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Optimal hoist scheduling of a deep level mine twin rock winder system for demand side management

Werner Badenhorst *, Jiangfeng Zhang, Xiaohua Xia

Centre of New Energy Systems, Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretora 0002, South Africa

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ABSTRACT

This paper presents a near optimal hoist scheduling and control program for rock winders found in South African deep level mines in the context of demand side management and time-of-use (TOU) tariffs. The objective is to achieve a set hoist target at minimum energy cost within various system constraints. The development of a discrete dynamic and constrained mixed integer linear programming model for a twin rock winder system is presented on which a half-hourly model predictive control (MPC) algorithm containing an adapted branch and bound methodology is applied for near optimal scheduling. Simulation results illustrate the effectiveness of the control program by minimising the energy costs through scheduling according to the TOU tariff and controlling output and ore levels within their boundaries even in the case of significant random delays in the system. Scheduling according to the TOU tariff shows a possible 30.8% reduction in energy cost while approximately 6 h of delays in the system resulted in a mere 14% increase in energy cost.

1. Introduction

In order to meet the projected global electricity demand increase of 45% between 2006 and 2030,[1] supply capacity needs to be increased and the rate of demand increase needs to be reduced through the effective demand side management (DSM) of the electricity market. The need for additional future generation capacity can however be reduced through a reduction of demand [1,2]. In light of the aforementioned Africa's largest electricity supplier, Eskom, has made DSM a priority with respect to the efficient use of electricity and reducing peak demand by 3000 MW between April 2007 and April 2011 and a further 5000 MW by March 2026.[2]

Industry in South Africa consumes approximately 65% of all electrical energy of which the mining sector is the largest representing approximately 24% of industry consumption and 16% of total electrical energy consumption in South Africa.[3] DSM projects that have successfully been implemented in deep level gold mines are primarily on the underground pumping, cooling and lighting systems [3]. Another potential system identified for DSM in deep level mines

is the rock winders, which are responsible for hoisting ore and rock from underground to the surface where the gold is then extracted.

The primary objective of the problem presented in this paper is the development of a near optimal half hourly hoist control scheduling program for a deep level mine twin rock winder system in order to achieve a set hoist target at minimum energy cost based on a time-of-use (TOU) tariff while operating within various physical and operational constraints.

The paper first presents a brief summary in Section 2 of relevant literature applied in achieving the primary objective. Section 3 formulates the problem through the development of a physically based, discrete dynamic and constrained mixed integer linear programming (MILP) model of the rock winder system. Section 4 presents a model predictive control (MPC) framework with a receding horizon method ensuring that future scheduled hoists are updated in the event of delays and other system changes affecting system constraints and conditions. Section 5 describes the closed loop MPC algorithm in which measured system states and delays are used as feedback every 30 min. The algorithm also incorporates an adapted branch and bound methodology to obtain a near optimal mixed integer hoist schedule solution. Finally in Section 6 the program simulates and plots hoist schedules for the winders indicating the number of hoists and predicted system state levels for each half hour period over a set number of periods. The effectiveness of the MPC algorithm is illustrated by comparing the impact of using a TOU versus flat rate tariff as well as the inclusion delays into the winder system. The results presented focuses on the near optimality of the integer solution and the ability of the MPC algorithm

* Corresponding author. Tel.: +27 12 420 2587; fax: +27 12 362 5000.
  E-mail address: werner.badenhorst@eng.up.ac.za (W. Badenhorst).

to absorb the primary dynamic factor in the system, namely the uncertainty of duration and time of occurrence of system delays.

2. Literature research

The first significant contribution towards DSM in deep level mines was made in [4] in which an integrated electricity end-use planning methodology in deep level mines is proposed. It is shown in [4] that rock winders represent approximately 15% of the energy consumption on a deep level goldmine making this focussed study very relevant and applicable within large mining industry in South Africa.

The modelling of the rock winder system in this paper is primarily based on the physically based models such as presented in [5] for scheduling load for a flour mill to minimise electricity costs; providing an optimal load management strategy for an air conditioning plant utilising load shifting [6]; for peak-load management in steel plants [7] and for an optimal control model for load shifting of a colliery conveyer transport system [8].

Closed loop control is achieved through the application of MPC. MPC's ability to handle constraints and simple models along with its robustness and closed-loop stability has made MPC one of the most widely used multivariable control algorithms in many industry applications [9–11] including power systems. Amongst other MPC is applied in [12] for load shifting of a water pumping scheme using binary integer programming optimisation taking into account both TOU and maximum demand charges. Another example includes an MPC approach to the dynamic economic dispatch problem of generators with ramp rate constraints in [13]. A number of applications combine MPC with minimisation or optimisation problems of which a few include variable-air-volume boxes [14], fluidised furnace reactors [15], boiler start-ups [16] and in the design and operation of distributed energy resources [17].

In contrast to existing studies, the rock winder problem presented in this paper applies the above mentioned techniques in a mining environment with the added requirement of an integer solution to the optimised scheduling problem. The formulation of the rock winder problem into a constrained integer linear programming model along with a solution methodology using an adapted Branch and Bound methodology without feedback control was first introduced in [18]. The reformulation of the problem in [18] for application in a MPC algorithm has been presented in [19]. These two papers provide preliminary results for the complete and concise formulation of the study presented in this paper in which the adapted Branch and Bound methodology is incorporated within a MPC algorithm to obtain a near optimal hoist schedule for a twin rock winder system.

3. Problem formulation

3.1. Nomenclature

For better comprehension of the problem formulation to follow a nomenclature is provided first.

