DESIGNING, MODELLING AND OPTIMISING OF AN INTEGRATED RELIABILITY REDUNDANT SYSTEM

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(Dedicated to Prof K. Adendorff on his 80th birthday)

ABSTRACT

The reliability of a system is generally treated as a function of cost; but in many real-life situations reliability will depend on a variety of factors. It is therefore interesting to probe the hidden impact of constraints apart from cost – such as weight, volume, and space. This paper attempts to study the impact of multiple constraints on system reliability. For the purposes of analysis, an integrated redundant reliability system is considered, modelled and solved by applying a Lagrangian multiplier that gives a real valued solution for the number of components, for its reliability at each stage, and for the system. The problem is further studied by using a heuristic algorithm and an integer programming method, and is validated by sensitivity analysis to present an integer solution.

OPSOMMING

Die betroubaarheid van 'n sisteem word normaalweg as 'n funksie van koste beskou, alhoewel dit in baie gevalle afhang van 'n verskeidenheid faktore. Dit is dus interessant om die verskuilde impak van randvoorwaardes soos massa, volume en ruimte te ondersoek. Hierdie artikel poog om die impak van meervoudige randvoorwaardes op sisteem-betroubaarheid te bestudeer. Vir die ontleding, word 'n geïntegreerde betroubaarheid-sisteem met oortolligheid beskou, gemodelleer en opgelos aan die hand van 'n Lagrange-vermenigvuldiger. Die problem word verder bestudeer deur gebruik te maak van 'n heuristiese algoritme en heeltalprogrammering asook gevalideer by wyse van 'n sensitiwiteitsanalise sodat 'n heeltaloplossing voorgehou kan word.

1. INTRODUCTION

The reliability of a system can be increased by keeping redundant units, or by using components of greater reliability, or by employing both methods simultaneously [3,4]. Either of them consumes additional resources. Optimising system reliability, subject to resource availability such as cost, weight, and volume, is considered. Generally, reliability is treated as a function of cost; but when applied to real-life problems, the hidden impact of other constraints like weight, volume, etc, will have a definite impact on optimising reliability. The novel application of a redundant reliability model with multiple constraints is considered to optimise the proposed system.

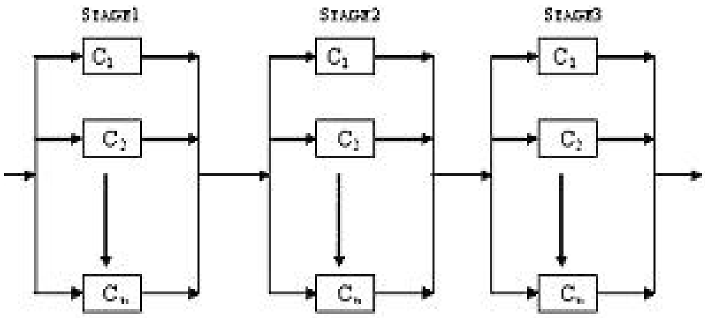

Figure 1: Series-parallel system

The problem considers the unknowns – that is, the number of components (x_j), the component reliabilities (r_j), and the stage reliability (R_j) at each stage for a given multiple of constraints to maximise the system's reliability, which is called an integrated reliability model (IRM) [9]. In the literature, integrated reliability models are optimised using cost constraints where there is an established relation between cost and reliability. The novelty of the proposed work is its consideration of weight and volume as additional constraints, along with cost, to design and optimise the redundant reliability system for a series-parallel system configuration.

2. ASSUMPTIONS AND NOTATION

- All the components in each stage are assumed to be identical – i.e., all the components have the same reliability.
- The components are assumed to be statistically independent – i.e., the failure of a component does not affect the performance of other components in any system.

R_s = System reliability
R_j = Reliability of stage j, $0< R_j < 1$
r_j = Reliability of each component in stage j, $0< rj < 1$
x_j = Number of components in stage j
c_j = Cost coefficient of each component in stage j
w_j = Weight coefficient of each component in stage j
v_j = Volume coefficient of each component in stage j
C_o = Maximum allowable system cost
W_o = Maximum allowable system weight
V_o = Maximum allowable system volume
a_j = Constant
b_j = Constant

