CHAPTER 6

Discussion and Evaluation

6. DISCUSSION AND EVALUATION

6.1 TEST PIECE

An aluminium beam was invoked as the test piece throughout the study. It was deemed advisable to start with a relatively simple structure but unexpectedly gave rise to some difficulties.

The fact that the test piece was a lightly-damped structure required special effort concerning the excitation function used in determining the frequency response functions. In the end a chirp-sine function was used, resulting in much sharper and well-defined peaks of the frequency response functions. The low damping was also mentioned as probable cause for difficulties experienced with the experimental modal analysis.

Another factor that might have influenced the accuracy of the force estimates is that the modes for the beam were very well-separated, and at any given frequency, the response of the beam was dominated by the two nearest modes.

In view of the above-mentioned it may be justifiable to consider a more complex structure with less well-separated modes and with higher damping. In a structure with higher damping the neighboring modes will have a larger contribution to the response of the structure at any given frequency and might not exhibit the same degree of ill-conditioning noted with this test piece.

6.2 APPLIED FORCES

6.2.1 Unknown Forces Locations

Throughout the experimental studies it was assumed that the exact forcing locations were known. In addition, only the columns associated with these input locations were included in the frequency response function matrix. Conversely, for the modal coordinate transformation method only the rows associated with the input locations were included in the reduced modal matrix of equation (4.16). Let's assume that the number of forces and the input locations were unknown.

a) Frequency Response Function Method

The maximum number of forces one can correctly predict will be equal to the rank of the frequency response function matrix, which in turn is equal to the number of significant participating modes (Fabunmi, 1986). For the case of the hinged-hinged beam, the analysis of the 'full' frequency response function matrix would allow the estimation of maximum six forces, except at frequencies where the frequency response function matrix renders rank deficient. An important prerequisite is that the actual force inputs should be among the locations included in the frequency response function matrix. If not, the estimated forces will be erroneous pseudo-forces. The forces that produce a certain response can then be determined from the pseudo-inversion of the frequency response function matrix. The force amplitudes of the non-forcing locations will be zero (Fregolent and Sestieri, 1990).

b) Modal Coordinate Transformation Method

The same argument can be followed in the case of the modal coordinate transformation method. The modal matrix is generally rectangular with the number of responses exceeding the number of modes within the analysis frequency band. In this case the number of force estimates should be smaller than or equal to the rank of the modal matrix (refer to Section 4.1.2). Proper choice of the set of response locations and orthogonal mode shapes will ensure that the modal matrix is well-conditioned and will not exhibit the ill-conditioning at the resonant frequencies of the structure. Thus the number of force estimates remain constant for the entire frequency range considered. Once again the actual force locations should form part of the set of response locations of the reduced modal matrix, to infer the excitations accurately.

6.2.2 Distributed Forces

Both the frequency response function and modal coordinate transformation methods are discrete representations of continuous functions. The frequency response function, modal matrix and the responses can usually be measured only at a finite number of discrete points and as a result the force identification problem is restricted to the determination of forces at the discrete points. In the case of a distributed loading the responses are used to determine a number of equivalent discrete forces, which are intended to represent the original distributed load. As mentioned before, the number of modes in the frequency response function or modal matrix restrains the number of discrete dynamic forces. Unfortunately, these discrete forces will always be in error, even with a high number of modes and a high matrix dimension (Fregolent and Sestieri, 1990).

6.2.3 Random Forces

Chapter 5 dealt with harmonic force determination only. An attempt to extend the force identification to determine random dual forces was unsuccessful due to shortcomings of the experimental setup. The force spectrums presented in Section 5.3 and 5.4 exhibited some correlation despite the uncorrelated drive signals supplied to the shakers. The correlation was the result of the interaction of the excitation system with the structure and will tend to be greater at the beam's resonant frequencies. Two random forces were applied to the beam and the accelerations were measured. Figure 6.1 presents the decomposition of the directly-measured random force spectral density function matrix obtained from the measurement procedure. The number of distinct non-zero eigenvalues, as well as the ratio between adjacent eigenvalues can be used to ascertain the degree of correlation of the inputs (Maia *et al.*, 1997). It is worth noting that in the vicinity of the resonant frequencies of the beam the ratio between the eigenvalues was high, indicating only a single independent force. In the rest of the frequency range the eigenvalues were of similar magnitude. This shows that there were two independent forces. The reason for the results noted can be explained as follows. Two rather small electromagnetic exciters were used to excite the structurally stiff beam. For the case where a particular exciter was exciting the structure at or near one of the resonant frequencies the other exciter was 'pulled' in phase with the response of the resonant mode. The use of larger electromagnetic shakers, or for that matter even hydraulic shakers, might not exhibit the same behaviour. In addition a more flexible beam might be employed as test piece.