C cost vector for model horizon N
c_k energy cost per kWh during period k
D the number of days within the control horizon H
E_x energy consumption per hoist for winder x
ff_x friction factor accounting for the friction load for winder x such that $0 \leq ff < 0.3$
g gravitational acceleration as 9.81 m/s^2
H the control horizon and the number of periods therein calculated as $H = 48D$

h_x vertical winding depth or hoist height for winder x in meters
J energy cost and linear objective function
M_{blast} one day's target production or ore bearing rock in tons to be blasted in the reefs
M_{min} daily minimum required tons to be hoisted to surface
m_1^k, m_2^k tons stored at the start of period k in respectively the change-over and orepass system.
m_{in}^k feed in rate of tons of rock from the reefs into the orepass system during period k.
N model horizon
η_x winder efficiency of winder x measured as the ratio of shaft output power required over total electrical input power required
P prediction horizon
R_x set skip payload per hoist for winder x in tons
T_{mx} number of half hourly periods to complete the maintenance or test period of winder x
T_{sx} starting time at either top or bottom of the hour for the maintenance or test period of winder x
u vector containing the number of scheduled hoists for all rock winders for each period in H
u_x^k number of hoists for winder x during period k.

3.2. Ore transport and rock winder system overview

The schematic layout in Fig. 1 is of a typical deep level ore transport system of which the most critical component is the rock winder. The blasted rock from the reefs is transported to the shaft area via orepasses, crushers and conveyor belts feeding flasks that weigh off a set payload to be loaded into the skips in which the rock is hoisted to surface by the rock winder.[4] The ore is then stored on surface in stockpiles or silos awaiting transport to the gold plant where the gold is extracted from the ore-bearing rock.

The twin rock winder system used in this study consists of an underground and a surface winder as indicated in the diagram of Fig. 2. Each day's blasted rock is transported from the reefs and stored in the underground orepass system. From the orepass system the rock is conveyed into a flask where the rock is weighed to R_g = 13.5 tons, before being emptied into the underground winder's skip. The loaded skip is then hoisted by the underground winder in the sub-shaft and emptied into a change-over. From the change-over the rock is conveyed into a flask where the rock is now weighed to R_s = 23.5 tons, before being emptied into the surface winder's skip. The loaded skip is then hoisted to surface where it is in turn emptied onto a conveyance transporting the rock to a surface stockpile with a capacity that will for the purposes of this study be assumed to be infinite.

There are two primary uncertainties or dynamic factors within the rock winder system. The first and least significant is the feed-in rate of ore from the reefs into the orepass system that differs from day to day. The feed-in rate can however be averaged over 1 h periods within a 24 h period based on historical records and the 24 h cyclic operation schedule of a mine with a high level of certainty. The second factor having by far the biggest impact on the operational hours of the rock winder system is that of unscheduled or unplanned system delays. This is because of the uncertainty in duration and time of occurrences of these delays which can occur at any time and averaged at 152 and 206 min per day for the surface and underground winder respectively during this study. If the MPC algorithm can therefore absorb the effects of such delays, the

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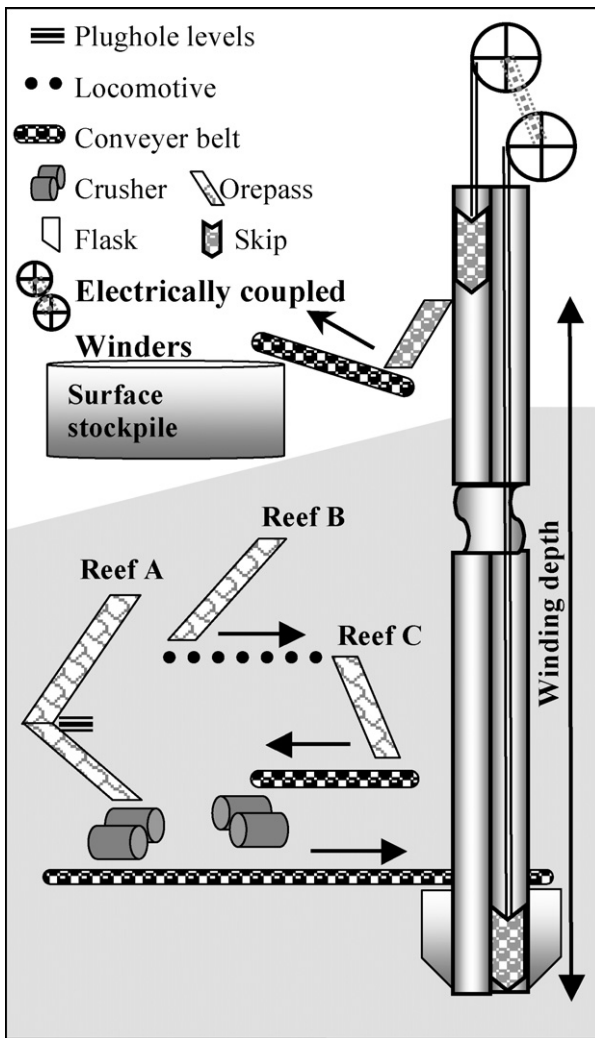[4] AngloGold Ashanti, AngloGold Ashanti Virtual Mine Tour, <http://www.anglogold.co.za>.

Fig. 1. Schematic layout of a typical deep level ore transport system.

Fig. 2. Process flow diagram of a twin rock winder system.

algorithm will easily be able to absorb other minor disturbances within the system.

3.3. Objective function

As the primary objective is to minimise the energy cost of the winders, the objective function will take the form of an energy cost function similar to that developed in [5–8]. The first part of the objective function entails the energy consumption per hoist calculated using Eq. (1):

$$E_x = \frac{(1 + ff_x)R_x \times g \times h_x}{\eta_x \times 3600} [\text{kWh}] \qquad (1)$$

The payload, hoist height and efficiency of each winder can safely be assumed to be constant meaning that the energy consumption per hoist can also be assumed constant. Therefore the energy consumption of a winder during period k can be calculated as the product of the number of hoists during that period and the energy consumption per hoist. For the surface winder this energy consumption during period k can be written as $u_s^k E_s$ and for the underground winder $u_g^k E_g$.

