A large factor model for forecasting macroeconomic variables in South Africa

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\textbf{Abstract}

This paper uses large Factor Models (FMs), which accommodate a large cross-section of macroeconomic time series for forecasting the per capita growth rate, inflation, and the nominal short-term interest rate for the South African economy. The FMs used in this study contain 267 quarterly series observed over the period 1980Q1–2006Q4. The results, based on the RMSEs of one- to four-quarter-ahead out-of-sample forecasts from 2001Q1 to 2006Q4, indicate that the FMs tend to outperform alternative models such as an unrestricted VAR, Bayesian VARs (BV ARs) and a typical New Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) model in forecasting the three variables under consideration, hence indicating the blessings of dimensionality.

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\textbf{Keywords:} Large factor model; VAR; BV AR; NKDSGE model; Forecast accuracy

\section{1. Introduction}

For a long time, policy makers, the general public and academics have all been interested in producing accurate forecasts of economic variables, for various different reasons. Model builders have exploited recent developments in computation to develop models, both simple and complex, that simulate reality with high degree of accuracy. There is increasingly a need for information to mimic economic relationships. However, traditional economic models, such as univariate time series and multivariate VAR models, are limited, in that they cannot accommodate large numbers of time series. Although the VAR is more popular than traditional macroeconometric models because of its forecasting ability, it has serious limitations, with the most conspicuous being that it cannot accommodate a large panel of series without the risk of running short or having too many degrees of freedom. Thus, given, the issue of overparameterization, the main problem of small-scale models lies in the decision regarding the correct choice of the variables

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to include. In practice, however, forecasters and policymakers often extract information from many series other than those that can be included in a VAR.

As Bernanke and Boivin (2003) eloquently argue, central banks monitor and analyze literally thousands of observations from various sources. Since central banks pay the costs of analyzing a wide range of data sources in order to improve their decisions, econometric models should take into account the marginal benefits that increasing information brings to forecasting. In recent decades, considerable progress into the accommodation of large panels of time series in forecasting using factor models has been made. The initial contributions in this area were the work of Geweke (1977) and Sargent and Sims (1977), who introduced the dynamic factor approach to macroeconomics. They exploited the dynamic interrelationships between the variables, and then reduced the number of common factors even further. However, the approach followed by Geweke (1977) and Sargent and Sims (1977) is too restrictive, in that it imposes orthogonality on the idiosyncratic components, while Chamberlain (1983) and Chamberlain and Rothschild (1983) admit the possibility of weakly cross-sectional correlation of the idiosyncratic components.

However, these large factor models have recently been improved by accounting for serial correlation and weakly cross-sectional correlation of idiosyncratic components, through advances in estimation techniques proposed by Forni, Hallin, Lippi, and Reichlin (2005), Kapetanios and Marcellino (2009) and Stock and Watson (2002b). This progress, in turn, has generated an increasing amount of interest in the usage of these models in academia, international organizations, central banks, and governments, simply because they can accommodate a large panel of time series when forecasting economic variables. However, there is still a considerable degree of divergence in opinion as to whether or not factor models with large cross-sections of time series tend to outperform traditional econometric models with limited numbers of variables. On the one hand, studies such as those of Cristadoro, Forni, Reichlin, and Veronese (2005), Forni, Hallin, Lippi, and Reichlin (2001), Forni et al. (2005), Giannone and Matheson (2007), Kabundi (2004), Schneider and Spitzer (2004), Stock and Watson (1989, 1991, 1999, 2002a,b) and van Nieuwenhuize (2005) provide evidence of improvements in the forecasting performances of macroeconomic variables using factor analysis, while on the other hand, Angelini, Henry, and Mestre (2001), Gosselin and Tkacz (2001), Schumacher (2007) and Schumacher and Dreger (2004) found only minor or no improvements in forecasting ability. These conflicting results have led to a fascinating debate as to whether or not the victory claimed by the proponents of large models was premature. Some attribute the success of large models to the different circumstances. For example, Banerjee, Marcellino, and Masten (2005) find that small models forecast macroeconomic variables better than factor models. In addition, they also find that the performances of factor models differ between countries. Factor models are comparatively good at forecasting real variables in the US relative to the euro area, while the euro area nominal variables are easier to predict than the US nominal variables, using factor models. Furthermore, Boivin and Ng (2006) claim that the composition of the data set and the dimensions of the cross-section are important in producing better forecasts from factor models.

Against this backdrop, this paper exploits the information contained in the large-dimensional factor model framework developed by Forni et al. (2005) (henceforth FHLR) and Stock and Watson (2002b) (henceforth SW), for forecasting the per capita growth (percentage change in real per capita GDP), inflation (percentage change in the implicit GDP deflator) and a measure of the short-term nominal interest rate (91-day Treasury bill rate) for South Africa, over the out-of-sample horizon spanning the period from 2001Q1 to 2006Q4, with an in-sample period from 1980Q1 to 2000Q4. The forecasting performances of the FMs, estimated under alternative assumptions with regard to the interaction between the factors and the variables of interest, are evaluated and compared with the performances of three other alternative models, namely an unrestricted classical VAR, an optimal Bayesian VAR2 (BVAR) and a New-Keynesian Dynamic Stochastic General Equilibrium (NKDSGE) model, on the basis of the Root Mean Squared Error (RMSE) of the out-of-sample forecasts. Although Kabundi (2009) used the DFM to assess the

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2 See Section 5 for further details regarding the issue of the optimality of BVARs.
synchronization of South Africa and the US, and the channels through which the US supply and demand shocks are transmitted, to the best of our knowledge this is the first attempt to use a large FM to forecast key macroeconomic variables in South Africa. Moreover, it must be noted that, with the exception of Wang (2009), comparisons between a FM and a DSGE model are rare, but worthy of discussion, especially in the context of a developing economy like that of South Africa. Note that allowing for a NKDSGE model as an alternative forecasting framework helps us to compare various atheoretical models, such as the FM, VAR and BVARs, with a microfounded theoretical model. The remainder of the paper is organized as follows. Section 2 lays out the large FM, while Section 3 discusses the data used to estimate the FM. Section 4 outlines the basics of the VAR, BVAR and NKDSGE models, and Section 5 presents the results from the forecasting exercise. Finally, Section 6 concludes.

