CHAPTER 1

Introduction

1. INTRODUCTION

1.1 PREAMBLE

The location and magnitude of self-generated or input forces on a structure, or for that matter any piece of equipment, may prove to be very important for the proper evaluation at the design and modification phases, as well as in the case of control and fatigue life predictions. The identification of the input forces has also attracted a great deal of interest in machine health monitoring and troubleshooting (Shih *et al.*, 1989). The location of the excitation forces can reveal the possible causes of vibration, while the force amplitude determines the severity of the vibration condition.

In cases where the direct measurement of these forces is possible, it is usually accompanied by structural changes to accommodate the attachment of force sensing equipment, and may as a result change the dynamic characteristics of the system.

However, there are many situations where the direct measurement of the excitation forces is not possible or feasible. For example:

- Shock/impact loads on ship hulls (Dubowski and Dobson, 1985).
- Engine torque pulses and shaking forces are difficult to measure, since these forces are distributed throughout the engine (Starkey and Merrill, 1989).
- Forces transmitted from machinery, such as compressors, to the foundations.
- Stress analysis on a finite element model of a structure can be performed by applying prescribed displacements. In the case of structural modifications, the stress analysis would require knowledge of the excitation forces.
- Propellor-induced pressure fluctuations on a ship hull (Stevens, 1987).
- Determining of acoustic loads where the environment does not permit the use of microphones to measure the acoustic field (Elliott *et al.*, 1988).
- Explosive loading or force input within hostile environments (Dubowski and Dobson, 1985)
- The indirect computation of the flow-induced forces in a piping system or petrochemical reactor.

Instead of being able to measure the force inputs directly, some other quantity, e.g. the response, is usually measured from which the forces can be determined indirectly. The aim of the present work is to show that it is possible to estimate dynamic forces by measuring the responses, be they acceleration, displacement or strain, of a linear structure subjected to those forces. In essence the structure becomes the force transducer.

Theoretically, it is possible to determine the forces by simply reversing the process of calculating the responses of a system subjected to known forces, but this procedure is known to be ill-posed and sensitive to noise, and may contribute to meaningless results.

1.2 FORMULATION OF DIRECT AND INVERSE PROBLEM

In this section the classification of the force identification as an inverse process and an ill-posed problem is motivated and the principal difficulties in such a procedure are discussed.

The direct problem (also referred to as the forward problem) is to find the response of the structure from knowledge of the excitation force through the use of a transfer function, which gives the relationship between the measured response and the excitation forces:

$$\{F(\omega)\} \Rightarrow \{X(\omega)\} \tag{1.1}$$

Conversely, the force identification problem is to accurately infer the excitation force from knowledge of the vibration responses via some transfer function:

$$\{X(\omega)\} \Rightarrow \{F(\omega)\} \tag{1.2}$$

The latter problem involves the recovery of the force (cause) given the incomplete and noise-contaminated response (effect) and system matrix, whence the name inverse problem.

Two important observations regarding the differences between the direct and inverse problem can be made (Karlsson, 1996):

Firstly, the excitation force in the direct problem is known all over the structure. These forces are usually concentrated on limited portions of the structure, while the rest of the structure can be regarded as non-loaded. This is typically the procedure followed in the finite element analysis of a structure in order to obtain its dynamic response as a result of the applied forces/loads.

However, in the inverse problem, a non-zero vibration response is in most cases present all over the structure. Furthermore, the responses can usually be measured only at a finite number of discrete points, and there is no information available regarding the responses between these points. Thus, the entire response can be solved for the direct problem, whereas the full excitation forces cannot be determined in the inverse problem. As a result, the solution to the direct problem is straightforward while the inverse problem is left without a unique solution.

The second observation to note is that even if the responses could be measured at closely spaced positions, the force identification problem is ill-posed and ill-conditioned. A well-posed problem can be stated as a problem that satisfies the three conditions of Hadamard. These are the existence, uniqueness and stability of the solution (Hashemi and Hammond, 1996). If any of these conditions are violated, the problem is said to be ill-posed. On the other hand, ill-conditioning refers to the phenomena where small measurement errors (noise) are mapped onto unboundedly large errors in the force estimates. For an in-depth discussion of ill-posed problems, as well as ways to regularise these problems to be well-behaved, refer to Sarkar *et al.* (1981).

