Location Model for Reaction Vehicles used by the Private Security Industry of South Africa

by

Hendrik Johannes Kellerman

28114745

Submitted in partial fulfillment of the requirements for the degree of

BACHELORS OF INDUSTRIAL ENGINEERING

in the

FACULTY OF ENGINEERING, BUILT ENVIRONMENT AND INFORMATION TECHNOLOGY

UNIVERSITY

OF PRETORIA

October 2011

Executive Summary

Crime in South Africa is a continuous concern and various organizations, businesses and security services have been developed to combat crime. Their goals are to prevent crime and to protect the society. Unfortunately very little research and development have been done to improve these services. This includes adapting to South African environment (rules, laws and regulations) in this regard.

Various location models for facilities and emergency services have been developed and implemented internationally with great success. Examples of these include locations of police stations, fire departments, libraries and ambulances. A new location model can be developed for reaction vehicles to be used by the private security industry in South Africa. This can be achieved by extracting core ideas from existing location models and altering them to suit the needs of South Africa. It is expected that this new location model will improve the services provided by security companies, giving them a competitive advantage. The consumer will also greatly benefit from this in lieu of better service that will be provided.

1 Table of Contents

2 List of Figures and Tables

3 Glossary

SAIDSA	South African Intruder Detection Services Association
RV	Reaction Vehicle
PSIRA	Private Security Regulatory Authority
EPR	Elect Protect Response

4 Introduction and Background

4.1 EPR Company Background

EPR Security is a security company based in the West Rand, Gauteng. They have more than 10 000 customers in three major areas namely Randfontein, Krugersdorp and Westonaria.

The company was originally established in 1994 and has grown and expanded considerably over the years. EPR is currently the largest security supplier in Randfontein and has a reputation for stability and delivering products and services with exceptional quality.

EPR provides services and products to the domestic, commercial and industrial sectors. EPR is one of the few companies to provide their services to customers in rural areas on the West Rand. All of their officers are equipped with 9mm firearms and undergo continuous firearm training. The vehicles used by EPR are monitored by real-time satellite tracking to ensure the best reaction times as conditions may permit.

Company's vision statement: "Committed to the community, due to commitment from the community "

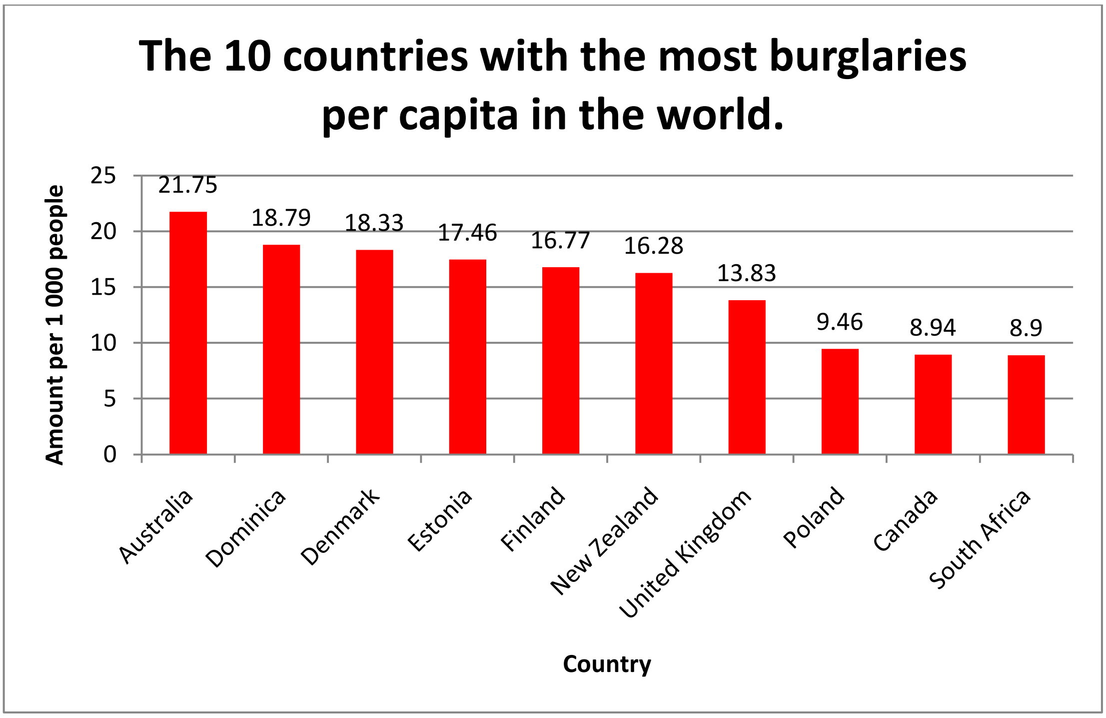
Company's mission statement: "Providing clients with the best and most efficient security service and advice, growing the current client base, understanding the needs of clients and always providing a personal service at an affordable rate to the end user."

From the company's mission and vision statements one can clearly see that this company cares about the community by giving competitive rates and personal customer service. One way of doing this is by keeping up to date with the latest technologies and strategic plans. This project will further help the company to gain a competitive advantage by using strategic measures to determine the fixed locations of their vehicles in residential and industrial areas.

4.2 Crime in South Africa

South Africa is one of the countries in the world with the highest number of burglaries per capita. South Africa ranks tenth in the world with approximately 8.89764 burglaries per 1000 people (see figure 1 below). This is the highest amount of burglaries per capita in Africa. The problem with crime is that there is no way to predict when or where it will take place, although there are certain areas with higher crime rates than others. One can only take steps to help prevent burglaries, one of these will be to get an efficient alarm system that is installed and maintained by a registered security company.

Figure 1 Burglaries per capita



4.3 Private Security Industry

The private security industry of South Africa is a very competitive environment. According to the PSIRA annual report for the period 2009/2010 the registered active security businesses have increased from 6 392 to 7 459. Due to the large number of security businesses and the competitiveness of the industry, it is essential to continuously improve the methods and processes of the business, as well as to comply with industry standards and be registered with the relevant associations.

EPR is a member of the following governing bodies: SAIDSA, PSIRA and SASSETA. In order to become a registered member, a company must comply with the requirements and regulations required by each of the relevant governing bodies.

SAIDSA is an association consisting of security providers. These security providers can deliver various products and/or services ranging from basic alarms to sophisticated electronic intruder detection systems. The objective of SAIDSA is to continuously develop and improve the skills and services provided by the private security industry by providing sufficient quality standards.

SASSETA is an educational authority that provides training to workers in various industries. By being a member of SASSETA, EPR ensures their customers that all employees have received the necessary training and has the relevant skills to perform their jobs.

4.4 Reaction Vehicles (RV)

According to SAIDSA a RV is a vehicle that is solely dedicated to the purpose of responding in a predetermined area to clients. The only requirements for RV's with regards to performance stated by SAIDSA are:

- A minimum of 2 vehicles must be fully equipped, manned and available 24-hours a day with a minimum of one fully equipped backup.
- The areas allocated to each vehicle must be predetermined and clearly marked on a map, which must be maintained for inspection.
- A log must be kept for all cases where the reaction time exceeds 15 minutes. If 10% of the occurrences in a specific area exceed 15 minutes, the situation must be reviewed and required steps must be taken to comply to the requirements of a 15 minute maximum reaction time.

The guidelines by SAIDSA also clearly state that "there can be no guarantee that a reaction service will arrive at a site within a specific time period". This is understandable because one cannot exactly predict where burglaries or crimes will take place, but companies can take strategic actions to minimize the response time when service is required.

Currently, the positions of the RV's in the residential areas are determined by managers of the company without using a specific tool or technique. This project will focus on developing a model that can be used to find strategic stationary positions for RV's in residential areas in order to give the company a competitive advantage with regards to response times.

5 Project Aim

The aim of this project is to find the best strategic locations for RV's in residential, commercial and industrial areas to:

- Minimize the response time to the point where service is required.

By doing this the following will also be achieved:

- The average distance travelled by RV's will decrease; and

- The total traveling costs of the company will decrease.

6 Project Scope

As a first step a literature review will be conducted to determine whether previous studies have been done on location models for reaction vehicles and whether they are applicable to the private security industry of South Africa. Time will also be spent with managers from the company to understand the operational processes and all of the relevant rules and regulations to which the company must adhere to when assigning RV's to fixed locations in the different types of areas.

Statistical analysis and forecasting will be done on the residential burglaries and call-outs of the suburbs on which the project will be done to help forecast future occurrences. One must take into account both actual burglaries and call-outs/false alarms because it is compulsory for the RV's to respond when an alarm is triggered.

A location model for the RV's will be developed through combining the relevant existing location models. The model will be versatile so that it can be used effectively for different business cases, for example:

- The determination of fixed locations of RV's during the day.
- The determination of fixed locations of RV's during the evening.

The mathematical model will be solved by using Lingo (version 8), it will also be integrated with a spreadsheet program (Microsoft Excel) to obtain inputs for the model and deliver results. Programming will also be done in Microsoft Excel (Visual Basic programming language) to determine the shortest routes, distances between locations and other necessary calculations.

7 Literature Review

7.1 Network Models

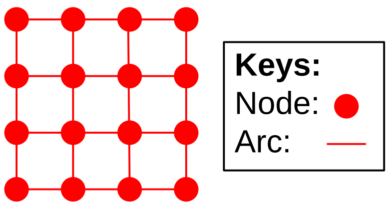
It is important to know the different definitions with regards to networks, because it will be used throughout this document. The following network definitions have been compiled in accordance with the definitions as stated by Winston et al. (2003) and Kasana et al. (2004).

Network: A network consists of nodes and arcs. The nodes are connected by arcs.

Node: Note that the nodes of a network can also be called vertices.

Arc: An arc is a pair of nodes that is linked, each arc has its own capacity. In this project the capacity will be distance, weight, etc. An arc may also represent a possible direction of motion between the two nodes.

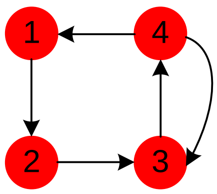
Figure 2 Graphical presentation of nodes and arcs



Chain: A sequence of arcs such that every arc has exactly one node in common with a previous arc is called a chain.

Path: A path is a chain where the end node of each arc is exactly the same as the beginning node of the following arc.

Figure 3 Graphical presentation of a path and chain



In figure 3 above (1,2)-(2,3)-(4,3) is a chain but not a path, but (1,2)-(2,3)-(3,4) is both a chain and a path.

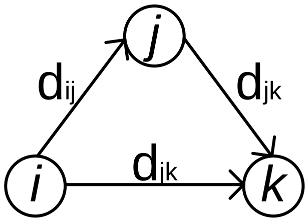
7.1.1 Floyd's Algorithm for Shortest Path

Floyd's Algorithm is an algorithm that is often used to find the shortest path between all of the nodes within a network. It differs from Dijkstra's algorithm in the sense that when using Dijkstra's algorithm one must specify a node to which the shortest distance must be determined in advance. Floyd's algorithm determines the shortest path between all of the nodes in the network at once, so you don't need to specify any nodes in advance when using the algorithm.

Floyd's algorithm works on a network with *n*-number of nodes. The network is viewed as a square matrix, this means it has exactly the same number of rows as it has columns. Entry

(i, j) of the matrix gives the distance d_{ij} from node i to node j. This entry in the matrix is finite if i and j is connected directly, otherwise the entry is set as infinite.

Figure 4 Logic behind Floyd's algorithm



Floyd's algorithm is based on the following logic: if three nodes i, j and k is given (figure 4 above) with the distances displayed on the three arcs. It would be shorter to reach k from i passing through j if $d_{ij} + d_{jk} < d_{ik}$. If this is the case it would be better to replace the direct route from node i to node k with the indirect path going through node j. This principle is applied to the whole network in a systematic way.

Steps for the algorithm as described by Kasana et al. (2004):

Step 0: Define the starting distance matrix D^0 as given subsequently. The diagonal elements given are marked with a very large number (10^{15} to indicate that they are blocked. Set $k = 1$.