2. The model

This study uses a large FM to extract components that are common between macroeconomic series, then uses these common components to forecast output growth, the inflation rate, and nominal interest rates. In the VAR models, since all variables are used in forecasting, the number of parameters requiring estimation depends on the number of variables $n$. With such a large information set, $n$, the estimation of a large number of parameters leads to what is termed the ‘curse of dimensionality’ In the FM, the information set is accounted for by only a few factors $q \ll n$, which transforms the curse of dimensionality into the blessing of dimensionality.

The FM expresses individual time series as the sum of two unobserved components: a common component, which is driven by a small number of common factors, and an idiosyncratic component, which is specific to each variable. The relevance of this method is that the FM is able to extract the few factors that explain the comovement of all South African macroeconomic variables. SW and FHLR demonstrate that when the number of factors is small relative to the number of variables and the panel is heterogeneous, the factors can be recovered from the present and past observations.

Consider a $n \times 1$ covariance stationary process $X_t$, which is standardized to have a zero mean and unit variance. Under FM, $X_t$ is described by a factor model, which can be written as the sum of two orthogonal components:

$$x_{it} = b_i(L) f_i + \xi_{it} = \lambda_i F_i + \xi_{it},$$

or, in vector notation:

$$X_{it} = B(L) f_i + \xi_{it} = \Lambda F_i + \xi_{it},$$

where $f_i$ is a $q \times 1$ vector of dynamic factors, $B(L) = B_0 + B_1 L + \cdots + B_q L^q$ is an $n \times q$ matrix of factor loadings of order $s$, $\xi_{it}$ is a $n \times 1$ vector of idiosyncratic components, and $F_i$ is a $r \times 1$ vector of factors, with $r = q(s + 1)$. However, in more general framework, $r \geq q$, rather than the more restrictive $r = q(s + 1)$. In a DFM, $f_i$ and $\xi_{it}$ are mutually orthogonal stationary processes, while $\chi_{it} = B(L) f_i$ is the common component.

In factor analysis jargon, $X_t = B(L) f_t + \xi_{it}$ is referred to as the dynamic factor model, and $X_t = \Lambda F_t + \xi_{it}$ is the static factor model. Similarly, $f_t$ is regarded as the vector of dynamic factors, while $F_t$ is the vector of static factors. Since dynamic common factors are latent, they need to be estimated. It is important to point out that the estimation technique used is important for factor forecasts. This paper uses two leading methods in the literature of large FMs, namely those proposed by SW and FHLR. SW use the static principal component approach (PCA) on $X_t$. The factor estimates are therefore the first principal components of $X_t$, i.e. $\hat{F}_t = \hat{\Lambda} X_t$, where $\hat{\Lambda}$ is the $N \times r$ matrix of the eigenvectors corresponding to the $r$ largest eigenvalues of the sample covariance matrix $\hat{\Sigma}$.

On the other hand, FHLR proposed a weighted version of SW’s principal components estimator based on dynamic PCA, which exploits information on the leads and lags of variables when time series are converted to the frequency domain. However, the dynamic PC is a two-sided filter. This causes a problem at the end of the sample, making it difficult to estimate and forecast the common component since no future observations are available. FHLR solve this problem by proposing a two-step approach. The first step relies on the dynamic approach in the estimation of the covariance matrices of the common and idiosyncratic components (at all leads and lags), through an inverse Fourier transformation of the spectral density matrices. It involves
estimating the eigenvalue and eigenvector decompositions of the spectral density matrix of $X_t$, $\hat{\Sigma}(\theta)$, which has rank $q$, corresponding to the $q$ largest eigenvalues. For each frequency $\theta$, the spectral density matrix of $X_t$, which is estimated using the frequency $-\pi < \theta < \pi$, can be decomposed into the spectral densities of the common and idiosyncratic components, $\Sigma(\theta) = \Sigma_X(\theta) + \Sigma_e(\theta)$. Hence, the spectral density matrix of the common component $\hat{\Sigma}_X(\theta)$ is estimated. In the second step, this information is used to compute the $r$ factors $\hat{F}_t^{FHLR} = Z'X_t$ by solving a generalized eigenvalue problem. $Z$ are the $r$ generalized eigenvectors (with eigenvalues in descending order) of $\hat{\Sigma}_X(0)$ with respect to $\hat{\Sigma}_e(0)$ under normalization that $Z_i|\hat{\Sigma}_e(0)|Z_i = 1$ if $i = j$ and zero otherwise. The $\hat{F}_t^{FHLR}$ are the so-called generalized principal components, and have the desirable property of reducing the idiosyncratic noise in the common factor space to a minimum, by selecting the variables with the highest common/idiosyncratic variance ratio. Importantly, this one-sided approach is only used to estimate and forecast the common component.

We use the models of both SW and FHLR because of the conflicting results in the literature. For example, Eickmeier and Ziegler (2008) find that the FHLR model tends to outperform that of SW in forecasting output. However, Schumacher (2007) finds minor improvements of the FHLR model over that of SW in predicting output, while for inflation, Eickmeier and Ziegler (2008) find the reverse. In contrast, Formi, Hallin, Lippi, and Reichlin (2003) find that the FHLR does better for both output and inflation. Finally, Stock and Watson (1999), Boivin and Ng (2005); D’Agostino and Giannone (2006) only find modest differences between the two methods.