As a result it is impossible to calculate the entire force distribution on a structure, simply from response measurements. To make the force identification solvable, additional information regarding the force distribution is needed *a priori*. The force identification problem can be regularised to a well-posed problem by limiting the excitation forces to a finite number of discrete points on the structure. An $(n \times m)$ frequency response function, $[H]$, is deduced consisting of both response and force identification points. With this *a priori* information the force identification problem is reduced to the determination of only the unknown force amplitudes at the discrete points.

This approach can be justified, since in many applications the exact forcing locations are known, for instance at an engine mount or bearing support. In the case of, for example, distributed acoustic loads the vicinity of the excitation forces needs to be known, since the force identification procedure will determine the pseudo- or equivalent forces that will result in the same response of the structure, but with quite a different spatial distribution than the actual applied forces.

1.3 LITERATURE SURVEY

Three commonly used 'domains' are employed in the indirect force identification process, i.e. the frequency, time and modal domains. The frequency domain models utilise the frequency response function, which gives the linear relationship between the measured response and the excitation force. As for the time domain models, these methods produce force estimates as a function of time. The modal models are defined as a set of natural frequencies with corresponding mode shapes and modal damping factors and can be used for either frequency or time domain models.

Stevens (1987) has given an overview of the general problems involved in the force identification process, while Dobson and Rider, (1990) reviewed some of the different techniques and applications reported in the literature.

1.3.1 Frequency Response Function Method

The frequency response function can be defined as the linear relationship between the measured response and the excitation forces and gives rise to a set of linear equations normally formulated in the frequency domain as

$$\{X(\omega)\}=[H(\omega)]\{F(\omega)\} \tag{1.3}$$

where

$\{X(\omega)\}$ is the $(n\times 1)$ response vector,
$[H(\omega)]$ is the $(n\times n)$ frequency response function matrix, and
$\{F(\omega)\}$ is the $(n\times 1)$ force vector.

The frequency response function matrix can be measured experimentally, reconstructed from an experimental modal analysis, or obtained from a finite element model.

The unknown forces can be reconstituted by taking the inverse of equation (1.3) as follows

$$\{F(\omega)\}=[H(\omega)]^{-1}\{X(\omega)\} \tag{1.4}$$

However, the frequency response function matrix proves to be singular and ill-conditioned at frequencies close to and at resonance (Desanghere, 1983).

In an attempt to improve the condition of the inverse problem, the formulation of an over-determined problem is proposed, which allows the use of more equations than unknowns. The advantage of using redundant information minimises the consequences of measurement errors (Hillary, 1983). If the number of responses, n, exceeds the number of force estimates, m, the frequency response function becomes rectangular. Adopting a least-squares solution of the force estimates are given by:

$$\{F(\omega)\}=[H(\omega)]^{+}\{X(\omega)\} \tag{1.5}$$

where

$[H(\omega)]^{+}$ is known as the pseudo-inverse of the $(n\times m)$ rectangular matrix $[H(\omega)]$.

The added effect of noise, as encountered in experimental measurements, will further degrade the inversion process. Hillary (1983) and Okubo *et al.* (1985) both investigated the influence of noise contaminating the response and frequency response functions on the accuracy of the force identification. The results show that noise in the frequency response function, and more specifically in the modal matrix, lead to gross errors in the force estimates. The noise contaminating the frequency response function matrix also reduces the number of significant figures in the matrix and, consequently reduces the rank of the matrix (Fregolent and Sestieri, 1990). This in turn reduces the number of forces that can be correctly estimated (Fabunmi, 1986). The quality of the frequency response function matrix can be improved by measuring the frequency response functions by means of a shaker excitation, rather than with an impact hammer (Hendricx, 1994). Even when taking great care hitting the test object in a perpendicular way, the impact hammer may exert in-plane forces. This in turn results in unreliable pseudo-driving point frequency response functions, which are detrimental to force estimates.

Mas *et al.* (1994) published an excellent article in which other causes for unreliable force estimates are explored, other than the poor conditioning of the frequency response function matrix usually associated with indirect force identification. It is shown that the error propagation in the inversion process is proportional to the condition number of the frequency response function matrix to be inverted. Over-determination can improve the condition number and as a result reduce the errors of the force estimates. The damping in the system can have an influence on the force identification, since the condition number varies with damping. Starkey and Merrill (1989) have suggested that the condition number of the frequency response function matrix should be used as an 'indication' of the expected accuracy of indirectly measured force amplitudes at a given frequency.

