Structural Breaks and GARCH Models of Stock Return Volatility: The Case of South Africa*

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ABSTRACT

This paper investigates the empirical relevance of structural breaks in forecasting stock return volatility using both in-sample and out-of-sample tests applied to daily returns of the Johannesburg Stock Exchange (JSE) All Share Index from 07/02/1995 to 08/25/2010. We find evidence of structural breaks in the unconditional variance of the stock returns series over the period, with high levels of persistence and variability in the parameter estimates of the GARCH(1,1) model across the sub-samples defined by the structural breaks. This indicates that structural breaks are empirically relevant to stock return volatility in South Africa. However, based on the out-of-sample forecasting exercise, we find that even though there structural breaks in the volatility , there are no statistical gains from using competing models that explicitly accounts for structural breaks, relative to a GARCH(1,1) model with expanding window. This could be because of the fact that the two identified structural breaks occurred in our out-of-sample, and recursive estimation of the GARCH(1,1) model is perhaps sufficient to account for the effect of the breaks on the parameter estimates. Finally, we highlight that, given the point of the breaks, perhaps what seems more important in South Africa, is accounting for leverage effects, especially in terms of long-horizon forecasting of stock return volatility.

Keywords: Stock return volatility, structural breaks, in-sample tests, out-of-sample tests, GARCH Models.

JEL classification: C22, C53, G11, G12

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1. Introduction

When volatility is interpreted as uncertainty, it becomes a key input to many investment decisions and portfolio creations. Given that, investors and portfolio managers have certain bearable levels of risk, proper forecast of the volatility of asset prices over the investment holding period is of paramount importance in assessing investment risk. Further, volatility is the most important variable in the pricing of derivative securities. To price an option, we need to know the volatility of the underlying asset from now until the option expires. So, a volatility forecast and a second prediction on the volatility of volatility over the defined period is needed to price derivative contracts. In addition, financial risk management has taken a dominant role since the first Basle Accord was established in 1996, making volatility forecasting a compulsory risk-management exercise for financial institutions around the world. Banks and trading houses have to set aside reserve capital of at least three times that of value-at-risk (VaR), estimates of which, in turn, are readily available given volatility forecast, mean estimate, and a normal distribution assumption for the changes in total asset value. Even when the normal distribution assumption is violated, volatility is still needed in the simulation process used to produce the VaR figures. Finally, financial market volatility, as witnessed during the recent “Great Recession”, as well as earlier, can have a wide repercussion on the economy as a whole, via its effect on public confidence. Hence, market estimates of volatility can serve as a measure of for the vulnerability of financial markets and the economy, and help policy makers to design appropriate policies.

The impact of structural breaks on the accuracy of volatility forecasts has largely been ignored in previous research. This is because researchers in using the generalised autoregressive conditional heteroscedastic (GARCH) model of Engle (1982) and Bollerslev (1986) often assume (both implicitly and explicitly) the existence of a stable GARCH process in volatility forecasting. As a result most researchers use a fixed or expanding window when estimating GARCH models used to generate out-of-sample volatility forecasts. This affects the accuracy of volatility forecasts using GARCH processes in several ways.

Failure to account for structural breaks in the unconditional variance of stock market returns can lead to sizeable upward biases in the degree of persistence in estimated GARCH models (Andreou and Ghysels, 2002; Mikosh and Starica, 2004; Hillebrand, 2005; building on earlier work by Diebold, 1986; Hendry, 1986; Lamoureux and Lastrapes, 1990). With structural breaks, GARCH models do not accurately track changes in the unconditional variance leading to forecasts that underestimate or overestimate volatility on average for long stretches. This is because the fixed or expanding window mechanism as used under stable GARCH processes does not perform well in the presence of structural breaks (West and Cho, 1995). Again neglecting structural breaks in the unconditional variance may lead to over persistent GARCH models which have adverse effects on volatility forecasts (Rapach and Strauss, 2008; Rapach *et al.*, 2008). Consequently long-horizon forecasts of stock return volatility generated by GARCH(1,1) models that allow for periodic changes in the unconditional variance of stock returns have been found to yield better results than forecasts that assume parameter stability (Stărică and Granger, 2005).

Despite extensive work on volatility forecasting of asset returns, hardly any work is specific to South Africa in terms of forecasting the volatility of stock market returns. The only study we are aware of is by Samouilhan and Shannon (2008), who use a small data set of 682 observations (01/02/2004-28/09/2006) of daily data for the TOP40 index of the Johannesburg stock exchange (JSE). The authors investigated the comparative ability of three types of volatility forecasts namely different autoregressive conditional heteroscedasticity (ARCH) by Engle (1982), and as generalised by Bollerslev (1986) on one hand, a Safex Interbank Volatility Index (SAVI) for the options market, and measures of volatility based purely on historical volatility using a random walk (naive) and 5-day moving average forecasts. Samouilhan and Shannon (2008) found that the GARCH (2, 2) specification provided the best in-sample fit of all the symmetric GARCH models. For their out-of-sample results the GARCH(1,1) specification provided the best forecast of all the symmetric models as compared to GARCH (1, 2), (2, 1) and (2, 2) models.

However, Samouilhan and Shannon (2008) assume the existence of a stable GARCH process in volatility forecasting and do not take into consideration the impact of structural breaks on the accuracy of volatility forecasts. Additionally only one period ahead forecasts, whether using daily data or averaged out daily data to compute weekly data, were used in their paper to ascertain the accuracy of the three different volatility forecasting approaches.

To address these gaps in the South African literature, we investigate the empirical relevance of structural breaks for GARCH(1,1) models of stock return volatility in South Africa using in-sample and out-of-sample tests. We again differ from Samouilhan and Shannon (2008) by using multi-period horizons to ascertain the accuracy of different forecasting approaches as compared to a one period ahead approach by Samouilhan and Shannon (2008). Note, we did not consider GARCH (*p*, *q*) models, because the GARCH(1,1) model is essentially treated as the canonical specification in the literature on asset returns volatility. Further, the GARCH(1,1) specification has been found to be sufficient in practice for such studies, even though the GARCH (*p*, *q*) model might be of theoretical interest (Bollerslev *et al.* 1992). More importantly, in our case, we found the GARCH(1,1) model to fit the data better than the GARCH (2, 2) model, both in- and out-of-sample.[1] Given that, Samouilhan and Shannon (2008) observed the so-called "leverage effect" in the volatility of returns of the TOP40 index using the Glosten *et al.*, (1983)-GARCH (GJR-GARCH) model, we too look into the issue by considering not only the GJR-GARCH(1, 1) model, but also the Markov Switching-GARCH (MS-GARCH) framework (Klassen, 2002; Haas *et al.*, 2004) in terms of forecasting relative to our benchmark GARCH(1,1) model. Note, the so-called leverage effect refers to the situation where negative returns shocks are correlated with larger increases in volatility than positive returns shocks. The rest of the paper is organised as follows; section 2 details the econometric methodology, section 3 the empirical results for the in-sample and out-of-sample tests. Section 4 concludes.

