



The design of stable pillars in the Bushveld Complex mines: a problem solved?

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Synopsis

This paper gives an overview of the difficulties associated with determining the strength of hard-rock pillars. Although a number of pillar design tools are available, pillar collapses still occur. Recent examples of large-scale pillar collapses in South Africa suggest that these were caused by weak partings that traversed the pillars. Currently two different methods are used to determine the strength of pillars, namely, empirical equations derived from back analyses of failed and stable cases, and numerical modelling tools using appropriate failure criteria. The paper illustrates that both techniques have their limitations and additional work is required to obtain a better understanding of pillar strength.

Empirical methods based on observations of pillar behaviour in a given geotechnical setting are popular and easy to use, but care should be exercised that the results are not inappropriately extrapolated beyond the environment in which they are established. An example is the Hedley and Grant formula (derived for the Canadian uranium mines), which has been used for many years in the South African platinum and chrome mines (albeit with some adaptation of the K -value). Very few collapses have been reported in South Africa for layouts designed using this formula, suggesting that in some cases it might yield estimates of pillar strength that are too conservative.

As an alternative, some engineers strongly advocate the use of numerical techniques to determine pillar strength. A close examination unfortunately reveals that these techniques also rely on many assumptions. An area where numerical modelling is invaluable, however, is in determining pillar stresses accurately and for studying specific pillar failure mechanisms, such as the influence of weak partings on pillar strength.

In conclusion, it appears that neither empirical techniques nor numerical modelling can be used solely to provide a solid basis for conducting pillar design. It is therefore recommended that both these techniques should be utilized to obtain the best possible insight into a given design problem. Owing to the uncertainties regarding pillar strength and loading stiffness, monitoring in trial mining sections and in established mining areas is also an essential tool to test the stability of pillar layouts in particular geotechnical areas.

Keywords

pillar design, bord and pillar mining, stable pillars.

Introduction

Appropriate pillar design is a fundamental building block of mine design to ensure the safe and economic extraction of valuable national resources. It is therefore worthwhile

to take a critical look at the tools currently available to rock engineers to conduct these designs. This paper focuses only on stable pillars and does not address the issues associated with crush/yield pillars. The design of stable pillars is currently very topical in the shallow hard-rock mining sector in South Africa, and this paper will highlight a number of examples from this country. Hard-rock pillar design nevertheless appears to be of universal interest, as shown by the examples described below.

Zipf¹ describes collapses of room and pillar mines in the USA. The term catastrophic pillar failure or CPF is used to describe the mechanism whereby a few pillars fail initially; their load is then transferred to adjacent pillars, which also fail. This may result in a 'pillar run' and hundreds of pillars may fail in the process. A number of examples of CPF collapses in 'metal' mines are given in Zipf's paper, and apparently at least four such examples have occurred in the USA since 1972. One of the more recent examples is a large pillar collapse in a room and pillar base metal mine². Figure 1 illustrates the area of the collapse. The failure began in four centrally located pillars and spread rapidly to include almost 100 pillars. The pillar width was 8.5 m and the room width was 9.7 m. The pillar height was about 12 m, resulting in a width-height ratio of 0.70. The extraction ratio was approximately 78%. Based on the examples available to him, Zipf¹ made the comment that mines experiencing CPF generally exhibit the following characteristics (quoted directly from his paper):

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The design of stable pillars in the Bushveld Complex mines: a problem solved?

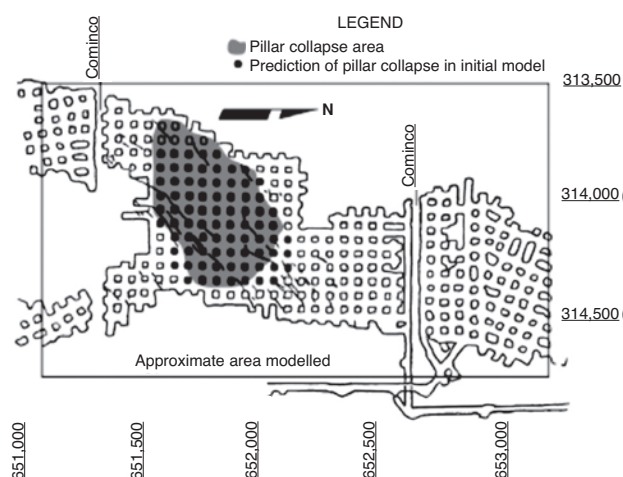


Figure 1—Collapse of a number of pillars in a base metal mine in Missouri (after Dismuke *et al.*² and also published in Zipf¹)

- (1) Extraction ratios are usually more than 60%. A high extraction ratio will put pillar stress close to peak strength and provide ample expansion room for the failed pillar material.
- (2) Width-height ratios of pillars are always less than 3 for coal mine failures, usually much less than 1 in metal-mine failures, and less than about 2 for nonmetal mine failures. A low width-height ratio ensures that the failed pillar material can easily expand into the surrounding openings and that the failed pillar will have little residual load-bearing capacity.
- (3) The number of pillars across the panel width is always at least five and usually more than 10, which typically ensures that pillars have reached their full tributary area load. Minimum panel widths for CPF are at least 80 m.
- (4) Substantial barrier pillars with width-height ratios more than 10 are absent from the mine layout.
- (5) Although CPF seems more prevalent in shallow mines less than 100 m deep, this may be only a reflection of the prevalence of shallow room-and-pillar coal mines.'

An extensive database of hard rock pillar failures was compiled by Lunder and Pakalnis³, who analysed 178 case histories from hard-rock mines of which 98 were located in the Canadian Shield. Of the pillars investigated, 76 were classed as stable; 62 were classed as failed; and 40 were classed as unstable. Many of these pillars were rib or sill pillars from steeply dipping ore bodies. Lunder and Pakalnis proposed that the pillar strength can be adequately expressed by two factor-of-safety (FOS) lines. Pillars with a FOS < 1 fail, while those with a FOS > 1.4 are stable. The region 1 < FOS < 1.4 is referred to as unstable and pillars in this region are prone to spalling and slabbing, but have not completely failed. The data collected by these authors is given later in this paper.

Esterhuizen⁴ conducted an evaluation of the strength of slender pillars. Figure 2 illustrates published case histories of failed pillars from hard-rock metal mines. This figure illustrates that the pillar strength becomes highly variable as the width to height ratio decreases. For a ratio of 1, the pillar strength is not expected to exceed 65% of the laboratory

strength (but it can also be significantly lower than this value). Esterhuizen⁴ suggests that a larger factor of safety is required for these pillars to account for the increased variability in strength. Numerical modelling indicated that for slender pillars, the difference between the pillar load at the onset of spalling and the ultimate pillar strength can be small. This implies that slender pillars are at or near the point of failure at the onset of brittle spalling.

Although a large number of additional papers on pillar design can be found in the literature, the objective of this paper is not to give an overview of all of these, but rather to highlight the difficulties associated with pillar design in South Africa and to emphasize the need for additional research. Some examples of pillar failures in South Africa are therefore described below, and the commonly used pillar design methodologies are critically examined.

The Coalbrook disaster and the evolution of pillar design in South Africa

Although this paper focuses on pillar design in hard rock mines, it is worthwhile to investigate the evolution of pillar design in South Africa in general. This will shed some light on the apparent lack of appropriate pillar design tools for shallow hard rock mines.

Initially, pillar dimensions and mining spans in South African coal mines were based on experience obtained through trial and error. This resulted in a number of collapses. The first report of a coal pillar collapse in South Africa was in 1904 at the Witbank colliery⁵. Since that date, 81 pillar collapses have been recorded in 31 collieries in the former Transvaal and the Free State. Between 1904 and 1965, there were 50 pillar collapses⁶. The research into coal pillar strength in South Africa gained momentum after the Coalbrook disaster. As described by Van der Merwe⁷, the major need for coal mine research in South Africa was identified when the Coalbrook disaster occurred on 21 January 1960. In total, 437 workers lost their lives when pillars over an area of 324 hectares collapsed (Figure 3). Following the disaster, the South African government sponsored research into coal mine safety by forming the Coal Mines Research Controlling Council (CMRCC). Research into

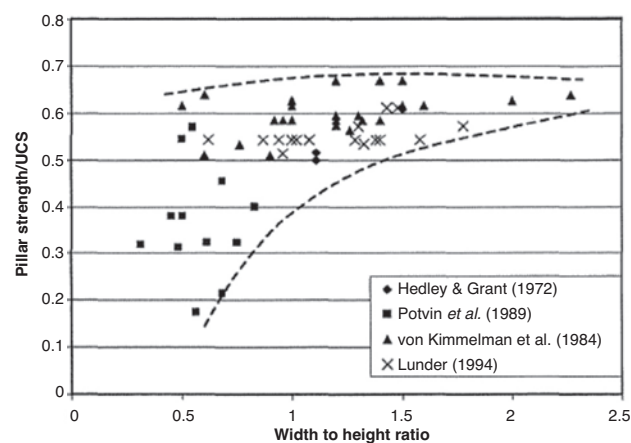


Figure 2—Pillar stability graph showing examples of failed pillars from hard-rock mines (after Esterhuizen⁴)

The design of stable pillars in the Bushveld Complex mines: a problem solved?

