CHAPTER 4

TRACK SETTLEMENT MODELLING

Although it is known that track geometry generally deteriorates due to repetitive loading from passing traffic, the mechanism governing this phenomenon is rather complex. If every point of the track were to settle by the same amount, no irregularities in the vertical space curve would develop. However, these settlements are generally far from uniform due to variations in the track support and the wheel load distribution. These deviations cause differential track settlement due to plastic deformation of the support in the wavelengths experienced by rolling stock.

Various approaches to predict such track settlement are discussed in literature. The most common approach to predict track settlement is the use of the logarithmic track settlement law. Some research work however looks at simplified predictions using a statistical distribution of track properties or a so-called damage factor. More comprehensive approaches consider localised discrete irregularities or extended spatially distributed irregularities. A review in this respect is given in Appendix A. Although a large portion of the research reviewed, focuses on laboratory and field tests intended to determine the rate of track deterioration, very few authors have considered differential track settlement due to dynamic wheel loads and spatially varying nonlinear track stiffness. References in this respect have already been given in Chapter 2.

In this chapter a new approach to determine differential track settlement due to dynamic wheel loading and spatially varying track support conditions is formulated. The first section of this chapter outlines the basic assumptions that were made. The second section describes the basic methodology used to predict track settlement and in the last section a validated Modified Settlement Algorithm is developed for later implementation into the Track Deterioration Prediction Model.

4.1 ASSUMPTIONS

As the objective of this research is to predict and analyse track settlement due to dynamic wheel loads and spatial track stiffness variations alone, numerous factors which could also influence track deterioration are not accounted for. The basic assumptions are:

- Only settlement in the ballast layer is considered. It is assumed that the sub-ballast and subgrade stays undisturbed and continues on its relatively low settlement trend, while the top ballast layer is loosened by tamping. The contribution of ballast settlement is thus more significant. This assumption would for example be invalid if the moisture content of the subgrade would change.
- The ballast layer thickness is constant over the test section.
- No ballast degradation is considered.
- Track settlement due to vibrations, transmitted by the track superstructure to the ballast, is not included in the settlement model. These vibrations are typically caused by rail joints, rail corrugations and wheel flats.
- Environmental changes due to the weather are not included.

4.2 PREDICTION OF TRACK SETTLEMENT

A schematic of the basic methodology to predict track settlement is shown in Figure 4.1. Figure 4.1 shows the various components and the feedback mechanism of the proposed interactive dynamic track settlement model. Using the initial vertical track profile, the spatial track stiffness variations and the static wheel load, the track definition module calculates the loaded vertical track profile and the effective linearised track stiffness under the given static wheel load. The initial loaded track profile together with the spatial track stiffness variations are then used as input into the vertical vehicle/track model, where the vertical dynamic wheel/rail interaction forces are calculated. Together with the effective linearised track stiffness, the dynamic forces between the wheel and the rail are used as input into the track settlement model. The predicted track settlement is then added to the loaded track profile which is then used as a new excitation input into the vehicle/track model where a different dynamic reaction will result in different dynamic loads and subsequently a change in the differential settlement of the track.