Step k: Define row k and column k as pivot row and pivot column. Apply the triangle operation to each entry d_{ij} in D^{k-1}, for all i and j if the condition $d_{ik} + d_{kj} < d_{ij}$ $(i \neq k, j \neq k, i \neq j)$ is satisfied, the following change needs to be made:

- Construct D^k by replacing d_{ij} in D_{k-1} with $d_{ik} + d_{kj}$;

Set $k = k + 1$, and repeat step k till no changes are given by triangle operation.

Thus, after n steps, we can determine the shortest distance between nodes i and j from matrix D^n with the use of the following rule:

- From D^n, d_{ij} gives the shortest distance between nodes i and j.

7.2 Forecasting Crimes (Burglaries)

Forecasting is a method of examining historical data for patterns and trends, which is likely to occur again. Forecasting helps planners to handle uncertainties of future situations, by giving a description of a possible future reoccurrence. There are various approaches of forecasting for example simple graphics, manual calculations and sophisticated techniques.

A major problem with forecasting is that it holds a high level of uncertainty - this means that one is not certain whether an event will take place or what the result will be. Willis (1987) suggests three possible ways on how an organization can adapt to uncertainty:

- Increase the ability to pre-plan.
- Increase the flexibility to adapt to the inability to pre-plan.
- Increase the level of performance required for continued viability.

Forecasting forms part of pre-planning to help reduce uncertainty. Uncertainty can also be reduced by having better and timelier information on the current situation. This means that the data that is used for forecasting should be from a relevant time period, for example over the last ten years and not fifty years ago, and the data should also be updated on regular intervals.

If uncertainty cannot be reduced, the organization or planner must take the necessary steps to counteract the effects of uncertainty. Willis (1987) suggests the following methods to counteract uncertainty:

- Redundant resources. This can be done by using safety stock or other safety margins, some organizations will also use various approaches with the hope that at least one will be successful. In the case of the security company analyzed during this project it will mean using more reaction vehicles, but this will be very costly and ineffective.
- System flexibility. This approach suggests that elements must be designed into the system so that it will be less sensitive to uncertainty.

Forecasting is basically a statistical analysis of both historical and present data to help determine future outcomes.

During this project, two forecasting methods will be used namely: simple linear regression and moving averages.

Simple linear regression is a statistical method that helps the user to develop a mathematical relationship between the variables - this will enable the user to predict the value of a variable based on another variable. Linear regression uses a straight-line relationship to determine the value of the Y variable, when you know the value of the X variable.

Simple moving average is the average of the number of sets being studied. This means that the forecasted value for the next period will be the average value of the past periods.

7.3 Location Problems

7.3.1 Background of Location Problems

Location problems play an important part in the strategic planning of companies. This location that needs to be determined can be new stores, equipment in a manufacturing plant, police stations, fire stations, libraries and even schools. Numerous indepth studies have been undertaken and various mathematical models have been developed to help solve location problems. Unfortunately many of these models are extremely difficult to solve and require integer programming formulations.

7.3.2 Static and Deterministic Location Problems

Since most location models are very hard to solve due to their complexity, most of the studies focus on static, deterministic problems. In these problems all of the inputs that is required are taken to be fixed quantities that is determined before the model is solved. These input values might be demands at certain points, distances travelled and traveling times.

In this project a statistic and deterministic location model needs to be developed, because predetermined fixed locations need to be allocated to each RV. These locations and areas should be displayed at the company at all times as stated by SAIDSA, for this reason it would be impractical to use other models. The inputs of the model will still need to be updated frequently to ensure an effective model and accurate results.

7.3.3 The P-median problem

According to Owen (1998) the P-median problem was first introduced by Hakimi - the aim of this model is to minimize the distance travelled from a specific point to a predetermined demand point. If the demand at one point is higher than the demand at another point, the model would minimize the travel distance to the point where the demand is highest.

Owen (1998) describes the model as follows:

We let:

$i \triangleq$ Index of demand node

$j \triangleq$ Index of potential facility site

$h_i \triangleq$ Demand at node i

$d_{ij} \triangleq$ Distance between demand node i and potential facility site j

$P \triangleq$ The number of facilities to be located

$$X_j \triangleq \begin{cases} 1 \text{ if we locate a potential facility site } j \\ 0 \text{ if not} \end{cases}$$

$$Y_{ij} \triangleq \begin{cases} 1 \text{ if demand at node } i \text{ are served by a facility at node } j \\ 0 \text{ if not} \end{cases}$$

The formulation is given as

$$Minimize \sum_i \sum_j h_i d_{ij} Y_{ij} \qquad (1)$$

Subjected to:

$$\sum_j X_j = P \qquad (2)$$

$$\sum_j Y_{ij} = 1 \quad \forall i \qquad (3)$$

$$Y_{ij} - X_j \leq 0 \quad \forall i, j \qquad (4)$$

$$X_j \in_{\{0,1\}} \quad \forall j \qquad (5)$$

$$Y_{ij} \in_{\{0,1\}} \quad \forall i, j \qquad (6)$$

Explaining the objectives:

(1) Minimize the total demand-weighted distance between customers and facilities.

(2) Exactly P facilities must be located.

(3) Every demand must be assigned to some facility.

(4) Allows assignment only to sites where facilities have been located.

(5) & (6) Binary requirements for problem variables.

This mathematical formula only allows facilities to be located at certain sites in the network, these are the nodes of the network.

The P-median location problem will be used as the base for the development of a location model for RV's. The only major difference between the P-median location model and the location model for reaction vehicles is that the P-median services demand points, which are nodes on the network. In the location model for RV's the points that need to be served (the customers) lie on the arcs on the network, not on the nodes.

7.3.4 Covering Problems

P-median problems focus on minimizing the average distance travelled, but in some situations the problem can't be solved by minimizing the average distance travelled. For emergency services such as hospitals and fire stations the distance from the location to the point where service is required must be either minimized or within acceptable distance. This acceptable distance is often set by regulatory organisations such as SAIDSA, which specifies that response to a point of service must be within 15 minutes. If a vehicle fails to respond within this time limit, it must be documented. At the end of each month the managers of the company needs to analyse the response times. If more than 10% of the call-outs are longer than 15 minutes, steps needs to be taken to minimize the response times.

In covering problems, the demand is said to be covered if the facility servicing it is within a specific distance. This is due to response times being directly proportionate to the distance travelled. Two types of covering problems have been developed, they are the location set covering problem and the maximal covering problem. Both of these have a relationship with the P-median location problem.

7.3.5 Location Set Covering Problem

The objective of the location set covering problem is to minimize the cost of the facility location, whilst still obtaining the specified level of coverage.

The mathematical model of the location set covering problem as described by Owen (1998) are as follows:

We let:

$c_j \triangleq$ fixed cost of siting a facility at node j

$S \triangleq$ maximum acceptable service distance (or time)

$N_i \triangleq$ set of facility sites j within acceptable distance of node i

$$X_j \triangleq \begin{cases} 1 \text{ if we locate a potential facility site } j \\ 0 \text{ if not} \end{cases}$$

The formulation is given as

$$Minimize \sum_j c_j X_j \qquad (1)$$

Subjected to:

$$\sum_{j \in N_i} X_j \geq 1 \quad \forall i \qquad (2)$$

$$X_j \in \{0,1\} \quad \forall j \qquad (3)$$

Explaining the objectives:

The objective (1) of the model is to minimize the cost associated with allocating facilities to specific locations. If the cost to allocate a facility to a location is equal for all the locations the model will try to minimize the number of facilities. Constraint (2) ensures that all of the demand points have at least one facility within acceptable distance. Constraint (3) is a binary requirement for the variables.

A problem with this model is that it doesn't take demand size at specific nodes into consideration. This might lead to one facility servicing only 1 demand whilst another facility might need to service 100 demands. Another problem is that if the acceptable service distance is very small, a large number of facilities may be situated in very close proximity to one another.

7.3.6 Maximal Covering Problem

Due to the problems associated with the location set covering problem, the maximal covering problem has been developed. The maximal covering problem seeks to maximize the amount of demand covered within the acceptable service distance by locating a fixed number of facilities.

The following set of additional variables is necessary for the maximal covering problem:

$$Z_i = \begin{cases} 1 \text{ if node } i \text{ is covered} \\ 0 \text{ if not} \end{cases}$$

The formulation is then given as

$$Maximize \sum_i h_i Z_i \qquad (1)$$

Subjected to:

$$Z_i \le \sum_{j \in N_i} X_j \quad \forall i \qquad (2)$$

$$\sum_j X_j \le P \qquad (3)$$

$$X_j \in \{0,1\} \quad \forall j \qquad (4)$$

$$Z_i \in \{0,1\} \quad \forall i \qquad (5)$$

Explaining the objectives:

The objective (1) of this mathematical model is to maximize the amount of demand covered. The constraint (2) is used to determine which of the demand nodes are covered within the acceptable service distance. Node i can only be considered covered when the distance to the closest facility at j is smaller than the maximum acceptable service distance. Constraint (3) states how many facility locations should be determined, due to limited resources. Constraints (4) and (5) are binary requirements.

8 The New Location Model for Reactive Vehicles

In defining variables, we let:

$\bar{I}, \bar{J}, \bar{K} \in \{1..n\}$, where n is the number of nodes in the network

$$X_k \triangleq \begin{cases} 1 \text{ if node should be a fixed location for RV's, where } k \in \bar{K} \\ 0 \text{ otherwise} \end{cases}$$

$$S_{kij} \triangleq \begin{cases} 1 \text{ if the arc between nodes } i \text{ and } j \text{ is serviced by reaction vehicle/s located at node } k, \\ \quad \text{where } i \in \bar{I}, j \in \bar{J} \text{ and } k \in \bar{K} \\ 0 \text{ otherwise} \end{cases}$$

$$N_{ij} \triangleq \begin{cases} 1 \text{ if nodes i and j is linked with an arc (a street on the map of the area),} \\ \quad \text{where } i \in \bar{I} \text{ and } j \in \bar{J} \\ 0 \text{ otherwise} \end{cases}$$

$P \triangleq$ the number of fixed positions to be determined

$G \triangleq$ the maximum allowed distance between the location and demand point to be serviced

$T \triangleq$ the maximum number of clients that can be serviced by a reaction vehicle

$W_{ij} \triangleq$ weight allocated to arc between nodes i and j, where $i \in \bar{I}$ and $j \in \bar{J}$

$C_{ij} \triangleq$ the number of clients situated between nodes i and j, where $i \in \bar{I}, j \in \bar{J}$

$AD_{kij} \triangleq$ the average distance from node k to arc i, j where $i \in \bar{I}, j \in \bar{J}$ and $k \in \bar{K}$

$D_{ki} \triangleq$ the shortest distance from node k to node i, where $i \in \bar{I}$ and $k \in \bar{K}$

$D_{kj} \triangleq$ the shortest distance from node k to node j, where $j \in \bar{J}$ and $k \in \bar{K}$

