

**Improvement of Coal Tonnages With Minimum Capital Investment**

**At Transnet Freight Rail Coal Export Line**

**By**

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## **Executive Summary**

### **Improvement Of Coal Tonnages With Minimum Capital Investment At Transnet Freight Rail Coal Export Line.**

Transnet Freight Rail (Formerly Spoornet) the largest division of Transnet limited intend to increase the amount of export coal through Richards Bay Coal Terminal from 65Mtpa to 81Mtpa with a minimum of capital expenditure .Various projects were initiated with the purpose of supporting this improvement. Currently TFR does not reach the capacity that the export line could actually carry per annum. The project will focus on one of the aspects and will aim to perform in depth research about the problems faced by the loading operations at the drop-off mines which includes Uitkyk which is one of the busiest mechanical loading sites.

The project will aim to improve the tonnages shipped per period by developing an optimal routing model based on the hypothetical analysis of the process that will aid the users to schedule and control planned as well as unplanned operations. The use of operations research and simulation-based procedures will aim to develop a solution to increase the amount of tonnage and also ensure that all capital investments required are minimized. Basic Resource allocation techniques will be used to accomplish maximum loads with minimum resources (i.e. Locomotives and wagons)

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## **List Of Abbreviations**

CAPEX-Capital Expenditure

GFB-General Freight Business

LOCO-Locomotive

OR-Operations Research

RBCT-Richards Bay Coal Terminal

SOC-Satellite Operating Centre

TAT-Turnaround Time

TFR-Transnet Freight Rail

TRAIN-locomotive with wagons

WAGON SET-100 wagons

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# Chapter 1:

## 1.1. Introduction and Background

**Transnet Freight Rail** is part of Transnet Limited, a public company with the South African government as its shareholder. Transnet, operating and controlling South Africa's major transport infrastructure is also responsible for ensuring that the country's transport industries operate according to world-class standards. Transnet forms an integral part of the Southern African economy.

Transnet Freight Rail strategic plan aligns with Transnet's intent and vision. This includes a focus on key segments of the market, disposal of non-core and non-freight businesses in order to drive future growth, enhanced accountability, governance and operational efficiency. The company has further redefined its strategic intent to reflect a renewed focus on customers. This will be achieved by reducing the cost of doing business, building capacity, operating safely and improving efficiently.

## 1.2. Problem Description

Currently Transnet Freight Rail does not meet the required annual demand for coal export. The increase in capacity will include both infrastructural as well as procedural improvements but the number of rolling stock resources will stay the same. As far as the project is concerned, apparently TFR wants to improve coal tonnages from 65mt to 81mt per annum by minimizing the overall capital investment.

Given the estimated times to be spent and the demand that have been determined, the variable demand that changes week after week and the available limited resources (locomotives and wagons), TFR wants to know how to determine to which mine must trains be sent in order to get the quickest cycle time and thus highest capacity.

Therefore Transnet Freight Rail wants to improve coal tonnages shipped per annum at the coal export line. The main aim of the project is to use operations research, modeling techniques and perform time-based planning scheduling that will help to improve the operational functionalities and planning thus improving coal tonnages by tremendously reducing turnaround time, whilst minimizing the number of resources used while maximizing resource utilization and also minimizing capital investment and expenditure.

Various projects were initiated with the same purpose of improving coal tonnages from 65mtpa to 81mtpa. The proposed project is in support of the initiatives proposed by TFR.

### **1.3. Project Aim**

The project will aim to improve the tonnages shipped per period by performing analysis, recommend and develop a hypothetical scheduling model that will aid the TFR managers, planners, schedulers to manage and control planned as well as unplanned operations. The use of operations research will aim to develop hypothetical multiple solution that will enhance the increase of tonnages and also ensure that all capital investments required are minimized. The question that rises is, how must the schedule be set up to have the least number of locomotive movements required and bring all loads at home in a three-day period?

## 1.4. Project Scope

The focus of the project will only be in one area, the drop off mines where trains are loaded mechanically by the front end loaders as compared to the Rapid Loaders equipped with loading shuts (see figure 9) where loads are left behind (i.e. drop-off) without transition power to be loaded. In this area improvement will be made on improved planning when scheduling loads in order to minimize light locomotives moves and synchronise locomotives trips, total loading time and turnaround time.

Industrial techniques to be applied in the project includes, integer programming by building a hypothetical integer programming for assigning resources which will enhance in determining where locomotives and wagon sets should be sent at a mine to minimize cycle time, operating distance and maximize capacity; analysis of schedules and planning methods which will ensure that available resources are assigned to relevant loads. Also apply the use of simulating-based procedures with an aim of establishing the optimal route and schedule.

### **Deliverables:**

- The project will result with an optimal schedule and an integer program model built on lingo for assigning locomotives and wagon-sets at the mines based on the hypothetical structure(see figure 5) of locomotive movements and wagon sets with an optimal train schedule and thus helps improve scheduling and executing of locomotives.

### **Benefits of the project:**

- Least number of resources and maximum usage of resources
- Maximum number of loads within a three-day period

## **Chapter 2:**

### **2. Literature Review**

#### **2.1. Introduction and Background**

As a result of clients requiring high service delivery, TFR was enforced to find ways of improving service delivery to clients. The operations manager division for TFR deals with all the planning of shipments. TFR uses the usual method of scheduling, which is to set up time slots for trains that have to be sent into the mines at the particular times. TFR's current approach method is considered being ineffective as it has resulted into higher cycle times, insufficient capacity and insufficient service to clients.

Railroad scheduling involves the setting up of train timetables, determining the arrival and departure times in its direction. However the scheduled times may change as a result of unplanned outages.

#### **2.2. Historical Review**

According to operations research experts, planning and scheduling are the main aspects of every organization. Planning and scheduling has evolved over the years simply from reacting to the available amount of resources to complete activities. Based on the review, to perform time-based planning scheduling, nowadays emphasis on the ability to plan and schedule operations prior executing activities. The most essential areas of planning where scheduling principles have been applied includes chemical companies, distributors, railways, airlines and steel manufacture companies.[10]

The VRP problem is a difficult optimization problem with no exact polynomial algorithm, thus researchers and mathematicians classified it as Non-deterministic class problem. *Figure 2* demonstrate the complex structure of solving a VRP problem. However the introduction of computer systems; softwares have a huge impact on solving problems of this kind.

## **2.3. Scheduling and Planning**

### **2.3.1. Scheduling**

In the classical railway environment that is setting up the time slots (especially on single track lines) to get as many as possible trains through safely and with the minimum waste of time. Schedule specifies information about timing for every possible occurrence of the service to be performed at specific period of time. This includes Departure times from the origin, arrival times at the final destination which are the most relevant parameters for this project, and intermediate stops if there is any. TFR trains still operates under the “go when full” rule which ensure that trains leave when they are full, even when schedules are prepared. Earliest departures and arrivals are scheduled to compensate for uncertainties.

### **2.3.2. Planning**

Planning requires human manpower, material resources and that demonstrate complex relationships and tradeoffs among the several decisions as well as management policies having effects on the company’s elements. Railroad Planning can be categorized according as Strategic, Tactical, and Operational. After the study has been done the following planning stages were identified: [6]

- **Strategic Planning:** it is typically the planning over long-term. Strategic planning involves the high management level and also requires huge capital investments over long term range. The element of strategic decisions involves the running of huge long term projects, determining specific policies and predominantly smoothing the operating strategies of the whole system. These includes the design or re-design of relevant networks, the location of greater importance facilities(e.g. location of terminals, design of new wagons, acquiring of new locomotives and the acquisition of major resources ) [10]
- **Tactical Planning:** It occurs over medium-term planning. It aims to resolve efficient allocation and the utilization of resources to accomplish the best possible performance from the system. Typical tactical decisions concern the design of the service network and may include issues related to the determination of the routes and types of service to operate, service and schedules of the fleet for use in the next planning period. [10].
- **Operational Planning:** Planning characterized over short term. Operational planning is usually performed by local management team, team responsible for control. The relevant operational decisions entails, the implementation of new schedule plans, adjustments where possible, management of maintenance activities, dispatching of trains and crew allocation.