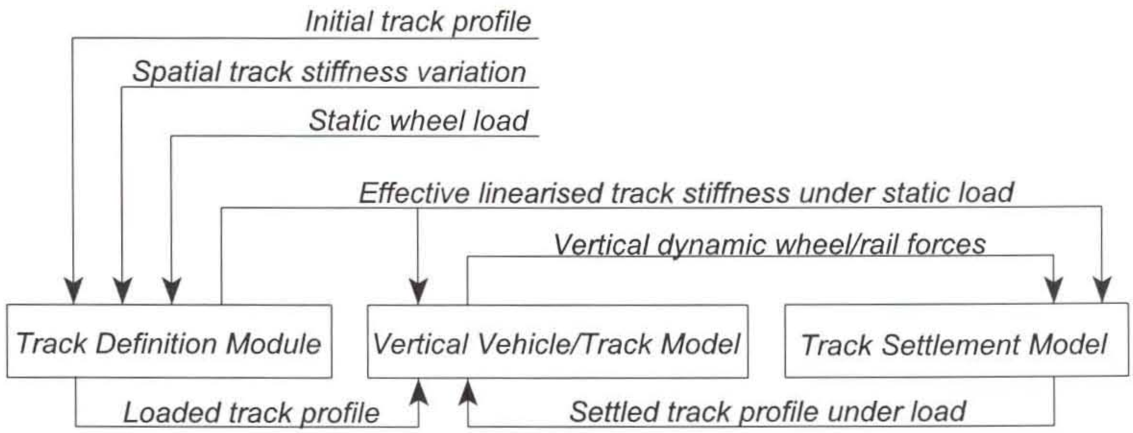

Figure 4.1: Interactive dynamic settlement methodology.

Before describing the development of the Modified Settlement Algorithm, some detail with respect to the mechanisms of track settlement is given. In principle, the methodology as developed by Stewart and Selig (1982) was used as the foundation on which to build the proposed interactive dynamic settlement model. The sequence to determine the permanent differential settlement of the ballast due to dynamic wheel loads and spatial track stiffness variations is given in Figure 4.2. The main components of the dynamic track settlement model are now discussed in detail.

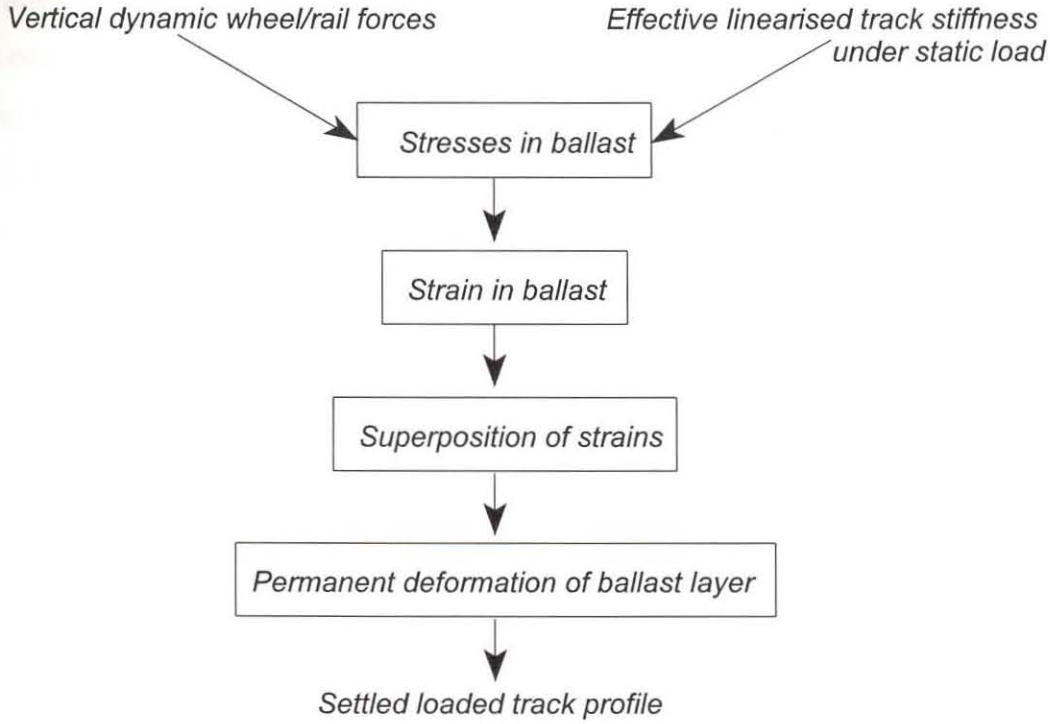

Figure 4.2: Dynamic track settlement model.