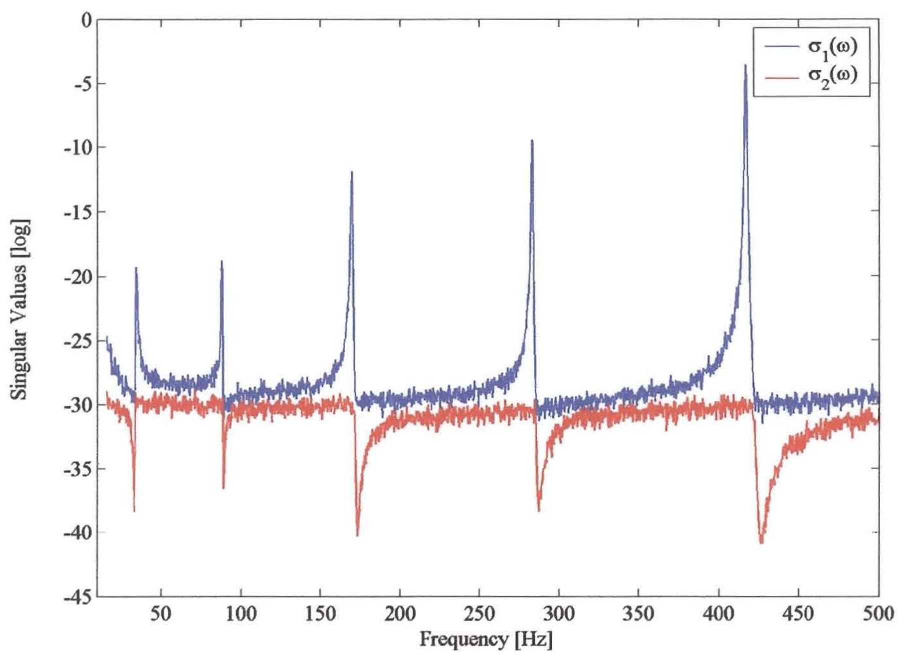

Figure 6.1 – Decomposition of force spectral density function matrix

Instead harmonic forces were applied to the beam, with forcing frequencies more or less in between the structural resonant frequencies.

6.3 THE FREQUENCY RESPONSE FUNCTION METHOD

The formulation of the frequency response function method was described in Chapter 3. No modal parameter estimation is required, although an experimental modal analysis can be performed to 'filter' some of the measurement noise from the frequency response functions.

The number of forces one can attempt to determine is limited by the number of significant participating modes at any given frequency line.

The ill-conditioning of the force predictions at the system's resonant frequencies was illustrated through the use of a numerical simulation. The various modal parameters and responses were perturbated with random errors to resemble experimental measurements. The results indicated that the perturbation of the mode shapes had the most significant affect on the accuracy of the forces. In addition, it was noted that the pseudo-inverse of the frequency response function matrix needs to be computed at each frequency line within the analysis frequency range.

The force predictions might be ameliorated at the structural natural frequencies by truncating very small singular values of the frequency response function matrix from the pseudo-inversion, since the inclusion of these small values might lead to erroneous force estimates.

The condition number of the frequency response function matrix furnishes as a tool to ascertain the expected ill-conditioning of the force predictions. A numerical study highlighted the factors influencing the value of this parameter.