In accounting for the cost component of the objective function it was noted that the mine operated on a time-of-use (TOU) tariff
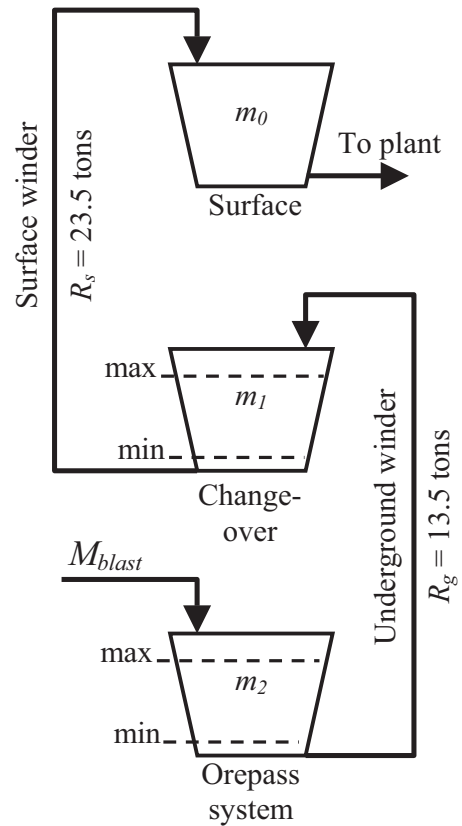
package known as Megaflex[5] of which the time intervals and energy costs are stated in Table 1. Although the shortest time interval in Table 1 is 1 h, the demand costs are calculated over half hourly integration periods. Therefore the minimisation of the energy cost through near optimal hoist scheduling will also be done in half hourly periods thereby inherently minimising the network demand charge applicable during standard and peak periods. A cost vector is hence defined as $C = [c_0 \ c_1 \ c_2 \ \dots \ c_{H-1}]^T$ over a control horizon H such that the elements in C correspond to the values in Table 1 as indicated in Table 2 for the case of $D = 1$ day or $H = 48$.

The energy cost in period k for the surface and underground winder therefore equates to $c_k u_s^k E_s$ and $c_k u_g^k E_g$ respectively. The summation of these two terms over the whole of H results in the energy cost and linear objective function in (2) that is to be minimised:

$$\min J = \min \left[\sum_{k=0}^{H-1} c_k(u_s^k E_s + u_g^k E_g) \right] \qquad (2)$$

3.4. System constraints

The objective function in (2) is subject to four constraints similar to the storage, production and process constraints found in [5–8]. The first constraint puts a limitation on the number of hoists that can be achieved by each winder during a 30-min period. The hoist constraint is applicable to each period k over H and can be stated as $0 \le u_s^k \le 11$ for the surface winder and $0 \le u_g^k \le 17$ for the underground winder.

Table 1
Megaflex energy costs in c/kWh for 2008.

Season/period	Peak c_p	Standard c_s	Off-peak c_o
Jan–MaySept–Dec	20.52	12.77	9.10
Jun–Aug	72.05	19.04	10.38
Time of day: $t \in$	$[7, 10) \cup [18, 20)$	$[6, 7) \cup [10, 18) \cup [20, 22)$	$[0, 6) \cup [22, 24)$

Table 2
Assigned values for the discrete cost function C.

$t \in$	[0,6)	[6,7)	[7,10)	[10,18)	[18,20)	[20,22)	[22,24)
c_k	$c_0, \ldots, c_{11} = c_o$	$c_{12}, c_{13} = c_s$	$c_{14}, \ldots, c_{19} = c_p$	$c_{20}, \ldots, c_{35} = c_s$	$c_{36}, \ldots, c_{39} = c_p$	$c_{40}, \ldots, c_{43} = c_s$	$c_{44}, \ldots, c_{47} = c_o$

The second constraint is with respect to the upper (maximum) and lower (minimum) boundaries for the change-over and orepass system ore levels as indicated in Fig. 2. The level inequality constraints can be formulated as $m_{1\min} \leq m_1^k \leq m_{1\max}$ and $m_{2\min} \leq m_2^k \leq m_{2\max}$.

A third constraint puts a lower boundary on the tons to be hoisted over the control horizon and is formulated in (3) as a horizon target constraint.

$$\sum_{k=0}^{48D-1} u_s^k R_s \geq D \times M_{\min} \quad \text{for } k = 0, 1, 2, \ldots, H-1 \quad (3)$$

Finally the fourth constraint takes into account mandatory routine daily winder maintenance or tests to be carried out on all winders during which time no hoisting of rock is allowed. For the surface winder it can be formulated as $u_s^{48d+j} = 0$ for $j = T_{ss}, T_{ss} + 1, \ldots, T_{ss} + T_{ms} - 1$ and $d = 0, 1, \ldots, D$. An equivalent formulation can be written for the underground winder in terms of u_g, T_{sg} and T_{mg}.