2. Econometric Methodology

2.1 In-Sample Tests

For the in-sample tests we employ a modified version of the iterated cumulative sum of squares algorithm (Inclán and Tiao, 1994) to test for the possibility of structural breaks in the unconditional variance of the daily Johannesburg Stock Exchange (JSE) all share index from 1995 to 2010.

Let Z_t = 100[ln(R_t) −ln(R_{t-1})], be the returns on a stock index from time t-1 to period *t*. R_t denotes the value of the stock index at time *t* and $z_t = Z_t - \mu$, where μ is the constant (conditional and unconditional) mean of Z_t. Supposing z_t can be observed for *t* = 1… *T*, the cumulative sum of squares statistic given by

$$\text{IT} = sup_k \left| (T/2)^{0.5} D_k \right| \qquad (1)$$

[1] The results from the GARCH(2, 2) model is available upon request from the authors.

that tests the null hypothesis that the unconditional variance z_t is constant for k =1… T, against the alternative hypothesis of a break in the unconditional variance at some point in the sample. D_k= (C_k / C_T) - (k/T) and $C_k = \sum_{t=1}^{k} z_t^2$ for k = 1… T. When the null hypothesis is rejected, the value of k that maximises $\left|(T/2)^{0.5} D_k\right|$ serves as the estimate of the break date. When z_t ~*iid* $N\ (0, \sigma_z^2)$, Inclán and Tiao (1994) show that under the null hypothesis the asymptotic distribution of the IT statistic is given by $sup_r \left|W^*(r)\right|$ where $W^*(r)$ = $W(\ r\ ) - r\ W(1)$ is a Brownian bridge and $W(\ r\ )$ is a standard Brownian motion. Finite-sample critical values of IT are then generated using simulation methods.

Several studies (see Andreu and Ghysels, 2002: de Pooter and van Dijk, 2004; Sansó *et al.*,. 2004) have shown that the IT statistic can be substantially oversized when z_t follows a dependent process such as a GARCH process. This is because the IT statistic is designed for *i.i.d* processes. Following Kokoszka and Leipus (1999), Lee and Park (2001) and Sansó *et al.*,. 2004, we address this deficiency of the IT statistic, and allow z_t to follow a variety of dependent processes under the null hypothesis, including GARCH processes, by using a nonparametric adjustment based on the Bartlett Kernel applied to the IT statistic. Formally, the adjusted IT (AIT) statistic is given by:

$$\text{AIT} = sup_k \left| T^{-0.5}\ G_k \right| \qquad (2)$$

where $G_k = \hat{\lambda}^{-0.5}\ [\ C_k - (k/T)\ C_T)]$, $\hat{\lambda} = \hat{\gamma}0 + 2\sum_{l=1}^{m}[1 - l(m+1)^{-1}]\ \hat{\gamma}l$,

$\hat{\gamma}l = T^{-1} \sum_{t=l+1}^{T}(z_t^2 - \hat{\sigma}^2)(z_{t-1}^2 - \hat{\sigma}^2)$, $\hat{\sigma}^2 = T^{-1}C_T$. The lag truncation parameter m is selected using the procedure in Newey and West (1994). The asymptotic distribution of AIT is also given by $sup_r \left|W^*(r)\right|$ under general conditions, and finite-sample critical values can again be generated by simulation methods.

The IT statistic can also be used to test for multiple breaks in the unconditional variance using an iterative cumulative sum of squares (ICSS) algorithm also developed by Inclán and Tiao (1994). To avoid the size distortions that results with the use of the IT statistic, the ICSS procedure can alternatively be based on the AIT statistic in order to allow z_t to follow dependent processes under the null hypothesis. We then use a 5% level of significance to test for structural breaks in the unconditional volatility of the daily stock returns series for the JSE All Share Index.

The GARCH(1,1) model for z_t with mean zero (conditional and unconditional) is expressed as

$$z_t = h_t^{0.5}\ \varepsilon_t \tag{3}$$

$$h_t = \omega + \propto z_{t-1}^2 + \beta h_{t-1} \tag{4}$$

where h_t represents the conditional volatility of z_t and ε_t is *i.i.d.* with mean zero and unit variance. $\propto + \beta$ measures the persistence of the GARCH (1,1) model and $\propto + \beta < 1$ for the process to be covariance-stationary. When $\propto + \beta = 1$ we have the integrated GARCH(1,1) of model of Bollerslev (1986). In equation (4), β is unidentified and set to zero when α= 0, so that $h_t = \omega$ and z_t is characterised by conditional homoscedasticity. For the GARCH(1,1) process to be stationary the unconditional variance of z_t is given by ω/ (1 - α - β). The Quasi Maximum Likelihood Estimation (QMLE) is often used to estimate the GARCH(1,1) because QMLE parameter estimates have been shown to be consistent and asymptotically normal (Berkes *et al*. 2003; Jensen and Rahbek, 2004; Straumann, 2005). It is however assumed that $e_t \sim N$ (0.1), and the restrictions $\omega > 0$ and $\propto, \beta \geq 0$ imposed. The in-sample tests enable us to analyse the empirical relevance of structural breaks in unconditional volatility for the JSE All share index and the effect of structural breaks on GARCH(1,1) models. The in-sample tests also provide a framework for analyzing the out-of-sample tests results.

2.2 Out-of-Sample Tests

To compare the out-of-sample forecasts of stock return volatility, we first divide the sample of stock returns into two portions; in-sample and out-of-sample, where the in-sample portion contains the first *R* observations and the out-of-sample portion contains the last *P* observations. Following Rapach and Strauss (2008), we use three benchmark models and five competing models to compare the out-of-sample forecasts. Note that, while the decision to use the RiskMetrics model as a second benchmark emanates from its popularity in the literature on out-of-sample volatility forecasting exercises, the choice of the FIGARCH(1,*d*,1) model as a third benchmark is motivated out of the well-known fact that the autocorrelations of squared returns for many financial assets decay slower than exponentially, as implied by GARCH models, so that conditional heteroskedasticity might be better represented by a long-memory process as captured by the FIGARCH(1,*d*,1) specification.[2] The first benchmark model is a GARCH(1,1) model estimated using an expanding window. The first out-of-sample forecast at the 1-period horizon (s = 1) is given by $\hat{h}_{R+1 \mid R,EXP} = \hat{\omega}_{R,EXP} + \hat{\alpha}_{R,EXP}\, z_R^2 + \hat{\beta}_{R,EXP}\hat{h}_{R,EXP,}$ where $\hat{\omega}_{R,EXP}$, $\hat{\alpha}_{R,EXP}$, $\hat{\beta}_{R,EXP}$ and $\hat{h}_{R,EXP,}$ are the estimates of $\omega,\ \alpha,\ \beta$ and h_R respectively obtained from equation (4) using QMLE and data from the first observation through to observation R. For the second out-of-sample forecast R+2, we expand the estimation window by one observation using data from the first observation through observation R+1, $\hat{h}_{R+2 \mid R+1,EXP}$. We continue this way through to the end of the available out-of-sample period, yielding a series of *P* one-step ahead out-of-sample forecasts given by $\{ \hat{h}_{t \mid t-1,EXP} \}_{t=R+1}^{T}$.

