Modelling efficiency with farm-produced inputs: dairying in KwaZulu-Natal, South Africa[1]

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Abstract

This paper models dairy farms in KwaZulu-Natal, South Africa, emphasising the complexities unique to this multi-product industry. Net and gross output approaches to measuring production are discussed and then tested using panel data from 37 dairy farms in KwaZulu-Natal from 1999 and 2007. Production functions for the three outputs: milk production, animals and farm-produced feed, are fitted as a simultaneous system to model the farms' production activities. This simultaneous model is complemented by a single equation reduced form that is fitted as a frontier, which allows estimation of the relative efficiencies of the individual farms. The results show that, with data this detailed, it is possible to refine the model until it fits very tightly. Indeed, in the gross output model that includes cows there is nothing left to call inefficiency and what was clearly a frontier becomes a mean response function.

Keywords: Dairy farms; production; frontiers; efficiency

1. Introduction

This object of this paper was to fit production functions and frontiers to a panel of dairy farms in KwaZulu-Natal, South Africa. For these farmers the basic output is milk production, but in order to produce milk, cows must lactate and reproduce to supply the calves to maintain the herd. Then, both the male calves and the cows that are past breeding age are sold, the former to ranchers to raise as beef animals and the latter for low-quality meat or as lobola, which is the dowry paid by the groom in African weddings. Thus, dairying is a dual output activity. A third output is feed, as although farmers purchase feed, they also grow much of their own cattle feed and sell any

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surpluses. These secondary outputs are not major income sources, but they are too large to simply ignore when measuring farm level efficiencies and therefore need to be included in estimates of the production frontiers. There is also an issue of whether to count only final outputs as they leave the farm, or to include farm-produced inputs that are consumed on the farm. Thus, the herd, or changes in the herd, and farm-produced and used feed may or may not be included.

The next section provides a background to the dairy industry in South Africa, and in KwaZulu-Natal in particular. Section three develops these joint production relationships further to produce a system of equations that takes account of the farm-produced intermediate outputs. In section four, the conceptual framework is applied to an unbalanced panel of data for 37 farms in the Midlands district of KwaZulu-Natal, from 1999 to 2007. The system of equations establishes that the structure of production is supported, but to obtain farm-level efficiencies it is necessary to estimate a single reduced form equation. A stochastic frontier model allows the simultaneous estimation of the production function and an explanation of farm-level inefficiencies using the characteristics of the individual farms. This section includes specification tests to determine the preferred model and the results, which are analysed with a concentration on the effects of farm size on efficiency. The final section provides a conclusion.

2. Background to the dairy industry in South Africa

In many developed countries, productivity in the dairy industry has increased substantially. This can be explained by a number of factors, including new management practices and veterinary and biological technologies such as artificial insemination and embryo transfer procedures, improved quality feed, better pasture management, machine milking, and higher levels of animal disease control. All of these have contributed significantly to the increase in yield per cow. For example, milk production increased by 22% in the Organisation for Economic Cooperation and Development (OECD) countries between 1992 and 1996 (OECD, 1993). This was achieved by a 49% increase in productivity, which was more than sufficient to compensate for the steady decline in the number of cows over this period.

2.1 South African dairying

This trend is reflected in South Africa, where milk production has been increasing while the national dairy herd has been declining (FAO, 2005; MPO, 2008). South Africa has a relatively poor resource base. Only about 15 million hectares, or 12% of the land area, is under cultivation, and only about 10% of this is under irrigation. Furthermore, the climate is unstable. Generally, the best rainfall is in the Western Cape surrounding Cape Town, along the coast of KwaZulu-Natal, in the Eastern Cape and in Mpumalanga. The rest of the country is relatively dry, and much of the arid Northern Cape is suitable only for grazing sheep (McKenzie & Vink, 1989; Vink, 2003).

The dairy industry is the fourth largest agricultural industry in South Africa, representing 6% of the gross value of agricultural production (WESGRO, 2004). During the 2000/2001 season, the dairy industry was one of the fastest growing agricultural sectors in South Africa, increasing by 17% compared to a decline in gross income of 9% in the red meat industry (Coetzee, 2002). The gross value of milk produced during the 2002/2003 production season (March-February), including milk for own consumption on farms, was estimated at nearly R4 billion (Republic of South Africa, 2003:54). However, the retail value of the total dairy industry is estimated at around R7 billion annually. More than 65% of dairy products are distributed through hypermarkets, supermarkets and smaller local stores. The dairy industry is also important to the South African economy in terms of employment, with more than 4 000 milk producers directly employing about 60 000 farm workers and indirectly a further 40 000 people (WESGRO, 2004).

