Production Lags And Growth Dynamics In An Overlapping Generations Endogenous Growth Model

Rangan Gupta, University of Pretoria, South Africa

ABSTRACT

This paper analyzes growth dynamics in an endogenous growth overlapping generations model characterized by production lags in the firm-specific and average economy-wide capital inputs, with the growth process being endogenized by allowing for a production externality. We show that endogenous convergent fluctuations emerge, with the convergence being faster for higher values of the marginal product of labor, given the initial value of the gross growth rate - a result, otherwise impossible, if the production is a function of contemporaneous capital stock. Finally, when production is a function of lagged labor, as well as lagged capital inputs, steady-state is infeasible.

Keywords: Endogenous fluctuations; Overlapping generations; Production lags

INTRODUCTION

paper by Chetty and Ratha (1994) highlighted the crucial role that time-structure in the production process could play in the determination of growth process.[1] The authors indicated that in a Diamond (1965)-type overlapping generations model with production lags, growth is feasible if borrowing is for capital services and capital productivity is sufficiently high. However, if labor services is to be financed as well - in other words, wages are to be paid in advance - then steady growth is infeasible, with capital being fully owned by the old.

Against this backdrop, this paper revisits the importance of time structure in production, using an overlapping generations model characterized by endogenous growth and, in turn, analyzes the resulting growth dynamics. In this case, the endogeneity in the growth process is obtained from a Romer (1986)-type production technology exhibiting increasing returns.[2] The motivation to extend the analysis of Chetty and Ratha (1994), whose Solow (1956)-type economy exhibited exogenous growth (no growth; i.e., stationary steady-state), captured by growth rate of the population (when there was no growth in population), in the steady-state, into an endogenous growth model, was mainly to account for empirical evidence of non-zero endogenously determined per-capita growth rates experienced by economies in general. It must be noted that though the results of Chetty and Ratha (1994) were independent of the assumptions on the production structure, they were essentially concerned with existence of the steady-state in a Solow (1956)-type model, and hence, no implications for endogenous growth models were accounted for. To the best of our knowledge, this is the first attempt to look at the role played by timing in the production structure in an overlapping generations framework yielding balanced growth in the steady-state.

---

[1] Also refer to Chetty (1990) and Cherty and Ratha (1991) for some extensions in the context of two-sector models.
[2] It is, however, easy to show that our results also hold for a Barro (1990)-type production function, where the growth process is endogenized via a productive role for public expenditures.

ECONOMIC ENVIRONMENT

Time is divided into discrete segments and is indexed by $t = 1, 2,...$ There are three theaters of economic activities:

1. Each two-period lived overlapping generations household (consumer/worker) is endowed with one unit of labor when young, with the agent retiring when old. Thus, at each date t, there are two coexisting generations of young and old. N people are born at each time point $t \geq 1$. At $t = 1$, there exist N people in the economy, called the initial old, who live for only one period. The young earn their wage by inelastically supplying one unit of the labor endowment and deposits it into banks for future consumption. The agent consumes only when old[3].
2. The bank simply converts one period of deposit contracts into loans. No resources are assumed to be spent in running the banks.
3. Each infinitely-lived producer is endowed with a production technology to manufacture a single final good using the inelastically-supplied labor and physical capital (financed by credit provided by financial intermediaries) and economy-wide per-capita capital stock.

The sequence of events can be outlined as follows: When a young household works and receives pre-paid wages, the agent deposits it into banks. A bank provides a loan to a goods producer, which subsequently manufactures the final good and returns the loan with interests. Finally, the banks pay back the deposits, with interest, to the households at the end of the first period and the latter consumes it in the second period.

Consumers

Each consumer possesses a unit of time endowment (n_t) which is supplied inelastically earning a real-wage (w_t) which is saved as bank deposits (d_t). The consumer is retired when old and consumes (c_{t+1}) out of the investment of young age savings. Consumers are alike, so there exists a representative consumer in each generation.

Formally, the representative young agent's problem (born in period $t \geq 1$) can be described as follows:

$$\max_{c_{t+1}} U_t = u(c_{t+1}) \tag{1}$$

where U is assumed to be, though not necessary, twice differentiable; $u' > 0$; $u'' < 0$ and $u'(0) = \infty$. This utility function is maximized, subject to the following constraints:

$$d_t \leq w_t n_t \tag{2}$$

$$c_{t+1} \leq (1 + r_{dt+1})d_t \tag{3}$$

with r_{dt} being the real interest rate on deposit at time t.