Hillary and Ewins (1984) have employed both accelerometers and strain gauges to determine two simultaneous sinusoidal forces on a uniform cantilever beam. The strain responses gave more accurate force estimates than the accelerations. This is because the strain responses are influenced more by the higher modes at low frequencies, and therefore the frequency response functions are more complex in shape and hence obtain better force predictions. Han and Wicks (1990) also studied the application of displacement, slope and strain measurements. From both these studies it is evident that proper selection of the measurement type can improve the condition of the frequency response function matrix.

Another paper by O'Callahan and Piergentili (1996) have noted that regardless of the number of response locations chosen, the force prediction result is excellent if the response locations coincide with actual force locations. For the case where the actual force locations are excluded from the response locations the forces are distributed to all response locations in the set. The amount of distribution to each of the surrounding response locations depends on the direction and distance of that particular response from the actual force location. Again it is emphasised that the response locations should be concentrated within the vicinity of the force locations. The analysis was conducted on a finite element model of a plate and as a result did not include experimental verification or the effect of measurement noise.

Following the argument of Fabunmi, it is not possible to determine more than a single force for a lightly-damped structure in the vicinity of the resonant frequencies. This may prove to be highly undesirable, since the energy flows are usually a maximum in these regions. Lewit (1993) suggested the calculation of an equivalent force or forces, which will result in the same vibration output as the original force inputs. The total input power into the structure can then be calculated from the equivalent forces at the resonant frequencies.

Different matrix decomposition methods are used when dealing with ill-conditioned problems. Some of these include:

- The Moore-Penrose pseudo-inverse (Brandon, 1988; Hillary, 1983),
- QR decomposition (Fregolent and Sestieri, 1990),
- Singular Value Decomposition (Maia, 1991; Brandon, 1988),
- Second Order Epsilon Decomposition (Ojalvo and Zhang, 1993) and many more.

Singular Value Decomposition (SVD) of the frequency response function matrix can be used to improve the conditioning of the pseudo-inverse matrix. Powell and Seering (1984) calculated a threshold value from the coherence function corresponding to the measured frequency response functions. The singular values smaller than the threshold were truncated from the pseudo-inversion. Although the truncation reduced the resolution of the inputs, it prevented the prediction of large spurious forces.

Numerous regularisation methods are also available, which give a stable approximate solution to an ill-conditioned problem, e.g. Tikhonov regularisation (Sarkar *et al.*, 1981; Hashemi and Hammond, 1996).

Most of the successful applications that deal with the problem of multiple excitation forces have been for the least-squares method. These include work done by Flannely and Bartlett (1979), which produced results that can be compared to directly-measured harmonic forces acting on the hub of a laboratory model of a helicopter. The forces were accurately determined for combinations of two orthogonal forces from measurements of fourteen responses at three different frequencies.

In a similar case, Okubo *et al.* (1985) applied the least-squares technique to a number of applications. These include the identification of the cutting forces of a machine tool, the generated forces on mounts of an automobile engine and the transmitted forces to the piping system of an air conditioner. The frequency response functions were measured in advance at a stationary state, while the responses of the structure were measured under operating conditions. Although the frequency response functions often admittedly differ from those at operating conditions, this approach is more desirable since the frequency response function is usually very noisy under operating conditions.

When dealing with random forces, equation (1.3) can be defined in terms of the spectral density functions as

$$[G_{xx}(\omega)]=[H(\omega)][G_{ff}(\omega)][H(\omega)]^{*} \tag{1.6}$$

where

$[G_{xx}(\omega)]$ is the $(n\times n)$ response matrix,

$[G_{ff}(\omega)]$ is the $(m\times m)$ spectral matrix of the forces,

$[H(\omega)]$ is the $(n\times m)$ rectangular frequency response function matrix, and $[\cdot]^{*}$ denotes the complex conjugate transpose of $[H(\omega)]$.

The pseudo-inverse of equation (1.6) results in

$$[G_{ff}(\omega)]=[H(\omega)]^{+}[G_{xx}(\omega)][H(\omega)]^{T+} \tag{1.7}$$

It is generally accepted that in the case of statistically uncorrelated forces, the cross-spectral density terms (off-diagonal terms) become equal to zero and the above equation reduces to:

$$\{G_{ff}(\omega)\}=[|H(\omega)|^{2}]^{+}\{G_{xx}(\omega)\} \tag{1.8}$$

where

$\{G_{ff}(\omega)\}$ and $\{G_{xx}(\omega)\}$ are column vectors of the diagonal terms of the force and response spectral density matrices, respectively.