The formulation is then given as

$$Minimize \sum_i \sum_j \sum_k W_{ij} AD_{kij} S_{kij} \qquad (1)$$

Subjected to:

$$\sum_k X_k = P \qquad (2)$$

$$0 \leq \left[\left(\sum_k S_{kij} \right) - 1 \right] + M_1 \times y_{ij} \qquad \forall i, j \qquad (3)$$

$$N_{ij} \leq M_2 (1 - y_{ij}) \qquad \forall i, j \qquad (4)$$

$$y_{ij} = \{0,1\} \qquad \forall i, j \qquad (5)$$

$$\sum_k S_{kij} \leq 1 \qquad \forall i, j \qquad (6)$$

$$S_{kij} - X_k \leq 0 \qquad \forall k, i, j \qquad (7)$$

$$\sum_i \sum_j C_{ij} \times S_{kij} \leq T \qquad \forall k \qquad (8)$$

$$AD_{kij} = \frac{D_{ki} + D_{kj}}{2} \qquad \forall i, j, k \qquad (9)$$

$$AD_{kij} \times S_{kij} \leq G \qquad \forall i, j, k \qquad (10)$$

$$X_k \in \{0,1\} \qquad \forall k \qquad (11)$$

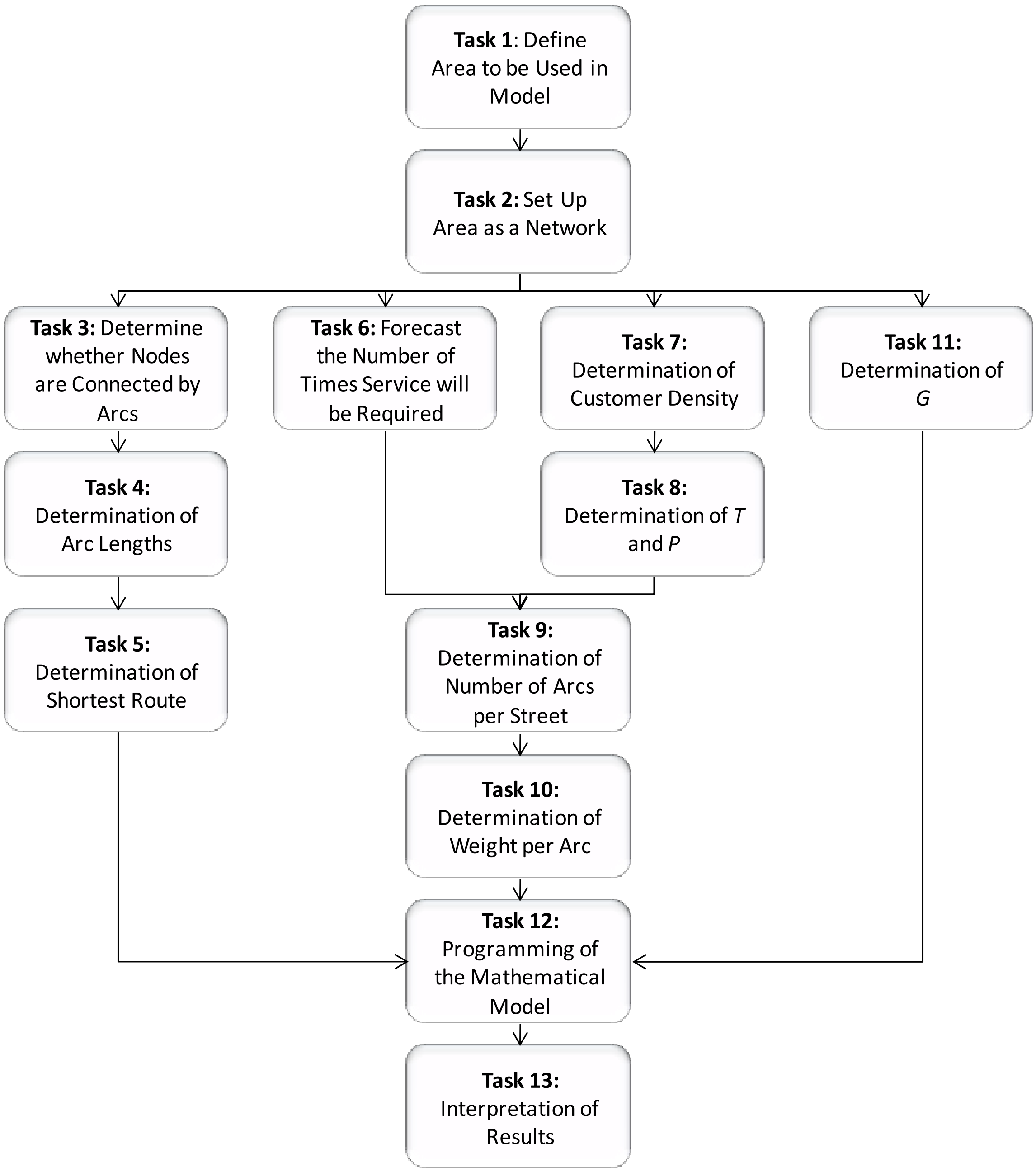
$$S_{kij} \in \{0,1\} \qquad \forall k, i, j \qquad (12)$$

Explaining the objectives:

The objective (1) as stated above is to reduce the average time/distance travelled from a fixed location to the point where the service is needed. Since the weight is fixed for each arc the model would allocate reaction vehicles closer to arcs with a higher weight. Constraint (2) states that only a certain amount of positions for reaction vehicles must be determined. Constraints (3),(4) and (5) states that if the nodes are connected by an arc they must be serviced by at least one reaction vehicle. Constraint (6) makes sure that only one vehicle is servicing each street. Constraint (7) allows assignment only to sites to which reaction vehicles have been allocated. Constraint (8) ensures that each reaction vehicle can only service a predetermined number of customers. Formula (9) is a calculation to determine the average distance from a node to an arc in the network. Constraint (10) places a limit on the maximum distance from the reaction vehicle's location to the demand point. Constraints (11) and (12) are binary constraints.

9 Implementation Framework for the New Location Model for Reaction Vehicles

Figure 5 Implementation Framework for the New Location Model for RV's



10 Practical Example of the New Location Model for Reaction Vehicles and Application of the Implementation Framework

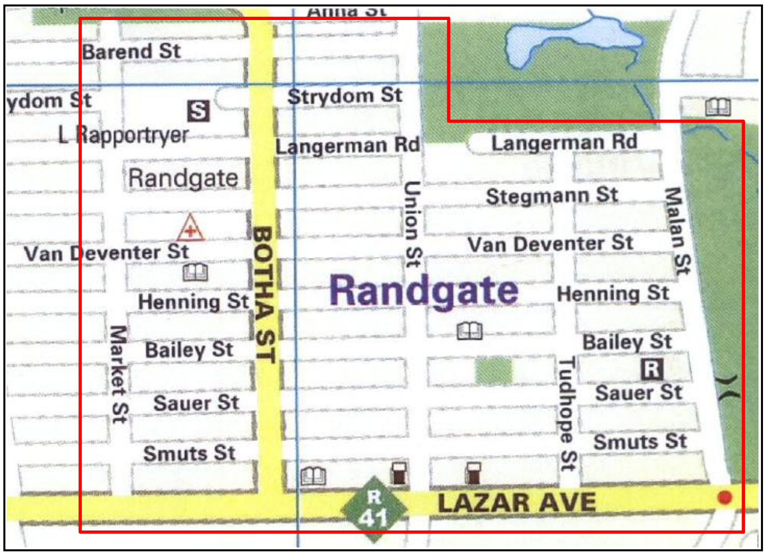
Task 1: Define Area to be used in the model

It is the responsibility of management to determine the area for which the model will be used. The model can either be used on a small area, where the number of vehicles and fixed locations to be determined is already known, or the model can be used on a larger area, where the number of vehicles and locations must still be determined. When making the decision with regards to area size, management must keep in mind the capability of the linear programming software and the hardware capability, not all of the different linear programming solver software and versions allow the same number of constraints and variables.

A map of the area must be obtained, which will be used to reference specific positions, areas and streets in the tasks below.

Randgate, a suburb in Randfontein, will be used as the area to which the mathematical model will be applied. The area is bordered by the red lines in figure 6.

Figure 6 Area to be studied

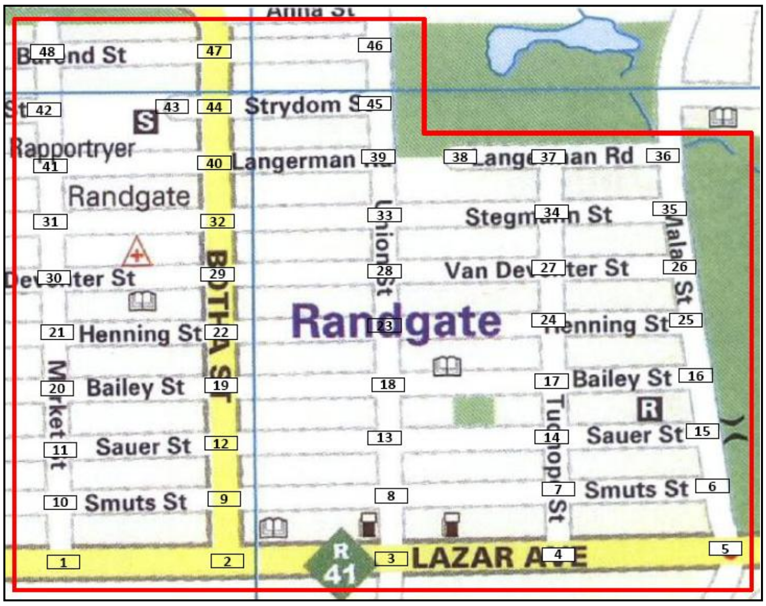


Task 2: Set Up Area as a Network

For an area to be used in a mathematical model, it must be set up as a network consisting of nodes and arcs. Software such as Microsoft Visio or Microsoft PowerPoint can easily be used to easily label all of the intersections within the area as nodes. When the intersections on the map are connected via a road it is said to be an arc. One must be careful when labeling the nodes on the map, some of the roads may be part of a residential development and this may not be part of the area that needs to be studied.

The area being studied was labeled in order to represent a network. All of the intersections were labeled as nodes (as can be seen in figure 7 below) using Microsoft Visio.

Figure 7 Area as a Network



Task 3: Determine whether Nodes are connected by Arcs

When two nodes in the network are connected with an arc (two labeled intersections connected with a road) it must be labeled in the matrix (N_{ij}) with a 1 otherwise it must be labeled with a 0. Only the portion above the diagonal of the matrix needs to be completed. All values below the diagonal of the matrix must be 0, since when the value of N_{ij} is 1 the mathematical model will make sure that arc ij is serviced. Arc ij refers to the same street section as arc ji, thus only arc ij or arc ji needs to be serviced.

Matrix N_{ij} (as per table 1 below) is an extract of the matrix that was completed in Microsoft Excel - note that all of the values below the diagonal are all equal to 0. For the complete matrix refer to Appendix 1.

Table 1 Matrix Nij

	1	2	3	4	5	6	7	8	9	10
1	0	1	0	0	0	0	0	0	0	1
2	0	0	1	0	0	0	0	0	1	0
3	0	0	0	1	0	0	0	1	0	0
4	0	0	0	0	1	0	1	0	0	0
5	0	0	0	0	0	1	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0
7	0	0	0	0	0	0	0	1	0	0
8	0	0	0	0	0	0	0	0	1	0
9	0	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0

Task 4: Determination of Arc Lengths

In order to determine the travelling distances of the vehicles and the distance from the fixed location to the point where service is required, the distance between the various nodes in the network is required. There are various methods that can be used to measure street lengths:

- Physical measurement of road maps using a ruler, the maps must be drawn according to scale.
- Internet services such as Google Earth have a function that can be used to measure street distances.

All arc lengths must be captured in a matrix format. The easiest way for doing this is by using spreadsheet software i.e. Microsoft Excel. The matrix will be symmetrical around the diagonal as long as there is no one-way streets within the area and have a size of $n \times n$, where n is the number of nodes present in the network. When two nodes are not connected with an arc, the cell must contain a very large value (a value of 10^{15} is sufficient). The values captured in this matrix will be used as the inputs to the next step, in order to determine the shortest route.

In the current example the lengths of the various arcs present in the area being studied, have been physically measured using a ruler. The map was drawn according to scale 1cm:200m (The map insert in Tasks 1 and 2 is not drawn according to scale). The distance between arcs, not directly connected, was given a value of 10^{15}. The measured distances were used to complete the matrix "Floyd Input" of which an extract can be seen in table 2 below (for a complete matrix, refer to Appendix 2).

Only the part above the diagonal of the matrix has been completed, a macro has been developed to automatically complete the part of the matrix below the diagonal. For more information on the macro, refer to Appendix 3.