Financial Intermediaries

The banks perform a simple pooling function. We are implicitly assuming that consumers individually

---

[3]This assumption makes computation easily tractable and is not a bad approximation of the real world (see Hall, 1988; Bhattacharya and Qiao, 2007). Note, this makes the analysis independent of the consumers utility function. However, our results continue to hold even if we allow the consumer to consume in both periods and derive utility from a Constant Relative Risk Aversion (CRRA)-type function.

cannot finance the firms' investment needs directly, as it requires a threshold level of loan supply. At the start of each period, the financial intermediaries accept deposits and loans out to the producers with a goal of maximizing profits subject to the feasibility constraint. At the end of the period, they receive their interest income from the loans made and they meet the interest obligations on the deposits. Given such a structure, the intermediaries obtain the optimal choice for loans by solving the following problem:

$$\max_{l,d} \pi_b = r_{lt}l_t - r_{dt}d_t \tag{4}$$

$$s.t.: l_t \le d_t \tag{5}$$

where π_b is the profit function for the financial intermediary; l_t is the size of real loans, and; r_{lt} is the real interest rate on loans. Free entry drives profits to zero and we have:

$$r_{lt} - r_{dt} = 0 \tag{6}$$

since profit maximization implies $l_t = d_t$; i.e., no leftover deposits.

Firms

We will first consider the standard case of output being produced by contemporaneous input requirements. All firms are identical and produce a single final good using a Romer (1986)-type production technology with output (y_t) being produced using current labor and current capital input, both firm-specific and per-capita economy-wide, such that:

$$y_t = Ak_t^\alpha (n_t \overline{k_t})^{(1-\alpha)} \tag{7}$$

where A > 0; $0 < \alpha((1-\alpha)) < 1$, is the elasticity of output with respect to lagged capital (labor or average economy-wide lagged capital), with k_t, n_t and $\overline{k_t}$ respectively denoting capital at time t, labor at time t and per-capita average capital stock at t. At time t, the final good can either be consumed or stored. We assume that producers are able to convert bank loans l_t into fixed capital formation such that $p_t i_{kt} = L_t$, where i_t denotes the investment in physical capital. In each of the respective technologies, the production transformation schedule is linear so that the same technology applies to both capital formation and the production of the consumption good, and hence both investment and consumption goods sell for the same price, p. We follow Diamond and Yellin (1990) and Chen *et al.* (2008) in assuming that the goods producer is a residual claimer; that is, the producer uses up the unsold consumption good in a way which is consistent with lifetime value maximization of the firms. This assumption regarding ownership avoids the "unnecessary" Arrow-Debreu redistribution from firms to consumers and simultaneously retains the general equilibrium structure of the models.

The representative firm, at any point of time t, maximizes the discounted stream of profit flows subject to the capital evolution and loan constraint. Formally, the problem of the firm can be outlined as follows:

$$\max_{k_{t+1},n_t} \sum_{i=0}^{\infty} \rho^i [Ak_t^\alpha (n_t \overline{k_t})^{(1-\alpha)} - w_t n_t - (1+r_{lt})l_t] \tag{8}$$

$$k_{t+1} \le (1-\delta_k)k_t + i_{kt} \tag{9}$$

$$i_{kt} = l_t \tag{10}$$

where ρ is the firm owners (constant) discount factor and δ_k is the (constant) rate of capital depreciation. The firm solves the above problem to determine the demand for labor and investment.

The firm's problem can be written in the following recursive formulation:

$$V(k_t) = \max_{n,k}[Ak_t^\alpha(n_t\overline{k_t})^{(1-\alpha)} - w_t n_t - (1+r_{lt})(k_{t+1} - (1-\delta_k)k_t)] + \rho V(k_{t+1}) \tag{11}$$

The upshot of the above dynamic programming problem are the following first order conditions:

$$k_{t+1} : (1+r_{Lt}) = \rho V'(k_{t+1}) \tag{12}$$

$$n_t : A(1-\alpha)k_t^\alpha n_t^{-\alpha}\overline{k_t}^{(1-\alpha)} = w_t \tag{13}$$

And the following envelope condition:

$$V'(k_t) = [A\alpha k_t^{(\alpha-1)}(n_t\overline{k_t})^{(1-\alpha)} + (1+r_{lt})(1-\delta_k)] \tag{14}$$

Optimization leads to the following efficiency condition, besides (17), for the production firm:.

$$(1+r_{lt}) = \rho[\alpha A k_{t+1}^{(\alpha-1)}(n_{t+1}\overline{k_{t+1}})^{(1-\alpha)} + (1+r_{lt+1})(1-\delta_k)] \tag{15}$$

Equation (15) provides the condition for the optimal investment decision of the firm. The firm compares the cost of increasing investment in the current period with the future stream of benefit generated from the extra capital invested in the current period, and equation (13) simply states that the firm hires labor up to the point where the marginal product of labor equates the real wage.