For forecasting purposes, we use a small VAR which contains the variable(s) of interest, augmented by common factors extracted using the Stock and Watson (2002a) approach. This approach is similar to the univariate Static and Unrestricted (SU) approach of Boivin and Ng (2005). Therefore, the forecasting equation for predicting $y_t$ is given by

$$\left(\frac{y_{T+h|T}}{\hat{F}_{T+h|T}}\right) = \hat{\phi}(L) \left(\frac{y_T}{\hat{F}_T}\right).$$

where $y_t$ is either a small vector of time series to be predicted or a scalar value.

We also consider another forecasting approach, namely the Bayesian estimation of Eq. (1), called the BFAVAR. As Boivin and Ng (2005) clearly put it, AR and VAR are special cases of Eq. (1). When the factors and the parameters are known, the FAVAR approach should produce smaller mean squared errors. However, in practice one does not observe the factors, and thus they should be estimated. In addition, the forecasting equation should be specified correctly.

Finally, for the FHLR model we adopt the Dynamic-Nonparametric approach (DN), as was discussed by Boivin and Ng (2005). Furthermore, we estimate the idiosyncratic component with AR($p$) processes, also following a suggestion by Boivin and Ng (2005). The forecasting equation is:

$$y_{T+h|T} = \chi_{T+h|T} + \hat{\varphi}(L)\hat{\xi}_{T+h},$$

where $\varphi(L)$ is estimated from equation $\hat{\xi}_{T+h} = \hat{\varphi}(L)\hat{\xi}_t + \zeta_t$, and $\chi_{T+h|T}$ is obtained by artificially projecting $\chi_{T+h|T}$ on $\hat{F}_{t}^{FHLR}$, such that $\chi_{T+h|T} = \hat{\chi}_h(\hat{\Sigma})(\hat{Z}'\hat{Z})^{-1}\hat{Z}'X_t$, $Z$ is the $r$ generalized eigenvectors of $\hat{\Sigma}_X(0)$ with respect to $\hat{\Sigma}_e(0)$ under normalization $Z'\hat{\Sigma}_e(0)Z = 1$, and $\hat{\xi}_t$ and $\hat{\xi}_t$ are covariance matrices of the common and idiosyncratic components at different leads and lags, since $\hat{F}_{t}^{FHLR} = Z'X_t$, then $\chi_{T+h|T} = \hat{\chi}_h(\hat{\Sigma})(\hat{Z}'\hat{Z})^{-1}\hat{F}_{t}^{FHLR}$.

3. Data

In an approximate factor analysis, it is imperative to extract the common components from a data rich environment. After extracting the common components of the output growth, inflation rate, and nominal interest rates, we produce out-of-sample forecasts for one, two, three, and four quarters ahead.

The data set contains 267 quarterly series from South Africa, in the real, nominal, and financial sectors. We also have various intangible variables, such as confidence indices. In addition to national variables, the paper also uses a set of global variables such as the commodity industrial inputs price index and crude oil prices. The data also include series from major trading partners such as Germany (GE), the United Kingdom (UK), and the United States of America (US). The in-sample period contains data from 1980Q1 to 2000Q4. All series are seasonally adjusted and are covariance stationary. The more
powerful DF-GLS test of Elliott, Rothenberg, and Stock (1996), rather than the most popular ADF test, is used to test the degree of integration of all series. All nonstationary series are made stationary via differencing. The Schwarz information criterion is used in the selection of the appropriate lag length, such that no serial correction is left in the stochastic error term. Where there were doubts about the presence of a unit root, the KPSS test proposed by Kwiatowski, Phillips, Schmidt, and Shin (1992), with the null hypothesis of stationarity, was applied. All series are standardized to have a mean of zero and a constant variance. The details of the statistical treatment of all data are available upon request. The in-sample period contains data from 1980Q1 to 2000Q4, while the out-of-sample set is 2001Q1–2006Q4.

We consider the following FM specifications:

- FHLR: this is a FM based on FHLR’s specification, including two dynamic common factors and five static common factors;
- UFAVAR: this is a SW FM which includes one of the variables of interest and five common static factors;
- MFAVAR: this is a SW FM which includes all three variables of interest and five common static factors;
- UBFAVAR: this is a SW FM using one of the variables of interest and five common static factors, which, in turn, are estimated based on the Bayesian restrictions discussed below in Section 4;
- MBFAVAR: this is a SW FM, with a specification similar to that of the MFAVAR, except that the current model applies Bayesian restrictions to the lag of the variables, as is discussed in the next section.

4. Alternative forecasting models

In this study, FMs, in their various forms, are our benchmark models. However, to evaluate the forecasting performances of the FMs we require alternative models as well. In our case, these are the unrestricted classical VAR, BVARS, and a NKDSGE model which was developed recently by Liu, Gupta, and Schaling (2009) for forecasting the South African economy, besides the naïve random walk (RW) model with a drift. This section outlines the basics of these competing models.

An unrestricted VAR model, as suggested by Sims (1980), can be written as follows:

\[ y_t = c + \sum_{i=1}^{p} A_i y_{t-i} + \epsilon_t, \]

where \( y_t \) is a \((n \times 1)\) vector of the variables being forecasted; \( A_i, i = 1 \ldots p \), are \((n \times n)\) autoregressive matrices; \( c \) is a \((n \times 1)\) vector of constant terms⁴; and \( \epsilon \) is a \((n \times 1)\) vector of white noise error terms.