p_j = Constant
q_j = Constant
u_j = Constant
v_j = Constant

3. MATHEMATICAL MODEL

The objective function and the constraints of the model

Maximize $$R_S = \prod_{j=1}^{n} [1 \quad (1 \quad r_j)^{x_j}] \tag{1}$$

subject to the constraints

$$\sum_{j=1}^{n} c_j . x_j \le C_0 \tag{2}$$

$$\sum_{j=1}^{n} w_j . x_j \le W_0 \tag{3}$$

$$\sum_{j=1}^{n} v_j . x_j \le V_0 \tag{4}$$

non-negative restriction that x_j is an integer and r_j, R_j >0

4. MATHEMATICAL FUNCTION

To establish the mathematical model, the most commonly-used function is considered for the purpose of reliability design and analysis. The proposed mathematical function

$$c_j = a_j . \exp \frac{b_j}{(1 \quad r_j)} \tag{5}$$

where a_j, b_j are constants

System reliability for the given function

$$R_S = \prod_{i=1}^{n} R_j \tag{6}$$

The number of components at each stage X_j is given through the relation

$$x_j = \frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)} \tag{7}$$

The problem under consideration is

Maximize $$R_S = \prod_{j=1}^{n} [1 \quad (1 \quad r_j)^{x_j}] \tag{8}$$

subject to the constraints

$$\sum_{j=1}^{n} a_j.\exp \frac{b_j}{(1 \quad r_j)} . \frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)} \quad C_0 \le 0 \tag{9}$$

$$\sum_{j=1}^{n} p_j.\exp \frac{q_j}{(1 \quad r_j)} . \frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)} . \quad W_0 \le 0 \tag{10}$$

$$\sum_{j=1}^{n} u_j.\exp \frac{v_j}{(1 \quad r_j)} . \frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)} . \quad V_0 \le 0 \tag{11}$$

5. THE LAGRANGIAN METHOD

Solving the proposed formulation using the Lagrangian method:

$$F = R_s + \lambda_1\left[\sum_{j=1}^{n}\left[a_j.\exp\left[\frac{b_j}{(1 \quad r_j)}\right].\frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)}\right] \quad C_0\right] + \lambda_2\left[\sum_{j=1}^{n}\left[p_j.\exp\left[\frac{q_j}{(1 \quad r_j)}\right].\frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)}\right]. \quad W_0\right] +$$
$$\lambda_3\left[\sum_{j=1}^{n}\left[u_j.\exp\left[\frac{v_j}{(1 \quad r_j)}\right].\frac{\ln(1 \quad R_j)}{\ln(1 \quad r_j)}\right]. \quad V_0\right] = 0 \tag{12}$$

where λ1, λ_2, λ_3 are Lagrangian multipliers.

The number of components in each stage (x_j), the optimum component reliability (r_j), the stage reliability (R_j), and the system reliability (R_s) are derived by using the Lagrangian method. The method provides a real valued solution with reference to cost, weight, and volume.

i) Reliability design relating to cost

Stage	r_j	R_j	x_j	c_j	$c_j.\ x_j$
01	0.4697	0.8626	3.2	374	1197
02	0.4481	0.9205	4.3	512	2201
03	0.6000	0.9900	5.1	314	1602
Total cost					5000

Table 1: Cost constraint details

ii) Reliability design relating to weight

Stage	r_j	R_j	X_j	W_j	$W_j.\ X_j$
01	0.4697	0.8626	3.2	548	1755
02	0.4481	0.9205	4.3	641	2757
03	0.6000	0.9900	5.1	576	2938
Total weight					7450

Table 2: Weight constraint details

iii) Reliability design relating to volume

Stage	r_j	R_j	X_j	V_j	$V_j .X_j$
01	0.4697	0.8626	3.2	374	1197
02	0.4481	0.9205	4.3	512	2201
03	0.6000	0.9900	5.1	314	2602
Total volume					5000

Table 3: Volume constraint details

System reliability = R_s = 0.7860

6. HEURISTIC METHOD

Since the Lagrangian method gives a real valued solution, a heuristic approach is applied to derive an approximate integer solution.

6.1 Heuristic algorithm

Step 1: Initialise and enter the values of required input parameters.

Step 2: Enter the maximum number of components (n).

Step 3: Initialise the number of components, set to 1 for the 1st stage, and calculate the values of system cost (C), weight (W), volume (V) for the 1st stage.

Step 4: Initialise the number of components, set to 1 for the 2nd stage, and calculate the values of system cost(C), weight(W), volume(V) for the 2nd stage.

