

ON THE SPHERICALLY SYMMETRIC EINSTEIN-YANG-MILLS-HIGGS EQUATIONS IN BONDI COORDINATES

CALVIN TADMON^{1,2*}

¹Department of Mathematics and Computer Science,
University of Dschang, P. O. Box 67, Dschang, Cameroon

²Department of Mathematics and Applied Mathematics,
University of Pretoria, Pretoria 0002, South Africa

SOPHONIE BLAISE TCHAPNDA^{3†}

³Department of Mathematics, University of Yaounde 1,
P. O. Box 812, Yaounde, Cameroon

Abstract

We revisit and generalize, to the Einstein-Yang-Mills-Higgs system, previous results of D. Christodoulou and D. Chae concerning global solutions for the Einstein-scalar field and the Einstein-Maxwell-Higgs equations. The novelty of the present work is twofold. For one thing the assumption on the self-interaction potential is improved. For another thing explanation is furnished why the solutions obtained here and those proved by Chae for the Einstein-Maxwell-Higgs decay more slowly than those established by Christodoulou in the case of self-gravitating scalar fields. Actually this latter phenomenon stems from the non-vanishing local charge in Einstein-Maxwell-Higgs and Einstein-Yang-Mills-Higgs models.

AMS Subject Classification: [2000] 35L15, 35L70, 46E35, 46J10, 81T13, 83C10, 83C20.

Keywords: Global solution, Einstein-Yang-Mills-Higgs equations, spherical symmetry, Bondi coordinates.

1 Introduction

The Yang-Mills-Higgs (YMH) field equations arise in elementary particle physics. The Yang-Mills equations appear as the generalization of the classical Maxwell equations where ordinary derivatives are replaced by covariant derivatives. The YMH equations are nonlinear partial differential equations (PDE) that are conformally invariant and gauge invariant. This latter property has been exploited by Eardley and Moncrief [13, 14] to make a remarkable contribution to the area of PDE in proving global existence for these equations in 4-dimensional Minkowski space.

When the YMH equations are coupled to gravity, such a global existence result is yet to be proven without any symmetry assumption. Since this problem is a very difficult one, it makes sense to start by investigating it with some symmetry assumption. In this paper we prove a global existence result for the Einstein-Yang-Mills-Higgs (EYMH) equations under the assumption of spherical symmetry.

The issue of proving global existence results for gravitational and matter field equations is of interest in mathematical relativity and in the area of (PDE). In general relativity, the global existence problem is important since it is a reformulation of the cosmic censorship conjecture rendering this more amenable to direct analytic attack (see [15]).

*E-mail address: tadmonc@yahoo.fr; calvin.tadmon@up.ac.za

†E-mail address: sophonieblaise@yahoo.com

Much work has been done in the past concerning the EYM equations. Let us mention for instance that the spontaneous compactification of space in EYM model has been studied in [10]. A canonical formulation of the spherically symmetric EYM system for a general gauge group has been found in [9]. Regular localized solutions of the electric or magnetic type were found in [8]. Stimulated by the discovery of globally regular solutions of the EYM equations, numerically by Bartnik and McKinnon [1], many authors published a considerable number of papers dealing with the static spherically or axially symmetric EYM equations. Most of these papers concern asymptotically flat, smooth particle-like and black hole solutions (see e.g. [3, 17, 20, 21, 23, 29]) although some cosmological solutions have been found for instance in [2, 16].

As far as rigorous existence proofs are concerned, Künzle and Oliynyk [22] used differential geometry techniques to investigate the static spherically symmetric EYM system. Local existence to the characteristic Cauchy problem for EYM without any symmetry assumption has been proved recently by Dossa and Tadmon [11, 12]. More recently, Vacaru [28] has written the Einstein field equations in variables adapted to nonholonomic $2+2$ splitting and has applied this geometric technique for constructing off-diagonal exact solutions of EYM equations.

In the present paper, as mentioned above, we investigate the initial value problem for the spherically symmetric EYM equations. We write the system of equations in the so-called Bondi coordinates so as to obtain a characteristic Cauchy problem, then we prove global existence and some decay property of solutions. This setting has been used before by Christodoulou [7] for the Einstein-scalar field system and by Chae [4, 5] respectively for Einstein-Klein-Gordon (EKG) and Einstein-Maxwell-Higgs (EMH) equations. To reach our goal, we take advantage of the tools set up in [7] to reduce the problem to that of the resolution of a nonlinear evolution system of two PDE. By comparison with the works in [5, 7] where a single evolution equation were dealt with, supplementary difficulties arise: here the number of terms to estimate is much more higher and some additional terms have to be handled meticulously. By dint of arduous calculations combined with several handy mathematical tools, we implement a fixed point method to arrive, under a more general assumption on the self-interaction potential, at a global existence and uniqueness result for the spherically symmetric EYM system with appropriate initial data. Furthermore we show that this solution decays more slowly than the one obtained in [7]. A thorough examination reveals that this latter phenomenon stems from the presence of the local charge Q . Some questions raised by Chae [5] are thus answered. It could be interesting to see whether the decay properties of the solution mentioned above can be explained by using compactification techniques due to Penrose [24]. In final, we obtained a generalization and improvement of the results in [5, 7]. The main result of the present paper encompasses the EYM system with zero self-interaction potential and the EKG system as well. It is worth noting that this paper is the corrected and detailed version of the Note [25].

The present work is organized as follows. Section 2 is devoted to the derivation of EYM equations under the spherical symmetry assumption. In section 3 we show that the equations reduce to a nonlinear evolution system. In the last section we state and prove the main result of the present investigation.

2 The Einstein-Yang-Mills-Higgs system

2.1 Equations of motion

Throughout the paper, unless otherwise is stated, Einstein convention is used, e.g., $w_c^d h^c = \sum_c w_c^d h^c$. We concentrate on the $\mathfrak{su}(2)$ -EYM and assume that the Yang-Mills field is in the adjoint representation of $\mathfrak{su}(2)$ while the Higgs fields is in the fundamental representation of $\mathfrak{su}(2)$. The basic elements of the $\mathfrak{su}(2)$ -EYM system consist of a quadruplet $(\mathcal{M}, g, A, \Phi)$, where \mathcal{M} is a $4D$ space-time manifold equipped with a lorentzian metric g ; A is a 1-form, called the Yang-Mills potential, defined on \mathcal{M} with values in the Lie algebra $\mathfrak{su}(2)$ of the Lie group $SU(2)$; Φ is a scalar-multiplet field, called the Higgs

field, defined on \mathcal{M} . The fields equations for the $\mathfrak{su}(2)$ –EYMH model read (see [6, 11, 12, 25])

$$\begin{aligned} R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R &= T_{\alpha\beta}, \\ g^{\lambda\mu}(\nabla_\lambda F_{\mu\alpha} + [A_\lambda, F_{\mu\alpha}]) &= J_\alpha, \\ g^{\lambda\mu}\widehat{\nabla}_\lambda\widehat{\nabla}_\mu\Phi &= V'(\Phi^\dagger\Phi)\Phi. \end{aligned} \quad (2.1)$$

Here $g_{\alpha\beta}$ are the components of the space-time metric. $R_{\alpha\beta}$ and R are, respectively, the Ricci tensor and the scalar curvature relative to the space-time metric. $(F_{\alpha\beta})$ represents the Yang-Mills strength field F , which is a $\mathfrak{su}(2)$ -valued antisymmetric 2–form of type Ad , defined on \mathcal{M} . F is related to the unknown Yang-Mills potential A as follows

$$F_{\alpha\beta} = \nabla_\alpha A_\beta - \nabla_\beta A_\alpha + [A_\alpha, A_\beta]. \quad (2.2)$$

∇ denotes the covariant derivative w.r.t. the space-time metric g , $[,]$ denote the Lie brackets of the Lie algebra $\mathfrak{su}(2)$. $\widehat{\nabla}_\alpha\Phi$ is the gauge covariant derivative of the complex doublet Higgs field, defined by

$$\widehat{\nabla}_\alpha\Phi = \frac{\partial\Phi}{\partial x^\alpha} - iA'_\alpha \frac{\sigma_I}{2}\Phi, \quad (2.3)$$

where $(\sigma_I)_{I=1,2,3}$ are the conventional Pauli spin matrices. Φ^\dagger denotes the hermitian conjugate of Φ , i.e., Φ^\dagger is the transpose of Φ^* , where Φ^* is the complex conjugate of Φ . V is a real function defined on $[0, \infty)$, often called the self-interaction potential, with derivative V' . $T_{\alpha\beta}$ are the components of the energy-momentum-stress tensor, given by

$$\begin{aligned} T_{\alpha\beta} &= g^{\rho\sigma}F_{\alpha\rho}\cdot F_{\beta\sigma} - \frac{1}{4}g_{\alpha\beta}g^{\rho\lambda}g^{\sigma\mu}F_{\rho\sigma}\cdot F_{\lambda\mu} \\ &\quad + (\widehat{\nabla}_\alpha\Phi)^\dagger\widehat{\nabla}_\beta\Phi + (\widehat{\nabla}_\beta\Phi)^\dagger\widehat{\nabla}_\alpha\Phi - g_{\alpha\beta}\left(g^{\sigma\rho}(\widehat{\nabla}_\sigma\Phi)^\dagger\widehat{\nabla}_\rho\Phi + V(\Phi^\dagger\Phi)\right), \end{aligned} \quad (2.4)$$

where the dot “ \cdot ” denotes the Ad -invariant non degenerate scalar product of $\mathfrak{su}(2)$ defined by

$$f\cdot k = \sum_{I=1}^3 f^I k^I, \quad f \in \mathfrak{su}(2), \quad k \in \mathfrak{su}(2). \quad (2.5)$$

