

# Loan portfolio conditional loss estimation using an error-correcting macroeconometric model

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## Abstract

Active credit portfolio management is becoming a central part of capital and credit management within the banking industry. A credit portfolio manager must be able to identify the interdependencies between exposures in a portfolio and be able to relate credit risk to tangible portfolio effects on which specific actions can be taken. To these ends, this paper draws on the macroeconometric vector error correcting model (VECM) developed by De Wet et al. (2009) and applies the proposed credit risk modelling methodology of Pesaran, Schuermann, Treutler and Weiner (2006) to a dummy credit portfolio within the South African economy. It illustrates that it is possible to link macroeconomic factors to a South African credit portfolio and that scenario analysis can be performed. These results can be used for practical credit portfolio management and value enhancing applications.

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## 1. INTRODUCTION

At the heart of structural default and credit risk models lies the Merton (1974) approach to default risk. The intuition behind the structural approach can be summarised by the idea that a firm is expected to default if the underlying asset value falls below a specific threshold level. The threshold level will be determined by the underlying callable liabilities and will essentially change over time with changes in liability, equity and asset value. The fundamental insight of Merton (1974) was that equity could be seen as a call option on the value of the assets of the firm. If the value of the firm fell below the value of debt, equity holders' call option would expire worthless and the debt holders would effectively take ownership of the company. Typically, default is defined by banks and rating agencies as a non-payment of any coupon or interest payment or any principal due, although stricter definitions such as delisting activities are sometimes also applied for bank internal usage. Merton (1974) was therefore able to translate valuing debt into the already familiar option valuation space for which increasingly more sophisticated valuation techniques were becoming available. The option value view of debt has since been applied in both credit and interest rate risk management techniques and is a fundamental part of financial risk management and portfolio management techniques and models (see, for example, Van Deventer, Imai and Mesler, 2004).

Similar to the option valuation space, quantification of default risk therefore requires modelling of three aspects:

- The evolution of the underlying firm value;
- The default threshold of the specific firm; and

- The degree to which firm value is correlated with other companies and the macroeconomic environment – the asset-value correlation or volatility.

Although equity value data is readily available for listed companies, the underlying asset value of companies is not readily available. In fact, most Merton-type models uses a proprietary database in which firm asset value is estimated from balance sheet and equity data. From this database, asset-value correlation and volatility is estimated while the default thresholds are taken to be some function of the short-term and long-term debt in each period. In this framework, a distance to default variable for each company is estimated as the number of standard deviations the underlying firm value is from the default threshold. Default probability is then inferred through historical observations of the distance to default and actual default experiences.

Clearly, such a framework is heavily dependent on the proprietary data underlying both the asset-value evolution as well as the distance to default and default probability estimation. The methodology proposed by Pesaran, Schuermann, Treutler and Weiner (2006) (hereafter PSTW (2006)) has made a significant advancement in credit risk modelling in that it avoids the usage of proprietary balance sheet and distance to default data, instead focussing on credit ratings which are more freely available. Linking this adjusted structural default model to a structural global econometric (GVAR) model, credit risk analysis and portfolio management can be done through the use of a conditional loss distribution estimation and simulation process.

This analysis draws on the macroeconometric vector error-correcting model (VECM) developed by De Wet et al. (2009) and applies the proposed methodology of PSTW (2006) to a dummy portfolio of corporate bank loans. The analysis illustrates that it is not only possible to link macroeconomic factors to a South African-specific credit portfolio, but that scenario and sensitivity analysis can also be performed within the credit portfolio model.

## **2. LINKING MACROECONOMIC DYNAMICS TO FIRM DEFAULTS THROUGH THE PSTW METHODOLOGY**

Similar to a default-only model, a credit portfolio model has to include some mechanism through which a firm can be classified as being in default or not. However, to be able to perform any meaningful analysis and/or value enhancing activities, this mechanism has to be flexible enough to incorporate the correlation between exposures as well as the correlation of exposures with the macroeconomy or some financial market instrument.

Borrowing from PSTW (2006), consider a firm  $j$  in country  $i$  having a total asset value of  $V_{ji,t}$  at time  $t$  while the underlying debt obligation is represented by  $D_{ji,t}$ . Using the Merton (1974) approach, default occurs at maturity date of debt,  $t+H$ , if the firm's assets are less than the value of the debt, i.e. default occurs if  $V_{ji,t+H} < D_{ji,t+H}$ . This default definition is analogous to a European put option which can only be exercised at maturity. The first passage model proposed by Black and Cox (1976) allows default to occur the first time that  $V_{ji,t} < D_{j,t}$  over the time period  $t$  to maturity date,  $t+H$ . The default probability is therefore determined by the probability

distribution of asset values at the terminal date,  $t+H$ , or over the period  $t$  to  $t+H$  in the Merton and first passage models, respectively. Although the method proposed by PSTW (2006) can be adapted to suite both approaches, it is applied here to the Merton European put option specification.

Using the accounting definition equation, the value of a firm should equal the value of debt and equity, i.e.:

$$V_{ji,t} = D_{ji,t} + E_{ji,t}, \text{ with } D_{ji,t} > 0. \quad (1)$$

Dividing both sides by the value of debt, equation 1 can alternatively be represented by:

$$\frac{V_{ji,t}}{D_{ji,t}} = 1 + \frac{E_{ji,t}}{D_{ji,t}}. \quad (2)$$

Therefore, default will take place at time  $t+H$  if:

$$V_{ji,t+H} \leq D_{ji,t+H} \text{ or if } \frac{E_{ji,t+H}}{D_{ji,t+H}} \leq 0. \quad (3)$$

From equation 3, default will only occur if the equity value of a firm is negative. As such, this is a very restrictive condition and not necessarily realistic in practice. Often the management of a firm act pro-actively and put the firm up for receivership before the equity value of the firm hits zero. Several studies have also shown that equity owners receive some compensation even though debt holders have not been paid in full (Eberhart and Weiss, 1998 and Longhofen, 1997) and data suggests that equity values stay positive even for insolvent firms (Betker, 1995, Franks and Torous, 1991 and LoPucki and Whitford, 1991). From a bank perspective, various loan conditions allows the bank to force the firm into default if equity values fall below a specific non-negative threshold (Garbade, 2001). As argued by PSTW

(2006), the value of equity does not only take into account the firm asset value but also includes an option that the firm may recover before creditors take control of the assets. On the other hand borrowers often work out refinancing arrangements if, for example, one or two coupon payments have been missed, in this way effectively avoiding bankruptcy. As such, PSTW (2006) assumes that default takes place if:

$$0 < E_{ji,t+H} < C_{ji,t+H} \quad (4)$$

$C_{ji,t+H}$  now represents some positive default threshold which can vary over time and differ between firms depending on firm-specific characteristics such as sector or industry classification, leverage, profitability, firm size or age and qualitative factors such as management style. Clearly, accounting-based factors such as leverage are measurable and obtainable through data vendors. However, although new accounting standards are increasingly trying to improve the quality of data provided through financial statement disclosures, such data is still noisy and includes information asymmetries between firm management and investors (Wittenberg, 2006).

In order to perform credit portfolio management one implicitly encounters the same measurement and information asymmetry problems as those in a default model. However, PSTW (2006) propose to overcome these problems through the use of credit ratings of firms ( $R$ ).  $R$  takes the values usually depicted by either Moody's (Aaa, Ba, Baa, ..., Caa) or Standard And Poor's (AAA, AA, BBB, ..., CCC) rating notation. The use of ratings facilitates the estimation of default thresholds in order to obtain the default probabilities of each firm. As argued by PSTW (2006), most rating agencies go through a rigorous process of interviews with firm officials, analysis of financial statements and observable market data to assign a particular rating to a firm. Moreover, rating agencies are explicit in their commitment to

assigning consistent ratings between firms and also over time so that comparisons can be performed. It is therefore reasonable to assume that the information contained in the ratings outcome,  $R$ , contains estimates of current balance sheet and equity return data, historical return data and firm-specific confidential information which tries to bridge the information asymmetry gap and information on all firms in the past which have been given similar ratings.