## 2.4. Current method on Scheduling and Planning

TFR uses the classical method of scheduling, which is setting up time slots especially on the single track lines which connects the mine fields and the Ermelo. The main purpose is to use the schedule in getting as many trains in the tracks through safety and with the minimum

waste of time. (e.g. “a train may leave one hour before planned departure if full and conditions down the line are appropriate”).

Things that are considered here are the headway required (The length of the section in which only one train may be at a time) time waited, where to wait, where to cross and so forth. In this scenario the first step is to do the train schedule, the second is to adjust it for planned disruptions, this include maintenance called occupations on the line.

The last thing is where this is done dynamically as trains run behind or ahead schedule. This method is not yet used in South Africa but well used in big passenger train environment like India, china, US and other countries.

#### **2.4.1. Factors affecting scheduling at Transnet**

- Break downs
- Locomotive down times
- Shortage of resources
- Drivers' Performance
- Shift changes
- Relationship between Transnet and the mines
- Communication

## **2.5. Optimization and Mathematical Approach on Scheduling**

**The following Optimization aspects could be used solving the problems:**

### **2.5.1. Deterministic/Probabilistic Dynamic Programming**

Dynamic programming is a technique that can be used to solve optimization (maximization or minimization) problems involving decision variables. All models have states which are often grouped into stages. The sequential decision process with the transition occurring on each stage to the next. Dynamic programming can be used to solve pseudo-polynomial time problems like routing problems in an attempt to accomplish the optimal route at minimum cost and cycle time. [4]

### **2.5.2. Mixed Integer Programming**

According to Winston, Integer programming is an LP problem that comprises mathematical variables which all of them are positive. [3] Some LP's programs are found to contain, these LP's are called mixed integer programming. In most cases there are several solutions to an IP model. The solution that is arrived at depends on the method used to solve it. To find solutions to an IP model, there exist many polynomial algorithms. [13]



### 2.5.3. Vehicle Routing Problem

The vehicle routing problem is one of the popular methods used to solve transportation and distribution problems[6]. Though VRP is a complex integrative optimization where a number of clients need to be served by a fleet of a vehicle (in this case by train) with known demands and capacities. To solve the problem one must design an optimal route that will result to the reduction of cycle time thus maximizing coal tonnages shipped from the mines to Ermelo eventually to Richards's bay for exports. The demand is known as well as the capacity and the demand can not exceed the capacity that could be shipped by the available resources.

### 2.5.4. Branch and Bound Algorithmic

A lot of scheduling problems has been solved using the branch and bound algorithm method in the past. [2] (Xuesong Zhou) solved a transportation problem by developing a model that could ensure that a specific truck finds its scheduled track. [2]

*Winston* states that IP's problems are solved by using the technique of branch and bound. Branch and bound methods find the optimal solution to an IP by efficiently enumerating the points in a sub problem's feasible region [3].

Steps that are followed to accomplish optimal results using branch and bound method. [9]

#### ■ Step 1: (Initialization)

Create a new node, in which contains the first task of all trains. Set the departure time for this train and insert this node into active node list (L).

**■ Step 2: (Node Selection)**

Select an active node from L according to a given node selection rule.

**■ Step 3: (Stopping criterion)**

If all of active nodes in L have been visited, then terminate.

**■ Step 4: (Conflict set construction)**

Update the schedulable set in the selected node

## 2.6. Conclusion on Literature Review

From the literature that has been done it is clear that there are various methods that exist in optimal routing and schedule. The magnitude and complexity of the problem will determine which method shall be used to solve the problem. The problem that TFR currently experiences is not too complex using the Resource allocation, simulation-based procedures and literal thinking methods under Operational Planning will make the problem simpler. To perform time-based planning scheduling, nowadays emphasis on the ability to plan and schedule operations prior executing activities.

The integer programming is a best alternative because the TFR shipment system constitutes of more than one variable and thus a integer program will be much beneficial together with literal thinking techniques to accomplish the optimal route within a reasonably cycle time. [15]

## 2.7. Development of Tools, Techniques and Methodology

A linear integer programming could be used to help accomplish the optimal schedule. Demand, resources and other relevant parameters will be considered to be deterministic under tactical method of planning. Under tactical planning, then railroad determine how to move trains to the mines using available resources. The results tactical planning are operating plans, including: [14]

- **Blocking plans:** Dictating which loads should be (i.e. groups of shipments to be classified as units) should be built at each yard and which traffic should be assigned to each block;
- **Train Schedule Plans:** Specifying loads scheduled to each train, and train routes and arrival/departure times at yards;
- **Power Schedule:** Assigning crews to trains

In operational planning, railroads specifying in great detail the daily activity, an implementation plan, and an execution schedule.

- **Train Timetable:** Determining arrival and departure time of each train at stations in its itinerary.

The focus of this project is to develop a mixed integer program, which involves multiple variables with an objective of accomplishing a robust solution. [6]

## 2.8. Resource Assignment

Accomplishing the desired schedule involves the assignment of relevant resources; this is done to complete tasks as to attain the level performance acceptable to the decision makers [20]. This part aims to present an integer programming model for locomotive assignment. To achieve intended performance the locomotives should be assigned to mines in order to maximize loads which are considered as revenues. These revenues depend on the amount of demand available. The limited amount of resources (locomotives and wagons) constraint the model.

## 2.9. Simulation-based Procedure

Banks [19] defines simulation as: .Simulation is the imitation of the operation of a real world process or system over time. Simulation involves the generation of an artificial history of the system and the observation of the artificial history to draw inferences concerning the operating characteristics of the real system that is presented.

Simulation may be used to describe existing systems; analyse the behaviour of the systems; do sensitivity analyses on the systems (ask what-if questions); and it can be used as a tool to design new systems.

Thus, simulation endeavours to mimic real-world systems or processes as closely as possible so that system behaviour (and by implication, performance) may be predicted over time and so that the behaviour may be analysed.

### **2.9.1. Literal Thinking**

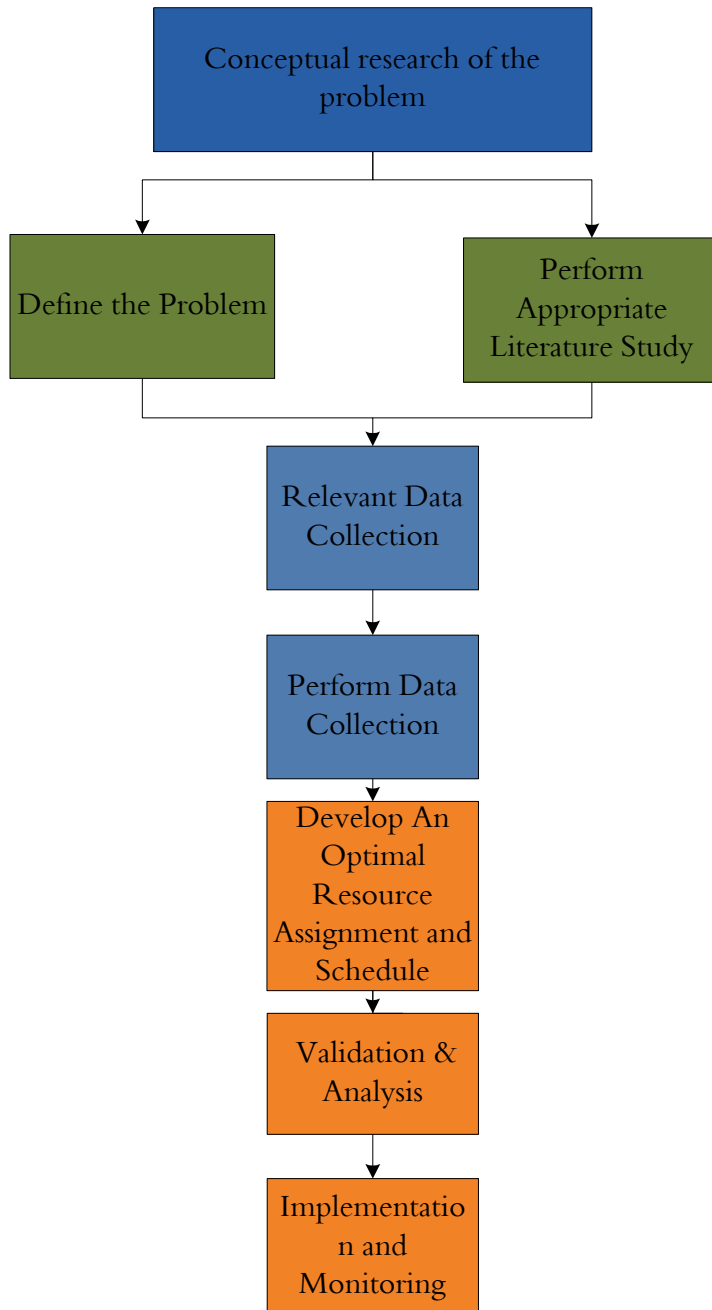
Designing an optimal route and schedule for the trains in such a way that the overall cycle time is minimized. Least number of locomotives used the number loads collected would be maximized per given period. The procedure need to be followed and performed to find out how to minimize the amount of resources used and how to maximize usage of the available resources. To perform time-based planning scheduling, `nowadays emphasis on the ability to plan and schedule operations prior executing activities.