One drawback of VAR models is overparameterization, which in turn leads to large out-of-sample forecasting errors. A popular alternative is to use a Bayesian VAR (BVAR) model. Instead of eliminating longer lags, the Bayesian method imposes restrictions on these coefficients. As is described by Doan, Litterman, and Smis (1984), Litterman (1981, 1986a,b); Spencer (1993); Todd (1984), the “Minnesota”⁵ prior means on variable \( j \) in equation \( i \) at lag \( m \) take the following form:

\[ E(A_{ijm}) = \begin{cases} 1 & \text{if } i = j, k = 1 \\ 0 & \text{otherwise}. \end{cases} \]

The specification of \( S(A_{ijm}) \), the standard deviation of the distribution of the prior imposed on variable \( j \) in equation \( i \) at lag \( m \), is given as follows:

\[ S(A_{ijm}) = u g(m) f(i, j) \frac{\sigma_i}{\sigma_j}, \quad (3) \]

where:

\[ f(i, j) = \begin{cases} 1 & \text{if } i = j \\ k_{ij} & \text{otherwise}, \quad 0 \leq k_{ij} \leq 1, \end{cases} \]

and \( g(m) = m^{-d} \) for \( d > 0 \). The term \( w \) measures the standard deviation of the first own lag, and also indicates the overall tightness. A decrease in the value of \( w \) results in a tighter prior. The function \( g(m) \) measures the tightness of lag \( m \) relative to lag 1, and is assumed to have a harmonic shape with a decay of

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³ Formally, \( y_t = c + y_{t-1} + \epsilon_t \), with \( \epsilon_t \) being a white noise.

⁴ Note that we explicitly specify a vector of constants, \( c \), when setting up the VAR, unlike in Eqs. (1) and (2), where the constant is allowed for through the appropriate specification of the lag polynomial, \( \phi(L) \). This allows us to better explain the different prior structures imposed on the parameters associated with the lags of the variables and the constant, as is done below.

⁵ The prior outlined here is referred to as the “Minnesota prior” because it was developed at the University of Minnesota and the Federal Reserve Bank at Minneapolis.
An increase in \( d \) tightens the prior as the number of lags increases.\(^6\) \( f(i, j) \) represents the tightness of variable \( j \) in equation \( i \) relative to variable \( i \), and thus reducing the interaction parameter \( k_{ij} \) tightens the prior. \( \hat{\sigma}_i \) and \( \hat{\sigma}_j \) are the estimated standard errors of the univariate autoregression for variables, \( i \) and \( j \), respectively. In the case of \( i \neq j \), the standard deviations of the coefficients on the lags are not scale invariant (Litterman, 1986b, p. 30). The ratio \( \frac{\hat{\sigma}_i}{\hat{\sigma}_j} \) in Eq. (3) scales the variables so as to account for any differences in the units of magnitude of the variables. Note that the a priori assumptions on the coefficients \( A_1, \ldots, A_p \) are that they are independent and normally distributed. In addition, the covariance matrix of the residuals is diagonal and known. Finally, the prior on \( c \) is a diffuse one.

Finally, the motivation for using a NKDSGE model as a competing forecasting model for the FMs in addition to the VAR and the BVARs, emanates from a recent study by Liu et al. (2009). In this paper, the authors used a NKDSGE model\(^7\) along the lines of Ireland (2004), and forecasted the growth rate, inflation, and the 91-day Treasury bill rate for the South African economy over the period 2001Q1–2006Q4. The results indicated that, in terms of out-of-sample forecasting, the NKDSGE model outperformed both the unrestricted VAR and the BVARs for inflation, but not for the per capita growth or the nominal short-term interest rate. However, the differences in the RMSEs across models were not significant. Given that South Africa moved to an inflation targeting framework in February 2000, the ability of the NKDSGE model to outperform the VAR and the BVARs for forecasting inflation gives the model tremendous economic importance. We therefore decided to take the NKDSGE model used by Liu et al. (2009) as one of the alternative models to the large FM. However, we do use a different estimation period: Liu et al. (2009) used 1970Q1 to 2000Q4, while we started in 1980Q1, which is the same quarter as for the FM.

Formally, the NKDSGE model is described by the following eight equations\(^8\):

\[
\begin{align*}
\hat{x}_t &= E_t \hat{x}_{t+1} - \left( \hat{r}_t - E_t \hat{\pi}_{t+1} \right) \\
&+ \left( 1 - \frac{1}{\eta} \right) \left( 1 - \rho_d \right) \hat{a}_t \\
\hat{\pi}_t &= \beta E_t \hat{\pi}_{t+1} + \psi \hat{x}_t - \hat{\theta}_t / \phi, \quad \psi = \eta \left( \frac{\theta - 1}{\phi} \right) \\
\hat{r}_t &= \rho_r \hat{r}_{t-1} + \rho_{\pi} \hat{\pi}_t + \rho_g \hat{g}_t + \rho_x \hat{x}_t + \epsilon_{rt}, \\
\epsilon_{rt} &\sim iid(0, \sigma_r^2) \\
\hat{y}_t &= \hat{y}_{t-1} - \frac{1}{\eta} \hat{a}_t \\
\hat{g}_t &= \hat{y}_{t-1} + \hat{z}_t \\
\hat{a}_t &= \rho_a \hat{a}_{t-1} + \epsilon_{at}, \quad 0 \leq \rho_a < 1, \epsilon_{at} \sim iid(0, \sigma_a^2) \\
\hat{\theta}_t &= \rho_\theta \hat{\theta}_{t-1} + \epsilon_{\theta t}, \quad 0 \leq \rho_\theta < 1, \epsilon_{\theta t} \sim iid(0, \sigma_\theta^2) \\
\hat{z}_t &= \epsilon_{zt}, \quad \epsilon_{zt} \sim iid(0, \sigma_z^2).
\end{align*}
\]