It is worth noting that the Ad -invariant non degenerate scalar product “ \cdot ” defined in (2.5) enjoys the following property

$$f\cdot [k, l] = [f, k]\cdot l, \quad \forall f, k, l \in \mathfrak{su}(2). \quad (2.6)$$

J_α are the components of the Yang-Mills current, given by

$$J'_\alpha = \Phi^\dagger S^I \widehat{\nabla}_\alpha\Phi - (\widehat{\nabla}_\alpha\Phi)^\dagger S^I\Phi, \quad I = 1, 2, 3, \quad \alpha = 0, 1, 2, 3; \quad (2.7)$$

where

$$S^I = i\frac{\sigma_I}{2}.$$

We work within the basis $(T_I)_{I=1,\dots,3}$ of $\mathfrak{su}(2)$, with

$$T_1 = i\frac{\sigma_1}{2}, \quad T_2 = -i\frac{\sigma_2}{2}, \quad T_3 = i\frac{\sigma_3}{2}.$$

2.2 The spherically symmetric ansätze and fields equations

We will work in a Bondi coordinates system $(x^\alpha) = (u, r, \theta, \varphi)$ on \mathbb{R}^4 , used in series of works by D. Christodoulou [7] and D. Chae [4, 5], where u is a retarded time coordinate and r is a radial coordinate. In this coordinates system the most general form for the spherically symmetric metric could be written as follows

$$ds^2 = -e^{2\nu}du^2 - 2e^{\nu+\lambda}dudr + r^2(d\theta^2 + \sin^2\theta d\varphi^2), \quad (2.8)$$

where v and λ are real functions of u and r only.

The general form for the spherically symmetric Yang-Mills potential in the adjoint representation of $\mathfrak{su}(2)$ could be written as follows (see [6, 11, 12])

$$A = aT_3 du, \quad (2.9)$$

i.e., $A_0 = aT_3$, $A_1 = A_2 = A_3 = 0$, where a is a function of u and r only.

We will use the following ansatz for the spherically symmetric Higgs field in the fundamental representation of $\mathfrak{su}(2)$

$$\Phi = \begin{pmatrix} 0 \\ \psi + i\xi \end{pmatrix}, \quad (2.10)$$

where ψ and ξ are real functions of u and r only.

As noted in [27], writing roughly the EYMH fields equations in the coordinates system (u, r, θ, ϕ) yields a system of PDE which is very difficult to handle. Ahead of overcoming this toughness we introduce, as in [7], a null tetrad $(e_\alpha)_{\alpha=0,\dots,3}$ defined by $e_0 = e^{-v} \frac{\partial}{\partial u} - \frac{1}{2} e^{-\lambda} \frac{\partial}{\partial r}$, $e_1 = e^{-\lambda} \frac{\partial}{\partial r}$, and (e_2, e_3) is a locally defined orthonormal frame on the unit 2-sphere. Denote by $\dot{\cdot}$ differentiation with respect to u and by $'$ differentiation with respect to r , e.g., $\dot{\lambda} = \frac{\partial \lambda}{\partial u}$, $v' = \frac{\partial v}{\partial r}$. After many lengthy calculations (see [27]), the relevant Einstein fields equations $R(e_\alpha, e_\beta) - \frac{1}{2} Rg(e_\alpha, e_\beta) = T(e_\alpha, e_\beta)$ are the following

$$\begin{aligned} \frac{1}{2}(\lambda' + v') - 2e^{\lambda-v} \dot{\lambda} &= 2re^{2(\lambda-v)} \left[\left(\dot{\psi} - \frac{1}{2} a \dot{\xi} \right)^2 + \left(\dot{\xi} + \frac{1}{2} a \dot{\psi} \right)^2 \right] \\ &\quad - 2re^{\lambda-v} \left[\left(\dot{\psi} - \frac{1}{2} a \dot{\xi} \right) \psi' + \left(\dot{\xi} + \frac{1}{2} a \dot{\psi} \right) \xi' \right] \\ &\quad + \frac{1}{2} r \left[(\psi')^2 + (\xi')^2 \right], \end{aligned} \quad (2.11)$$

$$v' - \lambda' + r^{-1} (1 - e^{2\lambda}) + r \left(\frac{1}{2} e^{-2v} (a')^2 + e^{2\lambda} V \right) = 0, \quad (2.12)$$

$$\lambda' + v' = r \left[(\psi')^2 + (\xi')^2 \right], \quad (2.13)$$

$$\begin{aligned} v'' + (v' - \lambda') (v' + r^{-1}) - e^{\lambda-v} \left[\left(\dot{\lambda} \right)' + \left(\dot{v} \right)' + 2 \left(\dot{\psi} - \frac{1}{2} a \dot{\xi} \right) \psi' + 2 \left(\dot{\xi} + \frac{1}{2} a \dot{\psi} \right) \xi' \right] \\ - \frac{1}{2} e^{-2v} (a')^2 + \left[(\psi')^2 + (\xi')^2 \right] + e^{2\lambda} V = 0. \end{aligned} \quad (2.14)$$

The relevant Yang-Mills equations are the following

$$a'' + (2r^{-1} - \lambda' - v') a' - e^{\lambda-v} \left(\left(\dot{a} \right)' - \dot{\lambda} - \dot{v} \right) = e^{2\lambda} \left[\psi \dot{\xi} - \xi \dot{\psi} + \frac{1}{2} a (\psi^2 + \xi^2) \right], \quad (2.15)$$

$$a'' + (2r^{-1} - \lambda' - v') a' = e^{\lambda+v} (\psi \xi' - \xi \psi'). \quad (2.16)$$

The Higgs equations are found to be equivalent to the following system

$$\begin{aligned} 2 \left(\dot{\psi} \right)' - a \xi' - \frac{1}{2} a' \xi + 2r^{-1} \left(\dot{\psi} - \frac{1}{2} a \dot{\xi} \right) - e^{v-\lambda} [\psi'' + \psi' (2r^{-1} + v' - \lambda')] &= -\psi V' e^{\lambda+v}, \\ 2 \left(\dot{\xi} \right)' + a \psi' + \frac{1}{2} a' \psi + 2r^{-1} \left(\dot{\xi} + \frac{1}{2} a \dot{\psi} \right) - e^{v-\lambda} [\xi'' + \xi' (2r^{-1} + v' - \lambda')] &= -\xi V' e^{\lambda+v}. \end{aligned} \quad (2.17)$$

Remark 2.1. (i) It is shown in [26] that if the YM equations are satisfied, then the energy-momentum-stress tensor given by (2.4) is divergence-free, i.e., $g^{\alpha\sigma} \nabla_\sigma T_{\alpha\beta} = 0$. Setting, as in [7], $E_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} Rg_{\alpha\beta} - T_{\alpha\beta}$, and using the contracted Bianchi identity $g^{\alpha\sigma} \nabla_\sigma (R_{\alpha\beta} - \frac{1}{2} Rg_{\alpha\beta}) = 0$, it easily follows that

$$g^{\alpha\sigma} \nabla_\sigma E_{\alpha\beta} = 0, \quad (2.18)$$

provided that the YMH equations are satisfied.

(ii) It is straightforward to check by using (2.18) that, if the Einstein equations (2.12) and (2.13), i.e., $E(e_1, e_1) = 0$ and $E(e_0, e_1) = 0$, are satisfied as well as the YMH equations (2.15), (2.16) and (2.17), then the Einstein equation (2.14) is satisfied, i.e., $E(e_2, e_2) = E(e_3, e_3) = 0$. Furthermore $E(e_0, e_0)$ solves the following PDE

$$[E(e_0, e_0)]' + 2(v' + r^{-1})E(e_0, e_0) = 0,$$

which yields

$$E(e_0, e_0)(u, r) = \left(\frac{r_0}{r}\right)^2 E(e_0, e_0)(u, r_0) \exp[2(v(u, r_0) - v(u, r))].$$

This implies $E(e_0, e_0) = 0$ if regularity is assumed at the center.

(iii) From the above observations we conclude that the Einstein equations (2.12) and (2.13), together with the YMH equations (2.15), (2.16) and (2.17), are equivalent to the full set of EYMH equations (2.23 – 2.29).

3 Reduction of the EYMH equations to a non linear evolution system

We adapt the tools set up and implemented in [7]. Two new functions are introduced as follows:

$$h = (r\psi)', \quad k = (r\xi)'. \quad (3.1)$$

Then

$$\psi = \bar{h} = \frac{1}{r} \int_0^r h(s) ds, \quad \xi = \bar{k} = \frac{1}{r} \int_0^r k(s) ds, \quad \psi' = \frac{h - \bar{h}}{r}, \quad \xi' = \frac{k - \bar{k}}{r}. \quad (3.2)$$

The Einstein equation (2.13) then reads

$$\lambda' + v' = \frac{1}{r} \left[(h - \bar{h})^2 + (k - \bar{k})^2 \right],$$

and the solution, which satisfies the asymptotic condition $\lambda + v \rightarrow 0$ as $r \rightarrow \infty$, is

$$\lambda + v = - \int_r^{+\infty} \frac{1}{s} \left[(h - \bar{h})^2 + (k - \bar{k})^2 \right] ds. \quad (3.3)$$