Stresses in the ballast. The methodology for predicting permanent track deformation starts by determining the stress state at the top of the ballast layer. These stresses comprise the initial vertical geostatic stresses that are due to the weights of the track superstructure and the soil, as well as the incremental stresses, that is those due to the imposed wheel loads. The incremental stresses are determined using the three-dimensional elastic multi-layer computer model GEOTRACK (Chang *et al.*, 1980) and are added to the initial geostatic stresses to determine the final three-dimensional stress state. These three-dimensional stresses are then converted into an equivalent triaxial stress state. The axial stress is defined by the difference between the major principle stress, σ_1, and the confining stress that is the minor principle stress, σ_3 in the loaded state, and is used to determine permanent track deformation under the instantaneous dynamic wheel load. The axial or vertical stress is also known as the principle stress difference, $(\sigma_1 - \sigma_3)$, or deviator stress.

The major principle stress and the minor principle stress of the loaded track are both computed using the GEOTRACK program. An extract out of a GEOTRACK output file is given in Appendix C. The slope of a measured axial stress-strain curve is defined as Young's modulus as long as the lateral stress is constant. One of the most versatile and useful laboratory tests for soil stress-strain and strength properties is the triaxial compression test (ORE Q C116 (Report 8), 1977).

Strain in the ballast. Once the stress in the ballast has been computed for the various dynamic wheel loads, the next step is to determine the permanent strains that would be expected to develop under the applied loads. This is generally done by using the following equation for permanent ballast strain as derived by British Rail from laboratory test results (ORE Q D71 (Report 10), 1970).

$$\epsilon_N = \epsilon_1 (1 + C \log N) \tag{4.1}$$

In this equation ϵ_N is the permanent axial strain after N load cycles, ϵ_1 is the permanent strain caused by the first load cycle, and C is a material constant between 0.2 and 0.4 (Selig and Waters, 1994). Although based on a limited number of tests, the following relationship between the deviator stress (given in terms of [kgf]), the porosity of the ballast n_p and the deformation produced by the first load cycle ϵ_1, was proposed.

$$\epsilon_1 = 0.082 (100 n_p - 38.2) (\sigma_1 - \sigma_3)^2 \tag{4.2}$$

Equation (4.2) can thus be used to relate the permanent axial strain to the ballast condition (porosity) and the number and magnitude of the applied axial load cycles. Typical values for porosity are between 0.4 and 0.5. For slag the porosity value is 0.34, for granite it is 0.26 and for limestone it is 0.40 (Stewart and Selig, 1982). The law governing the settlement of ballast is thus based on the assumption that the settlement of the track is proportional to the logarithm of the total tonnage moving over the section.

Superposition of strains. The cumulative strains due to a mix of wheel loads is based on a cumulative relationship similar to Miner's rule for structural fatigue analysis.

The procedure for superimposing strains to account for a mix of wheel loads at a particular sleeper is illustrated in Figure 4.3 for a two-load-level case. For each load level, the stresses in the ballast and the equivalent triaxial stress paths lead to different first cycle strains, ϵ_1. The higher load will cause a higher first cycle strain than the lower load. Knowing the respective first cycle strains and the material constant, C, the permanent strain, ϵ_N, that is expected to develop after N constant magnitude load cycles can be determined using Equation (4.1).

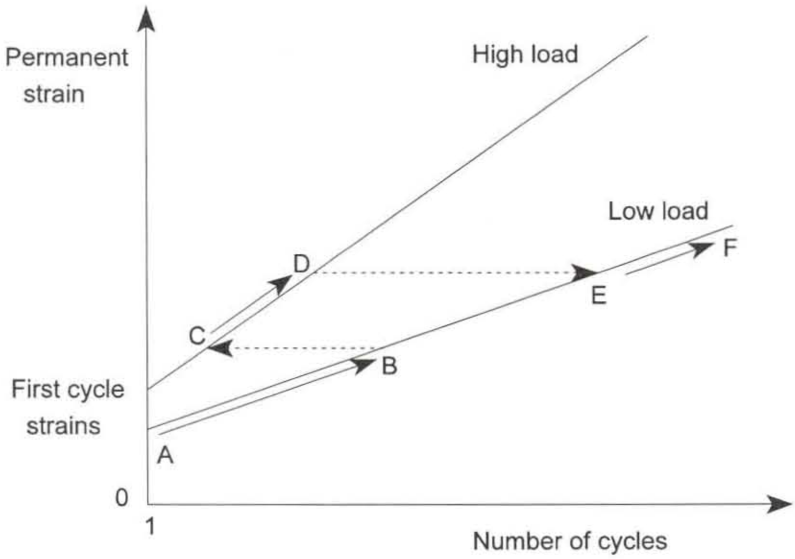

Figure 4.3: Ballast strain superposition for mixed loading.