The RiskMetrics model is the second benchmark model based on an expanding window. It is easier to implement because it does not involve the estimation of any parameters. It is given by the exponential weighted moving average $\hat{h}_{t+1 \mid t} = (1-\lambda)\sum_{k=0}^{t-1} \lambda^k z_{t-k}^2$ where $\lambda = 0.94$ as recommended by the RiskMetrics Group (1996) for daily data. Consistent with the usual practice, we set the s-step-ahead forecast for *s* > 1equal to the 1-step-ahead forecast for the RiskMetrics model. The *s*-step-ahead out-of-sample volatility forecasts for the RiskMetrics model is given by $\{ \hat{h}_{t \mid t-s,RM} \}_{t=R+1}^{T}$.

The fractionally integrated GARCH(1,*d*,1) or FIGARCH(1,*d*,1) model is the third benchmark model also estimated using an expanding window (see Baillie *et al.* 1996).

The FIGARCH(1,*d*,1) specification is given by

$$h_t = \omega + \beta h_{t-1} + [1 - BL - (1-\phi L)(1-L)^d] z_t^2 \tag{5}$$

where L is lag operator and $(1-L)^d$ = $1 - dL - [d(1-d)/2]L^2 - [d(1-d)(2-d)/6]L^3 - \ldots$ is fractional differencing operator. The parameter vector $(\omega, \beta, \emptyset\ d)$ is estimated using QMLE under the assumption that $\omega > 0,$ $0 \le d \le 1 - 2\emptyset,$ and $0 \le \beta \le \emptyset + d$ to ensure that the conditional variance is positive. The

[2] See Baillie *et al.*, (1996) for further details.

FIGARCH(1,*d*,1) model is considered a relevant benchmark in forecasting volatility of asset returns. This is because autocorrelations of squared or (absolute) returns for many financial assets decay slower than exponentially as implied by GARCH models. Thus conditional heteroscedasticity may be better described by a long memory process as captured by the FIGARCH(1,*d*,1) specification. The forecasts generated by the FIGARCH(1,*d*,1) model is denoted by $\{\hat{h}_{t\ |\ t-1,FI}\}_{t=R+1}^{T}$.

The first forecasting competing model is a GARCH(1,1) 0.5 rolling window model. This model generates forecasts using a rolling estimation window equal to one-half of the size of the in-sample period. The forecasts are generated similar to the GARCH(1,1) expanding window model, except that the parameter estimates for the first out-of-sample forecast are based on observations (0.5*R*+1, ……, *R*) and for the second out-of-sample forecast are based on observations (0.5*R*+2, ……, *R*+1) and so on. The forecasts for the GARCH(1,1) 0.5 rolling window model is denoted by $\{\hat{h}_{t\ |\ t-1,ROLL(0.5)}\}_{t=R+1}^{T}$. The second competing model is a GARCH(1,1) 0.25 rolling window. This model generates forecasts using a rolling estimation window equal to one-quarter of the size of the in-sample period so that the first out-of-sample forecast are based on observations (0.75*R*+1, ……, *R*) and for the second out-of-sample forecast are based on observations (0.75*R*+2, ……, *R*+1) and so on. We denote the forecasts for the GARCH(1,1) 0.25 rolling window model by $\{\hat{h}_{t\ |\ t-1,ROLL(0.25)}\}_{t=R+1}^{T}$.

As in Mittnik and Paolella (2000), the third competing forecasting model is a GARCH(1,1) model estimated using an expanding window and a weighted maximum likelihood procedure. This model is known to better handle structural instabilities in GARCH parameters (Mittnik *et al.* 2000). In forming the likelihood function used to estimate the GARCH(1,1) model parameters, declining weights are assigned to observations in the more distant past. For the first out-of-sample forecast using data through R observations, a weight of ρ^{R-t} is attached to observation t = 1,…, R in the log-likelihood function used to estimate the GARCH(1,1) parameters. To generate the second out-of-sample forecast the window is expanded by one observation and a weight of ρ^{R+1-t} attached to observations t = 1,..., R + 1 in the log-likelihood function. This procedure is continued through to the end of the available out-of-sample period. Mittnik *et al.* (2000) recommend $\rho = 0.994$, which they find work well in out-of-sample volatility forecasts. The forecasts generated by the GARCH(1,1) weighted maximum likelihood model is denoted as $\{ \hat{h}_{t|t-1,WML} \}_{t=R+1}^{T}$

In the fourth competing model the modified ICSS algorithm is used to select the estimation window for the GARCH(1,1) model. The modified ICSS algorithm is first of all applied to observation one through R. If there is evidence of one or more structural breaks, and the final break is expected to occur at time T_F, the GARCH(1,1) model is estimated using observations

$T_F + 1$ through R to form an estimate of h_{R+1}. One the other hand if no evidence of a structural break is found the GARCH(1,1) model is then estimated using observations one through R to form an estimate of h_{R+1}. For the second out-of-sample forecast, the modified ICSS algorithm is applied to observations one through R+1and the same procedure as in the first out-of-sample forecast is followed. We proceed in this manner through the end of the available out-of-sample period, producing a series of forecasts corresponding to the GARCH(1,1) with breaks model, given by $\{ \hat{h}_{t|t-1,BREAKS} \}_{t=R+1}^{T}$. The modified ICSS algorithm that determines the size of the estimation window only uses data available at the time of the forecast formation. As a result there is no "look ahead" bias involved in the generation of the forecasts for the GARCH(1,1) with breaks model.

The final competing forecasting model is a simple moving average model that uses the average of the squared returns over the previous 250 days to form the volatility for day ***t*** : $\hat{h}_{t|t-1,MA} = (1/250)\sum_{i=1}^{250} z_{t-i}^2$. This model has been found to outperform GARCH(1,1) models when forecasting daily stock return volatility over longer horizons especially for industrialised countries and also very useful in accommodating structural breaks (Stărică and Granger, 2005). Following Rapach and Staruss (2008), we set the s-step-ahead forecast for $s > 1$ equal to 1-step-ahead forecast for the moving average model. We denote the sequence of s-step-ahead out-of-sample forecasts for the moving average model by $\hat{h}_{t|t-1,MA} \}_{t=R+s}^{T}$.