South Africa produced some 2.37 billion litres of milk in 2007 (MPO, 2008). Most production takes place on a pasture-based system in the Western Cape, Eastern Cape and KwaZulu-Natal, although nearly five million litres of milk and 10 million kilograms of concentrated milk and powdered milk were imported in 2007 (MPO, 2007). More than one fifth of milk production is from KwaZulu-Natal (500 million litres) (MPO, 2008).

The total number of producers of fresh milk in South Africa declined from 5 348 at the end of 2001 to 3 665 by January 2008 (MPO, 2008). The largest decrease was in the Northern Cape (74.4%), while the Free State had the lowest decrease in the number of producers (23.7%), which can be attributed to an intensive campaign of producer registration (MPO, 2003). Figure 1 shows the concentration of milk production in the provinces.

Table 1: Number of producers per province, 1997 to 2008

Province	Number of producers					% change 1997-2008
	1997	2003	2006	2007	2008	
Western Cape	1 577	973	878	827	815	-48.3
Eastern Cape	717	481	422	420	407	-43.2
Northern Cape	133	67	39	37	34	-74.4
KwaZulu-Natal	648	449	402	385	373	-42.4
Free State	1 204	1 250	1067	987	919	-23.7
Northwest	1 502	819	649	596	549	-63.4
Gauteng	356	282	275	245	228	-36
Mpumalanga	866	477	407	357	302	-56.1
Limpopo	74	58	45	45	38	-48.6
Total	**7 916**	**4 856**	**4 184**	**3 899**	**3 665**	**-48.2**

Adapted from: MPO statistics

The geographical distribution of milk production is shown in Table 2. There has been a clear movement of milk production from inland to the coastal areas (KwaZulu-Natal, Western Cape and Eastern Cape). There are a number of reasons for this concentration of dairy farms along the coastal areas. Firstly, these areas are close to ports and this lowers the transportation costs of imported inputs relative to the more inland areas. Secondly, the coastal areas are more suitable for dairying, as mild temperatures and good rainfall result in high-quality natural and cultivated pastures (Republic of South Africa, 2003:54). However, the major processing plants and markets are inland (Coetzee, 2002; Ndambi & Hemme, 2009).

Table 2: Geographical distribution of milk production, 1997 and 2007

Province	% of production	
	1997	2007
Western Cape	22.9	25.3
Eastern Cape	13.8	21.8
Northern Cape	1.2	0.7
KwaZulu-Natal	15.7	21.1
Free State	18.0	12.8
Northwest	12.6	7.1
Gauteng	4.4	3.1
Mpumalanga	11.0	7.6
Limpopo	0.4	0.5
Coastal areas	52.4	68.2
Inland areas	47.7	31.8
Total	**100**	**100**

Source: MPO (2008) estimate; authors' calculations

2.2 The KwaZulu-Natal dairy industry

This study uses data from the KwaZulu-Natal Midlands. There were 381 milk producers registered with the milk producers' organisation of KwaZulu-Natal (KZNMPO) in 2007, dramatically lower than the 648 of 1997 (MPO, 2007). This is an indication of the small margins to be made out of dairying, with fewer farms producing more milk from more cows. Of the milk producers in KwaZulu-Natal, one from a previously disadvantaged background is registered with the KZNMPO, four with the national milk-recording scheme and an estimated 20 other producers are in the informal market. Most farms are largely grazing based, with irrigated ryegrass (predominantly annual, but some perennial) and dryland kikuyu, supplemented by maize silage and hay. Dairy meal is fed at the rate of an average of 7 kg per cow in milk daily, although not all farms do this (Penderis & Penderis, 2004).

Most of the milk is produced in the Mooi River, Howick, Boston, Bulwer, Underberg, and Ixopo areas, all of which are in the Midlands region, making it the most important milk-producing area in the province. This concentration of dairy farms in the Midlands is due to excellent conditions, with lower temperatures and higher rainfall, which are conducive to high growth of kikuyu in summer and ryegrass under irrigation in winter.

Since the deregulation of the industry in the early 1990s there has been radical restructuring of both the dairy production and processing sectors in an effort to improve global competitiveness. The South African dairy industry was gradually deregulated and this process was completed following the Marketing of Agricultural Products Act of 1996 (Act 47 of 1996) (Vink & Kirsten, 2000). One measure of the success of this restructuring of the processing sector, in particular, is the greater confidence in the industry. Evidence of this is the recent substantive investment of multi-nationals like Parmalat and Clover/Danone in large South African dairy companies, and the continuing presence of Nestlé.