The permanent strains due to the low load applications are determined by moving along the low load curve, Point A to Point B, in Figure 4.3 using the first cycle strain and the given number of low load cycles. Additional strains due to higher loads are calculated by first finding an "equivalent" number of cycles on the high load curve that would have caused the same strain as developed by the lower load. The equivalent number of high load cycles are found at Point C on the high load curve. The next step is to add the additional cycles at the higher load to these "equivalent" number of load cycles. The superimposed strain is then calculated by effectively following the high load curve, that is Point C to Point D. For subsequent lower loading, a similar procedure is used to return to the low load deformation curve, that is Point D to Point E for the equivalent number of load cycles, and then Point E to Point F for the increase in strain after an additional number of low load cycles. Finally the calculated strain in the ballast layer is multiplied by the thickness of the ballast layer to get the actual permanent settlement of the ballast layer.

4.3 MODIFIED SETTLEMENT EQUATION

As part of the research a model for the complex relationship between vehicle and track parameters, the dynamic response of the vehicle, and the track settlement behaviour had to be found. It was soon realised that the basic settlement equation as given in Equation (4.1) did not give results that could be compared to the measured differential track settlement. The reason for this was that Equation (4.1) was developed from controlled laboratory results. A new approach thus had to be taken.

Finding a relationship between the dynamic wheel loads, the spatial variation of the track stiffness and the resulting differential track settlement was difficult. To determine whether the dynamic wheel load, or the spatial variation of the track stiffness was dominant, and by how much, or to determine whether an average stiffness, the seating stiffness or the contact stiffness should be used and how the measured track modulus of elasticity relates to stress in the track was a challenge. An example of the measured dynamic wheel load, track settlement, track geometry and track stiffness over the 150 sleeper test section is given in Figure 4.4. From Figure 4.4 it can be seen that there is no dominant relationship between the measured parameters. Filtering over a certain number of sleepers was also tried as a means to identify possible correlations between measured vehicle and track parameters.