The current fallout of the financial crisis has raised questions with respect to the consistency and accuracy of rating agencies' methodologies. The extent of the loss in credibility of rating agencies and the significant challenge it entails to the investment, financial system and banking community at presents is still not certain – and need a significant degree of research and understanding, which falls outside of the scope of this paper. However, it would be unfair to attribute the financial crisis entirely to rating agency methodological weakness. In fact, various parties have contributed to the crisis – including, but not limited to, rating agency methodological weaknesses, lack of regulatory oversight and global integration, failure in internal risk management of financial institutions, global macroeconomic imbalances and consumer greed.

One of the major flaws found in methodologies such as the securitization rating methodologies is that the simulation models used in the assessments have been generally a-theoretical in nature and did not include the necessary theoretical underpinning and correlations that a model as presented in De Wet et al. (2009) and PSTW (2006) provides. This paper and the methodology proposed by PSTW (2006)

and Pesaran et al. (2004) should therefore add value in subsequent debates and methodologies.

Consider then a particular  $R$ -rated firm at time  $t$  and assume that when arriving at its rating the credit rating agency uses the standard geometric random walk model of equity values as assumed in fundamental financial pricing models such as the capital asset pricing model (CAPM):

$$\ln(E_{R,t+1}) = \ln(E_{Rt}) + \mu_R + \sigma_R \eta_{R,t+1}, \quad \eta_{R,t+1} \sim IID(0,1) \quad (5)$$

with a non-zero drift  $\mu_R$  and an idiosyncratic Gaussian innovation,  $\sigma_R$ , with a zero mean and a fixed volatility. In practice, rating agencies try to rate firms “through the cycle”, taking a longer term view on their ratings in order to provide investors with more stable ratings over time. Therefore equation 5 becomes:

$$\ln(E_{R,t+H}) = \ln(E_{Rt}) + H\mu_R + \sigma_R \sum_{s=1}^H \eta_{R,t+s}.$$

Using the information in equation 4, default will therefore occur if:

$$\ln(E_{R,t+H}) = \ln(E_{Rt}) + H\mu_R + \sigma_R \sum_{s=1}^H \eta_{R,t+s} < \ln(C_{R,t+H}) \quad (6)$$

or using a log equity threshold, default occurs if the  $H$ -period return falls below the log-threshold equity ratio:

$$\ln\left(\frac{E_{R,t+H}}{E_{Rt}}\right) < \ln\left(\frac{C_{R,t+H}}{E_{Rt}}\right). \quad (7)$$

Equation 7 illustrates that over the horizon  $H$ , the relative decline in firm value must be big enough to result in default. If firm equity values follow (5),  $\ln(E_{R,t+H}/E_{Rt})$  can be approximated by the cumulative returns so that equation 7 becomes:

$$H\mu_R + \sigma_R \sum_{s=1}^H \eta_{R,t+s} < \ln\left(\frac{C_{R,t+H}}{E_{Rt}}\right).$$



Therefore the default probability of an  $R$ -rated firm at the terminal date  $t+H$  is given by:

$$\pi_R(t, H) = \Phi\left(\frac{\ln(C_{R,t+h}/E_{Rt}) - H\mu_R}{\sigma_R \sqrt{H}}\right), \quad (8)$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function. Now, denote the  $H$ -period forward log threshold equity ratio to be  $\lambda_R(t, H) = \ln(C_{R,t+H}/E_{Rt})$  so that  $\lambda_R(t, H) = H\mu_R + Q_R(t, H)\sigma_R \sqrt{H}$  where  $Q_R(t, H) = \Phi^{-1}[\pi_R(t, H)]$  is the quantile associated with the default probability  $\pi_R(t, H)$ . Essentially an estimate of  $\lambda_R(t, H)$  can be obtained using past observations of equity returns,  $r_{R_t} = \ln(E_{R_{t+H}}/E_{R_t})$ , and the empirical default frequencies,  $\hat{\pi}_R(t, H)$ , of  $R$ -rated firms over a given time period say  $t=1, 2, \dots, T$ . Not surprisingly, a source of heterogeneity between firms will come from different bankruptcy laws, financial market sophistication, and regulations such as exchange controls that are found in countries across the globe. The use of rating agency data allows one to overcome these heterogeneities, since in assessing each firm, a specific rating is only assigned after these factors are taken into account, thus leaving the underlying default risk after adjustment for these factors to be displayed by the specific rating. Using empirical estimates for the mean return,  $\hat{\mu}_R$  and standard deviation,  $\hat{\sigma}_R$  for  $R$  rated firms over the sample period the following is obtained:

$$\hat{\lambda}_R(t, H) = H\hat{\mu}_R + \hat{Q}_R(t, H)\hat{\sigma}_R \sqrt{H}, \quad (9)$$

where

$$\hat{Q}_R(t, H) = \Phi^{-1}[\hat{\pi}_R(t, H)]. \quad (10)$$

Admittedly the estimate of  $\hat{\pi}_R(t, H)$  may not be a reliable estimate of  $\pi_R(t, H)$  since defaults in higher rating categories are not very common and estimates will be

based on very few defaults over any particular horizon  $(t, t+H)$ . Therefore to make estimates more robust an average estimate of  $\lambda_R(t, H)$  can be obtained from a reasonably long time period (10 to 20 years on a rolling basis). Therefore, based on a sample period of  $t=1, 2, \dots, T$  equation 9 would be estimated using  $\hat{Q}_R(H)$ , which is given by:

$$\hat{Q}_R(H) = T^{-1} \sum_{t=1}^T \left\{ \Phi^{-1}[\hat{\pi}_R(t, H)] \right\}. \quad (11)$$

Assume for example a one-year horizon used by rating agencies when assessing a firm,  $H=4$  quarters. The PSTW (2006) framework allows one to obtain estimates of the default-equity threshold ratios by credit rating. Also, if sufficient data is available one can estimate different default frequencies for specific countries or regions and even firms over particular rating categories. However, as already stated, default is a rare event and in the absence of multiple defaults and sufficient regional default experiences, individual default frequency estimates are impossible. For this reason the following reasonable identification condition is made:

$$\left( \frac{C_{jiR, t+H}}{E_{jiRt}} \right) = \left( \frac{C_{R, t+H}}{E_{Rt}} \right), \quad \text{for all } j \quad (12)$$

where  $E_{jiRt}$  and  $C_{jiR, t+H}$  are the equity and default threshold values of firm  $j$  in country  $i$ , with credit rating  $R$ , at time  $t$ . Essentially condition (12) states that at any given time, any number of firms that have received a similar rating will have the same default equity-threshold ratio. As such, the condition allows different threshold levels, allowing for heterogeneity between firm equity growth-paths, but assuming the same ratio.

Data permitting, other factors can also be used to solve the identification problem, for example, it can be assumed that all firms with the same credit rating have the same distance to default ( $DD$ )-ratio as apposed to an equity threshold level where  $DD = [\lambda_R(t, H) - H\mu_R] / \hat{\sigma}_R \sqrt{H}$ . Moreover, one can group firms into homogenous sub-groups by using other criteria such as rating category by industry or sector. However, these different criteria depend on data availability as well as the fact that a reasonable sample of data is required within each sub-group to allow reliable estimations to be obtained.

