_{\eta,s}^2}{(1-\delta\beta)^2} \end{bmatrix} \quad s \in (1,2). \quad (3.3)$$

Each regime in equation 3.3 is denoted by  $s \in (1,2)$ . In this two-regime system of equations, there are now six unknowns  $(\beta, \delta, \sigma_{\eta,1}^2, \sigma_{\varepsilon,1}^2, \sigma_{\eta,2}^2, \sigma_{\varepsilon,2}^2)$ , while there are also six equations in the two reduced-form covariance matrices. It should be clear that the equations are identified if they are independent, i.e. if the structural-form innovations across regimes are not correlated, the number of equations matches the number of structural parameters to retrieve. Figure 3.1 gives an intuitive explanation of the identification methodology. Assume it is known that there is an increase in the variance of variable  $y_i$ . During this period the realisations along the curve for variable  $y_i$  are going to widen. This allows one to identify the slope of the equation for variable  $y_i$ . The method is similar to that of an instrumental variable that allows one to identify an equation.

**Figure 3.1: Identification through heteroscedasticity**



Two assumptions are critical for the equations to be identified:

1. The structural innovations should not be correlated.
2. The parameters are stable across the heteroscedasticity regimes.

These two assumptions are not controversial and are standard in much of the applied macro economic research (Rigobon, 2003). If the two assumptions are satisfied, the equations will be identified. Rigobon (2003) provides the following proposition for identification:

*Proposition 3.1: The system as described in equation 3.1, where the parameters are stable and where the disturbance terms that have finite variance are not correlated and exhibit heteroscedasticity that can be described by two regimes, will be identified if the covariance matrices satisfy*

$$\det \left| \Sigma_2 - \frac{\mathfrak{w}_{11,2}}{\mathfrak{w}_{11,2}} \Sigma_1 \right| \neq 0.$$

The condition is similar to testing the rank condition when the order condition has been satisfied.

In the case where there are more than two regimes, the equations in the system may still be identified. If there are multiple regimes,  $s \in (1, \dots, S)$ , the data has to exhibit multiple finite heteroscedastic regimes. For each regime the covariance matrix is

$$\hat{\Sigma}_s \equiv \begin{bmatrix} \varpi_{11,s} & \varpi_{12,s} \\ \cdot & \varpi_{22,s} \end{bmatrix} = \begin{bmatrix} \frac{\delta^2 \sigma_{\varepsilon,s}^2 + \sigma_{\eta,s}^2}{(1-\delta\beta)^2} & \frac{\delta\sigma_{\varepsilon,s}^2 + \beta\sigma_{\eta,s}^2}{(1-\delta\beta)^2} \\ \cdot & \frac{\sigma_{\varepsilon,s}^2 + \alpha^2\sigma_{\eta,s}^2}{(1-\delta\beta)^2} \end{bmatrix} \quad s \in (1, \dots, S). \quad (3.4)$$

The system has  $3 \times S$  equations (3 equations per regime) and  $2 \times S + 2$  unknowns to solve ( $S$  times the two structural variances for each regime plus two parameters).

It is also possible to extend this identification framework to the case of a multivariate system where common shocks occur. The inclusion of common shocks in the system of equations is equivalent to relaxing the assumption that the structural innovations are correlated. Continuing with a system of variables, assume there are  $M$  variables in the system determined endogenously, with  $K$  common shocks. There are still  $s \in (1, \dots, S)$  possible volatility regimes. The structural-form is denoted as:

$$\underset{M \times M}{\Lambda} \begin{bmatrix} y_{1,t} \\ \vdots \\ y_{M,t} \end{bmatrix} = \underset{M \times K}{\Pi} \begin{bmatrix} z_{1,t} \\ \vdots \\ z_{K,t} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{M,t} \end{bmatrix} \quad (3.5)$$

$\begin{matrix} M \times 1 & & K \times 1 & & M \times 1 \end{matrix}$

where  $y_{M,t}$  are the endogenous variables,  $z_{K,t}$  the common shocks and  $\varepsilon_{M,t}$  the structural innovations. The common shocks are assumed to be independent of one another, such that

$$\begin{aligned} E[z_{1,t}, z_{n,t}] &= 0 \quad \forall 1 \neq n, \quad 1, n \in (1, K) \\ E[\varepsilon_{1,t}, z_{n,t}] &= 0 \quad \forall 1 \neq n, \quad 1 \in (1, M), \quad n \in (1, K). \\ E[\varepsilon_{1,t}, \varepsilon_{n,t}] &= 0 \quad \forall 1 \neq n, \quad 1, n \in (1, M) \end{aligned} \quad (3.6)$$

Furthermore, matrix  $A$  contains the contemporaneous parameters from the system where normalisation is already imposed. Matrix  $\Pi$  contains the parameters of the common shocks to the system with normalisation of unity on the first equation:

$$\Lambda_{M \times M} = \begin{bmatrix} 1 & \alpha_{12} & \cdots & \alpha_{1m} \\ \alpha_{21} & 1 & \cdots & \alpha_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{m1} & \alpha_{m2} & \cdots & 1 \end{bmatrix} \quad (3.7)$$

$$\Pi_{M \times K} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{m1} & \pi_{m2} & \cdots & \pi_{mk} \end{bmatrix} \quad (3.8)$$

In order for the system described in equation 3.5 to be identified, Rigobon (2003) provides the following proposition:

Proposition 3.2: *In the multivariate system of  $M$  endogenous equations with  $K$  common shocks, the equations are identified if and only if, for  $M > 1$ ,*

1. *the number of states ( $S$ ) satisfies*

$$S \geq 2 \frac{(M+K)(M-1)}{M^2 - M - 2K}$$

2. *there is a minimum number of endogenous variables that satisfies*

$$\frac{N^2 - N}{2} > K$$

3. *and the covariance matrices constitute a system of equations that are linearly independent.*

From the above it should be clear that in the case where there are no common shocks, only two regimes are required to identify the system. If the common shocks are larger than zero, the regimes required to identify the system will also be larger than two.

### **3.3 EMPIRICAL STUDIES USING IDENTIFICATION THROUGH HETEROSCEDASTICITY**

Relatively new research has been conducted extending the intuition first developed by Wright (1928) and Wright (1921). This has been extended to non-linear models, ARCH and GARCH models and models that are partially identified.

There is a considerable amount of interest in the relationship between different asset prices, monetary policy and the interaction between them. There is also great interest in the feedback effect between asset prices between countries, especially in times of financial crisis. Since the interaction between these types of variables is simultaneous, the problem of identifying a structural model in order to solve the system simultaneously prevails. It is therefore not surprising that most of the applied research that incorporates heteroscedasticity to identify a system focuses on asset prices in and across countries as well as the effect of monetary policy on assets. Although the application and purpose of the methods differ in the various papers, they all share the same method of identification. By identifying heteroscedasticity, equations are added to the system after some covariance restrictions have been imposed. There have also been some recent developments in structural GARCH models using the same method.

Sentana (1992) and Sentana and Fiorentini (2001) studied the problem of estimation in a factor regression model when there is conditional heteroscedasticity. They were interested in the contemporaneous effects between different asset prices and the factors that drive them.

They study the case where heteroscedasticity is achieved when the common latent factors exhibit heteroscedasticity. They find that if the variation of conditional moments is explicitly recognised in estimation, identification problems are often alleviated. They apply their results to dynamic arbitrage pricing theory (APT) models to show that a system can be identified through heteroscedasticity.

Caporale, Cipollini and Demetriades (2000) evaluated whether tight monetary policy was successful in defending the exchange rate from speculative pressures during the Asian crisis. The challenge they faced was to distinguish between monetary policy exogenous shocks and monetary policy actions that to some extent respond to current developments in the economy. There is thus an identification scheme needed to solve the simultaneity problem between policy instruments and other endogenous variables, such as exchange rates to which monetary policy reacts. They employ a structural VAR to model movements in interest rates and exchange rates simultaneously, and identify the system through heteroscedasticity in the data. They find that by increasing interest rates, the central banks generated an adverse effect that led to a greater depreciation of the countries' currencies and thereby magnifying the crisis.

Dungey and Martin (2001) developed a multivariate GARCH model to identify the contemporaneous flows between Asian countries, Australia and the US during the Asian crisis. Their model is a latent factor model that allows them to decompose the relative contribution of alternative factors to the volatility in financial markets. Their identification of the contemporaneous coefficients is also based on identification through heteroscedasticity. They find strong results that volatility in currency markets was primarily driven by volatility in equity markets, with the main channel linking these markets being spillovers from the equity market to the currency markets. The empirical results in this paper provide strong support for modelling currency and equity markets simultaneously.

Rigobon and Sack (2003) looked at how monetary policy reacts to changes in the stock market. The impact of stock markets on the macro economy comes primarily through two

channels. The first is the wealth channel and the second the balance sheet channel. Because of the importance of the stock market, monetary policy will react to changes in stock prices. The problem in estimating this effect lies in the fact that the policy reaction function and stock prices react simultaneously. In order to overcome this problem, Rigobon and Sack (2001) use heteroscedasticity in the data to identify a reaction function for the monetary authorities in the United States. They specify stock prices as a function of a short-term interest rate, while the short-term interest rate is also a function of stock prices. They also identify some common factors that influence both variables. They estimate a reduced-form VAR using the two response functions. In order to identify the system, they divide the sample into sub-periods. This allows for the covariance matrices in each regime to add equations to the system in order to identify the structural parameters. In related research, Rigobon and Sack (2004) use a similar approach to identify how asset prices react to monetary policy. They demonstrate that the response of asset prices and market interest rates to changes in monetary policy can be estimated using heteroscedasticity as an instrument for identification.

Rigobon (2002) developed a multivariate GARCH model to identify the structural relationship between yields on sovereign debt between Mexico and several countries. He finds that there is a significant change in the risk associated with a country before and after the country receives an upgrade from rating agencies. His contribution to the literature concerning identification is the methodology that he applies. This GARCH model offers a solution to the problem of simultaneous equations when data suffer from conditional heteroscedasticity.

### **3.4 CONCLUSION**

This chapter introduces an alternative method to identify a system of equations when traditional restrictions cannot be defended on economic or statistical grounds. The intuition behind the identification procedure is straightforward. If the data exhibit heteroscedasticity,

the information can be used to add equations to the system in order to recover the structural parameters. Traditional restrictions placed on reduced-form estimations to recover structural parameters are in these cases not always defensible. This identification method is ideal in the case of high frequency financial data that often exhibit conditional heteroscedasticity. Although the concept of identifying a system of equations through the heteroscedasticity has been around for some time, it is not until recently that it has been applied in the context of high-frequency data. The applied studies using this methodology are therefore limited.

Identification through heteroscedasticity is used in conjunction with a multivariate GARCH model in the proposed methodology to identify a system of equations. The identification of the structural parameters is crucial to decompose the volatility in the system of variables into the endogenous volatility generated between variables and the exogenous volatility generated by structural innovations.

## Chapter 4

### MULTIVARIATE GARCH MODELS

#### 4.1 INTRODUCTION

The purpose of the research is to analyse the structural composition of a system of variables both in the first and second moments. Since the proposed methodology makes use of multivariate Generalised Autoregressive Conditional Heteroscedasticity (MGARCH) models, it is necessary to discuss existing MGARCH models to highlight their uses as well as shortcomings. More specifically, the survey on existing models will show that the majority of existing GARCH models do not attempt to obtain structural relationships between variables but rather focus on reduced-form estimates. Although sufficient to forecast volatility, reduced-form estimates provide little information on what drives the volatility of a variable.

The introduction of Autoregressive Conditional Heteroscedasticity (ARCH) models to econometrics by Engle (1982) allowed researchers to detect behaviour in financial data that may not be linear in nature. The ARCH model allowed econometricians to model volatility behaviour in financial data, which was previously extremely difficult. Following the success of ARCH modelling, Bollerslev (1986) introduced the now widely used GARCH model. This type of model explicitly models a time-varying conditional variance as a linear function of past squared residuals and of its own past values. The ARCH and GARCH models have been applied with great success in various situations but more predominantly in financial market research.

Since the introduction of ARCH and GARCH models to the literature, many different types of GARCH models have been developed. These types of models, as introduced by Engle (1982) and Bollerslev (1986) are all univariate in nature. That is, univariate models assume asset movements are independent from one another. Many comprehensive surveys exist on the univariate models (see e.g. Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and

Nelson (1994), Pagan (1996)). However, to some extent temporal dependence in second moments of assets does exist. In order to understand and predict the movements of different asset volatilities over time, it is necessary to recognise this dependence, which is captured in MGARCH models. Therefore, because MGARCH models incorporate the dependence of volatilities they provide a tool for better decision making in financial analysis. Examples where MGARCH models have been applied successfully include asset pricing models, option pricing, portfolio selection, and value-at-risk (Bauwens, Laurent and Rombouts (2003)). Not many comprehensive surveys exist on the available multivariate GARCH models. However, Bauwens, Laurent and Rombouts (2003) provide a fairly up to date survey of MGARCH models. This chapter is based on their survey and covers the most common used MGARCH models.

## 4.2 AN OVERVIEW OF MGARCH MODELS

Given a vector stochastic process  $\{y_t\}$  of dimension  $N \times 1$  and  $\theta$  a finite vector of parameters, we can write the process as

$$y_t = \mu_t(\theta) + \varepsilon_t. \quad (4.1)$$

In equation 4.1  $\mu_t(\theta)$  is the conditional mean vector and  $\varepsilon_t = H_t^{1/2}(\theta)z_t$ .  $H_t^{1/2}(\theta)z_t$  is a  $N \times N$  positive definite matrix and  $z_t$  is a  $N \times 1$  random vector to be i.i.d. with its first and second moments  $E(z_t) = 0$  and  $\text{var}(z_t) = I_N$  respectively.  $I_N$  is an identity matrix of order  $N$ .  $H_t$  is the positive definite conditional variance matrix of  $y_t$  and is given by

$$\text{Var}(y_t/I_{t-1}) = \text{Var}_{t-1}(y_t) = \text{Var}_{t-1}(\varepsilon_t) = H_t \quad (4.2)$$

where  $I_t$  is the information matrix available at time  $t$ .  $\Sigma$  is the unconditional variance of the matrix, i.e.  $\Sigma = E[H_t]$ .

When estimating  $H_t$  the usual trade-off between general models and parsimonious models apply. Some models become intractable if the number of time series included in the model becomes too large (usually more than 4). The MGARCH models therefore differ in the number of parameters to be estimated in  $\theta$ . A second problem when estimating MGARCH models, is that  $H_t$  has to be positive definite<sup>2</sup>. Several models ensure this condition under very loose conditions. The purpose and use of each MGARCH model differ and it is therefore difficult to define the “best” model. Ranking the MGARCH models therefore depends on the specific problem at hand and the application of the model.

MGARCH models can be divided into three broad classes. In the subsequent section each of these classes will briefly be discussed.

#### 4.2.1 VEC and BEKK models

The VEC model proposed by Bollerslev, Engle and Wooldridge (1988) has a fairly general formulation. The model stacks the lower triangular portion of a  $N \times N$  matrix as a  $N(N+1)/2 \times 1$  vector. The VEC(p,q) model can be defined by

$$h_t = c + \sum_{j=1}^q A_j n_{t-j} + \sum_{j=1}^p G_j h_{t-1} \quad (4.3)$$

where  $h_t$  is  $\text{vech}(H_t)$  and  $n_t$  is  $\text{vech}(\epsilon_t \epsilon_t')$ . In the specification  $\text{vech}(\cdot)$  is an operator that stacks the lower triangular portion of a  $N \times N$  matrix as a  $N(N+1)/2 \times 1$  vector. In the bivariate case, the (p,q) model will be

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<sup>2</sup> A matrix is said to be positive definite if the characteristic roots of that matrix is positive.

$$h_t = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1}^2 \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix}. \quad (4.4)$$

This specification of the VEC model contains 21 parameters to estimate (because of its generality, i.e. the structure of  $h_t$  is not constrained). If the specification is higher than the (1,1) specification in equation 4.4, the model becomes too complex to estimate in practice.

To overcome this problem the same authors have introduced some simplifying assumptions. Bollerslev, Engle and Wooldridge (1988) suggest a diagonal VEC (DVEC). In this specification, it is assumed that the  $A_j$  and  $G_j$  matrices in equation 4.3 are diagonal. This implies that the off-diagonal elements are zero, which greatly reduces the number of parameters to be estimated. The variance depends now only on past values and its own squared errors. In equation 4.4 the number of parameters to be estimated reduces to only 9.

It is difficult to guarantee that the variance matrix in the VEC model is positive-definite without imposing strong restrictions on the parameters. In order to overcome this problem, Engle and Kroner (1995) proposed a new specification for  $H_t$  that easily imposes its positivity, i.e. the BEKK model (after Baba, Engle, Kraft and Kroner).

The BEKK (p,q,K) model is defined as:

$$H_t = C'C + \sum_{k=1}^K \sum_{j=1}^q A'_{jk} \varepsilon_{t-j} \varepsilon'_{t-j} A_{jk} + \sum_{k=1}^K \sum_{j=1}^p G'_{jk} H_{t-j} G_{jk} \quad (4.5)$$

where  $C, A_{jk}$  and  $G_{jk}$  are  $N \times N$  matrices but  $C$  is upper triangular.

For the bivariate case the BEKK model is:

$$\begin{aligned}
 \begin{bmatrix} h_{t,11} & h_{t,12} \\ h_{t,21} & h_{t,22} \end{bmatrix} &= \begin{bmatrix} c_{11} & 0 \\ c_{21} & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{21} \\ 0 & c_{22} \end{bmatrix} \\
 + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} &\begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \varepsilon_{1,t-1}\varepsilon_{2,t-1} & \varepsilon_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}. \tag{4.6} \\
 + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} &\begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}
 \end{aligned}$$

In this specification of the BEKK model, there are 11 parameters to be estimated, compared to the 21 in the VEC specification. It can also be shown that the BEKK model is a special case of the VEC model. A further reduction of the number of parameters to be estimated can be achieved by estimating a diagonal BEKK model (Engle and Kroner, 1995). This is also a DVEC model but less general than the specification suggested by Bollerslev, Engle and Wooldridge (1988).

#### 4.2.2 Factor and orthogonal models

BEKK and VEC models both require a high number of parameters to be estimated (even after imposing some restrictions). The BEKK and VEC models are therefore not often used when estimating models with large numbers of series. To overcome this problem, factor and orthogonal models impose a common dynamic structure on all elements of the conditional covariance matrix ( $H_t$ ). This results in a model with less parameters to be estimated.

Engle, Ng and Rothschild (1990) proposed a factor model where  $H_t$  is determined by a small number of common underlying variables, called factors. The common underlying variables are supposed to be a small number of factors that drives the underlying volatility across variables. The factor model can also be expressed as a special case of the BEKK model. The BEKK(p,q,K) model is a factor model, denoted by F-GARCH(p,q,K), if for each

$k=1,2,\dots,K$ ,  $A_{jk}$  and  $G_{jk}$  have rank of unity and have the same left and right eigenvectors,  $\lambda_k$  and  $\omega_k$ , i.e.

$$A_{jk} = \alpha_{kj}\omega_k\lambda'_k \text{ and } G_{jk} = \beta_{kj}\omega_k\lambda'_k. \quad (4.7)$$

In equation 4.7,  $\alpha_{jk}$  and  $\beta_{jk}$  are scalars, and  $\lambda_k$  and  $\omega_k$  are  $N \times 1$  vectors with

$$\omega'_k\lambda_i = \begin{cases} 0 & \text{for } k \neq i \\ 1 & \text{for } k = i \end{cases} \text{ and } \sum_{n=1}^N \omega_{kn} = 1. \quad (4.8)$$

Using equation 4.5 of the standard BEKK model to substitute equation 4.7 and 4.8 into, it is possible to obtain:

$$H_t = \Omega + \sum_{k=1}^K \lambda_k\lambda'_k \left( \sum_{j=1}^q \alpha_{kj}^2 \omega'_{jk} \varepsilon_{t-j} \varepsilon'_{t-j} \alpha_{kj} + \sum_{j=1}^p \beta_{kj} \omega'_k H_{t-j} \omega_{kj} \right). \quad (4.9)$$

The K-factor GARCH model implies that the time-varying part of  $H_t$  has reduced rank K, but  $H_t$  remains of full rank because of  $\Omega$ . In this model  $\lambda_k$  is called the k-th factor loading and  $\omega'_k \varepsilon_t$  the k-th factor.

The orthogonal models are a specific class of factor models. Orthogonal models are based on the assumption that the observed data can be obtained by a linear transformation of a set of uncorrelated components as expressed in equation 4.10

$$y_t = \lambda_1 \delta_{1t} + \lambda_2 \delta_{2t} + e_t. \quad (4.10)$$

In equation 4.10  $\delta_{it}$  ( $i = 1, 2$ ) are the factors and  $e_t$  represents idiosyncratic shocks with a constant variance which is uncorrelated with the two factors. These factors or components are chosen to be the principal components of the data. Alexander and Chibumba (1997) as well as Alexander (2001) first proposed this model. In these models the  $N \times N$  time-varying variance is generated by  $m$  univariate GARCH models, where  $m < N$  determined using principal component analysis is. The orthogonal GARCH models are based on the factor GARCH models and are thus nested in the class of BEKK-GARCH models.

#### 4.2.3 Conditional-Correlation models

When estimating a conditional-correlation GARCH model, the first step is to choose a model for each conditional variance. Each conditional variance may follow a different process. For example, one variance may follow a GARCH process while another series may follow an exponential GARCH (E-GARCH) process. In the second step, based on the conditional variances, the conditional correlation matrix is modelled. This conditional matrix should also be positive definite across the whole sample. Two classes of conditional-correlation models exist. The first is the constant conditional-correlation model and the second the dynamic conditional-correlation model.

Bollerslev (1990) proposed a class of MGARCH models where the conditional correlations are constant across time. This restriction greatly reduces the number of parameters to be estimated. The Constant Conditional Correlation (CCC) model is defined as:

$$H_t = D_t R D_t = (\rho_{ij} \sqrt{h_{iit} h_{jjt}}) \quad (4.11)$$

where

$$D_t = \text{diag}(h_{11t}^{1/2} \dots h_{NNt}^{1/2}). \quad (4.12)$$

In equation 4.11 and equation 4.12,  $h_{iit}^{1/2}$  can be defined as a univariate GARCH model, and

$$R = (\rho_{ij}) \quad (4.13)$$

which is a symmetric positive definite matrix with  $\rho_{ii} = 1, \forall i$ .

For example, in the first step one would take the GARCH(p,q) process for each conditional variance in  $D_t$  (Bauwens, Laurent and Rombouts, 2003)

$$h_{iit} = \omega_i + \sum_{j=1}^q \alpha_{ij} \varepsilon_{i,t-j}^2 + \sum_{j=1}^p \beta_{ij} h_{ii,t-j} \quad i = 1, \dots, N. \quad (4.14)$$

If all the conditional variances are positive and  $R$  is positive definite,  $H_t$  will be positive.