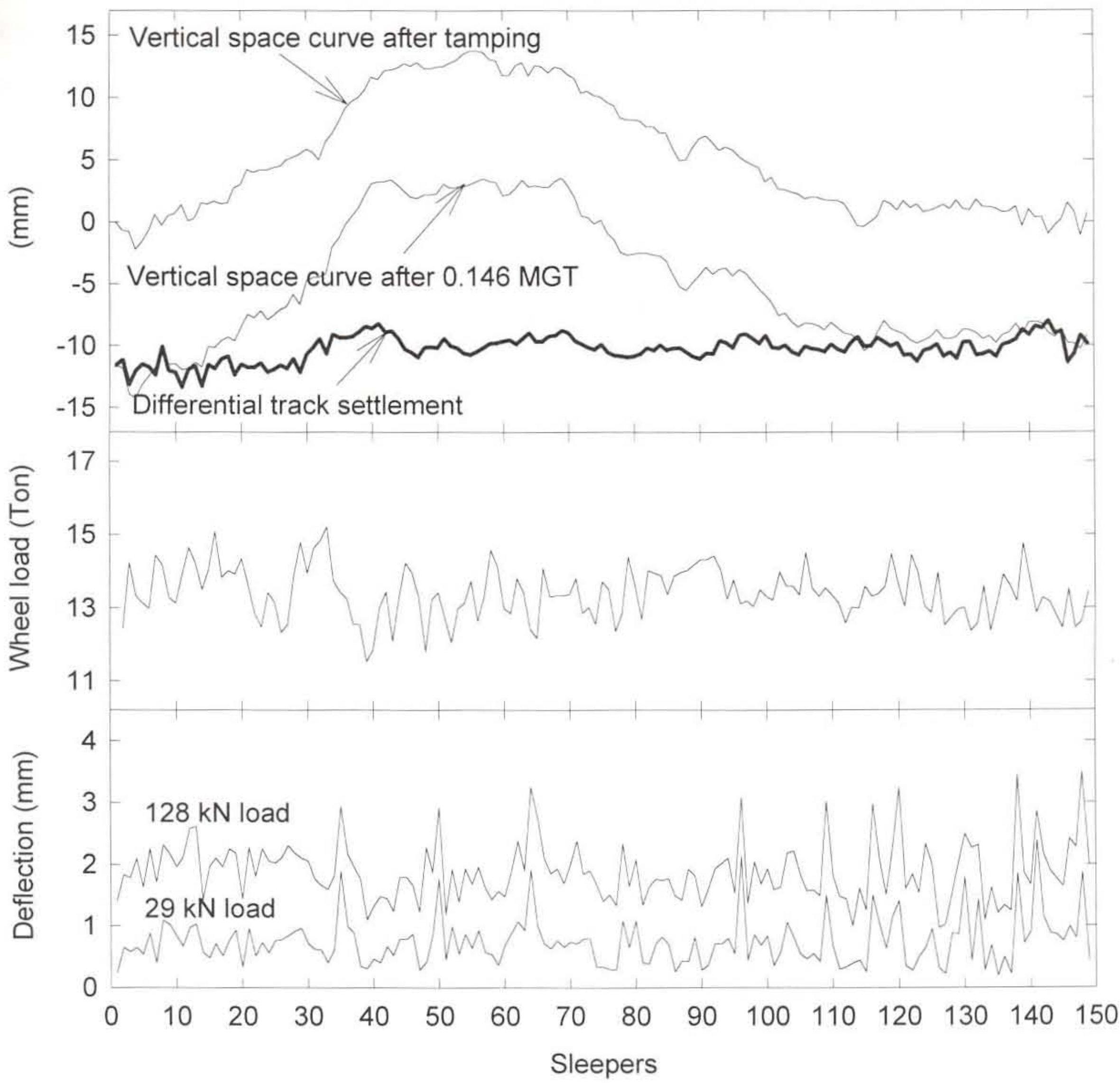

Figure 4.4: Measured vertical track space curve, differential track settlement, wheel load and track deflection.

Eventually it was realised that the differential settlement of the track is dominated by the spatial variation of the track stiffness and the modified settlement equation was developed. The remaining part of this section gives a chronological discussion of the development of the modified settlement equation. In principle the permanent deformation of the ballast is still modelled along the lines set out in the literature (Selig and Alva-Hurtado, 1982; Stewart and Selig, 1982; Fröhling *et al*, 1996a) and as discussed in the previous section. Before outlining the development of the modified settlement equation, the assumptions and simplifications that were made are listed.

- As a method for continuously measuring the spatially varying dynamic track stiffness is not yet available, the discretely measured static track stiffness values were used.
- It was assumed that the track stiffness as measured after the rate of settlement had decreased significantly, is a representative signature of the spatially varying track stiffness.
- As structural damping can only be determined from dynamic track stiffness measurements, the track damping was assumed to be linear and constant over the entire test section.
- Although there is a mixture of freight traffic on the line, the dominant 26 ton axle loading was used in the development of the model.
- Although the modulus of elasticity of the various sub-structure layers was measured only at one particular sleeper, this set of values was used to estimate the permanent strain behaviour of the entire test section. It was thus assumed that the modulus of elasticity of the top layer, that is that of the ballast, changes as a function of the measured static track stiffness.
- An equivalent linear but spatially varying track stiffness was used in the program GEOTRACK (Chang *et al*, 1980; Li and Selig, 1995) to be able to calculate the deviator stress in the track structure for each individually measured track stiffness. The equivalent linear track stiffness is defined as the prevailing wheel load divided by the prevailing track deflection.
- Only loading in the vertical direction was considered and the actual dynamic wheel loads were represented by equivalent quasi-static loads in GEOTRACK.