Additionally we consider a multi period volatility forecast over the out-of-sample period for horizons of 1, 20, 60 and 120 days with the aim of exploring the effects of structural breaks on volatility forecasting and the usefulness of various forecasting methods designed to accommodate potential structural breaks. Based on information available at period $t - s$ we denote the model i forecast[3] of h_t formed at period $t - s$ by $\hat{h}_{t \mid t-s,i}$ yielding a series of P - (s - 1) s-step ahead out-of-sample forecasts given by $\{\hat{h}_{t \mid t-s,i}\}_{t=R+s}^{T}$. We then iterate forward by generating $\hat{h}_{t \mid t-s,i}$ for s > 1 using the fitted GARCH(1,1) process and the iterative procedure given by the equation

$\hat{h}_{t+s \mid t} = \hat{\omega} + (\hat{\alpha} + \hat{\gamma}/2 + \hat{\beta})\ \hat{h}_{t+s-1 \mid t}$ from Franses and van Dijk (2000).

To compare volatility forecasts across models we employ two loss functions; an aggregated version of the mean square forecast error (MSFE) metric by Stărică and Granger (2005) and the value-at-risk (VaR) by González-Rivera and Mishra (2004). The MSFE metric is given by

$$MSFE_{s,i}^{*} = [P - (s-1)]^{-1} \sum_{t=R+s}^{T} (\tilde{z}_t^2 - \tilde{\hat{h}}_{t \mid t-s,i})^2 \quad (6)$$

where $\tilde{z}_t^2 = \sum_{j=1}^{s} z_{t-(j-1)}^2$ and $\tilde{\hat{h}}_{t \mid t-s,i} = \sum_{j=1}^{s} \hat{h}_{t-(j-1) \mid t-s,i}$. Aggregation provides a more useful metric for comparing volatility forecasts because it reduces the idiosyncratic noise in squared returns at horizons beyond one period (Andersen and Bollerslev, 1998). The MSFE loss function produces a consistent empirical ranking of forecasting models when squared returns serve as a proxy for measuring latent volatility (Awartani and Corradi, 2004; Hansen and Lunde, 2006). Thus using the aggregated MSFE metric we analyse volatility forecasts at horizons of 1, 20, 60 and 120 days (s = 1, 20, 60, 120).

With respect to the VaR loss function, let $VAR_{t \mid t-s,i}^{0.05}$ be the forecast of the 0.05 quantile of the cumulative distribution function for the cumulative return $\tilde{z}_t = \sum_{j=1}^{s} z_{t-(j-1)}$ generated by model i and formed at time $t - s$. The VaR loss mean function, as in González-Rivera *et al.*, (2004) is given by

$$MVAR_{s,i} = [P - (s-1)]^{-1} \sum_{t=R+s}^{T} (0.05 - d_{t,i}^{0.05})(\tilde{z}_t - VAR_{t \mid t-s,i}^{0.05}) \quad (7)$$

where $d_{t,i}^{0.05} = 1(\tilde{z}_t < VAR_{t \mid t-s,i}^{0.05})$ and 1(.) is the indicator function that takes a value of unity when the argument is satisfied. Compared to the "hit or no hit" VaR-based loss functions, this is an asymmetric loss function which is more sophisticated. In this regard, note that, assuming

[3] Where i = EXP, RM FI, ROLL(0.5), ROLL(0.25), WML, BREAKS, MA}.

normal markets and no trading in the portfolio, VaR is defined as a threshold value such that the probability that the mark-to-market loss on the portfolio over the given time horizon exceeds this value is the given probability level. Thus, VaR is essentially *a percentile of the predictive probability distribution* for the size of a future financial loss (Jorion, 2006).[4] When $\tilde{z}_t < VAR^{0.05}_{t\,|t-s,i}$, a large weight of 0.95 is attached by the loss function to the absolute value of the difference between $\tilde{z}_t$ and $VAR^{0.05}_{t\,|t-s,i}$, indicating a relatively high cost associated with large losses. Conversely a smaller weight of 0.05 is attached by the loss function to the difference between $\tilde{z}_t$ and $VAR^{0.05}_{t\,|t-s,i}$ when $\tilde{z}_t > VAR^{0.05}_{t\,|t-s,i}$. Although the weight is smaller in this case it is still positive thereby enabling the loss function to reflect the opportunity costs of the capital held to cover the potential losses indicated by $VAR^{0.05}_{t\,|t-s,i}$. The advantage of the $MVAR_{s,i}$ criterion is that it is motivated by the VaR as a risk management tool and does not require observations of the latent volatility h_t. By assuming that $z_t \sim N(0, 1)$ we calculate $VAR^{0.05}_{t\,|t-s,i}$ as the simulated cumulative returns

($\tilde{z}^*_t = \sum_{j=1}^{s} z^*_{t-(j-1)}$) of a simulated sequence of returns based on estimates of the conditional volatility process available at the time of forecast formation ($\{z^*_{t-(j-1)}\}_{j=1}^{s}$). This process is repeated 2000 times yielding an empirical distribution of simulated cumulative returns. The 100th element of the ordered simulated cumulative returns is the $VAR^{0.05}_{t\,|t-s,i}$.

Besides ranking the forecasting models using the $MSFE^*_{s,i}$ and $MVAR_{s,i}$ loss functions, we test the null hypothesis that none of the competing models has superior predictive ability over the benchmark model in terms of expected loss, against the alternative one sided (upper-tail) hypothesis that at least one of the competing models has superior predictive ability over the benchmark model. That is we check whether the expected loss of the forecasts generated by at least one of the five competing models is significantly less than that generated by a given benchmark model using the White (2000) test. The loss at time *t* for forecasting model *j* relative to benchmark model *i* is defined as $f_{t,i,j} = L_{t,i} - L_{t,j}$, where L_t is given by the expression

[4] Suppose that if a portfolio of stocks has a one-day 5 percent VaR of South African Rand (ZAR) 1 million, there is a 0.05 probability that the portfolio will fall in value by more than ZAR 1 million over a one day period if there is no trading. Informally, a loss of ZAR 1 million or more on this portfolio is expected on one day out of twenty.

after the summation operator in equation (6) or (7) for each loss function, and $\bar{f}_{i,j}$ = $[P-(s-1)]^{-1}\sum_{t=R+s}^{T} f_{t,i,j}$. The White (2000) statistic for l competing models is given by

$$\bar{V}_l = max_{k=1\ldots l}[P-(s-1)]^{0.05}(\bar{f}_{i,1},\ldots\ldots.\bar{f}_{i,j}) \quad (8)$$

with $l = 5$ in this paper.