### **2.1. Firm-specific return dynamics**

The discussion above illustrates that the fundamental problem faced by the credit portfolio manager in terms of answering practical questions (e.g. “How does my portfolio react to interest rate changes?”) lies in the ability to link the firm value to tangible factors such as the macroeconomic variables in the PSW (2004) GVAR model. Indeed, the fact that most commercially-available credit portfolio tools use a-theoretical risk factors as drivers for firm value implies that portfolio managers are almost always left with making crude approximations to represent the possible impact of tangible economic factors. If it is possible to link return behaviour to these factors, a direct link to performing macroeconomic credit portfolio modelling is obtained. As outlined in De Wet et al. (2009) and PSW (2004), the uniqueness properties of the GVAR and the South African-specific VECM extension of the GVAR model allows one to model firm specific return behaviour not only as a function of one global risk factor but of any set of global and foreign macroeconomic variables. Using the arbitrage pricing theory (APT) it is possible to include more

variables in the return specification without loss of theoretical underpinning. As such, the proposed framework and methodology encompass the usual single factor models allowing a flexible mechanism through which single and multifactor models could be applied in a credit portfolio model context. Moreover, the framework is able to provide a direct link between macroeconomic and credit risk behaviour. As illustrated in equation 13, a firm's change in value,  $r_{ji,t+1} = \ln(E_{ji,t} / E_{ji,t-1})$  can be assumed to be a function of correlated systematic factors from the South African-specific VECM model of De Wet *et al.* (2007), say  $k_i$  country specific,  $\mathbf{x}_{i,t+1}$  and  $k_i^*$  foreign,  $\mathbf{x}_{i,t+1}^*$  macroeconomic variables, and a firm-specific idiosyncratic shock,  $\eta_{ji,t+1}$ :

$$r_{ji,t+1} = \alpha_{ji} + \beta_{ji}' \Delta \mathbf{x}_{i,t+1} + \beta_{ji}^{*'} \Delta \mathbf{x}_{i,t+1}^* + \eta_{ji,t+1}, \quad (13)$$

where  $\eta_{ji,t+1}$  is normally distributed and has a mean of zero and a constant time invariant variance  $\omega_{\eta,ji}^2$ , i.e.  $\eta_{ji,t+1} \sim \text{IIN}(0, \omega_{\eta,ji}^2)$ . The extent to which this specification is able to predict returns will depend on the factor loading,  $\beta_{ji}'$ . Although the model allows for an operational procedure of relating excess returns to macroeconomic factors, it is not supposed to represent perfect predictability. As such, if actual returns deviate from those predicted by the model, it is possibly an indication of time varying risk premia rather than market inefficiencies.

## 2.2. Conditional loss simulation

Based on the log-equity default threshold ratio,  $\hat{\lambda}_R(t, H)$  as defined above, and the value change process of firm  $j$  which is driven by the outcomes of the GVAR model, it is possible to define the expected loss to firm  $j$  at time  $T$ . Given the information set available to the bank at time  $T$ , say  $\Omega_T$ , PSTW (2006) show that the conditional expected loss i.e.  $E(L_{ji,T+1}) = E(L_{ji,T+1}|\Omega_T)$ , can be defined as:

$$E(L_{ji,T+1}) = E(L_{ji,T+1}|\Omega_T) = \Pr(r_{ji,t+1} < \hat{\lambda}_R(t,1)|\Omega_T) \times E_T(\chi_{ji,T+1}) \times E_T(S_{ji,T+1}) + \left[1 - \Pr(r_{ji,t+1} < \hat{\lambda}_R(t,1)|\Omega_T)\right] \times \tilde{L}, \quad (14)$$

where  $\chi_{ji,T+1}$  is the exposure at default assuming no recoveries (usually assumed to be the face value of the loan and referred to as the exposure at default (EAD)) and is known at time  $T$ ,  $S_{ji,T+1}$  is the percentage of exposure that is not recoverable in the event of default, also known as the loss severity or loss given default (LGD) and  $\tilde{L}$  is the loss in event of no default (usually assumed to be zero).

$S_{ji,T+1}$  will typically not be known at time  $T$  and takes the form of a random variable assumed to be between 0 and 1. In commercial and other applications  $S_{ji,T+1}$  is usually drawn from a beta distribution where the smoothness parameters are set to match a predefined central tendency obtained from empirical observations. As such,  $S_{ji,T+1}$  is assumed to be uncorrelated to or independent of the default probability.

Now, using the firm-return dynamics which are simulated from the GVAR and multifactor models and setting  $\tilde{L} = 0$ , the following specification for the expected loss of an exposure is obtained:

$$E(L_{ji,T+1}) = \pi_{ji,T+1|T} \times E_T(\chi_{ji,T+1}) \times E_T(S_{ji,T+1}), \quad (15)$$

where:

$$\pi_{ji,T+1|T} = \Pr(\alpha_{ji} + \Gamma'_{ji} \Delta y_{T+1} + \eta_{ji,T+1} < \hat{\lambda}_R(T,1) | \Omega_T).$$

Equation 15 can be interpreted as the conditional default probability at time  $T$  for the time period  $T+1$ . The expected loss of a bank credit portfolio can finally be computed by aggregating the expected losses over the different loans in the portfolio, i.e.:

$$E_T(L_{T+1}) = \sum_{i=0}^N \sum_{j=0}^{nc_i} \pi_{ji,T+1|T} \times E_T(\chi_{ji,T+1}) \times E_T(S_{ji,T+1}), \quad (16)$$

where  $nc_i$  is the number of loans in the portfolio in region or country  $i$  (since this analysis focuses on a South African-specific credit portfolio  $N=1$ ).

In the stochastic simulation model framework using this information at time  $T+1$ , together with the loan face value,  $FV_{ji,T}$ , and the estimates of loss severity, the conditional loss of an exposure can be simulated as:

$$(L_{T+1}^{(r)}) = \sum_{i=0}^N \sum_{j=0}^{nc_i} I(r_{ji,T+1}^{(r)} < \hat{\lambda}_R(T,1)) \times FV_{ji,T} \times (S_{ji,T+1}^{(r)}), \quad (17)$$

where the conditional default probability is represented by the default indicator such that:

$$\begin{aligned} I(r_{ji,t+1} < \hat{\lambda}_R(t,1)) &= 1 \quad \text{if} \quad r_{ji,t+1} < \hat{\lambda}_R(t,1) \Rightarrow \text{Default} \\ I(r_{ji,t+1} < \hat{\lambda}_R(t,1)) &= 0 \quad \text{if} \quad r_{ji,t+1} \geq \hat{\lambda}_R(t,1) \Rightarrow \text{No Default.} \end{aligned} \quad (18)$$

In practice the estimation of the loss severity (LGD) is normally done by drawing random events from a beta distribution with fixed parameters to approximate a historically observed loss distribution. The limitation of this methodology in the

South African context is that corporate loss data and experience have been significantly limited. Moreover, the methodology does not recognize the inherent correlation between the default probability and the loss severity experienced, i.e. it may be that the same underlying macroeconomic factors that drives the default experience drives the loss severity as well. To this extent a LGD-logit transformation model can be developed (data permitting) to capture these underlying correlations. Given the limitation of the beta distribution methodology outlined above and the fact that the development of a LGD-logit transformation model is beyond the scope of this paper, we apply a 45 per cent LGD which is the parameter proposed by the Bank of International Settlement (BIS) under the foundation internal ratings based approach of Basel II (BIS 2006) for loss severity of senior unsecured loans.

