Chapter 11

Appendices

Appendix A

Systematic Approach for SFRC Mixes Proportioning

The method was developed by Hanna [3] in 1977. It basically suggests a mechanistic approach to adjust SFRC mixes made of pre-designed plain concrete mixes and steel fibers. It takes into account the fact that both mix parameters and fibers parameters are affecting the final product of the SFRC

The amount of paste required to coat the fibers was determined experimentally. Cement mixes prepared with different water-cement ratios were added to pre-weighed amounts of fibers and vibrated to produce a uniform mix. The fiber -paste mix was then placed on a No.4 sieve, shaked by hand to drain the excess paste, and weighed. The amount of the paste coating the fibers was determined by subtracting the weigh of the sieve and the uncoated steel fibers from the total weigh. Test results showed that the relationship between the weight of steel fibers and the required paste could be expressed as given in equation A-1.

After adjusting the plain concrete mix for specific compressive strength using plain concrete systematic mix proportioning methods, the adjustment of water and cement for the SFRC could possibly be calculated by using equations A-2 to A-4.

For straight steel fibers (0.25x0.56x0.254 mm), the values of A and B were obtained experimentally as described before. Test results provided values of A = 3.0 and B = -1.4. Although the straight steel fibers are no longer in use, the values of A and B above can give an indication of their expected values when applying the same approach to the hook-ended steel fibers.

This method does however not go further in determining the flexural strength after the addition of the steel fiber (which is normally the aim of the addition of the steel fiber). Moreover, the extra cement for the steel fiber coating is expected to increase the compressive strength of the final product, which is not considered by the method. The author's view is that, the method can be used successfully to optimize the aggregate blend and minimize the cement content while assuring adequate paste content, which is typically higher for SFRC. Flexural strength could be predicted for the specific type of steel fibers by doing trail mixes. The steel fiber suppliers formulas and tables in appendix B could be used successfully to give an indication prior preparing trial mixes.

$$Log\left(\frac{w_f}{w_p}\right) = A\frac{w}{c} + B \qquad \Rightarrow \text{Eq. } A-1$$

$$w_{cf} = w_{co}\left[1 - 0.01p - \frac{pC\left(0.318 + \frac{w}{c}\right)}{10^{\left(A\frac{w}{c}\right)}\left(1 + \frac{w}{c}\right)}\right] + \frac{\lambda_w pc}{10^{\left(A\frac{w}{c}\right)}\left(1 + \frac{w}{c}\right)} \qquad \Rightarrow \text{Eq. A - 2}$$

$$w_{af} = w_{ao}\left[1 - 0.01p - \frac{pC\left(0.318 + \frac{w}{c}\right)}{10^{\left(A\frac{w}{c}\right)}\left(1 + \frac{w}{c}\right)}\right] \qquad \Rightarrow \text{Eq. A - 3}$$

$$C = \frac{\gamma_f}{\gamma_w 10^B}$$

$$w_{wf} = \frac{w}{c} w_{cf} \qquad \Rightarrow \text{Eq. A - 4}$$

Where :

w_f = weight of steel fibers

w_p = weight of paste (water and cement)

$\frac{w}{c}$ = water - cement ratio by weight

A, B and C = parameters dependent on the characteristics of the fiber material. A and B are obtained from the slope and intercept of the line defined in Eq. A - 1 can be calculated as in Eq. A - 4.

w_{cf} = cement content in SFRC mix

w_{co} = cement content in plain concrete mix

w_{af} = aggregate content in SFRC mix

w_{ao} = aggregate content in plain concrete mix

p = fiber content in volume percentage.

c = parameter dependent on fiber characteristics

γ_f = density of fibers

γ_w = density of water

w_{wf} = water content for SFRC mix

Appendix B

Flexural Strength Ratios:

Steel fibers manufacturer provides formulas and table to estimate the equivalent flexural strength ratios for different fibers parameter and dosage. The following equations were provided for the hook-ended steel fibers of two different manufacturers.

$R_{e,3}$ values in table B-1 are for hook-ended steel fibers generated from third-point loading test according to the JSCE-SF4 method [11].

Table B-1: Equivalent Strength Ratios for Various Steel Fiber Dosages

Dosage (kg/m^3)	RC-8/60-BN	*RC-65/60-BN	*RL-45/50-BN
15	42	38	-
20	52	47	38
25	60	56	45
30	68	63	52
35	75	69	58
40	80	75	63
45	86	80	68
50	90	85	72
55	95	89	77
60	99	93	80
65	102	97	84

*RL for Round section and loose fibers
*RC for Round section Collated fibers

Another estimation for the equivalent flexural strength ratio is given by equation B-1. The equivalent flexural strength ratio and equivalent flexural strength for a mixture with a compressive strength of 32 MPa is given in table B-2. [87].

$$R_{e,3} = \frac{180WLD^{1/4}}{(180C)+\left(WLD^{1/4}\right)} \quad \Rightarrow \text{Eq. B - 1}$$

Where :

D = fiber diameter (mm)

W = fiber dosage (kg/m^3)

L = fiber aspect ratio

C = constant ≈ 16

Table B-2: Equivalent Flexural Strength Characteristics

Steel fiber dosage (kg/m^3)	$f_{e,3}$	$R_{e,3}$
15	2.4	50
20	2.9	61
25	3.3	71
30	3.7	79
35	4.0	85
40	4.3	92

Appendix C

First Crack Determination Technique:

There is no unique definition for the first crack. It may be defined as follows:

- The point at which the load reaches its first maximum point.
- The point from which a series of 20 consecutive data points (over total deformation of 0.01 mm or more) have a slope at least 5% less than the average slope of the load-deflection curve between 45% and 70% of the peak load.
- The point at which the load-deflection curve first become non-linear (ASTM C: 1018).