Interestingly, production of milk per producer has been increasing on average in the province (Coetzee, 2002). The MPO (2008) estimates that 89% of the total milk produced in 2006 was sold in the formal market, and that 3% was sold informally. The residual was used for own consumption and for feeding calves on the farm. Although production per farmer has increased, costs also increased from 2001 to 2003 by an average of 44% in real terms (MPO, 2003) and this trend is continuing. Historically, it has been reported that, for the majority of dairy farmers in KwaZulu-Natal, the highest cost items were feed and labour (Gordijn, 1985).

3. Data

The data were provided by Alan Penderis of Tammac Consulting cc, Ixopo, a firm providing Midlands dairy farmers with production and marketing services. The farms are highly specialised dairy producers deriving more than 90% of their income from dairying, and the data comes from an unbalanced panel of 37 farms for 1999 to 2007, with a total of 293 farm-years.

Variables

Most of the data were in values of current Rand. These were deflated using the appropriate deflators (such as price indices and 2001 values for milk price) from the Abstract of Agricultural Statistics (Republic of South Africa, 2007), to put the series in constant 2001 Rand.

Outputs: There are three outputs, the most important of which is product income (milk), other income (sales of farm-produced fodder and other minor crops) and trading income (culled cows plus male calves, minus occasional purchases). On average, product income accounts for 87.4% of total income, followed by trading income, which is 9.2%, leaving 3.4% from other income.

Inputs: Land (in hectares) and labour, both as the number of full-time equivalent employees and the wage bill, were available. The wage bill, which quality adjusts labour, proved to have more explanatory power. Feed is a combination of farm-produced and purchased fodder. Purchased feed is the aggregate of feeds bought for cows, heifers and calves. The farm-produced feed is not actually measured, so here we used the cost of production, which is the value of seed, fertiliser, herbicides, pesticides and other costs, such as transport. Total veterinary costs include veterinary visits, medicines, artificial insemination costs, semen and other miscellaneous costs. Capital is the service flows emanating from the capital stocks, and these were constructed for two types of physical capital, namely milking machinery and equipment and other machinery, such as tractors. These are the depreciation on the capital stocks plus the running costs. Depreciation is assumed to be straight line over ten years. Running costs for the milking machinery included expenditure on electricity, repairs and maintenance of fixed improvements, such as milking sheds, insurance and other miscellaneous costs, while for other machinery these included fuel, lubricants, tractor repairs and maintenance, implements repair and maintenance, and other miscellaneous running costs.

The descriptive statistics for these data are in Table 3. Although there is plenty of variation to allow estimation, the differences are not huge, which is not surprising, as the farms are heterogeneous and from the same area. As noted in the table, the largest farm is less than six times the size of the smallest, although there is more variation in most of the other variables.

Table 3: Descriptive statistics (in 2001 Rand) of the variables used in the analysis

Variable	Mean	Std. deviation	Minimum	Maximum
Value of output (Rand)	266,529	138,658	266,529	138,658
Land (hectares)	205	76	205	76
Labour (full-time equivalent workers)	21	7	21	7
Labour (total wages – Rand)	23,704	11,932	23,704	11,932
Feed (rand)	166,288	149,515	166,288	149,515
Veterinary services and medicines (Rand)	31,561	40,728	31,561	40,728
Tractors and equipment (Rand)	37,781	57,958	37,781	57,958
Milking machinery (Rand)	24,559	35,556	24,559	35,556
Herd size	289	103	289	103
% Cows not in milk	18	8	0	44

4. Modelling the efficiency of dairy farms

4.1 Literature review

The literature on production frontiers and efficiency in agricultural economics was reviewed by Battese (1992), who lists a formidable number of studies dating back to the 1960s. Seven of the empirical papers are applications to dairy farming. In chronological order they are Russell and Young (1983) and Dawson (1985), who studied farms in the north-west of England, Battese and Coelli (1988), who studied a panel of Australian farms, Bravo-Ureta (1986) and Bravo-Ureta and Rieger (1990), who studied farms in New England, USA, and Bailey *et al*. (1989), who studied Utah dairying and farms in Ecuador. Two countries have been added since this survey, with Hallam and Machado's (1996) study on Portugal and Alvarez and Arias (2003) on Spain.

A common feature of these studies is an interest in economies of scale and an ambivalent attitude towards specification of the frontier. This often seems to be determined by the data. For instance, the last two papers use the EU Farm Analysis Data Network (FADN), which reports milk output only and inputs of cows, land, labour and purchased feed. Russell and Young (1983) show most concern with the issue of other outputs, using the aggregate value of crop and livestock outputs, by-product and forage output, changes in livestock and crop valuations and miscellaneous revenue. Again, the determining factor is the data, which allows this type of gross output basis with on-farm crop and livestock production included. The inputs vary in a similar manner, sometimes including even fewer variables than the FADN studies. For instance, only labour, feed concentrate and forage feed (per hectare) are included by Bravo-Ureta and Rieger (1990), whereas Ahmad and Bravo-Ureta (1995) add cows and animal, crop and other expenses.