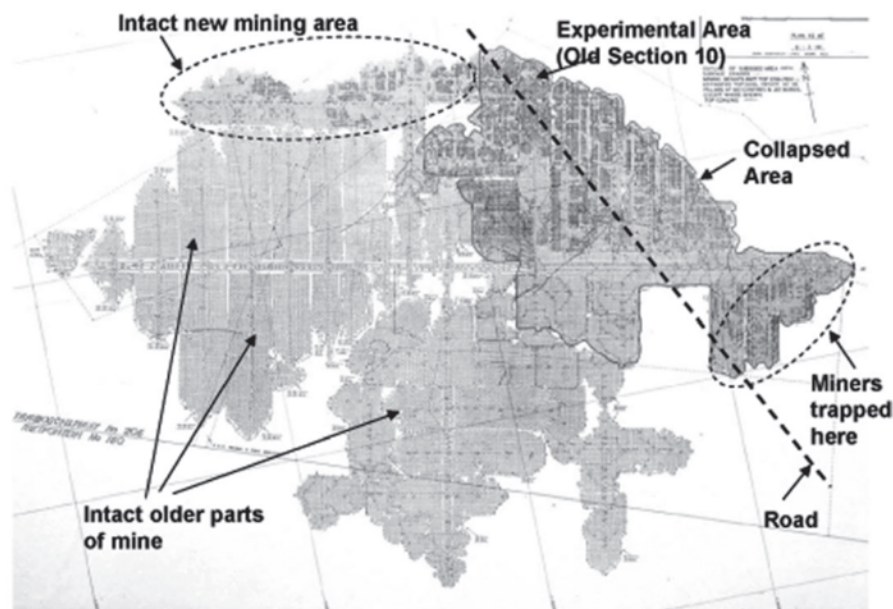


Figure 3—Coalbrook at the time of collapse. The collapsed area is the outlined area in the eastern part of the mine (after Van der Merwe⁷)

coal pillar strength received top priority from the Council for Scientific and Industrial Research (CSIR) and the Chamber of Mines Research Organization (COMRO). A large *in situ* testing programme to determine pillar strength was conducted by the CSIR⁸ and tests were also conducted by the Chamber of Mines⁹.

A key study on coal pillar strength was conducted by Salamon and Munro¹⁰ and the findings as described in their paper are still being used today. It was postulated that the strength of the pillars can be expressed, in the given range of dimensions, as a power function of the height and width. Of interest is that this ‘power law’ was originally proposed by Greenwald, Howard, and Hartmann¹¹. Salamon and Munro¹⁰ derived their equation by estimating the value of the constant K and the powers of width and height by the method of maximum likelihood. The derivation was empirical and was based on data from stable and collapsed cases. The analysed data included 125 cases of which 27 were collapsed pillars. It is noteworthy that they warned in the paper against the following mistake commonly made in rock engineering:

‘The work described in this paper is essentially empirical, and the results, therefore, should not be extrapolated beyond the range of the data which were used to derive them.’

Although the formula is well known, it is repeated below for completeness:

$$Strength_{(coal)} = K \frac{w^a}{h^b} \quad [1]$$

where K reflects the fitted ‘strength’ of a metre cube of coal (7.2 MPa), w is the width of the (square) pillar, and h is the height in metres. The parameters a and b are equal to 0.46 and 0.66 respectively. The pillar volume is given by $V = w^2h$. Defining the width:height ratio, $R = w/h$, Equation [1] can be expressed in the alternative form¹²:

$$Strength_{(coal)} = KV^{(a-b)/3} R^{(a+2b)/3} = KV^{-0.0667} R^{0.5933} \quad [2]$$

It can be seen from Equation [2] that if $a = b$, the pillar strength is independent of the pillar volume whereas if $b > a$, as in Equation [1], the pillar strength is predicted to decrease as the pillar size is increased even if the pillar shape is unchanged.

Did the use of Equation [1] improve pillar designs? Wagner and Madden¹³ reported that since 1967, approximately 1100 million tons of coal had been mined in South Africa and they estimated that 1.2 million pillars were left underground during that time. During the same period, 13 cases of pillar collapse were reported, involving a total of about 4000 pillars. This corresponded to a probability of failure of only 0.003. In spite of the satisfactory performance of the pillar design procedure, three areas requiring further research were identified. These were:

- Regional differences in coal seam strength
- Effect of mining method on pillar strength
- Strength of squat coal pillars.

Regarding the strength of the squat pillars, the original database used by Salamon contained no pillar with a width to height ratio greater than 3.8. Evidence collected in the field suggested that beyond a critical width to height ratio, the pillar strength exceeds that suggested by Equation [1]. Salamon¹⁴ proposed therefore that when the width to height ratio exceeds a critical ratio, the pillar strength formula should be replaced by the following:

$$Strength_{(coal)} = KV^{-0.0667} R_o^{0.5933} \left\{ \frac{0.5933}{\epsilon} \left[\left(\frac{R}{R_o} \right)^\epsilon - 1 \right] + 1 \right\} \quad [3]$$

where

K = the strength of a unit cube of coal

V = the pillar volume (m^3)

R = the pillar width to height ratio

R_o = the critical width to height ratio

ϵ = rate of pillar strength increase.

The design of stable pillars in the Bushveld Complex mines: a problem solved?

From field data, no evidence was available of a collapse of a pillar with a width to height ratio greater than 4. Therefore the critical width to height ratio was selected as 5. A value of 2.5 was chosen for ϵ as it was considerably lower than that obtained from laboratory tests on sandstone.

Regardless of the apparent success of the coal pillar power strength formula, some criticism can be raised regarding its applicability. Virtually all laboratory and field evidence indicates that the width-height strengthening curve has a zero or positively upwards curvature¹⁵. The power formula forces downward curvature and this leads to the inelegant form of the squat pillar formula. An objection raised by Bieniawski¹⁶ is that according to the power law formulation, the cube strength ($w = h$) would continue to decrease indefinitely with side length. This is considered unreasonable¹⁷.

Based on these criticisms, an alternative 'linear' equation, with no volumetric size effect, was proposed which directly expresses the strengthening effect of the width-height (w/h) ratio¹⁶.

$$\text{Strength}_{(coal)} = K \left(0.64 + 0.36 \frac{w}{h} \right) \quad [4]$$

As noted above, Equation [1] implies that pillars with self-similar dimensions (the same ratio of w to h) will have different strengths and that the predicted strength decreases as the pillar volume is increased. By contrast, Equation [4] has no size effect in that the strength depends only on the ratio of w to h . York and Canbulat¹⁸ compared the relative goodness-of-fit power formula versus linear forms for both coal and hard-rock materials, and concluded that the latter behaved at least as well.

Further work regarding coal pillar design was conducted by other workers (e.g. Van der Merwe⁷), but it is beyond the scope of this paper and the reader is referred to this reference for additional information. In conclusion, Ryder and Jager¹⁵ state:

'The power law and its derivatives are perhaps too entrenched in coal engineering to warrant withdrawing from them at this time, but in hard rock engineering, the simpler and probably more realistic linear forms are advocated for general use.'

Regarding the gold mining industry in South Africa, the pillar problem was a completely different issue owing to the great depth of these mines and the associated seismicity. Leaving small pillars or remnants in stopes is in fact detrimental to stability as these may become a source of seismicity. This was a key motivation for the early adoption of the longwall mining method as it minimized the formation of remnants¹⁹. These longwalls remained vulnerable to seismicity and one method to reduce this risk was to leave strips of ground behind as strike stabilizing pillars. A key function of these pillars is to reduce the levels of energy release rate¹⁵. Owing to the size of the gold mining industry and the role it played in the South African economy, significant research into the behaviour of these pillars was conducted over the years. Their widths typically vary from 30 m to 40 m and at a typical stoping width of 1.5 m, this gives a width to height ratio of at least 20. These pillars are therefore considered 'indestructible'. Seismic problems are nevertheless experienced when these pillars 'punch' into the

footwall, and a simple empirical formula is used during design to ensure that the average stress on these pillars does not exceed a specified multiple of the strength of the rock in the footwall or hangingwall. So-called bracket pillars are also used to clamp seismically active geological structures in the gold mines. As the nature and function of the bracket and stabilizing pillars are different to those found in shallow hard-rock mines, these topics are not discussed in this paper and the reader is referred to Ryder and Jager¹⁵ for additional information.