Table 2 Floyd Input Matrix

	1	2	3	4	5	6	7	8	9	10
1	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150
2	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15
3	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15
4	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15
5	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15
6	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15
7	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15
8	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15
9	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250
10	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0

Task 5: Determination of the Shortest Route

In order to determine the shortest route from one node in the network to another node in the network, Floyd's algorithm must be used. Floyd's algorithm can be effectively used since it determines the distances between all nodes in the network. The distances between the different nodes that are interconnected, have already been determined in task 4 above. For more information on how Floyd's algorithm is applied, refer to paragraph 7.1.1.

A macro was developed to use the inputs in matrix "Floyd Input" and then to calculate the distances between all of the nodes present in the network, using Floyd's Algorithm (see table 3 below for the Pseudo code).

Table 3 Pseudo code for Floyd's Algorithm

```
Sub Macro2()
   Dim i, j, k, n
   ActiveWorkbook.Sheets("Floyd Input").Activate
   n = ActiveWorkbook.Sheets("Floyd Input").Cells(1, 16).Value

   'Clear output sheet
   For i = 1 To n
      For j = 1 To n
         Sheets("Dij").Cells(1 + i, 1 + j).Value = Null
      Next j
   Next i

   'Copy weights to output sheet
   For i = 1 To n
      For j = 1 To n
         Sheets("Dij").Cells(1 + i, 1 + j).Value = Sheets("Floyd Input").Cells(2 + i, 1 + j)
         If Sheets("Floyd Input").Cells(2 + i, 1 + j).Value = "" Then Sheets("Dij").Cells(1 + i, 1
+ j).Value = 10 ^ 15
      Next j
   Next i

   'Run algorithm
   Sheets("Dij").Activate
```

```
    For k = 1 To n
        For i = 1 To n
            For j = 1 To n
                If (Sheets("Dij").Cells(1 + i, 1 + j) > (Sheets("Dij").Cells(1 + i, 1 + k) + Sheets("Dij").Cells(1 + k, 1 + j))) Then
                    Sheets("Dij").Cells(1 + i, 1 + j) = Sheets("Dij").Cells(1 + i, 1 + k) + Sheets("Dij").Cells(1 + k, 1 + j)
                End If
            Next j
        Next i
    Next k
End Sub
```

The output of the algorithm was used to compile matrix D_{ij}. An extract of matrix D_{ij} can be viewed in table 4 below (For a complete matrix, refer to Appendix 4).

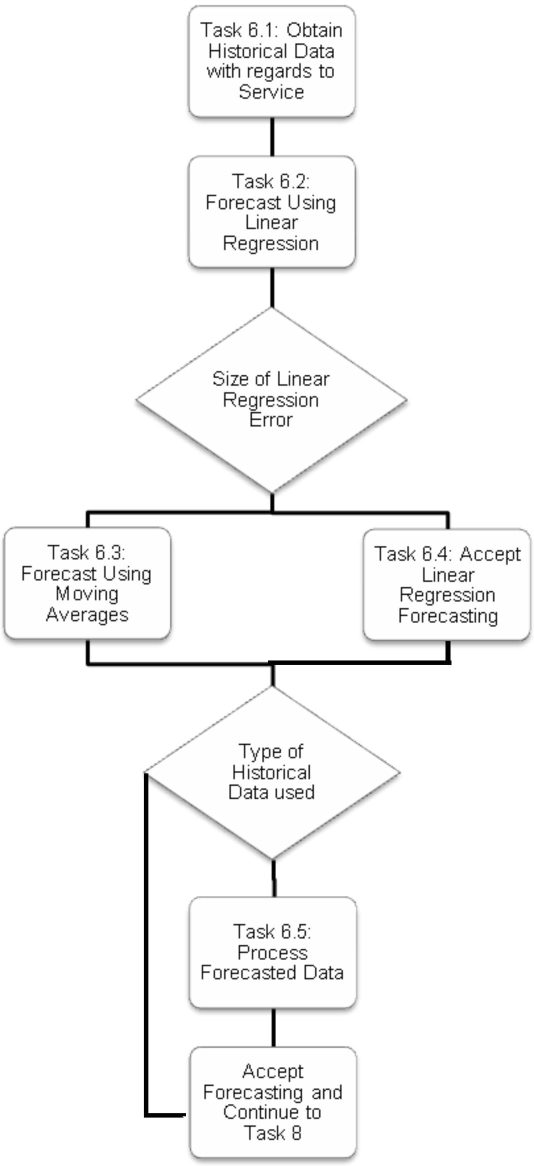
Table 4 Floyd's Algorithm Output (Matrix Dij)

	1	2	3	4	5	6	7	8	9	10
1	0	250	500	750	1000	1150	900	650	400	150
2	250	0	250	500	750	900	650	400	150	400
3	500	250	0	250	500	650	400	150	400	650
4	750	500	250	0	250	400	150	400	650	900
5	1000	750	500	250	0	150	400	650	900	1150
6	1150	900	650	400	150	0	250	500	750	1000
7	900	650	400	150	400	250	0	250	500	750
8	650	400	150	400	650	500	250	0	250	500
9	400	150	400	650	900	750	500	250	0	250
10	150	400	650	900	1150	1000	750	500	250	0

Task 6: Forecast the Number of Times Service will be required

In order for the company to strategically position their vehicles in areas, they must attempt to do a predict/forecast of where the service will be required. It would be advantageous for a company to position vehicles closer to either (i) an area where more customers are situated and/or (ii) where more call-outs or burglaries are likely to occur (the mathematical model takes both of these factors into consideration). From a service point of view the company can't distinguish between call-outs and actual burglaries since they are obliged by law to respond to any situation (alarm triggered). The following flowchart (see Figure 8 below) can be used as a guideline for effective forecasting by a security company:

Figure 8 Forecasting Flowchart



Task 6.1: Obtain Historical Data with regards to Service

All private security companies in South Africa keep record of the information when a service is delivered. This information includes how frequently a vehicle responds to a specific area or to a specific customer. Some companies store this data on an area-level, where other may store it on street-level. To obtain better results from the mathematical model, it is best for the company to record the information on street-level, but processing the area-level information it can also be used effectively.

The number of times service was required for the previous periods must be obtained, if more periods are used, the results of the forecasting will be more accurate. A minimum of three periods will be sufficient for the purposes of this model.

In the below example, the frequency that service was required by customers for the previous three months has been captured in a spreadsheet, this variable will be used in steps 6.2 to 6.5 below to do forecasting. The spreadsheet extract with the data can be viewed in table 5 below.

Table 5 Historical Data - Service Requirement

Street Name	Historical Data Number of Times Service was required during month X		
	1	2	3
Bailey Street	1199	1154	1357
Barend Street	886	860	964
Botha Street	209	217	268
Henning Street	484	112	151
Langerman Street	997	868	763
Malan Street	728	1374	1472
Market Street	113	447	201

Sauer Street	0	0	0
Smuts Street	0	0	0
Stegmann Street	434	647	960
Strydom Street	992	991	994
Tudhope Street	764	1432	858
Union Street	333	260	173
Van Deventer Street	85	92	26

Task 6.2: Forecast Using Linear Regression

The historical data captured in Task 6.1 must now be used as inputs to forecast the number of times service will be required in the future period. The regression sum of squares (SSR), error sum of squares (SSE) and the total sum of squares (SST) must also be calculated. It is used to determine the ratio of the regression sum of squares to the total sum of squares (r^2).

The value of r^2 will always be less than 1. If the value of r^2 is exactly 1 it means that a perfect straight line relationship is obtained between the x and y values of the linear equation. If the value of r^2 is very small it means that the data doesn't form a straight line and another method of forecasting must be used.

The formulas for linear regression can be viewed in Appendix 5.

The linear forecasting and linear forecasting errors for the example can be viewed in table 6 below.

Table 6 Linear Regression Calculations

Linear Regression								
Calculations		Slope	Y Intercept	Forecasted Value for Next	Linear Regression			
SSXY	SSX	b1	b0	Y4	SSR	SSE	SST	r^2
158	2	79	1078.666667	1395	12482	10250.66667	22732.66667	0.549077686
78	2	39	825.3333333	981	3042	2816.666667	5858.666667	0.519230769
59	2	29.5	172.3333333	290	1740.5	308.1666667	2048.666667	0.849576961
-333	2	-166.5	582	-84	55444.5	28153.5	83598	0.663227589
-234	2	-117	1110	642	27378	96	27474	0.996505787
744	2	372	447.3333333	1935	276768	50050.66667	326818.6667	0.846854933
88	2	44	165.6666667	342	3872	56066.66667	59938.66667	0.064599368
0	2	0	0	0	0	0	0	#DIV/0!
0	2	0	0	0	0	0	0	#DIV/0!
526	2	263	154.3333333	1206	138338	1666.666667	140004.6667	0.988095635
2	2	1	990.3333333	994	2	2.666666667	4.666666667	0.428571429
94	2	47	924	1112	4418	257094	261512	0.016894062
-160	2	-80	415.3333333	95	12800	32.66666667	12832.66667	0.997454413
-59	2	-29.5	126.6666667	9	1740.5	888.1666667	2628.666667	0.662122749

Task 6.3: Forecast Using Moving Averages

If the value of r^2 is less than 0.7, the forecasted value would not be accurate enough for this model and moving averages should instead be used as a more accurate forecasting method.

The formulas for moving averages can be viewed in Appendix 5.

The forecasting for the example by using moving averages can be viewed in table 7 below.

Table 7 Moving Averages Calculation Results

Street Names	Forecasted Value for Next Period
Bailey Street	1236.666667
Barend Street	903.3333333
Botha Street	231.3333333
Henning Street	249
Langerman Street	876
Malan Street	1191.333333

Market Street	253.6666667
Sauer Street	0
Smuts Street	0
Stegmann Street	680.3333333
Strydom Street	992.3333333
Tudhope Street	1018
Union Street	255.3333333
Van Deventer Street	67.66666667

Task 6.4: Accept Linear Regression Forecasting

If the r^2 is greater than 0.7, the forecasted value by using linear regression would be accurate enough for this model. In table 8 below, all the values in the blocks that have a green background have a r^2 of greater than 0.7.

Table 8 Selecting Forecasting Method

Street Name	Linear Regression Forecasted Value for Next Y4	Error r^2	Moving Averages Forecasted Value for Next Y4	Forecasted Value to be Used
Bailey Street	1395	0.549077686	1236.666667	1237
Barend Street	981	0.519230769	903.3333333	903
Botha Street	290	0.849576961	231.3333333	290
Henning Street	-84	0.663227589	249	249
Langerman Street	642	0.996505787	876	642
Malan Street	1935	0.846854933	1191.333333	1935
Market Street	342	0.064599368	253.6666667	254
Sauer Street	0	#DIV/0!	0	#DIV/0!
Smuts Street	0	#DIV/0!	0	#DIV/0!
Stegmann Street	1206	0.988095635	680.3333333	1206
Strydom Street	994	0.428571429	992.3333333	992
Tudhope Street	1112	0.016894062	1018	1018
Union Street	95	0.997454413	255.3333333	95
Van Deventer Street	9	0.662122749	67.66666667	68

Task 6.5: Process Forecasted Data

The forecasted data only needs to be processed further if the data that was used as inputs were captured on an area-level. For the model to work efficiently all data needs to be on a street-level. The following formula must be used to transform the data from area-level to street-level:

$$\text{Forecasted Value at Street-Level} = \frac{\text{Forecasted Value at Area-Level}}{\text{Number of Customers in Area}} \times \text{Number of Customers in Street}$$

In the current example, all of the data was captured on a street-level, thus no further processing is needed.

Task 7: Determination of Customer Density

The customer density refers to the number of customers living in each street. There are various ways to determine this number: the company can either obtain the data from a database if the function is available or it can be physically counted by using a complete customer list that contains information about customer addresses.

A list of streets together with the number of customers situated in each street was captured in table 9 below.