There is a sense in which it is better to be limited by the data in this way. The farm accounting data used in this study allow so many possible approaches that the specification becomes a major concern. All the final outputs can be included, which is a positive, but what of farm-produced outputs that are consumed on the farm? The breeding herd on a dairy farm comes into this category. Cows, in some form, such as gross investment, would be included if a gross output approach is taken.

4.2 Net and gross output and input approaches

The literature reviewed above do not discuss the issue of matching up outputs and inputs on an annual basis as accurately as possible, but the literature on the national income accounts for agriculture that are used in productivity analysis do this. The fullest account of the handling of different categories of outputs and inputs is in Thirtle and Bottomley (1992). The key concept in the net output approach is that, when an input purchased off the farm crosses the farm gate it is recorded as an input, whereas outputs are only recorded when the product leaves the farm to enter the non-farm economy. Thus, the rules of double entry bookkeeping guide the specification of the dairy production function.

Animal feed produced on the farm and consumed by farm animals does not appear either as an input or an output. It is the milk or meat resulting from the feed consumption that later cross the farm gate that is an output, which is useful, as feed produced and consumed on the farm is usually not recorded. Thus, in this study only surplus feed that is sold and so leaves the farm is recorded as an output. The other item affected is cows, which, apart from occasional purchases, are a farm-produced capital stock that is consumed on the farm rather than sold. The culled cows do sell for small sums, but this is really just scrap value at the end of the service life of the animals. All the inputs used to produce and maintain the herd are already counted, so cows should definitely not be included under the net outputs accounting approach. For one thing, to do so would mean double counting and second, the herd is a capital stock and the appropriate input would be the flow of services, which is depreciation at 10% per annum, for a ten-year life with straight line depreciation.

However, the net accounts were correct without this item, so if the gross approach is used instead, this extra depreciation item on the input side of the accounts must be matched by farm-produced output on the other side of the account. In any year, the value of the new cows added to the herd is gross investment. Net investment, which is the change in the value of the herd, is gross investment minus depreciation, as equation 1) shows.

$$\Delta Herd_t = Gross\ Investment_t - Depreciation_t \qquad (1)$$

Thus, it can never be correct to simply include the number of cows as an input and, if the depreciation of the herd were included, then the value of gross investment should be included as an output. It is because cows are a farm-produced input that this balancing term is needed, whereas it is not needed for a purchased capital item like machinery.

There is another convention that should be observed if the calculations are to be accurate. Since all the data are recorded on an annual basis, if an input or output crosses the boundary from one reporting year to the next, this should be taken into account, or items will not match up in the panel. Thus, it will transpire that veterinary services and other machinery have a greater impact on output when lagged one year. Perhaps the veterinary expenses produce an animal that does not begin producing until the next year and the tractor contributes to a crop that is feed for the next year.

The question of which approach to follow is not one that can be answered in theoretical terms. The net approach has the advantage of being simpler and less subject to errors regarding farm-produced outputs that are also inputs. However, there is something unsatisfactory about not including the size or value of the dairy herd as an input in the production of milk. Thus, although including the herd as an input results in double counting unless it is also included as an output, it may be preferable to take this pragmatic step in the modelling. All of these issues will be addressed in the process of estimation and interpretation that follows.

4.3 Specification of outputs and inputs of dairy farms: a framework and some estimates

In this analysis of dairying, these concepts can be applied to show the different models that result. The variables were aggregated, as described in Section 3, as the production functions need to be limited to a reasonably small number of outputs and inputs. To begin with the net approach, the categories that proved best in estimation are in Table 4, with their inter-relationships presented in Figure 1.

Table 4: Output and inputs – net modelling approach

Outputs	Inputs		
	Basic	**Intermediate**	**Capital**
Milk and milk products	Land	Veterinary services/ medicines	Running costs of milking machines
Surplus feed and other crops	Labour	Purchased feed	Running costs of other machinery
Meat and culled animals			