3.5. Discrete dynamic equation

Underlying the winder system model is a discrete time dynamic system based on the basic dynamic programming model defined in [20]. This dynamic model is applied to the winder system's two state variables, m_1 and m_2, of which the result is formulated in the two discrete dynamic equations in (4).

$$\begin{aligned} m_1^{k+1} &= m_1^k + R_g u_g^k - R_s u_s^k \\ m_2^{k+1} &= m_2^k - R_g u_g^k + m_{in}^k \end{aligned} \quad \text{for } k = 0, 1, 2, \ldots, H-1 \quad (4)$$

4. Linear integer MPC formulation

The above minimisation problem can be solved using linear programming after formulating the problem as a linear programming model (LPM). This formulation will require the objective function, cost vector and constraints to be explicitly stated at the beginning of each sampling instant k within H. As presented in [8] the problem can be written in the form of formulations (5a) through (5d).

$$\min_u f^T \cdot u \quad (5a)$$

$$lb \leq u \leq ub \quad (5b)$$

$$A \cdot u \leq b \quad (5c)$$

$$Aeq \cdot u = beq \quad (5d)$$

The product of the coefficient vector f and the optimal real solution vector u forms the objective energy cost function in (2) that is to be minimised. The two vectors lb and ub respectively represents the lower and upper hoist constraint boundaries for $u \cdot A$ and Aeq are matrices representing respectively the inequality and equality constraint coefficients along with vectors b and beq containing real values thus forming the ore level and target constraints.

The linear programming formulation of (5) is combined with the MPC methodology presented in [21,22] referred to as dynamic matrix control. The objective of the MPC control calculations over H is to determine a sequence of manipulated inputs u, $\{u(k+j-1), j = 1, 2, \ldots, H\}$, such that a set of predicted outputs over P, $\{\hat{y}(k+j), j = 1, 2, \ldots, P\}$, reaches a target in an optimal manner [22]. The nature of this particular problem requires P to equal H. Control calculations are based on multiple j-step ahead predictions of future outputs $\hat{y}(k+j)$, current measurements including actual outputs y, and on optimizing the objective function within a constant model horizon of $2H \leq N \leq 3H$. The model horizon includes the effect of past, current and future control and uncontrolled actions [22]. Defining the problem in terms of the MPC methodology in [21,22] results in the following reformulations.

The dynamic state formulation in (5) is reformulated in (6):

$$\begin{aligned} \hat{m}_1^{k+j} &= m_1^k + \sum_{i=1}^{j}(R_g u_g^{k+j-i} - R_s u_s^{k+j-i}) \\ \hat{m}_2^{k+j} &= m_2^k - \sum_{i=1}^{j} R_g u_g^{k+j-i} + \sum_{i=1}^{j} m_{in}^{k+j-i} \end{aligned} \quad \text{for } j = 1, 2, 3, \ldots, P \quad (6)$$

The objective function in (2) separated into the vectors f and u in (5a) is formulated in Eqs. (7) and (8) respectively:

$$f^T = \begin{bmatrix} c_{k'+j-1}E_s & c_{k'+j-1}E_g & c_{k'+j}E_s & c_{k'+j}E_g \\ c_{k'+j+1}E_s & c_{k'+j+1}E_g \ldots & c_{k'+j+H-1}E_s & c_{k'+j+H-1}E_g \end{bmatrix}$$
$$\text{for } j = 1, 2, 3, \ldots, P \quad (7)$$

$$\begin{aligned} u &= [\, u_1 \quad u_2 \quad u_3 \quad u_4 \quad u_5 \quad u_5 \quad \cdots \quad u_{(2H-1)} \quad u_{2H}\,] \\ &= [\, u_s^k \quad u_g^k \quad u_s^{k+1} \quad u_g^{k+1} \quad u_s^{k+2} \quad u_g^{k+2} \quad u_s^{k+H-1} \quad u_g^{k+H-1}\,] \end{aligned} \quad (8)$$

where $k' = k - 48 \left\lfloor \frac{k}{48} \right\rfloor = k - 48k_{tr}$ for $k = 0, 1, 2, \ldots$ and $\lfloor \rfloor$ denotes rounding down to the nearest integer.

Eq. (7) requires the cost vector C to be reformulated to allow N to move the required $(H-1)$ elements into the future. The reformulation of C is given in (9) based on the values and notation of Tables 1 and 2.

$$\begin{aligned} C &= [\, c_1' \quad c_2' \quad c_3' \quad \cdots \quad c_{2D}'\,]^T \\ c_n' &= [\, c_0 \quad c_1 \quad c_2 \quad \cdots \quad c_{47}\,]^T \end{aligned} \quad (9)$$

The first constraint, the hoist limits, can be written in the form of inequality (5b) such that ub is defined as in (10) and that lb contains $2H$ zero elements.

$$\begin{aligned} ub &= [\, ub_1 \quad ub_2 \quad ub_3 \quad ub_4 \quad \cdots \quad ub_{(2H-1)} \quad ub_{2H}\,]^T \\ &= [\, ub_s \quad ub_g \quad ub_s \quad ub_g \quad \cdots \quad ub_s \quad ub_g\,]^T \\ &= [\, 11 \quad 17 \quad 11 \quad 17 \quad \cdots \quad 11 \quad 17\,]^T \end{aligned} \quad (10)$$

The second constraint, the level inequality boundaries, can be formulated as in inequality (5c), which when written in the form

presented in [22] results in the four inequalities given in (11) for the change over and orepass system levels.

$$\sum_{i=1}^{j}(R_g u_g^{k+j-i} - R_s u_s^{k+j-i}) \le (m_{1\,\text{max}} - m_1^k)$$

$$\sum_{i=1}^{j}(-R_g u_g^{k+j-i} + R_s u_s^{k+j-i}) \le (-m_{1\,\text{min}} + m_1^k)$$

$$\sum_{i=1}^{j}(-R_g u_g^{k+j-i}) \le \left(m_{2\,\text{max}} - m_2^k - \sum_{i=1}^{j}m_{in}^{k+j-i} \right)$$

$$\sum_{i=1}^{j}R_g u_g^{k+j-i} \le \left(-m_{2\,\text{min}} + m_2^k + \sum_{i=1}^{j}m_{in}^{k+j-i} \right)$$

$$\text{for } j = 1, 2, 3, \ldots, P \tag{11}$$

Writing the horizon target constraint in inequality (3) in the form presented in [22] over a model horizon of N periods results in:

$$-\sum_{i=1}^{j}R_s u_s^{k+j-i} \le \left(\sum_{i=j+1}^{N}R_s u_s^{k+j-i} - \frac{N}{48}M_{\text{min}} \right)$$

$$\text{for } j = 1, 2, 3, \ldots, P \tag{12}$$

The formulation in (12) can be read as follows: "The tons to be hoisted during the next j period(s) must be equal to or greater than the difference between the actual tons already hoisted to surface during the past $(N–j)$ periods and the product between the number of days within N, $(N/48)$, and M_{min}." From the summation on the right of the inequality in (12) it is clear that $N–1$ known past control actions are required. For simulation purposes (12) has to be implemented in phases until $k \ge N - 1$ in order to create this history of known past control actions.