Integrating the Yang-Mills equation (2.16) yields

$$a(u, r) = \int_0^r e^{\lambda+v} \frac{Q(u, s)}{s^2} ds, \quad (3.4)$$

where Q is the local charge function defined by

$$Q(u, r) = \int_0^r s (\bar{h}k - \bar{k}h)(u, s) ds. \quad (3.5)$$

Now we put the Einstein equation (2.12) under the form

$$(v' - \lambda') e^{v-\lambda} + r^{-1} e^{v-\lambda} = r^{-1} e^{\lambda+v} - r \left(\frac{1}{2} e^{-\lambda-v} (a')^2 + e^{\lambda+v} V \right),$$

which is integrated to give

$$e^{v-\lambda} = \frac{1}{r} \int_0^r e^{\lambda+v} \left[1 - \frac{1}{2} \frac{Q^2}{s^2} - s^2 V \right] ds. \quad (3.6)$$

From (2.17) and (3.1) we recast the Higgs equations into the following non linear evolution system

$$\begin{aligned} \dot{h} - \frac{1}{2} e^{v-\lambda} h' &= \frac{1}{2r} (e^{\lambda+v} - e^{v-\lambda}) (h - \bar{h}) - \frac{Q^2}{4r^3} e^{\lambda+v} (h - \bar{h}) + \frac{Q}{4r} \bar{k} e^{\lambda+v} \\ &\quad - \frac{1}{2} [r(h - \bar{h}) V e^{\lambda+v} + r \bar{h} V' e^{\lambda+v} - a k], \\ \dot{k} - \frac{1}{2} e^{v-\lambda} k' &= \frac{1}{2r} (e^{\lambda+v} - e^{v-\lambda}) (k - \bar{k}) - \frac{Q^2}{4r^3} e^{\lambda+v} (k - \bar{k}) + \frac{Q}{4r} \bar{h} e^{\lambda+v} \\ &\quad - \frac{1}{2} [r(k - \bar{k}) V e^{\lambda+v} + r \bar{k} V' e^{\lambda+v} + a h]. \end{aligned} \quad (3.7)$$

We write system (3.7) in matrix form with unknown vector function $W = \begin{pmatrix} h \\ k \end{pmatrix}$ as follows

$$DW = \frac{1}{2r}(g - \tilde{g})(W - \bar{W}) - \frac{Q^2 g}{4r^3}(W - \bar{W}) + \frac{Qg}{4r}\sigma_1 \bar{W} + \frac{a}{2}i\sigma_2 W - \frac{r}{2}[Vg(W - \bar{W}) + V'g\bar{W}], \quad (3.8)$$

where

$$D = \frac{\partial}{\partial u} - \frac{\tilde{g}}{2} \frac{\partial}{\partial r}, \quad g = e^{\lambda+v}, \quad \tilde{g} = e^{v-\lambda}. \quad (3.9)$$

Remark 3.1. (i) It is worth emphasizing that the integration of the Einstein equations is achieved under the asymptotic condition $\lambda + v \rightarrow 0$, $r \rightarrow \infty$. The Yang-Mills equation (2.16) is integrated easily by classical tools (change of unknown function and variation of the constant) to yield the solution $a(u, r)$ that exists for all $r \in [0, +\infty)$.

(ii) For the Yang-Mills field it is easy to see that it vanishes at spatial infinity i.e., $F(u, r) \rightarrow 0$, $r \rightarrow +\infty$. Actually, by simple calculation, all the components of the Yang-Mills field F vanish except F_{01} which is given by $F_{01} = -a'T_3$. Using the expression of $a(u, r)$ and estimating the local charge $Q(u, r)$ one easily gains $F_{01}(u, r) \rightarrow 0$, $r \rightarrow +\infty$.

(iii) It could be of interest to investigate the case of an asymptotic (anti) de Sitter metric.

4 Existence and uniqueness of global classical solutions

This section aims at stating and proving a global existence and uniqueness result for the initial value problem associated with the non linear evolution system (3.8). In order to do this we begin by introducing the spaces of functions used and some preliminary notations.

4.1 Functional framework, notations, and statement of the main result

For a vector function $W = \begin{pmatrix} h \\ k \end{pmatrix}$ defined on $[0, \infty) \times [0, \infty)$ with the real functions h and k belonging both to $C^1([0, \infty) \times [0, \infty))$, we will just write $W \in C^1([0, \infty) \times [0, \infty))$ instead of $W \in [C^1([0, \infty) \times [0, \infty))]^2$. We will also use the notation $|W| := |h| + |k|$. Consider the initial value problem for system (3.8) with initial datum $W_0(r) = W(0, r)$. Following the works in [5] we define the Banach function spaces $(\mathcal{X}, \|\cdot\|_{\mathcal{X}})$, $(\mathcal{X}_0, \|\cdot\|_{\mathcal{X}_0})$, and $(\mathcal{Y}, \|\cdot\|_{\mathcal{Y}})$ by

$$\begin{aligned} \mathcal{X} &= \{W = W(u, r) \in C^1([0, \infty) \times [0, \infty)) : \|W\|_{\mathcal{X}} < \infty\}, \\ \mathcal{X}_0 &= \{v = v(r) \in C^1([0, \infty)) : \|v\|_{\mathcal{X}_0} < \infty\}, \\ \mathcal{Y} &= \{W = W(u, r) \in C^1([0, \infty) \times [0, \infty)) : W(0, r) = W_0(r), \quad \|W\|_{\mathcal{Y}} < \infty\}, \end{aligned} \quad (4.1)$$

where

$$\begin{aligned} \|W\|_{\mathcal{X}} &:= \sup_{u, r \geq 0} \left\{ (1+r+u)^2 |W(u, r)| + (1+r+u)^3 |W'(u, r)| \right\}, \\ \|v\|_{\mathcal{X}_0} &:= \sup_{r \geq 0} \left\{ (1+r)^2 |v(r)| + (1+r)^3 |v'(r)| \right\}, \\ \|W\|_{\mathcal{Y}} &:= \sup_{u, r \geq 0} \left\{ (1+r+u)^2 |W(u, r)| \right\}. \end{aligned}$$

We are now in the position to state the main result of this paper which is the generalization, as we have mentioned from the outset, of the results obtained in [5, 7].

Theorem 4.1. *Assume for the self-interaction potential V that $V \in C^2([0, \infty))$, and there exists a constant $K_0 \geq 0$ such that*

$$|V(t)| + t|V'(t)| + t^2|V''(t)| \leq K_0 t^{p+1} \quad \forall t \geq 0, \text{ for some } p \geq \frac{3}{2}. \quad (4.2)$$

Suppose for the initial datum W_0 that

$$W_0 \in C^1([0, \infty)), \quad W_0(r) = O(r^{-2}) \quad W_0'(r) = O(r^{-3}). \quad (4.3)$$

Then there exists $\varepsilon > 0$ such that if $\|W_0\|_{x_0} < \varepsilon$, then there exists a unique global classical solution $W \in C^1([0, \infty) \times [0, \infty))$ of (3.8) such that $W(0, r) = W_0(r)$. In addition this solution fulfills the decay property

$$|W(u, r)| \leq C(1 + u + r)^{-2}, \quad |W'(u, r)| \leq C(1 + u + r)^{-3}, \quad (4.4)$$

where $C > 0$ is an increasing continuous function of K_0 . Moreover, the corresponding space-time is time-like and null geodesically complete toward the future.

Remark 4.2. (i) Theorem 4.1 was stated and proved, under more restrictive assumptions, by D. Chae [5] for the EMH model. Notably, in [5], the assumption on p was $p \geq 3$. It is our aim, in this paper, to provide a generalization and improvement of the results obtained in [5, 7] by extending them, under assumption (4.2), to the EYMH model. Note also that, unlike the solution obtained by D. Christodoulou [7] for the spherically symmetric Einstein-Scalar field system, the solution obtained in the present work decays more slowly than that of [7]. This latter fact is attributed, as it will be shown thereafter, to the non-vanishing of the local charge Q .

(ii) Note that assumption (4.2) is satisfied for $V \equiv 0$. Theorem 4.1 then applies to provide global existence and uniqueness of classical solutions of the spherically symmetric EYMH system with vanishing self-interaction potential.

(ii) If Φ is a real doublet of the form $\Phi = \begin{pmatrix} 0 \\ \Psi \end{pmatrix}$, i.e., $\xi \equiv 0$, then the local charge Q as well as the Yang-Mills potential A and the Yang-Mills strength field F vanish (see (3.1), (3.5), (3.4) and (??)). In this case we can choose $V(t) = \frac{t^{p+1}}{p+1}$, $t \geq 0$, so that the Higgs equations reads

$$g^{\lambda\mu} \nabla_\lambda \nabla_\mu \Psi = \Psi^{2p+1}. \quad (4.5)$$

(4.5) is the non linear Klein-Gordon equation that can be reduced to the form

$$Dh = \frac{1}{2r} (g - \tilde{g}) (h - \bar{h}) - \frac{rg}{2} \left[(h - \bar{h}) \frac{(\bar{h})^{2p+2}}{p+1} + (\bar{h})^{2p+1} \right],$$

where

$$\tilde{g} = \frac{1}{r} \int_0^r g \left[1 - s^2 \frac{(\bar{h})^{2p+2}}{p+1} \right] ds.$$

Theorem 4.1 then applies in this situation to encompass global existence and uniqueness of classical spherically symmetric solutions of the non linear EKG system. Moreover, in this case, it turns out that the solutions obtained possess the same order of decay estimates as those obtained by Christodoulou [7].

The same statement holds true if Φ is of the form $\Phi = \begin{pmatrix} 0 \\ i\xi \end{pmatrix}$, i.e., $\Psi \equiv 0$.