The unconditional variances are then obtained through:

$$\sigma_{ii} = \frac{\omega_i}{1 - \sum_{j=1}^q \alpha_{ij} - \sum_{j=1}^p \beta_{ij}}. \quad (4.15)$$

The assumption that the conditional correlations between assets are constant may be an unrealistic assumption in many cases of applied research. Tse and Tsui (2002) and Engle (2001) proposed a generalisation of the CCC model where the conditional correlation matrix is time dependent. The Dynamic Conditional Correlations (DCC) model of Engle (2001) is defined by:

$$H_t = D_t R_t D_t \quad (4.16)$$

where  $D_t$  is defined in equation 4.12. Once again,  $h_{iit}$  can be defined as any univariate GARCH model, and  $R_t$  as:

$$R_t = (\text{diag}Q_t)^{-1/2} Q_t (\text{diag}Q_t)^{-1/2}. \quad (4.17)$$

$Q_t$  is a  $N \times N$  symmetric positive definite matrix given by:

$$Q_t = (1 - \sum_{l=1}^L \alpha_l - \sum_{s=1}^S \beta_s) \bar{Q} + \sum_{l=1}^L \alpha_l \mu_{t-1} \mu'_{t-1} + \sum_{s=1}^S \beta_s Q_{t-s} \quad (4.18)$$

where  $\mu_{it} = \varepsilon_{it} / \sqrt{h_{iit}}$ ,  $\bar{Q}$  is the  $N \times N$  unconditional variance matrix of  $u_t$ , and

$\alpha_l (\geq 0)$  and  $\beta_s (\geq 0)$  are scalar parameters satisfying  $\sum_{l=1}^L \alpha_l + \sum_{s=1}^S \beta_s < 1$ .

The DCC models can be estimated consistently in two steps, which make this approach attractive when the number of variables ( $N$ ) is large. The DCC models also allow for more complex specifications, using  $N$  univariate specifications for the  $N$  variables.

### 4.3 CONCLUSION

MGARCH models allow for the simultaneous estimation of time-varying volatilities of different variables. Time-varying volatility allow for the better estimation of measures of risk and therefore asset allocation. The great practical drawback with MGARCH models is that the number of parameters to be estimated increases greatly as the number of variables increases. In most instances, more than four variables in the models make the number of parameters too many to be estimated. To overcome this problem many different MGARCH models have been developed. Depending on the problem at hand, the researcher will apply a different, but relevant MGARCH model. With MGARCH models, the trade-off is between

generality (i.e. including as much information on many variables as possible) and the number of parameters to be estimated.

MGARCH models can be broken down into three broad types of models. The first group is the VEC and BEKK specifications. The VEC models are very general in specification but require a lot of parameters to be estimated. The BEKK model constrains the number of parameters to be estimated at the expense of generality. Secondly there are the Factor and Orthogonal GARCH models. These models allow for many variables to enter into the GARCH models without increasing the number of parameters to be estimated too much. More recently the Conditional Correlation model has been developed that uses a two-step procedure to estimate the parameters. This procedure allows for a fairly general specification of the conditional covariance matrix. Table 4.1 presents a summary of the MGARCH models discussed above.

All the models discussed are reduced-form models. It is not possible to recover any structural parameters from the multivariate set-up. As mentioned in the beginning of the chapter, this is simply because most of these models are only concerned with forecasting the volatility of variables. For forecasting purposes the reduced-form estimates are sufficient. However, when the purpose of the research is to explain the underlying structure of the volatility in individual, as well as the volatility between different variables, traditional MGARCH models are not sufficient. In this case another methodology is necessary.

In the next chapter a methodology is introduced that determines the structural characteristics of the volatility in and between variables. This methodology makes use of multivariate GARCH models and the heteroscedasticity in the data to obtain the structural estimates of the volatility.

**Table 4.1: Summary of MGARCH models**

	$H_t$	# of parameters for N=2, 3, 4.
VEC(1,1)	$h_t = c + \sum_{j=1}^q A_j n_{t-j} + \sum_{j=1}^p G_j h_{t-1}$	$\frac{N(N+1)(N(N+1)+1)}{2}$  21, 78, 210
BEKK(1,1)	$H_t = C'C + \sum_{k=1}^K \sum_{j=1}^q A'_{jk} \varepsilon_{t-j} \varepsilon'_{t-j} A_{jk} + \sum_{k=1}^K \sum_{j=1}^p G'_{jk} H_{t-j} G_{jk}$	$\frac{N(5N+1)}{2}$  11, 24, 42
F-GARCH(1,1,1)	$H_t = \Omega + \sum_{k=1}^K \lambda_k \lambda'_k \left( \sum_{j=1}^q \alpha_{kj}^2 \omega'_{jk} \varepsilon_{t-j} \varepsilon'_{t-j} \alpha_{kj} + \sum_{j=1}^p \beta_{kj} \omega'_k H_{t-j} \omega_{kj} \right)$	$\frac{N(N+5)}{2}$  7, 12, 18
CCC	$H_t = D_t R D_t = (\rho_{ij} \sqrt{h_{iit} h_{jtt}})$	$\frac{N(N+5)}{2}$  7, 12, 18
DCC(1,1)	$H_t = D_t R D_t = (\rho_{ij} \sqrt{h_{iit} h_{jtt}})$	$\frac{(N+1)(N+4)}{2}$  9, 14, 20

Source: Bauwens, Laurent and Rombouts (2003)

## Chapter 5

### A STRUCTURAL GARCH MODEL

#### 5.1 INTRODUCTION

If the researcher is interested in not only forecasting the volatility of variables but also in explaining the structure of the volatility, traditional MGARCH models are not sufficient. That is, if the researcher is interested in decomposing the conditional covariance of a system of equations into the endogenous conditional covariance generated inside the system and the exogenous conditional covariance generated by innovations or latent factors outside the system, it is necessary to find the structural parameters to identify the structural equations. However, in order to recover the structural equations from the reduced-form estimation, some identification restrictions are necessary. When modelling with high-frequency data (like financial data), traditional constraints are not always valid to identify a system (see chapter 3). Some alternative identification methodology is necessary that doesn't impose a structure on the model that is invalid.

This chapter proposes a two-step structural GARCH model as opposed to the traditional "reduced-form" MGARCH models. The structural model estimates the structural equations as well as the conditional covariance matrix. The estimation methodology is divided into two parts:

1. The first step identifies the system of equations and estimates the conditional covariance matrix of the structural innovations. A "structural" GARCH model developed by Rigobon (2002) and Rigobon and Sack (2003) is utilised to
  - a.) identify the structural parameters

- b.) estimate the conditional covariance of structural innovations resulting from outside the system (the exogenous volatility).
2. The second step recovers the variation of the variables explained within the system and estimates the endogenous conditional covariance. For this step any standard MGARCH model (as explained in chapter 4) can be employed to estimate the conditional covariance matrix of variables determined inside the system (the endogenous volatility).

As equation 5.1 shows, the sum of the two conditional covariance matrices gives the total conditional covariance matrix for the system of variables:

$$H_{t,\text{total}} = H_{t,\text{endogenous}} + H_{t,\text{exogenous}} \quad (5.1)$$

The traditional MGARCH models determine the conditional covariance matrix ( $H_{t,\text{total}}$ ) without decomposing the conditional covariance matrix into separate parts. When decomposing the total conditional covariance matrix as in equation 5.1, it is possible to determine which part of the variance is determined by other variables inside the system ( $H_{t,\text{endogenous}}$ ) and which part of the variance is explained by variable specific structural innovations ( $H_{t,\text{exogenous}}$ ). It is also possible to take into account the effect of the structural parameters on movements in volatility going forward. This is not possible when estimating reduced-form parameters or traditional MGARCH models.

The chapter is outlined as follows. In section 5.2 the first step in decomposing the conditional covariance to obtain  $H_{t,\text{exogenous}}$  is discussed. This section provides a detailed discussion of the “structural” GARCH model developed by Rigobon (2002) and Rigobon and Sack (2003), used in the first step. The following section explains the second step of the proposed estimation methodology. This step employs a standard MGARCH specification

discussed in chapter 4 to model  $H_{t,\text{endogenous}}$ . Finally in section 5.4 some concluding remarks are made.

## 5.2 STEP 1: ESTIMATING THE EXOGENOUS CONDITIONAL COVARIANCE MATRIX DRIVEN BY THE STRUCTURAL INNOVATIONS IN THE SYSTEM

A bivariate “structural” GARCH model has recently been developed by Rigobon (2002) and also extended into a multivariate model (Rigobon and Sack, 2003). These models have the advantage of recovering the structural parameters from the reduced-form, while also restricting the number of parameters to be estimated to a reasonable size. The models are derived from structural equations and follow a VEC specification as in equation 4.3. They are useful in that they give more information on what the movement of variables will be, following a structural innovation to a certain variable in the system of variables.

The bivariate GARCH model of Rigobon (2002) is an ARCH model that achieves identification of the structural parameters through conditional heteroscedasticity in the data. Given the bivariate model in equation 5.2 with endogenous variables, the system will be under-identified according to the rank and order conditions discussed in chapter 2

$$\begin{aligned} y_{it} &= c_i + \delta y_{jt} + \eta_{it} \\ y_{jt} &= c_j + \beta y_{it} + \varepsilon_{jt} \end{aligned} \quad (5.2)$$

The structural innovations are assumed to follow the following ARCH process:

$$\begin{aligned} \eta_{it} &= \sqrt{h_{\eta,t}} \cdot v_{\eta,t} \\ \varepsilon_{it} &= \sqrt{h_{\varepsilon,t}} \cdot v_{\varepsilon,t} \end{aligned} \quad (5.3)$$

where

$$\begin{aligned}
 E(v_{\varepsilon,t}) &= 0 \\
 E(v_{\varepsilon,t}^2) &= 1 \\
 E(v_{\eta,t}) &= 0 \\
 E(v_{\eta,t}^2) &= 1 \\
 E(v_{\varepsilon,t}, v_{\eta,t}) &= 0
 \end{aligned} \tag{5.4}$$

The most important assumption in the ARCH model is the zero correlation between the structural innovations ( $E(v_{\varepsilon,t}, v_{\eta,t}) = 0$ ). As indicated in chapter 2, it is this covariance restriction plus the heteroscedasticity in the data that allows for identification of the parameters.

It is assumed the conditional variance satisfy an ARCH process of:

$$\begin{pmatrix} h_{\eta,t} \\ h_{\varepsilon,t} \end{pmatrix} = \begin{pmatrix} \zeta_{\eta} \\ \zeta_{\varepsilon} \end{pmatrix} + \begin{pmatrix} \lambda_{\eta\eta} & \lambda_{\eta\varepsilon} \\ \lambda_{\varepsilon\eta} & \lambda_{\varepsilon\varepsilon} \end{pmatrix} \begin{pmatrix} \eta_{t-1}^2 \\ \varepsilon_{t-1}^2 \end{pmatrix} \tag{5.5}$$

Equations 5.2 to 5.5 describe the structural model relationships between the two variables. The objective is to measure the ARCH effects of the structural innovations as well as the parameters in equation 5.2<sup>3</sup>. The ARCH specification in equation 5.5 includes only one lag, but can easily be extended to more lags. From the structural equations a reduced-form ARCH model can be derived<sup>4</sup>. The reduced-form residuals are given by:

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<sup>3</sup> The structural model described in equations 5.2 to 5.5 is equivalent to a latent factor model. Both models have the same problem of identification.

<sup>4</sup> See Appendix A for detailed derivation of the reduced-form model

$$\begin{aligned}\omega_{i,t} &= c'_i + \frac{(\beta\varepsilon_t + \eta_t)}{1-\delta\beta} \\ \omega_{j,t} &= c'_j + \frac{(\varepsilon_t + \delta\eta_t)}{1-\delta\beta}\end{aligned}\quad (5.6)$$

In equation 5.6,  $\omega_{i,t}$  and  $\omega_{j,t}$  have zero means. The conditional moments are given by

$$\Sigma_{\omega,t} = \begin{bmatrix} \omega_{i,t}^2 & \omega_{i,t}\omega_{j,t} \\ \cdot & \omega_{j,t}^2 \end{bmatrix}. \quad (5.7)$$

Unlike the structural innovations, the reduced-form residuals have a covariance that is different from zero. This is because the structural parameters  $\beta$  and  $\delta$  are non-zero. If the expected conditional covariance matrix of reduced-form is given by,

$$E\Sigma_{\omega,t} = \begin{bmatrix} h_{i,t} & h_{i,t}h_{j,t} \\ \cdot & h_{j,t} \end{bmatrix} \quad (5.8)$$

it can be shown that they follow an ARCH specification as in equation 5.9

$$\begin{bmatrix} h_{i,t} \\ h_{ij,t} \\ h_{j,t} \end{bmatrix} = \begin{bmatrix} \zeta_i \\ \zeta_{ij} \\ \zeta_j \end{bmatrix} + \frac{1}{1-(\delta\beta)^2} A \begin{bmatrix} \omega_{i,t-1}^2 \\ \omega_{j,t-1}^2 \end{bmatrix}. \quad (5.9)$$

$\zeta_i$ ,  $\zeta_j$  and  $\zeta_{ij}$  are constants while matrix A is given by:

$$A \equiv \begin{bmatrix} \left[ \beta^2\lambda_{\varepsilon\eta} + \lambda_{\eta\eta} \right] - \delta^2 \left[ \beta^2\lambda_{\varepsilon\varepsilon} + \lambda_{\eta\varepsilon} \right] & -\beta^2 \left[ \beta^2\lambda_{\varepsilon\eta} + \lambda_{\eta\eta} \right] + \left[ \beta^2\lambda_{\varepsilon\varepsilon} + \lambda_{\varepsilon\eta} \right] \\ \left[ \beta\lambda_{\varepsilon\eta} + \delta\lambda_{\eta\eta} \right] - \delta^2 \left[ \beta\lambda_{\varepsilon\varepsilon} + \delta\lambda_{\eta\varepsilon} \right] & -\beta^2 \left[ \beta\lambda_{\varepsilon\eta} + \delta\lambda_{\eta\eta} \right] + \left[ \beta\lambda_{\varepsilon\varepsilon} + \delta\lambda_{\varepsilon\eta} \right] \\ \left[ \lambda_{\varepsilon\eta} + \delta^2\lambda_{\eta\eta} \right] - \delta^2 \left[ \lambda_{\varepsilon\varepsilon} + \delta^2\lambda_{\eta\varepsilon} \right] & -\beta^2 \left[ \lambda_{\varepsilon\eta} + \delta^2\lambda_{\eta\eta} \right] + \left[ \lambda_{\varepsilon\varepsilon} + \delta^2\lambda_{\varepsilon\eta} \right] \end{bmatrix}. \quad (5.10)$$

The reduced-form ARCH model is defined by equations 5.6 to 5.10. From the reduced-form estimates, the structural parameters can be obtained. In matrix A there are six equations, while there are also six structural coefficients. This is a restricted multivariate model, where the restrictions result from the fact that the structural innovations are assumed to be uncorrelated.

Using the same methodology, Rigobon and Sack (2003) extend the bivariate ARCH model described above to a multivariate GARCH model that allows for the estimation of the structural contemporaneous parameters within the GARCH model. In the model, they estimate the conditional covariance matrix between three financial assets. Their general structural-form model assumes the dynamics between the three financial assets to be described by:

$$\mathbf{B} \underset{3 \times T}{y} = \psi + \varphi(L)y_t + \phi(L)g_t + \eta_t. \quad (5.11)$$

In this model  $y_t$  contains the variables of interest,  $\psi$  is a vector of constants,  $\varphi(L)y_t$  contains lags of the endogenous variables and  $\phi(L)g_t$  represents other exogenous variables that may influence the system. The system can also be extended to contain lags of the endogenous variables and additional exogenous variables that may influence the system, like commodity prices. In this set-up, the matrix  $\mathbf{B}$  captures the contemporaneous relationship amongst the endogenous variables. The matrix is normalised to have the following form:

$$\mathbf{B}_{3 \times 3} \equiv \begin{bmatrix} 1 & b_{12} & b_{13} \\ b_{21} & 1 & b_{23} \\ b_{31} & b_{32} & 1 \end{bmatrix}. \quad (5.12)$$

Equation 5.11 is once again equivalent to a latent factor model where  $\eta_t = (\eta_{1,t}, \eta_{2,t}, \eta_{3,t})$  represents the “structural innovations” or latent factors that drive asset movements in the system. Given the assumption that the structural innovations that represent changes in fundamental factors, have zero mean and conditional cross moments, the following characteristics apply across time:

$$\begin{aligned} E_t(\eta_i) &= 0 \quad \text{where } i = 1, 2, 3 \\ E_t(\eta_1 \eta_2) &= 0 \\ E_t(\eta_1 \eta_3) &= 0 \\ E_t(\eta_2 \eta_3) &= 0 \end{aligned} \quad (5.13)$$

Furthermore, these structural innovations are assumed to exhibit a GARCH(1,1) behaviour equivalent to

$$h_t = \psi_h + \Pi h_{t-1} + \Lambda \eta_{t-1}^2 \quad (5.14)$$

The conditional variances are then given by  $h_t \equiv (h_{1,t}, h_{2,t}, h_{3,t})'$ . This implies that structural innovations evolve from their lagged values, the magnitudes of the most recent shocks and a constant. The matrices  $\Pi$  and  $\Lambda$ , which determine the dependence of the conditional variances on their lagged values and on lagged shocks, are subject only to the restrictions that their elements are positive and have finite second moments.

Identification of the system in equation 5.11 can be achieved if there is conditional heteroscedasticity in the data. The intuition behind the identification is based on the movement of structural innovations and the movement of the conditional covariances between them. As explained in chapter 2, the heteroscedasticity adds equations to the system, thereby making identification of the structural parameters possible. These movements depend on the contemporaneous responsiveness to one another.

The reduced-form model (which is being estimated for purposes of obtaining the reduced-form residuals used in the GARCH model) is given by

$$y_t = c + F(L)y_t + q(L)g_t + v_t \quad (5.15)$$

where all the variables are premultiplied by the inverse of matrix  $B$ . This implies that the reduced-form residuals and the structural innovations exhibit the following relationship:

$$v_t = \begin{pmatrix} v_{1,t} \\ v_{2,t} \\ v_{3,t} \end{pmatrix} = B^{-1} \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix}. \quad (5.16)$$

The reduced-form coefficients can be estimated consistently using ordinary least squares (OLS). Thus, the structural coefficients can be recovered if matrix  $B$  is identified. The reduced-form residuals  $v_t$  will exhibit GARCH behaviour if the structural innovations exhibit GARCH behaviour. The second moments of the reduced-form residuals will satisfy

$$v_t \sim N(0, H_t) \quad (5.17)$$

$$H_t = \begin{bmatrix} H_{11,t} & H_{12,t} & H_{13,t} \\ & H_{22,t} & H_{23,t} \\ & & H_{33,t} \end{bmatrix}$$

with

$$\begin{bmatrix} H_{11,t} \\ H_{12,t} \\ H_{22,t} \\ H_{13,t} \\ H_{23,t} \\ H_{33,t} \end{bmatrix} = \underset{6 \times 3}{C_f} \cdot \underset{3 \times 1}{\Psi_h} + \underset{6 \times 3}{C_f} \cdot \underset{3 \times 3}{\Pi} \cdot (\underset{3 \times 3}{C^2})^{-1} \begin{bmatrix} H_{11,t} \\ H_{22,t} \\ H_{33,t} \end{bmatrix} + \underset{6 \times 3}{C_f} \cdot \underset{3 \times 3}{\Lambda} \cdot (\underset{3 \times 3}{C^2})^{-1} \begin{bmatrix} v_{1,t-1}^2 \\ v_{2,t-1}^2 \\ v_{3,t-1}^2 \end{bmatrix} \quad (5.18)$$

where

$$C \equiv \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \equiv B^{-1} \quad (5.19)$$

and

$$C_f = \begin{bmatrix} c_{11}^2 & c_{12}^2 & c_{13}^2 \\ c_{11}c_{21} & c_{12}c_{22} & c_{12}c_{23} \\ c_{21}^2 & c_{22}^2 & c_{23}^2 \\ c_{11}c_{31} & c_{12}c_{32} & c_{13}c_{33} \\ c_{21}c_{31} & c_{22}c_{32} & c_{23}c_{33} \\ c_{11}^2 & c_{11}^2 & c_{11}^2 \end{bmatrix}. \quad (5.20)$$

From equation 5.18 the structural parameters can be obtained. It shows that the structural-form GARCH specification imposes restrictions on the evolution of the conditional variance-covariance matrix of the reduced-form innovations. These restrictions once again result from the fact that the conditional covariances between the structural innovations are assumed to be zero. The structural-form GARCH model in equation 5.18 contains 27 parameters to be estimated, consisting of 3 constants, 6 coefficients in matrix B and 9 coefficients each in matrix  $\Pi$  and  $\Lambda$ . If the structural innovations were allowed to have

conditional covariances different from zero, the model becomes an unrestricted multivariate GARCH model with 60 parameters, which proves extremely difficult to estimate.

Unlike most multivariate GARCH models that are “reduced-form” models, this model enables one to recover the structural parameters. In cases where models have attempted to recover the structural parameters, the restrictions placed on the model were mostly on an ad hoc basis and not derived as in Rigobon and Sack (2003). Although this “structural” GARCH model recovers the structural parameters, it is still a reduced-form model in the sense that it does not distinguish explicitly between the conditional variances generated endogenously within the system and conditional variances generated exogenously by structural innovations.

Step 1 of the proposed methodology uses equation 5.14 and the parameters estimated in equation 5.18 to obtain:

- a.) The conditional variance of the structural innovations to each variable in the system, i.e. the exogenous part of the conditional covariance is modelled. These variables have no covariance as they are assumed to be independent in order for the equations to be identified. Since these are structural innovations, this assumption is not restrictive as it is generally assumed in macroeconomics for fundamental shocks to be independent.
- b.) The structural parameters of the system. These structural parameters (matrix B) allow one to determine the variation of a variable explained endogenously within the system of assets and the variation explained exogenously by external structural innovations.

Once the conditional covariance matrix of the structural innovations<sup>5</sup> is retrieved from the model in equation 5.18, the second step can be performed. This requires the estimation of the conditional covariance matrix of the endogenous explained variation of the variables ( $H_{t,\text{endogenous}}$ ).

### 5.3 STEP 2: ESTIMATING THE ENDOGENOUS CONDITIONAL COVARIANCE MATRIX OF VARIABLES IN THE SYSTEM

Utilising matrix B in equation 5.16 and equation 5.19 the variation of the variable explained by other variables in the system can now be determined. This is done by simply substituting the structural parameters into the equation for each variable. Depending on the data generating process of the resulting series of the explained variation it is possible to model its volatility across time. If the series exhibit GARCH behaviour, then a multivariate GARCH model can be used to model the conditional covariance matrix of this endogenous variation. In the case of modelling with financial data, it is likely that the endogenous variation of a variable will exhibit GARCH behaviour, for it is simply a linear combination of individual series that exhibit GARCH behaviour. Therefore, any applicable multivariate GARCH model (e.g. a BEKK specification) described in chapter 4 can be used to model the endogenous variation of the variables.