For the purpose of this research, the last definition is adopted. Differentiation techniques were used to estimate the point of the first crack. Ideal elastic behaviour was not found for all tests conducted and some engineering judgment was used. The following steps were followed:

- Spot readings at 10 KN intervals are considered and used to plot the load-deflection diagram.
- The first portion of the curve (curved portion) is omitted by deducting the deflections due to seating at the start of loading.
- The zone of the load-deflection curve at which the first crack point could lie is initially estimated (the most straight portion).
- The first two readings of the straight portion are used to draw the load-deflection curve. Similar curves are generated by adding one reading each time, and the correlation factor (R^2) was computed.
- The relation between the load and the R^2 is established. The load at which the Load-R^2 relation has its first deviation is deemed to be the first crack point.

As an example, the above steps were followed to determine the first crack point for the corner (at the Plain Concrete slab).

Step 1:

The data as recorded from the closed-loop servo system is plotted as in figure C-1. As it can be seen the data is noisy because of the foam concrete sub base used in the experiment (compressible) and the high recording rate used. Also it is obvious that the very first portion of the curve is not linear (due to seating).

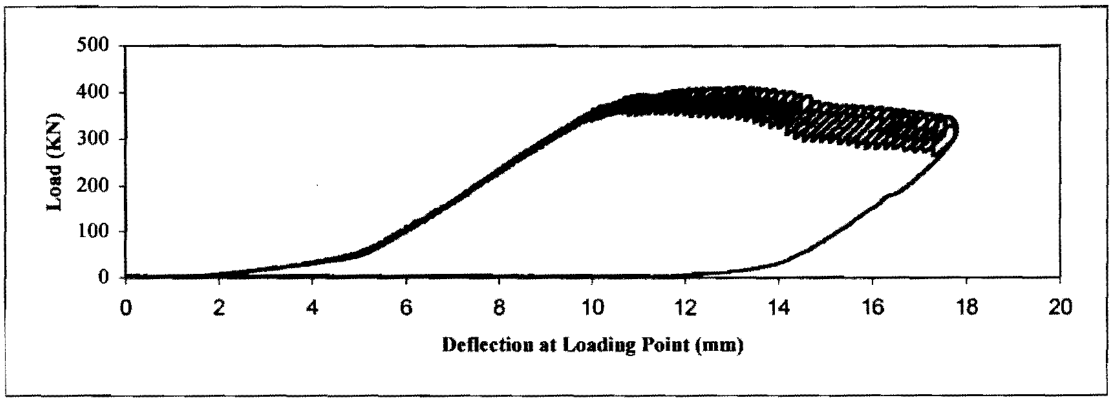

Figure C-1: Load-Deflection Diagram for the Plain Concrete Slab Corner

Step 2: The data is normalized by taking a spot reading at 10 KN intervals and the first portion of the curve is cut off by deducting the seating deflections. Figure C-2 shows the plot of the normalized data.

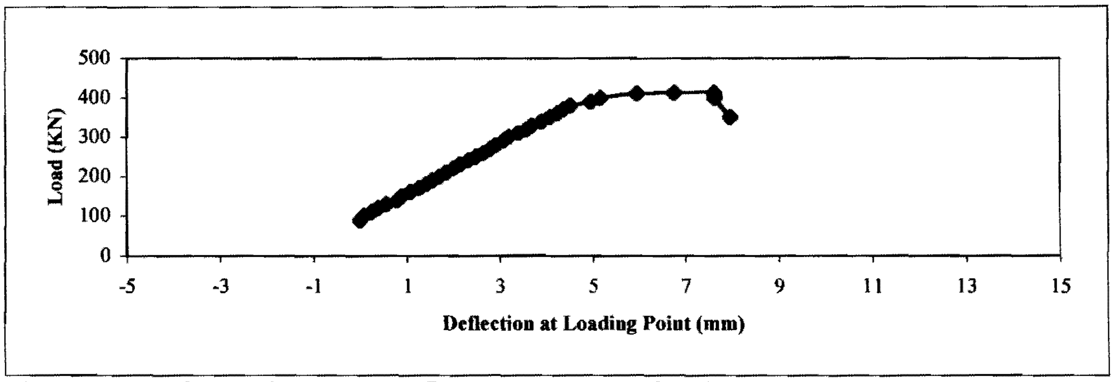

Figure C-2: shows the Load-Deflection Diagram for the Normalized Data

Step 3: The region of the load-deflection curve at which the crack point may occur was estimated as between 170 KN and 230 KN. The readings beyond 170 KN were investigated by adding a portion of these readings and draw the best-fit line. Several trials were conducted by adding readings each trial till the last line is drawn from the full range of readings of which the first crack estimated to fall in. The correlation factor (R^2) is found from each trial in step and a plot of load-R^2 is developed as in figure C-3. The point at which the curve has the first deviation is deemed to be the first crack point. From figure C-3, it is obvious that the first crack strength is approximately 200 KN.

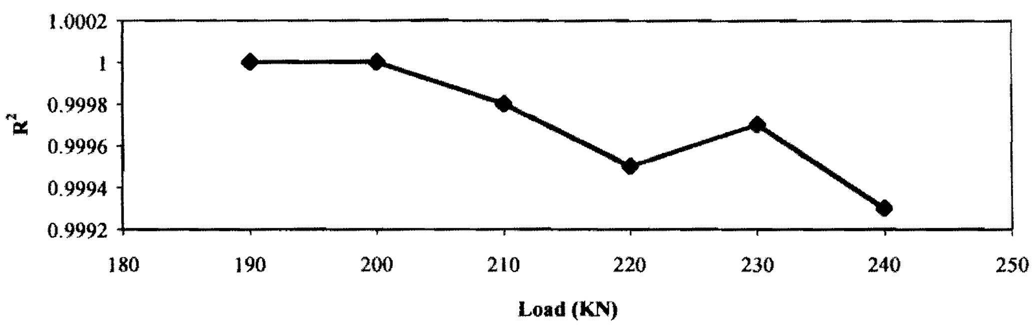

Figure C-3: Load-R^2 Diagram

<u>Step 4:</u> Further investigation on the data between 200 and 210 KN revealed that the first crack point is better estimated at load equal to 202 KN.