Conversely, it has been shown that this assumption is inadequate when solving the inverse problem for random forces. The cross-spectral densities of the response should be included with the real valued auto-spectral densities, since the former carries the phase information that in turn, establishes the correlation requirements among time variables (Varoto and McConnell, 1997).

Elliott *et al.* (1988) employed the SVD technique to predict acoustic forces on a thin panel. Force estimates were calculated from a strain frequency response function (SFRF) matrix and the corresponding strain response matrix. The measured strain response matrix was heavily influenced by noise. The measurement noise increased the rank of the strain responses, which in turn circumvented the force predictions. By applying singular value decomposition the noise contaminating the input measured strain can be reduced, but not completely rejected. It is shown that improvement in the force predictions was obtained by truncating singular values so that the rank of the measured strain response matrix resembles its true rank, without the effect of noise.

1.3.2 Modal Coordinate Transformation Method

The modal coordinate transformation method (also referred to as the modal model method) is based on the modal transformation theory. The system is expressed in terms of its modal parameters [i.e. the natural frequencies, modal damping factors and modal (eigen)vectors], which can be obtained from various experimental modal parameter estimation methods, widely used by the modal analysis community. The orthogonality criterion of the mass-normalised modal vectors is used to establish the transformation basis, and can be expressed as:

$$[\Phi]^T[M][\Phi]=[I] \quad \text{and} \quad [\Phi]^T[K][\Phi]=[\Lambda]$$
$$[\Phi]^T[C][\Phi]=[\beta] \qquad (1.9)$$

where

$[M]$, $[K]$ and $[C]$ are the mass, stiffness and damping matrices, respectively;

$[\Phi]$ is the modal matrix;

$[I]$ is the identity matrix;

$[\Lambda]$ is the diagonal modal stiffness matrix with $\Lambda_r = \omega_r^2$, and ω_r the natural circular frequency of the r-th mode;

Assuming some type of proportional damping it follows that $[\beta]$ is the diagonal modal damping matrix, and $\beta_r = 2\zeta_r \omega_r$ with ζ_r the modal damping factor of the r-th mode.

Utilising the orthogonality criterion results in a set of uncoupled equations of motions and the response can be written in the frequency domain as follows:

$$\{X(\omega)\} = [\Phi]\left[-\omega^2[I] + i\omega[\beta] + [\Lambda]\right]^{-1}[\Phi]^T\{F(\omega)\} \tag{1.10}$$

The physical forces acting on the system are determined by an inverse coordinate transformation:

$$\{F(\omega)\} = \left[\Phi^T\right]^+\left[-\omega^2[I] + i\omega[\beta] + [\Lambda]\right][\Phi]^+\{X(\omega)\} \tag{1.11}$$

In the equation above, the forces are computed by transforming the operating response to the modal response. The modal forces are determined and then transformed back to the forces acting on physical coordinates of the system by an inverse coordinate transformation (Desanghere and Snoeys, 1985).

The modal coordinate transformation technique might just as well be implemented in the time domain (Genaro and Rade, 1998).

$$\{f(t)\} = \left[\Phi^T\right]^+\left([I]\{\ddot{p}(t)\} + [\beta]\{\dot{p}(t)\} + [\Lambda]\{p(t)\}\right) \tag{1.12a}$$

with

$$\{\ddot{p}(t)\} = [\Phi]^+\{\ddot{x}(t)\} \qquad \{\dot{p}(t)\} = [\Phi]^+\{\dot{x}(t)\}$$

$$\{p(t)\} = [\Phi]^+\{x(t)\} \tag{1.12b}$$

where

$\{\ddot{p}(t)\}$, $\{\dot{p}(t)\}$ and $\{p(t)\}$ are the modal (generalised) acceleration, velocity and displacement vectors.

The frequency response function method previously considered has two major drawbacks (Desanghere and Snoeys, 1985):

- The first is the ill-conditioned behaviour of the force estimates near and at the system's resonances.
- Secondly, the frequency response function matrix needs to be inverted at each frequency line and thus, increase the computational time required.