Once steps 1 and 2 have been completed, the two parts can be summed to give the total conditional covariance matrix of the system of variables as in equation 5.1. Where the multivariate GARCH models from chapter 4 only determine the total conditional covariance matrix ( $H_{t,\text{total}}$ ) the two steps allow for a more detailed breakdown of the structure of the volatility.

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<sup>5</sup> This is the conditional covariance matrix " $H_{t,\text{exogenous}}$ " from equation 5.1, which is a diagonal matrix since the structural innovations are assumed independent.

## 5.4 CONCLUSION

Decomposing the variance of a system of endogenous variables necessitates the identification of the system of equations. This is required to distinguish between the portion of the variable explained endogenously within the system and the portion of the variable explained exogenously from outside the system. However, given that such a system is not identified, in order to overcome this problem, heteroscedasticity is used to identify the equations. By employing two multivariate GARCH models the system is decomposed into two parts. The first is the conditional covariance matrix of the structural innovations to the variables. The second part is the conditional covariance matrix of the explained variation of the variables.

This two-step methodology for decomposing the covariance of a system of endogenous variables provides more information than under traditional reduced-form GARCH models. It allows one to determine the amount of volatility generated by other variables, and the amount of volatility generated by structural innovations. It also allows for the retrieval of the structural parameters of equations, without imposing invalid constraints on the system. Lastly it allows for structural analysis of the conditional variances of individual variables or combinations of them.

Traditional GARCH models discussed in chapter 4 focuses only on the reduced-form without recovering the structural parameters (matrix B). They do not measure the contemporaneous interactions between variables. These models therefore have to specify the conditional heteroscedasticity directly in terms of the reduced-form innovations, rather than in terms of the structural-form innovations as in the case with the Rigobon-Sack model. The two-step decomposition allows for a more tractable analysis of how volatility is generated between different variables.

## Chapter 6

### LITERATURE REVIEW ON EMPIRICAL RESEARCH

#### 6.1 INTRODUCTION

Economists have long debated the effect that financial variables have on one another. This debate was fuelled anew after several financial crises hit the world economy during the 1990s. These crises spread very fast across regions and within domestic economies. Understanding how domestic financial variables influence one another has therefore become an important focus in financial research. Much research has focused on exchange rates, monetary policy and international stock markets. The effect they have on one another, i.e. their structural dependence (through the mean) as well as through the second moments have become equally important. Since the variables of interest in this study are the ALSI, the South African Rand /US Dollar exchange rate and the South African 90-day Treasury bill interest rate, the literature review will cover some of the findings and empirical techniques applied to estimate the relationships between these variables.

As mentioned, these variables are determined within a system. Furthermore, given the nature of high-frequency data, problems arise in identifying the structural relationships between these variables. For these reasons, almost all of the applied studies have focused on reduced-form estimates between these variables. In the cases where structural relationships were determined, they tend to be single-equation estimations. Nevertheless, an overview of existing research will be informative in understanding these relationships and provide more information on the relationships to expect when estimating the structural parameters.

For expositional reasons, this chapter is divided into three parts. The first section discusses studies that analysed relationships between stock prices and the exchange rate. Section 2

discusses studies covering the relationship between stock markets and monetary policy<sup>6</sup>. The third and final section covers empirical findings on the relationship between the exchange rate and monetary policy or short-term interest rates.

## 6.2 STOCK PRICES AND THE EXCHANGE RATE

The theoretical link between stock prices and exchange rates can be explained by two different approaches (Yang, 2003). The first is flow-oriented models of exchange rates or goods market approaches. This approach focuses on the current account or the trade balance. Changes in the exchange rate affect international competitiveness and the resulting trade balance influences real domestic income and output. The stock prices, generally interpreted as the present value of future cash flows of firms, react to exchange rate changes and form the link between future income, interest rate innovations, and current income and consumption decisions. Innovations in the stock market then affect aggregate demand through wealth and liquidity effects, thereby influencing money demand and exchange rates (Gavin, 1989).

The second approach is stock-oriented models of exchange rates, or the so-called portfolio-balance approach (e.g. Branson, 1983; Frankel, 1983). These models view exchange rates as equating the supply and demand for assets, such as stocks and bonds. This approach gives the capital account an important role in determining exchange rate dynamics. Since the values of financial assets are determined by the present values of their future cash flows, expectations of relative currency values play a considerable role in their price movements especially for international held financial assets. Therefore, stock price innovations may affect, or be affected, by exchange rate dynamics.

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<sup>6</sup> In the context of this study analysing monetary policy and a short-term interest rate is equivalent for they are closely related through monetary policy in South Africa.

Early empirical studies have focused on the contemporaneous relation between stock returns and exchange rates. Aggarwal (1981) used monthly data for US stock markets and a trade weighted exchange rate for the Dollar for the period 1974 – 1978. He found a positive relationship between stock prices and the exchange rate. Soenen (1988) found a strong negative correlation between U.S. stock markets and a weighted Dollar exchange rate. Ma (1990) explained these contradicting results by looking at the structure of the economy. For an export-dominant economy, a currency appreciation has a negative effect on the stock market, while a currency appreciation boosts the stock market for import-dominant economies.

More recent studies have focused on the direction of causality between exchange rates and stock prices for major industrial economies. Bahmani-Oskooee and Sohrabian (1992) showed that there is a bi-directional causality between stock prices in the US and the effective exchange rate of the Dollar. Ajayi and Mougoue (1996) found short-run and long-run feedback between the two variables in eight industrial economies. Their results show that if the exchange rate appreciates it has a short-run negative effect and long-run positive effect on the stock market. Ajayi, Friedman and Mehdian (1998) provided evidence of unidirectional causality from the stock market to the currency market for advanced economies and no consistent relations in emerging markets.

Using cointegration techniques Harasty and Roulet (2000) model stock prices of 17 developed countries. They argue that theory explains long-run movements in stock prices while other variables will drive stock prices in the short run. They find that the main drivers of stock prices in the long-run are earnings and long-term interest rates. However, in the short-run variables like short-term interest rates and the exchange rate tend to determine stock prices.

Attempts have also been made to analyse the possibility that the transmission of volatility spillover effects can exist between the stock market and currency markets. Most of this literature examines the stochastic behaviour of stock prices and exchange rates employing

ARCH and GARCH specifications. For example, Hamao, Masulis and Ng (1990) investigated the price and volatility spillovers in three major stock markets while Koutmos and Booth (1995) found asymmetric spillovers across stock markets and the exchange rate. Yang (2003) adopted a bivariate EGARCH framework and investigated the volatility spillovers between stock and exchange rate for G8 countries. He found that movements in the stock prices affect future exchange rate movements, but that changes in the exchange rates have less direct impact on future changes in the stock prices. His results also pointed to significant volatility spillovers between some of the stock markets and the currencies of the G8 countries.

There exists a significant amount of research on the behaviour between the stock markets and exchange rates. Depending on the structure of the economy, an exchange rate can either have a positive or negative effect on the stock market. The movements of the stock market in turn also affect the exchange rate. However, not much empirical research is available on the volatility spillovers across the two variables for emerging markets. The available research indicates that in some instances there exist volatility spillovers.

### **6.3 STOCK PRICES AND THE INTEREST RATE**

Theory posits that stock prices equal the expected present value of future net cash flows. Therefore, any evidence that a monetary tightening is expected should have a decreasing effect on stock prices by decreasing future cash flows or by increasing the discount factors at which those cash flows are capitalised ((Thorbecke, 1997).

To examine the relationship between stock prices and monetary policy, a variety of empirical techniques have been employed. These differ from single-equation estimation, VAR's and impulse responses to variance decompositions. The standard methods employed can be categorised into three techniques. The first applies simultaneous equations in the form of vector autoregressive (VAR) estimations. These studies employ impulse responses and study

mainly the reduced-form parameters. The second makes use of narrative accounts by looking at monetary authorities' reaction to stock prices and vice versa. These studies do not attempt to obtain magnitudes of the relationships. The last category employs event studies using high-frequency data to estimate the reaction of stock prices to monetary policy. The general finding of most papers is that expansionary monetary policy increases an asset's return.

Relevant to the first category, i.e. VAR's, Bernanke and Blinder (1992) used monthly data for the federal funds rate and employed a VAR approach to measure monetary policy. Evidence from variance decomposition and Granger causality tests indicated that the funds rate adequately predicted stock returns over the period 1959 to 1989. The evidence indicates that there should be some reaction in stock prices when interest rates change. Christiano, Eichenbaum and Evans (1994) applied a monthly VAR amongst other variables, the federal funds rate and stock prices. Orthogonalised innovations in the funds rate were used to measure monetary policy. In similar fashion, Thorbecke (1997) also used a VAR approach applied to monthly data to measure the impact of monetary policy on stock prices. However, unlike Christiano et al., who found that the funds rate did not predict movements in stock prices, Thorbecke found evidence of the federal funds rate influencing stock return.

Zhou (1996) studied the relationship between interest rates and stock returns using regression analysis. He found that interest rates have an influence on stock prices over longer horizons, but that this relationship is not so strong in the short run.

The narrative approach to identify monetary effects on stock prices was pioneered by Friedman and Schwartz (1963). They used Federal Reserve statements and other historical documents over the 1867 – 1960 period to identify exogenous changes in monetary policy and the responses of real variables. Romer and Romer (1989) extended this research to include the period after 1960 up until 1988. Both studies found that monetary policy innovations such as changes in the federal fund rate are highly correlated to changes in the stock market. The approach was also employed in other research, e.g. Boschen and Mills (1995) and White (1984).

The third category investigates the relationship between monetary policy and stock returns and uses daily data. The studies that use daily data tend to be event studies. If the monetary authorities control a short-term interest rate in setting monetary policy very closely, market participants are able to discern a change in the fund rate target on the day it occurs (Cook and Hahn, 1989). By collecting anticipated rate changes in the financial media, actual rate changes are easily identified. By regressing a change in stock prices on the change in the policy interest rate over the period of change, the relationship between the variables is measured. Using event studies, significant negative relationships between policy induced changes in the interest rate and changes in stock prices are found (see for example Thorbecke (1997), Thorbecke (1995), Jones (1994), Bradsher (1994), Risen (1994), Grant (1992) and Cook and Hanh (1989)).

Different to the methods applied for high-frequency data, Rigobon and Sack (2003) employed a “structural” GARCH model to determine the contemporaneous effects and spillovers between US stock prices, the long-term interest rate and the short-term interest rate in the US. They found significant contemporaneous effects and volatility spillovers between these variables. The methodology used in this study is an extension of their methodology. Although Rigobon and Sack (2003) estimate the structural parameters, they do not decompose the variance into exogenous and endogenous parts.

This section gave a brief summary of the three different methods that are generally used to determine the short run reaction of stock prices on changes in monetary policy as defined by a change in a short-term interest rate. Where the effects were measured using systems of equations, the structural coefficients are mostly not recovered. When working with high-frequency data (i.e. event studies), the estimates obtained are mostly single equation estimates, ignoring the contemporaneous effects between variables.

As far as South Africa is concerned, van Rensburg (1998) used bivariate Granger causality tests and correlations to study relationships between stock returns and macroeconomic variables. Although he doesn't estimate the relationship he found that various interest rates

(including a short-term interest rate) do influence stock returns on the Johannesburg Stock Exchange (JSE). Barr (1990) also models returns on the JSE as a function of macroeconomic variables. Barr follows a factor-analytical approach and identifies short-term interest rates as one factor out of four that influence the stock returns on the JSE.

#### **6.4 THE EXCHANGE RATE AND THE INTEREST RATE**

Research on the relationships between these two economic variables can be dated back to the study of the interest rate parity condition. The existing literature, however, focuses mainly on the long-run equilibrium relationships between interest rates and exchange rates. The short-run relationships between these two markets are often ignored. Theoretically, it is true that the equilibrium relationships between the interest rate and the exchange rate should be a long-run concept; nevertheless, often short-run changes in the exchange rates are observed after changes in the interest rate. Apart from price movements, the relationship between higher moments of the two markets also deserves an examination because the variance is also a source of information.

Since the Asian financial crisis the high frequency relationship between the exchange rate and the short-term interest rate has been at the centre of a hot policy debate. The questions raised are whether an increase in the interest rate results in an appreciation of a currency, or whether sharp rises in the interest rate destabilise the currency (by increasing the risk of bankruptcy). Given the monetary approach to exchange rate determination, an increase in the interest rate should result in an appreciation of the exchange rate. Tight monetary policy strengthens the exchange rate by sending a signal that authorities are committed to maintaining a strong currency, thereby increasing capital inflows (Backus and Driffill, 1985). Also, depending on the monetary policy setting of a country, a depreciation of the currency will result in an increase in the short-term interest rate via possible inflationary pressures imported into the domestic economy. A number of economists (e.g. Radelet and Sachs, 1998, Feldstein, 1998 and Stiglitz, 1999) argued against the signaling value of monetary policy

by considering the positive effect of the interest rates on the likelihood of bankruptcy for highly leveraged borrowers.

The empirical evidence on the issue is mixed. Some empirical studies (based on reduced-form VAR specifications) support the traditional view of interest rates and exchange rates (i.e. the monetary approach). Dekle et al. (1998), using weekly data, found that in the case of Korea an increase in the interest rate differential helped to appreciate the Korean Won. Tanner (1999) also used a VAR approach and found that tight monetary policy helps to reduce exchange rate market pressures.

A number of empirical studies support the view that increases in interest rates might not lead to exchange rate appreciation. Goldfajn and Baig (1998) used a VAR approach and impulse responses based on weekly data and found a perverse effect of monetary tightening on the exchange rate for six emerging markets countries. They found that during periods of high volatility the exchange rates are not significantly affected by changes in the interest rate in any of the countries examined. Ohno, Shirono and Sisly (1999) found similar results for daily data for seven Asian countries. Caporale et al. (2000) also evaluated whether tight monetary policy was successful in defending the exchange rate from depreciation during the Asian financial crisis. They applied their analysis to 5 Asian countries utilising a bivariate VAR model and identified the structural parameters using the heteroscedasticity in the data. Their empirical evidence shows that tight monetary policy did not help to stabilise the currencies under investigation.

However, in determining the interaction between the interest rates (monetary policy) and exchange rates, there are important challenges. The main challenge is the issue of identification of monetary policy exogenous shocks as distinct from monetary policy actions. An identification scheme is needed to solve the simultaneity problem between policy instruments (i.e. the interest rate) and other endogenous variables, such as the exchange rate to which monetary policy systematically reacts (Caporale et al., 2000). Using VAR analysis does not explicitly recognise this feedback between the two variables. By identifying the

structural relationships between the variables, it is possible to identify the exogenous reaction of policy to movements in the exchange rate, and the exchange rate's reaction to changes in policy.

Juselius and McDonald (2000) empirically examine the joint determination of a number of key parity conditions for Germany and the US using monthly data. They consider the German mark – US dollar exchange rate, prices, short term interest rates and long term interest rates. They use a cointegrated VAR model to define long-run stationary relationships as well as common stochastic trends. They find that long term bond rates in both the US and Germany that drives exchange rates. However, they also found that the short-term interest rate was an important driver of movements in the purchasing power exchange rate.

The methodology applied in chapter 7 to estimate a structural GARCH model for the South African case, allows for the identification of exogenous changes in the interest rate (i.e. policy shocks) that do not come from movements in the South African Rand/US Dollar exchange rate or the South African stock exchange. This methodology also allows for the measurement of the effect of an interest rate change on the exchange rate in both the mean and variance.

## **6.5 CONCLUSION**

Current empirical literature on determining the relationship between the exchange rate, the short-term interest rate and the stock market vary greatly in terms of the methodology applied. The focus tends to estimate the relationship on terms of high-frequency data such as daily, weekly and monthly data. The majority of the research relates to reduced-form estimations mainly in the form of vector auto regression (VAR) analysis. It therefore ignores the structural relationships that exist between the variables of interest. Furthermore, the literature indicates that there exist contemporaneous effects amongst the three variables discussed, which makes it important to estimate the parameters simultaneously. In most cases where the system is solved simultaneously, the structural parameters are not recovered. In the cases where structural relationships are estimated, the estimates tend to be either single-

equation estimates (i.e. not solved simultaneously) or ad hoc restrictions are placed on the system (such as long-run constraints) to identify the structural parameters. Therefore, given the necessity of solving the equations simultaneously, and recovering the structural parameters without placing unnecessary constraints on the system, the approach and methodology employed in this research are essential. In applying these techniques in chapter 7 this methodology allows for the simultaneous estimation of the contemporaneous structural parameters. The identification methodology also allows for the determination of the volatility due to endogenous reactions and the volatility due to exogenous structural innovations. This study encompasses previous literature in that it measures structural relationships between these variables as well as volatility spillovers in the system of equations.

## Chapter 7

### ESTIMATING A STRUCTURAL GARCH MODEL

#### 7.1 INTRODUCTION

When modelling high-frequency data, the contemporaneous effects between variables may differ significantly from their long-run behaviour. It is likely that there are contemporaneous effects across all variables that need to be determined. However, such a system with endogenous variables is not identified (see chapter 2). When interested in determining how volatility is generated in a system, the identification problem will yield serious problems in the analysis. First of all, only the reduced-form parameters are observable. In order to recover the structural parameters, restrictions need to be imposed on the reduced-form parameters. Most of these restrictions are long run in nature and cannot always be justified when using high-frequency data. Secondly, if the structural parameters are not observable, it is impossible to decompose the system into variability explained within the system and variability due to external structural innovations.

In this chapter, the proposed two-step methodology (outlined in chapter 6) is implemented to decompose the conditional covariance matrix of a system of financial variables for South Africa. This two-step approach allows one to identify the system, and determine the “endogenous” conditional covariance matrix as well as the “exogenous” conditional covariance matrix. The approach utilises two multivariate GARCH models to obtain the results. In the first step, a multivariate GARCH model developed by Rigobon and Sack (2003) is utilised. This model solves the identification problem using heteroscedasticity as instrument, while an estimate for the conditional covariance matrix of the external structural innovations can also be recovered from the model. The second step utilises a standard BEKK model. The BEKK specification is used to estimate the conditional covariance matrix of the “endogenous” variation from within the system. Once the two steps are completed, the conditional covariance matrices can be summed in order to get the conditional

covariance matrix for the total system. The two-step methodology allows for analysis of the variances that is not possible with traditional multivariate GARCH models.

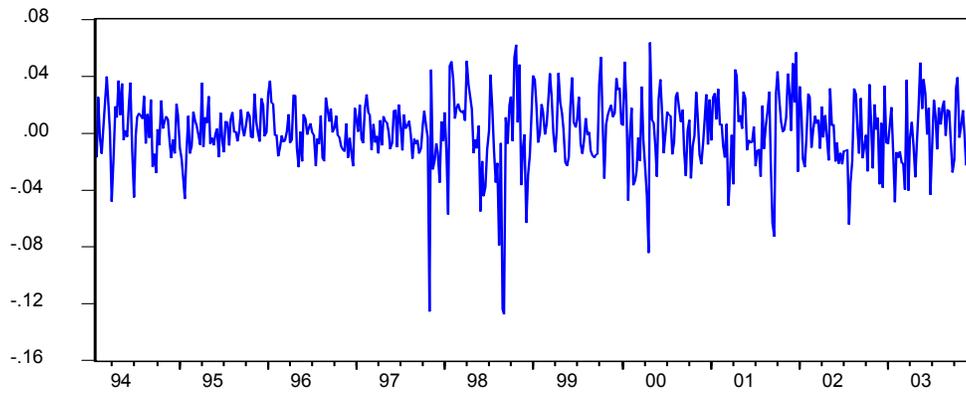
## 7.2 THE DATA

The analysis utilises three financial variables for South Africa. However, financial variables are exposed to the problem of simultaneity for their movements are determined within a broader integrated financial system causing two sets of problems. First, although certain unidirectional long-run relationships do exist amongst most financial variables, the short-run relationships often differ from what long-run theory suggest. Second, financial variables often exhibit heteroscedasticity, which makes this type of system ideal for implementing restrictions through heteroscedasticity in order to identify the structural parameters.

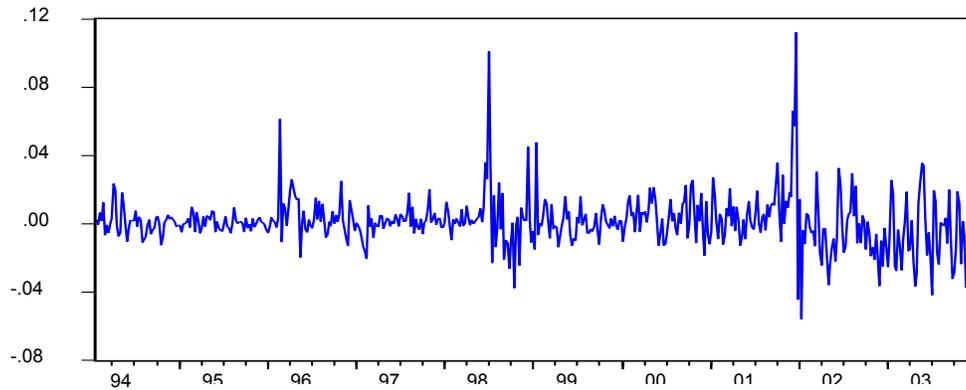
The first financial variable used is the return on the All Share index (ALSI) of the Johannesburg stock exchange in South Africa. The second variable is the change in the South African Rand/US Dollar exchange rate (R/\$). The third and final variable is the change in a short-term interest rate in South Africa, namely the 90-day Treasury bill interest rate (Tbill). These three variables were chosen for their importance in the economy mainly from a monetary policy perspective. Since these three variables are so closely linked and plays a significant role in determining inflation, understanding the high frequency relationship to one another is important not only from a portfolio point of view but also from a monetary policy perspective.

Weekly data for the three variables are used for the period January 1995 to December 2003. The reason for weekly data as opposed to daily data is that when analysing the volatility of these variables in terms of portfolios, it might be more useful to have a weekly analysis than daily analysis. It is not always possible (and feasible) to rebalance a portfolio on a daily basis. Figure 7.1 represents an exposition of the data for the three variables.