Appendix D

Example of Modulus of Elasticity Calculation

The sawn SFRC beam (IS) is considered as an example of the calculations of the modulus of elasticity. The followings are the characters of the beam:

Supported length = 450 mm   Section depth (d) =131 mm

Section width (b) =122 mm   Poisson's ratio (μ) = 0.15

Second moment of inertia (I) = $[122x(131)^3]/12 = 22.85x10^6$ mm^4

Equation D-1 is used to calculate the modulus of elasticity using data from a third-point loading test:

$$E(\text{M.Pa}) = \frac{23}{1296} * \frac{F}{\delta} * \frac{l^3}{I}\left[1 + \frac{216}{115} * \left(\frac{d}{l}\right)^2 (1+\mu)\right] * 10^3 \Rightarrow EqD-1$$

Where:

$\frac{F}{\delta}$ = the slope of the best - fit straight line drawn through the plotted points of the initial portion of the load - deflection curve (N/mm^2).

l = support span, (mm).

I = second moment of area of the section ($\frac{bd^3}{12}$).

b, d = width and depth of the prism section respectively (mm).

μ = poisson's Ratio

Equation D-1 further requires the slope of the straight portion (elasticity zone) of the load-deflection curve. Figure D-1 shows the load-deflection curve for the sawn SFRC beam and figure D-2 shows the best-fit line on the data with in the straight portion to compute the required slope.

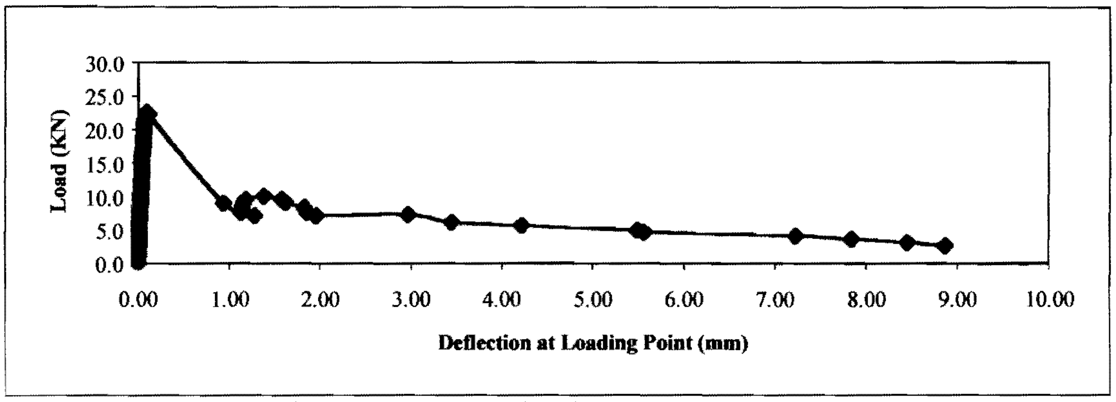

Figure D-1: Load-Deflection Diagram for the Sawn SFRC Beam (IS)

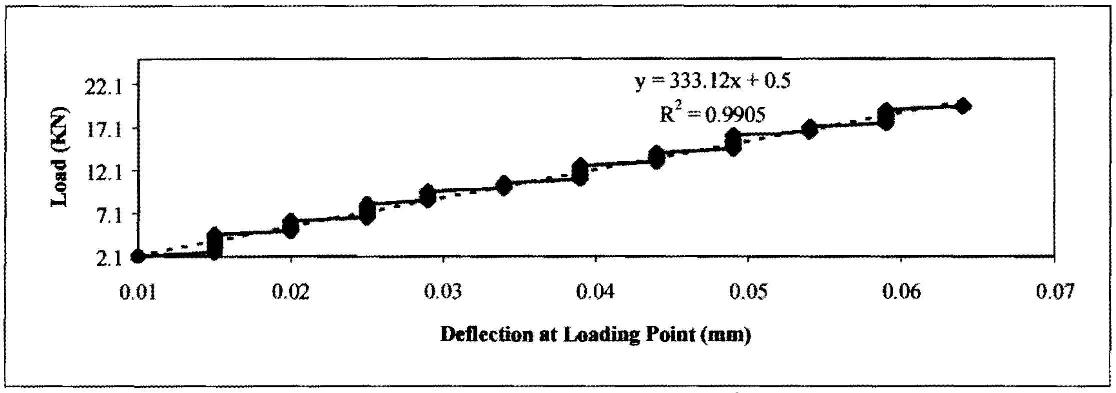

Figure D-2: Slope Determination for the Elastic Zone of Sawn SFRC Beam (IS)

Hence, the value of $(F/\delta) = 333\text{x}10^3$ N/mm.

Substituting in equation 2-5:

<u>$E = 27.8\text{x}10^3$ MPa</u>

Appendix E

Example of Theoretical Analysis Calculation

Interior Load

SFRC Slab :

Radius of relative stiffness $l = \left[\frac{27.5*10^3*(125)^3}{12\left(1-(0.15)^2\right)*0.3}\right]^{0.25} = 351.5\text{ mm} \rightarrow K = 0.3$

$= 367.9\text{ mm} \rightarrow K = 0.25$

Equivalent Radius of contact area $a = \left(\frac{100*100}{\Pi}\right)^{0.5} = 56.41\text{ mm}$

$b = \left[1.6*(56.41)^2 + (125)^2\right]^{0.5} - 0.675*125 = 59.56\text{ mm}$

Westergaard :

$$P_{FirstCrack} = \frac{4.8*(125)^2}{0.275(1+0.15)*\log_{10}\left(0.36*27500*(125)^3/0.25*(59.56)^4\right)} = 62.4\text{ KN}$$

Meyerhof :

$$M_0 = \frac{6.8*1000*(125)^2}{6}*10^{-6} = 17.71\text{ KN.m/m width}$$

$$P_{Ultimate} = 6*17.7*\left(1+\frac{2*56.41}{367.9}\right) = 138.8\text{ KN}$$

Falkner :

$$P_{Ultimate} = 62.4*\left[1+\left(\frac{0.25}{27500*(125)^3}\right)^{0.25}*3000*\frac{100}{125}\right]\left[1+\frac{1.9}{4.8}\right] = 395.4\text{ KN}$$