In comparison to the gold and coal mining industries in South Africa, the platinum and chrome mines were the proverbial 'stepchildren' and have never received the same attention in terms of research efforts and funding. Fortunately, the mines in the Bushveld Complex have never had catastrophic pillar failures analogous to the Coalbrook disaster or rockbursts to kick-start research, and it was only with the rapidly increasing price of platinum from 1999 onwards that research activity started to receive more attention. In terms of rock engineering knowledge, much was therefore 'borrowed' from the other mining sectors and overseas research findings. A contributing factor may also have been that pillar cutting in the hard-rock industry is not as precisely controlled as in the coal industry, and it is therefore rather difficult to repeat the statistical approach followed by Salamon and Munro¹⁰. An example is shown in Figure 4, which illustrates a mapped survey of typical pillar cutting in one of the Bushveld mines, where the pillars can be seen to have a wide variety of shapes. Gay *et al.*²⁰ stated that at that time:

'The design of pillar layouts in shallow to medium depth chromium and platinum mines has not reached the same advanced stage as has the design of pillars in coal mines.'

Three reasons for this were given in the paper namely:

- ▶ Very little is known about the strength of small pillars composed of chromitite or pyroxenitic platinum ore
- ▶ Because of the very competent and stiff strata in the hangingwall, it is difficult to determine the pillar loads accurately
- ▶ The presence of and shear along near vertical faults can change the loading condition from a stiff displacement controlled system to a soft load controlled system.

A large collapse involving pillars at the No 6 Shaft, Bafokeng North mine is described by Kotze²¹. The collapse occurred in February 1975. Luckily, nobody was injured during this collapse as it occurred overnight. This collapse is referred to as the 'Hospital collapse' by mine personnel as the mine's hospital was located on surface above the collapsed area and some damage was sustained by the hospital building owing to the resulting subsidence. The building was repaired and it is still being used today. Extraction of the reef in this area was by room and pillar methods, with rooms approximately 25 m wide and pillars nominally 5 m square, providing an extraction of 96%. It appears as if the pillars were cut smaller than this, as Kotze²¹ estimated from the mine plans that the equivalent 'average' pillar width in this area was 3.6 m with an associated extraction of about 98%. Unfortunately no mention is made in the paper of the pillar height. Local hangingwall support was provided by timber sticks and mat packs. The strike span of the stope at the time of the collapse was approximately 400 m, and the average

The design of stable pillars in the Bushveld Complex mines: a problem solved?



Figure 4—An example of poor pillar cutting in a platinum mine in the Bushveld Complex

depth of the overburden was 58 m. The extent of the collapse was of the order of 350 m on strike and 350 m on dip, and the outline of the collapsed region is shown in Figure 5. The upper and lower limits of the fall were bounded by two faults. Up to the time of the collapse there were no signs of pillar scaling and no timber poles were broken, suggesting that conditions were indeed stable. This condition, however, changed overnight with the entire rock mass to surface moving down between the two faults shown in Figure 5. No doubt this observation resulted in the hypothesis given by Gay *et al.*²⁰ that the presence of and shear along near vertical faults can change the loading condition from a stiff displacement controlled system to a soft load controlled system. Kotze²¹ used numerical modelling and stress measurements conducted by the CSIR in 1975 in pillars adjacent to the collapse area to estimate a Merensky pillar strength of 75 MPa (for a pillar with an effective width of 4.3 m). This value is of course applicable only to the particular width to height ratio of the pillars (the pillar height was given by Kotze²¹ as 1 m).

Empirical methods to estimate hard rock pillar strength

Martin and Maybee²² give a very good overview of the different empirical strength formulae that were developed to predict pillar strength. A comparison of these formulae is given in Figure 6. These curves were calculated for a pillar height of 5 m.



Figure 5—Extent of the collapse (dotted line) at No 6 Shaft, Bafokeng North mine

The design of stable pillars in the Bushveld Complex mines: a problem solved?

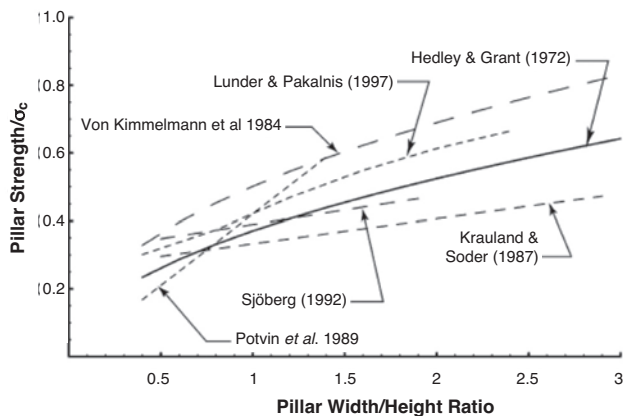


Figure 6—Comparison of empirical pillar strength formulae (after Martin and Maybee²²)

In South Africa, based on the success of the Salamon and Munro strength formula in the coal mines, it was natural for the shallow hard-rock mines to adopt a similar power-law strength formula. The necessary research to develop and calibrate a formula for local conditions was, unfortunately, never conducted. Instead, the Hedley and Grant formula²³ developed for the Canadian uranium mines was adopted. Only the K -value was modified to reflect local rock strengths. This approach seemed to work well and over the years it has become firmly entrenched. It would not be incorrect to state that it is currently the ‘industry accepted’ method for designing pillars in shallow hard-rock mines in South Africa. Ryder and Jager¹⁵ comment that this original formulation: ‘for want of local data, have subsequently been applied in

South African hard-rock mines’. This is a good example of what frequently happens in rock engineering. Over time, initial assumptions and interim solutions become entrenched as common practice and the original assumptions are rarely revisited or questioned. Based on this wide acceptance of the Hedley and Grant formulation and lack of local research into more appropriate formulations, it will be worthwhile to examine the assumptions made in the original paper.

The uranium mines in the Elliot Lake area used a stope and pillar layout to mine the orebody. Narrow pillars about 250 ft (76 m) long were left along dip (see Figure 7). The pillars which were chosen for analysis were typically conducted 10–20 ft wide (≈ 3 –6 m) and 8–20 ft high (≈ 2.5 –6 m). The width to height ratio of most of the pillars was close to 1 and only a very few (3 in the database) had a width to height ratio of 2.5. It is clear that this original formulation was derived for slender rib pillars and it can be questioned whether it is applicable to square pillars with a width to height ratio greater than 2.5. The original data used by Hedley and Grant is reproduced in Table I. It is immediately obvious that the dataset used was very small (28 pillars). This should be compared to the coal database of Salamon and Munro¹⁰, which included 125 pillars of which 27 were collapsed. The width to height ratio of the failed pillars varied from 1.1 to 1.5. Only 3 of these pillars were ‘crushed’ and 2 were ‘partially crushed’. Of further concern is that it is stated in the paper that: ‘The information on complete pillar crushing was obtained second-hand because it happened in mines which are closed.’ This work was conducted in the days before computer-based numerical modelling could be used to determine pillar stress. The approach followed was therefore to use tributary area theory, which relates the pillar stress to the pre-mining stress and the extraction ratio by:



Figure 7—Typical layout of a mine in the Elliot Lake district (after Hedley, Roxburgh and Muppalaneni²⁴)

The design of stable pillars in the Bushveld Complex mines: a problem solved?

