Testing for fractional integration in SADC real exchange rates

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Abstract

This paper utilises "a class test for fractional integration" associated with the seminal contribution of Hinich and Chong (2007) to appraise the possibility that South African Development Community (SADC) real exchange rates can be treated as long memory processes. The justification for considering fractional integration is that the general failure to reject the unit-root hypothesis in real exchange rates is caused by the restrictiveness of standard unit-root tests regarding admissible low-frequency dynamic behaviour. The paper presents evidence that a majority of SADC real exchange rates are fractionally integrated and therefore mean-reverting.

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1 Introduction

Recent work by Mokoena (2007) presents evidence that some of Africa's real exchange rates could be regarded as nonlinear globally ergodic processes, while others could be considered random walks. This paper employs "a class test for fractional integration" associated with the seminal contribution of Hinich and Chong (2007) to appraise the possibility that Southern African Development Community (SADC) real exchange rates can be treated as long memory processes. The justification for considering fractional integration arises from the general failure to reject the unit-root hypothesis in real exchange rates due to the restrictiveness of standard unit-root tests regarding admissible low-frequency dynamic behaviour. In allowing for only integer orders of integration in the series dynamics, the linear tests of nonstationarity were found by authors such as Diebold and Rudebusch (1990) to have low power against fractional alternatives.

The concept of long memory is gaining popularity in econometrics, because econometricians wish to ensure that a stationary process is not mistaken for a non-stationary or a fractionally integrated process. The introduction of the concept of long memory in time series econometrics is associated with the seminal work on fractional integration by Granger and Joyeux (1980) who, according to Smallwood (2005: 4), observed that "the spectral density function of the differenced process appeared to be overdifferenced, while the level of the series exhibited long run dependence that was inconsistent with stationary ARMA dynamics". In short, Granger and Joyeux (1980) developed the concept of long memory or fractional integration to fill the gap between a covariance stationary process and a linear autoregressive moving average (ARMA) process. The model of fractionally integrated time series allows for a fractional exponent in the differencing process of the time series, thereby avoiding the infinite-variance unit-root distinction while admitting persistence, or long memory, which characterises many macroeconomic time series. Some major works in fractional integration in the behaviour of exchange rates include Cheung (1993), Baillie (1996), Kapetanios and Shin (2003), Ballie and Kapetanios (2004), Robinson (2003), and Smallwood (2005).

The rest of the paper is organised as follows: Section 2 describes the concept of long memory. Section 3 describes the Hinich-Chong testing methodology used in the paper. Section 4 presents the results of the testing algorithm and Section 5 concludes.

2 Concept of fractional integration

According to the definition in Baillie (1996), a time series process y_t with autocorrelations ρ_k possesses a long memory if:

$$\lim_{n\to\infty}\sum_{k=-n}^{n}|\rho_k| \to \infty . \tag{1}$$

In addition, a process is integrated of order d , denoted $I(d)$, if

$$(1-L)^d y_t = u_t , \tag{2}$$

where u_t is ergodic and stationary when $-0.5 < d < 0.5$. It is noted that when u_t is $I(0)$ the process is covariance stationary. In addition, the series y_t has an invertible moving average representation when $d > -0.5$. When $d > 0$, y_t has long memory, and the autocovariances of y_t are not absolutely summable.

The autoregressive fractionally integrated moving average representation for a time series process y_t can be written as:

$$\varphi(L)(1-L)^d (y_t - \mu) = \theta(L)\varepsilon_t \tag{3}$$

where the characteristic polynomials $\varphi(z)=0$ and $\theta(z)=0$ have all their roots outside the unit circle. In this setting, $\{\varepsilon_t\}$ is a martingale difference sequence, d is a real number, and $(1-L)^d$ is the generalised factorial function of the form

$$(1-L)^d = \sum_{k=0}^{\infty}\Gamma(k-d)\{\Gamma(-d)\Gamma(k+1)\}^{-1}L^k$$

where $\Gamma(k-d) = \{(k-d-1)(k-d-2)...(2-d)(1-d)(-d)\}\Gamma(-d)$ (4)

According to Baillie (1996), the Wold representation of a fractional white noise process is given by the following equivalent relations:

$$y_t = \sum_{k=0}^{\infty} w_k \varepsilon_{t-k} \tag{5}$$

$$y_t = (1-L)^{-d}\varepsilon_t \tag{6}$$

$$y_t = \{1 + dL + d(d+1)L^2/2! + d(d+1)(d+2)L^3/3! + ...\}\varepsilon_t \tag{7}$$

As far as the testing strategy is concerned, the Hinich and Chong (2007) methodology is followed to test for long memory using what the authors call "a class test of fractional integration", a test that is able to distinguish fractionally integrated processes from other time series processes. In this context, the null hypothesis is that $\{y_t\}$ is a long memory process, while the alternative is that $\{y_t\}$ does not follow an $I(d)$ process, where $-0.5 < d < 0.5$.