As in White (2000) a p-value corresponding to $\bar{V}_l$ is generated using the stationary bootstrap method of Politis and Romano (1994). The White (2000) reality check is performed by comparing each of the benchmark models (GARCH(1,1) expanding window, RiskMetrics and FIGARCH(1,*d*,1) to the five competing models to check whether any of the five competing models performs better than the given benchmark model in terms of real-time volatility forecasting. Additionally, the Hansen (2005) version of the White (2000) test which has a higher power in determining superior predictive ability is also conducted. The Hansen (2005) studentised version of the $\bar{V}_l$ statistic, T_n^{SPA} is computed and the p-values again generated by the stationary bootstrap method of Politis and Romano (1994).

One drawback of the GARCH(1,1) model is its assumption that the response of the conditional variance to both positive and negative shocks is the same-symmetrical. Thus only the size and not the sign of the shock is relevant. There exists ample empirical evidence of the existence of leverage effects in stock returns, meaning negative return shocks results in higher volatility in subsequent periods than positive return shocks. We compare the performance of two asymmetric GARCH models in accurately capturing leverage effects in stock return volatility, namely the GJR-GARCH(1,1) model and the MS-GARCH(1,1). The GJR-GARCH(1,1) model is the asymmetric GARCH(1,1) model of Glosten *et al.* (1993) and is expressed as $z_t = \boldsymbol{h}_t\varepsilon_t$ where $\varepsilon_1 t\sim iid\ N($ 0, 1), and $\boldsymbol{h}_t = \omega + \propto z_{t-1}^2(1-I) + \gamma z_{t-1}^2 I + \beta\boldsymbol{h}_{t-1}$. The dummy *I* takes on the value of unity when ε_{t-1} < 0 and zero otherwise. The MS-GARCH(1,1) is the two-state Markov-switching GARCH(1,1) model of Haas et al. (2004) and is expressed as $z_t = \boldsymbol{h}_t\varepsilon_t$ where $\varepsilon_1 t\sim iid\ N($ 0, 1), with $\boldsymbol{h}_{s_t} = \boldsymbol{h}_{1,t} = \omega_1 + \propto_1 z_{t-1}^2 + \beta_1\boldsymbol{h}_{1,t-1}$ in state one, and in state two $\boldsymbol{h}_{s_t} = \boldsymbol{h}_{2,t} = \omega_2 + \propto_2 z_{t-1}^2 + \beta_2\boldsymbol{h}_{2,t-1}$, with transition probabilities given by $p_{i,j}$ = [P($s_1 t = (j \quad) \mid s_1(t-1) = i)]$ for j= 1, 2. The ratio of the mean loss for each model to the mean loss of the benchmark GARCH(1,1) expanding window model is compared under the $MSFE^{*}_{s,i}$ and $MVAR_{s,i}$ criteria, as above.

Recent literature has shown that comparing the relative predictive accuracy of different forecasting models need to take into consideration the relative sizes of the in-sample and out-of-sample periods (*P/R*), type of estimating window used (expanding, rolling or fixed) and whether the models being compared are nested or not. Since these requirements are not all necessarily satisfied in our application we report the bootstrapped p-values for the White (2000) $\bar{V}_l$ and Hansen (2005) T_n^{SPA} statistics as a crude guide to assessing statistical significance of the various models used in this paper.

3. Empirical Results

3.1 Data and Descriptive Statistics

Daily data on the Johannesburg Stock Exchange All Share Index is used in this paper. The sample period is from 07/02/1995 to 08/25/2010 consisting of 3788 observations. The daily stock returns are based on the closing prices. The descriptive statistics are reported in Table 1. These statistics include heteroscedastic and autocorrelation consistent standard errors for the mean, standard deviation, skewness, and excess kurtosis. The computation of these statistics is based on the procedure in West and Cho (1995). The mean is significantly different from zero at 5 % level of significance. Daily stock returns appear quite volatile and exhibit strong evidence of excess kurtosis. The modified Ljung-Box statistics are robust to conditional heteroscedasticity and show no evidence of autocorrelation of the daily stock returns. However there is strong evidence of serial correlations in the squared stock returns. The Lagrange multiplier statistics are significant at 1 % level confirming ARCH effects (Engle, 1982). These descriptives support the modelling of stock market returns in South Africa using GARCH processes.

Table 1: Summary statistics, SA JSE All Share Index returns (07/03/1995 to 08/25/2010)

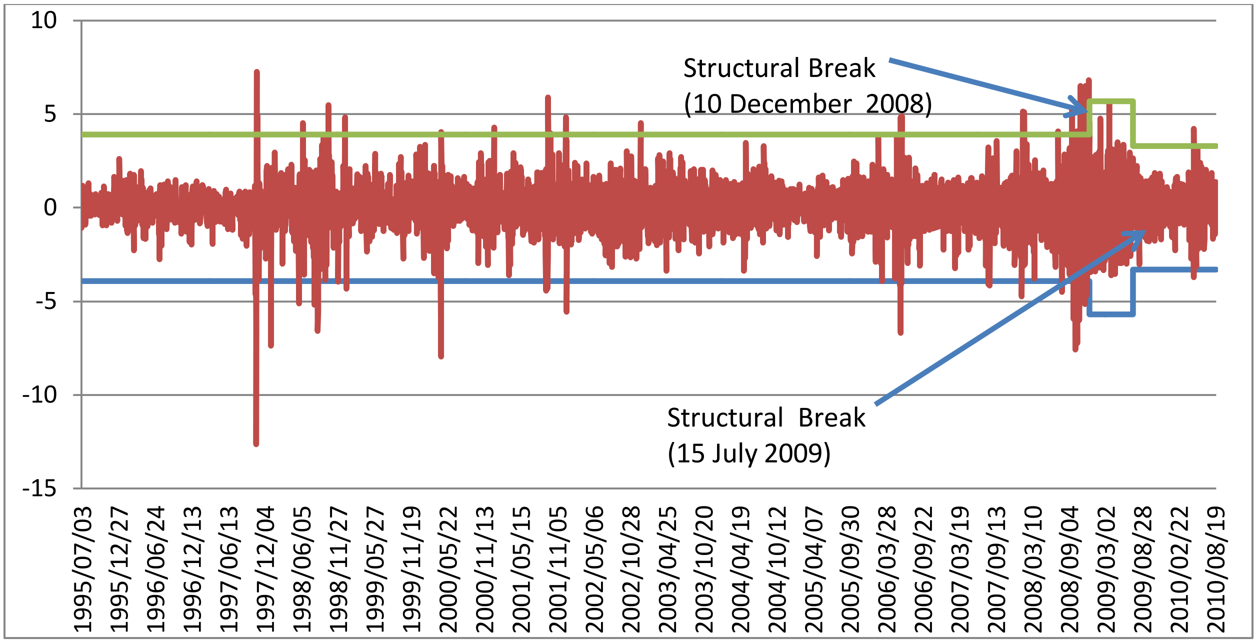
	Estimates	Std. err	p-values
Stock Market Price Return			
Mean	0.045	0.021	
Standard deviation	1.320	0.030	
Skewness	-0.479	0.253	
Excess kurtosis	5.944	1.852	
Minimum	-12.628		
Maximum	7.266		
Modified Ljung-Box (r=20)	26.137		0.161
Squared Stock Market Price Return			
Ljung-Box (r=20)	1781.218		0.000
ARCH Lagrange multiplier (q = 2)	411.880		0.000
ARCH Lagrange multiplier (q = 10)	560.927		0.000