Finally the mandatory maintenance equality constraints for the underground winder can be written in the form of (5d) in (13a) through (13c) as follows:

$$\sum_{d=0}^{D-1}\left(\sum_{i=T_{sg}}^{T_{sg}+T_{mg}-1} u_g^{i+48(d+k_{tr})} \right) = 0 \quad \text{for } 0 \le k' < T_{sg} \tag{13a}$$

$$\sum_{d=0}^{D-1}\left(\sum_{i=k'}^{T_{sg}+T_{mg}-1} u_g^{i+48(d+k_{tr})} + \sum_{i=T_{sg}}^{k'-1} u_g^{i+48(d+k_{tr}+1)} \right)$$

$$= 0 \quad \text{for } T_{sg} \le k' < T_{sg} + T_{mg} \tag{13b}$$

$$\sum_{d=0}^{D-1}\left(\sum_{i=T_{sg}}^{T_{sg}+T_{mg}-1} u_g^{i+48(d+k_{tr}+1)} \right) = 0 \quad \text{for } T_{sg} + T_{mg} \le k < 48 \tag{13c}$$

An equivalent formulation can be written for the surface winder in terms of u_s, T_{ss} and T_{ms}.

5. MPC integer solution algorithm

The Branch and Bound (BnB) methodology described in [23] is adapted and applied as explained partially in [18] within an MPC algorithm to obtain a near optimal mixed integer hoist schedule solution to the LPM formulated in Section 4. Integer values are required because the winders cannot be controlled to complete a

fraction of a hoist cycle within a 30-min period, but only complete cycles. A summary of the complete MPC algorithm including the adapted BnB methodology is provided below:

Step 1: Set $k = 0$ and define the constants and initial conditions listed below where m_x^0 refers to the respective ore levels at the start of the simulation period.

$$Es \quad Eg \quad M_{blast} \quad m_1^0 \quad m_{1\,\text{max}} \quad m_{1\,\text{min}} \quad N \quad T_{ss} \quad T_{sg}$$

$$R_s \quad R_g \quad M_{\text{min}} \quad m_2^0 \quad m_{2\,\text{max}} \quad m_{2\,\text{min}} \quad D \quad T_{ms} \quad T_{mg}$$

Step 2: Define f and u as in (7) and (8) and construct lb, ub, A, b, Aeq and beq for all $j = 1, 2, 3, \ldots, P$.

Step 3: Minimise $f^T \cdot u$ subject to $lb \le u \le ub$, $A \cdot u \le b$ and $Aeq \cdot u = beq$ in order to obtain an optimal real solution for u of (8).

Step 4: Using the first two elements u_1 and u_2 from the feasible optimal solution obtained for u, branch into four sub problems as illustrated in Fig. 3. Again $\lfloor \rfloor$ denotes rounding down and $\lceil \rceil$ rounding up to the nearest integer.

For each sub problem create and add two equality constraints to the Aeq matrix and beq vector $Aeq \cdot u = beq$ as defined in (14) using sub problem c as an example:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_s^k \\ u_g^k \end{bmatrix} = \begin{bmatrix} \lfloor u_1 \rfloor \\ \lceil u_2 \rceil \end{bmatrix} \tag{14}$$

Step 5: Re-minimise $f^T \cdot u$ for each of the four sub problems with their specific added equality constraints now included into $Aeq \cdot u = beq$.

Step 6: Select the values of u_s^k and u_g^k in the feasible sub problem having the lowest objective value that is not less than the optimal objective value obtained from the first minimisation in *Step 3*. Implement only these two values in period k.

Step 7: At the end of period k, record the actual number of hoists achieved by the surface winder $\hat{u}_s^k$ and the underground winder $\hat{u}_g^k$, as well as the actual feed-in rate $\hat{m}_{in}^k$, during period k. As mentioned earlier the actual feed-in rate will be replaced by an historical average estimation for the purposes of this study.

Step 8: From the recorded values, update the historical data vector for u_s required in (12) and calculate the new initial system state values for period $k + 1$:

$$m_1^{k+1} = m_1^k + R_g \hat{u}_g^k - R_s \hat{u}_s^k$$
$$m_2^{k+1} = m_2^k - R_g \hat{u}_g^k + \hat{m}_{in}^k$$

Finally increment k and repeat from *Step 2*.

It should be noted that the above algorithm differs quite substantially from commercially available solvers. If for example the control horizon is set for 2 days it results in 192 variables in H to be solved at the beginning of each period. However, whereas the conventional solvers would actually endeavour to obtain an integer solution for each of the 192 variables, the algorithm presented only branches on the first two variables in u irrespective of the magnitude of H as these will be the only two variables to be implemented in accordance to the MPC algorithm. This adapted BnB algorithm therefore drastically reduces the computational effort and time required for the conventional BnB solvers by having to solve for only 2 variables instead of 192 in the case of H being set to cover 2 days.