Finally the simulated expected loss is given by:

$$\tilde{L}_{R,T+1} = \frac{1}{R} \sum_{r=1}^R L_{T+1}^{(r)}. \quad (19)$$

According to PSTW (2006) for high enough values of  $R$  i.e. as  $R \rightarrow \infty$ ,  $\tilde{L}_{R,T+1} \xrightarrow{p} E_T(L_{T+1})$  and the simulated loss distribution is given by ordering the simulated values of  $L_{T+1}^{(r)}$ . Any percentile value of the loss distribution e.g. a capital point of the 99.9<sup>th</sup> percentile can be obtained from the simulated loss distribution. Although the discussion illustrates the conditional loss distribution over the horizon  $T+1$ , and as shown in PSTW (2006) similar results can be obtained for longer horizons, e.g.  $T+4$ .

The above discussion points out that the conditional credit loss relies essentially on three factors which need to be estimated in order to perform credit portfolio

management: a correlation model which relates macroeconomic variables with each other in a theoretically, as apposed to a statistically consistent way; a translation or transformation function which relates firm return behaviour in a correlated way with the macroeconomy; and the equity threshold levels per rating category which governs the default behaviour of exposures given specific macroeconomic states. As already highlighted above, the South African-specific VECM extension of the PSW (2004) framework serves as the macroeconomic correlation model in this analysis. Default threshold levels are assumed to be similar to those provided by PSTW (2006), but the transformation functions relating the macroeconomic factors to firm return behavior are provided through firm return models as discussed in section 4.

### **3. DEFAULT THRESHOLDS BY RATING CATEGORY**

From section 2 it follows that a company would default if the log equity threshold falls below the default threshold ratio. Adopting the identification condition from equation 12 the equity default threshold ratio allows different threshold levels, allowing for heterogeneity between firm equity growth paths, but assumes that the same ratio applies. This assumption therefore allows the use of historical data to estimate equity threshold ratios empirically from historical default observations over a sufficiently long time period. In the current application, default equity threshold ratios are taken as presented by PSTW (2006). Using S&P default and rating histories spanning 1981-1999, PSTW (2006) estimates the one to four quarter-ahead threshold equity ratios. Since default experiences and default data are significantly scarce, it is unlikely that a replication of the threshold ratio calculation would lead to significantly different threshold estimates. For this reason, the focus in this analysis



remains on using their results in a new and previously unexplored African bank credit portfolio context. Details of the estimation procedure followed can be obtained in PSTW (2006). Using this methodology, PSTW (2006) estimate the 1 to 4 quarter-ahead return,  $\hat{\mu}_R$ , volatility,  $\hat{\sigma}_R$ , and equity threshold levels  $\hat{\lambda}_R$ . The 1 quarter-ahead threshold levels are presented in table 1 for rating categories AAA to B. As expected, the results indicate that the average volatility increases monotonically as rating quality decreases over the rating spectrum. This is in line with rating agency criteria that place a premium on return stability over time. Intuitively it is also expected that higher-rated firms' returns are influenced much less by the average economic cycle while lower-rated firms (usually start-up companies) are impacted more directly by economic conditions.

**Table 1 One and four quarter-ahead return, volatility and default-threshold estimates per rating category**

| Rating Grade | $\hat{\mu}_R$<br>( $J=1000$ ) | $\hat{\sigma}_R$<br>( $J=1000$ ) | $\frac{\hat{\mu}_R}{\hat{\sigma}_R}$<br>Sharpe | $\hat{\lambda}_R(t,1)$ | $\frac{\hat{C}(t,1)}{E(t)}$ | # Obs  |
|--------------|-------------------------------|----------------------------------|--|------------------------|-----------------------------|--------|
| AAA          | 4.54%                         | 13.87%                           | 32.72%   | -0.588                 | 0.555                       | 1,177  |
| AA           | 4.06%                         | 15.16%                           | 26.76%   | -0.648                 | 0.523                       | 6,272  |
| A            | 4.13%                         | 15.31%                           | 26.99%   | -0.645                 | 0.525                       | 12,841 |
| BBB          | 3.80%                         | 17.38%                           | 21.86%   | -0.688                 | 0.503                       | 9,499  |
| BB           | 3.21%                         | 24.72%                           | 12.99%   | -0.870                 | 0.419                       | 7,002  |
| B            | 2.04%                         | 34.82%                           | 5.86%  | -0.908                 | 0.403                       | 6,493  |

Source: PSTW (2006).

From the Sharpe-ratio it is clear that over this longer investment horizon period (1981-2002), on a risk-adjusted basis, investors have not been compensated for increased risk. Moreover, the huge difference in this ratio between investment (BBB and above) and non-investment (BB and below) grade firms clearly indicates the huge divide between the two types of investments over time. Ignoring short-term

speculative returns, investors have not been adequately rewarded for higher default risk in these rating categories.

In order to forecast default in the credit portfolio environment, the equity default threshold level is of relevance. Relative to the four quarter ahead threshold levels, the one quarter ahead levels are much higher implying that given a bad performance of a particular firm, default is less likely to occur over the short term while a sustained period of bad performances e.g. one year is more conducive to default risk. In line with the rating agency methodology this implies that a firm is given time to recover from a bad short run performance and that default occurs if a firm's value deteriorates over a "through the cycle" period.

Return estimation and bank capital planning are usually calculated over a one year horizon, and as such, the four quarter ahead default thresholds are of particular concern. Admittedly this have been identified as a major shortcoming in the aftermath of the 2007-08 financial crisis and pressure from regulatory authorities such as the Bank of International Settlement have forced banks to explore ways to extend this time period. Examples of these initiatives include scenario analysis and stress testing through the Internal Capital Adequacy Assessment Process (ICAAP) and the so-called Pillar II stress testing process of the Basel II framework. While we focus on a four quarter ahead horizon the framework and methodology is theoretically applicable to longer-term horizons as well. As such it could serve as one methodology to extend and supplement existing practice to address many of the concerns identified in both rating agency methodology of securitized assets as well as the extension of Pillar II stress testing and ICAAP procedures of banks.

Intuitively one would expect the equity threshold ratios to be decreasing, implying that higher rated firms would need to suffer worse losses than their lower rated counterparts in order to default. However the equity default threshold ratios show little variation over rating categories. In fact, there is only a 7 per cent difference between the threshold levels of AAA and B firms. In order to understand the role of the threshold level, the variance of returns should be analysed concurrently. As an example, an AAA and B rated firm, each with an equity level of 100 today would be able to sustain a drop in value of  $0.55*100=55$  and  $0.403*100=40$  respectively before defaulting. However, the likelihood of such an event is driven by the variance of return. For the AAA-rated firm the likelihood is quite low relative to a B-rated firm since  $\hat{\sigma}_{AAA} = 13.87\%$  and  $\hat{\sigma}_B = 34.82\%$ . Clearly the B-rated firm would pierce the equity threshold more often than the AAA-rated company.

#### **4. INDIVIDUAL RETURN ESTIMATES**

##### **4.1. The sample portfolio**

In order to estimate the impact of the macroeconomy on a bank loan portfolio a fictitious South African-specific corporate loan portfolio is constructed. Only a small number of South African firms are currently rated by S&P and because firm rating is one of the major inputs for applying the identification condition in equation 12 to the proposed framework, this presents a severe constraint on the number of counterparts available for inclusion in the sample portfolio. In order to overcome this constraint proprietary ratings from FirstRand Bank as obtained through their credit rating

models and credit processes are used. Due to the fact that such rating information is highly confidential, this study will refrain from providing any link between firm and rating, focusing rather on the portfolio aggregates of a dummy portfolio. Firm ratings have been assigned and assessed as at 1 January 2007 (the beginning of the forecast period) and then mapped into the major rating categories as used in the portfolio model. In order to be in line with the calculation of equity threshold ratios, the cumulative dividend return data per firm over time is obtained from publicly available share data from the McGregor equity database.