Eqs. (4) and (5) model the expectational IS curve and the New Keynesian Phillips curve, respectively, while Eq. (6) presents the interest rate rule pursued by the monetary authority. Following Ireland (2004) and Liu et al. (2009), the terms of the NKDSGE model are defined as follows\(^9\): \( y_t \) measures the output; \( x_t \) is the output gap; \( r_t \) is the nominal short-term interest rate; \( \pi_t \) is the inflation rate; \( g_t \) is the output growth; \( \eta \) (\( \geq 1 \)) captures the degree of marginal disutility from labor; \( a_t \) is the preference shock; \( 0 < \beta < 1 \) is the discount factor; \( \theta_t \) is the cost-push shock; \( \psi \) governs the magnitude of the cost of price adjustment; \( \psi = \eta(\frac{\theta - 1}{\phi}) \); \( \epsilon_{rt} \) captures the monetary policy shock; \( \rho \) (\( \hat{\theta} = a, r, x, \pi, g, \theta \)) captures the persistence parameters; and \( z_t \) is the technology shock. Given this, Eqs. (7) and (8) define the stochastic process governing the deviations of the output-gap and growth rate, while Eqs. (9)–(11) outline the processes for

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\(^6\) In this paper, we set the overall tightness parameter \( w \) equal to 0.3, 0.2, and 0.1, and the harmonic lag decay parameter \( d \) equal to 0.5, 1 and 2. These parameter values are chosen to be consistent with those used by Liu et al. (2009) and Liu, Gupta, and Schaling (2010).

\(^7\) See also Liu and Gupta (2007) and Liu et al. (2010) for the usage of calibrated and estimated versions of the Hansen (1985) realistic business cycle model in forecasting the South African economy.

\(^8\) See Ireland (2004) and Liu et al. (2009) for details of the microfoundations of the model.

\(^9\) A hat above a letter indicates that it deviates from its steady state.
the preference, cost-push and technology shocks, respectively.

As far as estimation is concerned, the BVAR models are estimated using Theil’s (1971) mixed estimation technique, which involves supplementing the data with prior information on the distributions of the coefficients. For each restriction imposed on the parameter estimated, the number of observations and degrees of freedom is artificially increased by one. Therefore, the loss of degrees of freedom associated with the unrestricted VAR is not a concern in the BVAR. On the other hand, the NKDSGE model is in state space form and can be estimated via the maximum likelihood approach.\(^{10}\)

5. Results

In this section, we compare the one- to four-quarter-ahead RMSEs of the alternative models with those of the large FMs for the out-of-sample forecast horizon of 2001Q1 to 2006Q4. However, before we proceed to the forecasting results, a discussion as to the way in which the numbers of factors in the FMs were chosen is due. There are various statistical approaches for determining the number of factors in the FM. The two most commonly used in the factor models literature are those of Bai and Ng (2002) and Forni et al. (2001). The number of static factors \(r\) is determined using the Bai and Ng (2002) selection criteria, while we estimate the number of dynamic factors \(q\) using the method proposed by Forni et al. (2001).\(^{11}\) The second criterion suggests that the choice of \(q\) be based on the variance explained by the \(i\)th eigenvalue. Furthermore, there should be a substantial gap between the variances explained by the \(q\)th and \((q + 1)\)th eigenvalues. Forni et al. (2001) propose that factors be included as long as they explain at least a certain percentage of the total variance, such as 5%. As is indicated in Table 1, the approach of Bai and Ng (2002) proposes five static factors based on the \(IC_{p1}\) and \(IC_{p2}\) criteria, while the \(PC_{p1}\) and \(PC_{p2}\) criteria suggest seven factors. Following Bai and Ng (2002), we adopt the five factors based on \(IC_{p}\) over the \(PC_{p}\) criteria, since they are more desirable in practice and do not depend on the maximum number of factors included. Moreover, Bai and Ng (2007) also suggest five static factors and two primitive or dynamic factors. Similar to the latter method, the dynamic principal component technique, proposed by Forni et al. (2001), suggests two dynamic factors, as is indicated in the last column of Table 1. The first two dynamic principal components explain approximately 99% of the variation, while the eigenvalue of the third component is 0.005 < 0.05.\(^{12}\)

Given that we now know how we determine the number of factors, four points regarding the forecasting exercise must be emphasized: First, unlike the FMs, the small-scale VAR, the small-scale BVAR and the NKDSGE are estimated using data\(^{13}\) only

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10 For further details, please refer to Ireland (2004) and Liu et al. (2009, 2010). It must, however, be pointed out that the maximum likelihood technique developed by Ireland (2004) and used in this paper has two problems: first, the estimation is quite sensitive to the starting values, and second, due to the relatively large number of coefficients, convergence can often be a problem. Hence, one would ideally use the recently developed Bayesian Markov Chain Monte Carlo methods to estimate the DSGE models.

11 Hallin and Liska (2007) recently proposed a more robust method for selecting the number of dynamic factors when the latter are estimated by dynamic principal components.

12 A cumulated variance of 99% is rare in the factor analysis literature. However, a considerable number of studies in the literature have found 2 dynamic factors to be optimal (Forni et al., 2001; Forni & Reichlin, 1998; van Nieuwenhuyze, 2005).

13 Note that, to be consistent with the NKDSGE model, we estimate the VAR and the BVARs using demeaned data for the three variables of interest, and hence, stationarity is not an issue. This was also confirmed by the Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), Dickey-Fuller with GLS detrending (DF-GLS) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests of stationarity. In addition, given that the Bayesian approach is based entirely on the likelihood function, the associated inference does not account for nonstationarity specifically (Sims, Stock, & Watson, 1990).
the three variables of interest, with all of the data being obtained from the Quarterly Bulletins of the South African Reserve Bank (SARB), except for the population size, which is obtained from the World Development Indicators of the World Bank. Note that the RW model is estimated separately for each of the three variables. Second, even though the FMs incorporate global variables, given that the NKDSGE model is based on a closed economy, we, like Liu et al. (2009), use the percentage change in the GDP deflator as an appropriate measure of inflation rather than the CPI, simply to ensure the consistency of comparison between the alternative models. Third, the stable FAVAR and the BFAVARs were estimated with four lags, as determined by the unanimity of the sequential modified LR test statistic, the Final Prediction Error (FPE), the Akaike Information Criterion (AIC), the Schwarz Information Criterion (SIC), and the Hannan-Quinn Information Criterion (HQIC); while the M(B)FAVARs were estimated with 8 lags, as determined by the LR criterion, the FPE criterion, the AIC and the HQIC. Furthermore, the U(B)FAVARs for growth and inflation were also estimated with 8 lags, as decided by the LR criterion, the FPE criterion and the AIC for the former, and the LR criterion and the AIC by the latter. Finally, the U(B)FAVAR for the Treasury bill rate was based on one lag, as suggested by the SC and the HQIC.14