Note: Returns are defined as 100 times the log-differences of the stock price indices. Ljung-Box statistics correspond to a test of the null hypothesis that the r autocorrelations are zero. Modified Ljung-Box statistics are robust to conditional heteroskedasticity. ARCH Lagrange multiplier statistics correspond to a test of the null hypothesis of no ARCH effects from lag 1 through q. 0.000 indicates the p values less than 0.0005.

3.2 In-sample results

The modified ICSS algorithm employed for our in-sample test revealed two structural breaks in the unconditional volatility of stock market return in South Africa, specifically on 10th of December, 2008 and 15th of July, 2009. While, the first structural break represent an increase in volatility, the second one is characterized by a reduction of the same as indicated by the increased and reduced unconditional variances reported for the sub-samples 2 and 3, relative to sub-sample 1, in Table 2. Figure 1 below shows a plot of the stock returns series and three-standard-deviation bands defined by the structural breaks identified by the modified ICSS algorithm.

Figure 1: The JSE All Share Index and ± 3 standard deviation bands.



Most emerging markets like South Africa largely stood at the fringes of the global financial crisis for most part of 2007 and 2008, which led to very high volatility, in general. With the South African economy reaching the trough of the business cycle by the end of 2008 (Venter, 2011), we observe a sharp increase in volatility (as depicted in sub-sample 2 relative to sub-sample 1 in Table 2) indicating the uncertainty in the market, and hence, resulting likely in the structural break identified by the ICSS algorithm in early December of 2008. The importance of the impact of US stock returns on South African stock returns has recently been highlighted by Gupta and Modise (2012). In light of this, as the US economy started to show mild signs of revival, the decreased uncertainty is likely to have produced lower levels of volatility in the South African stock returns, following the hike in the variance. Further, as the US recession was officially called-off in the first quarter of 2009, a reduced volatility in the stock returns was observed in the early third quarter of 2009. Also, both the leading and the coincident indicator for South Africa had started to turn upwards in the first quarter of 2009 (Venter, 2011). In addition, as indicated by van Wyk de Vries *et al.*, (forthcoming), during the financial crisis, due to the global uncertainty, hedging demand by South African investors for domestic stocks were much less volatile, with a positive mean value, than hedging demands for US and UK stocks, with the mean value of the latter set of stocks being actually negative. Finally, evidence provided by Naraidoo and Raputsoane (2010) and Naraidoo and Ndahiriwe (forthcoming), indicated that the south African Reserve Bank, had systematically reacted to a financial conditions index (containing stock prices) during the recent financial crisis to minimize the forecasted volatility in the financial conditions index. Given these set of events, the two structural breaks in the South African stock returns volatility in December 2008 and July 2009, characterized by increased and reduced volatility respectively, seems to be logical.

Table 2 below shows the unconditional variance of the squared stock return series estimated using a standard QMLE GARCH(1,1) model both over the full sample period and sub-sample periods.

Table 2: Quasi Maximum Likelihood Estimation Results for GARCH(1,1) Models

	Estimates	Std.err
GARCH(1,1) full sample estimation results		
ω	0.020	0.005
α	0.111	0.011
β	0.878	0.011
$\omega/(1 - \alpha - \beta)$	1.746	0.166
GARCH(1,1) sub-sample 1 estimation results		
ω	0.020	0.005
α	0.115	0.012
β	0.873	0.012
$\omega/(1 - \alpha - \beta)$	1.711	0.182
GARCH(1,1) sub-sample 2 estimation results		
ω	3.581	0.367
α	0	0
β	0	0
$\omega/(1 - \alpha - \beta)$	3.581	0.367
GARCH(1,1) sub-sample 3 estimation results		
ω	0.058	0.041
α	0.089	0.036
β	0.863	0.307
$\omega/(1 - \alpha - \beta)$	1.191	0.307

Notes: Table 2 reports the GARCH(1,1) model estimations for the squared stock return series for the full sample and those for the different sub-samples defined by the structural breaks. The table also includes standard deviations of the estimates.

The sub-samples are defined by the structural breaks identified by the modified ICSS algorithm. The fitted full sample GARCH(1,1) model is highly persistent with an estimate of $\alpha + \beta$ of about 0.989. The first sub-sample GARCH(1,1) model also exhibits high persistence with an estimate of $\hat{\alpha}\propto + \hat{\beta}\beta = 0.984$. The second sub-sample GARCH(1,1) model shows absolutely no persistence, whiles the third sub-sample GARCH(1,1) model also shows high persistence with an estimate of $\hat{\alpha}\propto + \hat{\beta}\beta = 0.953$. These high levels of persistence show that the sub-samples are generally characterized by conditional heteroscedasticity, barring sub-sample 2 which is characterized by unconditional homoskedasticity. As discussed in Rapach *et al.,* (2008), this kind of sizable decreases in the persistence of the volatility process relative to the full-sample (as well as other sub-sample) estimates are a likely due to the upward biases in the persistence that results from failing to account for structural breaks. Table 2 also shows some significant[5] changes in the unconditional variance as reflected by $\hat{\omega}/(1 - \hat{\alpha} - \hat{\beta}\beta)$. These changes are due to the structural breaks bringing about substantial shift in the intercept defined by $\hat{\omega}\omega$ over the period under review. In addition the GARCH(1,1) parameter estimates vary across sub-samples defined by the structural breaks. These in-sample results show, in general, highly persistent conditional variance for the stock return and also confirm that structural breaks are an empirically relevant feature of stock market returns in South Africa.