Table 1
A reproduction of the original dataset used by Hedley and Grant²³ to calibrate their power law formulation for pillar strength

Depth ft	Dip deg.	Extraction %	Pillar dimensions		Estimated pillar properties	
			Width ft	Height ft	Stress psi	Strength psi
Stable pillars						
500	17	85	10	10	5000	14600
700	17	85	10	10	6400	14600
800	26	65	20	18	3800	13300
850	20	85	10	10	7600	14600
1000	22	65	20	18	4000	13300
1050	15	85	10	10	8500	14600
1200	18	85	10	10	9400	14600
1300	20	65	20	20	4600	12300
1600	20	60	20	18	4800	13300
1600	20	65	18	18	5400	12600
1600	22	75	20	14	7600	16000
1700	22	65	40	20	5800	17400
1700	22	60	22	20	5000	12900
1700	12	75	20	14	7600	16000
1800	5	75	20	14	8000	1600
1900	23	65	19	18	6400	13000
2200	25	65	20	20	7200	12300
2400	11	65	20	8	7600	24400
2500	9	65	20	8	7900	24400
2700	13	65	20	8	8600	24400
2900	12	70	15	9	10500	19400
2900	12	75	20	9	12600	22400
Partially failed pillars						
1400	20	85	10	10	11400	14600
2400	18	80	10	9	13400	15800
Crushed pillar						
2800	12	80	10	9	15200	15800
2900	12	80	10	9	15700	15800
3400	5	80	15	10	18500	17900

$$\sigma_p = \frac{S_o}{1 - e} \quad [5]$$

where

σ_p = pillar stress

S_o = pre-mining stress normal to the orebody

e = extraction ratio.

For workings inclined at an angle α to the horizontal, the normal stress S_o is a combination of the vertical stress component S_v and the horizontal stress component S_h :

$$S_o = S_v \cos^2 \alpha + S_h \sin^2 \alpha \quad [6]$$

The vertical stress was assumed to be simply a function of the weight of the overlying strata and the horizontal stress perpendicular to strike was assumed to be 3000 pound per square inch based on measurements in two mines. The stress given in the Table 1 for each pillar is therefore only a rough estimate. The methodology followed in the paper to derive the pillar strength formula can be summarized as follows:

The first step was to adopt the power law strength formulation used by Salamon and Munro¹⁰. In the notation of Hedley and Grant²³ it is given as:

$$Q_u = K \frac{w^a}{h^b} \quad [7]$$

where

Q_u = pillar strength (psi)

w = pillar width (ft)

h = pillar height (ft)

K = strength of 1 ft cube (psi)

a and b are constants

Hedley and Grant²³ acknowledge that this equation refers to square pillars, whereas those in the uranium mines are long and narrow. Their assumption was therefore that the strength of the slender pillars will not be much greater than a square pillar of width equalling the minimum width of the long pillar.

Secondly, from extrapolation of laboratory tests, it was estimated that the value of K is 26 000 psi for a 1-ft cube.

Thirdly, appropriate values for parameters a and b had to be derived. Three different sets of values were available to them at that stage in the literature. The value for a was relatively constant at 0.5 and Hedley and Grant²³ therefore decided to also adopt this value. As b varied more, a new value was computed and their approach was to focus on the three failed pillars in the database. For each of these pillars, the tributary area stress in the table was assumed to be the pillar strength. This value, as well as the K -value and $a = 0.5$, were substituted into Equation [7] and the value of b was solved for each pillar. The calculated values of b ranged from 0.736 to 0.768 with a mean of 0.75. This value was adopted and it resulted in the now familiar Hedley and Grant formulation.

Clearly the formulation above is based on a large number of assumptions, and the applicability of this formulation to the design of hard-rock pillars in the Bushveld Complex in South Africa becomes highly questionable. The first use of this formula in a South African mine is not clear, but Ozbay *et al.*²⁵ stated that it was 'popularized by Wagner and Salamon²⁶ as quoted by Kersten²⁷'. Kersten²⁷ used it to design pillars for Agnes Gold Mine. Subsequently, it has been used to design a large number of bord and pillar layouts in the country with an appropriate modification of the value of K . The rule frequently used in South Africa is to estimate K at between one-third and two-thirds of the Uniaxial Compressive Strength (UCS) of the pillar material (e.g. see Ozbay *et al.*²⁵). Almost no collapses in the Bushveld Complex have been reported to date using this formulation (except where weak clay layers are present in the pillar, see sections below), so the uncomfortable question therefore remains: Why does it work and are the current designs perhaps too conservative? One hypothesis is that the 'squat' behaviour of hard-rock pillars may occur significantly earlier than the w:h ratio of 5 assumed for coal pillars¹⁵.

As an alternative to the estimate of K being one-third of the UCS of the pillar material, Stacey and Page²⁸ and Stacey and Swart²⁹ suggested that the value of K be chosen as the design rock mass strength (DRMS). The DRMS takes into account the rock quality, unfavourable joint orientations, and the excavation method³⁰. Wesseloo and Swart³¹ conducted a study on LG6 Chromitite pillars to determine the constants K , a and b (see Equation [7]) more accurately for these pillar types. The study showed that the value of the exponents a and b suggested by Hedley and Grant²³ do not apply to these chromitite pillars. They indicated in the study that values of $a = 1$ and $b = 1$ may be more applicable to these pillars.

The design of stable pillars in the Bushveld Complex mines: a problem solved?

As a further attempt to develop a new formula for the South African mines, Watson *et al.*³² derived new values for the power law formulation given in Equation [7]. They used a maximum likelihood evaluation similar to that used by Salamon and Munro²³. Their database consisted of 179 Merensky Reef pillars of which 109 were stable. The width to height ratio of the pillars in the database ranged from 1 to 8, with the majority between 3 and 6. Only one pillar had a width to height ratio of less than 1. The values derived are $K = 86$ MPa, $a = 0.76$, and $b = 0.36$. It is interesting to note that the b parameter is much lower than for the Hedley and Grant formula. The formula predicts pillar strengths that are much greater than the traditional Hedley and Grant formulation, with K values assumed to be a third of the UCS (see Figure 8). Unfortunately, it is not known if this formula has been tested in any trial mining sections.

Corrections for rectangular pillars

Equation [7] is applicable to square pillars. Another awkward assumption commonly made when designing pillars is to calculate the 'effective width' for rectangular shaped pillars. Holland and Gaddy³³ state that only the minimum lateral dimension, w , affects the strength of a pillar, while the other dimension, L , has no effect. Wagner⁹ suggested that the effective width of a pillar should take the form:

$$w_{eff} = \frac{4A}{C} = \frac{2wL}{w+L}, \quad [8]$$

where A is the cross-sectional area of the pillar and C is the perimeter. This formula is based on observations made by Wagner during large scale underground tests, namely that: 'the strength of the circumferential portions of a pillar is virtually independent of the width-to-height ratio, whereas the strength of the centre increases with increasing ratio.' The effective width as defined by Equation [8] approaches a finite value of twice the minimum pillar width for very long and narrow pillars. It is not clear if this assumption is correct and Ryder and Ozbay³⁴ suggested an alternative shape strengthening factor of the form $f = 1.0/1.1/1.2/1.3$ for pillars having w_1/w_2 ratios of $1/2/4/\infty$. This implies that Wagner's perimeter rule may be overestimating the strengthening effect of very long pillars.

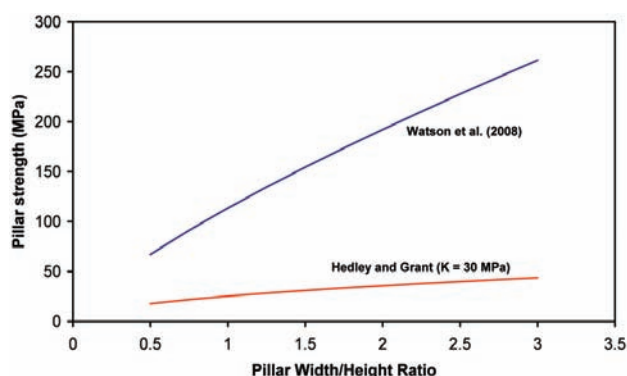


Figure 8—A comparison of the Merensky pillar strength predicted by the new Watson formula and the traditional Hedley and Grant using a K -value of 30 MPa (a third of the average UCS obtained from laboratory tests of Merensky Reef samples from Impala 2A Shaft)

Unfortunately no good experimental evidence is available regarding the effect of pillar shape on strength for hard-rock pillars in South Africa and this area requires further research. Somewhat concerning is that the whole empirical design philosophy rests on a huge number of unproven assumptions, and pillar strength is clearly an area that requires systematic research in future.

Recent examples of pillar failures

To illustrate the inherent dangers of using empirical design formulae for rock masses in environments for which they were not originally designed, consider three case studies of recent mine collapses in the South African Bushveld Complex.