Fourth, the optimality of the BFAVARs are based on the minimum average RMSEs16 for the one- to four-quarter-ahead forecasts, produced by combining the values of the hyperparameters defining the overall weight (w) and tightness (d). The main results, as reported in Tables 2–4, can be summarized as follows:

- Per capita growth rate. The MBFAVAR \((w = 0.2, \ d = 1)\) outperforms all other models, producing the lowest minimum average RMSEs. The “optimal” MBFAVAR is followed by the FHLR, the “optimal” BVAR, the “optimal” UBFAVAR, the unrestricted VAR, the FAVAR, the RW model, the NKDSGE, and ultimately the MFAVAR.

- Inflation. As with the per capita growth rate, the MBFAVAR \((w = 0.2, \ d = 1)\) outperforms all of the other models, followed by the FHLR. Unlike in the case of the per capita growth rate, where the “optimal” BVAR scores well among small-scale models, the classical VAR is the best performer in this case, with the RW model ranked last overall.

- 91-day Treasury bill rate. Unlike the above cases, the FHLR stands out in forecasting the Treasury bill rate, relative to other alternative models. The second and third best performers are, respectively, the “optimal” UBFAVAR \((w = 0.1, \ d = 1)\) and the “optimal” MBFAVAR \((w = 0.1, \ d = 2)\). The small-scale “optimal” BVAR performs the best amongst the four small-scale models. Note that the RW model beats the VAR, BVAR, NKDSGE, UBFAVAR and MFAVAR models.

In order to evaluate the models’ forecast accuracies, we perform the across-model test of the “optimal” FMs against the RW model, the VAR, “optimal” BVARs and the NKDSGE model. The across-model

| Quarter Mean Squared Errors (in %), 2001Q1–2006Q4: per capita growth. |
|-----------------|---------|---------|---------|---------|---------|
| RW              | 0.995   | 0.998   | 1.019   | 1.046   | 1.015   |
| VAR (4)         | 0.479   | 0.541   | 0.619   | 0.735   | 0.593   |
| BVAR \((w = 0.1, \ d = 2.0)\) | 0.385   | 0.403   | 0.495   | 0.641   | 0.481   |
| NKDSGE          | 0.9894  | 1.1259  | 1.7950  | 1.2081  | 1.2796  |
| FHLR            | 0.489   | 0.431   | 0.438   | 0.443   | 0.450   |
| UFAVAR          | 1.077   | 1.099   | 1.151   | 1.122   | 1.112   |
| MFAVAR          | 1.325   | 1.295   | 1.400   | 1.375   | 1.349   |
| UBFVAR          | 0.547   | 0.517   | 0.507   | 0.508   | 0.520   |
| MBFAVAR \((w = 0.2, \ d = 1)\) | 0.467   | 0.416   | 0.401   | 0.432   | 0.429   |

14 Stability was ensured because no roots were found to lie outside the unit circle.

15 Note that we required at least two of the lag-selection tests to agree on a particular lag-length, in order to decide on the “optimal” lag length.

16 Zellner (1986, p. 494) pointed out that “the optimal Bayesian forecasts will differ depending upon the loss function employed and the form of predictive probability density function”. In other words, Bayesian forecasts are sensitive to the choice of the measure used to evaluate the out-of-sample forecast errors. However, Zellner (1986) also indicated that the use of the mean of the predictive probability density function for a series is optimal relative to both a squared error loss function and the Mean Squared Error (MSE), and hence, the RMSE is an appropriate measure for evaluating the performances of forecasts, when the mean of the predictive probability density function is used. Thus, the paper uses RMSEs to evaluate the out-of-sample forecasting performances of alternative models.
test is based on the statistic proposed by Diebold and Mariano (1995), which is defined as follows. Let \( \{e'_t\}_{t=1}^{T} \) (with \( i = \text{RW, VAR, BVARs, NKDSGE} \)) denote the associated forecast errors from the alternative models and let \( \{e^{d}_t\}_{t=1}^{T} \) denote the forecast errors from the alternative forms of the FM. The test statistic is then defined as \( s = \frac{l}{\sigma_l} \), where \( l \) is the sample mean of the “loss differentials”, with \( \{l_t\}_{t=1}^{T} \) obtained by using \( l_t = (e'_t)^2 - (e^{d}_t)^2 \) for all \( t = 1, 2, 3, \ldots, T \), and where \( \sigma_l \) is the standard error of \( l \). The \( s \) statistic is asymptotically distributed as a standard normal random variable and can be estimated under the null hypothesis of equal forecast accuracy, i.e. \( l = 0 \). Thus, in this case, a positive value of \( s \) would suggest that the particular form of a FM outperforms the specific alternative model the comparison is made against in terms of out-of-sample forecasting. The results are reported in Table 5.

In general, and at the 10% level of significance at least, the “optimal” FMs tend to perform better than the alternative models in predicting the three variables in which we are interested, for each of the one- to four-step-ahead forecasts. In other words, Mariano and Diebold – based on the average RMSEs, and, more importantly, on the Diebold and Mariano (1995) test statistic, we have relatively strong evidence that there is a significant statistical gain from using the “optimal” FMs over other atheoretical and theoretical alternatives in forecasting key macroeconomic variables in South Africa for the majority of the one- to four-quarter-ahead forecast horizons.