Shentu :

$$P_{ultimate} = 1.72*2.2*(125)^2\left[\left(\frac{0.25*56.41}{27500}\right)*10^4 + 3.6\right] = 516.1\text{ KN}$$

Plain Concrete Slab :

Radius of Relative stiffness $l = \left[\frac{24*10^3*(150)^3}{12\left(1-(0.15)^2\right)*0.3} \right]^{0.25} = 389.5\ \text{mm} \rightarrow K = 0.3$

$= 407.4\text{mm} \rightarrow K = 0.25$

$b = \left[1.6*(56.41)^2 + (150)^2\right]^{0.5} - 0.675*150 = 64.9\ \text{mm}$

Westergaard :

$$P_{FirstCrack} = \frac{4.1*(150)^2}{0.275*(1+0.15)*\log_{10}\left(0.36*27500*(150)^3/0.25*(64.9)^4\right)} = 76.4\ \text{KN}$$

Meyerhof :

$$M_0 = \frac{4.1*1000*(150)^2}{6}10^{-6} = 15.4\ \text{KN.m/m width}$$

$$P_{Ultimate} = 6*15.4*\left(1+\frac{2*56.41}{407.7}\right) = 117.8\ \text{KN}$$

Falkner :

$$P_{Ultimate} = 76.4*\left[1+\left(\frac{0.25}{24000*(150)^3}\right)^{0.25}*3000*\frac{100}{150}\right] = 278.9\ \text{KN}$$

Shentu :

$$P_{Ultimate} = 1.72*2.2*(150)^2\left[\left(\frac{0.25*56.41}{24000}\right)*10^4 + 3.6\right] = 806.8\ \text{KN}$$

Edge and Corner Load

FRC Slab:

Westergaard:

Edge:

$$P_{FirstCrack} = \frac{4.8*(125)^2}{0.529*(1+0.54*0.15)*\log_{10}\left(0.2*27500*(125)^3/0.3*(59.56)^4\right)} = 38\,\text{KN}$$

Corner:

$$P_{FirstCrack} = \frac{4.8*(125)^2}{3*\left[1-1.41\left(12\left(1-(0.15)^2*\frac{0.3*(59.56)^4}{27500*(125)^3}\right)\right)^{0.25}\right]} = 32.4\,\text{KN}$$

Meyerhof:

Edge:

$M_0 = 17.7$ KN.m/m width

$$P_{Ultimate} = 3.5*17.7*\left[1+\frac{3*56.41}{351.5}\right] = 91.8\,\text{KN}$$

Corner:

$M_0 = 17.7$ KN.m/m

$$P_{Ultimate} = 2*17.7*\left[1+\frac{4*56.41}{351.5}\right] = 58.1\,\text{KN}$$

Plain Concrete Slab:

Westergaard:

Edge:

$$P_{FirstCrack} = \frac{4.1*(150)^2}{0.529*(1+0.54*0.15)*\log_{10}\left(0.2*24000*(150)^3/0.3*(64.9)^4\right)} = 46.3\,\text{KN}$$

Corner:

$$P_{FirstCrack} = \frac{4.1+(150)^2}{3*\left[1-1.41\left(12\left(\left(1-(0.15)^2\right)*\frac{0.3*(64.9)^4}{24000*(150)^3}\right)\right)^{0.25}\right]} = 40.2\,\text{KN}$$

Meyerhof:

Edge:

$$P_{Ultimate} = 3.5*15.4*\left[1+\frac{3*56.41}{389.5}\right] = 77.2\,\text{KN}$$

Corner:

$$P_{Ultimate} = 2*15.4*\left[1+\frac{4*56.41}{389.5}\right] = 48.6$$

Elastic Deflection

SFRC Slab :

Westergaard formula was used to calculate slab settlement beneath the loading point :

Interior load :

$$Z = \frac{62.6 * 10^3}{8 * 0.25 * (367.9)^2} = 0.22 \text{ mm.}$$

Edge :

$$Z = \frac{1}{\sqrt{6}}(1 + 0.4 * 0.15)\frac{38 * 10^3}{0.3 * (351.5)^2} = 0.44 \text{ mm}$$

Corner :

$$Z = \left(1.1 - 0.88\frac{212}{351.5}\right) * \frac{32.8 * 10^3}{0.3 * (351.5)^2} = 0.5 \text{ mm}$$

Plain Concrete Slab :

Interior :

$$Z = \frac{76.4 * 10^3}{8 * 0.25 * (407.7)^2} = 0.23 \text{ mm}$$

Edge :

$$Z = 0.433 * \frac{46.3 * 10^3}{0.3 * (407.7)^2} = 0.44 \text{ mm}$$

Corner :

Distance between loading point and corner $a_1 = 212$ mm

$$Z = \left(1.1 - 0.88\frac{212}{407.7}\right) * \frac{40.2 * 10^3}{0.3 * (407.7)^2} = 0.55 \text{ mm}$$

Appendix F

Properties of Concrete Used for the Slabs

Cube and beams specimens were taken from the mix used to cast the SFRC slab and the plain concrete slab. Compressive strength test, third-point loading test were conducted to assess the cube strength, Flexural strength.

Compressive Strength

Table F-1: Cube Compressive Strength for SFRC and Plain Concrete

Property / Age	7 days			28 days		
Specimen No.	C1	C2	C3	C4	C5	C6
SFRC Cubes Strength (MPa)	17.7	18.3	18.2	33.4	35	32.5
P.C. Cubes Strength (MPa)	18.1	16.8	17.3	34.8	31.9	34

7 days strength for SFRC = <u>18.1 MPa</u>.

28 days strength for SFRC = <u>33.6 MPa</u>.

7 days strength for plain concrete = <u>17.4 MPa</u>.