Although the analysis starts off with a sample of 696 firms, the final portfolio consists of 145 exposures, spanning the rating spectrum in South Africa. All firms included are listed on the Johannesburg Stock Exchange and were or have until present been listed for at least 16 quarters, i.e. 4 years. In order not to bias results to any particular rating category or introduce any undue large exposure concentration, exposure size has been assigned randomly for each counter party in the portfolio. In general, this portfolio can therefore be thought of as being representative of the benchmark or market portfolio of the South African corporate loan market. Details of the 145 exposure sample portfolio are illustrated in table 2. On a percentage-of-exposure basis the portfolio is concentrated within the BBB rating category and reflects the composition of the South African corporate rating spectrum which is perceived to be more risky relative to international standards. 38 per cent of the portfolio is concentrated in sub-investment grade ratings, with only 15 per cent of exposures obtaining single A rating status.

**Table 2      Sample portfolio composition**

| <b>Major rating category</b> | <b>Exposure % of total portfolio</b> | <b>% of total firms</b> |
|------------------------------|--------------------------------------|-------------------------|
| A                            | 14.75                                | 12.41                   |
| BBB                          | 47.44                                | 45.52                   |
| BB                           | 25.31                                | 28.28                   |
| B                            | 12.47                                | 13.79                   |
|                              | <b>100.00</b>                        | <b>100.00</b>           |

When using a fixed LGD assumption of 45 per cent together with the historical default probabilities (PD) per rating category as estimated by S&P, the expected portfolio loss (estimated as PD multiplied by LGD) over a one-year period is estimated at 0.53 per cent of exposure value. As such, this provides the benchmark for the estimated conditional portfolio loss from our simulated credit portfolio model. If the simulated conditional portfolio loss is estimated as being below this level it implies that there was positive migration over the forecast period while a higher expected loss would imply that the portfolio quality has deteriorated.

#### **4.2. Firm-specific return regressions**

The individual firm return or correlation equations are used to translate the macroeconomic conditions from the GVAR model into firm-specific return outcomes. These multifactor return equations can be estimated through least squares analysis assuming that the firm-specific and macroeconomic innovations are uncorrelated.

PSTW (2006) uses pooled mean-group estimators (MGE) to facilitate the estimation of return regressions. In commercially available models the MGE procedure and

estimates are usually applied as well. The only heterogeneity within such a framework is generated through the differentiation between countries' firms with respect to macroeconomic factor sensitivities. More differentiation and therefore diversification can be allowed through panel-fixed effects or random-effects modelling. In general, fixed- or random-effects heterogeneity implies that each firm would have a different intercept but that the sensitivity to macroeconomic factors would still be similar across all firms i.e. the factor loadings would be similar. Effectively, average returns per firm would be different but the sensitivity to macrofactors would still be the same. Such a generalisation would be applicable if firms have been classified into homogenous groups e.g. industries or sectors, where a group-specific multifactor panel model could be estimated for each homogenous group. Without such homogenous grouping, a panel estimation model could still overstate risk, as the differential sensitivity of each firm is not taken into account when estimating overall portfolio risk. The difference between a single risk factor versus a multi-factor model can also contribute to risk being overstated. This difference has indeed been criticised by various practitioners as being a major drawback of the single risk factor approach.

As a result, this analysis first estimates a pooled and then a fixed or random effects model to test overall economy-wide sensitivity to macroeconomic factors. This estimation is used as vantage point and to inform and guide expectation into the final firm-specific return estimates, as it may turn out that pooled estimation results indicate that pooling is inappropriate and render statistically insignificant and theoretically inconsistent parameter estimates. All macroeconomic variables in the VECM model of De Wet et al. (2009) are included as explanatory variables.

Following the estimation of a pooled model, the appropriateness of a fixed or random effects panel model is tested by evaluating the F- and Chi-squared tests for fixed and random effects and the Hausmann test for correlation between the individual random effects and the regressors in the random effects model. As explained above the result from these pooled models can be counter intuitive and may lead to a misrepresentation of the level of the underlying portfolio risk (either up or down). To overcome these shortcomings we propose the allowance of maximum diversification between the 145 firms and the estimation of each firm return model on an individual basis in section 4.3.

Short names and notation of variables in all estimation results are adopted from De Wet et al. (2009) but are used in log difference transformation form in order to obtain stationary representations of the factors as required in multifactor models. The variable description is as follows: the dependent variable or firm specific equity return variable is calculated as the log difference in the cum dividend quarterly equity price per firm;  $y$  represents domestic real gross domestic product ( $GDP$ );  $q$  the domestic real equity price index;  $\rho$  ( $\rho$ ) the nominal short-term domestic risk free rate (a proxy for the monetary policy rate);  $d$  the domestic household debt to income ratio;  $h$  the domestic real house price index and  $p$  the consumer price index. All real variables have been deflated using the consumer price index, while  $e$  represents the real effective exchange rate as constructed by De Wet et al. (2009).

Furthermore  $y^*$ ,  $q^*$  and  $\rho^*$  ( $\rho$ ) represents the global counterparts of the domestic variables. Global variables are constructed as trade-weighted aggregates, with the countries included representing 85 per cent of South African trade.

#### 4.2.1. Pooled model for portfolio-wide return estimates

As discussed above, the first step when deriving a specification for firm-specific return dynamics is to estimate a pooled multifactor model for the dummy portfolio. This estimation procedure assumes cross-section (firm) homogeneity, i.e. it is implicitly assume that all firms react similarly to macroeconomic factors. In line with commercially available factor model analysis the maximum number of firms is used to estimate the pooled model, i.e. all 696 firms are included. Since the model specification is used to represent the “average” South African firm, including more firms in the specification is more reflective of this average firm.

**Table 3 Pooled estimation results**

| Dependent Variable: Equity return           |             |                       |             |           |  |
|---|-------------|-----------------------|-------------|-----------|--|
| Method: Pooled Least Squares                |             |                       |             |           |  |
| Cross-sections included: 696                |             |                       |             |           |  |
| Total pool (unbalanced) observations: 26934 |             |                       |             |           |  |
| Variable                                    | Coefficient | Std. Error            | t-Statistic | Prob.     |  |
| <i>C</i>                                    | 0.421503    | 0.115481              | 3.649992    | 0.0003    |  |
| $\rho^*$                                    | 0.017514    | 0.023760              | 0.737121    | 0.4611    |  |
| <i>q</i> *                                  | -2.683237   | 0.257293              | -10.42874   | 0.0000    |  |
| <i>y</i> *                                  | -0.203933   | 0.032683              | -6.239804   | 0.0000    |  |
| <i>E</i>                                    | 0.213667    | 0.092027              | 2.321782    | 0.0203    |  |
| $\rho$                                      | 0.780490    | 0.029617              | 26.35260    | 0.0000    |  |
| <i>q</i>                                    | 0.164608    | 0.109128              | 1.508399    | 0.1315    |  |
| <i>y</i>                                    | 1.631373    | 0.130061              | 12.54315    | 0.0000    |  |
| <i>d</i>                                    | -0.463861   | 0.096354              | -4.814118   | 0.0000    |  |
| <i>h</i>                                    | 1.620473    | 0.183589              | 8.826623    | 0.0000    |  |
| <i>p</i>                                    | 3.018423    | 0.408633              | 7.386640    | 0.0000    |  |
| R-squared                                   | 0.054486    | Mean dependent var    |             | -0.004473 |  |
| Adjusted R-squared                          | 0.054169    | S.D. dependent var    |             | 0.386164  |  |
| S.E. of regression                          | 0.375559    | Akaike info criterion |             | 0.879570  |  |
| Sum squared resid                           | 3793.678    | Schwarz criterion     |             | 0.882618  |  |
| Log likelihood                              | -11823.29   | Hannan-Quinn criter.  |             | 0.880553  |  |
| F-statistic                                 | 1.792826    | Durbin-Watson stat    |             | 1.854387  |  |
| Prob(F-statistic)                           | 0.000000    |                       |             |           |  |



Estimation results are illustrated in Table 3 and show that only world interest rates and domestic equity prices are rejected as return factors on a pooled basis and that all other variables are significant determinants of firm returns. It seems unreasonable that global interest rates and in particular domestic equity prices should not be included in our specifications. Domestic equity market returns (the market factor) in particular should have a very strong positive correlation to the portfolio's equity dynamics. We therefore propose to move forward and estimate a panel regression model which allows more heterogeneity amongst firms in the estimation process and to address the a-theoretical estimation results in Table 3 above .