Spencer³⁵ reported on the failure of the pillar system and the subsequent closure of the Wonderkop Chrome Mine in May 1998 (stopping commenced only in September 1996). The mine is located close to Rustenburg and exploited the Lower Group chromitite seams, namely the LG6 and LG6A. An internal pyroxenite waste band is found between these two reefs and this results in the pillars having a multi-layered appearance (Figure 9) wherever these two reefs are mined. The Wonderkop Mine was the most easterly situated LG6 mining operation in the Rustenburg area and was situated adjacent to the Spruitfontein dome (an upfold structure which separates the Rustenburg section from the Marikana section). Its close proximity to the Spruitfontein dome has influenced the structure of the LG6 and LG6A seams and this has resulted in thick clay layers (up to 300 mm in some places) traversing the pillars in some areas (Figure 10). The position and thickness of this weak layer is highly variable (e.g. see Figure 11).

The original pillar design at the mine was conducted using the Hedley and Grant pillar strength formula. The pillar sizes were 12 m x 6 m giving an 'effective width' of 8 m according to Wagner's perimeter rule. K was assumed to be 27.3 MPa, which was a third of the laboratory strength of the rock (using samples obtained from another mining section). Owing to the complex multilayer structure of the pillars, it is not clear which of these layers were tested. The stopping width was 2 m, so the width to height ratio was at least 3 if the smallest dimension of the pillars is considered. When using the 'effective width' of the pillars as 8 m, the strength of the pillars was estimated to be 45.9 MPa.



Figure 9—Typical multi-layer composition of pillars in the Bushveld Complex mines exploiting the LG6 and LG6A chromitite layers

The design of stable pillars in the Bushveld Complex mines: a problem solved?



Figure 10—Presence of weak clay layers in the pillars at Wonderkop Mine. This photograph illustrates the presence of a clay layer between the LG6A chromitite and the pyroxenite below it

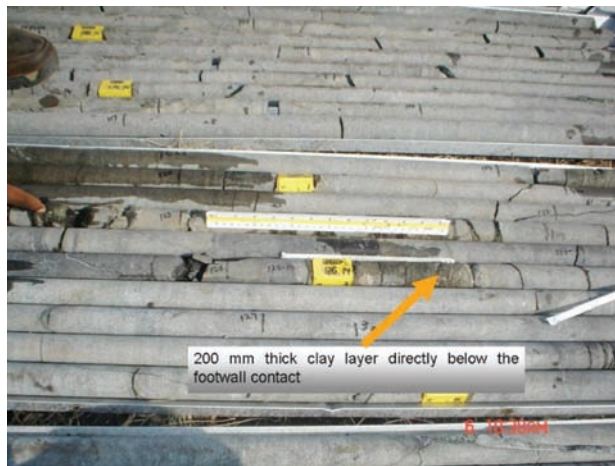


Figure 11—Presence of weak clay layers in proximity to the LG6/LG6A chromitite reefs at Wonderkop Mine

The first underground inspection of the pillars by the consulting rock engineer was conducted in July 1997. During this visit it was noted that some joints were beginning to open at the corners of some pillars. Some joints were also opening up at the sides of the pillars and in a few cases

sliding along the clay layer was noted. Figure 12 illustrates the layout and the positions where pillar failure was observed. Following these observations, steps were taken to introduce a system of barrier pillars (Figure 13) with a width to height ratio of at least 10 (it is generally believed that at this ratio the pillar becomes indestructible). During the following nine months, the condition of the pillars continued to deteriorate. To strengthen the pillars along the main dip belt and road declines, two strategies were adopted; namely waste stowing between the pillars and mesh and lacing of the pillars. The success of the mesh and lacing of the pillars appeared to be doubtful, as the drilling process introduced additional water into the clay which probably weakened the pillars further. During April 1998, the failure process accelerated and the rate of closure in some areas increased to 1.8 mm/day. Numerous falls of ground occurred and management decided to cease operations at this stage. Recently, a back analysis of this pillar failure was conducted by the authors using the TEXAN boundary element program³⁶, which can explicitly simulate the individual pillars and calculate the stresses on these pillars relatively accurately. The results are shown in Figure 14. Two face positions were simulated; namely July 1997 (Figure 12) and July 1998 (Figure 13). From this study, the Hedley and Grant formulation was used to back-calculate the K -value for the pillars. A value as low as 6 MPa was obtained, which is in agreement with earlier back analysis studies by Spencer³⁵, who estimated a value as low as 4.6 MPa. It should nevertheless be noted that a power law strength formula might not be applicable to these pillars owing to the presence of the clay layer. Although the mechanism of pillar failure needs further investigation, the low strength of the pillars can probably be attributed to the low friction angle of the clay layer, which allowed lateral deformation of the pillar and a reduction of confinement.

Two other large scale pillar collapses recently occurred in the Bushveld Complex. Detailed information regarding these collapses is not available in the public domain, and therefore these mines will only be referred to as Mine A and Mine B.

Mine A is a platinum mine located in the eastern portion of the Bushveld Complex. At this mine, a clay layer is also present at the hangingwall/pillar contact (Figure 15). The reef exploited in this area is the UG2. The original mine design was conducted using the Hedley and Grant pillar strength formulation with a K value of 35 MPa. The mining height was 2 m. In mid 2008, some concern was expressed regarding the stability of the pillars and a minor collapse occurred during this time. In an attempt to reinforce some of the pillars, many were supported using fibre-reinforced shotcrete. This did not stop the deterioration, however, as shown in Figure 16 with the cracked shotcrete clearly visible. During December 2008, operations were temporarily suspended at the mine when the decline was affected by the instability.

Similar to Wonderkop Mine, Mine B also exploits the LG6/LG6A chromitite reefs. The problem is essentially similar to that experienced at Wonderkop Mine with the presence of a clay layer in some of the pillars. This resulted in collapses in parts of the mine. Typical pillar failure at the mine is shown in Figure 17. Experience has indicated that increasing the pillar sizes does not necessarily work in these cases.

The design of stable pillars in the Bushveld Complex mines: a problem solved?