Let us now consider the case when output is produced using current labor, but the capital input, both firm-specific and per-capita economy-wide, has to be applied one year earlier, such that:

$$y_t = Ak_{t-1}^\alpha(n_t\overline{k_{t-1}})^{(1-\alpha)} \tag{16}$$

The firm's problem now takes the following recursive formulation:

$$V(k_{t-1}) = \max_{n,k}[Ak_{t-1}^\alpha(n_t\overline{k_{t-1}})^{(1-\alpha)} - w_t n_t - (1+r_{lt-1})(k_t - (1-\delta_k)k_{t-1})] + \rho V(k_t) \tag{17}$$

yielding the following set of optimal conditions:

$$n_t : A(1-\alpha)k_{t-1}^\alpha n_t^{-\alpha}\overline{k_{t-1}}^{(1-\alpha)} = w_t \tag{18}$$

$$k_t : (1+r_{lt-1}) = \rho[\alpha A k_t^{(\alpha-1)}(n_{t+1}\overline{k_t})^{(1-\alpha)} + (1+r_{lt})(1-\delta_k)] \tag{19}$$

EQUILIBRIUM

A valid perfect-foresight, competitive equilibrium for this economy is a sequence of real interest rates $\{r_{dt}, r_{lt}\}_{t=0}^{\infty}$, allocations $\{c_{t+1}, n_t, i_{kt}\}_{t=0}^{\infty}$, stocks of financial assets $\{d_t, l_t\}_{t=0}^{\infty}$ such that:

- Taking r_{dt+1} and w_t, the consumer maximizes utility (1) such that (2) and (3) holds.

- Banks maximize profits, taking r_{lt} and r_{dt} as given and such that (6) holds.

- The real allocations solve the firm's date–t profit maximization problem, given real interest rate, such that (13) and (15) and (18) and (19) holds given the production structures outlined by (8) and (16), respectively.

- The loanable funds market, the goods market, and the labor market clear for all $t0$;

- d_t, l_t, r_{dt} and r_{lt} must be positive at all dates.

GROWTH DYNAMICS

In this section, we analyze the (possible) growth dynamics obtained from the model under the two alternative production functions given by (7) and (16). Using equations (2), (5), (9), (10) and (13), and the fact that in equilibrium $n_t = 1$ and $k_t = \overline{k}_t$, we obtain the following expression for the gross (balanced) growth rate (Ω_{t+1}) under the production function with no lags in the capital input requirements:

$$\Omega_{t+1} = A(1-\alpha) + (1-\delta_k) \tag{20}$$

Clearly, as can be seen from (20), there exists a unique balanced growth equilibrium for the economy, with no growth dynamics. While, when we allow for lags in both the firm-specific and average economy-wide capital stock, the corresponding expression for Ω_{t+1} is given as follows:

$$\Omega_{t+1} = \frac{A(1-\alpha)}{\Omega_t} + (1-\delta_k) \tag{21}$$

where Ω_{t+1} is now captured by a function (f) of Ω_t, i.e, $\Omega_{t+1} = f(\Omega_t)$, with the function f satisfying: (a) $f'(\Omega) \left(= -\frac{A(1-\alpha)}{\Omega^2}\right) < 0$; (b) $|f'(\Omega)| < 1$; (c) $\lim_{\Omega \to 0} f'(\Omega) = \infty$; (d) $\lim_{\Omega \to \infty} f'(\Omega) = 0$; (e) $\lim_{\Omega \to 0} f(\Omega) = (1-\delta_k)$, and; (f) $\lim_{\Omega \to \infty} f(\Omega) = 0 > 0$. The balanced growth equilibrium (21) can be written in a quadratic form, with the roots: $\Omega = \frac{(1-\delta_k) \pm \sqrt{(1-\delta_k)^2 + 4A(1-\alpha)}}{2}$. Concentrating on only positive values for the gross growth rate, in steady-state, balanced growth will be equal to the positive root. Note, given that $f'(\Omega) < 0$ and $|f'(\Omega)| < 1$, the economy will exhibit convergent endogenous fluctuations with the convergence being faster for higher values of the marginal product of labor, given the initial value of the gross growth rate.