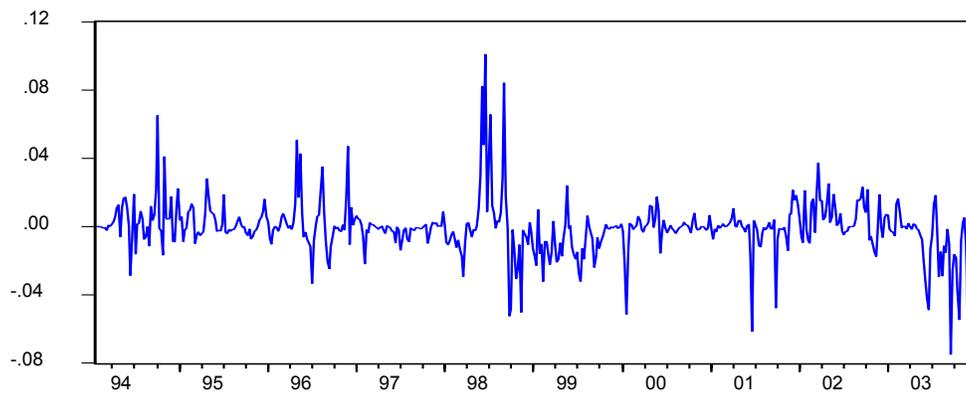
Figure 7.1: The three financial variables used in the estimation



Weekly return on the ALSI



Weekly changes in the R/\$ exchange rate



Weekly changes in the 90-day Treasury bill interest rate

Similar to Rigobon and Sack (2003) the system analysed is of the form:

$$B \underset{3 \times T}{y} = \psi + \varphi(L)y_t + \phi(L)g_t + \eta_t \quad (7.1)$$

Once again, in this model  $y_t$  contains the three variables of interest, i.e.  $y_t = (\text{ALSI}_t, R/\$, \text{Tbill}_t)$ .  $\psi$  is still a vector of constants,  $\varphi(L)y_t$  contains lags of the 3 endogenous variables and  $\phi(L)g_t$  represents other exogenous variables that may influence the system like commodity prices. It is important to notice that this system is not identified. The objective of this analysis is to analyse the “structural” volatility of this system by implementing the two-step methodology as proposed in chapter 5. It is expected that changes in these variables influence one another in the short-run, which perhaps differ from the long run.

Given the fact that one of the objectives is to recover the structural contemporaneous parameters in the system, a short discussion on their expected signs will be informative. A priori expectations are that a positive movement in the ALSI will result in an appreciation of the exchange rate (i.e. a decrease in R/\$). As the ALSI rise, it is likely that foreign investors will seek to gain from the increases in stock returns. The result is a higher demand for South African Rand. Also, a positive movement in the ALSI is expected to have a positive impact on the interest rate through the wealth effect in the economy. This can be seen as a high frequency monetary response effect.

A depreciation (increase) in the R/\$ exchange rate is expected to induce a positive effect on the ALSI. Commodity shares have the greatest market capitalisation on the Johannesburg stock exchange. Since the companies earn foreign currency, these stocks tend to be Rand-hedged shares. Therefore, if the R/\$ exchange rate depreciated (increased) the companies' Rand-profits are expected to increase, thereby pushing up the share prices. Furthermore, a depreciation in the R/\$ exchange rate is also expected to result in an increase in the interest rate. Since a depreciating Rand implies higher imported prices, and therefore inflation

pressures, it can be expected that monetary authorities may increase interest rates when the Rand depreciates, and decrease interest rates when the Rand appreciates.

As far as a change in the interest rate is concerned, the usual intuition applies. An increase in the interest rate is expected to have a negative effect on the ALSI. This effect can be thought of as the standard discount dividend model, where an increase in the interest rate (i.e. discount rate) results in lower stock prices. Lastly, a positive change in the interest rate is expected to have decreasing effect (appreciation) on the exchange rate. The monetary approach to exchange rate determination suggests that an increase in domestic interest rates relative to foreign rates will result in an appreciation of the domestic currency.

To summarise the expected causalities between the three variables:

- An increase in the ALSI is expected to result in an appreciation (decrease) in the R/\$ exchange rate (i.e. a negative relationship). On the other hand, a depreciation (increase) in the R/\$ exchange rate is expected to result in an increase in the ALSI (i.e. a positive relationship). The net effect between changes in any of the two variables will be either positive or negative depending on which direction dominates.
- A depreciation (increase) in the R/\$ exchange rate is expected to result in an increase in the Tbill (i.e. a positive relationship), while it is expected that an increase in the Tbill will result in an appreciation (decrease) in the R/\$ exchange rate (i.e. a negative relationship). Once again, the net effect between changes in any of the two variables will be either positive or negative depending on which direction dominates.
- An increase in the Tbill is expected to result in a decrease in the ALSI (i.e. a negative relationship), while an increase in the ALSI is expected to result in an increase in the Tbill (i.e. a positive relationship). The net effect between changes in any of the two variables will be either positive or negative depending on which direction dominates.

### 7.3 ESTIMATING THE CONDITIONAL COVARIANCE MATRIX OF THE SYSTEM

The methodology outlined in chapter 5 contains two steps. Each of the two steps is implemented in this section and a detailed discussion of the results is given. This empirical application puts the importance of structural analysis into perspective and highlights the possible mistakes than can be made when using only reduced-form estimates.

#### 7.3.1 Step 1: Estimating the exogenous conditional covariance matrix driven by the structural innovations in the system

The first step is to estimate the Rigobon and Sack (2003) GARCH model as in equation 5.18. The parameters of interest in this model are matrix  $C$ , matrix  $\Pi$  and matrix  $\Lambda$ . All three these matrices are contained in equation 5.18. This representation of the GARCH model allows one to retrieve the structural contemporaneous coefficients through equation 5.19. It also allows one to retrieve the GARCH behaviour through equation 5.14. For expositional reasons the equations are again presented below:

$$h_t = \Psi_h + \Pi h_{t-1} + \Lambda \eta_{t-1}^2. \quad (5.14)$$

$$\begin{bmatrix} H_{11,t} \\ H_{12,t} \\ H_{22,t} \\ H_{13,t} \\ H_{23,t} \\ H_{33,t} \end{bmatrix} = \underset{6 \times 3}{C_f} \cdot \underset{3 \times 1}{\Psi_h} + \underset{6 \times 3}{C_f} \cdot \underset{3 \times 3}{\Pi} \cdot \underset{3 \times 3}{(C^2)^{-1}} \begin{bmatrix} H_{11,t} \\ H_{22,t} \\ H_{33,t} \end{bmatrix} + \underset{6 \times 3}{C_f} \cdot \underset{3 \times 3}{\Lambda} \cdot \underset{3 \times 3}{(C^2)^{-1}} \begin{bmatrix} v_{1,t-1}^2 \\ v_{2,t-1}^2 \\ v_{3,t-1}^2 \end{bmatrix}. \quad (5.18)$$

$$C \equiv \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \equiv B^{-1}. \quad (5.19)$$

The estimation process is conducted along the following steps. First a VAR of lag order 1 is estimated to obtain consistent estimates for the reduced-form residuals. The lag length was tested using the Schwartz information (SC) criterion, the Hannan-Quinn Information criteria (HQ) and the Akaike Information criteria (AIC). One lag was selected based on two reasons. At 1 lag the residuals obtained are stationary, while this lag length also represent a trading week. The results are presented in table 7.1.

**Table 7.1: Test statistics and choice criteria for selecting the order of the VAR model**

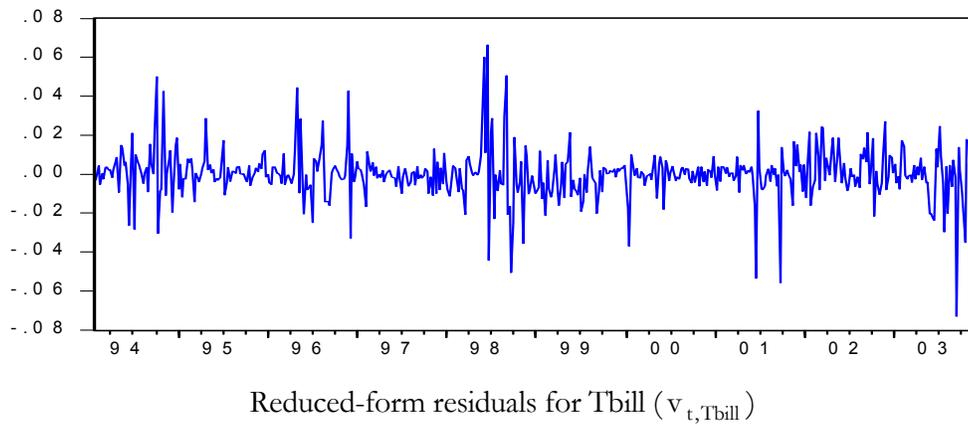
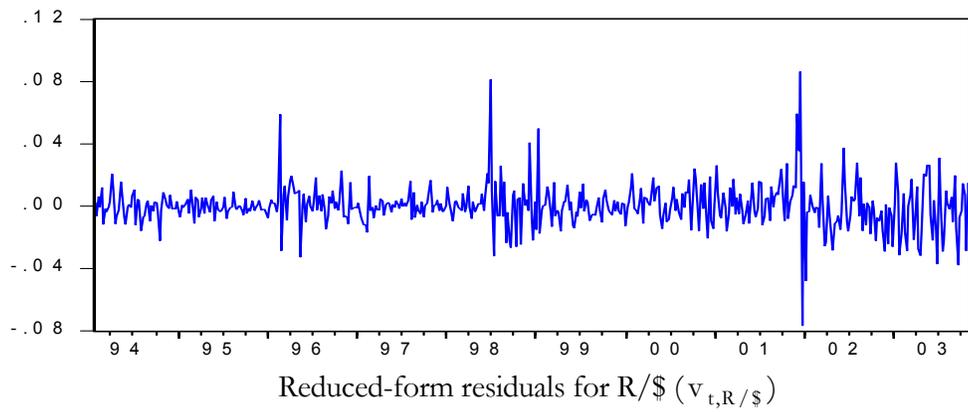
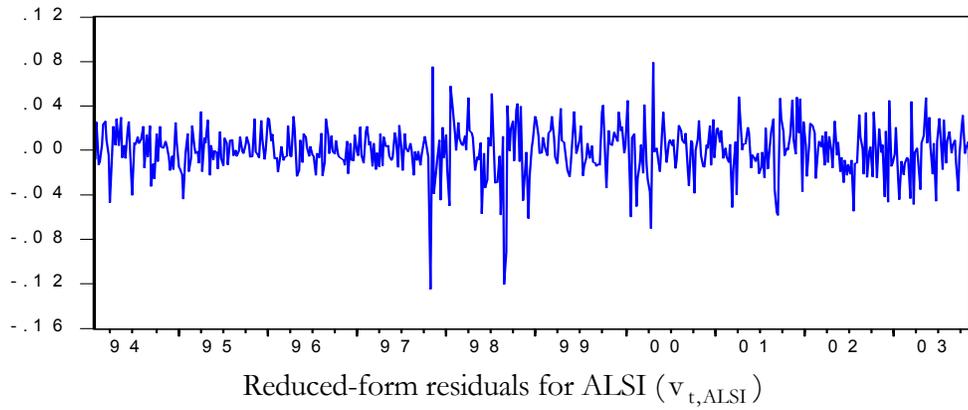
Lag	AIC	SC	HQ
0	-15.72468	-15.70003	-15.71502
1	<b>-17.19704</b>	<b>-17.09844</b>	<b>-17.15841</b>
2	-17.19512	-17.02257	-17.12750
3	-17.18351	-15.93700	-17.08692
4	-17.17511	-15.85466	-17.04955

Source: Own calculations

All three criteria indicate a lag length of one for the VAR<sup>7</sup>. From the VAR the reduced-form residuals ( $v_{i,t}$ ) are retrieved. Figure 7.2 shows the reduced-form residuals.

<sup>7</sup> The estimation results of the VAR are given in Appendix B.1.

Figure 7.2: The reduced-form residuals from the VAR estimation



Once the reduced-form residuals are recovered, the Rigobon and Sack (2003) GARCH model (equation 5.18) can be estimated<sup>8</sup>. The maximum likelihood estimation technique is employed using the BHHH logarithm. Before the matrices of interest are recovered, some of the results from the estimated GARCH model are discussed.

As mentioned in chapter 5, the estimated GARCH model is also a reduced-form model in the sense that the volatility of the structural innovations is not modelled separately. Although the model enables one to retrieve structural coefficients, it doesn't distinguish explicitly between the conditional covariance of the structural innovations and the conditional covariance endogenous to the system.

**Table 7.2: The structural coefficients from matrix B: Contemporaneous interaction between the financial assets (z-statistics in brackets)**

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ALSI =	0.0309R\$ (0.857)	-0.165Tbill (-7.094)
R\$ =	0.026ALSI (4.395)	0.002Tbill (0.342)
Tbill =	0.012ALSI (4.009)	0.061R\$ (2.709)

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Source: Own calculations

<sup>8</sup> The full table of the estimation results is given in Appendix B.2.

The matrix B that contains the contemporaneous interaction between the three variables is estimated and the values are contained in table 7.2. From the table it is evident that there are significant contemporaneous effects between the three variables. The first equation in table 7.2 is the ALSI equation. As a priori expectations would suggest, changes in the ALSI are influenced contemporaneously by changes in the short-term interest rate. This equation can be seen as an exchange rate augmented Gordon-type dividend discount model, where the 90-day Tbill is the discount rate. The R/\$ exchange rate does have a positive sign (which is counterintuitive), however the coefficient is not statistically significant. Given the fact that the ALSI is an index, it may be that in the short run the effect of the exchange rate is cancelled out at an aggregate level across the listed shares. This finding is also consistent with Yang (2003)<sup>9</sup>.

On a weekly basis the change in the Tbill is affected by the change in the R/\$ exchange rate as well as the ALSI. The interest rate equation in table 7.2 can be interpreted as a short-run monetary policy response equation. Due to the nature of monetary policy in South Africa, there is a definite increasing conditional correlation between short-term interest rates and the exchange rate.

From the contemporaneous influence in the exchange rate equation, movements in the interest rate do not influence the exchange rate. Although economic theory dictates that the interest rates determine movements in the exchange rate in the long run, this is not necessarily the case in the short run and specifically on a weekly basis. This also explains the asymmetric effect that South Africa experiences with regard to monetary policy. Short-term interest rates are much more likely to respond to changes in the exchange rate than the exchange rate to changes in the interest rate. A possible explanation for this phenomenon is that it is easier to anticipate short-term interest rate movements than exchange rate movements. Therefore, when the short-term interest rate changes, the movement has already been discounted in the exchange rate. However, because the exchange rate

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<sup>9</sup> The findings of Yang (2003) are briefly discussed in chapter 6.

movements are so difficult to anticipate, it is very difficult to discount the movements in the short-term interest rate.

From the estimated results it is also evident that changes in the ALSI influence the exchange rate contemporaneously, although by very small margins. However, the sign is counterintuitive. It indicates that a positive movement in the ALSI will result in an exchange rate depreciation that is contrary to a priori expectations. A possible explanation for the effect might lie in the relationship between the interest rate and the other two variables. When the interest rate decreases, the ALSI increases (for reasons explained above). Investors, who took advantage of higher interest rate differentials between South Africa and other countries, might decide to move their funds out of the country when the domestic interest rate decreases. Therefore, stocks and money market instruments are not seen as substitutes. The investors might not choose to invest their funds in the ALSI, anticipating an increase in stocks for various reasons. The reasons range from institutional guidelines that prohibit investment in certain stocks to portfolio balance considerations where only a certain amount of the portfolio might for example be invested in emerging market stocks. The resulting outflow of capital will result in a depreciation of the currency.

As far as the conditional covariance matrix of the structural coefficients is concerned, it can be recovered if matrix  $\Pi$  and matrix  $\Lambda$  are known. Equation 5.14 defines the form of the structural innovations. Table 7.3 gives the estimates of these matrices as recovered from the GARCH estimation. The coefficients were restricted to be positive and some of the parameters satisfy this constraint.

It should be clear that the structural innovations exhibit GARCH behaviour. The structural innovations to the ALSI are a function of past shocks to the ALSI itself, as well as some significant volatility spillovers from the Tbill. The structural innovations to the R/\$ exchange rate also exhibit GARCH behaviour, but there are no significant spillovers from other variables. Lastly, the Tbill has GARCH behaviour with some spillovers from structural innovations to the R/\$ exchange rate. This is perhaps not surprising due to the nature of

monetary policy in South Africa. Movements in the exchange rate are likely to induce movements in the interest rate via possible impacts on domestic inflation. Figure 7.3 shows the conditional variances of the structural innovations to the individual variables.

**Table 7.3: Estimates of conditional variance parameters of the structural innovations ( $H_{t,\text{exogenous}}$ )**

Parameters	Coefficient	Standard errors
$\Pi_{11}$	0.844	0.0326
$\Pi_{22}$	0.623	0.052
$\Pi_{33}$	0.198	0.028
$\Lambda_{11}$	0.206	0.046
$\Lambda_{13}$	0.503	0.088
$\Lambda_{22}$	0.236	0.045
$\Lambda_{32}$	0.011	0.006
$\Lambda_{33}$	0.139	0.096

Source: Own calculations

In equation form, the GARCH equations for the structural innovations are presented by:

$$h_{\text{ALSI},t} = 0.844 \cdot h_{\text{ALSI},t-1} + 0.206 \cdot \eta_{\text{ALSI},t-1}^2 + 0.503 \cdot \eta_{\text{TBill},t-1}^2$$

$$h_{\text{R/\$,}t} = 0.623 \cdot h_{\text{R/\$,}t-1} + 0.236 \cdot \eta_{\text{R/\$,}t-1}^2 \quad (7.2)$$

$$h_{\text{TBill},t} = 0.198 \cdot h_{\text{TBill},t-1} + 0.144 \cdot \eta_{\text{TBill},t-1}^2 + 0.011 \cdot \eta_{\text{R/\$,}t-1}^2$$

Figure 7.3 shows that the variance of the structural innovations picks up the major periods of high volatility in these variables. For example, during the Asian crisis in 1997/8 there were a series of huge structural innovations (i.e. external shocks) to the ALSI. This shows up in the conditional variance of structural innovations to the ALSI. The other two variables also show

an increase in volatility during this period. It is possible that all three variables experienced structural shocks and the contagion introduced by the crisis affected all three variables directly. Also, during the second half of 2001 South Africa experienced a large depreciation in its currency that was not due to movements in the interest rate or the ALSI. Therefore, in this system one would expect that the increase in the conditional volatility will show up in the volatility of the structural innovations to the R/\$ exchange rate. This is indeed the case and shows up in figure 7.3.

The Tbill, which follows monetary policy closely, is subject to movements in the repurchase rate set by the South African Reserve Bank. These policy movements can be seen as external shocks to the system that will be picked up by the variance of the structural innovations to Tbill.

One of the conditions for the system to be identified is that the structural innovations exhibit zero covariance. Therefore, the conditional covariance matrix of the structural innovations is a diagonal matrix. This assumption is not restrictive if the fact is considered that most macroeconomic applications assume that these shocks are uncorrelated.

Once the system has been identified, it is possible to recover the actual structural innovations ( $\eta_{i,t}$ ) from the reduced-form residuals based on the fact that the reduced-form shocks are a function of the structural innovations and the contemporaneous parameters<sup>10</sup>. The recovered structural innovations are presented in figure 7.4. It is furthermore possible to obtain the variation of the variable that is explained within the system by a structural equation. This portion for each variable is obtained by simply subtracting the structural innovations to each variable from the total change in each variable. The remainder should be what is explained of the variable by other variables. Figure 7.5 shows the endogenous explained variation of the variables. The difference between the lines in figure 7.1 and figure 7.4 gives figure 7.5 - the endogenous explained variation of the variables.

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<sup>10</sup> See equation 5.16 for the relationship between the contemporaneous equations and the reduced-form residuals from the system.