6. Simulation study and discussion

The study included various simulations investigating the impact of various factors on the hoist scheduling of which only a few will be summarised in this paper. By comparing the hoist schedules and related energy costs, the results presented in this paper will point out:
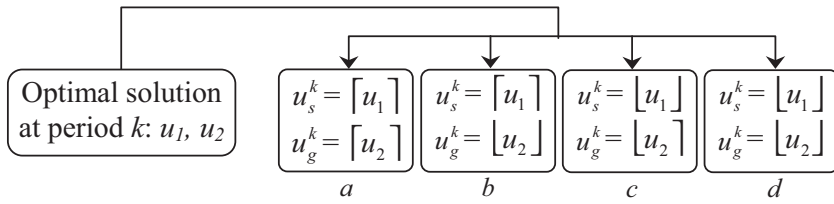
Fig. 3. Branching of the optimal solution at period k into four subproblems.

Table 3
Initial state conditions and constraint values.

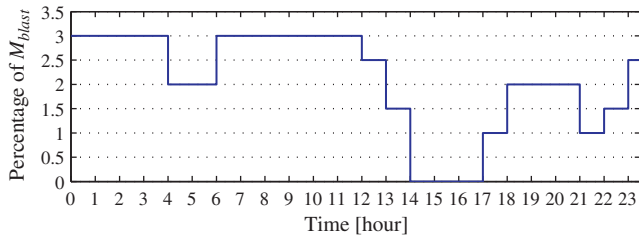|			
$E_s = 130.85$ kWh	$R_s = 23.5$ tons	$m_{1max} = 5$ ktons	$T_{ss} = 7$
$E_g = 42.91$ kWh	$R_g = 13.5$ tons	$m_{1min} = 500$ ktons	$T_{ms} = 8$
$N = 2H = 96D$	$M_{min} = 8013.5$ tons	$m_{2max} = 20$ ktons	$T_{sg} = 8$
$D = 2$	$M_{blast} = 8200$ tons	$m_{2min} = 7$ ktons	$T_{mg} = 4$

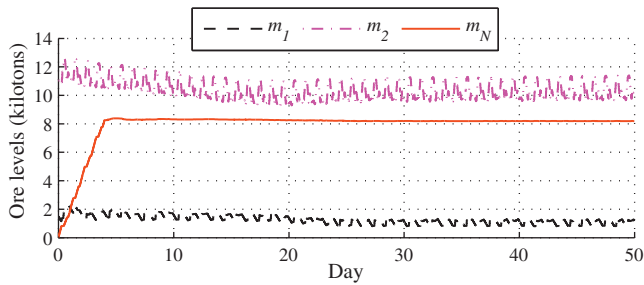**Fig. 4.** Percentage function for m_{in} as a function of M_{blast}.

Fig. 5. Ore levels and transient response to obtain steady state values.

the near optimality of the adapted BnB method by applying and not applying the adapted BnB methodology,

the response of the controller to a TOU versus a flat rate tariff,

the impact of introducing unplanned operational delays into the winder system.

Table 3 contains the values for the various constraints and initial conditions applied in the simulations below.

Fig. 4 shows the periodic percentage function for the feed-in rate of rock from the reefs into the orepass system m_{in}^k discussed in Section 3.1. The actual tons fed into the orepass system for each hour of the day is therefore estimated as the product of M_{blast} and the percentage at that time according to Fig. 4.

6.1. Obtaining steady state and historical hoist record values

Fig. 5 shows the results of an optimal solution obtained after running a single simulation for 50 days. This simulation was done for two reasons. First to create the historical data record of $N-1$ known past control actions required in (12) and denoted as u_{sm}. Secondly to determine the initial steady state conditions for m_1^0 and m_2^0 to be used in further simulations. The values for m_1^0 and m_2^0
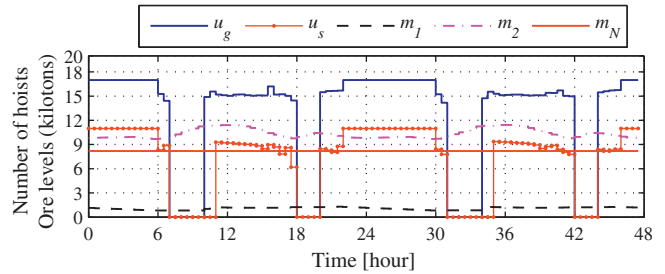

Fig. 6. An optimal hoist schedule with MPC applied and history taken into account.

were set to 1500 tons and 11 000 tons respectively at the start of the 50 day simulation.

The top broken line in Fig. 5 represents the ore-pass system ore level m_2 and the bottom broken line the change-over ore level m_1. The solid line represents m_N, which is defined as the average tons hoisted per 24 h over all model horizon windows N within the simulated period in accordance with (12). This running average m_N is calculated at the end of each and every period using (15) and is used as a measure as to how well the controller adheres to the hoist target constraint.

$$m_N^k = \frac{R_s}{N/48} \sum_{i=k-(N-1)}^{k} u_s^i \tag{15}$$

In steady state m_N has an average of 8194 tons, which is noticeably closer to M_{blast} than M_{min}. This is to be expected in view of the fact that what is blasted underground needs to be hoisted to surface to prevent the ore transport system from saturating. The steady state values for m_1 and m_2 in Fig. 5 are 1168 tons and 9806 tons respectively and taken as the initial values of m_1^0 and m_2^0 for future simulations. Though the actual schedule of u_s is not shown in Fig. 5 to avoid a cluttered graph, the last $N-1$ values were taken for constructing the historical hoist record u_{sm}.