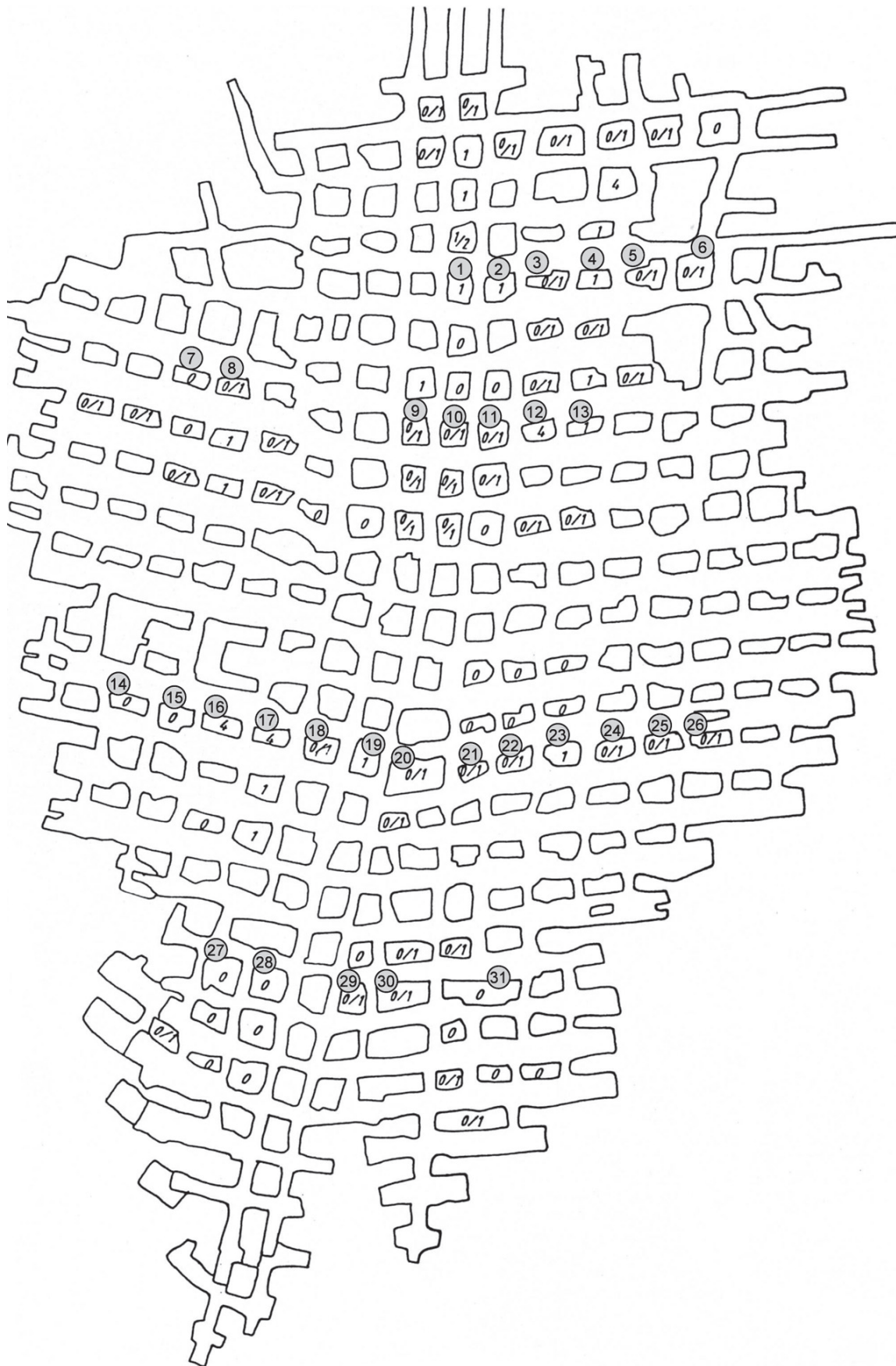


Figure 12—Failure condition of the pillars and the extent of mining during July 1997 (after Spencer³⁵). The failure codes used in this figure are as follows: 0 - No failure, 1 - opening of joints at the corners, 2 - opening of joints at the corners and along the sides, 3 - material slabbing off the corners and sides, 4 - horizontal movement occurring along the clay layer

The design of stable pillars in the Bushveld Complex mines: a problem solved?

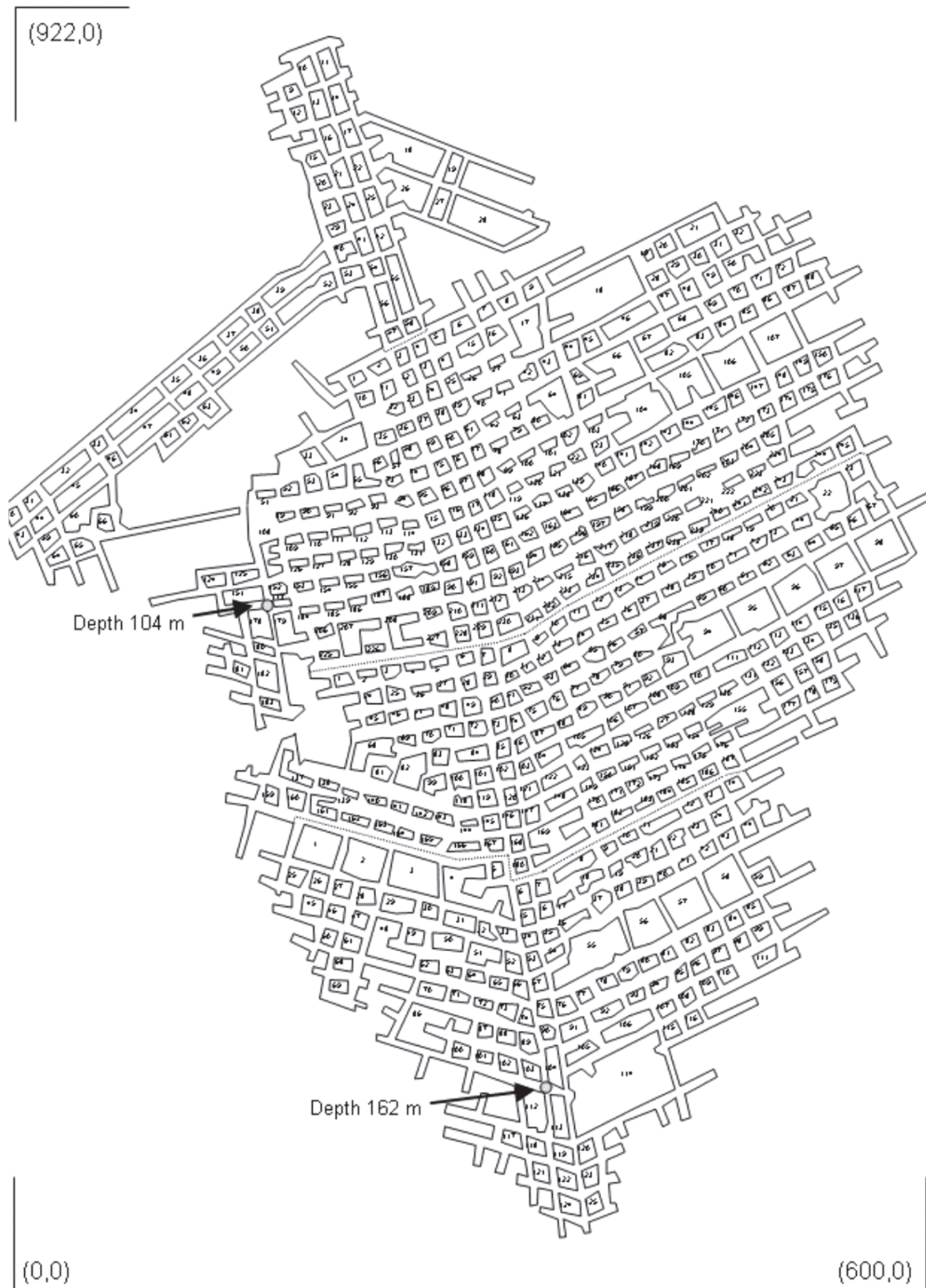


Figure 13—Extent of mining at Wonderkop Mine during July 1998

The design of stable pillars in the Bushveld Complex mines: a problem solved?

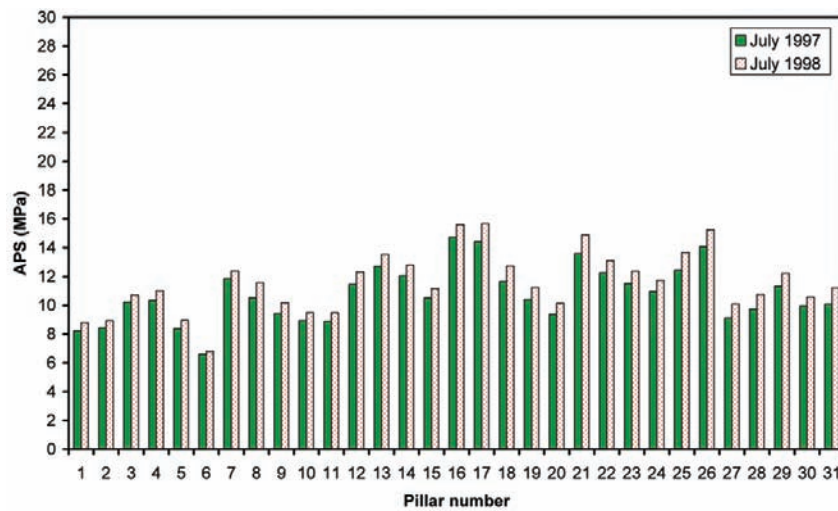


Figure 14—Simulated pillar stresses for selected pillars in the Wonderkop Mine (see Figure 12 for the positions of the pillars)



Figure 15—Typical pillar composition at Mine A (after Roberts and Clark-Mostert³⁷)



Figure 16—Pillar failure at Mine A after attempts to strengthen the pillars with shotcrete (photograph courtesy Dr Mike Roberts)

From these studies, the drawbacks of using empirical pillar strength formulae are obvious. The failure in all three cases is caused by the presence of clay layers which substan-



Figure 17—Interest mode of pillar failure at Mine B. For this pillar, a clay layer was found between the upper LG6A chrome and the pyroxenite below it. This slippery layer facilitates the fracturing of pyroxenite, causing it to scale out (left). The failures led to large amounts of convergence as can be seen in the photograph on the right

tially weaken the pillars. The original empirical formulae were developed for different rock types and the application of these formulae outside the limits for which they were developed resulted in the large mine collapses described here.

Numerical modelling to estimate pillar strength

From the sections above, it is clear that the applicability of the current empirical formulas in hard-rock mines is highly uncertain and additional verification and calibration work is required. The orebodies in the Bushveld Complex are very different to that for which the Hedley and Grant formulation²³ was derived. Furthermore, the restricted range of slender width to height ratios used when deriving the equation is unfortunate and the choice of the appropriate K value is undefined and highly uncertain.