#### **4.2.2. Fixed- and random-effects model for portfolio-wide return estimates**

The next step is to estimate a panel multifactor model for the portfolio of corporate loans allowing for firm-specific heterogeneity through either fixed or random effects panel data modelling.

From Table 4, the Chi-squared test for random effects indicates that the null hypothesis for correlation between the random effects and the regressors can clearly be rejected, pointing to endogeneity issues, and placing doubt on whether the technique is indeed applicable in this context. The F- and Chi-squared tests for fixed effects reject the null hypothesis that all fixed effects are jointly insignificant, i.e. firms in this portfolio context are indeed heterogeneous, and would react uniquely to changes in the global and domestic macroeconomic environment.

**Table 4 Random and fixed effects diagnostic testing**

| <b>Correlated Random Effects – Hausman Test</b> |                          |                     |              |
|---|--------------------------|---------------------|--------------|
| <b>Test cross-section random effects</b>        |                          |                     |              |
|   | <b>Chi-Sq. Statistic</b> | <b>Chi-Sq. d.f.</b> | <b>Prob.</b> |
| Cross-section random                            | 142.86                   | 10                  | 0.0000       |

| <b>Redundant Fixed-Effects Tests</b>    |                  |             |              |
|---|------------------|-------------|--------------|
| <b>Test cross-section fixed effects</b> |                  |             |              |
| <b>Effects Test</b>                     | <b>Statistic</b> | <b>d.f.</b> | <b>Prob.</b> |
| Cross-section F                         | 1.308            | (694,26)    | 0.0000       |
| Cross-section Chi-square                | 916.05           | 694         | 0.0000       |

The result of the fixed effects model is of particular importance and is entirely consistent with the motivation for estimating firm-specific return equations as apposed to a pooled model specification. Not controlling for heterogeneity disregards significant diversification benefits that are present in the portfolio and could lead to a-theoretical estimation results as shown above in the pooled model.

The estimation result for the fixed effects model is presented in Table 5. Despite allowing for more firm heterogeneity, real world equity, domestic interest rates and the real house price index are not significant determinants of returns within the dummy portfolio. Although all other variables are significant determinants of equity returns, the adjusted R-squared statistic indicates that only 6 per cent of the variance of the dependent variable is explained by variables included in the model specification.

While the results indicate that this specification allows for more heterogeneity than the pooled model, the specification does not sufficiently capture firm-specific return dynamics as it is highly questionable whether a variable such as the world equity

prices and domestic interest rates can not have any impact on the dummy portfolio's return dynamics in a well-developed capital market such as the one in South Africa .

**Table 5 Fixed effects estimation results**

| Dependent Variable: Equity return           |             |                       |             |           |
|---|-------------|-----------------------|-------------|-----------|
| Method: Pooled Least Squares                |             |                       |             |           |
| Cross-sections included: 696                |             |                       |             |           |
| Total pool (unbalanced) observations: 26934 |             |                       |             |           |
| Variable                                    | Coefficient | Std. Error            | t-Statistic | Prob.     |
| c   | -0.040796   | 0.008597              | -4.745442   | 0.0000    |
| $\rho^*$                                    | 0.237786    | 0.120050              | 1.980722    | 0.0476    |
| $q^*$                                       | 0.015331    | 0.024037              | 0.637816    | 0.5236    |
| $y^*$                                       | -2.327328   | 0.259455              | -8.970071   | 0.0000    |
| e   | -0.163703   | 0.032794              | -4.991889   | 0.0000    |
| $\rho$                                      | -0.029725   | 0.100489              | -0.295806   | 0.7674    |
| q   | 0.780752    | 0.029745              | 26.24841    | 0.0000    |
| y   | 0.234861    | 0.110277              | 2.129739    | 0.0332    |
| d   | 1.509952    | 0.134238              | 11.24831    | 0.0000    |
| h   | 0.086892    | 0.140674              | 0.617683    | 0.5368    |
| p   | 3.018423    | 0.408633              | 7.386640    | 0.0000    |
| R-squared                                   | 0.087182    | Mean dependent var    |             | -0.004473 |
| Adjusted R-squared                          | 0.062656    | S.D. dependent var    |             | 0.386164  |
| S.E. of regression                          | 0.373870    | Akaike info criterion |             | 0.896037  |
| Sum squared resid                           | 3662.492    | Schwarz criterion     |             | 1.110892  |
| Log likelihood                              | -11349.83   | Hannan-Quinn criter.  |             | 0.965334  |
| F-statistic                                 | 3.554703    | Durbin-Watson stat    |             | 1.854387  |
| Prob(F-statistic)                           | 0.000000    |                       |             |           |

The results therefore support our proposition that pooling slope and/or intercept parameters may be inappropriate when attempting to capture the underlying firm-specific characteristics within a portfolio. To address this issue it is therefore proposed that a significant enhancement is made to the PSTW (2006) methodology by estimating individual multifactor models on a name-by-name basis, allowing for maximum heterogeneity within the portfolio.

### **4.2.3. Firm-specific multifactor models**

As argued the multifactor MGE model estimation procedure is enhanced by estimating firm-specific multifactor models by running individual regressions using the variables from the South African-specific VECM model as possible risk factors.

While this process of modelling is intensive some of the intensity associated with searching for the correct specification in each of the 145 firm multifactor models can be overcome by utilising stepwise regression techniques. Stepwise regression techniques have been criticised by statisticians for a number of shortcomings. Most importantly  $p$ -values listed in the final regression output and all subsequent testing procedures do not account for the regressions run during the selection process which complicates interpretation of results. Other problems include an upwardly biased final R-squared value, potentially upwardly biased coefficient estimates, and narrow confidence intervals. It is also often pointed out that the selection methods themselves use statistics that do not account for the selection process.

However, it can be argued that since a set theoretical multifactor specification for each firm is not available and because it is time-intensive to search for and specify 145 individual specifications manually, step-wise regression provides the closest approximation for a specification procedure. A set of estimation procedures is proposed which aim to limit the risk of incorrect specification due to the shortcomings of the stepwise technique.

The “stepwise-forwards” technique is used to find the final estimation model for each firm-specific multifactor model. The stepwise-forwards methodology begins with no regressors in the regression, and then adds the variable with the lowest  $p$ -value as the first variable in the specification. The variable with the next lowest  $p$ -value, given that the first variable has already been chosen, is then added. Next both of the added variables are checked against the backwards  $p$ -value criterion. Any variable whose  $p$ -value is higher than the criterion is removed. Once the removal step has been performed, the next variable is added, i.e. the variable with the lowest  $p$ -value. With this, and each successive addition to the model, all the previously added variables are checked against the backwards criterion and possibly removed. The stepwise-forwards routine ends when the lowest  $p$ -value of the variables not yet included is greater than the specified forwards stopping criterion.