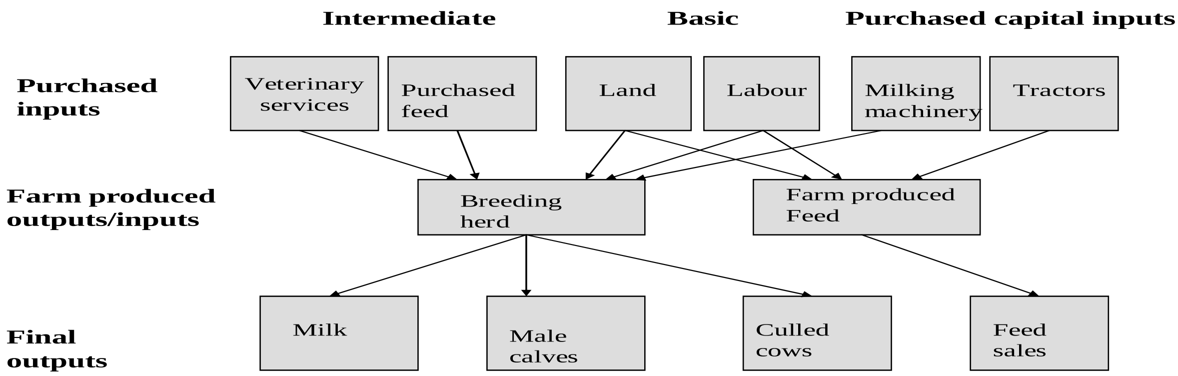

Figure 1: A Midlands dairy farm

The difference between the gross and net basis for the calculations is quite clear. Only if some kind of gross accounting approach is used should cows be treated as an input. If each of the three outputs is treated as exogenous, then the structure in Figure 1 can be expressed as a system of three equations, with a production function for each, as follows:

$$\text{Milk}_t = h(\text{cows}_{mt}, \text{land}_{mt}, \text{labour}_{m(t\text{-}1)}, \text{milk machinery}_{mt}, \text{veterinary}_{m(t\text{-}1)}, \text{feed}_{mt}) \quad (2)$$

$$\text{Cows}_t = g\left(\text{land}_{h(t-1)}, \text{labour}_{h(t-1)}, \text{machinery}_{ht}, \text{feed}_{ht}\right) \quad (3)$$

$$\text{Feed}_t = f(\text{land}_{f(t\text{-}1)}, \text{labour}_{ft}, \text{machinery}_{ft}, \text{fertiliser}_{ft}, \text{seed}_{ft}, \text{chemicals}_{ft}, \text{transport}_{ft}) \quad (4)$$

The independent variables here are based on expectations, and not all proved to be significant in the estimation. The feed equation is less well defined, as the value of feed produced on the farm was not recorded and had to be estimated from the expected consumption of the herd minus the purchased feed. This system was estimated using three stage least squares to allow for unobserved inter-linkages between the outputs.

The results are reported in Table 5. The milk equation has the greatest explanatory power, with 70% of the variance explained. As the variables are all in logarithms, the coefficients can be interpreted as output elasticities. All have the appropriate sign and are significantly different from zero in one-tailed tests, which are appropriate as the theory confines elasticities to be between zero and unity. Note that, with cows included, this is the variable with the dominant elasticity of 0.35. This means that a 1% increase in cows alone increases output by 0.35%. This is perhaps not surprising in milk production: if it is correct to include cows they emerge as the most important input, followed by land and then milking machinery. The proportion of cows that are dry, that is, not producing milk, has a significant but small negative effect. Last, the sum of the elasticities is 0.96, which is reassuringly close to unity, which would mean constant returns to scale.

Table 5: Results of estimating a three-equation dairy model

Regressors	Coefficient	t statistic	R^2
	Equation 2 - Dependent variable: Ln milk		
Ln land	0.2375	3.77	
Ln labour (-1)	0.0850	1.42	Sum of elasticities
Ln feed	0.0707	1.94	=0.958
Ln milking machinery	0.1482	8.58	
Veterinary costs (-1)	0.0594	3.41	
Ln cows	0.3572	1.84	
Ln dry cows	-0.0087	-3.45	
Constant	-2.6856	-3.67	0.7092
	Equation 3 - Dependent variable: Ln cows		
Ln land (-1)	0.1673	3.12	
Ln labour (-1)	0.3127	6.86	
Constant	1.6589	4.48	0.3192
	Equation 4 - Dependent variable: Ln feed		
Ln chemicals	0.0059	1.84	
Ln transport and other costs	0.1134	3.89	
Ln fertiliser	0.0184	5.82	
Ln land (-1)	0.3284	1.37	
Constant	4.3476	1.89	0.2632

In equation 3, with herd size as the dependent variable, land and labour, both lagged one period, alone explained 32% of the variance in herds, but machinery, veterinary costs and feed were all insignificantly different from zero. Equation 4 models feed production, but only succeeds in explaining 26% of the variance, perhaps because the constructed dependent variable is not accurate. As the land grows feed that is consumed in the next year, the lag on land is acceptable. Chemicals and fertiliser contribute to the feed crop and transport costs, especially fuel used in harvesting, but no lags could be established for these inputs. Last, labour and machinery were not significantly different from zero. Perhaps expecting to catch the labour allocation with a constructed dependent variable is too optimistic, but machinery may well fail because fuel has already picked up the effect of machinery use.