28 days strength for plain concrete = <u>33.6 MPa</u>.

Flexural Strength

SFRC Beams

Table F-2: Results from Third-point Loading Test (SFRC Beams)

Property		Beams	SF. 1	SF.2	SF.3	SF.4	SF.5	SF.6
At First Crack		Load (KN)	36.6	36.0	31.3	33.7	37.1	31.9
		Strength (MPa)	4.9	4.8	4.2	4.5	4.9	4.3
		Deflection (mm)	0.006	0.005	0.004	0.005	0.006	0.001
At Maximum Load		Load (KN)	41.7	41.3	36.4	38.9	42.8	37.8
		Strength (MPa)	5.6	5.5	4.85	5.2	5.7	5.05
		Deflection (mm)	0.009	0.009	0.006	0.006	0.009	0.004
Japanese Standard JSCE-SF4 Properties	Equivalent Load	Pe,3	14.71	16.5	9.9	15.1	14.0	13.5
	Equivalent Strength	f e,3	1.96	2.2	1.32	2.0	1.9	1.8
	Equivalent Strength	Re,3	35	40	27.2	38.5	33.3	35.6

The average first crack strength = 4.6 MPa.

The average MOR = 5.3 MPa.

The average equivalent strength = 1.86 MPa.

The average equivalent strength ratio = 34.9

Figure F-1 to figure F-6 show the load deflection diagrams for the SFRC beam specimens.

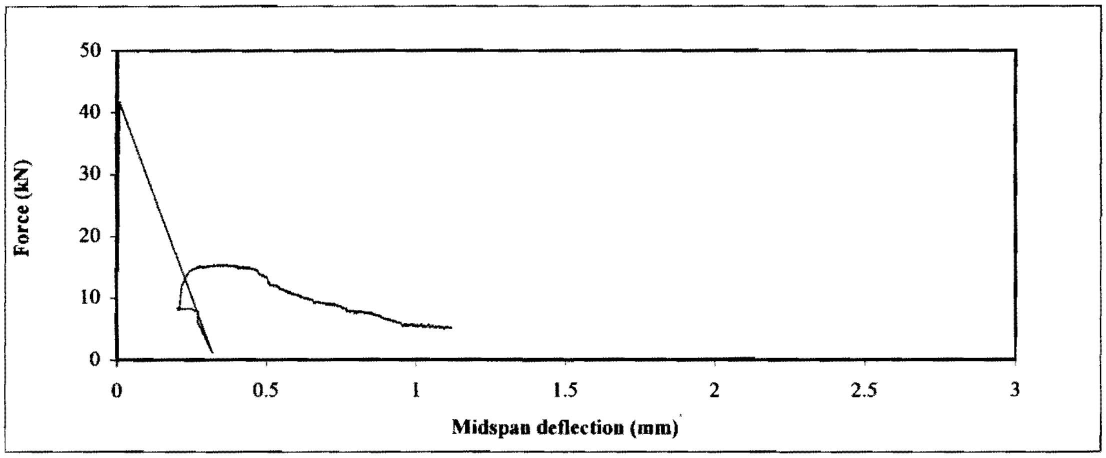

Figure F-1: Third-Point Loading Test: Load-Deflection Diagram (SF.1)

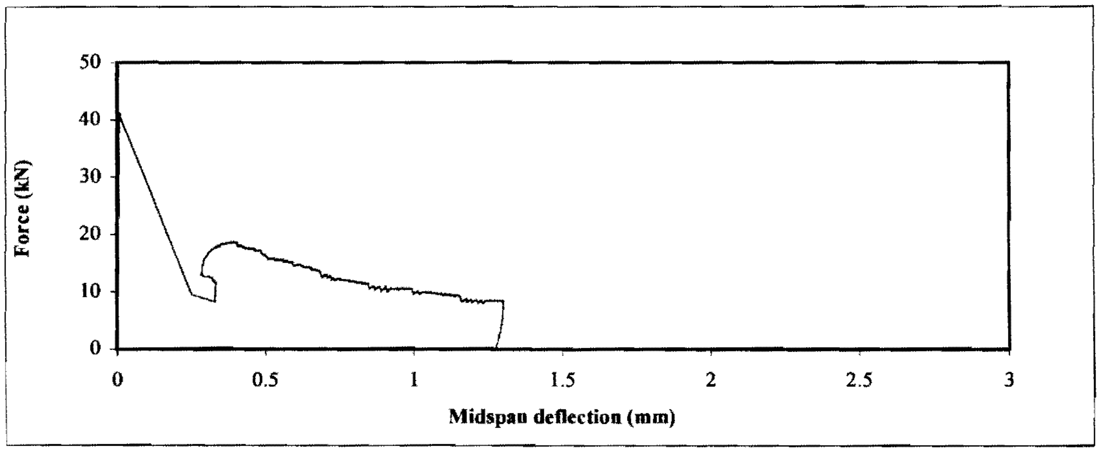

Figure F-2: Third-Point Loading Test: Load-Deflection Diagram (SF.2)

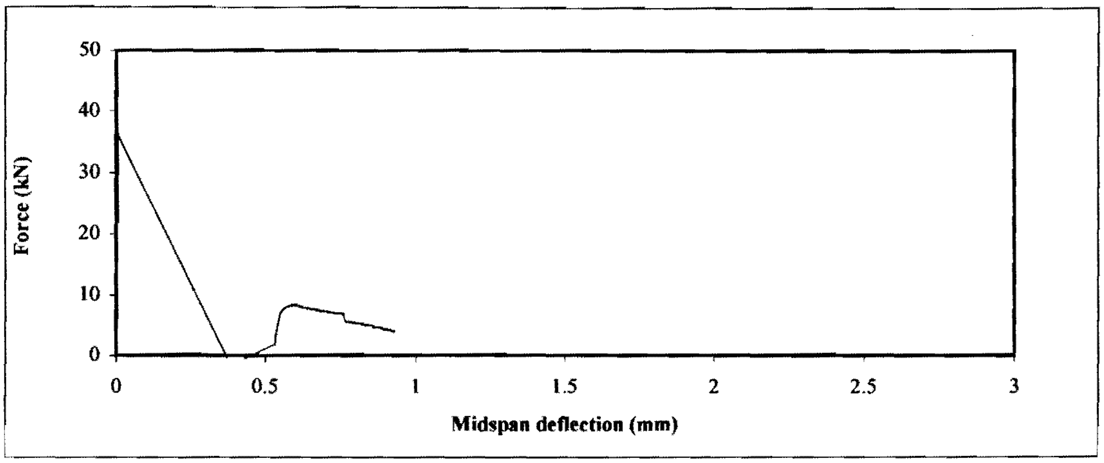

Figure F-3: Third-Point Loading Test: Load-Deflection Diagram (SF.3)