## **2.10. Development of Supplementary Method and Review of Related Literature**

### **2.10.1. Network Design**

Network design is a general class of problems involving the selection of set of location or set of movements to include in a network in order to flow commodities from their origins to their destinations and accomplish maximum profits; while satisfying level of service. Level of service requirements might include requirement to move commodities from origins to destinations within certain time frames or distances or requirements to maintain level of network connectivity. [5]

## 2.11. Research Strategy



**Figure 1: Research Strategy**

The above figure demonstrate the research strategy that will be followed in accomplishing the optimal way of routing and scheduling trains to the mines given the limited resources (locomotives,

wagons ), in order to determine which mine must trains be sent in order to get the quickest cycle time and thus the highest capacity.

## **2.12. Data Collection and Relevant Information**

Relevant data will be needed to enhance the accuracy of the model results; however a clear understanding of the current schedule will be required, (1) Mulvey et al (1995). Locomotives and wagons are the most important resources of the whole shipment system. Data will be gathered on the relevant section, between the drop-off mines and Ermelo, the important depot on coal exports as described on the scope.

## Chapter 3:

### 3.1. Model Formulation:

#### Load Plan

- Load plan will constitute the defining the origin and destination of the loads, however since all loads are aimed to be transported for exports, all loads should be shipped from a specific mine to Ermelo then eventually to Richards bay.
- No local loads will be incorporated.
- This part will constitute the assignment of locomotives to specific loads at the mine(locomotive-Load-Assignment)

#### Train schedule

- Train schedule will constitute of departure, arrival times and assumed delay times at the lines.
- The given cycle time

#### Trip Plan

- The trip plan will constitute of the amount time to be taken during the shipment and the distance which will be kept constant between the Ermelo depot and the mines.



### 3.1.1. Wagon-sets assignment

The first part of the model will aim to determine the wagon set-assignment procedure that between the depot and the mine. This part of the model will help to determine in which Mine must a set of wagons assigned to as to maximize loads within the given time. This part will be executed for all mines which are assigned to the wagons with which needs to be loaded. For assignment the objective is to maximize the number of loads within a period of three days with minimum locomotives. Thus the objective function (3.1) is formulated as follows:

Maximize:

$$\sum_{i=1}^5 * \sum_{j=1}^3 l_{ij} x_{ij} \quad (3.1)$$

Subject to:

$$\sum_{j=1}^3 x_{1j} = 4 \quad \forall i = 1 \dots 5 \quad (3.2)$$

$$\sum_{j=1}^3 x_{2j} = 3 \quad \forall i = 1 \dots 5 \quad (3.3)$$

$$\sum_{j=1}^3 x_{3j} = 3 \quad \forall i = 1 \dots 5 \quad (3.4)$$

$$\sum_{j=1}^3 x_{4j} = 3 \quad \forall i = 1 \dots 5 \quad (3.5)$$

$$\sum_{j=1}^3 x_{5j} = 3 \quad \forall i = 1 \dots 5 \quad (3.6)$$

$$x_{11} = 1 \quad (3.7)$$

$$x_{32} = 0 \quad (3.8)$$

$$x_{23} = 0 \quad (3.9)$$

$$x_{ij} \in \{0,1\} \quad \forall i = 1 \dots 5 ; j = 1..3 \quad (3.10)$$

Where:

$l_{ij} \triangleq$  number of loads to be maximized between depot and the mine

$$x_{ij} \triangleq \begin{cases} 1 & \text{if a set of wagons is assigned to a mine} \\ 0 & \text{Otherwise} \end{cases}$$

Constraint (3.2) ensures that mine 1 to 3 is assigned to wagon set #1 for loading. Constraint (3.3) ensures that mine 1 to 3 is assigned to wagon set #2 for loading. Constraint (3.4) ensures that mine 1 to 3 is assigned to wagon set #3 for loading. Constraint (3.5) ensures that mine 1 to 3 is assigned to wagon set #4 for loading. Constraint (3.6) ensures that mine 1 to 3 is assigned to wagon set #5 for loading. Constraint (3.7) ensures that wagon-sets #1 is assigned to mine #1 Constraint (3.8) states that wagon set #2 cannot be send mine #3. Constraint (3.9) states that wagon set #3 cannot be send mine #2. Constraint (3.10) ensures that the appropriate variable remains as binary variable.

### 3.1.2. Locomotives assignment

The second part of the model will aim to determine the locomotive–assignment procedure that could be followed between the depot and the mine. This part of the model will help to determine in which Mine must a locomotive assigned to as to maximize loads within the given time and minimize locomotive movements. This part will be executed for all mines which are assigned to the locomotive with which a load needs to be collected. For assignment the objective is to maximize the number of loads within a period of three days with minimum locomotives. Thus the objective function (3.11) is formulated as follows:

Maximize:

$$\sum_{i=1}^4 * \sum_{j=1}^3 l_{ij} x_{ij} \quad (3.11)$$

Subject to:

$$\sum_{j=1}^3 x_{1j} = 3 \quad \forall i = 1 \dots 4 \quad (3.12)$$

$$\sum_{j=1}^3 x_{2j} = 3 \quad \forall i = 1 \dots 4 \quad (3.13)$$

$$\sum_{j=1}^3 x_{3j} = 3 \quad \forall i = 1 \dots 4 \quad (3.14)$$

$$\sum_{j=1}^3 x_{4j} = 1 \quad \forall i = 1 \dots 4 \quad (3.15)$$

$$x_{11} = 1 \quad (3.16)$$

$$x_{21} = 1 \quad (3.17)$$

$$x_{32} = 1 \quad (3.18)$$

$$x_{43} = 1 \quad (3.19)$$

$$x_{ij} \in \{0,1\} \quad \forall i = 1 \dots 4; j = 1..3 \quad (3.20)$$

Where:

$l_{ij} \triangleq$  number of loads to be maximized between depot and the mine :

$$x_{ij} \triangleq \begin{cases} 1 & \text{if a locomotive is assigned to a mine} \\ 0 & \text{Otherwise} \end{cases}$$

Constraint (3.12) ensures that mine 1 to 3 is visited by locomotives #1 for load collection.

Constraint (3.13) ensures that mine 1 to 3 is visited by locomotives #2 for load collection.

Constraint (3.14) ensures that mine 1 to 3 is visited by locomotives #3 for load collection.

Constraint (3.15) ensures that mine 1 to 3 is visited by locomotives #4 for load collection.