The results of this system of equations are reassuring, as it seems the relationships do work well in combination. This system of multiple equations adds to our understanding and shows that it is possible to model the underlying gross output approach to estimation where the farm-produced inputs appear as outputs as well. If the feed equation is dropped, cows in milk and feed can be added to the cows equation. Without cows in milk, the R^2 is 0.4 and with it is 0.84.

However, whilst this line of enquiry is of interest, panel techniques and frontier production functions can give more information, especially about the efficiency of the individual farms, and this approach is used in the next section. The net approach is used first, as it presents less problems. Since it is a single equation model, the three outputs need to be aggregated, which is the cost of taking this route, but it is outweighed by the benefits. Frontiers are used for the final models because the tests indicate that they are appropriate, rather than mean response functions or ordinary least squares (OLS).

Without cows in the equation, the preferred translog model with farm effects was the random effects model, in which the R^2 for the within estimator was 0.59, while that for the between estimator was 0.75, giving an overall R^2 of 0.69. All six inputs were significantly different from zero at the 5% confidence level; the sum of these elasticities was 0.66 and the Wald statistic was 392.2. With cows added, the same model was still preferred, and it was improved. The R^2 for the within estimator was 0.57, while that for the between estimator was 0.89, giving an overall R^2 of 0.79. All inputs were again significant, with the sum of these elasticities being 0.996 and the Wald statistic 810.6. Thus, the theory says cows should not appear as an input, but all the statistical tests say that they improve the estimates. Particularly, the sum of the elasticities, which suggests constant returns, is far more reasonable than the hugely diminishing returns result when cows are not included.

A two-way error components model, which adds a period effect, retains the entirely acceptable results and give a small increment in terms of the statistical measures. Without cows, the adjusted R^2 is 0.86 and the likelihood statistic is 167.30, but with cows, the R^2 reaches 0.89 and the likelihood statistic is 177.74. The likelihood ratio test also favours the model that includes cows. These models are not reported more fully, as the frontier estimates are preferred, as the next section shows.

5. Stochastic frontier models

5.1 Theory

The measurement of firm-level technical efficiency has become commonplace with the development of frontier production functions. The approach can be deterministic, where all deviations from the frontier are attributed to inefficiency, or stochastic, where it is possible to discriminate between random errors and differences in inefficiency. The stochastic frontier model was originally proposed by Aigner *et al*. (1977), and extended by Battese and Coelli (1995) to include the characteristics of the firm that explain the inefficiency. This approach allows the use of panel data and is estimated such that the technical inefficiency effects are specified as factors that interact with the input variables of the frontier function. The theory is described in Coelli (1995) and Coelli *et al*. (1998), and many applications are discussed in Bravo-Ureta and Pinheiro (1997). The estimating equation is

$$y_{it} = f(x_{j,it}, t, \beta) + \varepsilon_{it} \quad where\ \varepsilon_i = V_{it} - U_{it},\ with\ U_{it} \sim |N(\mu_{it}, \sigma_U^2)|\ and\ V_{it} \sim N(0, \sigma_V^2) \qquad (5)$$

where f(.) is a suitable functional form, y_{it} is the output of farm *i* at time t, $x_{j,\,it}$ is the corresponding level of input *j*, and β is a vector of parameters to be estimated. The V_{it}'s are independently and identically distributed random errors and uncorrelated with the regressors, and the U_{it}'s are non-negative random variables associated with the technical inefficiency of farm *i* at period *t*. In the second part of the model, this inefficiency term, U_{it}, is made an explicit function of *k* explanatory variables, $z_{k,it}$, which represent the characteristics of the farms. The U_{it} are independently (but not identically) distributed as non-negative truncations of the normal distribution of the form

$$U_{it} \sim N\left[\delta_0 + \sum_{k=1}^{M} \delta_k z_{k,it}, \sigma^2\right] \qquad (6)$$

The technical efficiency of each farm is defined as the ratio of the observed output to the corresponding frontier output, conditional on the inputs used and expressed in terms of the errors

$$TE_{it} = E[\exp(-U_{it})|(V_{it} - U_{it})] \qquad (7)$$

which is the expectation of the exponentiated technical inefficiencies, conditional on the error, ε_{it}. Since U_{it} is a non-negative random variable, these technical efficiencies lie between zero and unity, where unity indicates that this farm is technically efficient. The variables are in logarithms, except the time trend, and mean differenced to allow direct estimation of the elasticities, evaluated here at the mean.