From a purely economic point of view, this paper, in conjunction with the work of Kabundi (2009), highlights the fact that not only can the FM be used to successfully analyze the degree of synchronization of South Africa with the US, but also the framework has tremendous potential for use as a forecasting tool relative to small-scale models, given its ability to handle large amounts of data on the wide range of variables that tend to affect a small open developing economy like that of South Africa, and specifically in our context the three key macroeconomic variables, namely the per capita growth rate, inflation, and the short-term interest rate. The fact that there exists a FM that tends to outperform both the naïve RW model and small-scale models that only account for the role of a particular variable or the interaction amongst the variables of interest, clearly highlights the possible model misspecification, in the sense that the latter set of models fail to utilize the effect of a large number of other variables which are not used in their estimation. Given that South Africa has targeted inflation since February 2000, having a model that can forecast inflation and the instrument used to achieve it is of paramount importance. Moreover, more accurate forecasts of the nominal interest rate and inflation implicitly imply better forecasts of the real interest rate — a key monetary policy variable. It is important to point out that the South African government has well-defined growth targets based on the Accelerated and Shared Growth Initiative for

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**Table 5**

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>RW</td>
<td>3.828</td>
<td>5.408</td>
<td>6.265</td>
<td>6.670</td>
<td>5.543</td>
</tr>
<tr>
<td>VAR (4)</td>
<td>0.262</td>
<td>0.313</td>
<td>0.370</td>
<td>0.434</td>
<td>0.345</td>
</tr>
<tr>
<td>BVAR</td>
<td>0.261</td>
<td>0.311</td>
<td>0.375</td>
<td>0.465</td>
<td>0.353</td>
</tr>
<tr>
<td>(w = 0.1, d = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NKDSGE</td>
<td>0.297</td>
<td>0.421</td>
<td>0.577</td>
<td>0.695</td>
<td>0.498</td>
</tr>
<tr>
<td>FHLR</td>
<td>0.303</td>
<td>0.269</td>
<td>0.275</td>
<td>0.272</td>
<td>0.280</td>
</tr>
<tr>
<td>UFAVAR</td>
<td>0.812</td>
<td>0.931</td>
<td>0.836</td>
<td>0.809</td>
<td>0.847</td>
</tr>
<tr>
<td>MFAVAR</td>
<td>1.077</td>
<td>1.291</td>
<td>1.329</td>
<td>1.417</td>
<td>1.278</td>
</tr>
<tr>
<td>UBFAVAR</td>
<td>0.277</td>
<td>0.307</td>
<td>0.297</td>
<td>0.289</td>
<td>0.293</td>
</tr>
<tr>
<td>(w = 0.2, d = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MBFAVAR</td>
<td>0.264</td>
<td>0.287</td>
<td>0.266</td>
<td>0.257</td>
<td><strong>0.268</strong></td>
</tr>
<tr>
<td>(w = 0.2, d = 1)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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**Table 4**

<table>
<thead>
<tr>
<th>Quarter ahead</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>RW</td>
<td>0.935</td>
<td>1.683</td>
<td>2.274</td>
<td>2.746</td>
<td>1.909</td>
</tr>
<tr>
<td>VAR (4)</td>
<td>0.921</td>
<td>1.769</td>
<td>2.505</td>
<td>3.113</td>
<td>2.077</td>
</tr>
<tr>
<td>BVAR</td>
<td>0.918</td>
<td>1.699</td>
<td>2.343</td>
<td>2.865</td>
<td>1.956</td>
</tr>
<tr>
<td>(w = 0.2, d = 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NKDSGE</td>
<td>1.130</td>
<td>1.979</td>
<td>2.622</td>
<td>3.991</td>
<td>2.430</td>
</tr>
<tr>
<td>FHLR</td>
<td>1.691</td>
<td>1.205</td>
<td>1.160</td>
<td>1.035</td>
<td><strong>1.273</strong></td>
</tr>
<tr>
<td>UFAVAR</td>
<td>1.320</td>
<td>1.925</td>
<td>2.503</td>
<td>2.837</td>
<td>2.146</td>
</tr>
<tr>
<td>MFAVAR</td>
<td>3.257</td>
<td>4.864</td>
<td>6.286</td>
<td>7.996</td>
<td>5.601</td>
</tr>
<tr>
<td>UBFAVAR</td>
<td>1.161</td>
<td>1.587</td>
<td>2.008</td>
<td>2.344</td>
<td>1.775</td>
</tr>
<tr>
<td>(w = 0.1, d = 2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MBFAVAR</td>
<td>1.142</td>
<td>1.581</td>
<td>2.023</td>
<td>2.366</td>
<td>1.778</td>
</tr>
</tbody>
</table>

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The exception to this is the one-quarter-ahead forecast of the short-term interest rate, relative to the RW model, the unrestricted VAR and the “optimal” BVAR.

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South Africa (ASGISA) program because the per capita growth rate is, perhaps, one of the simplest measures of the economic performance of a country. Besides this, as was suggested by Naraidoo and Gupta (in press), there is strong evidence in South Africa of the role played by the per-capita growth rate in the interest rate rule. Given this, the fact that FMs are better-suited for forecasting the per capita growth rate, over and above the interest rate and inflation, than the alternative models, makes them an attractive forecasting tool.

At this stage, it may be worth providing some of the possible economic explanations for the results obtained. First, given that the FMs, whether the MBFAVAR ($w = 0.1, d = 2$) or the FHLR, are the best performing models, the results clearly indicate the importance of the information contained in the factors, which, in turn, are derived from 267 quarterly series. Second, the fact that the MBFAVAR with the tightest priors tends to perform better than all of the other models for growth and inflation is an indication of the role that persistence plays in determining the future paths of these variables. On the other hand, with the FHLR, which exploits information relating to the leads and lags of variables, being the stand-out performer for forecasting the short-term interest rate, the role of past information as well as expected values of the factors cannot be denied. This is not surprising, especially given that South Africa has targeted inflation since February 2000, and hence, one would expect the short-run interest rate to be determined by both past and future values of important factors which explain most of the variation in the economy.