Constraint (3.16) ensures that locomotives #1 visits mine #1. Constraint (3.17) ensures that

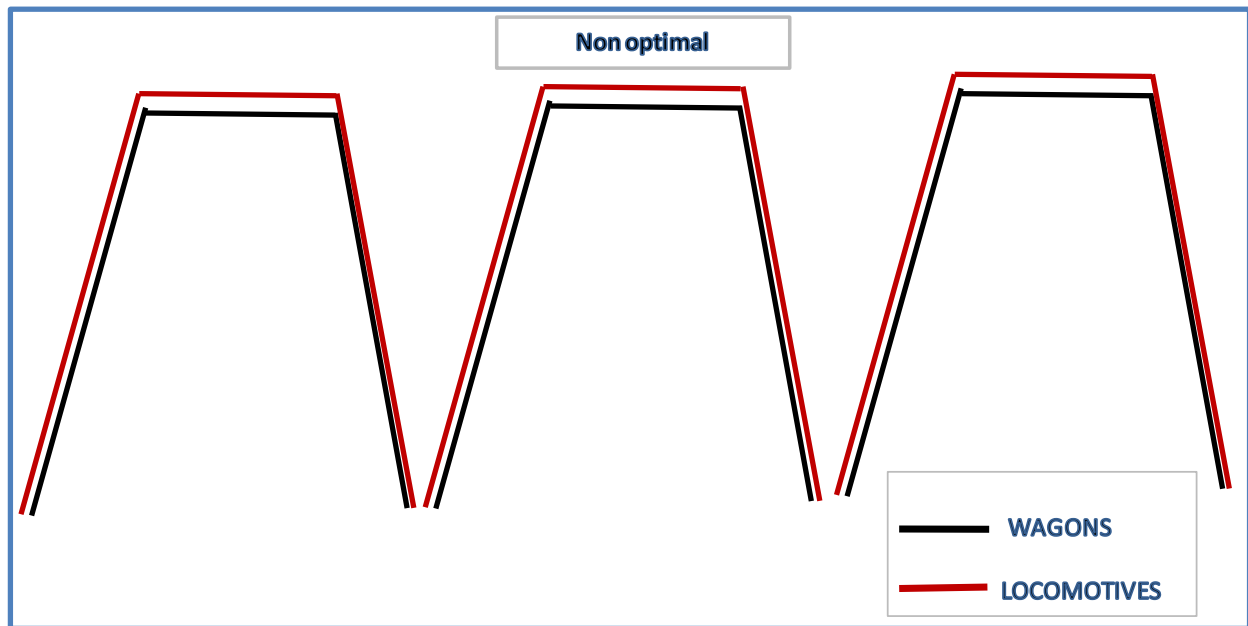
locomotives #1 visits mine #2. Constraint (3.18) ensures that locomotives #3 visits mine #2.

Constraint (3.19) ensures that locomotives #4 visits mine #3. Constraint (3.20) ensures that the

appropriate variable remains as binary variable.

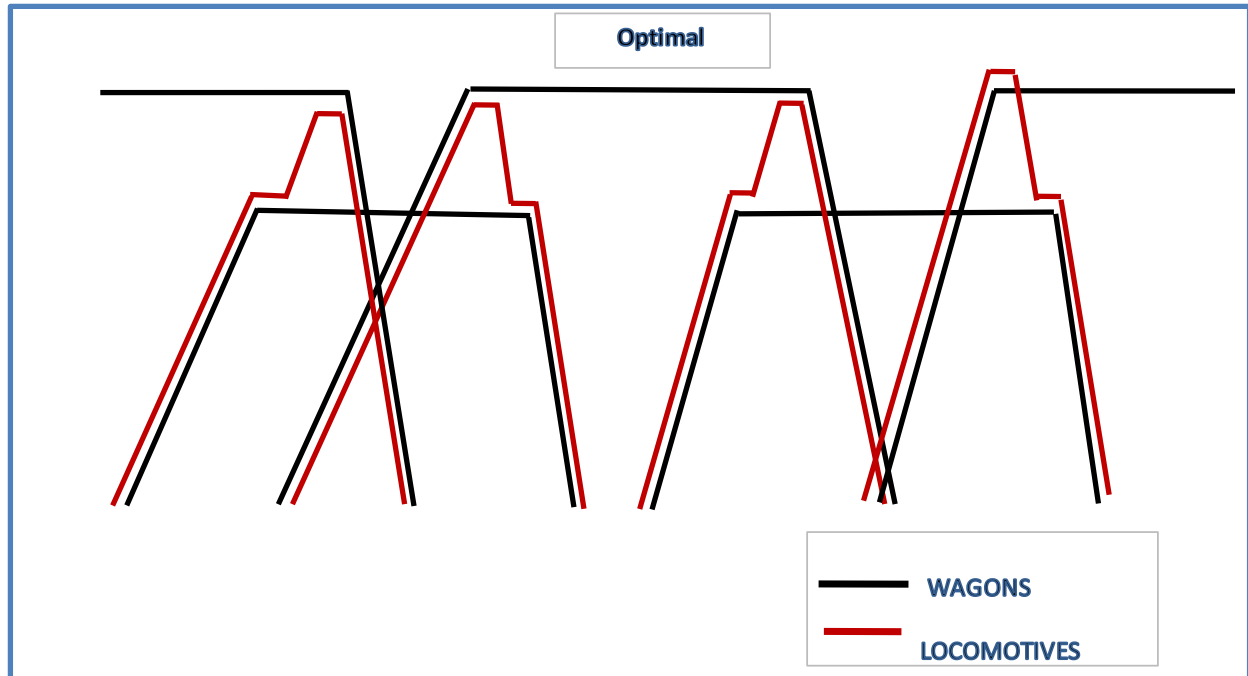
### 3.1.3. Routing and Scheduling

The second part of the model will aim to determine the optimal route that should be followed by the locomotive. This part of the model will help to determine the optimal schedule as to maximize loads within the given time and minimize locomotive movements. This part will be executed for every mine which is assigned to the locomotive with which a load needs to be collected. For routing the objective is to maximize the number of loads within a period of three days with minimum locomotives.



**Figure 2: Non optimal model of setting up schedules for locos and wagon sets**

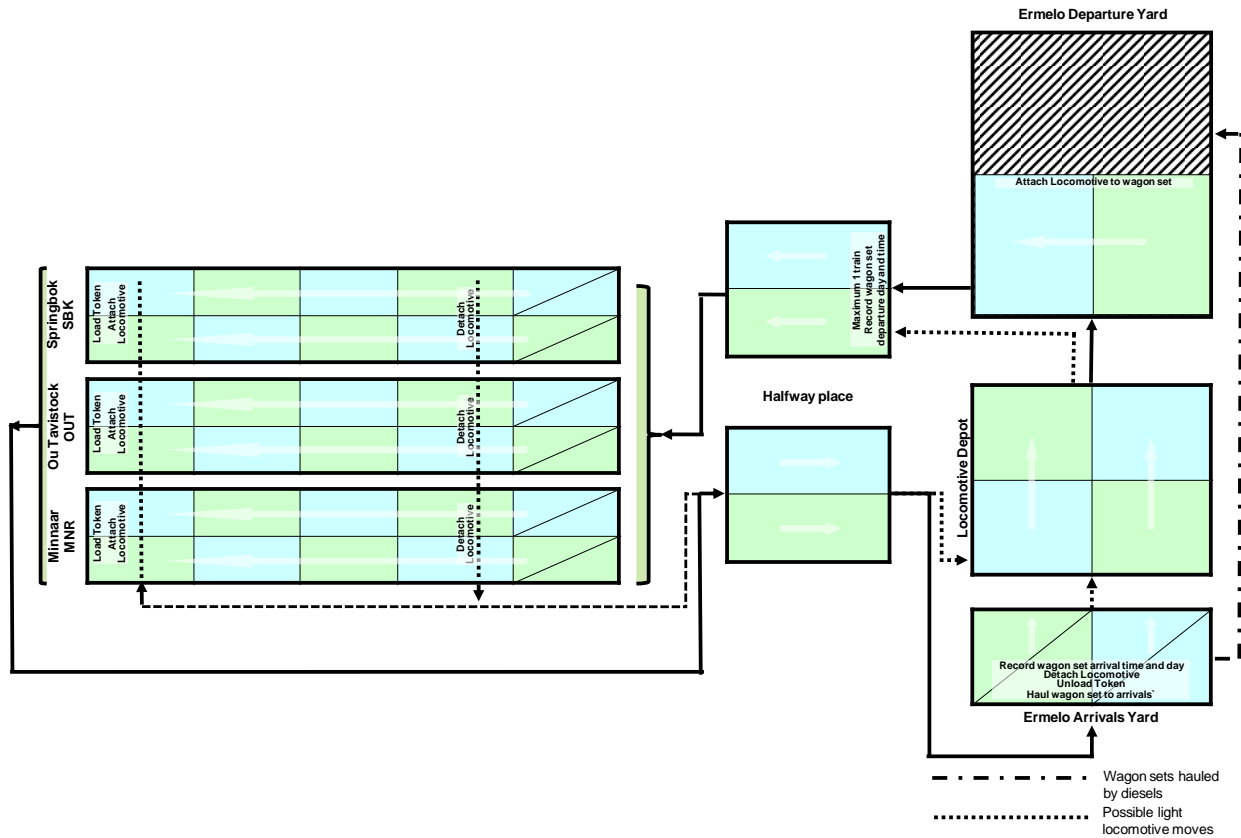
The above figure illustrates the non optimal way of sending out trains, where a locomotive is attached to wagon sets. This is not optimal because much time is wasted while a locomotive is waiting for the load to be full whereas the locomotive could be used somewhere else for load collection if schedule has been planned correctly.