5.2 Estimation and results

The model in equation (5) was estimated using both Cobb Douglas and translog specifications, with the model selection based on a series of hypothesis tests using generalised likelihood ratio (LR) tests. These results are available from the authors.

Table 6 reports the maximum likelihood estimates of the elasticities for the three selected models. These includes the translog frontier production function and the inefficiency terms, which will be discussed in conjunction with the inefficiencies themselves. For the net outputs approach in Model I, the frontier terms are all significant at the 5% confidence level or better, except for land, which is barely significant only at the 10% level. If herds are not included in the inefficiency model, the elasticity for land increases to 0.203 and its t statistic is 3.75. Thus, it seems fair to say that it is the double counting problem that is responsible for this one weak result. In addition, four squared terms and four cross products are significant, as seen in Table 6. The negative signs on the squared terms indicate decreasing returns to veterinary services and milking machinery, whereas there is evidence of increasing returns to tractors and feed. The elasticities on the direct frontier terms in Table 6 sum to 0.63, which suggests that the farms are subject to decreasing returns to scale. In fact, farm size has been increasing substantially for some time in the Midlands, due to the small margins on dairy production and consequent consolidation.

Four variables are included to explain the farm-level efficiencies. Firstly, the positive sign on the year variable indicates that time increases the inefficiency levels, although the coefficient is very small, at 0.04% per year. Not surprisingly, capital investment has a positive impact on these farms, while the size of the herd also contributes to higher levels of efficiency, which is contrary to the dubious returns to scale result reported above. Lastly, the proportion of the herd that is dry lowers efficiency, which is obvious, although clearly this is necessary to ensure the future structure of the future herd is maintained.

The gross output approach in Model II includes cows fully in the translog function. The direct terms now sum to 1.025, which supports the more sensible proposition of slightly increasing returns to scale. All the elasticities are significant, except that for land, which is rendered totally insignificant by the full inclusion of cows and the double counting this implies. Table 6 shows that the t test on the value of gamma indicates that this model was a frontier when no inefficiency terms were included.

Table 6: Stochastic frontier maximum likelihood estimates

	Model I No cows		Model II Cows, no inefficiencies		Model III, both, with cows	
Regressor	coefficient	t stat	coefficient	t stat	coefficient	t stat
Constant	0.3593	4.1895	0.2011	2.5593	0.3027	1.9480
Cows			0.3913	3.7710	0.0154	0.0733
Land	0.0765	1.2931	0.0638	0.9392	0.1255	2.3567
Labour	0.2092	3.6741	0.2102	3.4880	0.1072	2.2310
Feed	0.1496	3.6288	0.1702	3.7319	0.1293	3.8725
$Veterinary_{t-1}$	0.0694	3.2546	0.0705	3.3842	0.0530	3.0533
Milk machinery	0.0597	2.5874	0.0714	3.0756	0.0761	3.8374
Tractors	0.0605	2.1037	0.0472	1.6070	0.0301	1.3040
Inefficiency effects						
Constant					0.7611	2.3525
Year	0.0004	6.7884				
Capital investment	-0.0063	-1.3312			-0.0057	-1.6966
% dry cows in herd	0.0109	3.9589			0.0095	3.9910
Herd size	-0.0024	-4.3792			-0.0022	-3.1698
Likelihood statistic	21.6		52.9		65.3	
gamma	0.5200	3.4583	0.5429	1.6831	0.3865	1.9897

In Model III, a reduced translog specification was re-estimated using the inefficiency variables in Model I. The results are similar to those for Model II, except that land rather than cows is now significant, which is negated by having herd size as an inefficiency variable. However, the number of cows is important in explaining inefficiency, as well as the proportion of dry cows and capital investment, as in Model I. If herd size is left out of the inefficiency, the only result that changed significantly is the elasticity on cows, which rises to 0.488 and is significant, but land retains its elasticity and significance. Likelihood ratio tests showed that Model II is preferred to Model I, but Model III is preferred to both. The conclusion to the debate on including cows in the frontier can be resolved in this way. The herd size does not belong in the frontier, but it does an excellent job of improving the model when it is used in the inefficiency terms to establish that there are increasing returns to scale.

A summary of the farm-level efficiencies are reported by year in Table 7. The year coefficient in the inefficiency model for Model I in Table 6 indicates a negative time trend; however, it is now clear why this is the case, as there is not a monotonic increase in efficiencies over the period, although the overall trend is positive. The mean efficiencies have risen, except in 2002 and 2003, but the dispersion is greater, falling for the first four years and then rising for the second four. For example, the maximum value in 2000 was 90.4% efficiency, but this rose to 96.9%. This best practice farm only has scope for a 3.1% possible improvement by the end of the period.