4.2 Proof of Theorem 4.1

It is worth noting that some computational details can be found in [27]. Throughout this paragraph, given that $\{I_2, \sigma_1, i\sigma_2, \sigma_3\}$ is a basis for the real vector space of 2×2 real matrices, I_2 being the 2×2 identity matrix, we will use the notation

$$|N| = |n_0| + |n_1| + |n_2| + |n_3|,$$

if N is a 2×2 real matrix with

$$N = n_0 I_2 + n_1 \sigma_1 + n_2 (i\sigma_2) + n_3 \sigma_3.$$

We prove Theorem 4.1 by a contraction mapping argument as in [5, 7, 25] whilst improving and correcting at the same time some key estimates therein. We define the mapping $\mathcal{K} : W \rightarrow w = \mathcal{K}(W)$, where w is the solution of the first order linear initial value problem

$$\begin{aligned} Dw &= \frac{1}{2r} (g - \tilde{g}) (w - \bar{W}) - \frac{Q^2 g}{4r^3} (w - \bar{W}) + \frac{Qg}{4r} \sigma_1 \bar{W} + \frac{a}{2} i \sigma_2 w - \frac{r}{2} [Vg (w - \bar{W}) + V'g\bar{W}], \\ w(0, r) &= W_0(r). \end{aligned} \quad (4.6)$$

Our purpose is to show that the mapping \mathcal{K} defined above is a contraction from a non-empty closed ball of \mathcal{X} into itself. If this is done, then the standard fixed point theorem applies to yield the unique fixed point $W \in \mathcal{X}$ such that $\mathcal{K}(W) = W$, which is the solution of the nonlinear system (3.8) satisfying $W(0, r) = W_0(r)$.

4.2.1 \mathcal{K} is a mapping from a ball of \mathcal{X} into itself

Let $\rho > 0$ be a real number. B_ρ denotes the closed ball, in \mathcal{X} , of radius ρ centered at 0 i.e.,

$$B_\rho = \{W \in \mathcal{X} : \|W\|_{\mathcal{X}} \leq \rho\}.$$

We will prove that ρ can be chosen small enough such that $\mathcal{K} : B_\rho \rightarrow B_\rho$. Let $W \in B_\rho$. We have to estimate $\|\mathcal{K}(W)\|_{\mathcal{X}}$ in terms of $\|W\|_{\mathcal{X}}$. The characteristic system of ODE associated to the initial value problem (4.6) is

$$\begin{aligned} \frac{dr}{du} &= -\frac{1}{2}\tilde{g}, \\ \frac{dw}{du} &= \frac{1}{2r} (g - \tilde{g}) (w - \bar{W}) - \frac{Q^2 g}{4r^3} (w - \bar{W}) + \frac{Qg}{4r} \sigma_1 \bar{W} + \frac{a}{2} i \sigma_2 w - \frac{r}{2} [Vg (w - \bar{W}) + V'g\bar{W}], \end{aligned} \quad (4.7)$$

with initial data

$$r(0) = r_0, \quad w(0) = W_0.$$

Let $r(u) = \gamma(u, r_0)$ be the solution of the initial value problem

$$\frac{dr}{du} = -\frac{1}{2}\tilde{g}(u, r), \quad r(0) = r_0. \quad (4.8)$$

Then

$$r_1 = r_0 - \frac{1}{2} \int_0^{u_1} \tilde{g}(u, \gamma(u, r_0)) du, \quad (4.9)$$

where $r_1 = \gamma(u_1, r_0)$. Integrating the second ODE of (4.7) along γ we obtain

$$w(u_1, r_1) = \exp\left(\int_0^{u_1} [N(u, r)]_{\gamma} dv\right) W(0, r_0) + \int_0^{u_1} \left\{ \exp\left(\int_u^{u_1} [N(u, r)]_{\gamma} dv\right) \right\} [f]_{\gamma} du, \quad (4.10)$$

where the matrix function N and the vector function f are given by

$$\begin{aligned} N(u, r) &= \left(\frac{1}{2r} (g - \tilde{g}) - \frac{Q^2 g}{4r^3} - \frac{r}{2} Vg \right) I_2 + \frac{a}{2} i \sigma_2, \\ f(u, r) &= \left(\frac{Q^2 g}{4r^3} - \frac{1}{2r} (g - \tilde{g}) + \frac{rg}{2} [V - V'] \right) \bar{W} + \frac{Qg}{4r} \sigma_1 \bar{W}. \end{aligned} \quad (4.11)$$

We have to estimate N and f . Setting $\|W\|_{\mathcal{X}} = x$, we derive the following estimates as in [5]

$$|\sigma_1 \bar{W}(u, r)| = |\bar{W}(u, r)| \leq \frac{x}{(1+u)(1+u+r)}, \quad (4.12)$$

$$|W(u, r) - \bar{W}(u, r)| \leq \frac{xr}{2(1+u)(1+u+r)^2}. \quad (4.13)$$

(4.13) implies

$$\int_0^\infty \frac{|W(u, r) - \bar{W}(u, r)|^2}{r} dr \leq \frac{x^2}{24(1+u)^4}. \quad (4.14)$$

In view of (3.3) and the definition of g in (3.9), (4.14) yields

$$g(u, 0) \geq \exp\left(-\frac{x^2}{24(1+u)^4}\right). \quad (4.15)$$

From (4.13) we gain

$$|g(u, r) - \bar{g}(u, r)| \leq \frac{x^2 r^2}{12(1+u)^3(1+u+r)^3}. \quad (4.16)$$

In view of (3.5) and (4.12) we estimate the local charge to get

$$|Q(u, r)| \leq \frac{x^2 r^2}{2(1+u)^2(1+u+r)^2}. \quad (4.17)$$

Hence

$$\frac{1}{r} \int_0^r \frac{|Q(u, s)|^2}{s^2} ds \leq \frac{x^4 r^2}{12(1+u)^5(1+u+r)^3}. \quad (4.18)$$

The term containing the self-interaction potential is estimated using $\Phi^\dagger \Phi = (\bar{h})^2 + (\bar{k})^2$ and assumption (4.2) as

$$\left| \frac{1}{r} \int_0^r s^2 V(\Phi^\dagger \Phi) ds \right| \leq \frac{K_0 x^{2p+2} r^2}{3(1+u)^6(1+u+r)^4}. \quad (4.19)$$

From the definition of \tilde{g} in (3.9) and the estimates (4.16), (4.18), and (4.19) we get

$$|g(u, r) - \tilde{g}(u, r)| \leq \frac{(3 + 8K_0)(x^2 + x^4 + x^{2p+2})r^2}{24(1+u)^5(1+u+r)^3}. \quad (4.20)$$

In view of (4.15), (4.16), and (4.20) we find out that

$$\tilde{g}(u, r) \geq \exp\left(-\frac{x^2}{24}\right) - \frac{x^4}{24} - \frac{K_0 x^{2p+2}}{3}. \quad (4.21)$$

It is easy to see that the function $l = l(x) = \exp\left(-\frac{x^2}{24}\right) - \frac{x^4}{24} - \frac{K_0 x^{2p+2}}{3}$ has a unique positive root x_0 and $l(x) \in (0, 1]$ for all $x \in [0, x_0)$. We now estimate the function f defined in (4.11) as follows.

$$|f(u, r)| \leq \left| \frac{Q^2 g \bar{W}}{4r^3} \right| + \left| \frac{1}{2r} (g - \tilde{g}) \bar{W} \right| + \left| \frac{rg}{2} (V - V') \bar{W} \right| + \left| \frac{Qg}{4r} \sigma_1 \bar{W} \right|. \quad (4.22)$$

Since $0 < g \leq 1$, (4.12) and (4.17) give

$$\left| \frac{Q^2 g \bar{W}}{4r^3} \right| \leq \frac{x^5 r}{16(1+u)^5(1+u+r)^5}, \quad (4.23)$$

and

$$\left| \frac{Qg}{4r} \sigma_1 \bar{W} \right| \leq \frac{x^3 r}{8(1+u)^3(1+u+r)^3}. \quad (4.24)$$

Using (4.12) and (4.20) we gain

$$\left| \frac{1}{2r} (g - \tilde{g}) \bar{W} \right| \leq \frac{(3 + 8K_0)(x^3 + x^5 + x^{2p+3})r}{48(1+u)^6(1+u+r)^4}. \quad (4.25)$$

Assumption (4.2) and the estimate (4.12) imply

$$\left| \frac{rg}{2} (V(\Phi^\dagger \Phi) - V'(\Phi^\dagger \Phi)) \bar{W} \right| \leq \frac{K_0}{2} \left[\frac{x^{2p+3}}{(1+u)^6(1+u+r)^5} + \frac{x^{2p+1}}{(1+u)^4(1+u+r)^3} \right] \quad (4.26)$$

Inserting (4.23), (4.24), (4.25), and (4.26) into (4.22) yields

$$|f(u, r)| \leq \frac{(12 + 56K_0)(x^3 + x^5 + x^{2p+1} + x^{2p+3})}{48(1+u)^3(1+u+r)^2}. \quad (4.27)$$

For $r(u) = \gamma(u, r_0)$ it holds, in view of (4.8) and (4.21), that

$$r = r_1 + \frac{1}{2} \int_u^{u_1} \tilde{g}(s, r(s)) ds \geq r_1 + \frac{l(x)}{2} (u_1 - u). \quad (4.28)$$

Since $l(x) \in (0, 1]$ for all $x \in [0, x_0)$, (4.28) implies

$$1 + u + r \geq 1 + u + r_1 + \frac{l(x)}{2} (u_1 - u) \geq \frac{l(x)}{2} (1 + u_1 + r_1). \quad (4.29)$$

From (4.27) and (4.29) we gain the estimate

$$\int_0^{u_1} |[f]_\gamma| du = \int_0^{u_1} |f(u_1, r_1)| du \leq \frac{(12 + 56K_0)(x^3 + x^5 + x^{2p+1} + x^{2p+3})}{48(1+u_1+r_1)^2 l^2(x)}. \quad (4.30)$$