Chapter 5 evaluated the implementation of this method to infer the forces acting on an aluminium beam subjected to different boundary conditions. A single harmonic force was successfully identified on a free-free and hinged-hinged beam. The results of the dual force inputs applied to the free-free beam cannot be completely explained. However, a probable cause was mentioned. In the final experimental study, the frequency response function method was used to determine two harmonic forces on a hinged-hinged beam from both acceleration and strain measurements. Despite the influences of the boundary conditions on the frequency response functions noted, the method was sufficiently successful in determining the applied forces. One of the important issues resulting from the experimental studies was the contribution of the residual terms corresponding to the truncated modes, which affected the accuracy of the frequency response functions and consequently the force predictions. It was shown that the method has the ability to incorporate the influence of the residual terms in the force identification process. The residual terms corresponding to a particular reference position cannot be synthesised from the modal parameters extracted from the measurement of a single column or row of the frequency response function matrix. As a result the frequency response functions must be measured at all the expected force input locations.

6.4 THE MODAL COORDINATE TRANSFORMATION METHOD

Chapter 4 was concerned with the modal coordinate transformation method. An experimental modal analysis was a prerequisite for implementation of this method. In order to utilise a least-squares estimation of the forces the method requires the number of response locations to exceed the number of linear independent modes in the analysis frequency range, which in turn should preferably be greater than the number of forces one attempts to reconstruct. The results of a numerical study showed that the modal coordinate transformation method did not suffer from the ill-conditioning of the force estimates at the structural natural frequencies due to the inversion of the modal matrix. The successive integration of the acceleration to displacement signals was shown to amplify the errors in the low frequencies and might affect the force results.

This method was computationally much faster than the frequency response function method, since the pseudo-inverse of the modal matrix needed to be calculated only twice, regardless of the frequency resolution considered in the analysis.

A separate numerical study on a free-free beam indicated that the condition number of the modal matrix is a function of the set of response locations chosen and the modes included in the modal matrix.

The modal transformation is based on the assumption that the modal vectors are orthogonal and linearly independent functions. This assumption might be violated during the experimental modal analysis and a reciprocal modal vector was described as an alternative method of performing the modal transformation.

This method has the advantage of being able to identify forces at locations which may be inaccessible for frequency response function measurements. Based on the reciprocal theorem, another point on the structure can be artificially excited from which one can extract the modal parameters. Having measured only the response at the inaccessible force input location, the force estimate corresponding to this point can then be determined (Kim and Kim, 1997). The results of the experimental studies in Chapter 5 pointed out that this procedure is only valid when the values of the residual terms of the truncated modes outside the analysis frequency range are negligible. Although the residual terms corresponding to the low and high modes can be obtained from the experimental modal analysis, these terms have to be omitted when executing the modal coordinate transformation method. The findings of Section 5.1, where a free-free beam was used to predict a single harmonic force, indicated that the low frequency contribution, i.e. the RBM, had the most adverse affect on the accuracy of the reconstructed frequency response functions at the excitation frequency.

The frequency response function method for determining a single force resulted in a condition number of unity (the ratio of a single singular value by itself). However, the modal coordinate transformation method required the inversion of the modal matrix, and since more than one mode was included in the modal matrix, the value of the condition number was greater than unity.

In the second experimental study a hinged-hinged beam was investigated to exclude the effect of the RBM to the response of the beam. At first not enough modes were included in the analysis to infer the force estimate correctly at the excitation frequency. It was noted that more modes were required in the force identification process than the number of modes required in modeling of the response of the structure. This method also displays the truncation problem generally encountered in experimental modal analysis and are caused by the limited number of modes, limited number of measurement points and the lack of rotational degrees-of-freedom (Shih *et al.*, 1989). As jet, it is uncertain as to the number of modes, or the analysis frequency range required to guarantee success with the modal coordinate transformation method. This might prevent the method from becoming popular in cases where the vibration measurements are very expensive and/or labour intensive to perform and a second round of measurements are just not viable.

The final experimental evaluation was a hinged-hinged beam subjected to two simultaneous harmonic forces. Constraints imposed on the beam influenced the frequency response functions around the fifth mode. As a consequence, the frequency response functions were difficult to recreate from the modal parameters alone and spoiled the force identification.

This method was applied under the assumption of proportional damping and would require a more elaborate calculation procedure if this assumption was not fulfilled (Desanghere and Snoeys, 1985).

6.5 CONCLUSION

Two frequency domain force identification procedures were proposed in this work. In view of the above-mentioned advantages and disadvantages associated with each method it can be concluded that the frequency response function method is superior to the modal coordinate transformation method for the applications considered in this study.