Table 9 Customers per Street

Street Name	*Customers Per Street*
Bailey Street	30
Barend Street	7
Botha Street	21
Henning Street	3
Langerman Street	15
Malan Street	16
Market Street	22

Sauer Street	0
Smuts Street	0
Stegmann Street	12
Strydom Street	17
Tudhope Street	19
Union Street	25
Van Deventer Street	29

Task 8: Determination of *T* and *P*

T is the maximum number of clients that can be serviced by each location (note that this is not the quantity of vehicles). There is currently no legislation or standard that can be used to determine the maximum number of customers that each vehicle is required to service. Companies have their own unique policies that are compiled by their management. These policies may contain information with regards to the number of customers that is serviced by each individual vehicle. According to management of EPR, a good guideline is to use 1 vehicle to service an area containing approximately 350 customers.

One should keep in mind that in this model one defines the number of locations to be determined, not the number of vehicles to allocate. For safety reasons most private security companies allocates one RV to a fixed location during the day, but they allocate two RV's to a fixed location during the evening. This means that the value of *T* during nighttime is twice the value of *T* during daytime (See table 10 below).

Table 10 T Calculation Examples

Value of T during daytime	Value of T during the Evening
200	400
350	700

Table 11 and table 12 below is an extract from the spreadsheet - the user needs to give maximum number of customers each vehicle should service. The spreadsheet has been set up to automatically calculate the value of *T* that will be used during the evening.

Table 11 *T* Calculation Examples: During the Day

What is the maximum number of customers each vehicle should service?	***100***
Time of Day (Day or Night):	Day
Value of **T** for Model:	100

Table 12 *T* Calculation Examples: During the Evening

What is the maximum number of customers each vehicle should service?	***100***
Time of Day (Day or Night):	Night
Value of **T** for Model:	200

P is the number of fixed locations, where the RV/s will be allocated. The following formulae can be used to determine *P* (assuming that the company is attempting to use as few RV's as possible).

Table 13 Suggested P Value Formulas

During the day	$P = \frac{\text{Total Number of Customers in Area}}{T}$
During the night	$P = \frac{\text{Total Number of Customers in Area}}{T \times 2}$

These are only suggested figures, a company may also decide to use more vehicles if they have sufficient workforce that allow for it - in this instance the following formulas can be used to determine *P*.

Table 14 *P* Values, using own number of RV's

During the day	$P = \text{Total number of RV's to be used}$
During the night	$P = \frac{\text{Total number of RV's to be used}}{2}$

Tables 15 and16 below are extracts from the spreadsheet where all calculations for the mathematical model are done. The spreadsheet has been set up to automatically calculate the suggested value of *P*. The user can choose to overwrite the suggested *P* value by entering a different *P* value in the "P to be Used" block. The user should not use a *P* value lower than the suggested value since the model would malfunction because each vehicle will be required to service more customers than what is allowed.

Table 15 Using Suggested *P* Values

Number of Customers in Area:	216
Suggested number of Vehicles:	3
Suggested P:	3
P to be Used:	3

Table 16 Using own *P* values

Number of Customers in Area:	216
Suggested number of Vehicles:	3
Suggested P:	1
P to be Used:	3

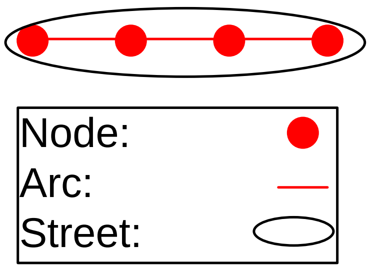
When an uneven number of vehicles are to be used during the evening, management needs to decide how they will group the vehicles. They can either allow one vehicle park on it's own at a fixed location, or they can allow three vehicles park together at a fixed location. *P* and *T* needs to be adjusted accordingly.

Task 9: Determination of Number of Arcs per Street

Since many of the streets involve very long distances (some extending over multiple areas) it may be possible for a vehicle to service one part of a street and not the other. This is why each individual street must be divided into different sections. Being a network model, this can easily be achieved since a street will consist of multiple arcs as illustrated in the diagram (figure 9) below. The following formula can be used as a guideline to determine the number of arcs per street:

$$\text{number of arcs in street} = \text{number of nodes} - 1$$

Figure 9 Number of Arcs Determination Diagram



The number of arcs contained in each street was physically counted and table 17 below was populated with the results.

Table 17 Arcs per Street

Street Name	***Arcs Per Street***
Bailey Street	4
Barend Street	2
Botha Street	9
Henning Street	4
Langerman Street	4
Malan Street	7

Market Street	9
Sauer Street	4
Smuts Street	4
Stegmann Street	4
Strydom Street	2
Tudhope Street	7
Union Street	90
Van Deventer Street	4

Task 10: Determination of Weight per Arc

The weight, in both the single facility location model and the multiple facility location model, is the number of trips between the facility and a specific point. In the location model for RVs, the number of trips between the fixed location and the point where service is required is unknown and a weight must be calculated by taking into account the forecasted number of occurrences where service will be required and the customer density.

The following formulae must be used to determine the weight of each arc:

$$\text{weight per arc} = \text{number of customers per arc} + \text{call-outs forecasted per customer} \times \text{number of customers per arc}$$

$$\text{call-outs forecasted per customer} = \frac{\text{call-outs forecasted per street}}{\text{customers in street}}$$

$$\text{number of customers per arc} = \text{Round}\left(\frac{\text{number of customers per street}}{\text{number of arcs in street}}\right)$$

The following matrices must be completed during this task:

- The number of customers per arc must be documented in matrix C_{ij}.
- The weight of each arc must now be documented in matrix W_{ij}.

The matrices that were completed during this task can be viewed in Appendix 6 and 7.

Only the portion above the diagonal of the matrices has been completed, a macro has been developed to automatically complete the portion of the matrices below the diagonal. For more information on the macro, refer to Appendix 3.

Table 18 Arc Weight Determination

Street Name	*Customers Per Street*	*Arcs Per Street*	*Customers Per Arc*	*Call-Outs Per Customer*	*Weight Per Arc*
Bailey Street	30	4	8	41	358
Barend Street	7	2	4	129	523
Botha Street	21	9	2	14	49
Henning Street	3	4	1	83	86
Langerman Street	15	4	4	43	187
Malan Street	16	7	2	121	258
Market Street	22	9	2	12	46
Sauer Street	0	4	0	#DIV/0!	#DIV/0!
Smuts Street	0	4	0	#DIV/0!	#DIV/0!
Stegmann Street	12	4	3	101	315
Strydom Street	17	2	9	58	539
Tudhope Street	19	7	3	54	181
Union Street	25	90	0	4	25
Van Deventer Street	29	4	7	2	43

The errors given in the blocks are due to the fact that the spreadsheet calculation attempts to divide a number by zero. This happens when an arc has no customers and no call-outs have been forecasted for the arc. When this occurs a weight of 0 must be allocated to the relevant arcs.

Task 11: Determination of *G*

G is defined in the mathematical model as the maximum distance that a vehicle is allowed to travel to the point where service is required. According to the standard set up by SAIDSA, all call-outs must serviced within 15 minutes, if the response exceeds 15 minutes it must be documented. If more than 10% of all the responses in a month exceed 15 minutes, corrective action must be taken to reduce the response time. The management of the company can decide what the maximum allowed response time of their vehicles to the point where service is required should be.

In order to determine the value of *G* the user must initially determine the average speed that a vehicle travels at. Due to the geography and street lengths differing from area to area, vehicles don't necessarily achieve an average speed that is similar to the speed limit. Private security companies keep record of the average response times and the average distance to the location where service is required for each of their vehicles. This information can be used to determine the average distance a vehicle can travel within the desired response time.

$$\text{Average Speed} = \frac{\text{Average Response Distance}}{\text{Average Response Time}}$$

$$\text{G (m)} = \text{Average Driving Speed } (km/h) \times \frac{1000}{60} \times \text{Desired Response Time (min)}$$

The historical data of the vehicles has been captured in table 19 below. The spreadsheet is set up to automatically do all of the relevant calculations. The user only needs to update the historical data periodically. Note that only two months of data is displayed in table 19 (month 2 is omitted), but the data of three consecutive months is used.

Table 19 G Value Calculations

	Month 1			Month 3		
	Average Response Distance (km)	Average Response Time (min)	Average Speed (km/h)	Average Response Distance (km)	Average Response Time (min)	Average Speed (km/h)
Vehicle 1	6.04	7.07	51.2588	6.24	6.43	58.22706065
Vehicle 2	8.12	7.55	64.52982	8.05	6.55	73.74045802
Vehicle 3	8.81	5.98	88.39464	7.54	5.42	83.46863469
Vehicle 4	11.83	11.18	63.48837	12.07	10.43	69.43432407
Vehicle 5	5.14	7.67	40.2086049	5.13	6.1	50.45901639
Average	39.94	39.45	60.7452471	39.03	34.93	67.04265674
Average Driving Speed (km/h):				64.63183003		
Desired Response Time (min):				1.5		
Value of ***G*** (m):				1616		

The value of desired response time is very small, because a small area is used for the example.

Task 12: Programming of the Mathematical Model

Appropriate linear programming software needs to be used to effectively solve the model. When choosing the software to be used, one must bear in mind the number of variables and constraints that will be used.

Table 20 below shows the generic programming for the RV location model when using Lingo. Data displayed in red are changes when the size and scope of the model changes.

Table 20 Lingo Programming Code

```
SETS:
!Set Members;
POSITIONS/1..Number of Nodes in Network/  : X;
FROM_NODE/1.. Number of Nodes in Network /: ;
TO_NODE/1.. Number of Nodes in Network /: ;
COMBINATIONS (POSITIONS, FROM_NODE, TO_NODE): AD, S;
ARCS (FROM_NODE, TO_NODE): N, y, W, C, D;
ENDSETS

DATA:
!POSITIONS = P1..P4; !P is hoeveel locations;
!ARCS =....;
!Attribute values;
P = The P value Calculated in Task 8;
T = The T value Calculated in Task 8;
G = The G value Calculated in Task 11;
N = Matrix Nij Calculated in Task 3;
C = Matrix Cij Calculated in Task 7;
D = Matrix Dij Calculated in Task 5;
W = Matrix Wij Calculated in Task 10;
ENDDATA
!1 Objective function: Reduce time/ distance travelled;
MIN = @SUM(COMBINATIONS(k.i.j):W(i,j)*AD(k,i,j)*S(k,i,j));
!2 Only a specific amount of vehicles must be determined;
@SUM(POSITIONS(k): X(k))=P;
!3-5 IF statement - if it is a road, it must be serviced;
!3;
@FOR (ARCS(i,j):
        0<= @SUM (POSITIONS(k):S(k,i,j))-1+9999*Y(i,j);
!4;
@FOR(ARCS (i,j):
        N(i,j)<=9999*(1-Y(i,j)));
!5;
```

```
@FOR (ARCS (i,j):
        @BIN(Y(i,j)));
!6 Only one vehicle is servicing each street;
@FOR(ARCS(i,j):
        @SUM(POSITIONS(k):
                S(k,i,j))<=1);
!7 Allows assignment only to sites to which reaction vehicles has been allocated;
@FOR (COMBINATIONS(k,i,j):S(k,i,j)-X(k) <=0);
!8 Each vehicle can only service a maximum number of T customers;
@FOR (POSITIONS (k):
        @SUM (ARCS (i,j):
                C(i,j)*S(k,i,j))<=T);
!9 Calculate average distance from node to arcs;
@FOR (COMBINATIONS(k,i,j):
        AD(k,i,j) = (D(k,i)+D(k,j))/2) );
!10 Maximum allowed distance from RV to customer;
@FOR (COMBINATIONS (k,i,j):
        AD(k,i,j)*S(k,i,j)<= G );
!11 Binary Constraints;
@FOR (POSITIONS(k): @BIN(X(k)));
!12 Binary Constraints;
@FOR (COMBINATIONS (k,i,j):
        @BIN(S(k,i,j)));
```

The model was solved using the following sets of data:

The value of G, N_{ij}, D_{ij}, C_{ij} , W_{ij} and the number of nodes (48) remained constant while the following changes were made:

Table 21 Business Case 1: Daytime with 3 RV's

Value of *T*:	100
Value of *P*:	3

Table 22 Business Case 2: Daytime with 4 RV's

Value of *T*:	100
Value of *P*:	4

Table 23 Business Case 3: Nighttime with 4 RV's

Value of *T*:	200
Value of *P*:	2

Task 13: Interpretation of Results

After the model has been solved by using the appropriate linear solving software, it remains the responsibility of the user or management to interpret the results.