**Figure 3: Optimal model of setting up schedules for locos and wagon sets**

**Figure 3** illustrates the optimal representation of how locomotives and wagon sets should be sent to eliminate time wasted, improve planning and schedule and subsequently determine the optimal routes to achieve the minimum possible cycle. Normally it takes eight hours to load 100 wagons, thus this figure depicts how one could the optimality of the schedule by departing a locomotive with in this case a locomotive always depart with a set of wagons, for instant a locomotive plus the wagons could be departed two hours before the wagons are being loaded to ensure that when the locomotive with empties reaches the Mine the load that is being loaded is full. The two hours left is incorporated for decoupling of the empties and coupling of the loaded wagon sets at the Mine.

**HYPHOTHETICAL REPRESENTATION:COAL FIELDS WAGON  
TURNAROUND SIMULATION**



**Figure 5: Coal fields turnaround model**

**3.2. General Procedure for executing locomotive moves**

Thirteen mines operate as drop-offs where locomotives are detached from the wagons for loading, but this model was build based on three operating drop-off mines at the coal fields in based in Witbank.

When a locomotive arrives in the Ermelo Departure Yard it can immediately be attached to a wagon set but it may only move to the Halfway place on the next time increment of time. This is done in order to simulate the preparation of a train in the yard.

The same principle applies at the end of the process at a drop-off mine. The load as well as the locomotive must be attached and only on the next two-hour cycle may be shipped the next stage of movement representing the loaded train.

When a train arrives at the Ermelo Arrival Yard, the load is shipped directly to Richards bay and the wagon set “hailed back by the diesels” to the departure yard. Only on the next cycle may the locomotive move to the first position in the Locomotive Depot.

As a train reaches the second block of a drop-off mine the locomotive may be detached to either wait for the wagon set it has just been detached from, or it can wait at another mine for a different wagon set.

It can also be sent back light to the Locomotive Depot at Ermelo. The route of a light locomotive is indicated by the short dashed line.

The blocks at the drop-off mines do not represent movement of the wagons but rather the passing of time. In this operation it takes eight hours for a wagon set to be fully loaded.



### **3.3. Generic Search of an Optimal Solution**

The search begins by First setting up an initial position of the locomotives and wagons then simulating the movements given the available number of resources. These moves are prohibited for three-day iteration where the aim is to achieve all loads in a minimum of three days. To accomplish the below optimal results shown at table 1, 2 and 3 the initial set up should be, a wagon set being loaded at mine number two, Ou Tavistock with only two hours left to finish loading, a train at the Halfway place, two trains available at the Ermelo departure yard and a locomotive at Ermelo arrival yard. In order to fully understand how the locomotive scheduling optimization is done, The optimal solution was obtained by simulating the moves of the locomotives with wagons based on the figure 5 by following the procedure.

### 3.2. Design, Results Interpretation and Analysis

The below tables, 1–3 will ensure that the user is able to execute light locomotives movements and locomotives with loads optimally. By examining the designed spreadsheet the user will be able to establish appropriate movements of locomotives and accomplish maximum amount of loads within a minimum cycle time of three days. Wagons and locomotives are designated as follows.

	DAY #											
	0	2	4	6	8	10	12	14	16	18	20	22
<b>ERMELO</b>	W3L3	W3L3	W4L4	W4L4	W5L1	W5L1	W1	W1L2	W2	W2	W3L3	W4
<b>DEPOT</b>	W4L4	W4L4		W5L1	W1	W1		W2		W3	W2	W2
HP <sub>(OUT)</sub>												
MINE11												
2												
3												
4												
5												
MINE21												
2												
3												
4												
5												
MINE11												
2												
3												
4												
5												
HP <sub>(RET)</sub>												
ARR												
LD1												
LD2												
<b>LOADS</b>												

Table 1: Routing and scheduling log table

**From the above table, referring to figure 5:**

$W \triangleq$  Represent wagon sets, (1...5)

$L \triangleq$  Represent locomotives, (1...4)

Ermelo depot  $\triangleq$  Represent Ermelo depot where numbers of resources are kept

HP (Out)  $\triangleq$  Represent the position of resource leaving the halfway place to the Mine

Mine 1  $\triangleq$  Represent the Minnar (MNR) Mine, where step 1 to 5 need be completed to finish loading.

Mine 2  $\triangleq$  Represent the Ou Tavistock (OUT) Mine where step 1 to 5 need be completed to finish loading.

Mine 3  $\triangleq$  Represent the Springbok (SBK) Mine where step 1 to 5 need be completed to finish loading.

HP (Ret)  $\triangleq$  Represent the position of returning the halfway place back loaded passing to the Ermelo depot.

ARR  $\triangleq$  Represent the Ermelo arrival yard, where loaded wagons are decoupled then eventually sent to Richards's bay.

LD  $\triangleq$  Represent the locomotive depot where only two moves are required for locomotives before are departed for the again.

The below optimal schedule **Table 4** was accomplished based on figure 5 by first simulating locomotive movements.

Refer to **Table 1**, day one schedule set up. The first column comprise of the different stages showed in Figure 5 namely, the Depot, Halfway place, Mines and the Locomotive depot. The first row depicts the elapse time with a difference of two hours interval.

By using the simulation procedure as described in (3.2) the user develops a 'feel' for what is happening in the model/network when different routes are taken. This is based on the performance measures of the different movements. By asking what if questions and analyzing the results, the effects or different decisions/scenarios are made clear to user. Furthermore the user could take ownership of the system output (since he was closely involved in the system). The system is accepted much easier since the user is able to work and develop outputs on how the system outputs are obtained, thus the user develops confidence in the system.

On the other hand, if the results from the iterations do not yield optimal solution the user loses confidence about the system without any further iteration done on the network. This part is due to the fact that the method used in this project results in a schedule that optimize the network performance measures.

Allowing the user to play with the model asking (i.e. what-if questions and performing sensitivity analysis) results in greater acceptance of outputs obtained by directly solving for the optimised schedule and also enable the user to have a confidence feel about the results obtained.

**DAY 1**

	<b>0</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>14</b>	<b>16</b>	<b>18</b>	<b>20</b>	<b>22</b>
<b>ERMELO</b>	W3L3	W3L3	W4L4	W4L4	W5L1	W5L1	W1	W1L2	W2	W2	W3L3	W4 W2
<b>DEPOT</b>	W4L4	W4L4		W5L1	W1	W1		W2		W3	W2	
HP(OUT)	W2L2		W3L3		W4L4		W5L1		W1L2			W3L3
MINE11						W4L4						
2							W4					
3	W1							W4				
4		W1L3							W4			
5			W1L2						L1	W4L1		
MINE21		W2L2						W5L1				
2			W2						W5			
3				W2						W5		
4					W2						W5	
5					L3	W2L3					L2	W5L2
MINE11				W3L3						W1L2		
2					W3						W1	
3						W3						W1
4							W3					
5							L4	W3L4				
HP(RET)				W1L2			W2L3		W3L4		W4L1	
ARR	L1				L2			L3		L4		L1
LD1		L1				L2			L3		L4	
LD2			L1				L2			L3		L4
<b>LOADS</b>					<b>1</b>			<b>2</b>		<b>3</b>		<b>4</b>