6.2. Near optimality of the adapted BnB method

Control is now continued in time for a further two days from where Fig. 5 ended without introducing delays. Excluding the adapted BnB methodology in *Step* 4, 5 and 6 in the algorithm of Section 5 results in the optimal non-integer hoist schedule and ore levels in Fig. 6. Including the adapted BnB methodology to the same scenario results in the near optimal schedule in Fig. 7, which clearly is very similar to that of the optimal solution in Fig. 6. The solid line remaining constant at approximately 8200 tons indicates the m_N average as in Fig. 5. The top solid line without markers represents the underground winder schedule u_g indicating the number of hoists to complete for each half-hourly period. Similarly the solid line with markers represents the surface winder schedule u_s. The broken lines are those representing m_1 and m_2 as in Fig. 5.

From the two figures above it is evident that no hoisting was scheduled during the expensive peak tariff periods. This happens provided that the hoist target or delays are not too high. Also, lim-

Table 4
Numerical results and comparison for optimal, near optimal integer, flat rate and delay impact schedules.

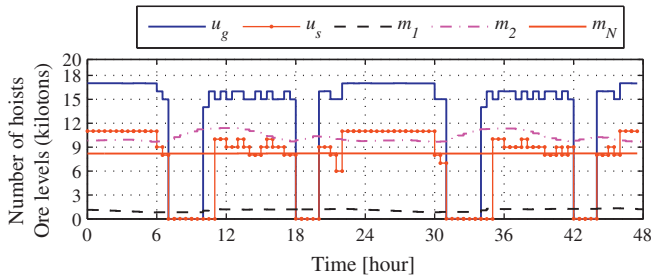| Solution:	Optimal (Fig. 6)	Near optimal (Fig. 7)	Flat Rate (Fig. 8)	Random delays (Fig. 9)
m_N at the end of day 2 [tons]	8193.20	8212.70	8209.10	8013.50
Average 2-day energy cost	R 10 664	R 10 727	R 15 504	R 12 254

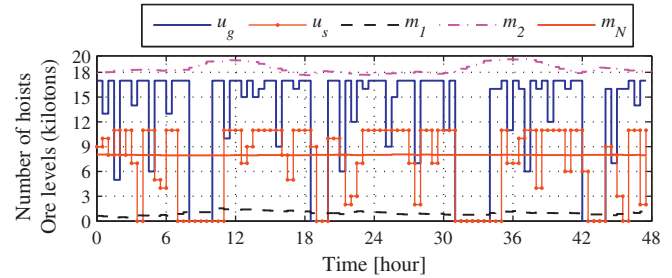**Fig. 7.** A near optimal integer hoist schedule with the adapted BnB applied.

Fig. 9. A near optimal integer schedule with random delays enforced.
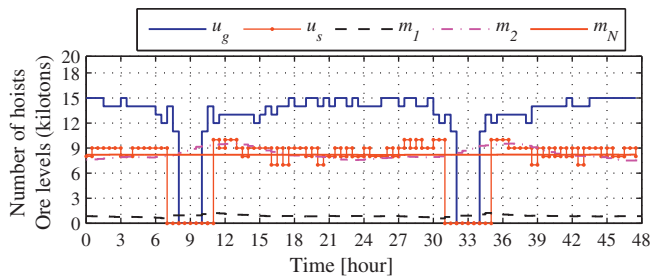

Fig. 8. A near optimal hoist schedule based on a weighted average flat rate tariff.

ited hoisting was scheduled during standard periods and maximum hoisting during the low cost off peak periods.

The near optimality of the adapted BnB solution is also supported by a numerical comparison of the results obtained for Figs. 6 and 7 in Table 4. A negligible difference is noted between the m_N averages and the objectives or energy costs for the two solutions. Note also that the energy cost of the near optimal integer solution is slightly higher than the optimal solution.

6.3. TOU versus flat rate tariff

The comparative study in this scenario shows what the energy cost based on the TOU active energy cost in Table 1 would be if the scheduling of both winders were done based on a flat rate tariff thereby essentially ignoring the TOU tariff structure. The schedule based on a flat rate tariff in Fig. 8 shows that hoisting was scheduled and distributed almost evenly across all periods of the day except during the mandatory maintenance and testing times.

A comparison of the numerical results for Fig. 8 to that of Fig. 7 in Table 4 shows a mere 3.6 ton reduction in the m_N average. However, applying the flat rate schedule on the TOU tariff resulted in an average 2-day energy cost increase of 44.5% from R 10 727 to R 15 504. It can also be stated that scheduling the hoists according to the TOU tariff results in an energy cost saving of (R 15 504–R 10 727)/R 15 504 = 30.8%.

6.4. Delay impact

In order to illustrate the impact of delays on the winder control system, approximately 145 min delay per day were randomly enforced on the surface winder and 190 min on the underground winder. These delays were applied to the same conditions in Fig. 7,

which resulted in the actual achieved hoisted schedule shown in Fig. 9.

Four significant differences are noted when comparing Fig. 9 with Fig. 7. First to note are the surface winder hoists scheduled during both evening and peak periods and the underground winder hoists during the morning peak period of the first day. Second to note is the increase in hoists scheduled on both days for both winders during standard periods. Thirdly the orepass system level is controlled at just below its upper limit of 20 ktons and finally the m_N average being controlled around the target of 8013.5 tons instead of above it at approximately M_{blast}. Simulations over longer periods showed that in time the change-over will also reach its upper boundary at 5 ktons at which time the m_N average will increase again to approximately M_{blast}.

Though not visible in Fig. 9, the m_N average does drop below M_{min} and m_1 below m_{1min} from time to time due to the unpredictability of the delays. The controller however continues to provide a sustainable hoist schedule by controlling the required levels around the boundaries. Therefore rather than seeing the boundaries as limits, they can be regarded as control set points.