When the value of X_k is equal to 1, it means that intersection *k* should be a fixed location for a RV/s. Sometimes it is physically impossible for a vehicle to be positioned at the exact calculated location, thus management must determine a position as close to the intersection or node as possible that will suffice.

When the value of S_{kij} is equal to 1 it means that the RV/s located at node or intersection *k* must service the arc between nodes *i* and *j*.

Figures 10 to 12 below show the results when the inputs determined during the previous steps are used.

Figure 10 Business Case 1: Results

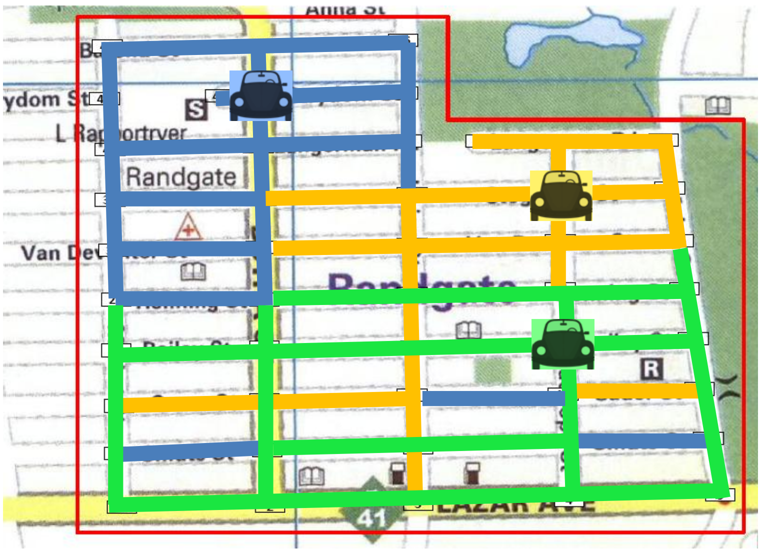


Figure 11 Business Case 2: Results

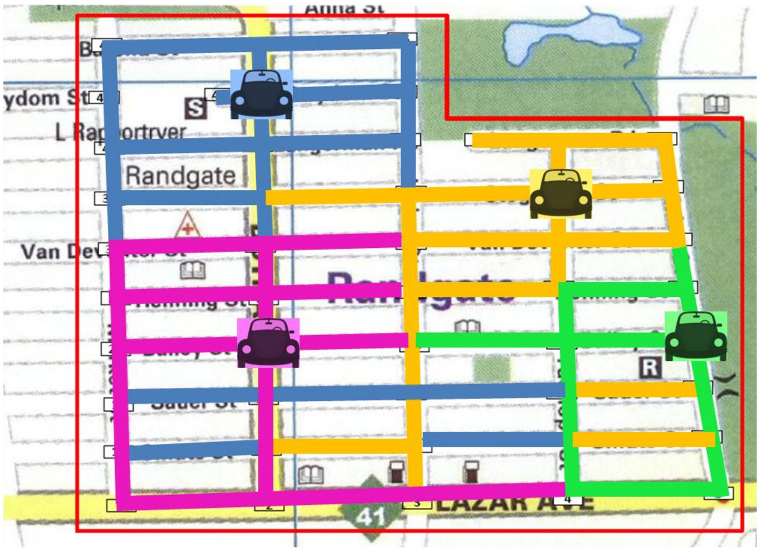


Figure 12 Business Case 3: Results

When the results given by the model is inspected, it may seem strange that most of the arcs serviced by the model are closely grouped together, but in each scenario there is a few arcs serviced by the vehicle that is not closely grouped with the rest. These arcs that do not form part of the close grouping are all part of Sauer and Smuts street. If one examines the weight allocated to the arcs of these two streets (see table 16) one will see that both have a weight of zero allocated to their arcs. This is due to the streets neither having any customers nor any future call-outs. When this is taken into consideration and one substitute the values in the objective function (see paragraph 8), the function will have a value of zero, independent of the vehicle that is servicing it. This means any vehicle can be used to service an arc with a zero weight allocated to it. Figures 13 to 15 below show the results when other vehicles are used to service the arcs in Sauer and Smuts street.

Figure 13 Business Case 1: Processed Results

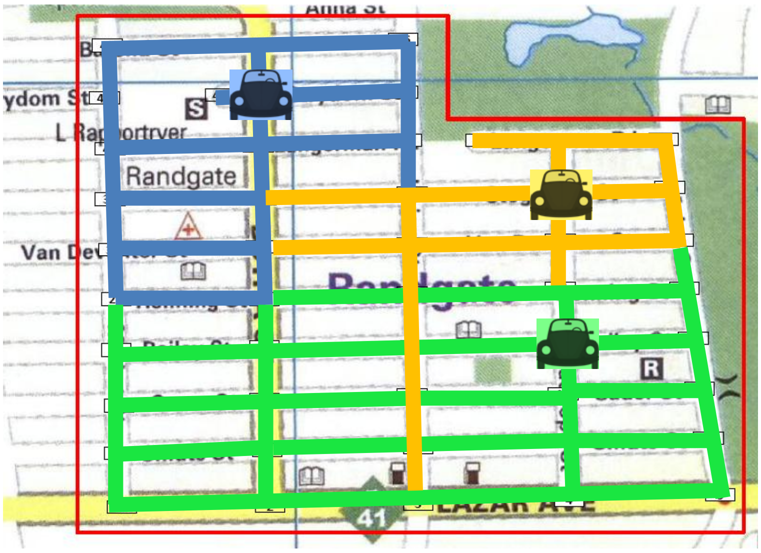


Figure 14 Business Case 2: Processed Results

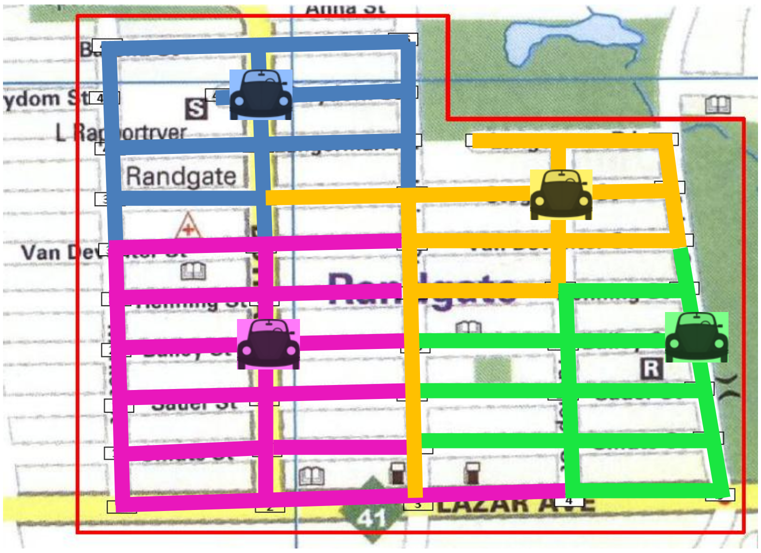


Figure 15 Business Case 3: Processed Results

11 How is the model influenced by change in service requirement and customer density?

The objective function of the mathematical model is given by the following formula:

$$Minimize \sum_i \sum_j \sum_k W_{ij} AD_{kij} S_{kij}$$

As the weight (W_{ij}) allocated to each arc is fixed, the only way for the model to minimize the formula is to minimize the average distance to the arc (AD_{kij}). For this reason the model will allocate the fixed positions of vehicles closer to arcs which have a higher weight (W_{ij}) allocated to them.

The weight allocated to each arc takes into consideration the number of customers situated in the arc and the number of call-outs forecasted in the arc for the following period. Since both the number of customers and the number of call-outs forecasted can change dramatically from one month to the next it is important to run the model on regular intervals (a monthly basis is advised). The data necessary for the model can easily be changed and modified by using the Excel Workbook that was developed for this project – The workbook allows the user to update the fields that can vary on a monthly basis. The lingo programming is imbedded in the Excel Workbook. The program reads the input fields from the spreadsheets and also delivers the output fields.

12 Conclusion

Operational research can enable security companies to make better strategic decisions with regards to the positioning of their vehicles. Other industries are already using models to determine the location of their facilities with great success.

Many South African companies are unaware of the potential of operational research and how they can benefit from it resulting very little research being completed in business segments that is unique to South Africa, such as the Private Security Industry.

Using the location model for RVs that has been developed in this project, security companies will be able to:

- Save on travelling expenses.
- Minimize incident response times.
- Gain a competitive advantage using strategic planning.

Further research can be undertaken to improve the location model for RVs, but this model can already be used with good results in the interim.

13 References

Brotcorne, L Laporte, G & Semet F 2003, ‘Ambulance location and relocation models’, European Journam of Operations Research, vol. 147, no. 3, pp. 451-463.

Bryan Rudolph 2011, South African Intruder Detection Services Association, South Africa, viewed 23 July 2011, < http://www.saidsa.co.za/>.

Francis, RL & White JA 1974, *Facility Layout and Location: An Analytical Approach,* Prentice Hall, Englewood Cliffs, N.J.

Kasana, HS & Kumar, KD 2004, *Introductory Operations Research: Theory and Applications*, Springer, Berlin.

Love, RF Morris, JG & Wesolowsky GO 1988, *Facilities Location: Models and Methods*, North-Holland New York, New York.

nTier Software Services 2007, *PSIRA Annual Report 2009/2010*, Private Security Industry Regulatory Authority, viewed 23 July 2011, <http://www.psira.co.za/joomla/pdfs/PSIRAAnnualReport2009-2010.pdf>.

Owen, HS & Daskin, MS 1998, ‘Strategic Facility Location: A Review’, *European Journal of Operations Research*, vol. 111, no. 3, pp. 423-447.

Rapid Intelligence 2005, NationMaster.com, Australia, viewed 23 July 2011, <http://www.nationmaster.com/red/graph/cri_bur_percap-crime-burglaries-per-capita&b_map=1>.

Rudolph, B 2011, *The SAIDSA Guide And Member Listing*, The South African Intruder Detection Services Association (SAIDSA), Benoni.

Willis, RE 1987, *A Guide to Forecasting for Planners and Managers*, Prentice Hall New York, Englewood Cliffs, N.J.