Chetty and Ratha (1994) showed that when the production structure involved lagged labor, besides lagged capital, there is no equilibrium possible due to the violation of the feasibility condition in the loans market. Note now that consumer savings should not only finance capital investment, but also the wage bill. Understandably, given equation (2), this is infeasible. In other words, irrespective of whether we have growth (exogenous or endogenous) or no-growth in the long-run equilibrium, this result of Chetty and Ratha (1994) continues to hold. The readers are referred to this paper for further details.

CONCLUSION

Chetty and Ratha (1994) highlighted the crucial role that time-structure in the production process could play in the determination of growth process in a Solow (1956)-type growth model. This paper revisits the issue, using an overlapping generations model characterized by endogenous growth and, in turn, analyzes the resulting growth dynamics. We show that when there exist production lags in both firm-specific and average economy-wide capital inputs, in an otherwise Romer (1986)-type increasing returns production function, endogenous convergent fluctuations emerge. Further, the speed of the convergence is determined by the marginal product of labor, given the initial value of the gross growth rate - a result, otherwise impossible, if production is a function of contemporaneous capital stock. Finally, when production is a function of lagged labor, as well as lagged capital inputs, there is no long-run balanced growth equilibrium for the economy due to the violation of the feasibility condition in the loanable funds market. Note that an immediate empirical application of the main result of this paper, relating to endogenous fluctuations, would be to test for the importance of lagged capital input relative to its concurrent counterpart in a production function, based on cross-country evidence. Such an empirical analysis might help one to understand the origin of often observed growth fluctuations (Bhattacharya and Qiao, 2007; Gupta and Vermeulen, 2010) by relating it to the role of lagged capital input.

ACKNOWLEDGEMENT

AUTHOR INFORMATION

Rangan Gupta is currently a Professor at the Department of Economics, University of Pretoria. After having completed my Ph.D. in May 2005 from the Department of Economics, University of Connecticut, he joined the Department of Economics, University of Pretoria, as a Senior Lecturer in August 2005 and got promoted to Associate Professor in July 2007. He secured a BSc (Honours) degree from the R.K.M.R. College, Narendrapur, and MSc degree from the University of Calcutta. His academic interests are mainly Monetary Theory and Policy, Business Cycles, and Time Series Econometrics. Professor, Department of Economics, University of Pretoria, Pretoria, 0002, South Africa. Phone: + 27 12 420 346, Email: Rangan.Gupta@up.ac.za.

REFERENCES

1.   Barro, Robert, 1990, Government Spending in a Simple Model of Endogenous Growth. *Journal of Political Economy*, 98(5), 103-126.
2.   Bhattacharya, Joydeep, and Xue Qiao, 2007, Public and Private Expenditures on Health in a Growth Model. *Journal of Economic Dynamics and Control*, 31(8), 2519-2535.
3.   Chen, Been-Lon, Yeong-Yuh Chiang, and Ping Wang, 2008, Credit Market Imperfections and Long-Run Macroeconomic Consequences. *Annals of Economics and Finance* 9(1), 151-175.
4.   Chetty, V.K., 1990, Debt, Taxes and Output in a Model with Fixed Capital. Discussion Paper, Department of Economics, Oakland University.
5.   Chetty, V.K., and Ratha, Dilip, 1991, Rationality, Capital Accumulation and Steady-States. Discussion Paper, Department of Economics, Oakland University.
6.   Diamond, Peter. A, 1965, National Debt in a Neo-Classical Growth Model. *American Economic Review* 55(5), 1126-50.
7.   Diamond, Peter, A and Yellin, Joel, 1990, Inventories and Money Holdings in a Search Economy. *Econometrica* 58(4), 929-950.
8.   Gupta, Rangan and Vermeulen, Cobus, 2010, Private and Public Health Expenditures in an Endogenous Growth Model with Inflation Targeting. *Annals of Economics and Finance*, 11(1), 139-153.
9.   Hall, Robert. E, 1988, Intertemporal Substitution in Consumption. *Journal of Political Economy*, 96(2), 339-357.
10.   Romer, Paul, 1986, Increasing Returns and Long-Run Growth. *Journal of Political Economy*, 94(5), 1002–1037.
11.   Solow, Robert, 1956, A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics* 70(1), 65-94.