Table 2: Day one Route set up

<b>DAY 2</b>												
	<b>0</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>14</b>	<b>16</b>	<b>18</b>	<b>20</b>	<b>22</b>
<b>ERMELO</b>	W2L4	W4	W4L1	W1	W5L2	W1	W1L3	W3	W3	W3L4	W2	W2L1
<b>DEPOT</b>	W4	W5	W5	W5	W1		W3		W2	W2	W4	W4
HP(OUT)		W2L4		W4L1		W5L2		W1L3			W3L4	W4L1
MINE11	W3L3						W5L2					
2		W3						W5				
3			W3						W5			
4				W3						W5		
5				L4	W3L4					L3	W5L3	
MINE21			W2L4						W1L3			
2				W2						W1		
3					W2						W1	
4						W2						W1
5						L1	W2L1					
MINE11					W4L1							W3L4
2						W4						
3							W4					
4	W1							W4				
5		W1L3						L2	W4L2			
HP(RET)	W5L2		W1L3			W3L4		W2L1		W4L2		W5L3
ARR		L2		L3			L4		L1		L2	
LD1	L1		L2		L3			L4		L1		L2
LD2		L1				L3			L4		L1	
<b>LOADS</b>		<b>5</b>		<b>6</b>			<b>7</b>		<b>8</b>		<b>9</b>	

Table 3: Day two Route set up

	DAY 3												DAY 4
	0	2	4	6	8	10	12	14	16	18	20	22	0
<b>ERMELO</b>	W4	W4L2	W5	W5L3	W1	W1L4	W3	W3 W2	W3L1	W2 W4	W2L2	W4	W4L3
<b>DEPOT</b>	W5	W5	W1	W1		W3			W2		W4	W5	W5
HP(OUT)	W2L1		W4L2		W5L3		W1L4			W3L1		W2L2	
MINE11		W2L1						W1L4					
2			W2						W1				
3				W2						W1			
4					W2						W1		
5					L2	W2L2					L1	W1L1	
MINE21				W4L2							W3L1		
2					W4							W3	
3						W4							W3
4							W4						
5	W1L4						L3	W4L3					
MINE11						W5L3							W2L2
2	W3						W5						
3		W3						W5					
4			W3						W5				
5			L1	W3L1					L4	W5L4			
HP(RET)		W1L4			W3L1		W2L2		W4L3		W5L4		W1L1
ARR	L3		L4			L1		L2		L3		L4	
LD1		L3		L4			L1		L2		L3		L4
LD2	L2		L3		L4			L1		L2		L3	
<b>LOADS</b>	<b>10</b>		<b>11</b>			<b>12</b>		<b>13</b>		<b>14</b>		<b>15</b>	

Table 4: Day three Route set up

Set Number	Departure Day	Departure Time	Arrival Day	Arrival Time	Throughput Time
1	0	16:00	1	08:00	16:00
2	0	22:00	1	14:00	16:00
3	1	02:00	1	18:00	16:00
4	1	06:00	1	22:00	16:00
5	1	10:00	2	02:00	16:00
1	1	14:00	2	06:00	16:00
2	1	20:00	2	12:00	16:00
3	2	00:00	2	16:00	16:00
4	2	04:00	2	20:00	16:00
5	2	08:00	3	00:00	16:00
1	2	12:00	3	04:00	16:00
2	2	18:00	3	10:00	16:00
3	2	22:00	3	14:00	16:00
4	3	02:00	3	18:00	16:00
5	3	06:00	3	22:00	16:00
1	3	10:00			
2	3	16:00			
3	3	20:00			

Table 5: Optimal schedule



## **Chapter 4:**

### **4.1. Recommendations:**

#### **4.1.1. Downtimes**

- Downtimes and breakdowns must be recorded and measured to ensure the shipping process is managed so it could operate optimally.

#### **4.1.2. Drivers**

- Drivers' availability should be recorded to ensure that the system operates optimally.
- All drivers should be well trained
- Drivers' performance should be recorded and best operating driver should be given an award encouragements

#### **4.1.3. Communication**

- Communication between clients and Transnet should be improved and revised to ensure quicker response times.
- Reports should be filled stating when each and every set arrived, departed, coupled and decouple. All faults related to shipment should be recorded accurately including delays if there is any.
- Store performance data and measure performance per period and thus data for future reference.

#### 4.2.4. Shift Changes

- Usually shift affect the schedules dramatically, shift changes time should always be kept at minimum.

## 4.2. Conclusion

Transnet has always been striving to solve the problem in resource scheduling and it became a huge challenge for years, much capital has been spent on trying to improve scheduling. As the schedule incorporate a number of resources which is why the whole system become very complex.

Various method of solving the problem were identified in the literature study, however applicability of some still has to be determined.

This project depicts and recommends possible methods that could be used to solve the scheduling problem. Firstly a model structure designating the routes of locomotives and trains was drawn, this structure enhance the user to understand the basic functions of the coal line export. The next step was to execute different moves till an optimal schedule is obtained that could help collecting all coal loads at possible minimum cycle time. The use of integer programming was applied for assigning resources which will enhance in determining where locomotives and wagon sets should be sent at a mine to minimize cycle time.

The merits of performing the behavior of the locomotives and wagons were also discussed. It was shown user acceptance of the model may be greatly enhanced if the users are allowed to input play around with the model, ask what if questions and compare the results with reality.

These results were arranged in such a way that the user could be able to use the set up together with the time slots to accomplish maximum loads with minimum locomotives.

An optimal schedule has been accomplished per three-day period. The user will be able to see when to depart a locomotive and what could be the expected arrival time

## Chapter 5

### 5.1. References

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## Chapter 6:

### 6.1. Appendices

#### 6.1.2. Appendix A

<b>Transnet Freight Rail Statistics</b>		
<b>WAGONS</b>		
	<b>Fleet Size</b>	<b>Available For Use</b>
GFB	90107	66451
Coal Line	8012	7327
Ore Line	4556	4071
<b>Total</b>	<b>102675</b>	<b>77849</b>

<b>Transnet Freight Rail Statistics</b>		
<b>LOCOMOTIVES</b>		
	<b>Fleet Size</b>	<b>Available For Use</b>
GFB	2012	1714
Coal Line	310	285
Ore Line	127	107
<b>Total</b>	<b>2449</b>	<b>2106</b>

Table 6: Available Resources for TFR

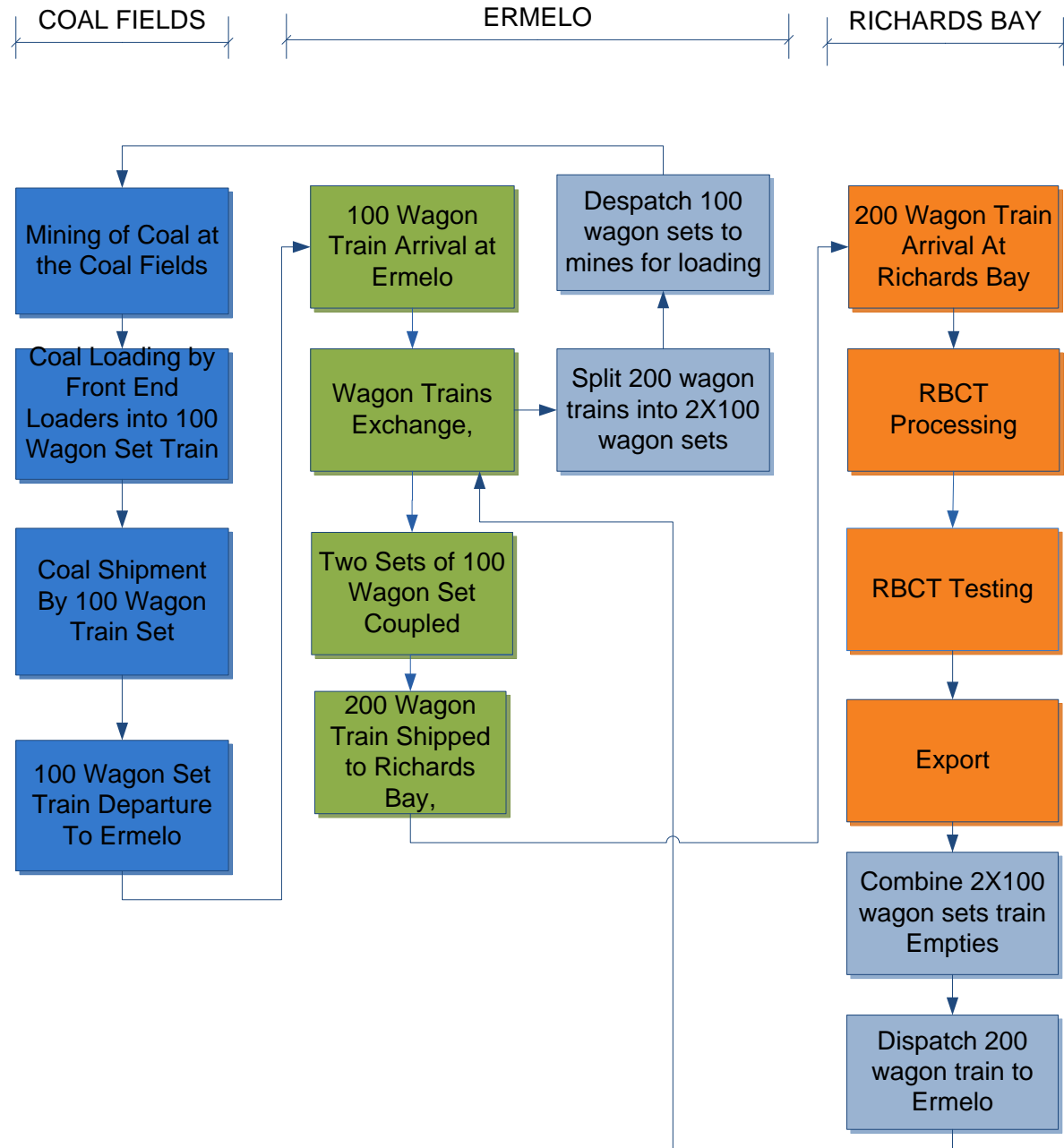
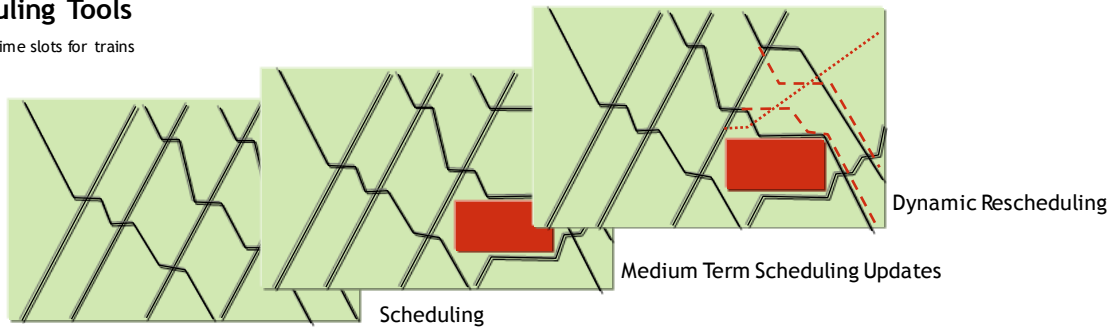