Winston, WL, 2003, *Introduction to Mathematical Programming: Operations Research*, Pacific Grove, CA

Appendix 1: Matrix *Nij*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Nij page 1 of 2

ELECT PROTECT RESPONSE (PTY) LTD t/a EPR

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Nij

page 2 of 2

ELECT PROTECT RESPONSE (PTY) LTD t/a EPR

Appendix 2: Floyd's Algorithm – Input Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
2	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
3	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
4	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
5	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
6	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
7	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
8	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
9	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
10	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
11	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
12	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
13	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
14	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
16	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15
17	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15
18	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15
19	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
20	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
21	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15
22	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15
23	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15
24	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150
25	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15
26	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250
27	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0
28	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	250
29	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
30	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
31	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
32	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
33	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
34	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150
35	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15
36	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
37	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
38	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
39	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
40	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
41	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
42	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
43	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
44	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
45	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
46	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
47	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
48	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
49	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
50	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15

	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
2	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
3	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
4	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
5	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
6	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
7	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
8	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
9	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
10	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
11	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
12	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
13	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
14	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
16	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
17	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
18	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
19	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
20	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
21	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
22	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
23	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
24	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
25	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
26	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
27	250	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
28	0	250	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
29	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
30	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
31	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
32	1E+15	150	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
33	150	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
34	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
35	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
36	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
37	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
38	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
39	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	0	250	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15
40	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	250	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
41	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
42	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15
43	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0	50	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
44	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	50	0	250	1E+15	150	1E+15	1E+15	1E+15
45	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	250	0	150	1E+15	1E+15	1E+15	1E+15
46	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	0	250	1E+15	1E+15	1E+15
47	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	250	0	250	1E+15	1E+15
48	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	150	1E+15	1E+15	1E+15	1E+15	250	0	1E+15	1E+15
49	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0	1E+15
50	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0

Appendix 3: Auto-Complete Macro

```
Sub Macro1()
'
' Macro1 Macro
' Matrix auto complete
'
' Keyboard Shortcut: Ctrl+j
'
    mynum = 1
    Do Until mynum = 590
        Cells(mynum + 1, mynum + 2).Select
        Range(Selection, Selection.End(xlToRight)).Select
        Selection.Copy
        Cells(mynum + 2, mynum + 1).Select
        Selection.PasteSpecial Paste:=xlPasteAll, Operation:=xlNone, SkipBlanks:= _
            False, Transpose:=True
        Application.CutCopyMode = False
        mynum = mynum + 1
    Loop
    Cells(1, 1).Select
End Sub
```

This macro has been designed in Visual Basic to help auto-complete matrices in Microsoft Excel, which will be less time consuming than completing all of the matrices by hand.

This macro assumes that the matrix used will be symmetrical, but it can also be used for matrices that are not symmetrical after some small alterations to either the macro or the matrix before it can be used.

Appendix 4: Matrix *Dij*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	250	500	750	1000	1150	900	650	400	150	300	550	800	1050	1300	1450	1200	950	700	450	600	850	1100	1350	1600
2	250	0	250	500	750	900	650	400	150	400	550	300	550	800	1050	1200	950	700	450	700	850	600	850	1100	1350
3	500	250	0	250	500	650	400	150	400	650	800	550	300	550	800	950	700	450	700	950	1100	850	600	850	1100
4	750	500	250	0	250	400	150	400	650	900	1050	800	550	300	550	700	450	700	950	1200	1350	1100	850	600	850
5	1000	750	500	250	0	150	400	650	900	1150	1300	1050	800	550	300	450	700	950	1200	1450	1600	1350	1100	850	600
6	1150	900	650	400	150	0	250	500	750	1000	1150	900	650	400	150	300	550	800	1050	1300	1450	1200	950	700	450
7	900	650	400	150	400	250	0	250	500	750	900	650	400	150	400	550	300	550	800	1050	1200	950	700	450	700
8	650	400	150	400	650	500	250	0	250	500	650	400	150	400	650	800	550	300	550	800	950	700	450	700	950
9	400	150	400	650	900	750	500	250	0	250	400	150	400	650	900	1050	800	550	300	550	700	450	700	950	1200
10	150	400	650	900	1150	1000	750	500	250	0	150	400	650	900	1150	1300	1050	800	550	300	450	700	950	1200	1450
11	300	550	800	1050	1300	1150	900	650	400	150	0	250	500	750	1000	1150	900	650	400	150	300	550	800	1050	1300
12	550	300	550	800	1050	900	650	400	150	400	250	0	250	500	750	900	650	400	150	400	550	300	550	800	1050
13	800	550	300	550	800	650	400	150	400	650	500	250	0	250	500	650	400	150	400	650	800	550	300	550	800
14	1050	800	550	300	550	400	150	400	650	900	750	500	250	0	250	400	150	400	650	900	1050	800	550	300	550
15	1300	1050	800	550	300	150	400	650	900	1150	1000	750	500	250	0	150	400	650	900	1150	1300	1050	800	550	300
16	1450	1200	950	700	450	300	550	800	1050	1300	1150	900	650	400	150	0	250	500	750	1000	1150	900	650	400	150
17	1200	950	700	450	700	550	300	550	800	1050	900	650	400	150	400	250	0	250	500	750	900	650	400	150	400
18	950	700	450	700	950	800	550	300	550	800	650	400	150	400	650	500	250	0	250	500	650	400	150	400	650
19	700	450	700	950	1200	1050	800	550	300	550	400	150	400	650	900	750	500	250	0	250	400	150	400	650	900
20	450	700	950	1200	1450	1300	1050	800	550	300	150	400	650	900	1150	1000	750	500	250	0	150	400	650	900	1150
21	600	850	1100	1350	1600	1450	1200	950	700	450	300	550	800	1050	1300	1150	900	650	400	150	0	250	500	750	1000
22	850	600	850	1100	1350	1200	950	700	450	700	550	300	550	800	1050	900	650	400	150	400	250	0	250	500	750
23	1100	850	600	850	1100	950	700	450	700	950	800	550	300	550	800	650	400	150	400	650	500	250	0	250	500
24	1350	1100	850	600	850	700	450	700	950	1200	1050	800	550	300	550	400	150	400	650	900	750	500	250	0	250
25	1600	1350	1100	850	600	450	700	950	1200	1450	1300	1050	800	550	300	150	400	650	900	1150	1000	750	500	250	0
26	1750	1500	1250	1000	750	600		1100	1350	1600	1450	1200	950	700	450	300	550	800	1050	1300	1150	900	650	400	150
27	1500	1250	1000	750	1000	850	600	850	1100	1350	1200	950	700	450	700	550	300	550	800	1050	900	650	400	150	400
28	1250	1000	750	1000	1250	1100	850	600	850	1100	950	700	450	700	950	800	550	300	550	800	650	400	150	400	650
29	1000	750	1000	1250	1500	1350	1100	850	600	850	700	450	700	950	1200	1050	800	550	300	550	400	150	400	650	900
30	750	1000	1250	1500	1750	1600	1350	1100	850	600	450	700	950	1200	1450	1300	1050	800	550	300	150	400	650	900	1150
31	900	1150	1400	1650	1900	1750	1500	1250	1000	750	600	850	1100	1350	1600	1450	1200	950	700	450	300	550	800	1050	1300
32	1150	900	1150	1400	1650	1500	1250	1000	750	1000	850	600	850	1100	1350	1200	950	700	450	700	550	300	550	800	1050
33	1400	1150	900	1150	1400	1250	1000	750	1000	1250	1100	850	600	850	1100	950	700	450	700	950	800	550	300	550	800
34	1650	1400	1150	900	1150	1000	750	1000	1250	1500	1350	1100	850	600	850	700	450	700	950	1200	1050	800	550	300	550
35	1900	1650	1400	1150	900	750	1000	1250	1500	1750	1600	1350	1100	850	600	450	700	950	1200	1450	1300	1050	800	550	300
36	2050	1800	1550	1300	1050	900	1150	1400	1650	1900	1750	1500	1250	1000	750	600	850	1100	1350	1600	1450	1200	950	700	450
37	1800	1550	1300	1050	1300	1150	900	1150	1400	1650	1500	1250	1000	750	1000	850	600	850	1100	1350	1200	950	700	450	700
38	1950	1700	1450	1200	1450	1300	1050	1300	1550	1800	1650	1400	1150	900	1150	1000	750	1000	1250	1500	1350	1100	850	600	850
39	1550	1300	1050	1300	1550	1400	1150	900	1150	1400	1250	1000	750	1000	1250	1100	850	600	850	1100	950	700	450	700	950
40	1300	1050	1300	1550	1800	1650	1400	1150	900	1150	1000	750	1000	1250	1500	1350	1100	850	600	850	700	450	700	950	1200
41	1050	1300	1550	1800	2050	1900	1650	1400	1150	900	750	1000	1250	1500	1750	1600	1350	1100	850	600	450	700	950	1200	1450
42	1200	1450	1700	1950	2200	2050	1800	1550	1300	1050	900	1150	1400	1650	1900	1750	1500	1250	1000	750	600	850	1100	1350	1600
43	1500	1250	1500	1750	2000	1850	1600	1350	1100	1350	1200	950	1200	1450	1700	1550	1300	1050	800	1050	900	650	900	1150	1400
44	1450	1200	1450	1700	1950	1800	1550	1300	1050	1300	1150	900	1150	1400	1650	1500	1250	1000	750	1000	850	600	850	1100	1350
45	1700	1450	1200	1450	1700	1550	1300	1050	1300	1550	1400	1150	900	1150	1400	1250	1000	750	1000	1250	1100	850	600	850	1100
46	1850	1600	1350	1600	1850	1700	1450	1200	1450	1700	1550	1300	1050	1300	1550	1400	1150	900	1150	1400	1250	1000	750	1000	1250
47	1600	1350	1600	1850	2100	1950	1700	1450	1200	1450	1300	1050	1300	1550	1800	1650	1400	1150	900	1150	1000	750	1000	1250	1500
48	1350	1600	1850	2100	2350	2200	1950	1700	1450	1200	1050	1300	1550	1800	2050	1900	1650	1400	1150	900	750	1000	1250	1500	1750
49	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15
50	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	1750	1500	1250	1000	750	900	1150	1400	1650	1900	2050	1800	1950	1550	1300	1050	1200	1500	1450	1700	1850	1600	1350	1E+15	1E+15
2	1500	1250	1000	750	1000	1150	900	1150	1400	1650	1800	1550	1700	1300	1050	1300	1450	1250	1200	1450	1600	1350	1600	1E+15	1E+15
3	1250	1000	750	1000	1250	1400	1150	900	1150	1400	1550	1300	1450	1050	1300	1550	1700	1500	1450	1200	1350	1600	1850	1E+15	1E+15
4	1000	750	1000	1250	1500	1650	1400	1150	900	1150	1300	1050	1200	1300	1550	1800	1950	1750	1700	1450	1600	1850	2100	1E+15	1E+15
5	750	1000	1250	1500	1750	1900	1650	1400	1150	900	1050	1300	1450	1550	1800	2050	2200	2000	1950	1700	1850	2100	2350	1E+15	1E+15
6	600	850	1100	1350	1600	1750	1500	1250	1000	750	900	1150	1300	1400	1650	1900	2050	1850	1800	1550	1700	1950	2200	1E+15	1E+15
7	850	600	850	1100	1350	1500	1250	1000	750	1000	1150	900	1050	1150	1400	1650	1800	1600	1550	1300	1450	1700	1950	1E+15	1E+15
8	1100	850	600	850	1100	1250	1000	750	1000	1250	1400	1150	1300	900	1150	1400	1550	1350	1300	1050	1200	1450	1700	1E+15	1E+15
9	1350	1100	850	600	850	1000	750	1000	1250	1500	1650	1400	1550	1150	900	1150	1300	1100	1050	1300	1450	1200	1450	1E+15	1E+15
10	1600	1350	1100	850	600	750	1000	1250	1500	1750	1900	1650	1800	1400	1150	900	1050	1350	1300	1550	1700	1450	1200	1E+15	1E+15
11	1450	1200	950	700	450	600	850	1100	1350	1600	1750	1500	1650	1250	1000	750	900	1200	1150	1400	1550	1300	1050	1E+15	1E+15
12	1200	950	700	450	700	850	600	850	1100	1350	1500	1250	1400	1000	750	1000	1150	950	900	1150	1300	1050	1300	1E+15	1E+15
13	950	700	450	700	950	1100	850	600	850	1100	1250	1000	1150	750	1000	1250	1400	1200	1150	900	1050	1300	1550	1E+15	1E+15
14	700	450	700	950	1200	1350	1100	850	600	850	1000	750	900	1000	1250	1500	1650	1450	1400	1150	1300	1550	1800	1E+15	1E+15
15	450	700	950	1200	1450	1600	1350	1100	850	600	750	1000	1150	1250	1500	1750	1900	1700	1650	1400	1550	1800	2050	1E+15	1E+15
16	300	550	800	1050	1300	1450	1200	950	700	450	600	850	1000	1100	1350	1600	1750	1550	1500	1250	1400	1650	1900	1E+15	1E+15
17	550	300	550	800	1050	1200	950	700	450	700	850	600	750	850	1100	1350	1500	1300	1250	1000	1150	1400	1650	1E+15	1E+15
18	800	550	300	550	800	950	700	450	700	950	1100	850	1000	600	850	1100	1250	1050	1000	750	900	1150	1400	1E+15	1E+15
19	1050	800	550	300	550	700	450	700	950	1200	1350	1100	1250	850	600	850	1000	800	750	1000	1150	900	1150	1E+15	1E+15
20	1300	1050	800	550	300	450	700	950	1200	1450	1600	1350	1500	1100	850	600	750	1050	1000	1250	1400	1150	900	1E+15	1E+15
21	1150	900	650	400	150	300	550	800	1050	1300	1450	1200	1350	950	700	450	600	900	850	1100	1250	1000	750	1E+15	1E+15
22	900	650	400	150	400	550	300	550	800	1050	1200	950	1100	700	450	700	850	650	600	850	1000	750	1000	1E+15	1E+15
23	650	400	150	400	650	800	550	300	550	800	950	700	850	450	700	950	1100	900	850	600	750	1000	1250	1E+15	1E+15
24	400	150	400	650	900	1050	800	550	300	550	700	450	600	700	950	1200	1350	1150	1100	850	1000	1250	1500	1E+15	1E+15
25	150	400	650	900	1150	1300	1050	800	550	300	450	700	850	950	1200	1450	1600	1400	1350	1100	1250	1500	1750	1E+15	1E+15
26	0	250	500	750	1000	1150	900	650	400	150	300	550	700	800	1050	1300	1450	1250	1200	950	1100	1350	1600	1E+15	1E+15
27	250	0	250	500	750	900	650	400	150	400	550	300	450	550	800	1050	1200	1000	950	700	850	1100	1350	1E+15	1E+15
28	500	250	0	250	500	650	400	150	400	650	800	550	700	300	550	800	950	750	700	450	600	850	1100	1E+15	1E+15
29	750	500	250	0	250	400	150	400	650	900	1050	800	950	550	300	550	700	500	450	700	850	600	850	1E+15	1E+15
30	1000	750	500	250	0	150	400	650	900	1150	1300	1050	1200	800	550	300	450	750	700	950	1100	850	600	1E+15	1E+15
31	1150	900	650	400	150	0	250	500	750	1000	1150	900	1050	650	400	150	300	600	550	800	950	700	450	1E+15	1E+15
32	900	650	400	150	400	250	0	250	500	750	900	650	800	400	150	400	550	350	300	550	700	450	700	1E+15	1E+15
33	650	400	150	400	650	500	250	0	250	500	650	400	550	150	400	650	800	600	550	300	450	700	950	1E+15	1E+15
34	400	150	400	650	900	750	500	250	0	250	400	150	300	400	650	900	1050	850	800	550	700	950	1200	1E+15	1E+15
35	150	400	650	900	1150	1000	750	500	250	0	150	400	550	650	900	1150	1300	1100	1050	800	950	1200	1450	1E+15	1E+15
36	300	550	800	1050	1300	1150	900	650	400	150	0	250	400	800	1050	1300	1450	1250	1200	950	1100	1350	1600	1E+15	1E+15
37	550	300	550	800	1050	900	650	400	150	400	250	0	150	550	800	1050	1200	1000	950	700	850	1100	1350	1E+15	1E+15
38	700	450	700	950	1200	1050	800	550	300	550	400	150	0	700	950	1200	1350	1150	1100	850	1000	1250	1500	1E+15	1E+15
39	800	550	300	550	800	650	400	150	400	650	800	550	700	0	250	500	650	450	400	150	300	550	800	1E+15	1E+15
40	1050	800	550	300	550	400	150	400	650	900	1050	800	950	250	0	250	400	200	150	400	550	300	550	1E+15	1E+15
41	1300	1050	800	550	300	150	400	650	900	1150	1300	1050	1200	500	250	0	150	450	400	650	800	550	300	1E+15	1E+15
42	1450	1200	950	700	450	300	550	800	1050	1300	1450	1200	1350	650	400	150	0	600	550	800	650	400	150	1E+15	1E+15
43	1250	1000	750	500	750	600	350	600	850	1100	1250	1000	1150	450	200	450	600	0	50	300	450	200	450	1E+15	1E+15
44	1200	950	700	450	700	550	300	550	800	1050	1200	950	1100	400	150	400	550	50	0	250	400	150	400	1E+15	1E+15
45	950	700	450	700	950	800	550	300	550	800	950	700	850	150	400	650	800	300	250	0	150	400	650	1E+15	1E+15
46	1100	850	600	850	1100	950	700	450	700	950	1100	850	1000	300	550	800	650	450	400	150	0	250	500	1E+15	1E+15
47	1350	1100	850	600	850	700	450	700	950	1200	1350	1100	1250	550	300	550	400	200	150	400	250	0	250	1E+15	1E+15
48	1600	1350	1100	850	600	450	700	950	1200	1450	1600	1350	1500	800	550	300	150	450	400	650	500	250	0	1E+15	1E+15
49	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0	1E+15
50	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	1E+15	0