The alternative to the empirical approach is to use numerical modelling with appropriate failure criteria to determine pillar strength. A vast amount of literature is available on attempts to simulate pillar failure, and it is not the objective of this paper to summarize all these findings. Focus will rather be placed on recent work that is applicable to the pillars in the Bushveld Complex.

The design of stable pillars in the Bushveld Complex mines: a problem solved?

Joughin *et al.*³⁸ presented a risk based approach to the design of chromitite pillars. Numerical modelling was conducted using the finite element program PHASE2 and a Hoek and Brown failure criterion. The variability of the strength properties, pillar dimensions, and spans were taken into account by applying the Point Estimate Method. Many numerical analyses were conducted with varying input parameters to obtain statistical distributions of pillar safety factors. Probabilities of failure were then determined from the distributions. Back analysis of an area with a large collapse and another in which only a few pillars had failed indicated good agreement with the actual number of pillar failures.

Day and Godden³⁹ presented a paper describing the design of panel pillars on Lonmin's platinum mines. They state that the validity of their approach is supported by extensive underground surveys and by computer back-analysis studies. According to the authors, over 300 pillars per month were cut at Lonmin at that stage and these pillars behaved as expected. The method apparently works well up to width to height ratios of 5.5, but not at greater values owing to the onset of squat pillar effects. This seems rather disappointing as the expectation is that a numerical method with an appropriate constitutive model will 'automatically' take care of the onset of squat pillar behaviour. Pillar strength was estimated by two-dimensional FLAC modelling using the original Hoek and Brown⁴⁰ failure criterion:

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^2)^{0.5} \quad [9]$$

where σ_c = uniaxial compressive strength (MPa) and m and s are constants that depend on the properties of the rock. The constant m was determined from laboratory testing, and s equals to 1 for intact specimens. *In situ* values for the constants m and s were derived by application of rock mass quality ratings using the equations of Priest and Brown⁴¹ for undisturbed rock masses:

$$m = m_i e^{\frac{RMR-100}{28}} \quad [10]$$

$$s = e^{\frac{RMR-100}{9}} \quad [11]$$

The authors³⁹ derived *in situ* values for m and s for the UG2 and Merensky reefs at Lonmin's Mines. Typical values used in the modelling are as follows: UG2; $m = 25.83$, $s = 0.51$ for a RMR of 94, Merensky (Type B); $m = 8.7$, $s = 0.57$. The resulting simulated pillar strengths seem plausible when the pillar width to height ratio is low. The authors nevertheless acknowledge that squat pillar behaviour is not correctly simulated by this approach and that this will probably result in pillars being over-designed at depths exceeding 700 m.

A further concern regarding this approach is that the failure criterion is still empirical and the unmodified Equations [10] and [11] may not be appropriate for the pillar material in the Bushveld Complex. Pells⁴² also expressed some concern about the Hoek and Brown failure criteria and quoted Mostyn and Douglas⁴³ which provided a detailed critique of this failure criterion for intact rock.

'...there are inadequacies in the Hoek-Brown empirical failure criterion as currently proposed for intact rock and, by inference, as extended to rock mass strength. The parameter m_i can be misleading, as m_i does not appear to be related to

rock type. The Hoek-Brown criterion can be generalized by allowing the exponent to vary. This change results in a better model of the experimental data.'

Martin and Maybee²² investigated the strength of hard rock pillars by using both empirical pillar strength formulae and numerical modelling using a Hoek-Brown failure envelope. Figure 18 illustrates a comparison between the pillar stability graph developed by Lunder and Pakalnis³, the Hedley and Grant equation and Phase 2 - Hoek and Brown modelling using different values of GSI. Figure 19 illustrates the same data using the Hoek and Brown brittle parameters. The conclusion reached by Martin and Maybee²² is that two-dimensional finite element analyses using conventional Hoek and Brown parameters for typical hard-rock pillars predicted rib pillar failure envelopes that did not agree with empirical pillar failure envelopes. It appears that the conventional Hoek-Brown failure envelopes over-predict the strength of hard rock pillars. This occurs because the failure process is fundamentally controlled by a cohesion loss process in which the frictional strength component is not mobilized. Their recommendation is that Hoek-Brown brittle parameters ($m_b = 0$ and $s = 0.11$) be used to simulate pillar strength.

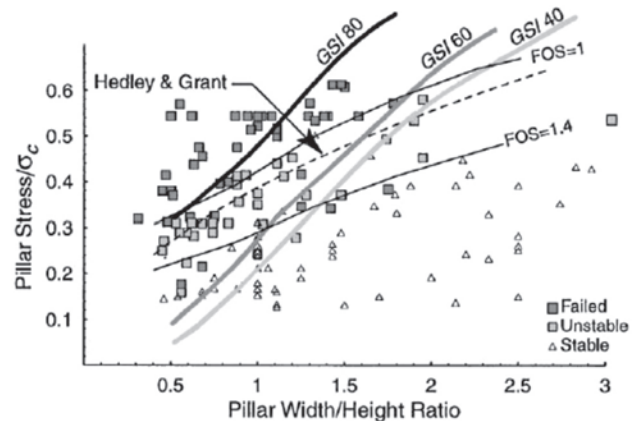


Figure 18—A comparison of the strength predicted by Hedley and Grant²³, the data and FOS lines from Lunder and Pakalnis³ and the Phase 2 modelling for various GSI values (after Martin and Maybee²²)

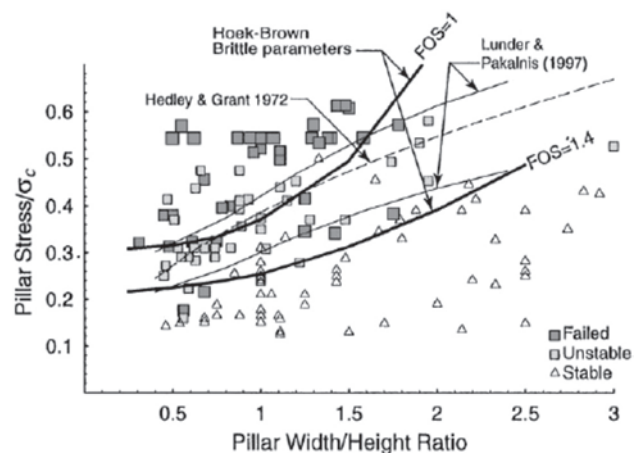


Figure 19—A comparison of the strength predicted by Hedley and Grant²³, the data and FOS lines from Lunder and Pakalnis³ and modelling results using the Hoek and Brown brittle parameters (after Martin and Maybee²²)

The design of stable pillars in the Bushveld Complex mines: a problem solved?

Hoek, Kaiser, and Bawden⁴⁴ summarized the rock mass conditions for which the Hoek-Brown failure criterion can be applied. The criterion is strictly applicable only to intact rock or heavily jointed rock masses that can be considered to be homogenous and isotropic. For cases in which rock mass behaviour is controlled by a single discontinuity or joint set, a criterion that describes the shear strength of joints should rather be used (e.g. see the pillar shown in Figure 20). The implication of this is that for the three case studies of pillar failure in the Bushveld Complex discussed above, explicit simulation of the clay layer will be required. An example was conducted for the authors by Dr John Ryder using the FLAC code, and this is presented below to illustrate the value of modelling.

The pillar composition simulated was the LG6/LG6A 'sandwich' shown in Figure 10. The qualitative effect of a strong pyroxenite layer within a chromitite pillar (with weak contacts, including weak hangingwall and footwall contacts) was therefore modelled in plane strain. A generic model was built to investigate the effect of an inhomogeneous pillar with weak interfaces. Estimated *in situ* strain-softening parameters were drawn directly from studies carried out in the Bushveld Complex. The hangingwall and footwall were assigned the same properties as the pyroxenite layer, and symmetry was assumed for both the vertical and horizontal centrelines in the following layout in the FLAC finite difference code (Figure 21). The grid size was 0.1 m × 0.1 m. By applying slow displacement loading (velocity 5×10^{-4} mm/step), complete stress-strain and lateral deformation curves could be modelled (Figure 22). Lateral deformations showed no dramatic effects owing to the presence of the 'strong' layer of pyroxenite in the pillar, possibly because the modelled contrasts in strength, Poisson's ratio and dilatancy were not particularly marked. (Note that the horizontal scale in Figure 22b is in millimeters whereas the vertical scale is in metres). Likewise, the presence of a weak interface between the layer and the body of the pillar had virtually no effect, even if the friction angle of interface 1 was set as low as 6°. In contrast, low friction angles on the hangingwall contact (interface 2) had a powerful effect, reducing the peak strength p of the pillar by allowing lateral deformation and reduction of confinement, and reducing also the residual strength (Figure 23).