3.3 Out-of-sample results

The out-of-sample period consists of the last 500 observations of the January 2 1995 to August 31 2010 full sample period and covers the September 2 2008 to August 31 2010 period for South Africa, which includes both the structural breaks. Table 3 reports the out-of-sample volatility forecasting results over horizons of 1, 20, 60 and 120 days. The first row in each panel of the table reports the mean loss for the GARCH(1,1) expanding window model, while the remaining

[5] Using a upper-tailed *F*-test, we were able to show that the unconditional variance for sub-sample 2 was significantly bigger than sub-samples 1 and 3, while, the unconditional variance of sub-sample 1 significantly exceeded the same for sub-sample 3. Note that the values of the *F*-statistic = (Unconditional Variance of sample 2(1))/ (Unconditional Variance of sample 1 or 3(3)) and were all greater than 1 (3.007, 2.0929 and 1.4366), and, given the sizes of sub-samples 1, 2 and 3 of 3362, 144 and 279 respectively, the null of the equality of the unconditional variances was rejected at one percent level of significance. The details of these results are available upon request from the authors.

rows present the ratio of the mean loss for each of the other models to the mean loss for the GARCH(1,1) expanding window model. The model with the lowest mean lost ratio under both the MSFE and MVaR criteria performs better than the other models in forecasting volatility. The table also reports *p*-values corresponding to the White (2000) $\bar{V}_l$ and Hansen (2005) T_n^{SPA} statistics with the GARCH(1,1) expanding window, RiskMetrics, and FIGARCH(1,d,1) expanding window models serving as the benchmark models and the two GARCH(1,1) rolling window, GARCH(1,1) weighted ML, GARCH(1,1) with breaks and moving average models serving as the competing models.

From the out-of-sample results in Table 3 based on the $\text{MSFE}_{s,i}^*$ loss function, within the benchmark models GARCH(1,1) with expanding window, RiskMetrics and FIGARCH(1,*d*,1), the RiskMetrics model outperforms the two other benchmarks at *s*=1, while, the FIGARCH(1,*d*,1) performs better than the other two benchmarks at *s*=60 and 120. This confirms the finding in the literature that the FIGARCH(1,*d*,1) model better captures conditional heteroscedasticity described by long memory processes, especially at longer horizons. At s= 20, the GARCH(1,1) with expanding window is best amongst the three benchmarks. As far as the competing models are concerned, the GARCH(1,1) 0.50 rolling window and the GARCH(1,1) with breaks models report the lowest mean loss ratio over the 1-day horizon compared to the benchmarks as well as the other competing models. The GARCH(1,1) 0.25 rolling window model performs better than all the competing models over the 20, 60 and 120 days horizons under the $\text{MSFE}_{s,i}^*$ criterion. This model, also does better than all the benchmark models, barring the 20 days ahead forecast, where it performs slightly worse than the GARCH(1,1) with expanding window. This proves that allowing for instabilities in GARCH(1,1) models has benefits in out-of-sample volatility forecasting**.**

The performance of GARCH(1,1) with breaks worsens over higher forecasting horizons but performs relatively better than the GARCH(1,1) weighted ML and the Moving average models. FIGARCH(1,*d*,1)Importantly though, the p-values corresponding to the Hansen (2005) T_n^{SPA} statistics and the White (2000) $\bar{V}_1$ statistics, in general, fails to indicate significant gains in predictive ability from the competing models, relative to benchmark models. There are however, some exceptions, based on Hansen (2005) T_n^{SPA} especially relative to the FIGARCH(1,d,1) model.

Using the $MVAR_{s,i}$ criterion, the benchmark GARCH(1,1) model with expanding window always performs better than the other two benchmarks for all horizons. Barring, at *s*=1, where the GARCH(1,1) 0.50 rolling window model delivers the lowest mean loss ratio, none of the competing models outperforms the GARCH(1,1) with expanding window. Hence, not surprisingly, the *p*-values corresponding to the Hansen (2005) T_n^{SPA} statistics and the White (2000) $\bar{V}_1$ statistics, fail to reject the null hypothesis consistently.

Following Rapach and Strauss (2008), we also tried forecast combinations based on the benchmark GARCH(1,1) with expanding window and the five other competing models, using mean of the six individual forecasts and trimmed mean (the mean of four individual forecasts remaining after discarding the highest and lowest individual forecasts) combination methods. In general, the trimmed mean combination method fairs well compared to the simple mean method. However, the combination methods fail to outperform the best competing model.

Table 3 also provides the out-of-sample forecasting results comparing the GJR-GARCH(1,1) expanding window model and MS-GARCH(1,1) models relative to the GARCH(1,1) with expanding window. Based on $\mathrm{MSFE}^*_{s,i}$, the GJR-GARCH(1,1) models and the MS-GARCH(1,1) model fails out outperform the GARCH(1,1) with expanding window at all forecast horizons, with the MS-GARCH(1,1) performing exceptionally poor beyond the one-day-ahead forecast. The performances are much better, based on $MVAR_{s,i}$, especially at *s*= 60 and 120, with the GJR-GARCH(1,1) performing consistently better relative to the five other competing models considered above beyond horizon of *s* = 2. The results also show that for shorter horizons, based on $MVAR_{s,i}$, the MS-GARCH(1,1) better captures the leverage effect in stock market return volatility than the GJR-GARCH(1,1). However for longer horizons, the GJR-GARCH(1,1) model performs better than the MS-GARCH(1,1) model in accurately capturing leverage effects in stock market return volatility.

Table 3: Summary of Out of Sample Forecasting Results

s = 1	MSFE			MVaR		
GARCH(1,1) expanding window	36.893	(0.879)	[0.372]	0.173	(0.768)	[0.558]
RiskMetrics	0.998	(0.867)	[0.570]	1.012	(0.736)	[0.382]
FIGARCH(1,*d*,1) expanding window	1.015	(0.693)	[0.097]	1.016	(0.705)	[0.281]
GARCH(1,1) 0.50 rolling window	**0.995**			**0.995**		
GARCH(1,1) 0.25 rolling window	1.011			1.007		
GARCH(1,1) weighted ML	1.029			1.017		
GARCH(1,1) with breaks	**0.995**			0.997		
Moving average	1.290			1.262		
Mean	1.003			1.011		
Trimmed Mean	0.999			0.999		
GJR-GARCH(1,1) expanding window	1.417			1.989		
MS-GARCH(1,1) expanding window	1.002			1.031		
s = 20						
GARCH(1,1) expanding window	2327.75	(0.962)	[1.000]	0.837	(1.000)	[1.000]
RiskMetrics	1.176	(0.689)	[0.260]	1.068	(0.617)	[0.106]
FIGARCH(1,d,1) expanding window	1.125	(0.787)	[0.254]	1.072	(0.682)	[0.111]
GARCH(1,1) 0.50 rolling window	1.044			1.032		
GARCH(1,1) 0.25 rolling window	**1.005**			**1.024**		
GARCH(1,1) weighted ML	1.635			1.046		
GARCH(1,1) with breaks	1.029			1.028		
Moving average	2.888			1.302		
Mean	1.065			1.061		
Trimmed Mean	1.010			1.028		
GJR-GARCH(1,1) expanding window	4.376			1.380		
MS-GARCH(1,1) expanding window	3704.21			***0.997***		
s = 60						
GARCH(1,1) expanding window	27263.1	(0.758)	[0.639]	1.504	(0.998)	[1.000]
RiskMetrics	1.204	(0.663)	[0.368]	**1.001**	(0.960)	[1.000]
FIGARCH(1,d,1) expanding window	0.954	(0.788)	[0.039]	1.038	(0.755)	[0.589]
GARCH(1,1) 0.50 rolling window	0.939			1.010		
GARCH(1,1) 0.25 rolling window	**0.861**			1.019		
GARCH(1,1) weighted ML	3.852			1.073		
GARCH(1,1) with breaks	1.049			1.037		
Moving average	1.944			1.153		
Mean	1.090			1.045		
Trimmed Mean	0.949			1.024		
GJR-GARCH(1,1) expanding window	2.661			***0.908***		
MS-GARCH(1,1) expanding window	2.414e+16			0.978		