The effectiveness of the MPC controller is revealed in the numerical results of Table 4 in that the m_N average is controlled very close to M_{min} while keeping the average 2-day energy cost increase to just over R 1500 from the R 10 727 for Fig. 7 to the R 12 254 for Fig. 9. This is quite significant in view of the fact that almost 3 h of delay was enforced on each of the winders.

7. Conclusion

This paper showed the development of a constrained MILP dynamic optimisation problem of a twin rock winder system in order to obtain a near optimal hoist schedule by achieving a set hoist target at the lowest possible energy cost under unstable and unpredictable operating conditions. The problem was solved through an MPC algorithm using an adapted BnB methodology to find a near optimal mixed integer solution to the hoist scheduling problem at the start of each control period. The near optimal integer solutions were shown to be very close to the optimal non-integer solutions. The effectiveness of the MPC algorithm was illustrated by comparing the impact of using a TOU versus flat rate tariff as well as the inclusion of random unplanned or unscheduled delays into the winder system. Application of the MPC algorithm provided a 30% reduction in energy costs when applied on a TOU tariff compared to a flat rate tariff schedule. Despite almost 3 h of delays being enforced on both winders, the MPC algorithm managed to control

the tons hoisted around the daily target while keeping ore levels within their boundaries at a mere 14% increase in energy cost. Most importantly, the MPC algorithm maintains the ore levels such as to avoid the system from either running empty or saturating thereby ensuring that maximum hoisting can be done during off-peak periods while minimal hoisting is scheduled during the more expensive periods. This not only bares a financial advantage to the mine, but also aids in the improvement of the utilities load factor as part of its DSM objectives.

References

[1] H. Nilsson, The many faces of demand-side management, Power Eng. J. 8 (5) (1994) 207–210.

[2] S.J. Redford, The rationale for demand side management, Power Eng. J. 8 (5) (1994) 211–217.

[3] R. Pelzer, E.H. Mathews, D.F. le Roux, M. Kleingeld, A new approach to ensure successful implementation of sustainable demand side management (DSM) in South African mines, Energy 33 (2008) 1254–1263.

[4] G.J. Delport, Integrated electricity end-use planning in deep level mines, Doctoral thesis, University of Pretoria, 1994.

[5] S. Ashok, R. Banerjee, An optimisation mode for industrial load management, IEEE Trans. Power Syst. 16 (4) (2001) 879–884.

[6] S. Ashok, R.R. Banerjee, Optimal cool storage capacity for load management, Energy 28 (2) (2003) 115–126.

[7] S. Ashok, Peak-load management in steel plants, Appl. Energy 83 (5) (2006) 413–424.

[8] A. Middelberg, J. Zhang, X. Xia, An optimal control model for load shifting – with application in the energy management of a colliery, Appl. Energy 86 (2009) 1266–1273.

[9] D.Q. Mayne, J.B. Rawlings, C.V. Rao, P.O.M. Scokaert, Constrained model predictive control: stability and optimality, Automatica 36 (6) (2000) 789–814.

[10] G. de Nicolao, L. Magni, R. Scattolini, Stability and robustness of nonlinear receding horizon control, in: F. Allgower, A. Zheng (Eds.), Nonlinear Model Predictive Control, Progress in Systems and Control Theory, Birkhauser Verlag, 2000, pp. 3–22.

[11] E.F. Camacho, C. Bordons, Model Predictive Control, 2nd ed., Springer, Berlin, 2004.

[12] A.J. van Staden, J. Zhang, X. Xia, A model predictive control strategy for load shifting in a water pumping scheme with maximum demand charges, in: IEEE Power Tech Conf., Bucharest, Romania, 28 June – 2 July, 2009.

[13] X. Xia, J. Zhang, A. Elaiw, A model predictive control approach to dynamic economic dispatch problem, in: IEEE Power Tech Conf., Bucharest, Romania, 28 June – 2 July, 2009.

[14] A. Kusiak, L. Mingyang, Reheat optimization of the variable-air-volume box, Energy (2010), doi:10.1016/j.energy.2010.01.014.

[15] S.S. Voutetakis, P. Seferlis, S. Papadopoulou, Y. Kyriakos, Model-based control of temperature and energy requirements in a fluidised furnace reactor, Energy 31 (13) (2006) 2418–2427.

[16] K. Krüger, R. Franke, M. Rode, Optimization of boiler start-up using a nonlinear boiler model and hard constraints, Energy 29 (12–15) (2004) 2239–2251.

[17] M. Houwing, A.N. Ajah, P.W. Heijnen, I. Bouwmans, P.M. Herder, Uncertainties in the design and operation of distributed energy resources: the case of micro-CHP systems, Energy 33 (10) (2008) 1518–1536.

[18] W. Badenhorst, J. Zhang, X. Xia, A near optimal hoist scheduling for deep level mine rock winders, in: IFAC Symp. Power Plants and Power Syst. Control, Tampere, Finland, 5–8 July, 2009.

[19] W. Badenhorst, J. Zhang, X. Xia, An MPC approach to deep level mine rock winder hoist control, in: IFAC Conf. Control Methodol. and Technol. for Energy Effic, Vilamoura, Portugal, 29–31 March, 2010.

[20] D.P. Bertsekas, Dynamic Programming and Optimal Control, 2nd ed., Athena Scientific, Belmont, Massachusetts, 2001.

[21] C.E. Garcia, D.M. Prett, M. Morari, Model predictive control theory and practice – a survey, Automatica 25 (3) (1989) 335–348.

[22] D.E. Seborg, T.F. Edgar, D.A. Mellichamp, Process Dynamics and Control, 2nd ed., John Wiley & Sons, New York, 2006.

[23] G.L. Thompson, D.L. Wall, A branch and bound model for choosing optimal substation locations, IEEE Trans. Power Appar. Syst. 100 (5) (1981) 2683–2688.