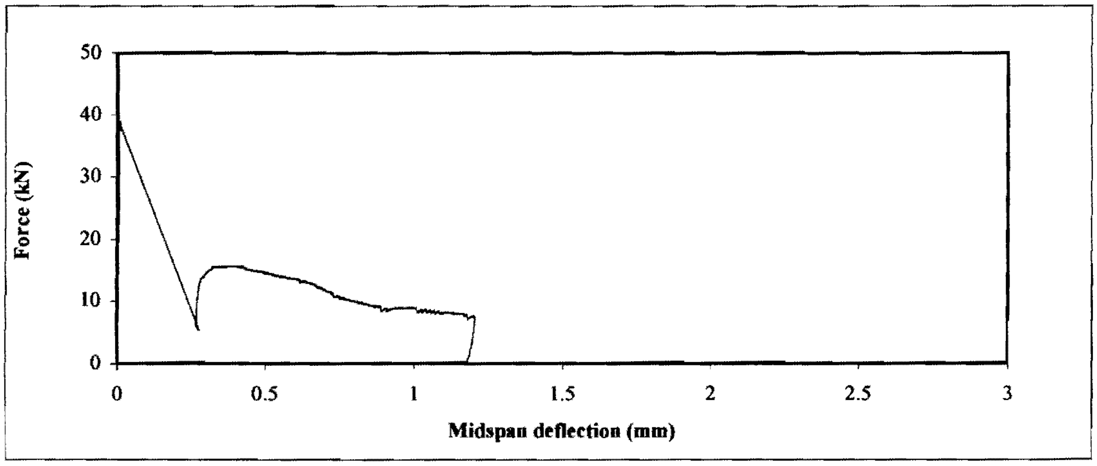

Figure F-4: Third-Point Loading Test: Load-Deflection Diagram (SF.4)

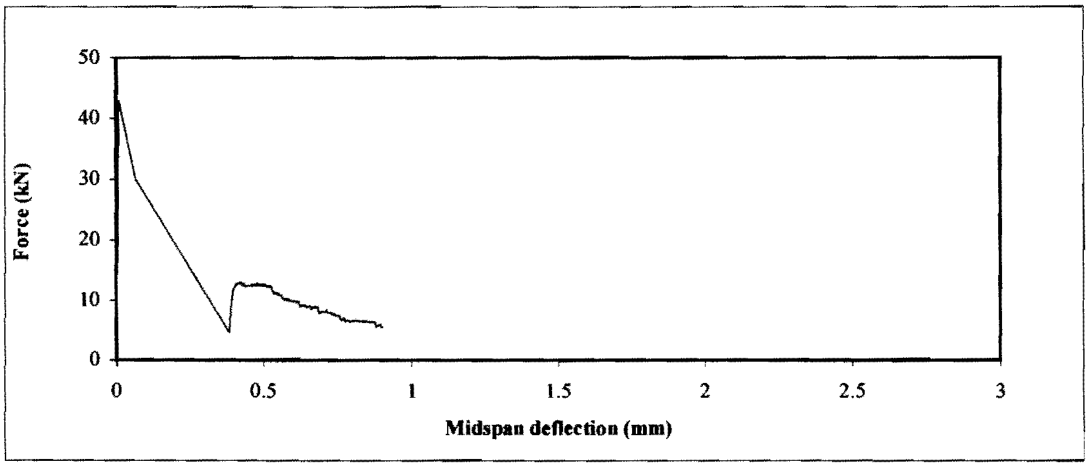

Figure F-5: Third-Point Loading Test: Load-Deflection Diagram (SF.5)

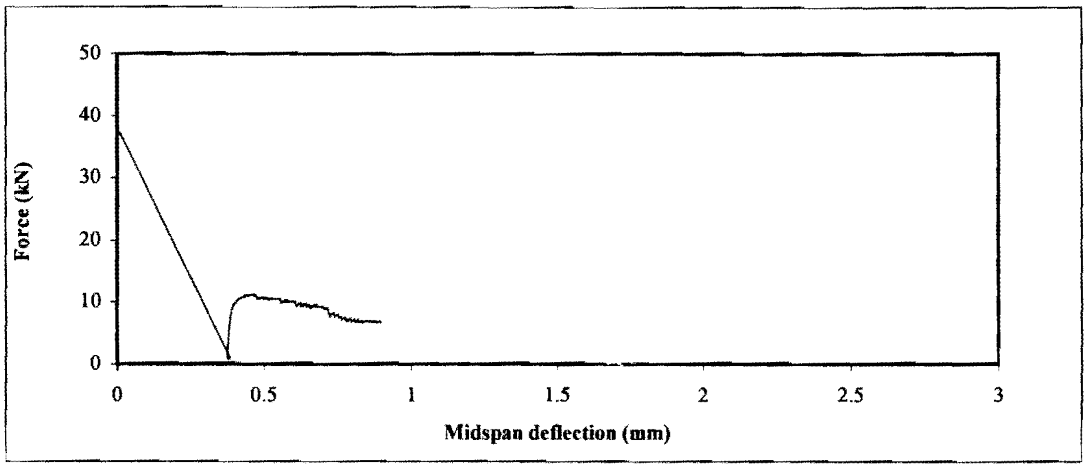

Figure F-6: Third-Point Loading Test: Load- Deflection Diagram (SF.6)