In Model II, the mean efficiencies are higher, which is not surprising, as the frontier function variables account for most of the performance and there is less dispersion across the farms. The lower end of the distribution is more efficient in this model, ranging from 70.4% in 2000 to 73.1% by the end of the period. Finally, the mean efficiency levels in Model III increase annually by close to 10%.

Table 7: Farm-level efficiency levels

	Model I				Model II				Model III			
Year	Mean	St. dev	Min	Max	Mean	St. dev	Min	Max	Mean	St. dev	Min	Max
2000	0.685	0.123	0.487	0.904	0.857	0.055	0.704	0.959	0.702	0.113	0.536	0.925
2001	0.707	0.113	0.471	0.897	0.868	0.044	0.751	0.933	0.725	0.101	0.545	0.929
2002	0.705	0.117	0.480	0.942	0.857	0.049	0.727	0.939	0.734	0.108	0.532	0.961
2003	0.704	0.105	0.467	0.952	0.858	0.051	0.708	0.918	0.736	0.093	0.547	0.966
2004	0.723	0.131	0.463	0.956	0.838	0.070	0.609	0.918	0.759	0.116	0.512	0.971
2005	0.731	0.144	0.435	0.962	0.839	0.067	0.670	0.950	0.768	0.133	0.509	0.976
2006	0.744	0.141	0.504	0.969	0.854	0.060	0.649	0.934	0.778	0.127	0.532	0.980
2007	0.770	0.147	0.497	0.969	0.854	0.055	0.731	0.937	0.807	0.131	0.550	0.982
Efficiency by farm size – measured by herd size												
	Mean		St. dev		Mean		St. dev		Mean		St. dev	
Small	0.6973		0.1144		0.8599		0.0494		0.7209		0.1049	
Medium	0.7193		0.1252		0.8464		0.0629		0.7575		0.1189	
Large	0.7454		0.1412		0.8508		0.0602		0.7761		0.1229	

Finally, the lower section of Table 7 examines whether the size of the farm, measured by herd size, has an impact on efficiency. The farms were ranked and the sample divided into three groups. The means and variances of their associated efficiencies were then computed. The results corroborate the increasing returns result in the inefficiency models, showing that, in Models I and III, the large farms on average are more efficient than the medium ones, and these are more efficient than the small ones. These two models include herd size in the inefficiency terms and pick up this effect. Model II does not have inefficiency variables and so shows no scale effect.

6. Conclusion

This paper examines the specification of production functions in dairying, using a sample of farms in the Midlands of KwaZulu-Natal, South Africa. In most empirical studies of efficiency in dairy farming, the output and input variables included have been determined largely by data availability. Often the only output is milk, and less important outputs like farm-grown feed, culled cows and male calves that are sold have been ignored. Perhaps the most interesting issue is the inclusion of the dairy herd as an input, which is done in some studies but not others, with the decision again based mostly on data availability. In this paper there is detailed farm accounting data for nine years on 37 dairy farms. The purchased inputs are land, labour, feed, veterinary expenses, milking machinery and other machinery. The final outputs are milk, surplus feed that is sold and beef animals, that is, the value of culled cows and male calves, minus the cost of any dairy cows purchased.

These variables are used in the net outputs approach, which avoids using farm-produced inputs. Thus, cows do not appear as an input, as although it is not possible to produce milk without them, they are both produced and consumed on the farm and so can be subtracted from both sides of the farm accounts. Cows would only appear in the specification if a gross output approach is used, and then the gross investment in the herd (new animals added) should also appear as an output and be balanced on the input side by depreciation of the herd, not the entire stock of animals.

The gross approach is modelled first as a system of three simultaneous equations, for milk, cows and feed. This is followed by single equation models that allow production frontiers to be fitted, which give efficiency estimates for each farm in every year. The statistical tests all favour the theoretically incorrect models that include cows as an input, although this is both double counting of inputs and mixing stocks and flows. The most sensible compromise between correct accounting on the one hand and better test statistics on the other is to opt for Battese and Coelli's (1995) inefficiency model, in which the herd size can be included as a variable that explains the inefficiencies, rather than in the frontier itself.

This approach results in translog models where so much is explained that what was clearly a frontier verges on becoming a mean response function because there is little residual left to define the inefficiencies. This is resolved by choosing a model in which all the inputs are significantly different from zero in the frontier estimates, while capital investment and herd size reduce the inefficiencies and the proportion of cows that are dry increases them. On the basis of these data it appears that the more efficient farms are those that are larger and have invested more in capital equipment, as well as having herds with fewer animals not in milk.

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