Let us now estimate the matrix function N defined in (4.11) as follows

$$|N(u, r)| \leq \left| \frac{1}{2r} (g - \tilde{g}) \right| + \left| \frac{Q^2 g}{4r^3} \right| + \left| \frac{r}{2} V g \right| + \left| \frac{a}{2} \right|. \quad (4.31)$$

In view of (4.20) it holds that

$$\left| \frac{1}{2r} (g - \tilde{g})(u, r) \right| \leq \frac{(3 + 8K_0)(x^2 + x^4 + x^{2p+2})}{48(1+u)^5}. \quad (4.32)$$

Since $0 < g \leq 1$, (4.17) yields

$$\left| \frac{Q^2 g}{4r^3}(u, r) \right| \leq \frac{x^4}{16(1+u)^4}. \quad (4.33)$$

From assumption (4.2), since $p \geq \frac{3}{2}$ and $0 < g \leq 1$, we have as in (4.26)

$$\left| \frac{r g(u, r)}{2} V(\Phi^\dagger \Phi) \right| \leq \frac{K_0 x^{2p+2}}{2(1+u)^5}. \quad (4.34)$$

Using (3.4) and (4.17) we estimate $\frac{a}{2}$ as

$$\left| \frac{a(u, r)}{2} \right| \leq \frac{x^2}{2(1+u)^3}. \quad (4.35)$$

Inserting (4.32), (4.33), (4.34), and (4.35) into (4.31) yields

$$|N(u, r)| \leq \frac{(30 + 32K_0)(x^2 + x^4 + x^{2p+2})}{48}. \quad (4.36)$$

From (4.32) we get

$$\int_0^{u_1} \left| \left[\frac{1}{2r} (g - \tilde{g}) \right]_\gamma \right| du \leq \frac{(3 + 8K_0)(x^2 + x^4 + x^{2p+2})}{192}. \quad (4.37)$$

(4.33) yields

$$\int_0^{u_1} \left| \left[\frac{Q^2 g}{4r^3} \right]_\gamma \right| du \leq \frac{x^4}{48}. \quad (4.38)$$

From (4.34) we have

$$\int_0^{u_1} \left| \left[\frac{rg}{2} V \right]_{\gamma} \right| du \leq \frac{K_0 x^{2p+2}}{8}. \quad (4.39)$$

Using (4.35) we gain

$$\int_0^{u_1} \left| \left[\frac{a}{2} \right]_{\gamma} \right| du \leq \frac{x^2}{4}. \quad (4.40)$$

It follows from (4.31), (4.37), (4.38), (4.39), and (4.40) that

$$\left| \int_0^{u_1} [N]_{\gamma} dv \right| \leq K_1 (x^2 + x^4 + x^{2p+2}), \quad (4.41)$$

where

$$K_1 = \frac{55 + 32K_0}{192}.$$

Using (4.9) it holds that, for $x \in [0, x_0]$,

$$1 + r_0 = 1 + r_1 + \frac{1}{2} \int_0^{u_1} \tilde{g}(s, r(s)) ds \geq 1 + r_1 + \frac{l(x)}{2} u_1 \geq \frac{l(x)}{2} (1 + r_1 + u_1). \quad (4.42)$$

Thus, by the definition of the Banach space $(X_0, \|\cdot\|_{X_0})$ (see (4.1)),

$$|W(0, r_0)| \leq \frac{\|W_0\|_{X_0}}{(1+r_0)^2} \leq \frac{4\|W_0\|_{X_0}}{(1+r_1+u_1)^2 l^2(x)}. \quad (4.43)$$

Considering (4.10), (4.30), (4.41), and (4.43), we finally arrive at the following estimate for the solution $w = \mathcal{K}(W)$ of the initial value problem (4.6)

$$|w(u_1, r_1)| \leq \frac{K_2 (x^3 + x^5 + x^{2p+1} + x^{2p+3} + \|W_0\|_{X_0}) \exp(K_1 (x^2 + x^4 + x^{2p+2}))}{(1+r_1+u_1)^2 l^2(x)}, \quad (4.44)$$

where

$$K_2 = \frac{204 + 56K_0}{48}.$$

(4.44) yields

$$\sup_{r, u \geq 0} \left[(1+u+r)^2 |w(u, r)| \right] \leq \frac{K_2 (x^3 + x^5 + x^{2p+1} + x^{2p+3} + \|W_0\|_{X_0}) \exp(K_1 (x^2 + x^4 + x^{2p+2}))}{l^2(x)}. \quad (4.45)$$

We also have to estimate $\sup_{r, u \geq 0} \left[(1+u+r)^3 |w'(u, r)| \right]$. Set

$$z(u, r) = w'(u, r), \quad \text{with } z(0, r_0) = W'(0, r_0).$$

Differentiation of (4.6) w.r.t. r gives

$$Dz = N_1 z + B_1 w + B_2 \bar{W} + B_3 \bar{W}', \quad (4.46)$$

where the matrix functions N_1 , B_1 , B_2 , and B_3 are given by

$$\begin{aligned} N_1 &= \left[\frac{\tilde{g}'}{2} + \frac{g-\tilde{g}}{2r} - \frac{Q^2 g}{4r^3} - \frac{rVg}{2} \right] I_2 + \frac{a}{2} i\sigma_2 = \frac{\tilde{g}'}{2} I_2 + N, \\ B_1 &= \left[\frac{(g-\tilde{g})'}{2r} - \frac{g-\tilde{g}}{2r^2} - \frac{QQ'g}{2r^3} + \frac{3Q^2 g}{4r^4} - \frac{Q^2 g'}{4r^3} - \frac{Vg+r(\Phi^\dagger \Phi)'V'g+rVg'}{2} \right] I_2 + \frac{a'}{2} i\sigma_2, \\ B_2 &= \left[\frac{QQ'g}{2r^3} - \frac{3Q^2 g}{4r^4} + \frac{Q^2 g'}{4r^3} - \frac{(g-\tilde{g})'}{2r} + \frac{(g-\tilde{g})}{2r^2} + \frac{Vg+r(\Phi^\dagger \Phi)'V'g+rVg'}{2} - \frac{gV'+r[V'g'+(\Phi^\dagger \Phi)'V''g]}{2} \right] I_2 \\ &\quad + \left(\frac{Q'g+Qg'}{4r} - \frac{Qg}{4r^2} \right) \sigma_1, \\ B_3 &= \left[\frac{Q^2 g}{4r^3} - \frac{1}{2r} (g-\tilde{g}) + \frac{r(V-V')g}{2} \right] I_2 + \frac{Qg}{4r} \sigma_1. \end{aligned} \quad (4.47)$$

Using the characteristics defined above, the solution of (4.46) reads

$$z(u_1, r_1) = \exp\left(\int_0^{u_1} [N_1]_\gamma dv\right) W'(0, r_0) + \int_0^{u_1} \left\{ \exp\left(\int_u^{u_1} [N_1]_\gamma dv\right) \right\} [f_1]_\gamma du, \quad (4.48)$$

where

$$f_1 = B_1 w + B_2 \bar{W} + B_3 \bar{W}'. \quad (4.49)$$

Using (3.6) we calculate

$$\frac{1}{2} \tilde{g}' = \frac{g - \bar{g}}{2r} - \frac{Q^2 g}{4r^3} - \frac{rVg}{2} + \frac{1}{4r^2} \int_0^r \left(\frac{Q^2 g}{s^2} \right) ds + \frac{1}{2r^2} \int_0^r s^2 V g ds. \quad (4.50)$$

From (4.16), (4.17), (4.18), (4.19), (4.34), and (4.50), we gain the estimate

$$\left| \frac{1}{2} \tilde{g}' \right| \leq \frac{(6 + 32K_0)(x^2 + x^4 + x^{2p+2})}{48}. \quad (4.51)$$

From (4.36) and (4.51), since $N_1 = \frac{1}{2} \tilde{g}' + N$, we gain

$$|N_1(u, r)| \leq \frac{36 + 64K_0}{48} (x^2 + x^4 + x^{2p+2}). \quad (4.52)$$

We now handle f_1 . From (3.3) and the definition of g in (3.9) we have

$$|g'| \leq \frac{x^2 r}{4(1+u)^2(1+u+r)^4}. \quad (4.53)$$

(4.51), and (4.53) give

$$\left| \frac{(g - \tilde{g})'}{2r} \right| \leq \frac{(6 + 16K_0)(x^2 + x^4 + x^{2p+2})}{24(1+u)^2(1+u+r)^3}. \quad (4.54)$$

(4.20) implies

$$\left| \frac{g(u, r) - \tilde{g}(u, r)}{2r^2} \right| \leq \frac{(3 + 8K_0)(x^2 + x^4 + x^{2p+2})}{48(1+u)^5(1+u+r)^3}. \quad (4.55)$$

In view of (3.5) and the definition of $\|W\|_X = x$, we have

$$|Q'(u, r)| \leq \frac{x^2 r}{(1+u)(1+u+r)^3}. \quad (4.56)$$

Since $0 < g \leq 1$, we deduce from (4.17) and (4.56) that

$$\left| \frac{QQ'g}{2r^3} \right| \leq \frac{x^4}{4(1+u)^3(1+u+r)^5}, \quad (4.57)$$

and

$$\left| \frac{3Q^2g}{4r^4} \right| \leq \frac{3x^4}{16(1+u)^4(1+u+r)^4}. \quad (4.58)$$

From (4.17) and (4.53) we obtain

$$\left| \frac{Q^2g'}{4r^3} \right| \leq \frac{x^6}{64(1+u)^6(1+u+r)^6}. \quad (4.59)$$