Inclusion of variables based on a  $p$ -value specification test which allows variables within the multifactor models with individual  $p$ -values smaller than 0.20 is allowed, although in general the final variables included in the multifactor models have individual  $p$ -values below 0.15. An intercept is included in all specifications in order to obtain the average return estimate for each company. In addition to the individual significance of variables, the overall combination of variables is evaluated using the F-statistic to ensure that the total set of variables is jointly significant in explaining firm returns. Residual diagnostic tests such as the White-heteroskedasticity and Breusch-Godfrey serial correlation tests are also performed on each model to test for validation of the Gaussian assumptions. Although the set of search variables may contain variables that are collinear, those variables are excluded from the search set upfront. In the case where two or more of the search variables are highly correlated

the variable listed first in the list of search variables is included. As per De Wet et al. (2009) we make the small open economy assumption for South Africa and as such propose that domestic equity dynamics would be dominated by global dynamics where-after the response of the domestic macroeconomic variables will also impact returns. It is therefore proposed that the sequence of evaluation of variables is as follows (all variables are used in log differenced form):  $\rho^*$ ,  $q^*$ ,  $y^*$ ,  $e$ ,  $\rho$ ,  $q$ ,  $y$ ,  $d$ ,  $h$  and  $p$ . Therefore, although stepwise regressions are used as a tool to expedite the estimation procedure, in the analysis several other model specification and diagnostic tests were performed on a name-by-name basis before a final choice for a multifactor model was made.

The process followed provides a total number of 145 firm-specific multifactor models, allowing the maximum degree of firm heterogeneity possible within the current framework. Presenting the model output and diagnostic estimates for 145 firm-return models falls beyond the scope of this paper, but it is sufficient to say that the model specifications differ across firms within the dummy portfolio. In general it is deemed acceptable if the estimation procedure followed has increased the percentage of variation explained by the models considerably with adjusted R-squared being around 25 per cent for most specifications (increasing to as high as 65 per cent). As a representative example the multifactor model for one of the larger cement producers in the country is provided (see Table 6) and the procedure followed to evaluate each multifactor model individually is discussed.

The estimation output indicates that 31 per cent of the variation in firm returns is explained by the model specification. Although not fully compatible with the

correlation assumptions in an asymptotic single risk-factor model which translates the R-squared estimate directly into an asset correlation, the R-squared estimate in the multi-factor model shows that a substantial proportion of firm returns can be attributed to the correlation with economic activities. This result again illustrates the importance and contribution made by this study in providing a mechanism through which credit risk portfolio managers can link the idiosyncratic component of firm risk to cyclical- or market-driven risk. The average quarterly return estimate (as illustrated by the significant constant term) is 3.8 per cent, which can loosely be translated into a 16 per cent annual return assuming no change in the underlying macroeconomic factors. Assuming an average annual inflation rate of 6 per cent, the estimate is in line with a target real return of 10 per cent to investors

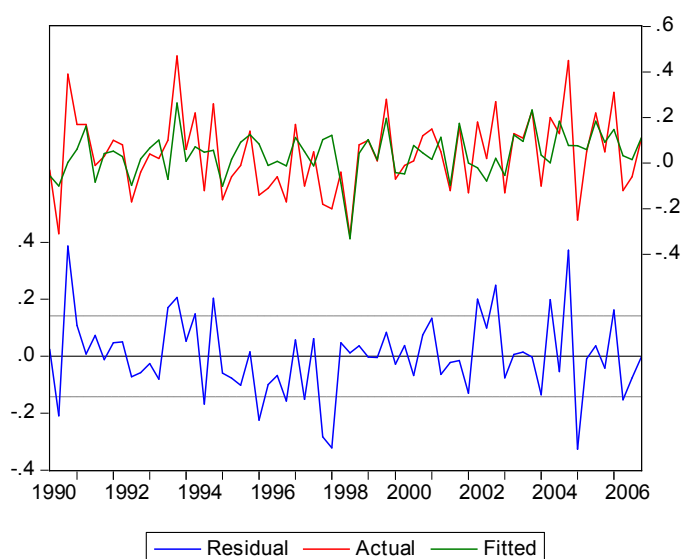
The multifactor model indicates that two factors are significant determinants of firm returns. As expected, the sign and magnitude of the coefficient of real domestic equity returns ( $q$ ), indicates that firm-specific returns are closely related to overall movements in equity markets. The second significant factor included in the multi-factor model is the effective exchange rate and illustrates that depreciation in the exchange rate leads to a loss in equity return implying that the import component of the cost base of the production process is sensitive to the effective price of imported goods.

The F-statistic for overall significance of the regressors confirms the individual regressors' high t-statistics (low p-values) and indicates that the independent variables included are jointly significant in explaining returns. Analysing the actual versus expected returns in Figure 1 shows that the model captures return dynamics

**Table 6 Example: Individual multifactor model**

| Dependent variable: Equity return<br>Method: stepwise regression |             |                       |             |           |
|--|-------------|-----------------------|-------------|-----------|
| Independent variables  | Coefficient | Std. Error            | t-Statistic | Prob.     |
| <i>c</i>   | 0.038278    | 0.017744              | 2.157206    | 0.0348    |
| <i>q</i>   | 0.876086    | 0.165017              | 5.309061    | 0.0000    |
| <i>e</i>   | -0.465580   | 0.222017              | -2.097052   | 0.0400    |
| R-squared  | 0.318509    | Mean dependent var    |             | 0.040455  |
| Adjusted R-squared   | 0.296875    | S.D. dependent var    |             | 0.169019  |
| S.E. of regression   | 0.141727    | Akaike info criterion |             | -1.025440 |
| Sum squared resid  | 1.265450    | Schwarz criterion     |             | -0.925911 |
| Log likelihood   | 36.83953    | Hannan-Quinn criter.  |             | -0.986111 |
| F-statistic  | 14.72221    | Durbin-Watson stat    |             | 2.320085  |
| Prob(F-statistic)  | 0.000006    |                       |             |           |

**Figure 1 Actual versus expected returns**



**Table 7 Diagnostic test results**

|  |          |                     |        |
|--|----------|---------------------|--------|
| <b>White heteroskedasticity test:</b>              |          |                     |        |
| F-statistic  | 0.948290 | Prob. F(5,60)       | 0.4567 |
| Obs*R-squared                                      | 4.833623 | Prob. Chi-Square(5) | 0.4365 |
| Scaled explained SS                                | 6.077828 | Prob. Chi-Square(5) | 0.2987 |
| <b>Breusch-Godfrey serial correlation LM test:</b> |          |                     |        |
| F-statistic  | 1.622092 | Prob. F(2,60)       | 0.2095 |
| Obs*R-squared                                      | 3.332849 | Prob. Chi-Square(2) | 0.1889 |
| <b>Normality test: Jarque-Bera</b>                 | 2.161070 | Prob.               | 0.3395 |



over the 17 year period and that the residual is well behaved and free of statistical problems. The diagnostic test results reported in Table 7 also indicates that there is no information not adequately accounted for and that residuals do not suffer from serial correlation or heteroskedasticity.

Following a similar process, multifactor return models for all firms in the dummy portfolio have been estimated and evaluated as being statistically and theoretically appropriate for inclusion in a stochastic simulation and forecasting model for determining firm-specific default dynamics. In conclusion it can be said that constructing individual return equations provides a significant enhancement over the pooled or even individual effects panel estimation techniques normally performed in the literature and commercially available models. The models in this analysis provide significant benefit in identifying true risk and return dynamics in a South African-specific credit portfolio model context.