Plain Concrete Beams

Table F-3: Results from Third-Point Loading Test (Plain Concrete Beams)

Property \ Beams		P.C. 1	P.C.2	P.C.3	P.C.4	P.C.5	P.C.6
At First Crack	Load (KN)	29.3	33.0	30.8	34.5	30.0	31.5
	Strength (MPa)	3.9	4.4	4.1	4.6	4.0	4.2
	Deflection (mm)	0.004	0.005	0.004	0.005	0.004	0.004
At Maximum Load	Load (KN)	33.2	42.6	41.6	42.6	37.4	45.0
	Strength (MPa)	4.43	5.68	5.5	5.7	5.0	5.6
	Deflection (mm)	0.006	0.007	0.007	0.007	0.007	0.007

The average first crack strength = 4.2 MPa.

The average MOR = 5.3 MPa.

Figure F-7 to figure F-12 show the load deflection diagrams for the plain concrete specimens.

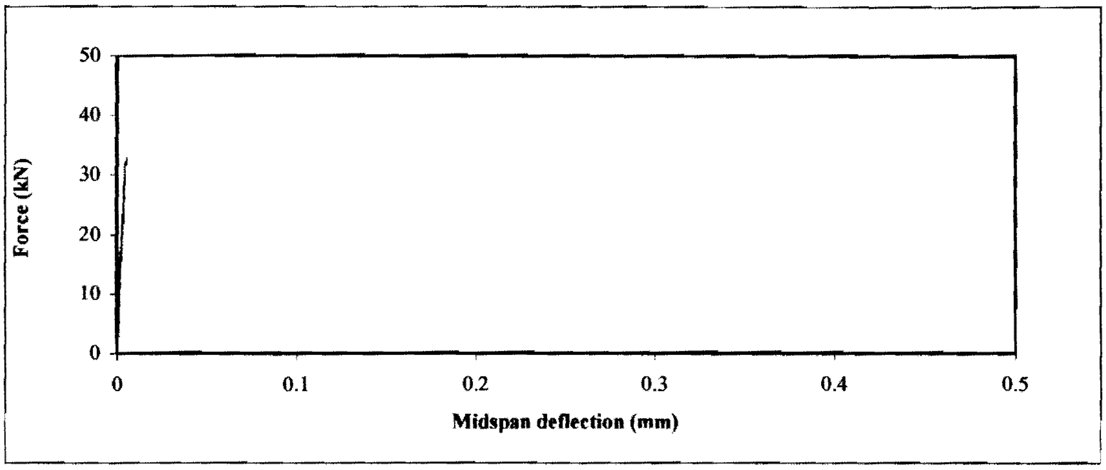

Figure F-7: Third-Point Loading Test: Load-Deflection Diagram (P.C.1)

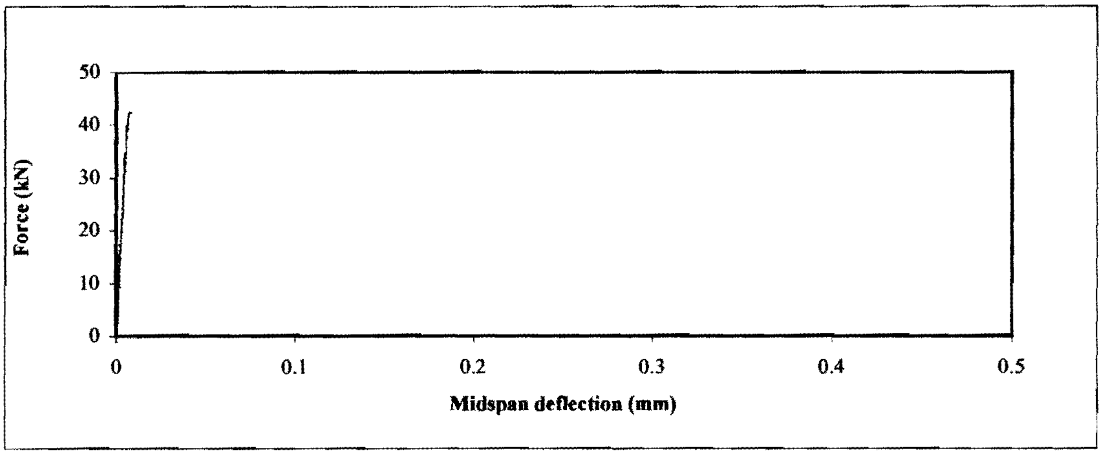

Figure F-8: Third-Point Loading Test: Load-Deflection Diagram (P.C.2)

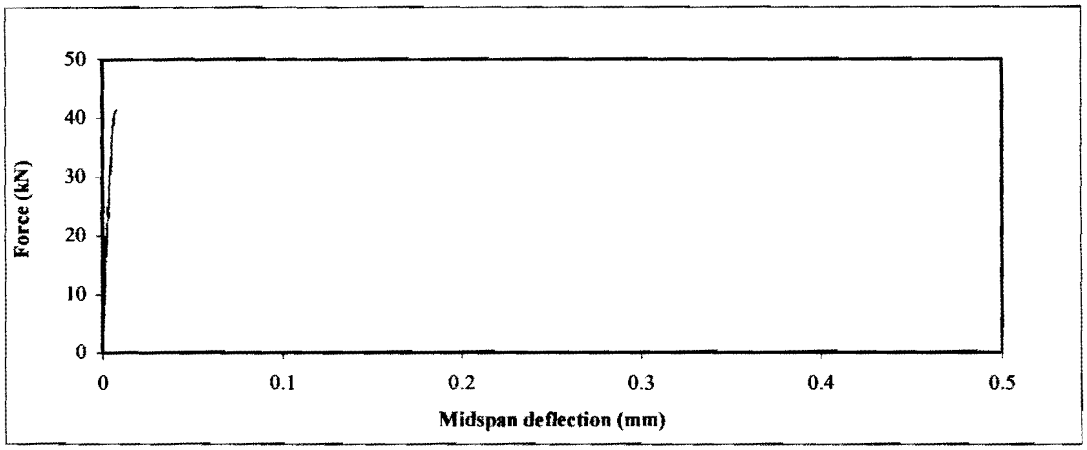

Figure F-9: Third-Point Loading Test: Load-Deflection Diagram (P.C.3)