Step 5: (i) Initialise the number of components, set to 1 for the 3rd stage, and calculate the values of system cost (C), weight (W), volume (V) for the 3rd stage.
(ii) Sum up all the values C, W, V for all three stages.
(iii) Calculate the system reliability (R_s).
(iv) Check the constraints.

Step 6: (i) If the constraints are satisfied, print the corresponding values of number of components and system reliability (R_s).
(ii) If the constraints are not satisfied, increment the number of components of stage three by 1 and go to Step 5.
(iii) Repeat the above step until the number of components in all three stages reaches the value less than or equal to the maximum number of components (n).

i) Reliability design relating to cost

Stage	r_j	R_j	x_j	c_j	$c_j. x_j$
01	0.4697	0.9209	4	374	1496
02	0.4481	0.9072	4	512	2048
03	0.6000	0.9360	3	314	942
Total cost					4486

Table 4: Reliability design relating to cost

Variation in total cost = 10.28%

ii) Reliability design relating to weight

Stage	r_j	R_j	X_j	W_j	$W_j. X_j$
01	0.4697	0.9209	4	548	2192
02	0.4481	0.9072	4	641	2564
03	0.6000	0.9360	3	576	1728
Total weight					6484

Table 5: Reliability design relating to weight

Variation in total weight = 12.96%

iii) Reliability design relating to volume

Stage	r_j	R_j	X_j	V_j	$V_j .X_j$
01	0.4697	0.9209	4	374	1496
02	0.4481	0.9072	4	512	2048
03	0.6000	0.9360	3	314	942
Total volume					4486

Table 6: Reliability design relating to volume

System reliability (R_s) = 0.7820
Variation in total volume = 10.28%
Variation in system reliability = 0.51%

6.2 Sensitivity analysis

It is observed that when the input data of constraints is increased by 10%, there is only a 4.09% increase in system reliability. When the input data is decreased by 10%, there is only an 8.3% decrease in system reliability. When one factor is varied, keeping all the other factors unchanged, the variation in system reliability is as shown in Table 7.

Variation in factors		**System reliability**
Cost	10% decrease	No change
	10% increase	No change
Weight	10% decrease	No change
	10% increase	No change
Volume	10% decrease	8.37% decrease
	10% increase	4.09% increase

Table 7: Sensitivity analysis

The analysis confirms that the volume factor is more sensitive to input data than are cost and weight.

7. INTEGER PROGRAMMING

The heuristic approach commonly provides a workable solution which is approximate one. To validate the established redundant reliability system, the IP method is applied.

i) Reliability design relating to cost constraint

Stage	r_j	R_j	x_j	c_j	$c_{j.}\ x_j$
01	0.4697	0.9209	4	374	1496
02	0.4481	0.9072	4	512	2048
03	0.6000	0.9360	3	314	942
Total cost					4486

Table 8: Reliability design relating to cost

Variation in total cost = 10.28%

ii) Reliability design relating to weight constraint

Stage	r_j	R_j	X_j	W_j	$W_{j.}\ X_j$
01	0.4697	0.9209	4	548	2192
02	0.4481	0.9072	4	641	2564
03	0.6000	0.9360	3	576	1728
Total weight					6484

Table 9: Reliability design relating to weight

Variation in total weight = 12.97%

iii) Reliability design relating to volume

Stage	r_j	R_j	X_j	V_j	$V_j . X_j$
01	0.4697	0.9209	4	374	1496
02	0.4481	0.9072	4	512	2048
03	0.6000	0.9360	3	314	942
Total volume					4486

Table 10: Reliability design relating to volume

System reliability (R_s) = 0.7820
Variation in total Volume = 10.28%
Variation in system reliability = 0.51%

8. CONCLUSIONS

This paper is focused on establishing a novel multiple constraint redundant reliability system, discussing the design and optimisation of the system in detail. The paper infers that the multiple constraints problem is first treated by the Lagrangian method, where this method provides a real valued solution, and as such may not be feasible for practical implementation. For this reason, the problem when treated heuristically proves to be a convenient and workable solution; and to present the exact solution, the proposed model is validated through the IP approach. The result of this work may not show any significant difference in deriving the solutions, but in many practical situations, the authors believe that the reported procedure is a scientific approach to deriving the solution for the series-parallel redundant reliability system with multiple constraints.

9. ACKNOWLEDGEMENT

V.S.S. Yadavalli is grateful to the National Research Foundation for its financial support.

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