Having defined the relevant assumptions, the development of the modified settlement equation is described in more detail. The first step in the development of the modified settlement equation was to take the statically measured spatially varying nonlinear track stiffness and linearise it to obtain the effective linearised track stiffness under the dominant wheel load. The linearised track stiffness is defined as the static wheel load divided by the prevailing track deflection. As the track stiffness randomly varies at a particular sleeper with accumulating traffic, the track stiffness as measured after the rate of track settlement had reduced considerably, that is after 2.84 Million Gross Ton (MGT), was used. It was found that this spatially varying track stiffness gives a better prediction with respect to the differential track settlement than if the track stiffness as measured directly after tamping would be used.

To be able to use GEOTRACK to calculate the stress state in the track sub-structure, in particular the stress state in the ballast layer, the varying modulus of elasticity of the various substructure layers are required. These moduli can be determined using GEOTRACK, but the differential deflections of the substructure layers are required. As the measurement of the differential deflection in the substructure layers is a costly and time consuming exercise only two Multi Depth Deflection Meters (MDDs), that is one on either side of Sleeper 76, were placed in position. See Figure B18 in Appendix B. Hence, only the deflection values in the various substructure layers at this particular sleeper were available to be used in GEOTRACK. The method of how to obtain the modulus of elasticity for the sub-structure layers is described in more detail in Appendix B Section 2.4.1 and calculated values are given in Table B4.

The next step was to use the modulus of elasticity values as determined at Sleeper 76 as input into GEOTRACK and determine the effective linear track stiffness and deviator stress. The relevant GEOTRACK input and an extract out of a typical output file are given in Appendix C. During the computations with GEOTRACK, it was found that the calculated track stiffness was on average 1.34 times lower than the actually measured linearised track stiffness at this particular sleeper. Subsequently this factor was incorporated into the settlement equation to relate measured track stiffness to the calculated deviator stress in the ballast layer.

As the modulus of elasticity values were only available at Sleeper 76, the assumption was made that the change in the elasticity of the top ballast layer from sleeper to sleeper is directly proportional to the change in the equivalent linear track stiffness. Using this assumption, the modulus of elasticity of the top layer was varied until a whole set of track stiffness values were determined, effectively covering the whole spectrum of measured track stiffness values. The stiffness factor of 1.34 between measured and calculated track stiffness was taken into consideration.

The result was that the deviator stress could now be calculated with GEOTRACK as a function of track stiffness by using different modulus of elasticity values. The relationship between the deviator stress and the track stiffness is shown in Figure 4.5 for the left and right hand side of the track. The fact that there is a dropping off of the curve below a track stiffness of 60MN/m means that the stress and thus effectively the differential plastic strain in the ballast layer is such that there would be no significant differential settlement at these low stiffness values. Hence, this part of the curve was ignored and a straight line was fitted through the data. It is assumed that this behaviour is due to the fact that actual changes in the properties of the other substructure layers were not considered. As the deflections of the substructure layers at only one sleeper are used for a given test site, there is no value in differentiating between the stress properties of the left and the right hand side of the track. Therefore the equation describing the relationship between the deviator stress and the track stiffness is the average curve fitted through both graphs in Figure 4.5. The resulting equation for the deviator stress [in kPa] is

$$\sigma_1 - \sigma_3 = K_1 + K_2 \ (k_{2i}) \qquad (4.3)$$

where k_{2i} [in kN/m] is the track stiffness at a particular sleeper as calculated by GEOTRACK. For the particular case under investigation K_1 is 194 kPa and K_2 is -1.96×10^{-3} m^{-1}. These values are site specific. Using the measured track stiffness, Equation (4.3) becomes

$$\sigma_1 - \sigma_3 = K_1 + K_2 \left(\frac{k_{2mi}}{K_3}\right) \tag{4.4}$$

where k_{2mi} is the measured track stiffness at a particular sleeper and K_3 is 1.34.