Figure 6: Shipment of Coal Wagons and Empties at the Coal Line Exports

## Scheduling and Allocation

### Scheduling Tools

Create time slots for trains



### Allocation Tools

Intelligent allocation of demand to slots

- Consolidation of “less than train load” flows in order to minimise *en route* shunting
- Allocate to pre-scheduled slots

MultiRail

- Variable demand for unit train block loads
- Allocate to pre-scheduled slots
- Deviation management based on tracking of actual movements

Plato-R

Figure 7: Current Schedule Allocation for TFR



### Single line schedule

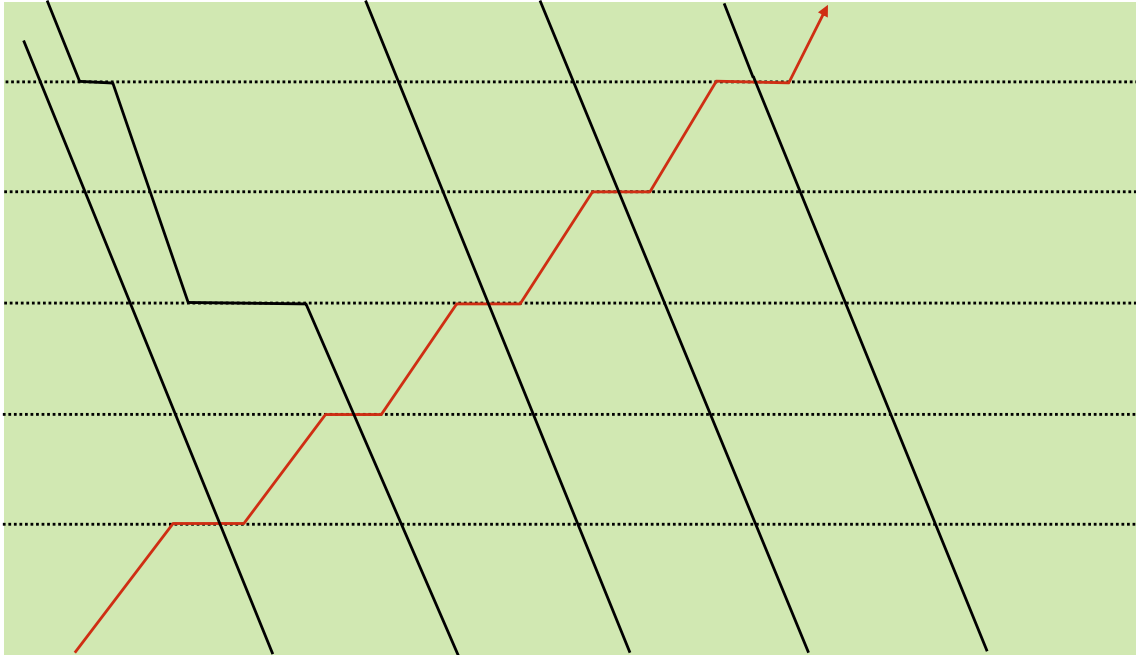


Figure 8: Single line Schedule for TFR



**Figure 9: Front End Loader loading a 100 wagon train**



Figure 10: loaded Set of wagons

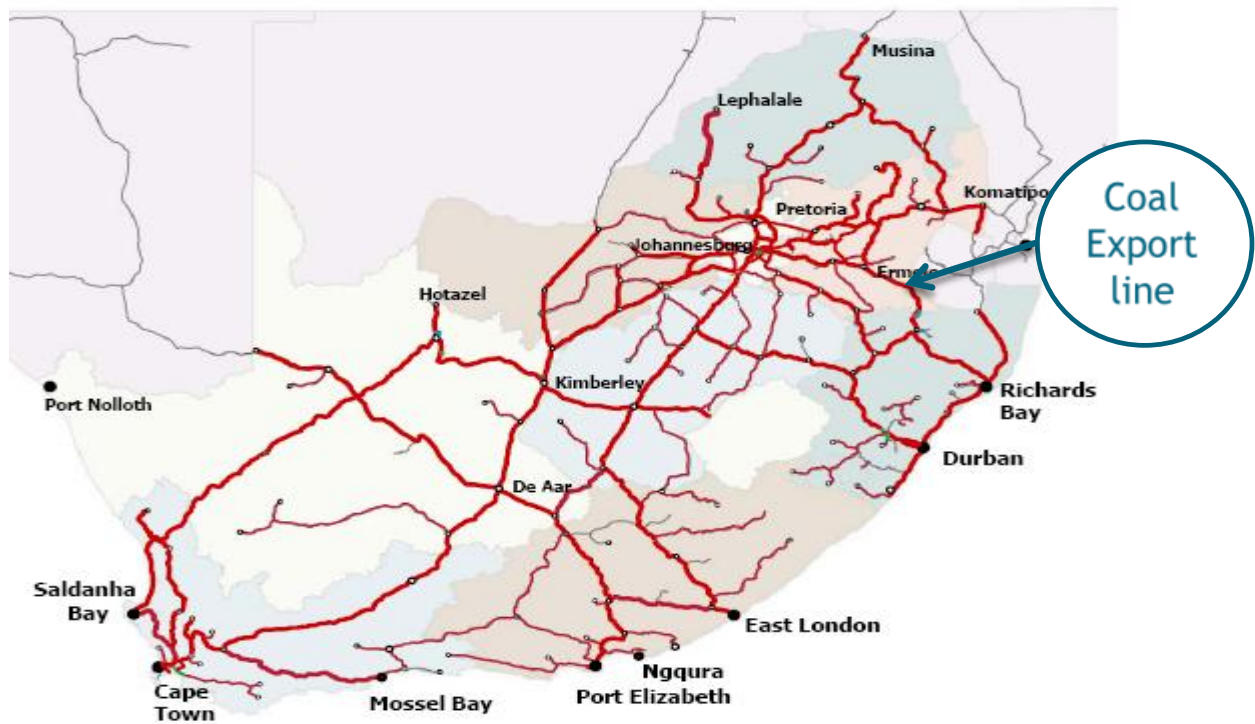


Figure 11: Coal export line



**Figure 12: Coal export line Rail Track**

## 6.2. Appendix B: Lingo Codes

### 6.2.1. Lingo codes: Wagon-sets Assignment

Model:

```
!Lingo code for wagon-sets assignment between available mines  
and wagon-sets;
```

```
!Creating two sets with 5 wagon-sets and 3 operating mines;
```