Both these limitations may be avoided by using the modal coordinate transformation method. The singularity problems are eliminated and the modal matrix needs to be inverted only twice.

Kim and Kim (1997) studied the effect of error propagation in the modal parameters on the force predictions. They concluded that, as was the case with the frequency response function technique, errors in the modal vectors are considered as the main source of error in the identified forces. A methodology is also proposed to recalculate the force estimates for inaccessible input locations.

Despite the advantages associated with the use of the modal coordinate transformation technique, Okubo *et al.* (1985) preferred to use the frequency response function technique, since the former requires the extraction of the modal parameters from the measured frequency response function. They argued that the modal parameters might be in error as a result of difficulties experienced by curve fitting algorithms, especially at resonances and anti-resonances. These modal parameters may in turn be detrimental to force identification. Recent advances in modal parameter extraction methods enables one to extract the modal parameters with greater accuracy, which makes the modal coordinate transformation technique more attractive.

Conversely, Desanghere and Snoeys (1985) found the method rather insensitive to measurement and curve-fitting perturbations. They successfully applied the modal coordinate transformation technique to identify the forces in a turbo compressor and a car-frame.

Hansen and Strakey (1990) extended the work of Starkey and Merrill (1989) by considering the condition number of this method. The findings revealed that the condition number, of the pseudo-inversion of the modal matrix can be ameliorated through proper selection of the sensor locations and the modes included in the analysis.

Most of the work done on modal coordinate transformation has been for the case where the locations of the input forces were known. The objective was then simply to resolve the amplitude and frequency content of these forces. Shih *et al.* (1989) proposed a method based on the modal coordinate transformation technique, where the number of forces, as well as the locations, is treated as unknowns. The modal response transformation (equation 1.10) is performed through a modal filter, calculated from frequency response function measurements and the modal parameters, rather than the pseudo-inverse of the modal matrix. The rank analysis of the modal force matrix (equation 1.11) can be evaluated to determine the number of incoherent force inputs to the structure. Once the number of excitation forces is known, their locations can be determined by various projection methods.

Genaro and Rade (1998) applied this technique on a simple numerically simulated structure and successfully identified harmonic and impact forces in the time domain. Another application consisted of a longitudinal beam of a car-frame where three random forces were accurately inferred (Desanghere and Snoeys, 1985). This method has also been demonstrated on a circular plate (Shih *et al.*, 1989; Zhang *et al.*, 1990).

Though the proposed method shows some success, the number of publications are limited. Accordingly it seems that further research is needed to clarify some of the difficulties to make the method more suitable for real-world applications.

1.3.3 Time Domain Methods

Since the focus of this work is primarily concerned with the assessment of force identification methods in the frequency domain, only some of the recent advances in the time domain methods will be summarised below. The time domain methods have the ability of exploring the transient behaviour of impulsive loads.

a) Sum of Weighted Accelerations Technique (SWAT)

This time domain method is the most widely known process in indirect force identification. As the name states, this method uses a sum of the weighted acceleration signals to experimentally predict the external forces, which excite the system.

$$f(t)=\sum_{i=1}^{n_a}\left(M_i \times A_i(t)\right) \tag{1.13}$$

where

$f(t)$ is the externally applied forces,

M_i is the i-th equivalent mass,

$A_i(t)$ is the i-th measured acceleration, and

n_a is the number of acceleration measurements.

The implementation of this method is confined to the computation of the sum of the external forces and moments about the center of mass of structures presenting free boundary conditions. The Rigid Body Modes (RBM) are explored in the preliminary tests to determine the optimal distribution of the weighting factors associated with an equivalent mass at each of the sensor locations. The weighting factors can be determined either from inverting the modal matrix or from the free-decay response of the structure (Carne *et al.*, 1992). Recently, the Max-Flat procedure was validated as an alternative for determining the weighting factors from frequency response functions and avoids possible errors resulting from mode shape estimation (Carne *et al.*, 1998). SWAT yielded excellent results in experiments conducted by Gregory *et al.* (1986) on a mass-loaded, free-free beam.