Figure 7.3: Conditional variances of the exogenous structural innovations

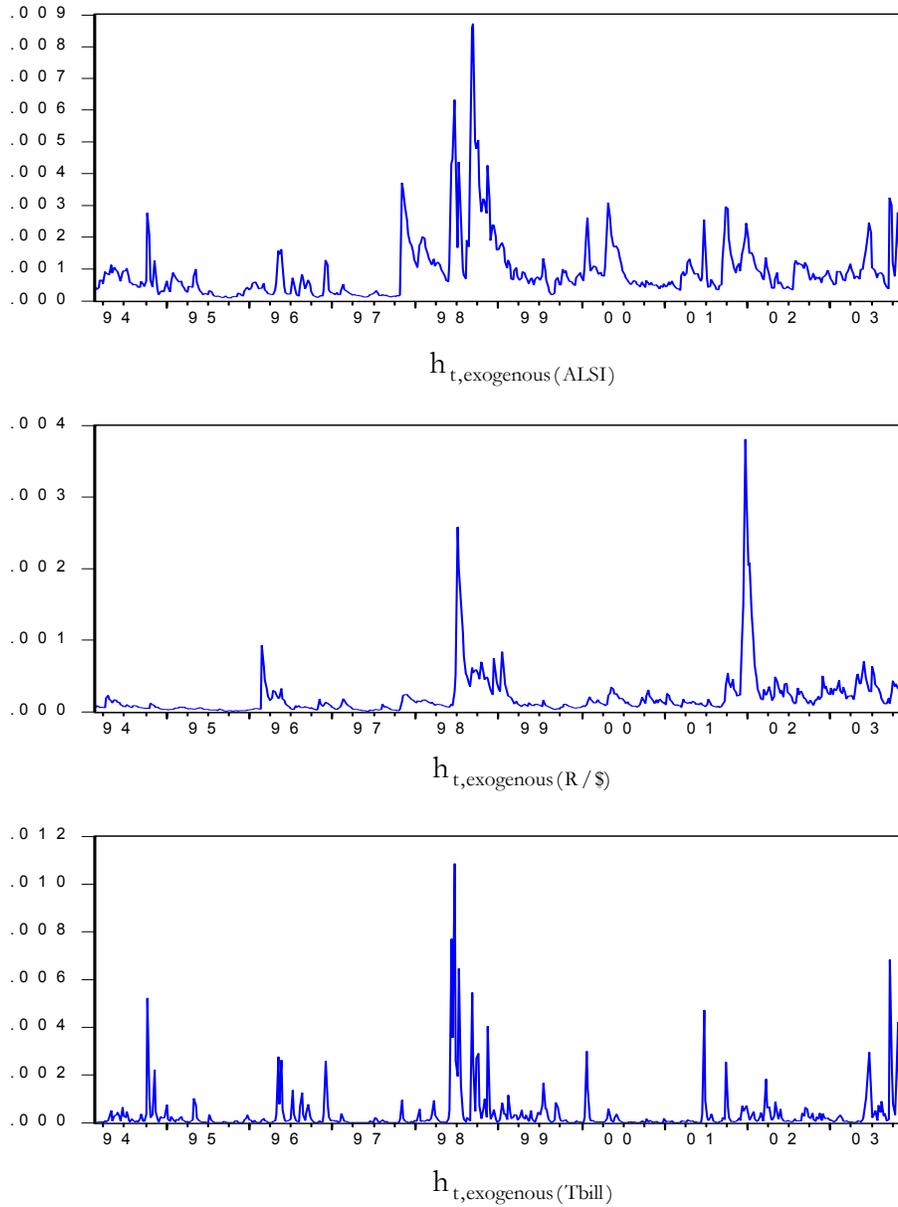
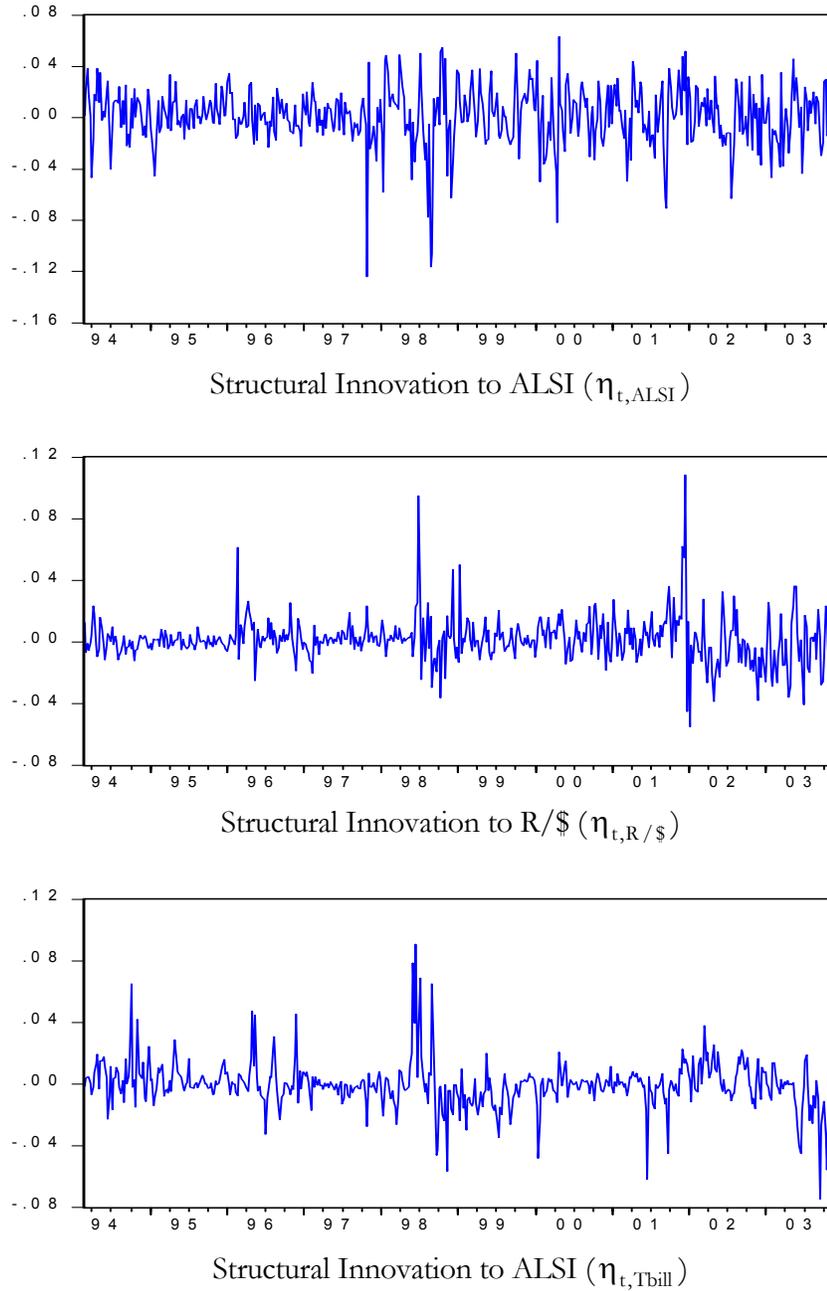
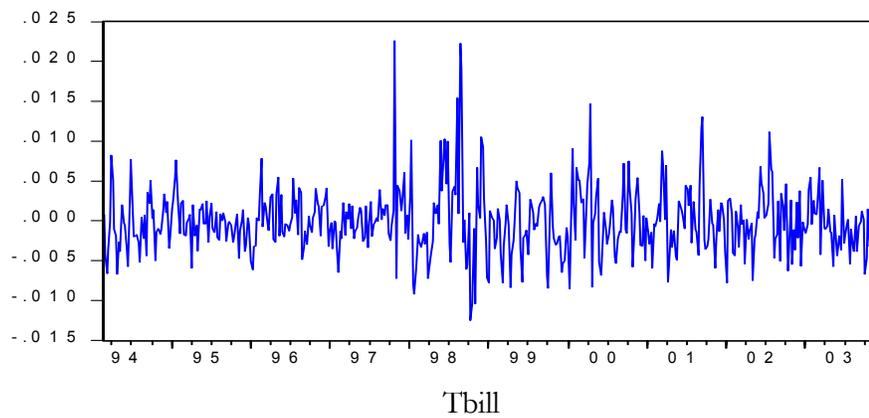
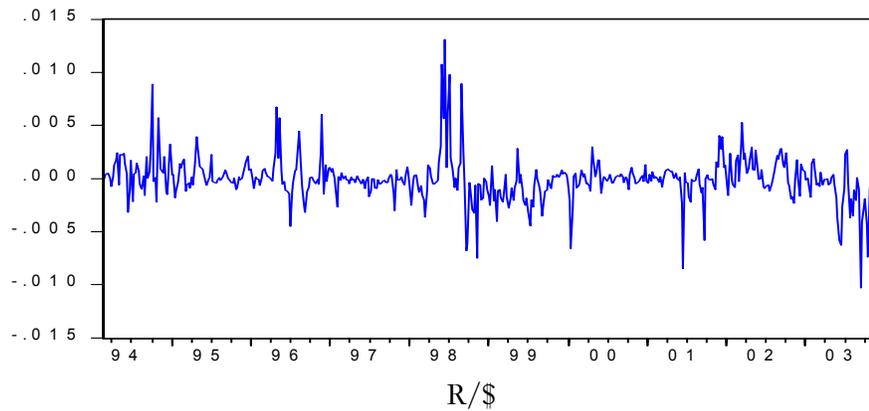
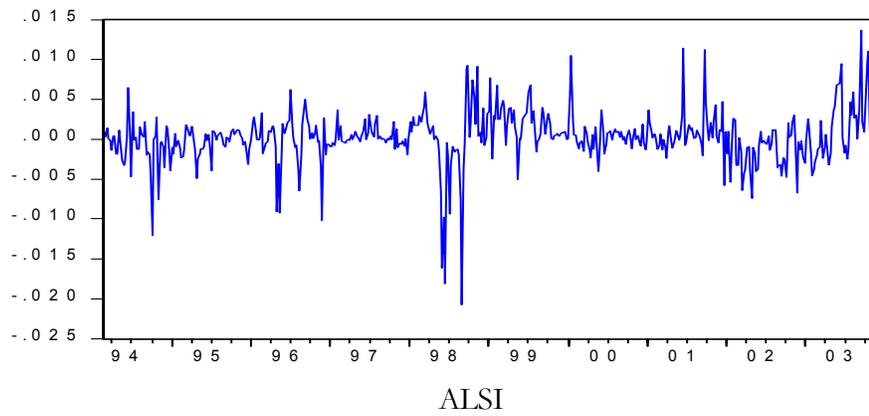


Figure 7.4: The exogenous structural innovations to the variables



**Figure 7.5: The endogenous explained variation: the difference between the total change in the variables and the structural innovations**



Once the system has been identified and the conditional covariance matrix of the structural innovations has been determined, it is possible to model the conditional covariance matrix of the endogenous explained variation (presented in figure 7.5) of the system.

### **7.3.2 Step 2: Estimating the endogenous conditional covariance matrix of variables in the system**

The second step of decomposing the conditional covariance matrix of the system is to model the endogenous explained variation of the conditional variances of the individual variables. The explained part of the variables has been recovered by subtracting the structural innovations from total variation of the variables. The data generating process of the explained variation will determine which process is used to model the conditional covariance matrix. Furthermore, the explained variation of the individual variables is likely to exhibit GARCH behaviour since a shock in one variable will result in increased or decreased movements in the other variables, resulting in clustered movements throughout the system.

The conditional covariance matrix of the explained variation of the system will not be diagonal. The contemporaneous parameters insure that there exists some correlation in movements between variables. It is therefore necessary to model this process through a multivariate set-up that captures this non-zero conditional covariance. Therefore, a restricted version of the traditional BEKK model (as outlined in section 4.2.1) is chosen to model the conditional covariance matrix of the explained portion of the system. The model is restricted so that the  $A_{jk}$  matrix and  $G_{jk}$  matrix in equation 4.5 are diagonal.

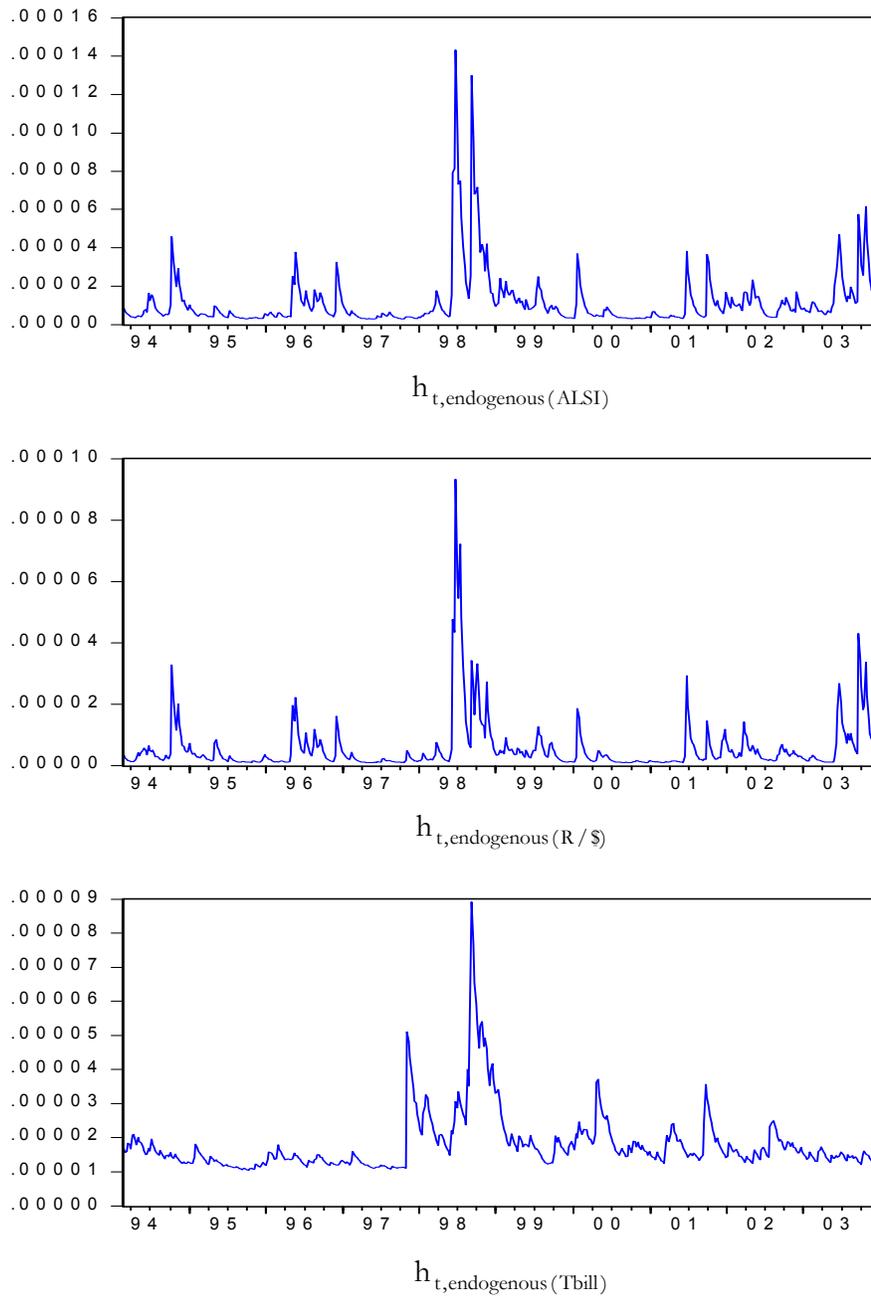
The BEKK model estimation is estimated through maximum likelihood using the BHHH logarithm. Using the notation as presented in equation 4.6, the estimation of the conditional covariance matrix is given in equation 7.3. It is clear that there exist significant GARCH effects in and between the variables<sup>11</sup>. Figure 7.6 shows the conditional variance of the endogenous explained variation of the variables while figure 7.7 presents the conditional covariance between the endogenous explained variations of three variables

$$\begin{aligned}
 & \begin{bmatrix} h_{t,ALSI} & h_{t,ALSI,R/\$} & h_{t,ALSI,TBill} \\ h_{t,R/\$,ALSI} & h_{t,R/\$} & h_{t,R/\$,TBill} \\ h_{t,TBill,ALSI} & h_{t,TBill,R/\$} & h_{t,TBill} \end{bmatrix} = \begin{bmatrix} 0.00088 & 0 & 0 \\ -0.000481 & 0.00029 & 0 \\ -0.00039 & -0.00093 & 0.000784 \end{bmatrix} \begin{bmatrix} 0.00088 & -0.000481 & -0.00039 \\ 0 & 0.00029 & -0.00093 \\ 0 & 0 & 0.000784 \end{bmatrix} \\
 & + \begin{bmatrix} 0.5103 & 0 & 0 \\ 0 & 0.6175 & 0 \\ 0 & 0 & 0.2811 \end{bmatrix} \begin{bmatrix} \varepsilon_{ALSI,t-1}^2 & \varepsilon_{ALSI,t-1}, \varepsilon_{R/\$,t-1} & \varepsilon_{ALSI,t-1}, \varepsilon_{TBill,t-1} \\ \varepsilon_{ALSI,t-1}, \varepsilon_{R/\$,t-1} & \varepsilon_{R/\$,t-1}^2 & \varepsilon_{R/\$,t-1}, \varepsilon_{TBill,t-1} \\ \varepsilon_{TBill,t-1}, \varepsilon_{ALSI,t-1} & \varepsilon_{TBill,t-1}, \varepsilon_{R/\$,t-1} & \varepsilon_{TBill,t-1}^2 \end{bmatrix} \begin{bmatrix} 0.5103 & 0 & 0 \\ 0 & 0.6175 & 0 \\ 0 & 0 & 0.2811 \end{bmatrix} \\
 & + \begin{bmatrix} 0.8365 & 0 & 0 \\ 0 & 0.8046 & 0 \\ 0 & 0 & 0.91215 \end{bmatrix} \begin{bmatrix} h_{t,ALSI} & h_{t,ALSI,R/\$} & h_{t,ALSI,TBill} \\ h_{t,R/\$,ALSI} & h_{t,R/\$} & h_{t,R/\$,TBill} \\ h_{t,TBill,ALSI} & h_{t,TBill,R/\$} & h_{t,TBill} \end{bmatrix} \begin{bmatrix} 0.8365 & 0 & 0 \\ 0 & 0.8046 & 0 \\ 0 & 0 & 0.91215 \end{bmatrix} \\
 & \tag{7.3}
 \end{aligned}$$

From figure 7.6 it is evident that the endogenous explained variance increase dramatically in periods of financial crisis. This observation is problematic from a portfolio management perspective. It implies that it will be very difficult to keep a portfolio diversified at a stable level across time. Figure 7.7 for the conditional covariance matrices confirms this and shows that the co-movement between the variables tends to be around zero in periods of relative tranquility. However, in periods of high volatility, there appears to be high co-movement between these variables that might result in less (more) diversification to a portfolio than in tranquil times. This is furthermore confirmed by the conditional correlation coefficients between the variables displayed in figure 7.8.

<sup>11</sup> The full table of the estimation results is given in Appendix B.3.

Figure 7.6: The conditional variance of the endogenous explained variation of the variables



According to figure 7.8 the conditional correlation between the ALSI and the R/\$ exchange rate tend to fluctuate in the region of -0.80. This implies that a positive change in the ALSI generally leads to a negative change (appreciation) in the R/\$ exchange rate. However, since 1998 there are greater and more frequent spikes in the conditional correlation towards zero. This suggests that the effect of R/\$ movements on the ALSI starts to dominate the effects of the ALSI on the R/\$ exchange rate. Because of the dominance of Rand-hedge shares (especially since 1998) on the Johannesburg stock exchange, a positive movement (appreciation) of the R/\$ exchange rate will result in a positive movement in the ALSI.

Figure 7.8 also indicates that the conditional correlation between the ALSI and the Tbill tend to fluctuate around -0.3, indicating that the changes in the Tbill are the dominant effect between the two variables. An increase (decrease) in the Tbill will result in a decrease (increase) in the ALSI. The conditional correlation between the Tbill and the R/\$ exchange rate tend to fluctuate around zero – except in times of high volatility (e.g. 1998). The conditional correlation then tends to be strongly positive. Monetary policy in South Africa reacts to movements in the exchange rate (because of possible inflation threats). A depreciation of the exchange rate therefore leads to an increase in the Tbill.

Figure 7.7: The conditional covariance between the explained portions of the variables

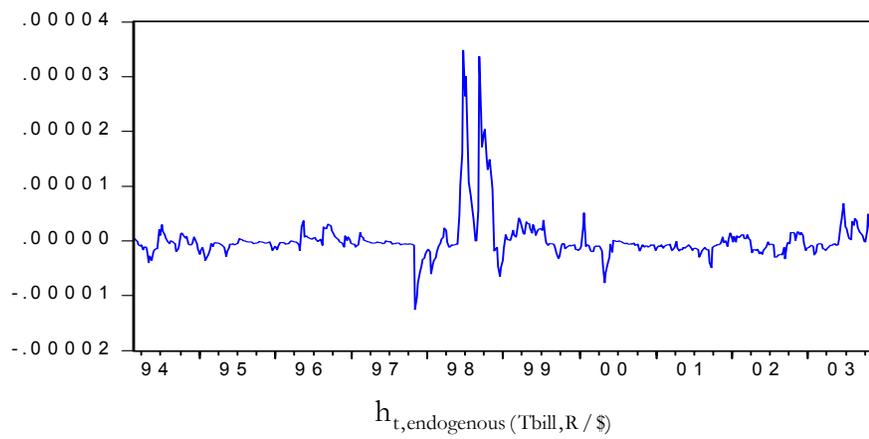
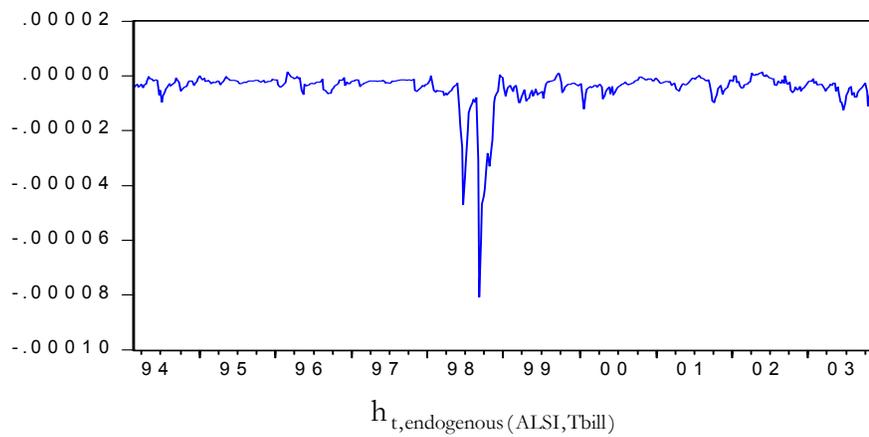
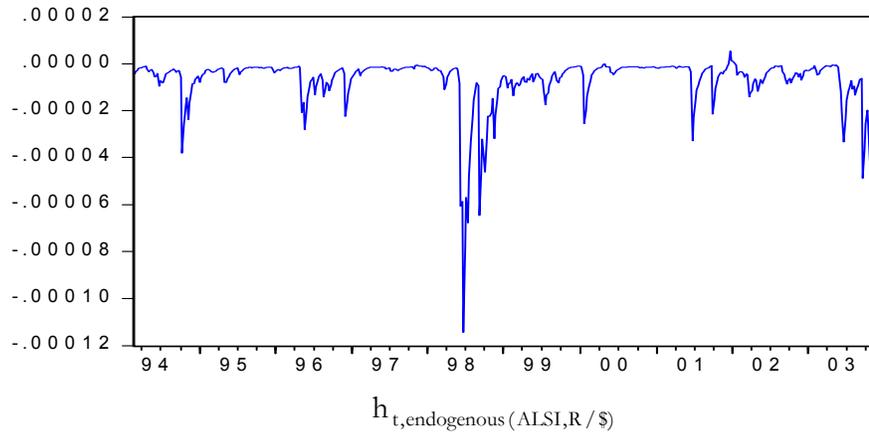
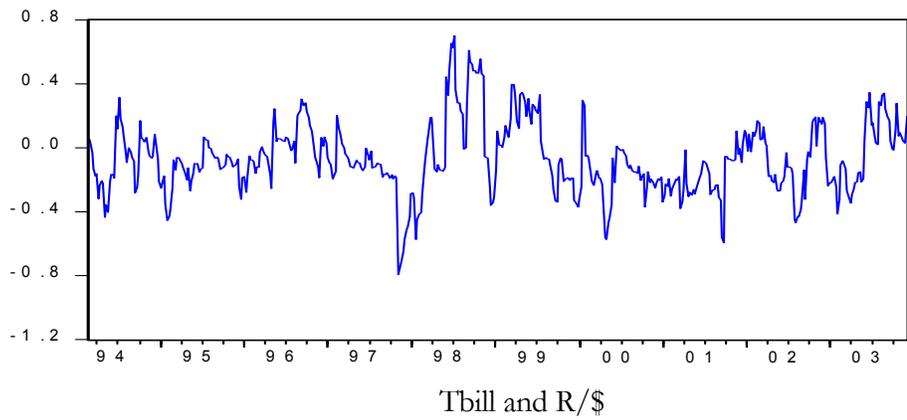
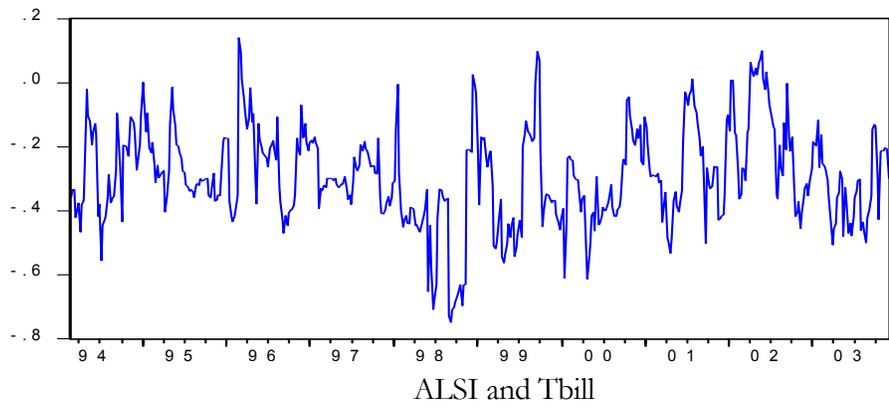
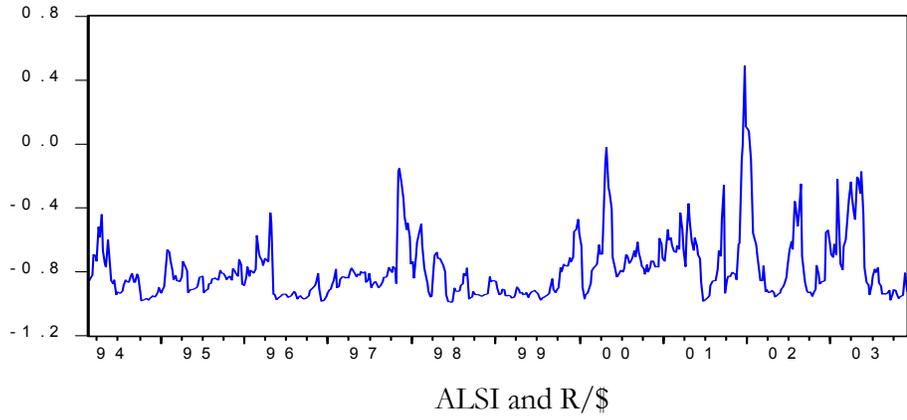