It is easy to see from (2.10) and (3.2) that

$$\left| (\Phi^\dagger \Phi)' \right| \leq \frac{2}{r} |\bar{W}(u, r)| |W(u, r) - \bar{W}(u, r)|. \quad (4.60)$$

From (4.2), (4.12), (4.13), and (4.60) we derive the following estimates

$$\left| \frac{Vg + r(\Phi^\dagger \Phi)' V'g + rVg'}{2} \right| \leq \frac{K_0 x^{2p+2}}{2(1+u)^5(1+u+r)^5} + \frac{K_0 x^{2p+2}}{2(1+u)^5(1+u+r)^5} + \frac{K_0 x^{2p+4}}{8(1+u)^7(1+u+r)^7}. \quad (4.61)$$

From (3.4) and (4.17) we have

$$\left| \frac{a'}{2} \right| = \left| \frac{Qg}{2r^2} \right| \leq \frac{x^2}{4(1+u)^2(1+u+r)^2}. \quad (4.62)$$

Inserting (4.54), (4.55), (4.57), (4.58), (4.59), (4.61), and (4.62) into the definition of B_1 in (4.47) we obtain

$$|B_1| \leq \frac{K_3(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})}{(1+u)^2(1+u+r)^2}, \quad (4.63)$$

where $K_3 > 0$ is an affine non-decreasing function of K_0 . (4.63) yields

$$|B_1 w(u, r)| \leq \frac{K_3(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})}{(1+u)^2(1+u+r)^4} \sup_{u, r \geq 0} \left[(1+u+r)^2 |w(u, r)| \right]. \quad (4.64)$$

We now estimate $|B_2 \bar{W}(u, r)|$. From (4.2), (4.12), (4.13), (4.53), and (4.60) we get

$$\left| \frac{gV' + r[V'g' + (\Phi^\dagger \Phi)' V''g]}{2} \right| \leq \frac{9K_0(x^{2p} + x^{2p+2})}{8(1+u)^3(1+u+r)^3}. \quad (4.65)$$

(4.17), (4.53), and (4.56) give

$$\left| \frac{Q'g + Qg'}{4r} - \frac{Qg}{4r^2} \right| \leq \frac{13(x^2 + x^4)}{32(1+u)(1+u+r)^2}. \quad (4.66)$$

Inserting (4.54), (4.55), (4.57), (4.58), (4.59), (4.61), (4.65) and (4.66) into the definition of B_2 in (4.47) we obtain

$$|B_2| \leq \frac{K_4(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})}{(1+u)(1+u+r)^2}, \quad (4.67)$$

where $K_4 > 0$ is an affine non-decreasing function of K_0 . (4.12) and (4.67) give

$$|B_2 \bar{W}| \leq \frac{K_4(x^3 + x^5 + x^7 + x^{2p+1} + x^{2p+3} + x^{2p+5})}{(1+u)^2(1+u+r)^3}. \quad (4.68)$$

We now handle $|B_3 \bar{W}'(u, r)|$. From (4.2), (4.17), (4.20) and the definition of B_3 in (4.47) we have

$$|B_3| \leq \frac{(3 + 14K_0)(x^2 + x^4 + x^{2p} + x^{2p+2})}{12(1+u)^2(1+u+r)}. \quad (4.69)$$

From (4.13) and (4.69) we gain

$$|B_3 \bar{W}'(u, r)| \leq \frac{(3 + 14K_0)(x^3 + x^5 + x^{2p+1} + x^{2p+3})}{24(1+u)^3(1+u+r)^3}. \quad (4.70)$$

Inserting (4.64), (4.68), and (4.70) into the definition of f_1 in (4.49), and using (4.45), we gain

$$\begin{aligned} |f_1(u, r)| &\leq \frac{K_2 K_3 (x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4}) (x^3 + x^5 + x^{2p+1} + x^{2p+3} + \|W_0\|_{x_0}) \exp(K_1(x^2 + x^4 + x^{2p+2}))}{(1+u)^2(1+u+r)^4 l^2(x)} \\ &\quad + \frac{K_5(x^3 + x^5 + x^7 + x^{2p+1} + x^{2p+3} + x^{2p+5})}{(1+u)^2(1+u+r)^3}, \end{aligned} \quad (4.71)$$

where $K_5 > 0$ is an affine non-decreasing function of K_0 . Using similar tools as in (4.30) we have

$$\int_0^{u_1} \left| [f_1]_\gamma \right| du \leq \frac{8K_2K_3(x^3+x^5+x^{2p+1}+x^{2p+3}+\|W_0\|_{X_0})(x^2+x^4+x^6+x^{2p}+x^{2p+2}+x^{2p+4}) \exp(K_1(x^2+x^4+x^{2p+2}))}{l^5(x)(1+u_1+r_1)^3} + \frac{8K_5(x^3+x^5+x^7+x^{2p+1}+x^{2p+3}+x^{2p+5})}{l^3(x)(1+u_1+r_1)^3}. \quad (4.72)$$

Since $N_1 = \frac{1}{2}\tilde{g}' + N$, it follows from (4.41) and (4.51) that

$$\left| \int_0^{u_1} [N_1]_\gamma dv \right| \leq K_6(x^2+x^4+x^{2p+2}), \quad (4.73)$$

where $K_6 > 0$ is an affine non-decreasing function of K_0 . By analogy with (4.43) (see (4.1) for the definition of the Banach space $(X_0, \|\cdot\|_{X_0})$), it holds that

$$|W'(0, r_0)| \leq \frac{8\|W_0\|_{X_0}}{(1+r_1+u_1)^3 l^3(x)}. \quad (4.74)$$

Considering (4.72), (4.73), and (4.74) we finally arrive at the following estimate for the solution z of the initial value problem (4.46)

$$\begin{aligned} & |z(u_1, r_1)| \\ & \leq \left[\frac{8\|W_0\|_{X_0}}{(1+r_1+u_1)^3 l^3(x)} + \frac{8K_2K_3(x^3+x^5+x^{2p+1}+x^{2p+3}+\|W_0\|_{X_0})(x^2+x^4+x^6+x^{2p}+x^{2p+2}+x^{2p+4})}{l^5(x)(1+u_1+r_1)^3} + \frac{8K_5(x^3+x^5+x^7+x^{2p+1}+x^{2p+3}+x^{2p+5})}{l^3(x)(1+u_1+r_1)^3} \right] \\ & \quad \times \exp(K_7(x^2+x^4+x^{2p+2})), \end{aligned} \quad (4.75)$$

where $K_7 > 0$ is an affine non-decreasing function of K_0 . From (4.75) we deduce that

$$\begin{aligned} & \sup_{u, r \geq 0} \left[(1+r+u)^3 |z(u, r)| \right] \\ & \leq \left[\frac{8\|W_0\|_{X_0}}{l^3(x)} + \frac{8K_2(x^3+x^5+x^{2p+1}+x^{2p+3}+\|W_0\|_{X_0})K_3(x^2+x^4+x^6+x^{2p}+x^{2p+2}+x^{2p+4})}{l^5(x)} + \frac{8K_5(x^3+x^5+x^7+x^{2p+1}+x^{2p+3}+x^{2p+5})}{l^3(x)} \right] \\ & \quad \times \exp(K_7(x^2+x^4+x^{2p+2})). \end{aligned} \quad (4.76)$$

Since $l(x) \in (0, 1]$ for all $x \in [0, x_0]$, it follows from (4.45) and (4.76) that

$$\begin{aligned} \|w\|_X & \leq \frac{K_2(x^3+x^5+x^{2p+1}+x^{2p+3}+\|W_0\|_{X_0}) \exp(K_9(x^2+x^4+x^{2p+2}))}{l^5(x)} \\ & \quad + \frac{K_8(x^3+x^5+x^7+x^{2p+1}+x^{2p+3}+x^{2p+5}+\|W_0\|_{X_0})(x^2+x^4+x^6+x^{2p}+x^{2p+2}+x^{2p+4}) \exp(K_9(x^2+x^4+x^{2p+2}))}{l^5(x)}, \end{aligned} \quad (4.77)$$

where

$$K_8 = (8 + 8K_4 + 8K_2)K_3, \quad K_9 = K_1 + K_7.$$

Setting

$$\Omega(x) = \frac{x l^5(x) \exp[-K_9(x^2+x^4+x^{2p+2})]}{K_2 + K_8(x^2+x^4+x^6+x^{2p}+x^{2p+2}+x^{2p+4})} - (x^3+x^5+x^7+x^{2p+1}+x^{2p+3}+x^{2p+5}), \quad (4.78)$$

one sees that

$$\Omega(0) = 0, \quad \Omega'(0) > 0, \quad \lim_{x \rightarrow +\infty} \Omega(x) = -\infty.$$

Hence there exists $x_1 \in (0, x_0)$ such that the function Ω is strictly monotonically increasing on $[0, x_1]$. We then choose $x \in (0, x_1)$ and $\|W_0\|_{X_0} < x$ to have $\|w\|_X \leq x$. This shows that \mathcal{K} is a mapping from B_ρ into itself for $\rho \in (0, x_1)$ and $\|W_0\|_{X_0} < \rho$.