A further illustration of the value of numerical modelling to investigate pillar strength is given by Esterhuizen⁴⁵. The

UDEC code was used to simulate pillars of different width to height ratios and the effect of jointing on these pillars. The most significant result obtained was that the effect of jointing on pillar strength decreases as the width to height ratio of the pillar increases. This indicated that when designing pillars, it would not be correct to simply determine the rock mass strength and use it in the pillar formulae regardless of the width to height ratio of these pillars. Further work was conducted by Esterhuizen *et al.*⁴⁶ to investigate on the effect of large discontinuities on pillar strength. UDEC simulations were used to investigate the effect of these large discontinuities on pillars with width to height ratios of 0.5 to 1.5. A revised empirical pillar strength formula was developed for the slender pillars in the stone mines in the USA. The form of the equation is:



Figure 20—An example of a pillar that contains a single joint dipping at almost 45° through the pillar. This joint will have to be modelled explicitly if the behaviour of this pillar is to be correctly simulated by a numerical modelling code

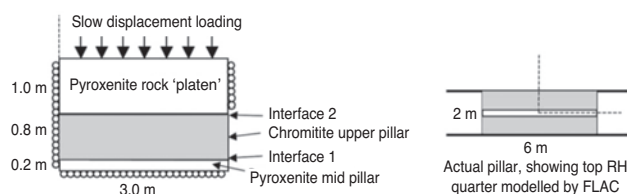


Figure 21—FLAC model to simulate the effect of weak interfaces in the pillar

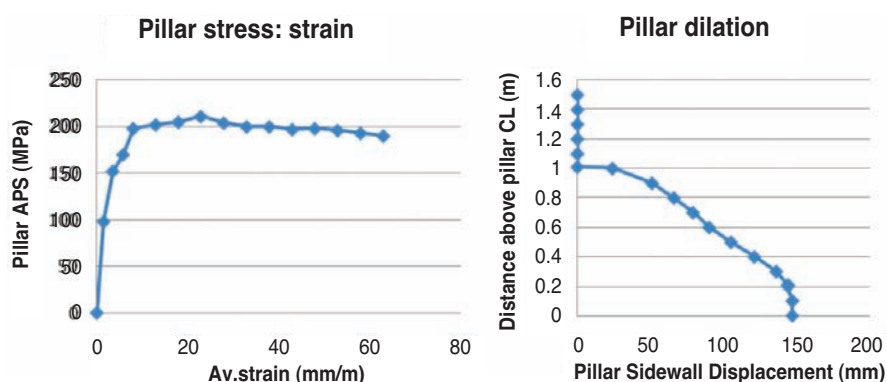


Figure 22—Pillar stress:strain and sidewall dilation (at peak strength point p), for interface friction angles of 30°

The design of stable pillars in the Bushveld Complex mines: a problem solved?

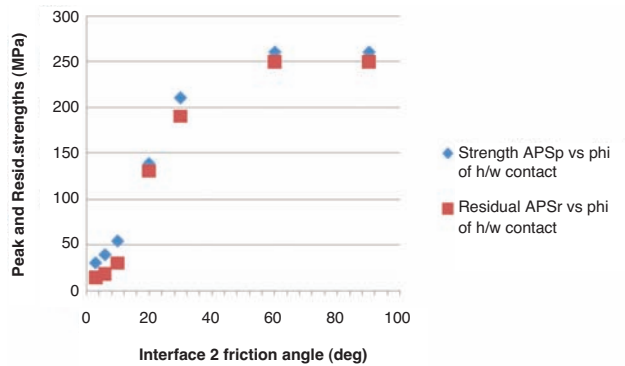


Figure 23—Peak and residual pillar strength versus hangingwall contact friction angle ϕ_2

$$\text{Strength} = 0.65 \times UCS \times \text{LDF} \times \frac{w^{0.3}}{h^{0.59}} \quad [12]$$

where UCS is the uniaxial compressive strength of the intact rock and LDF is a large discontinuity factor. The authors recommended that a factor of safety of at least 1.8 be used for designs where the pillar strength is assumed to be given by Equation [12]. This will ensure that the pillars remain within the limits of previously successful stone pillar layouts.

The need for monitoring in pillar mining sections

The report of the Government Mining Engineer⁴⁷, published five years after the Coalbrook disaster, made the following statements regarding instrumentation in the experimental section at the mine:

‘There is no evidence that during these operations any systematic inspections were made of the northern portions of No. 10 Section or that any measuring apparatus was installed to ascertain if stability was being affected by the reduction of the abutment support.’

‘It appears that no scientific controls were applied to the experiment nor was an attempt made to ascertain what reduction in their support strength was caused by cutting the pillars and increasing their height. The effects of the experiment on the roof and pillars were judged by eye and ear and the results were gauged by the tally board.’

Although monitoring is currently being conducted in some of the Bushveld Complex mines on a limited scale, routine monitoring is probably required in the many bord and pillar sections owing to the uncertainties associated with pillar strength and loads. One example of the type of monitoring that may be valuable is closure monitoring using sensitive instruments. It has been shown that in shallow mining areas, the rate of closure is essentially zero (less than the sensitivity of the instruments) in the stable back areas. Once an area becomes unstable, the rate increases from zero to a very small rate, e.g. 0.3 mm/day. This rate may persist for many days or even weeks⁴⁸. As this rate is so small, sensitive instruments are required to give early warning of impending instability. In pillar areas, compiling a photographic record of selected pillars over time will also be extremely useful to provide an unbiased opinion whether the pillars are scaling or not (care should be taken that the pillars

are appropriately marked and that the successive photographs are taken from the same positions).

As a final comment, in contrast to rock engineering, routine monitoring is commonly used in many of the other engineering disciplines. Civil engineers work with materials that are better characterized and better understood than rock masses, but still conduct routine monitoring of their structures. Structural health monitoring of civil infrastructure systems is a very active field and readers will be able to find a large number of references by searching for this topic on the internet. Many other examples can be quoted from other engineering disciplines, but in general the rule applies that if uncertainty is associated with an engineering design, monitoring should be used to reduce the risk associated with failure of the design.

Conclusions

This paper gives an overview of the difficulties associated with determining the strength of hard-rock pillars. Although a number of pillar design tools are available, pillar collapses still occur. Recent examples of large scale pillar collapses in South Africa were caused by weak partings that traversed the pillars. Currently, two different methods to determine the strength of pillars are used; namely, empirical equations derived from the back analysis of failed and stable cases and numerical modelling using appropriate failure criteria. Both techniques have their limitations and additional work is required to obtain a better understanding of pillar strength.

Empirical methods are popular and easy to use, but care should be exercised that the results are not extrapolated beyond the range of the data which were used to derive them. An example is the Hedley and Grant formula (derived for the Canadian uranium mines) that has been used for many years in the South African platinum and chrome mines (albeit with some adaptation of the K -value) to design pillar layouts in these mines. A careful study of the original publication by Hedley and Grant indicates that this formula was derived for conditions and rock types that are very different to those found in the South African mines. Nevertheless, very few collapses have been reported and in some cases it appears that the Hedley and Grant formula might underestimate pillar strength significantly.

As an alternative, some engineers strongly advocate the use of numerical techniques to determine pillar strength. A close examination unfortunately reveals that these techniques also rely on many assumptions, and extreme care needs to be exercised when using this approach. Pillar load is another unknown not discussed in this paper and care should also be exercised when this is estimated using numerical procedures (some difficulties are outlined in a companion paper in this issue). An area where numerical modelling is invaluable, however, is to study specific pillar failure mechanisms, such as the influence of weak partings on pillar strength. The modelling indicated that a likely failure mechanism in the case studies was that the presence of a parting with a low friction angle that allowed lateral deformation and a reduction of confinement in the pillars. This reduced the peak strength of the pillars considerably.

In conclusion, it appears that neither empirical techniques nor numerical modelling can be used solely to provide a solid basis to predict pillar strength. It is therefore recommended

The design of stable pillars in the Bushveld Complex mines: a problem solved?

that both these techniques be utilized when addressing pillar design problems in order to obtain the best possible insights. Owing to the uncertainties regarding pillar strength, pillar stress, and loading stiffness, monitoring in trial mining sections (and even in established mining areas) is considered to be an essential tool to assess the stability of pillar layouts in particular geotechnical areas. The need for additional research into pillar strength should also be emphasized strongly as this problem has clearly not yet been solved!

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