Figure 7.8: The conditional correlation between the variables

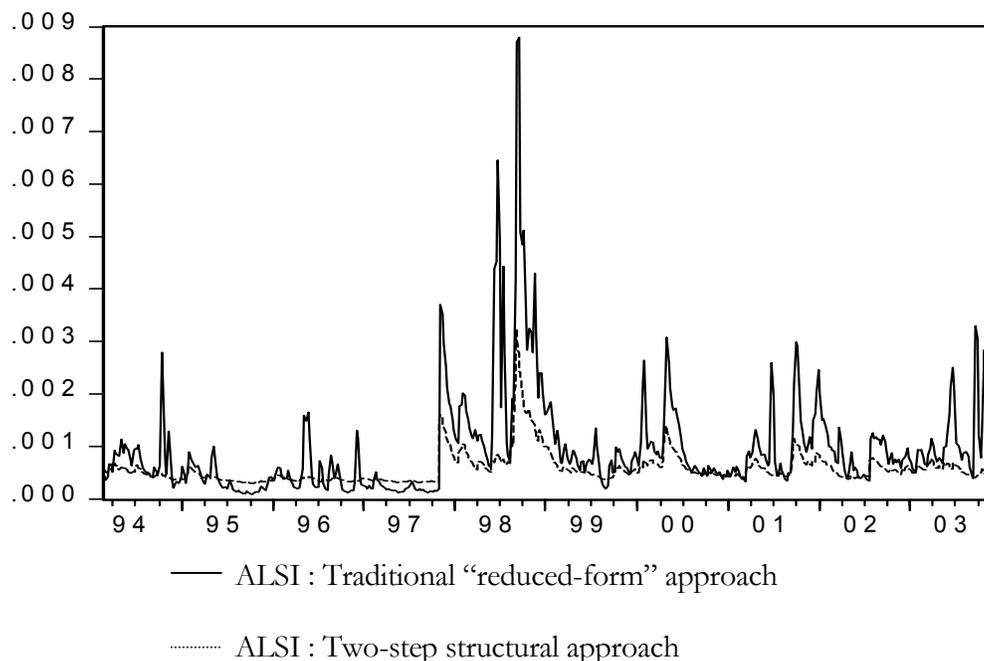


In order to calculate the total conditional covariance matrix of the system of assets, the endogenous explained conditional covariance matrix is added to the exogenous conditional covariance matrix of the structural innovation as explained in equation 7.4.

$$H_{t,\text{total}} = H_{t,\text{ednogenous}} + H_{t,\text{exogenous}} \quad (7.4)$$

A comparison between the two-step structural approach outline in this research, and the traditional “reduced-form” approaches (outlined in chapter 4) is made. Figure 7.9 shows the conditional variance of the three variables modelled under the two different approaches.  $H_{t,\text{total}}$  is calculated using the traditional “reduced-form” BEKK model as specified in equation 4.5. Then,  $H_{t,\text{total}}$  is calculated using the two-step structural approach (by first applying step 1 and then step 2). The conditional variance for each variable, as determined by the different methods, is displayed in figure 7.9.

**Figure 7.9: Total conditional variance – “two-step” structural approach vs. “reduced-form” approach**



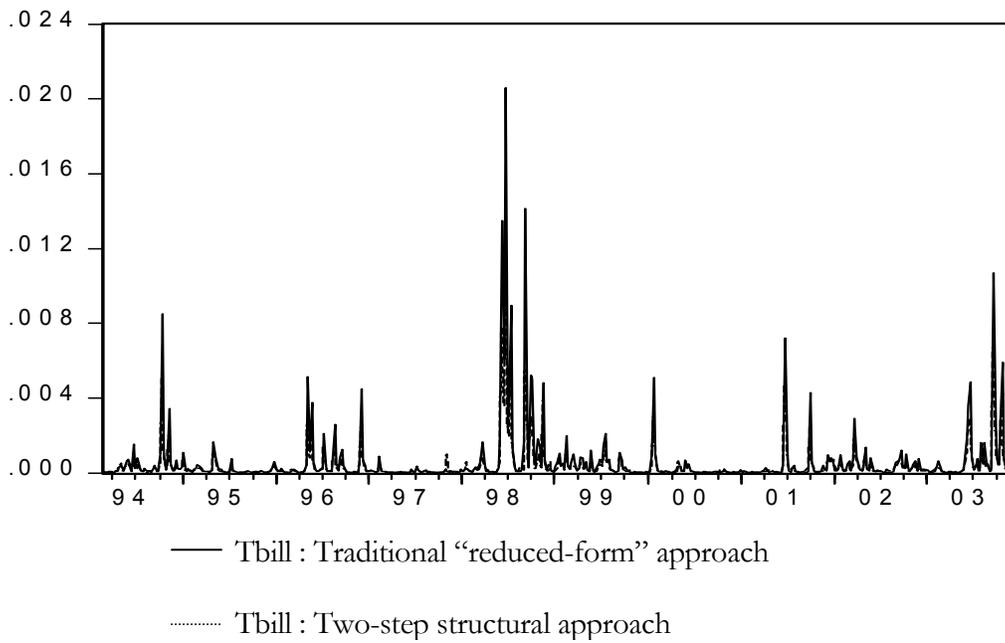
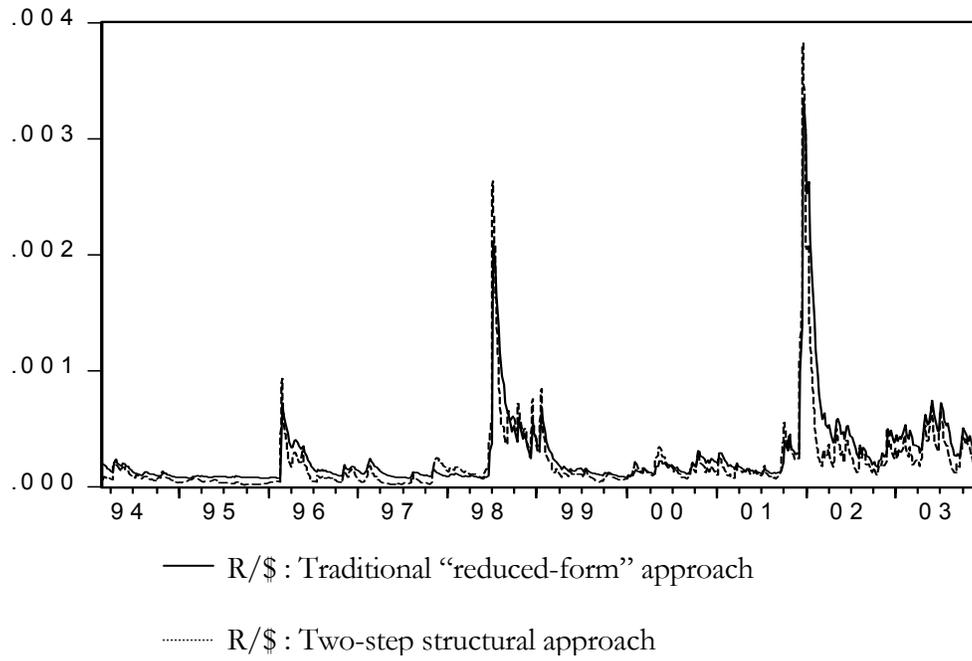
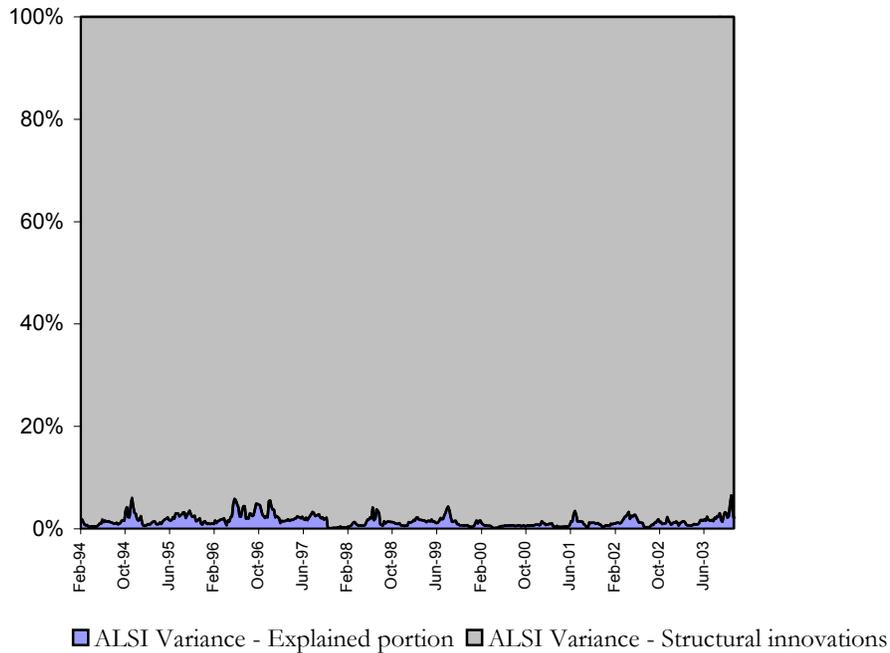


Figure 7.9 show that the conditional variance of both methods follows the same pattern. They pick up the same periods of high volatility and periods of tranquillity. However, with the two-step structural approach the estimation methodology provides more information than the “reduced-form” approaches. The two-step approach allows for the structural analysis of the volatility within and between variables. This additional information allows for analysis of the conditional covariance matrix of the system in a more complex manner than otherwise possible. For example, it might be informative to know which percentage of the volatility of a variable is determined within the system and which part outside the system by structural innovations. Figure 7.10 presents this breakdown based on the total conditional variance of each asset and the conditional variance of the structural innovations as estimated in the two-step approach.

**Figure 7.10: Total variance decomposition – structural (exogenous) vs. explained (endogenous)**



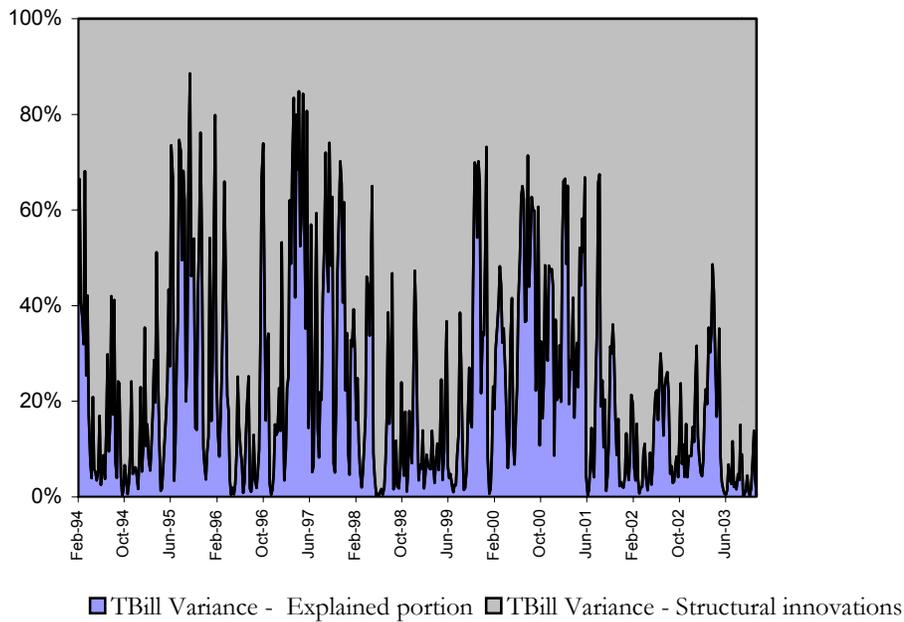
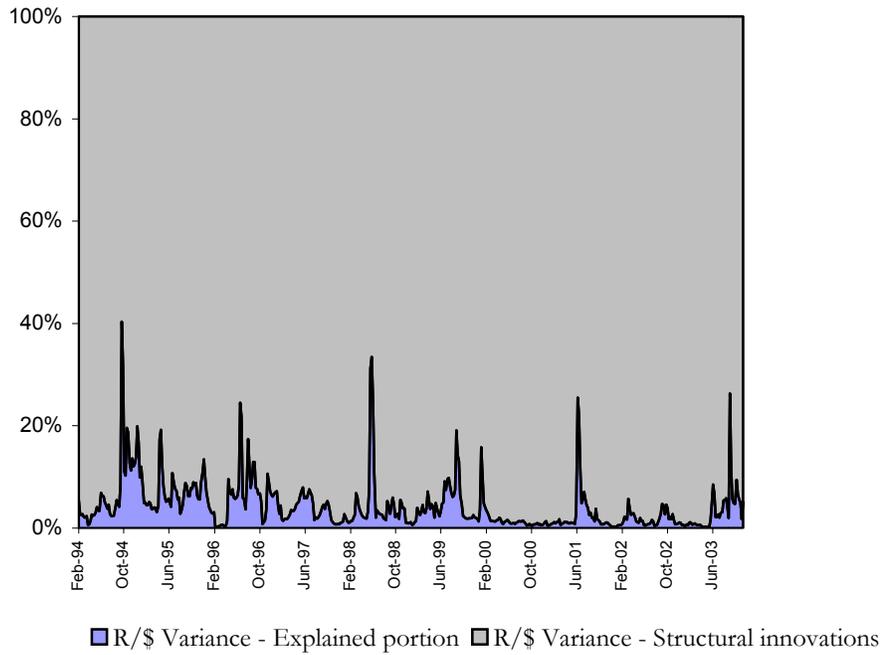


Figure 7.10 makes it clear that almost all the volatility in the ALSI is due to factors other than the R/\$ exchange rate and the Tbill rate. These factors are the latent factors in the model

explained by the structural innovations. They might include the gold price, GDP growth and company specific factors. There are three reasons for the possible small spillovers from the R/\$ exchange rate and the Tbill to the ALSI. First, given the fact that the ALSI is a broad index of shares that react differently to changes in the exchange rate and the interest rate, it might not be surprising that at an aggregate level, these two variables do not contribute a lot to volatility of the ALSI. Second, investors might only react to changes in the interest rate and exchange rate when they perceive them to be fundamental changes that are not short run in nature. Third, changes in the ALSI due to changes in the exchange rate and the interest rate might be mitigated using weekly data, i.e. daily volatility movements cancel one another out on a weekly basis.

As far as the contribution of the structural innovations to the total variance of the R/\$ exchange rate is concerned, one can see that the contribution is also small. The endogenous variables inside the system again do not contribute too much to volatility. The latent factors to the exchange rate, captured by the structural innovations, explain most of the volatility. These are factors like the demand for currency due to trade between South Africa and foreign countries. The interest rate and the ALSI represent two different asset classes. If there is money moving between these variables without flowing in or out of the country, their effect on the exchange rate should be relatively small, given the size and significance of the contemporaneous parameters. It could be seen in periods of high uncertainty when capital flowed out of South Africa; movements in the interest rate and the ALSI explain more of the volatility in the exchange rate<sup>12</sup>. This pattern of volatility is also consistent with the flow-oriented models of exchange rate as opposed to the stock-oriented or portfolio-balance approaches for the South African exchange rate. The portfolio-balance approach focuses on the capital account where the stock market and interest rates play an important role. In the flow-oriented models the focus is on the current account or the trade balance. If the stock market and interest rate were the dominant factors in determining the exchange

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<sup>12</sup>Periods of high uncertainty include the first free elections in South Africa and the following year (1994/5), the Asian crisis (1997/8), the Russian crisis (1999) and the attack on the World Trade Center (2001).

rate (as the portfolio-balance approach suggests), a higher degree of spillovers inside the system was likely.

Movements in the variables in the system explain a significant percentage of the total conditional variance of the Tbill rate. This contribution can be explained by the market's reaction to expected monetary responses (which is a structural innovation) and to changes in either the exchange rate or the ALSI. One can see that in the periods of uncertainty, spillovers from the other two variables to the Tbill increased dramatically. Since the effect of structural innovations (e.g. monetary policy) on the volatility of the Tbill is less, it is an indication that market participants view monetary authorities to be very proactive in acting on the new information. Movements in the ALSI and the exchange rate will be factored into the Tbill for anticipation of possible monetary policy reaction.

#### **7.4 CONCLUSION**

Most multivariate GARCH models estimate the total conditional covariance matrix between variables. These “traditional” models do not distinguish between external shocks (i.e. the structural innovations) and internal shocks (i.e. the explained changes). These models use reduced-form parameters in the estimation process.

This chapter gave an alternative methodology to estimate the total conditional covariance between variables. The methodology decomposes the conditional covariance matrix into a covariance matrix for the structural innovations and a covariance matrix for the endogenous explained variation of the system. The methodology also allows one to obtain the structural parameters from a system of endogenous equations without imposing any “invalid” restrictions on the system. Once the structural parameters in the system are identified, it is possible to distinguish between the structural innovations to a variable (the latent factors) and the explained portion for the variable determined within the system. The conditional covariance is then modelled separately using two different multivariate GARCH models.

Using the decomposition, more information is available to the researcher on the conditional covariance of the system.

The methodology was applied to a system of variables, including the All Share index on the Johannesburg stock exchange (ALSI), the South African Rand/ US Dollar exchange rate and the South African 90-day Treasury bill interest rate. Significant contemporaneous effects and volatility spillovers were identified between the variables, while GARCH effects were identified within both the structural innovations and the explained portion of the variables. From the results it was possible to determine the volatility generated in a specific variable, i.e. is it generated by exogenous structural shock or by endogenous interaction between variables? In the case of the ALSI very little volatility is generated because of movements in the R/\$ exchange rate or the Tbill interest rate. For the R/\$ exchange rate, it also appears as if latent factors to the model determine most of the volatility in the currency. These latent factors include demand for foreign currency because of trade. Finally, the volatility of the short-term interest rate appears to be driven to a large extent by movements in the R/\$ exchange rate and the ALSI. The interest rate equation represents a high-frequency monetary response function. It appears as if the market reacts to movements in the exchange rate and stock prices in anticipation of a monetary authority response.

## Chapter 8

### IMPULSE RESPONSES AND AN APPLICATION TO PORTFOLIO RISK MANAGEMENT

#### 8.1 INTRODUCTION

The proposed decomposition of the covariance matrix of a system with endogenous variables, utilising the GARCH models, provides more information than the traditional reduced-form GARCH models. Firstly, it is possible to identify the structural parameters. Secondly, it is possible to identify the structural innovations or latent factors and thirdly it is possible to identify the explained variation of variables. Given this information one can model the time-varying volatility of each part separately. This information is valuable, for a structural innovation or external shock to one variable will influence the behaviour of other assets differently through the structural contemporaneous parameters. The behaviour of the variance and covariance implied by the model can be more clearly understood by investigating impulse response functions. This chapter focuses on how the movement in variables reacts to structural innovations from outside the system. Firstly structural innovations are applied to variables one at a time. There after, an application to portfolio risk management is illustrated.

#### 8.2 IMPULSE RESPONSE FUNCTIONS

The structural innovations or latent factors are recovered from the model that was estimated in chapter 7. In each case a temporary two standard deviation shock is applied to the structural innovations of the variables. For expositional reasons the shock is introduced in the fourth period. The variance and covariance between the variables in the system are then simulated.

Figures 8.1, 8.2 and 8.3 show the variance of each variable in response to a shock. It can be seen that in each case the variance of the variables reacts greatly to a structural shock on themselves. The variables also react to shocks in other variables but to a lesser extent. The R/\$ and Tbill react greatly to shocks from outside, while the ALSI reacts to a lesser extent to structural innovations from outside. This is understandable if considered that the R/\$ is driven by many factors other than the ALSI and the interest rate – especially in the short run. The Tbill is also very prone to shocks from outside. Since it is a short-term interest rate, it is very sensitive to monetary policy responses.

To compare the conditional covariance between the variables, each variable is shocked in the same manner as before. The covariance is displayed in figures 8.4, 8.5 and 8.7. The covariance displays a wide range of patterns in response to the various identified shocks. The reactions in the covariance evolve from the contemporaneous interactions between variables identified in table 7.2. For expositional purposes, each of the covariance movements is discussed.