Kreitinger and Wang (1988) successfully applied this method to structures that exhibit non-linear behaviour. Other applications comprise of the impact force identification on nuclear shipping casks and energy absorbing noses (Bateman *et al*, 1991 and Bateman *et al.*, 1992).

b) Inverse William's Method

Ory *et al.* (1985 and 1986) analysed the reconstruction of transient loads from measured response-time histories on a beam. The use of an 'Inverse William's Method' improved the reconstructed force estimates. In the William's method, the response consists of a quasi-static component, which is superimposed on the dynamic component. The forcing functions were computed with a time-integration scheme. Providing that the stiffness matrix is known with sufficient accuracy, this matrix is combined with the measured displacements to produce the quasi-static forces. By extracting the quasi-static forces, the dynamic forces, which are the pure inertial forces pertaining to the significant modes, were reconstructed.

c) Central Difference Method

Among the applications is the work of Dubowski and Dobson (1985) where a central difference approach was applied to a cantilever structure suffering an impact load. The method yielded acceptable predictions of the excitation forces. However, in the post-shock period the method proved unstable as the force predictions continued to oscillate and diverged.

d) Time Domain Deconvolution Method

The convolution integral equation, which states the time domain relationship between the response of the structure and the applied forces, is given by Da Silva and Rade (1999) as:

$$\{x(t)\} = \int_0^t [h(t-\tau)]\{f(t)\}d\tau \tag{1.14}$$

where

$\{x(t)\}$ is the $(n \times 1)$ time response vector,

$\{f(t)\}$ is the $(m \times 1)$ time force vector, and

$[h(t)]$ is the corresponding $(n \times m)$ Impulse Response Function (IRF) matrix.

Deconvolution of equation (1.14) produces an estimate of the force inputs from the vibration response. Unfortunately, this procedure is known to be ill-conditioned and requires the implementation of regularisation schemes to stabilise computations. The problem can be regularised by employing Tikhonov regularisation (Fasana and Piombo, 1996), calculation of the inverse Markov parameters (Kammer, 1996) or application of the conjugate gradient method (Da Silva and Rade, 1999).

1.3.4 Continuous Systems

In the case of continuous systems one needs to solve partial differential equations from which solutions are available for every point on the structure. However, this limits the applications to relatively simple structures with well-defined boundary conditions. Some of the applications include a Timoshenko transformation technique, which was used to derive remote impact force-time histories from accelerations measured at remote locations (Whitson, 1984 and Jordan and Whitson, 1984). In another case, the impulse response functions for Euler-Bernouli and Timoshenko beams, subjected to transverse impact forces were also investigated (Park and Park, 1994). The impulse response functions, which state the relationship between the force and strain, were obtained by using the wave propagation approach in the time domain.

1.4 OUTLINE AND SCOPE OF THIS WORK

The force identification in this work will be performed in the frequency domain. The advantages of using the frequency domain and the theory relating to the frequency response function's formulation, measurement and modal parameter extraction are presented in Chapter 2.

The work presented in this dissertation can be roughly divided into three sections:

- Formulation of the frequency response function method as applied to the force identification process,
- Formulation of the modal coordinate transformation method, and
- An experimental study performed on a beam with different boundary conditions to assess the performance of the above mentioned methods.

Each section is dealt with in a separate chapter.

Chapter 3 presents the derivation of the frequency response function method. The limitations regarding the use of this method is highlighted as well as presenting some of the regularisation methods in dealing with the inverse problem. The results of a numerical study of a two degree-of-freedom system and Finite Element Analysis (FEA) of a beam are presented. The significance of the condition number on the force estimates is discussed.

Chapter 4 is entirely devoted to the modal coordinate transformation method. The numerical studies investigate the application of the method on a two degree-of-freedom system and the factors influencing the condition number of the modal matrix. This chapter also describes applying the modal filter to force reconstruction, which is validated by a numerical simulation.

The ultimate objective of this research is to implement these methods in an experimental investigation on a simple well-behaved structure, given the lack of experimental work pertaining to especially the modal coordinate transformation method. The aim is to determine a single harmonic force on an aluminium beam subjected to different boundary conditions. The work is then extended to predict two point sinusoidal forces from measured acceleration signals. Strain measurements have also been employed and the results noted.

In the work presented we will only focus on harmonic force inputs applied at discrete locations while reference is made to random forces, distributed loading and unknown forcing locations.

Assumptions:

- The frequency response functions measured at stationary state are the same as those at operating conditions.
- Discrete force inputs at known locations.
- An existing structure or representative scale model is already available for the acquisition of the frequency response functions and response measurement.