### 6. Conclusions

This paper assesses the forecasting performances of large-scale FMs, accommodating 267 quarterly series for South Africa, relative to those of the RW model, the VAR, BVARs and a typical NKDSGE model. The model extracts five static factors and two dynamic factors that explain most of the variation in the entire panel. These factors are then used to forecast the output growth, inflation and the nominal interest rate, based on large FMs estimated under both classical and Bayesian assumptions over the period 1980Q1–2000Q4.

The alternative models are evaluated based on the minimum average RMSEs for the one- to four-quarter-ahead forecasts over an out-of-sample horizon of 2001Q1–2006Q4. Overall, the results show that there exits a specific form of a FM, whether based on Bayesian assumptions or incorporating both static and dynamic factors, which tends to outperform all other

---

**Table 5**

Across-model test statistics.

<table>
<thead>
<tr>
<th>Quarters ahead</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Per capita growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHLR vs. RW</td>
<td>1.9511**</td>
<td>1.9532**</td>
<td>2.1231**</td>
<td>2.1561**</td>
</tr>
<tr>
<td>MBFAVAR vs. VAR</td>
<td>−1.4474</td>
<td>1.6752*</td>
<td>1.9752**</td>
<td>2.4474**</td>
</tr>
<tr>
<td>MBFAVAR vs. BVAR</td>
<td>−1.5995</td>
<td>−1.4465*</td>
<td>1.6917*</td>
<td>1.9251*</td>
</tr>
<tr>
<td>MBFAVAR vs. NKDSGE</td>
<td>1.9388*</td>
<td>1.7348*</td>
<td>3.3206***</td>
<td>2.5360**</td>
</tr>
<tr>
<td>(B) Inflation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHLR vs. RW</td>
<td>3.1193***</td>
<td>3.2515***</td>
<td>3.7511***</td>
<td>3.9919***</td>
</tr>
<tr>
<td>MBFAVAR vs. VAR</td>
<td>−1.4372</td>
<td>1.5061</td>
<td>1.6567*</td>
<td>1.7021*</td>
</tr>
<tr>
<td>MBFAVAR vs. BVAR</td>
<td>−1.4375</td>
<td>1.4969</td>
<td>1.6597*</td>
<td>1.7177*</td>
</tr>
<tr>
<td>MBFAVAR vs. NKDSGE</td>
<td>1.3574</td>
<td>1.5564</td>
<td>1.7487*</td>
<td>1.8448*</td>
</tr>
<tr>
<td>(C) Treasury bill</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FHLR vs. RW</td>
<td>−1.6512*</td>
<td>1.6501*</td>
<td>2.1552**</td>
<td>2.8061***</td>
</tr>
<tr>
<td>FHLR vs. VAR</td>
<td>−1.6721*</td>
<td>1.6866*</td>
<td>2.2401**</td>
<td>2.9112***</td>
</tr>
<tr>
<td>FHLR vs. BVAR</td>
<td>−1.6813*</td>
<td>1.6504*</td>
<td>2.2151**</td>
<td>2.8568***</td>
</tr>
<tr>
<td>FHLR vs. NKDSGE</td>
<td>−1.6068</td>
<td>1.6774*</td>
<td>2.4960**</td>
<td>2.9322***</td>
</tr>
</tbody>
</table>

* Significance at the 10% level.
** Significance at the 5% level.
*** Significance at the 1% level.
models in forecasting the three variables of interest, indicating the blessing of dimensionality.

At this stage, it is important to stress the following facts, and in the process identify future areas for research. (i) Practically speaking, a central bank, or for that matter any forecaster, would ideally want to include a large number of variables in the forecasting model, in order to obtain forecasts of the variables of key interest. In this regard, the VAR estimation is disadvantaged due to the curse of dimensionality. However, the BVAR can be considered a valid alternative to FM, as it is equally able to accommodate large numbers of variables, given that its estimation is based on Theil’s (1971) mixed estimation technique, which amounts to supplementing the data with prior information on the distribution of the coefficients, and therefore the loss of degrees of freedom associated with the unrestricted VAR is no longer a concern. (ii) In addition, due to fact that the problem associated with the degrees of freedom is no longer an issue for the FM and the BVAR, these models are also capable of forecasting a large number of time series simultaneously, beyond the possible key variables of interest. (iii) However, there are limitations to the use of the Bayesian approach. Firstly, the forecast accuracy depends critically on the specification of the prior, and secondly, the selection of the prior based on some objective function for the out-of-sample forecasts may not be “optimal” for the time period, beyond the period chosen for producing the out-of-sample forecasts. (iv) Finally, general to any traditional statistically estimated models, for example the large FM, the RW model, the VAR, and BV ARs used for forecasting at the business cycle frequencies, there are a couple of other concerns. Such procedures perform reasonably well as long as no structural changes are experienced in the economy, but changes of this nature, whether in or out of the sample, would then render the models inappropriate. Alternatively, these models are not immune to the ‘Lucas Critique’. Furthermore, the estimation procedures used here are linear in nature, and hence, they fail to take into account any nonlinearity in the data. In this regard, the role of microfounded DSGE models cannot be disregarded. The fact that the NKDSGE, based on the sample period used, is outperformed by all of the other models, calls mainly for a better DSGE model of the South African economy, by extending the current model to incorporate facts such as habit persistence and wage rigidities, the role of capital in the production process, and, perhaps more importantly, the role of external shocks, given South Africa’s small open economy structure.

However, as we show in this paper, whatever the limitations of the large FM, one cannot gainsay the importance of this kind of modeling strategy in forecasting three key variables, namely the per capita growth rate, inflation and the short-term interest rate for South Africa over the period 2001Q1–2006Q4. Clearly, the FM tends to perform well, relative to alternative popular forecasting methods, in predicting the South African economy.

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References