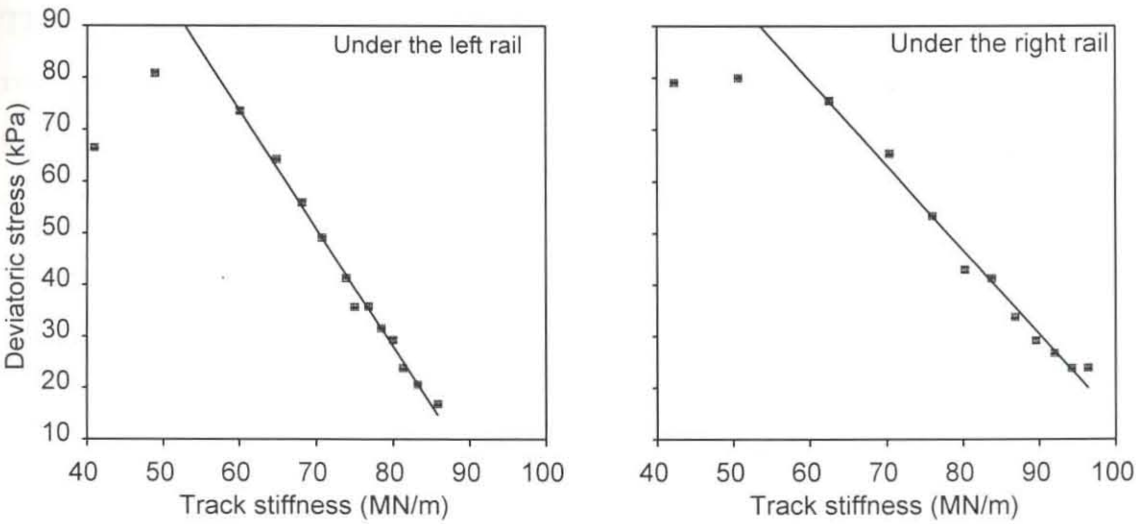

Figure 4.5: Deviator stress versus track stiffness.

To obtain the relationship between the dynamic wheel load and the settlement of the track, GEOTRACK was used once again. The procedure was to use a particular modulus of elasticity value and only change the static wheel load. The relationship between the resulting change in the deviator stress was approximately equal to the change in the dynamic wheel load relative to the static wheel load. The relationship between the deviator stress and the prevailing dynamic wheel load, P_{dyn}, is given as:

$$\sigma_1 - \sigma_3 = \left[K_1 + K_2 \left(\frac{k_{2mi}}{K_3}\right)\right] \frac{P_{dyn}}{P_{ref}} \tag{4.5}$$

where P_{ref} is a static reference wheel load. For this analysis the static reference wheel load was 13 tons.

It should be noted that Equation (4.5) is only valid for a measured linearised track stiffness below 132.6MN/m. In a section of track with a different stiffness range, the parameters K_1, K_2 and K_3 would change and could be re-determined using the procedure described above. It was however found, that Equation (4.5) can be used successfully for average track stiffness values between 60 and 132MN/m.

The next step was to obtain the logarithmic settlement behaviour as well as the relative differential settlement as a function of the dynamic wheel load and the spatial variation of the track stiffness after a number of load cycles. The final differential settlement equation [in mm] is thus given as

$$\epsilon_{N_i} = (\sigma_1 - \sigma_3)^w \log N \qquad (4.6)$$

that is

$$\epsilon_{N_i} = [[K_1 + K_2 (\frac{k_{2mi}}{K_3})] \frac{P_{dyn}}{P_{ref}}]^w \log N \qquad (4.7)$$

where w is the settlement exponent to give the best fit to the measured overall track settlement. For the chosen test site w is 0.3. Validated results and further analysis are given in Chapters 7 and 8.

Summary

In this chapter the assumptions and methodology to predict track settlement have been presented. The most important contribution is the development of the modified settlement equation. The constants of this settlement equation are dependent on the basic properties of a certain section of track and can systematically be determined for any typical section of track using the procedure as described in this chapter.