**Figure 8.1: Impulse response due to a shock to ALSI**

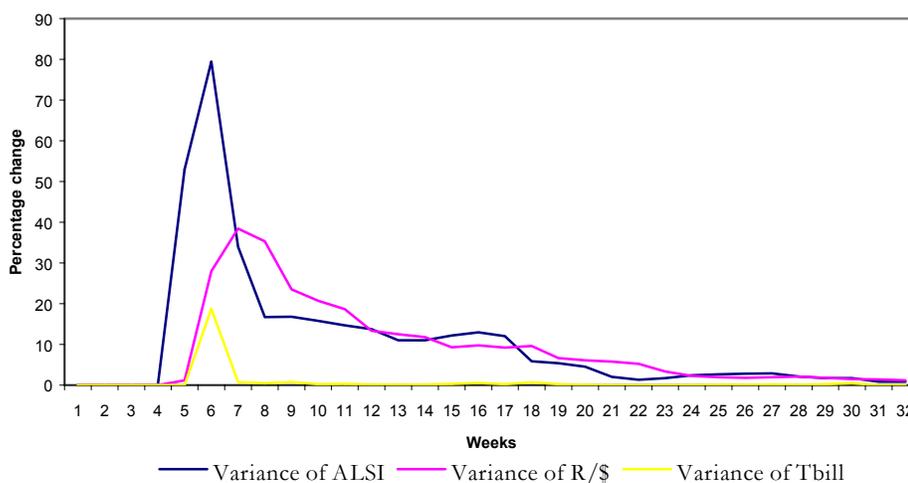


Figure 8.2: Impulse response due to a shock to R/\$

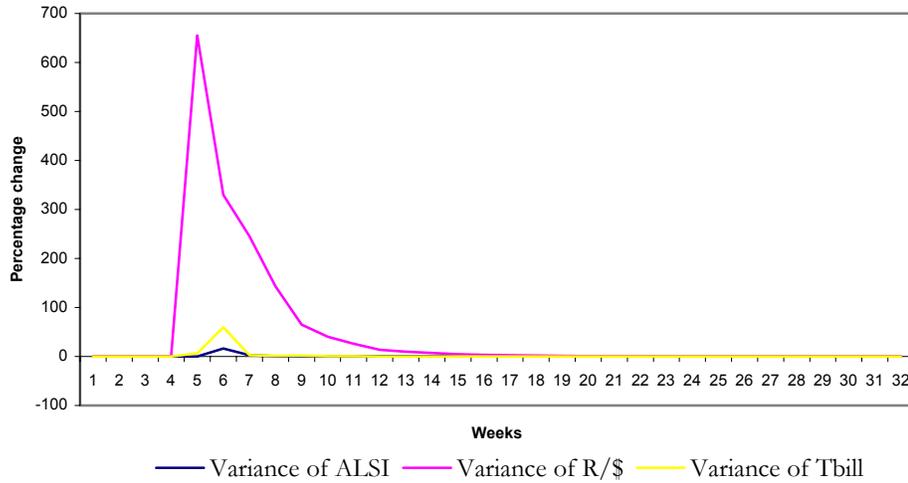
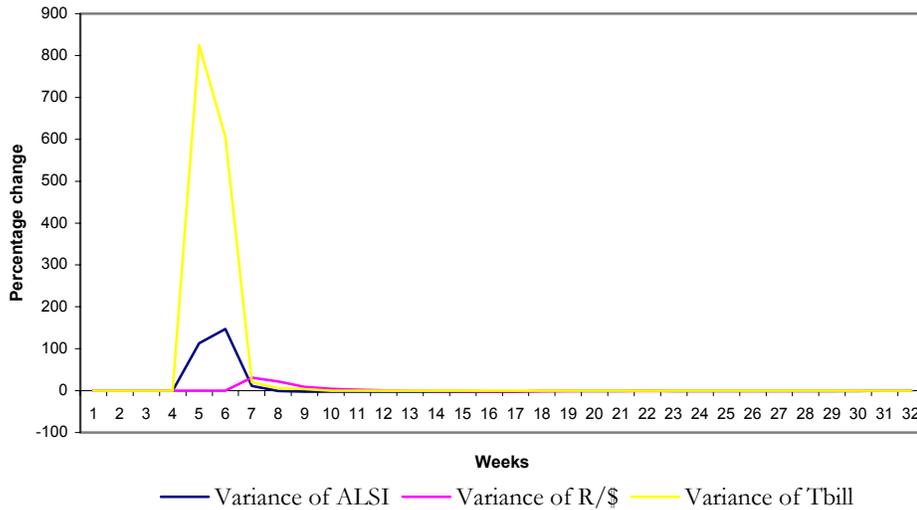


Figure 8.3: Impulse response due to a shock to Tbill



First of all the reaction of the covariance between the ALSI and the R/\$ exchange rate to each shock is discussed. Shocks to the ALSI tend to make the covariance between the ALSI and the R/\$ exchange rate more positive over the subsequent following weeks. This effect

arises because a positive shock to the ALSI tends to be followed by additional shocks to the ALSI, which have a positive impact on the R/\$ exchange rate. Positive shocks to the R/\$ exchange rate also make the covariance between the ALSI and the R/\$ positive, but to a much lesser extent. Once again is this because shocks to the R/\$ exchange rate are likely to be followed by more shocks. Through the contemporaneous parameters, the exchange rate has a positive effect on the ALSI, while the ALSI also impacts positively on the R/\$ exchange rate. By contrast, interest rate shocks tend to make the covariance between the ALSI and the R/\$ exchange rate more negative going forward. Shocks to the Tbill have a negative effect on the ALSI and an insignificant positive effect on the R/\$ exchange rate (through the contemporaneous parameters). The ALSI effect dominates the R/\$ effect, resulting in a negative covariance going forward.

**Figure 8.4: Impulse response due to a shock to ALSI**

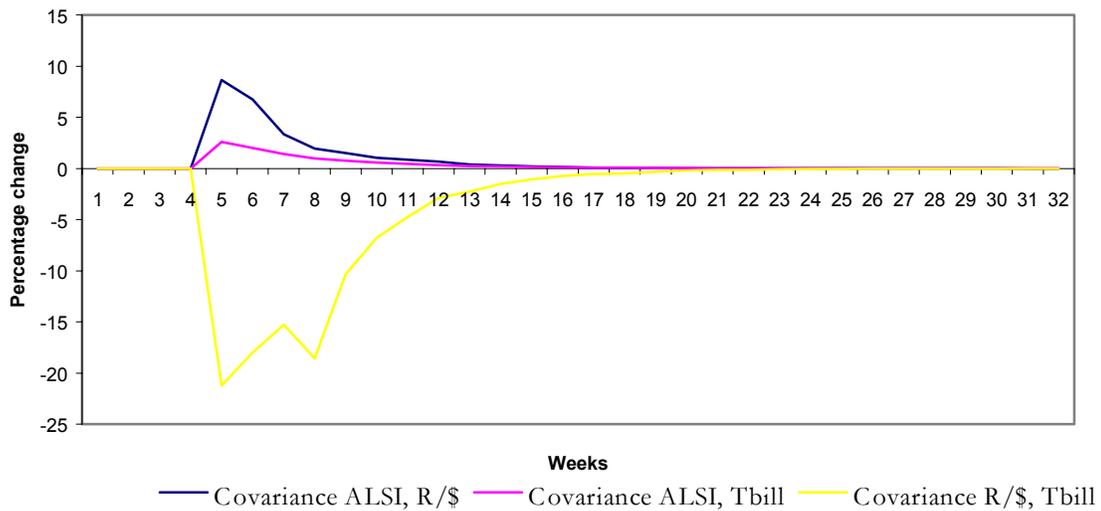


Figure 8.5: Impulse response due to a shock to R/\$

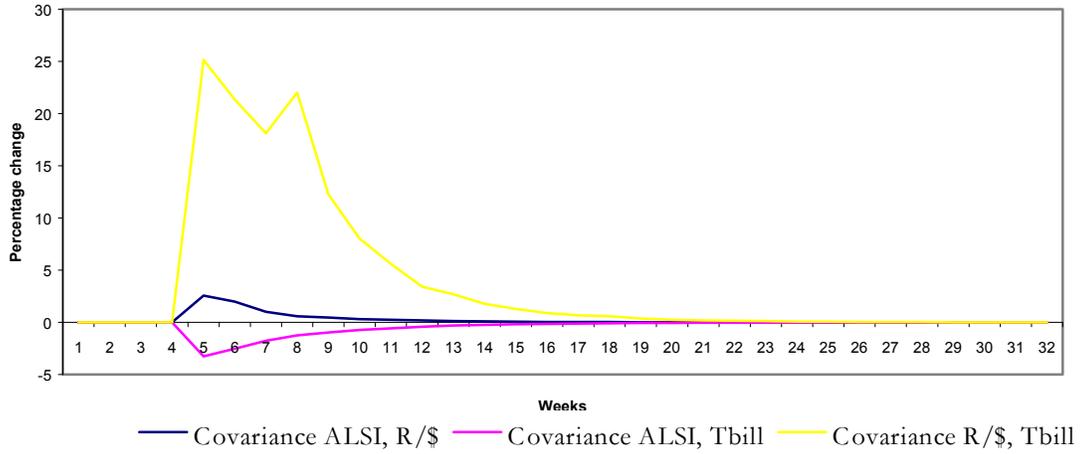
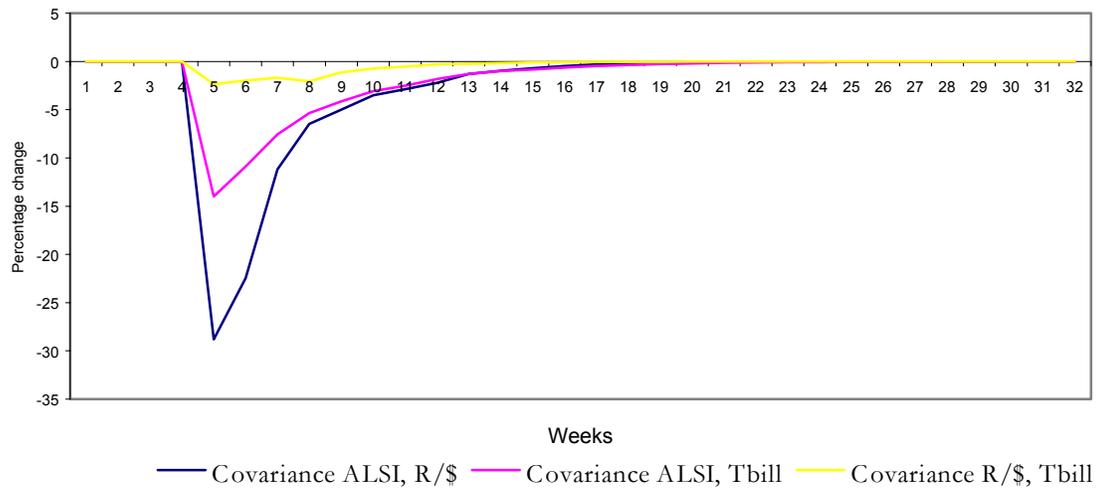


Figure 8.6: Impulse response due to a shock to Tbill



As far as the covariance between the ALSI and the Tbill is concerned, a similar pattern arises when there is a shock to the ALSI. Shocks to the ALSI make the covariance between the ALSI and the Tbill more positive in the following weeks. The effect of the shock is however much smaller due to the negative impact of the Tbill on the ALSI going forward. The negative contemporaneous effect between the ALSI and the Tbill mitigates the effect of shocks to the ALSI. The covariance between the ALSI and the Tbill becomes more negative following a shock to the exchange rate. A positive shock to the exchange rate increases both the ALSI and the Tbill. However, the Tbill increase dominates, increasing by almost twice as much as the ALSI. Through the Tbill's large negative contemporaneous effect on the ALSI, the covariance between them is negative going forward. After a shock to the Tbill, the covariance between the two variables becomes more negative, following directly from the large negative effect of the Tbill on the ALSI.

The conditional covariance between the Tbill and the R/\$ exchange rate is negative following a shock to the ALSI due to the contemporaneous interaction between the variables. After a shock to the R/\$ exchange rate, the conditional covariance between the Tbill and the exchange rate increases by a large amount. This is due to both variables having a positive contemporaneous effect on one another. When the shock is to the Tbill, the covariance becomes slightly negative going forward. Once again the large negative effect of the Tbill on the ALSI dominates, but is mitigated by the other positive contemporaneous effects between the variables.

Figures 8.1 through to 8.6 highlight the most important implications of identifying the contemporaneous parameters in the model. Understanding the source of the shock that drives a variable is crucial for accurately predicting the behaviour of assets going forward. Analysing the behaviour of a single variable in isolation could be misleading. Changes in a variable could be driven by an innovation to its own shock, or by endogenous responses to a shock to another variable. As has been seen in the figures, the sources of the shocks can have a very different implication for the second moments of the variables going forward. By recovering the contemporaneous parameters, the methodology allows one to determine the

source of the shock by looking at the contemporaneous movements in the other variables. Once the source of the shock is identified, the implication of the behaviour of variables going forward can be derived from the estimates.

To illustrate the possible miscalculation of movements between variables the traditional “reduced-form” BEKK specification, as estimated in chapter 7, is used. Each variable is shocked by the same magnitude as the shocks applied before. The same impulse responses are simulated using the BEKK estimation, and compared to the impulse responses from the two-step approach. For expositional reasons, only the covariance movement between the ALSI and the R/\$ exchange rate in response to different shocks is shown<sup>13</sup>.

From figure 8.7 can be seen that when one ignores the contemporaneous interaction between variables, the reaction of the conditional covariance matrix, due to a shock to the ALSI, is overestimated for some period into the future. The reason is that the effect of the ALSI shock has a positive effect on the Tbill, which in turn mitigates the effect on the ALSI through its negative coefficient. The traditional BEKK estimation ignores this effect. Figure 8.8 shows the reaction of the covariance between the ALSI and the R/\$ due to a shock to the R/\$. Once again the covariance is overestimated due to ignorance of the structural parameters. When the shock is to the Tbill, the covariance between the ALSI and the R/\$ is underestimated using the traditional BEKK specification.

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<sup>13</sup> For a comparison between the responses of the other conditional covariances, see Appendix C.1.

Figure 8.7: Comparison: Impulse response due to a shock to ALSI

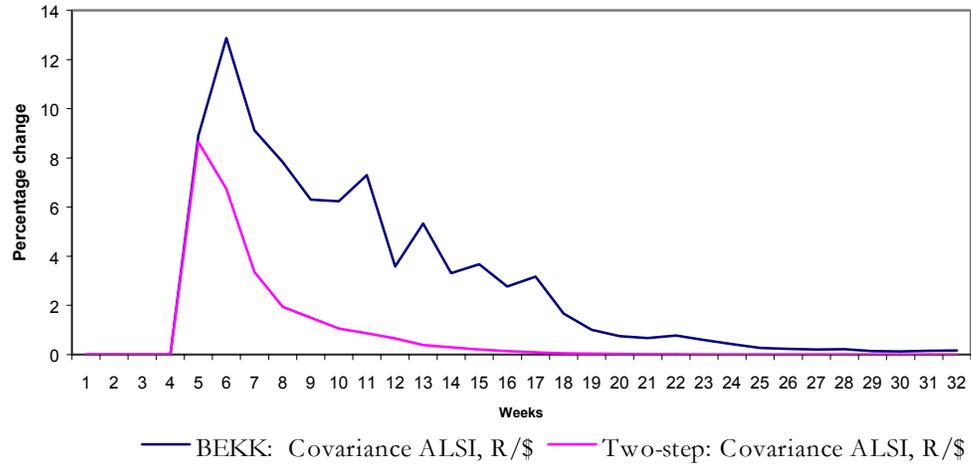
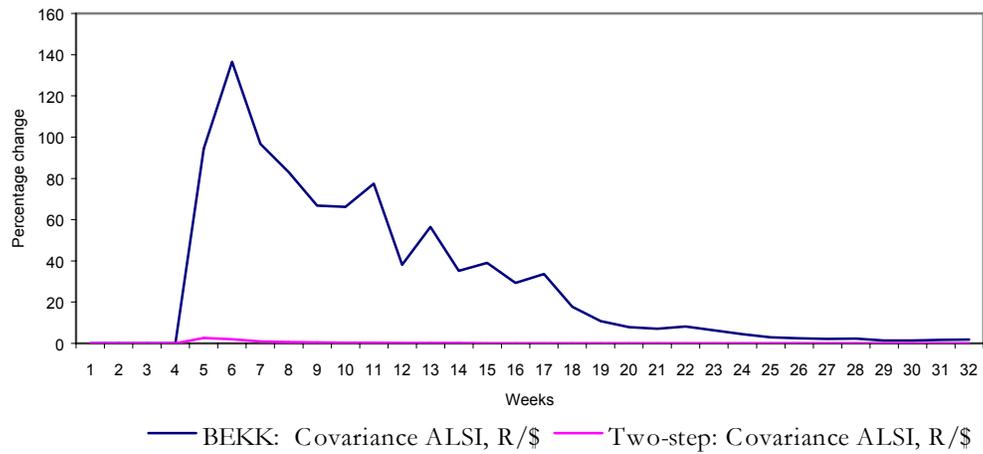
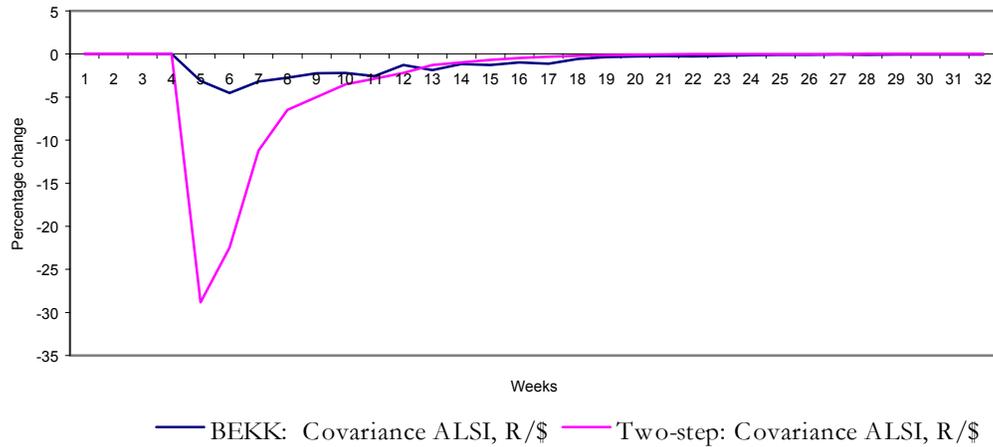


Figure 8.8: Comparison: Impulse response due to a shock to R/\$



**Figure 8.9: Comparison: Impulse response due to a shock to the Tbill**

### 8.3 AN APPLICATION TO PORTFOLIO RISK MANAGEMENT

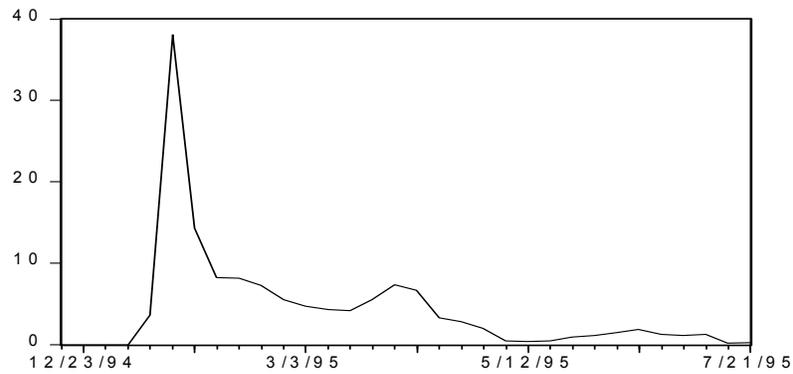
The previous section demonstrated that the conditional second moments of variables vary considerable over time as the relative volatilities of the underlying shocks shift. These observations would have important implications for forming portfolio decisions, managing risk and pricing derivative securities. To illustrate the practical implication of this, a simple risk management exercise is undertaken.

Consider a portfolio that is evenly split between the ALSI index and a 90-day Treasury note (the 90-day Treasury note have duration of 0.25 years). The portfolio suffers a 0.5 percent loss if equity prices fall by 1 percent or if the Tbill rate increase by 800 basis points. It is assumed that the investor uses the BEKK specification outlined in chapter 7 to estimate the conditional variance and covariance between the variables. The same exercise is repeated using the two-step decomposition methodology outlined and proposed in this study. The percentage is calculated by which the investor's estimate of the variance for the portfolio is mismeasured due to ignoring the contemporaneous effects between variables. Once again the

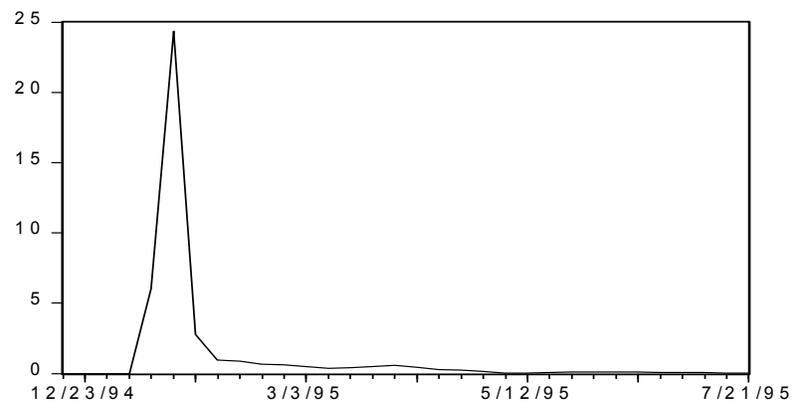
mismeasurement is calculated when there is a shock to the ALSI, the R/\$ exchange rate and the Tbill.

Figure 8.10, 8.11 and 8.12 show the mismeasurement of the portfolio when there is a shock to each variable. The figures give the percentage by which the investor overestimates or underestimates the portfolio variance relative to the two-step methodology.

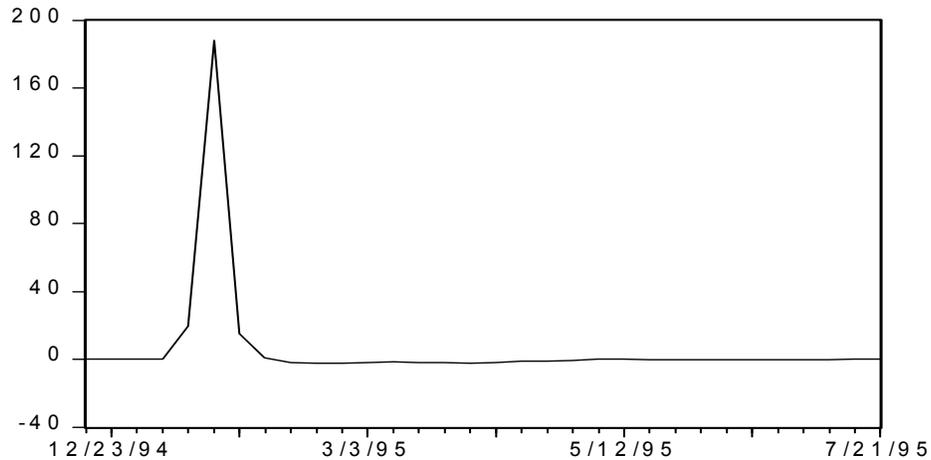
**Figure 8.10: Percent portfolio variance mismeasurement due to a shock to the ALSI**



**Figure 8.11: Percent portfolio variance mismeasurement due to a shock to the R/\$**



**Figure 8.12: Portfolio variance mismeasurement due to a shock to the Tbill (in percent)**



The figures show that when the contemporaneous effects between the variables are ignored, the variance of a portfolio is underestimated using the traditional BEKK specification. The effect is the greatest when the shock is to the interest rate, followed by shocks to the ALSI and lastly shocks to the exchange rate. The risk measurement implications of failing to account for spillovers across variables and contemporaneous effects between variables would likely be even more severe than the example suggests. The example assumes only a one period shock (i.e. a one week shock). If the structural innovations do appear for longer periods, the mismeasurement will be greater. Using traditional models the investor will be unable to measure the true impact, for he/she will be unable to identify the structural innovations.

It follows then directly that any risk measurement that uses the portfolio variance will be incorrect. Given measures like Value-at-Risk and Sharpe ratios that use total risk in their calculations, the investor will either over- or underestimate the risk of the portfolio depending on the variables and the direction of the shocks.

#### 8.4 CONCLUSION

In order to see how the second moments of the variables react to shocks in the structural innovations of variables, impulse responses were introduced. The impulse responses highlighted the fact that contemporaneous movements between variables constitute an important component of the behaviour of the variables. By ignoring the structural parameters in a system of variables, a researcher will be unable to recover the structural innovations to variables. These structural innovations are important for they determine how the second moments of assets will react going forward. Without this knowledge serious mismeasurement of portfolio variances are possible and as a result wrong investment decisions.

## Chapter 9

### SUMMARY AND CONCLUSION

#### 9.1 INTRODUCTION

The primary objective of this study was two-fold. The first objective was methodological in nature and the second empirical. The methodology proposed in this research was used to estimate the structural relationships, of both the first and second moments, between three financial variables in the South African economy. This analysis allows the researcher to better understand the drivers behind volatility of financial variables.