Sets:

```
wagonsets/1..5/;
```

```
mine/1..3/;
```

```
links(wagonsets,mine):load,assign;
```

endsets

```
!Data available represent the number of loads available at the  
mines;
```

data:

```
load=2 1 1
```

```
    1 2 0
```

```
    1 0 2
```

```
    1 1 1
```

```
    1 1 1;
```

enddata

```
!The Objective is maximise the number of loads;
```

```
max=@sum(links:load*assign);
```

```
!Subject to;;
```

```
!Wagon-sets #1 must be assigned to mine 1 to 3;
```

```
@for(wagonsets(i):  
    @sum(mine(j):  
        assign(1,j))  
        = 3);
```

```
!Wagon-sets #2 must be assigned to mine 1 to 3;
```

```
@for(wagonsets(i):  
    @sum(mine(j):  
        assign(2,j))  
        = 3);
```

```
!Wagon-sets #3 must be assigned to mine 1 to 3;
```

```
@for(wagonsets(i):  
    @sum(mine(j):  
        assign(3,j))  
        = 3);
```

```
!Wagon-sets #4 must be assigned to mine 1 to 3;
```

```
@for(wagonsets(i):  
    @sum(mine(j):  
        assign(4,j))  
        = 3);
```

```
!Wagon-sets #5 must be assigned to mine 1 to 3;
```

```
@for(wagonsets(i):  
    @sum(mine(j):  
        assign(4,j))
```

```

    = 3);
!Assign loco 1,2 3 to mine 1 respectively;

assign(1,1)=1;
assign(2,3)=0;
assign(3,2)=0;

!Binary viriable;
@for(links:@bin(assign));
end

```

## Solution:

Global optimal solution found at iteration: 0  
**Objective value:** 16.00000

Variable	Value
LOAD( 1, 1)	2.000000
LOAD( 1, 2)	1.000000
LOAD( 1, 3)	1.000000
LOAD( 2, 1)	1.000000
LOAD( 2, 2)	2.000000
LOAD( 2, 3)	0.000000
LOAD( 3, 1)	1.000000
LOAD( 3, 2)	0.000000
LOAD( 3, 3)	2.000000
LOAD( 4, 1)	1.000000
LOAD( 4, 2)	1.000000
LOAD( 4, 3)	1.000000
LOAD( 5, 1)	1.000000
LOAD( 5, 2)	1.000000
LOAD( 5, 3)	1.000000
<b>ASSIGN( 1, 1)</b>	<b>1.000000</b>
<b>ASSIGN( 1, 2)</b>	<b>1.000000</b>
<b>ASSIGN( 1, 3)</b>	<b>1.000000</b>
<b>ASSIGN( 2, 1)</b>	<b>1.000000</b>
<b>ASSIGN( 2, 2)</b>	<b>1.000000</b>
<b>ASSIGN( 2, 3)</b>	<b>0.000000</b>
<b>ASSIGN( 3, 1)</b>	<b>1.000000</b>
<b>ASSIGN( 3, 2)</b>	<b>0.000000</b>
<b>ASSIGN( 3, 3)</b>	<b>1.000000</b>
<b>ASSIGN( 4, 1)</b>	<b>1.000000</b>
<b>ASSIGN( 4, 2)</b>	<b>1.000000</b>
<b>ASSIGN( 4, 3)</b>	<b>1.000000</b>

---

<b>ASSIGN( 5, 1)</b>	<b>1.000000</b>
<b>ASSIGN( 5, 2)</b>	<b>1.000000</b>
<b>ASSIGN( 5, 3)</b>	<b>1.000000</b>

Row	Slack or Surplus
1	0.000000
2	-1.000000
3	-1.000000
4	-1.000000
5	-1.000000
6	-1.000000
7	-1.000000
8	-1.000000
9	-1.000000
10	-1.000000
11	-1.000000
12	-1.000000
13	-1.000000
14	0.000000
15	0.000000
16	0.000000
17	0.000000
18	0.000000
19	0.000000
20	0.000000
21	0.000000
22	0.000000
23	0.000000
24	0.000000
25	0.000000
26	0.000000
27	0.000000
28	0.000000
29	0.000000



## 6.2.2. Lingo Codes: Locomotives assignment

Model:

```
!Lingo code for locomotives assignment between available mines  
and locomotives;
```

```
!Creating two sets with 4 locos and 3 operating mines;
```

Sets:

```
loco/1..4/;  
mine/1..3/;  
links(loco,mine):load,assign;  
endsets
```

```
!Data available represent the number of loads available at the  
mines;
```

data:

```
load=2 1 1  
      2 1 1  
      1 2 1  
      1 1 2;
```

```
enddata
```

```
!The Objective is to maximise the number of loads;
```

```
max=@sum(links:load*assign);
```

```
!Subject to;;
```

```
!Locomotive #1 must visit mine 1 to 3;
```

```
@for(loco(i):
    @sum(mine(j):
        assign(1,j))
        = 3);

!Locomotive #2 must visit mine 1 to 3;

@for(loco(i):
    @sum(mine(j):
        assign(2,j))
        = 3);

!Locomotive #3 must visit mine 1 to 3;

@for(loco(i):
    @sum(mine(j):
        assign(3,j))
        = 3);

!Locomotive #4 must visit mine 1 to 3;

@for(loco(i):
    @sum(mine(j):
        assign(4,j))
        = 3);

!Assign loco 1,2 3 to mine 1 respectively;

assign(1,1)=1;
assign(2,1)=1;
assign(3,2)=1;
```

```

assign(4,3)=1;

!Binary viriable;

@for(links:@bin(assign));
end

```

## Solution:

Global optimal solution found at iteration: 0  
**Objective value:** 16.00000

Variable	Value	Reduced Cost
LOAD( 1, 1)	2.000000	0.000000
LOAD( 1, 2)	1.000000	0.000000
LOAD( 1, 3)	1.000000	0.000000
LOAD( 2, 1)	2.000000	0.000000
LOAD( 2, 2)	1.000000	0.000000
LOAD( 2, 3)	1.000000	0.000000
LOAD( 3, 1)	1.000000	0.000000
LOAD( 3, 2)	2.000000	0.000000
LOAD( 3, 3)	1.000000	0.000000
LOAD( 4, 1)	1.000000	0.000000
LOAD( 4, 2)	1.000000	0.000000
LOAD( 4, 3)	2.000000	0.000000
<b>ASSIGN( 1, 1)</b>	<b>1.000000</b>	<b>0.000000</b>
<b>ASSIGN( 1, 2)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 1, 3)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 2, 1)</b>	<b>1.000000</b>	<b>0.000000</b>
<b>ASSIGN( 2, 2)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 2, 3)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 3, 1)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 3, 2)</b>	<b>1.000000</b>	<b>0.000000</b>
<b>ASSIGN( 3, 3)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 4, 1)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 4, 2)</b>	<b>1.000000</b>	<b>-1.000000</b>
<b>ASSIGN( 4, 3)</b>	<b>1.000000</b>	<b>0.000000</b>

Row	Slack or Surplus	Dual Price
1	16.00000	1.000000
2	0.000000	0.000000
3	0.000000	0.000000
4	0.000000	0.000000
5	0.000000	0.000000
6	0.000000	0.000000
7	0.000000	0.000000
8	0.000000	0.000000
9	0.000000	0.000000

10	0.000000	0.000000
11	0.000000	0.000000
12	0.000000	0.000000
13	0.000000	0.000000
14	0.000000	0.000000
15	0.000000	0.000000
16	0.000000	0.000000
17	0.000000	0.000000
18	0.000000	2.000000
19	0.000000	2.000000
20	0.000000	2.000000
21	0.000000	2.000000