4.2.2 The mapping \mathcal{K} contracts in \mathcal{Y}

Let w_1 and w_2 be two solutions of (4.6) with $w_1(0, r) = w_2(0, r) = W_0(r)$, $w_j = \mathcal{K}(W_j)$, $W_j \in \mathcal{X}$, $j = 1, 2$. We use the following notations: $g_j = g(W_j)$, $Q_j = Q(W_j)$, $V_j = V(W_j)$, $z_j = z(W_j)$, $a_j = a(W_j)$. We assume

$$\max \{ \|W_1\|_{\mathcal{X}}, \|W_2\|_{\mathcal{X}} \} \leq x < x_1,$$

and set for convenience

$$w_1 - w_2 = \vartheta, \quad W_1 - W_2 = \Theta, \quad \|\Theta\|_{\mathcal{Y}} = y, \quad D_1 = \frac{\partial}{\partial u} - \frac{\tilde{g}_1}{2} \frac{\partial}{\partial r}.$$

Then we have the following non linear initial value problem with unknown ϑ

$$\begin{aligned} D_1 \vartheta &= N_2 \vartheta + B_4 \bar{\Theta} + B_5 (w_2 - \bar{W}_2) + B_6, \\ \vartheta(0, r) &= 0, \end{aligned} \tag{4.79}$$

where

$$\begin{aligned} N_2 &= \left(\frac{g_1 - \tilde{g}_1}{2r} - \frac{Q_1^2 g_1}{4r^3} - \frac{rV_1 g_1}{2} \right) I_2 + \frac{a_1}{2} i\sigma_2 \\ B_4 &= \left(\frac{Q_1^2 g_1}{4r^3} - \frac{g_1 - \tilde{g}_1}{2r} + \frac{r g_1 (V_1 - V_1')}{2} \right) I_2 + \frac{Q_1 g_1}{4r} \sigma_1 \\ B_5 &= \frac{(g_1 - \tilde{g}_1) - (g_2 - \tilde{g}_2)}{2r} - \frac{(Q_1^2 g_1 - Q_2^2 g_2)}{4r^3} - \frac{r(V_1 g_1 - V_2 g_2)}{2} \\ B_6 &= -\frac{r(V_1' g_1 - V_2' g_2)}{2} \bar{W}_2 + \frac{Q_1 g_1 - Q_2 g_2}{4r} \sigma_1 \bar{W}_2 + \frac{\tilde{g}_1 - \tilde{g}_2}{2} (w_2)' + \frac{a_1 - a_2}{2} i\sigma_2 w_2. \end{aligned} \tag{4.80}$$

Using the characteristic method as in the previous paragraph 4.2.2 we get

$$\vartheta(u_1, r_1) = \int_0^{u_1} \left\{ \exp \left(\int_u^{u_1} [N_2(u, r)]_{\gamma_1} dv \right) \right\} [f_2]_{\gamma_1} du, \tag{4.81}$$

where

$$f_2 = B_4 \bar{\Theta} + B_5 (w_2 - \bar{W}_2) + B_6, \tag{4.82}$$

and γ_1 is the characteristic defined by

$$\frac{dr}{du} = -\frac{1}{2} \tilde{g}_1(u, \gamma_1(u, r)), \quad r(0) = r_0. \tag{4.83}$$

We first estimate N_2 as in (4.41) to get

$$\left| \int_0^{u_1} [N_2]_{\gamma_1} dv \right| \leq K_1 (x^2 + x^4 + x^{2p+2}). \tag{4.84}$$

We now concentrate on estimating f_2 . Similarly to (4.27) it is straightforward to have

$$|B_4 \bar{\Theta}| \leq \frac{(12 + 56K_0) (x^2 + x^4 + x^{2p} + x^{2p+2}) y}{48(1+u)^3(1+u+r)^2}. \tag{4.85}$$

Using the mean value theorem and similar calculation as in (4.13) we have

$$|g_1 - g_2| \leq \frac{xy}{3(1+u)^2(1+u+r)^2}, \tag{4.86}$$

and

$$|\bar{g}_1 - \bar{g}_2| \leq \frac{xy}{3(1+u)^3(1+u+r)}. \tag{4.87}$$

In the same way, using the mean value theorem and similar calculation as in (4.12) we have

$$|Q_1 - Q_2| \leq \frac{xyr^2}{(1+u)^2(1+u+r)^2}. \tag{4.88}$$

From (4.17), (4.86), (4.87), and (4.88) we have

$$|Q_1^2 g_1 - Q_2^2 g_2| \leq \frac{13(x^3 + x^5)yr^4}{12(1+u)^4(1+u+r)^4},$$

which implies

$$\left| \frac{(Q_1^2 g_1 - Q_2^2 g_2)}{4r^3} \right| \leq \frac{13(x^3 + x^5)yr}{48(1+u)^4(1+u+r)^4}. \quad (4.89)$$

From (3.6) and the definition of \tilde{g} in (3.9) we have

$$|\tilde{g}_1 - \tilde{g}_2| \leq |\bar{g}_1 - \bar{g}_2| + \frac{1}{2r} \int_0^r \frac{|Q_1^2 g_1 - Q_2^2 g_2|}{s^2} ds + \frac{1}{r} \int_0^r s^2 |V_1 g_1 - V_2 g_2| ds. \quad (4.90)$$

(4.89) yields

$$\frac{1}{2r} \int_0^r \frac{|Q_1^2 g_1 - Q_2^2 g_2|}{s^2} ds \leq \frac{13(x^3 + x^5)yr^2}{72(1+u)^5(1+u+r)^3}. \quad (4.91)$$

Using the mean value theorem and assumption (4.2) we get

$$|V_1 - V_2| \leq \frac{4K_0 x^{2p+1} y}{(1+u)^4(1+u+r)^4}. \quad (4.92)$$

(4.2), (4.86), and (4.92) give

$$|V_1 g_1 - V_2 g_2| \leq \frac{(1+12K_0)(x^{2p+1} + x^{2p+3})y}{3(1+u)^4(1+u+r)^4}. \quad (4.93)$$

Thus

$$\frac{1}{r} \int_0^r s^2 |V_1 g_1 - V_2 g_2| ds \leq \frac{(1+12K_0)(x^{2p+1} + x^{2p+3})r^2 y}{9(1+u)^5(1+u+r)^3}. \quad (4.94)$$

It follows from (4.87), (4.90), (4.91), and (4.94) that

$$|\tilde{g}_1 - \tilde{g}_2| \leq \frac{(15+32K_0)(x+x^3+x^5+x^{2p+1}+x^{2p+3})y}{24(1+u)^3(1+u+r)}. \quad (4.95)$$

By analogy with (4.16) it holds that

$$|g_1 - g_2 - (\bar{g}_1 - \bar{g}_2)| \leq \frac{2xyr}{(1+u)^3(1+u+r)^2}. \quad (4.96)$$

Combination of (4.91), (4.94), and (4.96) yields

$$\left| \frac{(g_1 - \tilde{g}_1) - (g_2 - \tilde{g}_2)}{2r} \right| \leq \frac{(31+32K_0)(x+x^3+x^5+x^{2p+1}+x^{2p+3})y}{24(1+u)^3(1+u+r)^2}. \quad (4.97)$$

In view of (4.89), (4.93), (4.97), and the definition of B_5 in (4.80) we gain

$$|B_5| \leq \frac{(83+160K_0)(x+x^3+x^5+x^{2p+1}+x^{2p+3})y}{24(1+u)^3(1+u+r)^2}. \quad (4.98)$$

From (4.12) and (4.98) we obtain

$$|B_5(w_2 - \bar{w}_2)| \leq \frac{(83+160K_0)(x^2+x^4+x^6+x^{2p+2}+x^{2p+4})y}{12(1+u)^3(1+u+r)^4}. \quad (4.99)$$

We now estimate B_6 . Using once more the mean value theorem and assumption (4.2) we gain

$$|V'_1 - V'_2| \leq \frac{4K_0 x^{2p-1} y}{(1+u)^2 (1+u+r)^2}. \quad (4.100)$$

(4.2), (4.86), and (4.100) imply

$$|V'_1 g_1 - V'_2 g_2| \leq \frac{(1+12K_0)(x^{2p-1} + x^{2p+1})y}{3(1+u)^2 (1+u+r)^2}. \quad (4.101)$$

(4.12), and (4.101) yield

$$\left| \frac{r(V'_1 g_1 - V'_2 g_2)}{2} \overline{W}_2 \right| \leq \frac{(1+12K_0)(x^{2p} + x^{2p+2})y}{6(1+u)^3 (1+u+r)^2}. \quad (4.102)$$

From (4.12), (4.17), (4.86), and (4.88) we derive the following estimate

$$\left| \frac{Q_1 g_1 - Q_2 g_2}{4r} \sigma_1 \overline{W}_2 \right| \leq \frac{7(x^2 + x^4)y}{24(1+u)^3 (1+u+r)^2}. \quad (4.103)$$

In view of (4.95) we have

$$\left| \frac{\tilde{g}_1 - \tilde{g}_2}{2} (w_2)' \right| \leq \frac{(15+32K_0)(x^2 + x^4 + x^6 + x^{2p+2} + x^{2p+4})y}{48(1+u)^3 (1+u+r)^4}. \quad (4.104)$$

We use (3.4), (4.86), and (4.88) to gain

$$|a_1 - a_2| \leq \frac{19(x+x^3)y}{18(1+u)^3}. \quad (4.105)$$

Since $\max\{\|W_1\|_X, \|W_2\|_X\} \leq x < x_1$, (4.105) implies

$$\left| \frac{a_1 - a_2}{2} i \sigma_2 w_2 \right| \leq \frac{19(x^2 + x^4)y}{36(1+u)^3 (1+u+r)^2}. \quad (4.106)$$

Inserting (4.102), (4.103), (4.104), and (4.106) into the definition of B_6 in (4.80) we obtain

$$|B_6| \leq \frac{(187+384K_0)(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})y}{144(1+u)^3 (1+u+r)^2}. \quad (4.107)$$

Summing up (4.85), (4.99), and (4.107), considering the definition of f_2 in (4.82), we arrive at

$$|f_2| \leq \frac{(1219+2472K_0)(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})y}{144(1+u)^3 (1+u+r)^2}. \quad (4.108)$$