## **5. CONDITIONAL LOSS ESTIMATION AND SCENARIO ANALYSIS RESULTS**

With the conditional return dynamics defined through the multifactor models and the log-equity threshold levels determined from historical observations as provided by PSTW (2006), the conditional default and expected loss for the portfolio based on the macroeconomic VECM model as constructed and provided by De Wet et al. (2009) can now be simulated.

Expected loss and capital sensitivity analysis is generally conducted over a one-year period and therefore the conditional loss distribution estimates are only illustrated for the four quarters ahead forecast horizon (although one to three quarter estimates are also available). From a practitioner's perspective, the simulated loss distribution is of particular interest since it allows inference to be drawn with respect to the likelihood of various loss events taking place over the forecast horizon. Contrary to the analytical approach that provides estimated default and expected loss estimates of exposures and portfolios, a simulated loss procedure provides important information for credit portfolio managers. Since a simulation approach provides a complete loss distribution, it potentially includes information with respect to the magnitude of risks faced and the likelihood of such risks within the credit portfolio. While the methodology allows analytical estimates to be produced the focus here is on the simulated loss distribution and the practical application of such a conditional loss estimation process.

Using the methodology proposed by PSW (2004) and PSTW (2006), the conditional loss distribution of the dummy portfolio based on 95 000 simulations are generated. Since the main benefit of conditional loss simulation is to provide a direct way of estimating the impact of macroeconomic factors on portfolio loss, scenario analysis results are also constructed. In practice, portfolio expected loss scenario analysis is done based on either single factor stress tests or sensitivity analysis or on a combination of stresses applied to a set of risk drivers to assess the combined effects of such a scenario on the portfolio. These scenario analyses are usually done through a level movement in the underlying risk factors which would move the whole distribution of factor simulations up or down. We therefore present both these sets of

analysis: the sensitivity analysis is done by applying a 15 per cent up and down adjustment on the level of world equities in an attempt to assess the sensitivity of the portfolio to global effects. The scenario analysis focuses on South African specific factors and tests the combined effects of shocking two variables simultaneously to create an “upturn” and “downturn” scenario. The upturn scenario is characterised by a 4 per cent decrease in interest rates combined with a 15 per cent increase in equity prices while the downturn is based on the opposite but equal magnitude increase and decrease in interest rates and equity prices.

However, credit risk does not only originate from the level of variables but also from the volatility of variables over time. In fact, as indicated already by the log equity threshold levels above the absolute levels across rating categories are not necessarily that different, but the volatility and therefore the credit risk of the underlying assets increase as rating deteriorate. We therefore present a third scenario analysis, simulating a highly volatile economic environment over the forecast period by increasing and decreasing the standard deviation of the stochastic error terms of the exchange rate, South African and world equities equations in the VECM model by 1.5 and 0.5 times respectively.

Summary statistics of the conditional expected and unexpected (SD) loss estimate are provided in Table 8. As illustrated by the baseline distribution mean of 0.31 per cent (31 bps), the simulated one-year conditional expected loss of the portfolio indicates that the portfolio has experienced positive migration over the one year horizon from the 0.53 bps expected loss estimate provided by prior expectations and benchmark estimates.

The scenario expected loss estimates show the asymmetric behaviour of the portfolio; for the level adjustment analyses the higher risk environment sensitivity and scenario analysis expected loss and standard deviation increases over baseline is much higher than the less risky environment results. However, from the volatility scenario analysis one can see that the expected loss and standard deviation changes relative to the baseline is quite similar.

The benefit of the simulation approach is apparent in its ability to provide insight into the severity associated with tail events or the “body” of the distribution. As an example; the losses associated with a 1 in 10, 1 in 100 and 1 in 1000 year event (i.e. the 90<sup>th</sup>, 99<sup>th</sup> and 99.9<sup>th</sup> percentile values of the loss distribution) for each scenario are also presented in Table 8.

These results clearly illustrate the asymmetric reaction of the loss distribution for similar positive and negative shocks, i.e. an equal increase or decrease in the level of risk results in substantial higher increases in loss events in riskier environments than the decrease in loss in less riskier environments. Moreover, the difference between changes in scenario expected losses, over the baseline, increases marginally more the further in the tail one moves. However, an interesting result is that a decrease in volatility has a significantly more pronounced impact on expected losses than a similar increase in volatility. This might imply that the environment within which the portfolio is operating under the baseline scenario is significantly volatile already (i.e. the baseline simulations are already volatile) and that a reduction in this risk have a significant impact in credit riskiness of the portfolio.

**Table 8** Conditional expected loss simulation summary results \*

|                              | Mean            | SD              | 90 <sup>th</sup> | 99 <sup>th</sup> | 99.9 <sup>th</sup> |
|------------------------------|-----------------|-----------------|------------------|------------------|--------------------|
| <b>Baseline</b>              | 0.31            | 0.41            | 0.93             | 1.67             | 2.52               |
| <i>Sensitivity analysis</i>  |                 |                 |                  |                  |                    |
| <b>World equity decrease</b> | 0.49<br>(0.18)  | 0.51<br>(0.10)  | 1.17<br>(0.24)   | 2.08<br>(0.41)   | 2.92<br>(0.40)     |
| <b>World equity increase</b> | 0.29<br>(-0.02) | 0.35<br>(-0.06) | 0.73<br>(-0.20)  | 1.41<br>(-0.26)  | 2.15<br>(-0.37)    |
| <i>Scenario analysis</i>     |                 |                 |                  |                  |                    |
| <b>Downturn scenario</b>     | 0.52<br>(0.21)  | 0.47<br>(0.08)  | 0.99<br>(0.07)   | 1.73<br>(0.06)   | 2.70<br>(0.18)     |
| <b>Upturn scenario</b>       | 0.29<br>(-0.01) | 0.40<br>(-0.01) | 0.90<br>(-0.03)  | 1.62<br>(-0.05)  | 2.44<br>(-0.08)    |
| <i>Volatility analysis</i>   |                 |                 |                  |                  |                    |
| <b>High volatility</b>       | 0.43<br>(0.12)  | 0.50<br>(0.09)  | 1.08<br>(0.15)   | 2.05<br>(0.38)   | 3.16<br>(0.64)     |
| <b>Low volatility</b>        | 0.20<br>(0.11)  | 0.30<br>(0.11)  | 0.57<br>(-0.36)  | 1.19<br>(-0.48)  | 1.76<br>(-0.76)    |

\* Results presented in percentage terms with changes over baseline in brackets

## 6. CONCLUSION

In this paper we have highlighted the fact that credit portfolios are ultimately exposed to macroeconomic cycles even though idiosyncratic risk within a large corporate loan can be limited to some extent through diversification. In order to perform value enhancing and/or risk mitigation strategies the credit portfolio manager must be able to perform scenario analysis that links firm-specific dynamics to macroeconomic factors through statistical models. The end goal should be to translate it into actionable financial market strategies.

In this analysis the South African-specific VECM model provided by De Wet et al. (2009) is used as the macroeconomic engine or credit portfolio risk correlation model together with the Merton-type default model proposed by PSTW (2006), as evidence that conditional loss credit portfolio modelling is possible in South Africa. This credit-portfolio model provides stochastic simulation results and allows for

correlation between macroeconomic factors, the correlation of firms with these macro factors as well as correlation between firms. We extend the commercially available methodology in that an individual multifactor model for each exposure in the portfolio is provided. It can be argued that the enhancement allows for more diversification to be recognised in the portfolio than would be the case with normal asymptotic single risk factor type models. Scenario analysis results are also provided which confirm the asymmetric behaviour of credit risk. The methodology provides a theoretically consistent and direct method of estimating credit risk as well as performing scenario analysis that can be used in credit portfolio enhancement strategies.

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