Appendix 5: Forecasting Formulae

Formulae for linear regression:

$\hat{Y}_i$ = The predicted values of Y.

X_i = The independent X values.

Y_i = The dependant Y values.

$\bar{X}$ = The mean of the X values.

$\bar{Y}$ = The mean of the Y values.

n = The number of observations.

Determining the slope:

$$b_1 = \frac{SSXY}{SSX}$$

$$SSX = \sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n} X_i^2 - \frac{(\sum_{i=1}^{n} X_i)^2}{n}$$

$$SSXY = \sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = \sum_{i=1}^{n} X_i Y_i - \frac{(\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{n}$$

Determining the Y intercept:

$$b_0 = \bar{Y} - b_1 \bar{X}$$

Determining the Error:

$$r^2 = \frac{SSR}{SST}$$

$$SSE = \sum_{i=1}^{n} Y_i^2 - b_0 \sum_{i=1}^{n} Y_i - b_1 \sum_{i=1}^{n} X_i Y_i$$

$$SSR = \sum_{i=1}^{n}(Y_i - \bar{Y})^2$$

$$SST = SSR + SSE$$

Formula for Moving Averages:

$$Y_n = \frac{Y_{n-1} + Y_{n-2} + Y_{n-3}}{3}$$

Appendix 6: Matrix *Cij*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	1	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	2	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
7	0	0	0	3	0	1	0	1	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	2	0	0	0	0	0	1	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0
10	2	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0	0	0	0	0	2	0	0	0	0	0
12	0	0	0	0	0	0	0	0	2	0	1	0	1	0	0	0	0	0	2	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	3	0	0	0	0	0	1	0	1	0	3	0	0	0	0	0	0	0	0
15	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	8	0	0	0	0	0	0	0	2
17	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	8	0	8	0	0	0	0	0	3	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	0	8	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	8	0	8	0	2	0	0	0
20	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	8	0	2	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	1	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	0	0	0	0	1	0	1
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	1	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

ELECT PROTECT RESPONSE (PTY) LTD t/a EPR

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	7	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	7	0	7	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	7	0	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	7	0	7	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	7	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	2	0	3	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
32	0	0	0	2	0	3	0	3	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	3	0	0	0	0	0	3	0	3	0	3	0	0	0	0	0	0	0	0	0	0	0	0	0
35	2	0	0	0	0	0	0	0	3	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	2	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	3	0	4	0	4	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	2	0	0	0	0	0	0	4	0	4	0	0	2	0	0	0	0	0	0
41	0	0	0	0	0	2	0	0	0	0	0	0	0	0	4	0	2	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	2	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	9	0	9	0	2	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	4	0	4	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	4	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Appendix 7: Matrix *Wij*

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	0	1	0	0	0	0	0	0	0	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	1	0	0	0	0	0	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	1	0	0	0	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	1	0	181	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	258	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	258	0	5	0	0	0	0	0	0	0	258	0	0	0	0	0	0	0	0	0	0
7	0	0	0	181	0	5	0	5	0	0	0	0	0	181	0	0	0	0	0	0	0	0	0	0	0
8	0	0	25	0	0	0	5	0	5	0	0	0	25	0	0	0	0	0	0	0	0	0	0	0	0
9	0	49	0	0	0	0	0	5	0	5	0	49	0	0	0	0	0	0	0	0	0	0	0	0	0
10	46	0	0	0	0	0	0	0	5	0	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	46	0	5	0	0	0	0	0	0	0	46	0	0	0	0	0
12	0	0	0	0	0	0	0	0	49	0	5	0	5	0	0	0	0	0	49	0	0	0	0	0	0
13	0	0	0	0	0	0	0	25	0	0	0	5	0	5	0	0	0	25	0	0	0	0	0	0	0
14	0	0	0	0	0	0	181	0	0	0	0	0	5	0	5	0	181	0	0	0	0	0	0	0	0
15	0	0	0	0	0	258	0	0	0	0	0	0	0	5	0	258	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	258	0	358	0	0	0	0	0	0	0	258
17	0	0	0	0	0	0	0	0	0	0	0	0	0	181	0	358	0	358	0	0	0	0	0	181	0
18	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	358	0	358	0	0	0	25	0	0
19	0	0	0	0	0	0	0	0	0	0	0	49	0	0	0	0	0	358	0	358	0	49	0	0	0
20	0	0	0	0	0	0	0	0	0	0	46	0	0	0	0	0	0	0	358	0	46	0	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46	0	86	0	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49	0	86	0	86	0	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	86	0	86	0
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	181	0	0	0	0	0	86	0	86
25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	258	0	0	0	0	0	0	0	86	0
26	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	258
27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	181	0
28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0
29	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49	0	0	0
30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46	0	0	0	0
31	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
35	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
40	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
41	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

ELECT PROTECT RESPONSE (PTY) LTD t/a EPR

	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	0	0	0	0	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	0	0	0	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	0	0	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	0	181	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	258	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	0	43	0	0	0	0	0	0	0	258	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	43	0	43	0	0	0	0	0	181	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	0	43	0	43	0	0	0	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	0	0	43	0	43	0	49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	0	0	0	43	0	46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	0	0	0	0	46	0	315	0	0	0	0	0	0	0	0	46	0	0	0	0	0	0	0	0	0
32	0	0	0	49	0	315	0	315	0	0	0	0	0	0	49	0	0	0	0	0	0	0	0	0	0
33	0	0	25	0	0	0	315	0	315	0	0	0	0	25	0	0	0	0	0	0	0	0	0	0	0
34	0	181	0	0	0	0	0	315	0	315	0	181	0	0	0	0	0	0	0	0	0	0	0	0	0
35	258	0	0	0	0	0	0	0	315	0	258	0	0	0	0	0	0	0	0	0	0	0	0	0	0
36	0	0	0	0	0	0	0	0	0	258	0	187	0	0	0	0	0	0	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0	0	181	0	187	0	187	0	0	0	0	0	0	0	0	0	0	0	0
38	0	0	0	0	0	0	0	0	0	0	0	187	0	0	0	0	0	0	0	0	0	0	0	0	0
39	0	0	0	0	0	0	0	25	0	0	0	0	0	0	187	0	0	0	0	25	0	0	0	0	0
40	0	0	0	0	0	0	49	0	0	0	0	0	0	187	0	187	0	0	49	0	0	0	0	0	0
41	0	0	0	0	0	46	0	0	0	0	0	0	0	0	187	0	46	0	0	0	0	0	0	0	0
42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46	0	0	0	0	0	0	46	0	0
43	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	539	0	0	0	0	0	0
44	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49	0	0	539	0	539	0	49	0	0	0
45	0	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	0	539	0	25	0	0	0	0
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	25	0	523	0	0	0
47	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	49	0	523	0	523	0	0
48	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46	0	0	0	0	523	0	0	0
49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0