Thus, using the same tools as in (4.30), we deduce that

$$\int_0^{u_1} |[f_2]_{\gamma_1}| du \leq \frac{(1219+2472K_0)(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})y}{144(1+u_1+r_1)^2 l^2(x)}. \quad (4.109)$$

Insertion of (4.84) and (4.109) into (4.81) yields

$$|\vartheta(u_1, r_1)| \leq \frac{(1219+2472K_0)(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4})y \exp(K_1(x^2 + x^4 + x^{2p+2}))}{144(1+u_1+r_1)^2 l^2(x)}. \quad (4.110)$$

Hence

$$\|\vartheta\|_{\mathcal{Y}} \leq \Xi(x)y, \quad (4.111)$$

where

$$\Xi(x) = \frac{(1219 + 2472K_0)(x^2 + x^4 + x^6 + x^{2p} + x^{2p+2} + x^{2p+4}) \exp(K_1(x^2 + x^4 + x^{2p+2}))}{144l^2(x)}. \quad (4.112)$$

It is easy to see that the function Ξ given in (4.112) is strictly monotonically increasing on $[0, x_1]$ and $\Xi(0) = 0$. This shows that there exists $x_2 \in (0, x_1]$ such that $\Xi(x) < \frac{1}{2}$ for all $x \in (0, x_2]$. Thus, in view of (4.111), the mapping $W \rightarrow \mathcal{K}(W)$ contracts in \mathcal{Y} for $\|W\|_X \leq x_2$. This concludes the proof of the global existence and uniqueness of classical solution of (3.8).

The decay property (4.4) of the solution is a direct consequence of the definition (4.1) of the Banach spaces $(X, \|\cdot\|_X)$ and $(\mathcal{Y}, \|\cdot\|_{\mathcal{Y}})$.

Now, from (3.3) and (4.14) one deduces easily that, for each $r \geq 0$, $g \rightarrow 1$ if $u \rightarrow \infty$. In view of (4.20), this implies that, for each $r \geq 0$, $\tilde{g} \rightarrow 1$ if $u \rightarrow \infty$. So, as $u \rightarrow \infty$, the metric given in Bondi coordinates by (2.8) becomes the Minkowski metric. With this, we are done with the proof of Theorem 4.1.

Remark 4.3. (i) Theorem 4.1 was stated and proved, under the more restrictive assumption $p \geq 3$, by Chae [5] for the EMH system. Note that the solution obtained here decays more slowly than that of [7] concerning the spherically symmetric massless Einstein-scalar field system. We found out that this latter fact stems essentially from the estimate (see (4.17))

$$\left| \frac{Q}{r} \right| \leq \frac{x^2}{2(1+u)^2(1+u+r)},$$

due to the non-vanishing of the local charge Q . Some questions raised in [5] are thus answered. It would be interesting to find out whether one can use conformal compactification methods of Penrose [24] to explain slow decaying of the solution via extension by continuity to conformal null infinity.

(ii) Theorem 4.1 easily applies so as to encompass global existence and uniqueness of classical solutions of the spherically symmetric EYM system with vanishing self-interaction potential V and those of the non linear EKG system as well. Moreover, in the latter case, it turns out that the solutions possess the same order of decay estimates as those obtained in [7].

(iii) It is worth mentioning that assumption (4.2) is not fulfilled by the (classical) self-interaction potential $V(t) = t^2$.

Acknowledgement. C Tadmon wishes to express sincere thankfulness to Professor Mamadou Sango for warm welcome at the University of Pretoria where this work was finalized at the beginning of his postdoctoral fellowship.

References

- [1] BARTNIK, R. AND MCKINNON, J., Particlelike solutions of the Einstein-Yang-Mills equations, *Phys. Rev. Lett.* **61**, 141-144 (1988).
- [2] BREITENLOHNER, P., FORGÁCS, P. AND MAISON, D., Static cosmological solutions of the Einstein-Yang-Mills-Higgs equations, *Phys. Lett. B* **489**, 397-402 (2000).
- [3] BRIHAYE, Y. AND DESOIL, M., Gravitating (bi-)sphalerons, *Phys. Lett. A* **15**, 889-900 (2000).
- [4] CHAE, D., Global existence of spherically symmetric solutions to the coupled Einstein and non-linear Klein-Gordon system, *Class. Quantum Grav.* **18**, 4589-4605 (2001).
- [5] CHAE, D., Global existence of solutions to the coupled Einstein and Maxwell-Higgs system in the spherical symmetry, *Ann. Henri Poincaré* **4**, 35-62 (2003).

-
- [6] CHOQUET-BRUHAT, Y., Yang-Mills-Higgs fields in three space time dimensions, *Mém. Soc. Math. France* **46**, 73-97 (1991).
- [7] CHRISTODOULOU, D., The problem of a self-gravitating scalar field, *Commun. Math. Phys.* **105**, 337-361 (1986).
- [8] CLÉMENT, G., Einstein-Yang-Mills-Higgs solitons, *Gen. Rel. Grav.* **13**, 763-770 (1981)
- [9] CORDERO, P., Canonical formulation of the spherical symmetric Einstein-Yang-Mills-Higgs system for a general gauge group, *Ann. Phys.* **108**, 79-988 (1977).
- [10] CREMER, E. AND SCHREK, J., Spontaneous compactification of space in Einstein-Yang-Mills-Higgs model, *Nucl. Phys. B* **108**, 409-416 (1976).
- [11] DOSSA, M. AND TADMOM, C., The Goursat problem for the Einstein-Yang-Mills-Higgs system in weighted Sobolev spaces, *C. R. Acad. Sci. Paris, Ser. I* **348**, 35-39 (2010).
- [12] DOSSA, M. AND TADMOM, C., The characteristic initial value problem for the Einstein-Yang-Mills-Higgs system in weighted Sobolev spaces, *Appl. Math. Res. Express* **2010** (2), 154-231 (2010).
- [13] EARDLEY, D. M. AND MONCRIEF, V., The global existence of Yang-Mills-Higgs field in 4-dimensional Minkowski space. I. Local existence and smoothness properties, *Commun. Math. Phys.* **83**, 171-191 (1982).
- [14] EARDLEY, D. M. AND MONCRIEF, V., The global existence of Yang-Mills-Higgs field in 4-dimensional Minkowski space. II. Completion of the proof , *Commun. Math. Phys.* **83**, 193-212 (1982).
- [15] EARDLEY, D. M. AND MONCRIEF, V., The global existence problem and cosmic censorship in general relativity, *Gen. Rel. Grav.* **13**, 887-897 (1981).
- [16] FORGÁCS, P. AND REUILLON, S., Spatially compact solutions and stabilization in Einstein-Yang-Mills-Higgs theories, *Phys. Rev. Lett.* **95**, 061101 (2005).
- [17] GAL'TSOV, D. V. AND DONETS, E. E., Power-law mass inflation in Einstein-Yang-Mills-Higgs black holes, *C. R. Acad. Sci. Paris, Ser. B* **325**, 649-657 (1997).
- [18] HAJICEK, P., Wormhole solutions in the Einstein-Yang-Mills-Higgs system I. General Theory of the zero-order structure, *Proc. R. Soc. Lond. A* **386**, no. 1790, (1983) 223-240.
- [19] HAJICEK, P., Wormhole solutions in the Einstein-Yang-Mills-Higgs system II. Zero-order structure for $G = SU(2)$, *J. Phys. A* **16**, no. 6, (1983) 1191-1205.
- [20] HARTMANN, B., KLEIHAUS, B. AND KUNZ, J., Axially symmetric monopoles and black holes in Einstein-Yang-Mills-Higgs theory, *Phys. Rev. D* **65**, 024027 (2001).
- [21] KLEIHAUS, B. AND KUNZ, J., Monopole-antimonopole solutions of Einstein-Yang-Mills-Higgs theory, *Phys. Rev. Lett.* **85**, 2430-2433.
- [22] KÜNZLE, H. P. AND OLIYNYK, T.A., Spherical symmetry of generalized Einstein-Yang-Mills-Higgs fields, *J. Geom. Phys.* **56**, 1856-18774 (2006).
- [23] PATURYAN, V., RADU, E. AND TCHRAKIAN, D.H., Rotating regular solutions in Einstein-Yang-Mills-Higgs theory, *Phys. Lett. B* **609**, 360-366 (2005).
- [24] PENROSE, R., Conformal treatment of infinity, *Relativity, Groups and Topology*, 565-584 (1964); republished in *Gen. Relativ. Gravit.* **43**, 901-922 (2011).

- [25] TADMON, C., Global solutions, and their decay properties, of the spherically symmetric $\mathfrak{su}(2)$ -Einstein-Yang-Mills-Higgs equations, *C. R. Acad. Sci. Paris, Ser. I* **349**, 1067-1072 (2011).
- [26] TADMON, C., Construction of initial data associated to the characteristic initial value problem for the Einstein-Yang-Mills-Higgs system, *arXiv: 1203.2423v1 [gr-qc]* (2012).
- [27] TADMON, C. AND TCHAPNDA, S. B., On the spherically symmetric Einstein-Yang-Mills-Higgs equations in Bondi coordinates, *arXiv: 1203.3291v1 [gr-qc]* (2012).
- [28] VACARU, S., Decoupling of Einstein-Yang-Mills-Higgs equations, off-diagonal solutions and black ellipsoids and solitons, *arXiv: 1108.2022 v2 [gr-qc]* (2011).
- [29] WINSTANLEY, E. AND MAVROMAOS, N.E., Instability of hairy black holes in spontaneously broken Einstein-Yang-Mills-Higgs systems, *Phys. Lett. B* **352**, 242-246 (1995).