#### 9.2 METHODOLOGY

The methodology used in this research uses existing literature to solve some of the econometric problems encountered in modelling with financial variables. If one wants to analyse the structural relationships between variables, be it in the first or second moments, it is important to find consistent, efficient and unbiased estimates for the structural parameters.

First, when dealing with a system of endogenous variables, the system is not identified. Without imposing any restrictions on the estimated reduced-form parameters, it is impossible to retrieve the structural parameters. The literature has solved this problem by placing restrictions on the system, thereby indirectly increasing the number of equations in the system. These restrictions vary in nature and application. However, most of these restrictions cannot be justified when estimating models with high-frequency data. In the short run many financial variables react different than what economic theory would suggest. Therefore, to solve this problem of identification, identification through heteroscedasticity has been implemented to identify the structural contemporaneous parameters. Since financial data

often exhibit conditional heteroscedasticity, the identification methodology is well suited for this type of analysis.

Two structural GARCH models have been implemented in the proposed methodology. The first is the Rigobon and Sack (2003) model to identify the structural parameters and to obtain estimates for the conditional covariance matrix of the structural innovations. Once the system is identified, the portion of the volatility generated within the system is modelled using a multivariate BEKK specification.

This approach allows one to solve the system simultaneously and obtain structural parameters of the system. More information is available of the data generating process that drives the volatility between these variables. It enables one to determine to what extent the volatility is generated by variables inside the system and the extent to which volatility is driven by structural innovations or latent factors outside the system.

### **9.3 EMPIRICAL RESULTS**

The methodology outlined in this research is implemented to analyse a system of three financial variables in the South African economy. The All Share index of the Johannesburg stock exchange, the South African Rand / US Dollar exchange rate and the South African 90-day Treasury bill rate was analysed. The system was solved simultaneously and the conditional covariance matrix was analysed.

Significant contemporaneous effects were found between the three financial variables. The ALSI is significantly influenced by the interest rate, while the exchange rate is significantly influenced by the ALSI. However, the exchange rate is not influenced significantly by the interest rate in the short run. This is consistent with what is observed in the South African economy. There exists an asymmetric relationship between the exchange rate and short-term interest rates in the short run. When the interest rate goes up, the exchange rate do not seem

to react in the short run on these movements (although in the long run the relationship holds). However, when the exchange rate increases (i.e. depreciation) the interest rate reacts almost immediately<sup>14</sup>. This is supported by the empirical results in the research. The interest rate is positively influenced by both the ALSI and the exchange rate.

With regard to the second moments, significant GARCH behaviour was detected in both the exogenous structural innovations as well as the endogenous explained variation of the variables. In the system with three variables most of the total volatility of the variables was generated by latent factors or the structural innovations. In periods of uncertainty, like the Asian crisis, the volatility generated inside the system increased relative to volatility from structural innovations.

Impulse responses were simulated to detect how the different variances and covariances between the variables react. These impulse responses indicated that there might exist significant mismeasurement if the researcher ignores the contemporaneous effects between variables. Depending on the type of shock to the system, the covariance movements between variables will differ going forward.

Finally an application to portfolio management was implemented to highlight the possible dangers that exist in ignoring the contemporaneous parameters. The result was compared to the BEKK specification to show the differences in the estimation of a portfolio variance.

#### **9.4 CONCLUDING REMARKS**

This study developed an alternative method to analyse the structural relationships between variables that are determined contemporaneously in a system. It enables one to have a better understanding of the drivers of volatility inside and between variables. The empirical results

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<sup>14</sup> This reaction follows from the nature of monetary policy in South Africa. Since the inception of an inflation target by the South African Reserve Bank, short-term interest rates have been sensitive to changes in the exchange rate.

indicate that spillovers from one variable to another constitute an important component of the behaviour of financial variables. The Rigobon and Sack model makes it possible to quantify these effects that have been difficult to estimate previously. By extending their research, this study uses a second model to determine how variable behaviour is driven not only inside the system but also by latent factors outside the system.

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## Appendix A

### DERIVATION OF THE REDUCED-FORM ARCH MODEL

Given the reduced-form innovations from equation 5.6

$$\omega_{i,t} = c'_i + \frac{(\beta\varepsilon_t + \eta_t)}{1-\delta\beta}$$

$$\omega_{j,t} = c'_j + \frac{(\varepsilon_t + \delta\eta_t)}{1-\delta\beta}$$

and the structural relationship as described by equation 5.2 the second moments of the reduced-form can be written as

$$\omega^2_{i,t} = \frac{(\beta^2\varepsilon^2_t + \eta^2_t)}{(1-\delta\beta)^2}$$

$$\omega_{ij,t} = \frac{(\beta\varepsilon^2_t + \delta\eta^2_t)}{(1-\delta\beta)^2}$$

$$\omega^2_{j,t} = \frac{(\varepsilon^2_t + \delta^2\eta^2_t)}{(1-\delta\beta)^2}$$

Rigobon (2002) construct a VECH specification, where the expected conditional moments have a different structure. The expected conditional reduced-form residuals can be written in terms of  $h_{j,t}$  and  $h_{i,t}$ . Define  $h_{j,t} \equiv E\omega^2_{j,t}$ ,  $h_{i,t} \equiv E\omega^2_{i,t}$  and  $h_{ij,t} \equiv E\omega_{j,t}\omega_{i,t}$  then the conditional moments can be written as

$$h_{i,t} = \frac{(\xi_i + \eta_{t-1}^2[\beta^2\lambda_{\varepsilon\eta} + \lambda_{\varepsilon\varepsilon}] + \varepsilon_{t-1}^2[\beta^2\lambda_{\eta\eta} + \lambda_{\varepsilon\eta}])}{(1-\delta\beta)^2}$$

$$h_{ij,t} = \frac{(\xi_{ij} + \eta_{t-1}^2[\beta\lambda_{\varepsilon\eta} + \delta\lambda_{\varepsilon\varepsilon}] + \varepsilon_{t-1}^2[\beta\lambda_{\eta\eta} + \delta\lambda_{\varepsilon\eta}])}{(1-\delta\beta)^2}$$

$$h_{j,t} = \frac{(\xi_j + \eta_{t-1}^2[\lambda_{\varepsilon\eta} + \delta^2\lambda_{\varepsilon\varepsilon}] + \varepsilon_{t-1}^2[\lambda_{\eta\eta} + \delta^2\lambda_{\varepsilon\eta}])}{(1-\delta\beta)^2}$$

By writing  $\varepsilon_{t-1}^2$  and  $\eta_{t-1}^2$  as a function of only two out of the three moments of the reduced-form residuals gives

$$\eta_{t-1}^2 = \frac{1-\delta\beta}{1+\delta\beta}(\omega_{i,t-1}^2 - \beta^2\omega_{j,t-1}^2)$$

$$\varepsilon_{t-1}^2 = \frac{1-\delta\beta}{1+\delta\beta}(-\delta^2\omega_{i,t-1}^2 + \omega_{j,t-1}^2)$$

The restriction on the covariance of the structural innovation to be zero allows one to express the second moments as a function of only two reduced-form conditional moments. Given the above, the ARCH structure can be expressed as

$$\begin{bmatrix} h_{i,t} \\ h_{ij,t} \\ h_{j,t} \end{bmatrix} = \begin{bmatrix} \zeta_i \\ \zeta_{ij} \\ \zeta_j \end{bmatrix} + \frac{1}{1-(\delta\beta)^2} A \begin{bmatrix} \omega_{i,t-1}^2 \\ \omega_{j,t-1}^2 \end{bmatrix}$$

where the A matrix is given by

$$A \equiv \begin{bmatrix} \left[ \beta^2 \lambda_{\varepsilon\eta} + \lambda_{\eta\eta} \right] - \delta^2 \left[ \beta^2 \lambda_{\varepsilon\varepsilon} + \lambda_{\eta\varepsilon} \right] & -\beta^2 \left[ \beta^2 \lambda_{\varepsilon\eta} + \lambda_{\eta\eta} \right] + \left[ \beta^2 \lambda_{\varepsilon\varepsilon} + \lambda_{\varepsilon\eta} \right] \\ \left[ \beta \lambda_{\varepsilon\eta} + \delta \lambda_{\eta\eta} \right] - \delta^2 \left[ \beta \lambda_{\varepsilon\varepsilon} + \delta \lambda_{\eta\varepsilon} \right] & -\beta^2 \left[ \beta \lambda_{\varepsilon\eta} + \delta \lambda_{\eta\eta} \right] + \left[ \beta \lambda_{\varepsilon\varepsilon} + \delta \lambda_{\varepsilon\eta} \right] \\ \left[ \lambda_{\varepsilon\eta} + \delta^2 \lambda_{\eta\eta} \right] - \delta^2 \left[ \lambda_{\varepsilon\varepsilon} + \delta^2 \lambda_{\eta\varepsilon} \right] & -\beta^2 \left[ \lambda_{\varepsilon\eta} + \delta^2 \lambda_{\eta\eta} \right] + \left[ \lambda_{\varepsilon\varepsilon} + \delta^2 \lambda_{\varepsilon\eta} \right] \end{bmatrix}$$

## Appendix B.1

## VECTOR AUTOREGRESSION ESTIMATES

Table B.1: OLS Estimate of the reduced-form VAR

	ALSI	R/\$	Tbill
ALSI(-1)	0.1676 (0.190) [ 0.877]	-0.020 (0.046) [-0.429]	0.022 (0.101) [ 0.220]
R/\$(-1)	0.322 (0.769) [ 0.418]	0.003 (0.189) [ 0.018]	-0.116 (0.409) [-0.285]
Tbill(-1)	0.151 (0.337) [ 0.449]	0.116 (0.082) [ 1.405]	0.420 (0.179) [ 2.345]
C	-0.001 (0.003) [-0.587]	0.000 (0.000) [ 0.323]	0.002 (0.001) [ 1.372]
R-squared	0.055	0.070	0.184
Adj. R-squared	-0.046	-0.029	0.096
Sum sq. resids	0.007	0.000	0.002
S.E. equation	0.016	0.004	0.008
F-statistic	0.544	0.708	2.105
Log likelihood	87.311	132.195	107.530

Mean dependent	-0.001	0.000	0.003
S.D. dependent	0.016	0.004	0.009

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Standard errors in ( ) & t-statistics in [ ]

Source: Own calculations

**A p p e n d i x B . 2**

**RIGOBON AND SACK GARCH MODEL ESTIMATE**

Given the notation below, the GARCH model estimate is given in the table B.2.

$$\begin{bmatrix} H_{11,t} \\ H_{12,t} \\ H_{22,t} \\ H_{13,t} \\ H_{23,t} \\ H_{33,t} \end{bmatrix} = \begin{matrix} C_f \cdot \Psi_h \\ 6 \times 3 \quad 3 \times 1 \\ C_f \cdot \Pi \cdot (C^2)^{-1} \\ 6 \times 3 \quad 3 \times 3 \quad 3 \times 3 \end{matrix} \begin{bmatrix} H_{11,t} \\ H_{22,t} \\ H_{33,t} \end{bmatrix} + \begin{matrix} C_f \cdot \Lambda \cdot (C^2)^{-1} \\ 6 \times 3 \quad 3 \times 3 \quad 3 \times 3 \end{matrix} \begin{bmatrix} v_{1,t-1}^2 \\ v_{2,t-1}^2 \\ v_{3,t-1}^2 \end{bmatrix}$$

where

$$C \equiv \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \equiv B^{-1} = \begin{pmatrix} 1 & \text{beta}(2) & \text{beta}(3) \\ \text{beta}(4) & 1 & \text{beta}(6) \\ \text{beta}(7) & \text{beta}(8) & 1 \end{pmatrix}$$

and

$$C_f = \begin{bmatrix} c_{11}^2 & c_{12}^2 & c_{13}^2 \\ c_{11}c_{21} & c_{12}c_{22} & c_{12}c_{23} \\ c_{21}^2 & c_{22}^2 & c_{23}^2 \\ c_{11}c_{31} & c_{12}c_{32} & c_{13}c_{33} \\ c_{21}c_{31} & c_{22}c_{32} & c_{23}c_{33} \\ c_{11}^2 & c_{11}^2 & c_{11}^2 \end{bmatrix}$$

Table B.2: Maximum likelihood estimation: Rigobon and Sack model

	Coefficient	Std. Error	z-Statistic	Prob.
Beta(2)	0.030973	0.036115	0.857639	0.3911
Beta(6)	0.001594	0.004656	0.342379	0.7321
Beta(8)	0.061075	0.022546	2.708966	0.0067
Beta(3)	-0.164597	0.027008	-6.094441	0.0000
Beta(7)	0.012230	0.003051	4.009083	0.0001
Beta(4)	0.026636	0.006060	4.395411	0.0000
$\Lambda_{11}$	0.206196	0.046238	4.459472	0.0000
$\Lambda_{13}$	0.503403	0.088191	5.708123	0.0000
$\Lambda_{22}$	0.236836	0.045754	5.176245	0.0000
$\Lambda_{32}$	0.011506	0.069993	0.164389	0.8694
$\Lambda_{33}$	1.211745	0.096403	12.56964	0.0000
$\Pi_{11}$	0.843898	0.032641	25.85377	0.0000
$\Pi_{22}$	0.623898	0.052071	11.98167	0.0000
$\Pi_{33}$	0.198177	0.028450	6.965769	0.0000
Log likelihood	7140.497	Akaike info criterion	-27.62208	
Avg. log likelihood	13.83817	Schwarz criterion	-27.50688	
Number of Coefs.	14	Hannan-Quinn criter.	-27.57694	

Source: Own calculations

**A p p e n d i x B . 3**

**BEKK GARCH MODEL ESTIMATE**

Given the notation below, the GARCH model estimate is given in the table B.3:

$$\begin{aligned}
 & \begin{bmatrix} h_{t,ALSI} & h_{t,ALSI,R/\$} & h_{t,ALSI,TBill} \\ h_{t,R/\$,ALSI} & h_{t,R/\$} & h_{t,R/\$,TBill} \\ h_{t,TBill,ALSI} & h_{t,TBill,R/\$} & h_{t,TBill} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ 0 & c_{22} & c_{23} \\ 0 & 0 & c_{33} \end{bmatrix} \\
 & + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{ALSI,t-1}^2 & \varepsilon_{ALSI,t-1} \varepsilon_{R/\$,t-1} & \varepsilon_{ALSI,t-1} \varepsilon_{TBill,t-1} \\ \varepsilon_{ALSI,t-1} \varepsilon_{R/\$,t-1} & \varepsilon_{R/\$,t-1}^2 & \varepsilon_{R/\$,t-1} \varepsilon_{TBill,t-1} \\ \varepsilon_{TBill,t-1} \varepsilon_{ALSI,t-1} & \varepsilon_{TBill,t-1} \varepsilon_{R/\$,t-1} & \varepsilon_{TBill,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \\
 & + \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{bmatrix} h_{t,ALSI} & h_{t,ALSI,R/\$} & h_{t,ALSI,TBill} \\ h_{t,R/\$,ALSI} & h_{t,R/\$} & h_{t,R/\$,TBill} \\ h_{t,TBill,ALSI} & h_{t,TBill,R/\$} & h_{t,TBill} \end{bmatrix} \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix}
 \end{aligned}$$

Table B.3: Maximum likelihood estimation: BEKK model

	Coefficient	Std. Error	z-Statistic	Prob.
c <sub>11</sub>	0.0008	0.00005	16.648	0.0000
c <sub>21</sub>	-0.0004	0.00003	-14.714	0.0000
c <sub>22</sub>	0.0003	0.00002	9.990	0.0000
c <sub>31</sub>	-0.0004	0.00011	-3.478	0.0005
c <sub>32</sub>	-0.0009	0.00009	-10.273	0.0000
c <sub>33</sub>	0.0008	0.00013	5.806	0.0000
g <sub>11</sub>	0.8365	0.01144	73.123	0.0000
g <sub>22</sub>	0.8045	0.00835	96.282	0.0000
g <sub>33</sub>	0.9121	0.01567	58.177	0.0000
a <sub>11</sub>	0.5102	0.02241	22.765	0.0000
a <sub>22</sub>	0.6175	0.02094	29.484	0.0000
a <sub>33</sub>	0.2810	0.02517	11.163	0.0000
Log likelihood	4289.823	Akaike info criterion	-16.731	
Avg. log likelihood	8.394	Schwarz criterion	-16.606	
Number of Coefs.	15	Hannan-Quinn criter.	-16.682	

Source: Own calculations

Appendix C.1

IMPULSE RESPONSES AND THE COVARIANCE  
BETWEEN ALSI AND TBILL

Figure C.1.1: Comparison: Impulse response due to a shock to ALSI

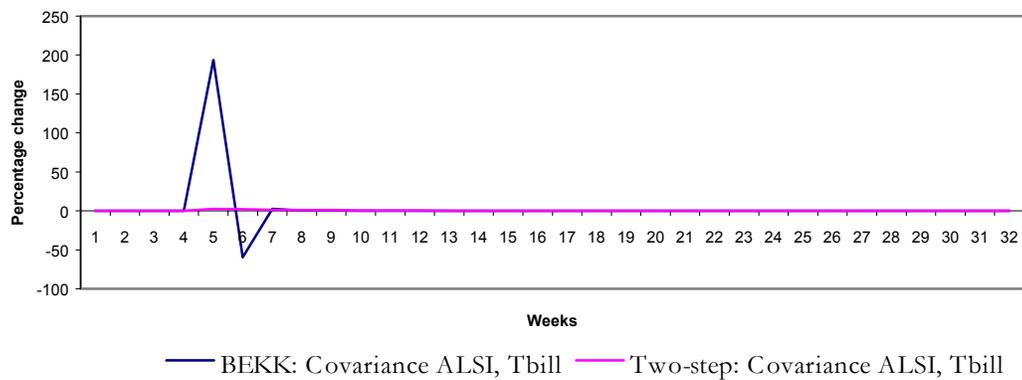


Figure C.1.2: Comparison: Impulse response due to a shock to R/\$

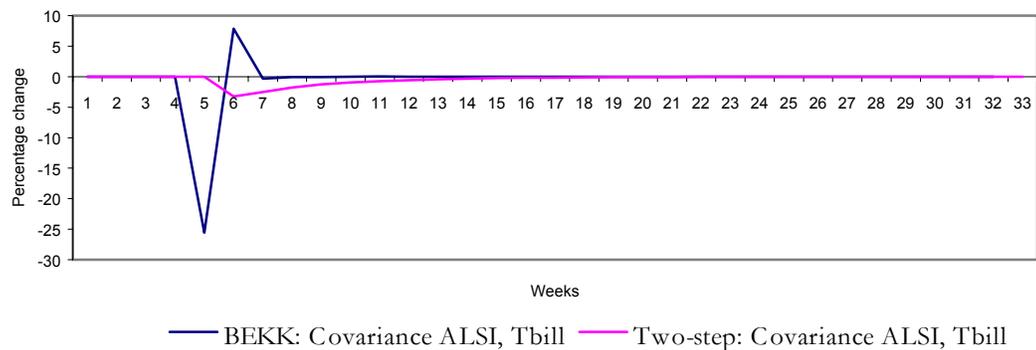
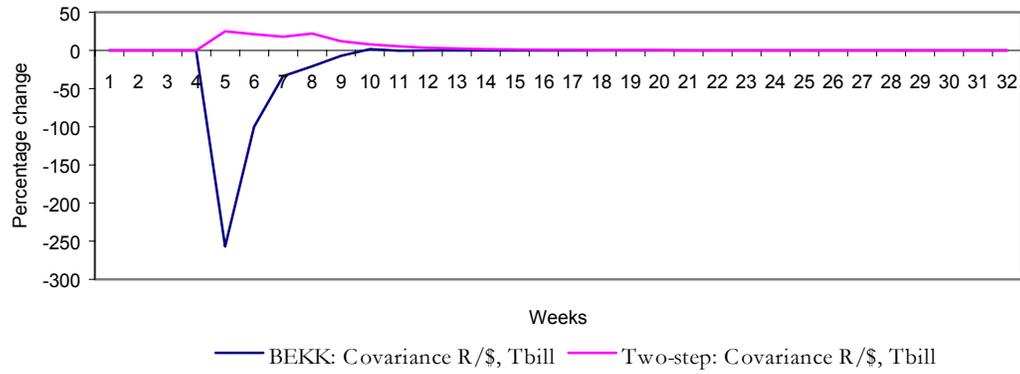


Figure C.1.3. Comparison: Impulse response due to a shock to Tbill



Appendix C.2

IMPULSE RESPONSES AND THE COVARIANCE  
BETWEEN TBILL AND R/\$

Figure C.2.1 Comparison: Impulse response due to a shock to ALSI

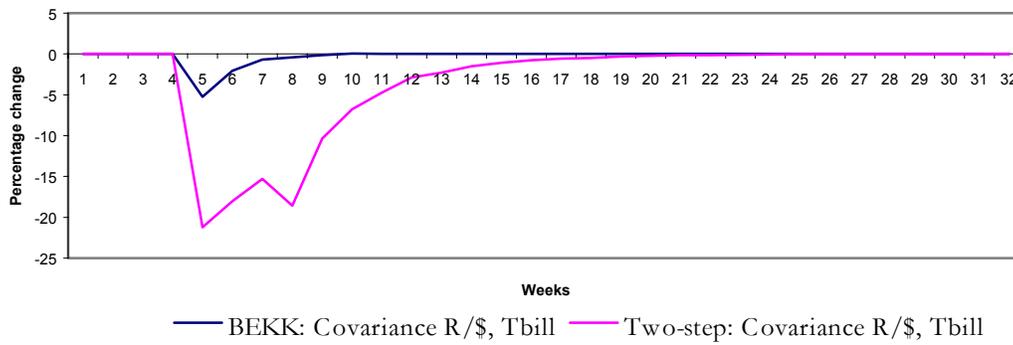


Figure C.2.2: Comparison: Impulse response due to a shock to R/\$

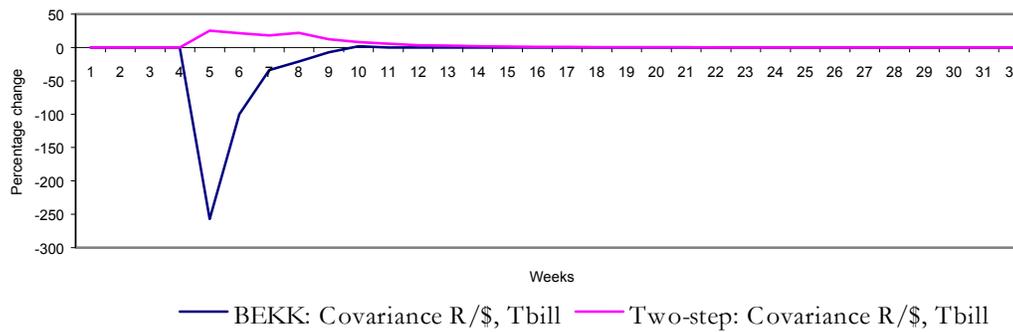


Figure C.2.3: Comparison: Impulse response due to a shock to Tbill