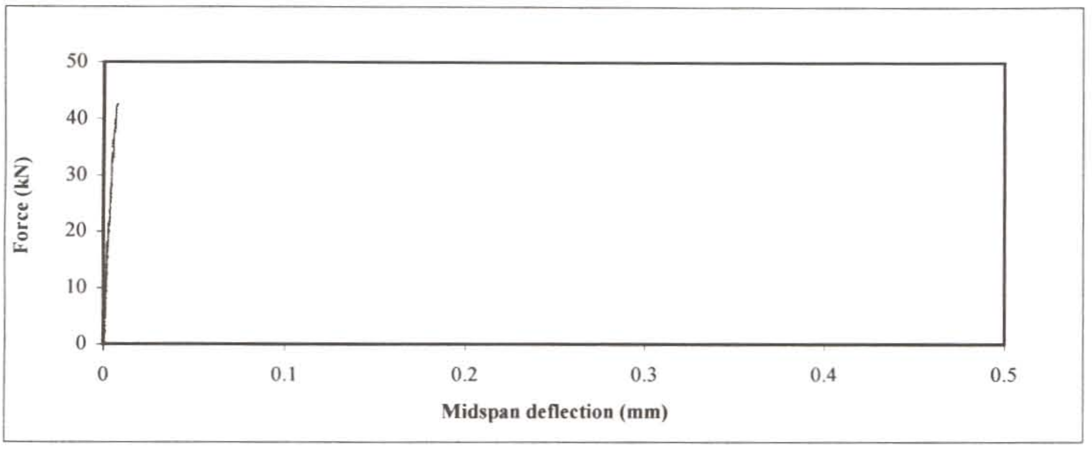

Figure F-10: Third-Point Loading Test: Load-Deflection Diagram (P.C.4)

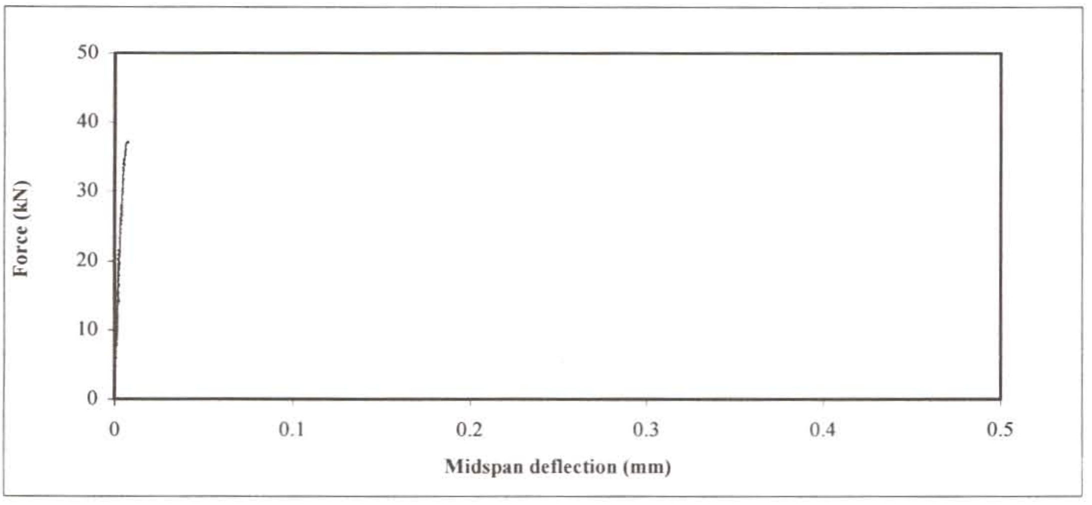

Figure F-11: Third-Point Loading Test: Load-Deflection Diagram (P.C.5)

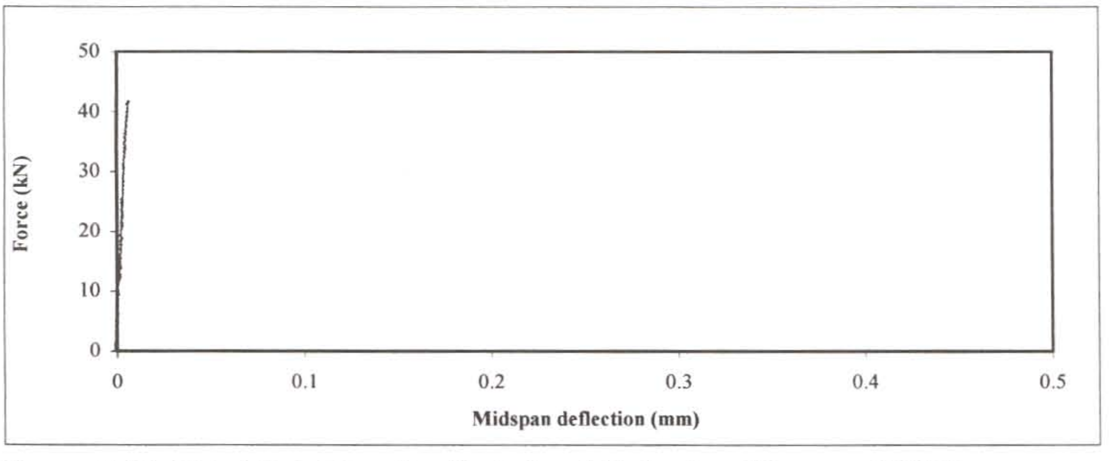

Figure F-12: Third-Point Loading Test: Load-Deflection Diagram (P.C.6)