	MSFE Ratio			**MVaR** Ratio		
s = 120						
GARCH(1,1) expanding window	92405	(0.635)	[0.107]	2.106	(1.000)	[1.000]
RiskMetrics	1.505	(0.503)	[0.088]	**1.003**	(0.928)	[1.000]
FIGARCH(1,d,1) expanding window	0.787	(0.699)	[0.000]	1.067	(0.450)	[0.008]
GARCH(1,1) 0.50 rolling window	0.682			1.010		

GARCH(1,1) 0.25 rolling window	**0.619**	1.016
GARCH(1,1) weighted ML	16.353	1.133
GARCH(1,1) with breaks	1.080	1.042
Moving average	1.888	1.188
Mean	1.566	1.059
Trimmed Mean	0.837	1.027
GJR-GARCH(1,1) expanding window	1.978	***0.935***
MS-GARCH(1,1) expanding window	9.09640e+36	0.966

Note: Entries for the GARCH(1,1) expanding window model give the mean loss for this model. Entries for the other models give the ratio of the mean loss for each model to the mean loss for the GARCH(1,1) expanding window model. Bold entries denote the model with the smallest mean loss among all of the models. P-values for the White (2000)) $\bar{V}_1$ (Hansen (2005) T_n^{SPA}) statistics are given in brackets (box brackets) and correspond to a test of the null hypothesis that none of the five competing models (two GARCH(1,1) rolling window, GARCH(1,1) weighted ML, GARCH(1,1) with breaks, and moving average models) has a lower expected loss than the benchmark model indicated on the left against the one sided (upper-tail) alternative hypothesis that at least one of the competing models have a lower expected loss than the benchmark model Mean and Trimmed Mean entries correspond to forecast combinations based on the GARCH(1,1) model with expanding window and the five other competing models using simple mean and trimmed mean combination methods.The bold-italic entries corresponds to the best model amongst the the GJR-GARCH(1,1) and the MS-GARCH(1,1) with expanding windows relative to the GARCH(1,1) model with expanding window. The ratios in the table are the mean loss for each model to the mean loss for the GARCH(1,1) expanding window model.

4. Conclusion

Proper forecast of volatility in stock returns is crucial for many investment decisions and portfolio creations. Further, volatility is the most important variable in the pricing of derivative securities, and has become a compulsory risk-management exercise for financial institutions around the world following the first Basle Accord. Finally, financial market volatility serves as a measure of for the vulnerability of financial markets and the economy, and help policy makers to design appropriate policies. Given that recent research has shown that, during the recent financial crisis, the South African Reserve Bank has been systematically reacting to a financial conditions index, which in turn, includes stock returns, predicting the volatility of stock returns accurately could be of paramount importance when designing the appropriate loss-function to obtain the future path of the financial conditions index to obtain the optimal monetary policy response. The impact of structural breaks on the accuracy of volatility forecasts has largely been ignored in previous research. Failure to account for structural breaks in the unconditional variance of stock market returns can lead to sizeable upward biases in the degree of persistence in estimated GARCH models - traditionally used to forecast volatility. With structural breaks, GARCH models do not accurately track changes in the unconditional variance leading to forecasts that underestimate or overestimate volatility on average for long stretches.

Despite extensive work on volatility forecasting of asset returns, hardly any work is specific to South Africa in terms of forecasting the volatility of stock market returns. The one that exists, namely, Samouilhan and Shannon (2008) assume the existence of a stable GARCH process in volatility forecasting and do not take into consideration the impact of structural breaks on the accuracy of volatility forecasts. Additionally only one period ahead forecasts were used in their paper to ascertain the accuracy of the three different volatility forecasting approaches. To address these gaps in the South African literature, we investigate the empirical relevance of structural breaks for GARCH(1,1) models of stock return volatility in South Africa using in-sample and out-of-sample tests on daily data from the JSE All Share index from 1995 to 2010.

Results from our in-sample tests using the modified ICSS algorithm identify two structural breaks (10^{th} of December, 2008 and 15^{th} of July, 2009), characterized by reduced volatility, in the unconditional volatility of the stock market return series in South Africa. Using an out-of-sample period of September 2 2008 to August 31 2010 period for South Africa, which includes both the structural breaks, we find that, though there are some gains in forecasting ability from using models that explicitly accounts for structural changes, statistically, these improved forecasting abilities relative to the benchmark GARCH(1,1) model with expanding window, are insignificant. Forecast combination too is not found to change the above results. Interestingly, when we try to capture leverage effects, the GJR-GARCH(1,1) model with expanding window shows some evidence of success relative to the benchmark GARCH(1,1) model with expanding window. Our results tend to suggest, that even though there structural breaks in the volatility of South African stock returns, there are no apparent gains from using competing models that explicitly accounts for structural breaks, relative to a GARCH(1,1) model with expanding window. We believe that this is most likely due to the fact that the two identified structural breaks occurred in our out-of-sample, and recursive estimation of the GARCH(1,1) model is perhaps sufficient to account for the effect of the breaks on the parameter estimates. Having said that, to vindicate our reasoning, it would entail conducting a similar analysis in the future whereby additional data would us to treat these two identified breaks within the in-sample, as was observed by Rapach and Strauss (2008), when they analyzed exchange rate volatility for eight US dollar-based exchange rates of industrialized countries. Further, based on our results, what seems more important in South Africa, is accounting for leverage effects, especially in terms of long-horizon forecasting of stock return volatility. But all in all, the GARCH(1,1) model with expanding window, seems to provide an accurate data-generating process of stock returns volatility in South Africa, even in the presence of structural breaks.

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