In the context of long memory and the purchasing power parity (PPP) puzzle as discussed by Rogoff (1996), an interesting question is whether deviations from the PPP are transitory or permanent. Thus, the empirical tests generally take the form of testing for stationarity of the real exchange rate. If deviations from PPP are transitory but persistent, the time series of the real exchange rate is a stationary series, meaning it is $I(-0.5 < d < 0.5)$. By contrast, if deviations from PPP are permanent, then the time series of the real exchange rate is nonstationary, which implies that the process is $I(d \geq 1)$. It should be noted that for $I(0.5 < d < 1)$ the process is mean-reverting, even though it is not covariance stationary, as there is no long-run impact of innovations on future values of the process.

3 Testing methodology

The development of the Hinich-Chong fractional integration test begins by supposing a regression of y_t on $y_{t-1}, y_{t-2}, ..., y_{t-n}$. In this context, the following is considered:

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_T \end{pmatrix}' \text{ and } Z_n = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ y_1 & 0 & \cdots & 0 & 0 \\ y_2 & y_1 & \cdots & 0 & 0 \\ \vdots & y_2 & & & 0 \\ \vdots & \vdots & & & 0 \\ \vdots & \vdots & & & y_1 \\ \vdots & \vdots & & & \vdots \\ y_{T-1} & Y_{T-2} & \cdots & & y_{T-n} \end{pmatrix} \quad (8)$$

$$\hat{\beta}(n) = (Z_n{}' Z_n)^{-1} Z_n{}' Y = (\hat{\beta}_{n1}, \hat{\beta}_{n2} \cdots \quad \hat{\beta}_{nn-1}, \hat{\beta}_{nn})' \quad (9)$$

According to the authors, a long memory process has a unique feature in the sense that if it is approximated by an $AR(n)$ model via a regression, then the probability limits of the autoregressive coefficient estimates are functions of d and n. It follows that if $T \to \infty$, then

$$\beta_{n,j} \xrightarrow{p} -\binom{n}{j} \frac{\Gamma(j-d)\Gamma(n-d-j+1)}{\Gamma(-d)\Gamma(n-d+1)} \quad (10)$$

where $\xrightarrow{p}$ represents convergence in probability and $\Gamma(d)$ is the Euler gamma function.

It is indicated that the estimated coefficients of y_{t-1} and y_{t-n} converge in probability to $nd(n-d)^{-1}$ and $d(n-d)^{-1}$, respectively, making it likely that the first estimate will be about n times the previous one. Therefore, to test whether the process follows an $I(d)$ process, a researcher can construct a test based on the relationship $\beta_{n,n}$ and $\beta_{n,1}$, such that

$$n\hat{\beta}_{n,n} - \hat{\beta}_{n,1} \xrightarrow{p} 0$$

The authors recommend that autoregressions $AR(2), AR(3),..., AR(n)$ be run to generate a test statistic of the form

$$W(d,n) = [B(n,1) - B(n,n)]\Lambda(n)\Omega(d)^{-1}[B(n,1 - B(n,n)]\Lambda(n)' \quad (11)$$

where $B(n,1) = (\hat{\beta}_{2,1} \quad \hat{\beta}_{3,1} \cdots \hat{\beta}_{n,1})$

$B(n,n) = (\hat{\beta}_{22} \quad \hat{\beta}_{3,3} \cdots \hat{\beta}_{n,n})$

$\Lambda(n) = diag(2 \quad 3 \cdots n)$ and

$\Omega(d) = E[(B(n,1) - b(n,n)\Lambda(n)'(B(n,1) - B(n,n)\Lambda(n)]$

Theorem 1 of Hinich and Chong (2007) establishes that

$$W(\hat{d},n) \xrightarrow{l} \chi^2(n-1) \quad (12)$$

where $\xrightarrow{l}$ represents weak convergence in distribution. To select a robust and consistent estimate $\hat{d}$, the authors recommend that a median of the following be taken:

$$\hat{d}_{j,1} = \frac{j\hat{\beta}_{j,1}}{j + \hat{\beta}_{j,1}} \tag{13}$$

$$\hat{d}_{j,j} = \frac{j\hat{\beta}_{j,j}}{j + \hat{\beta}_{j,j}} \tag{14}$$

whereby for $j = 1,2,3,...,n$, $\hat{d}_{j,1}, \hat{d}_{j,j}$ are arranged in an ascending order.

4 Results

This section utilises data from the International Monetary Fund's International Financial Statistics database to test for long memory. The real exchange rate is defined as

$$y_t = \ln Y_t = \ln S_t + \ln \overline{P}_t - \ln P_t \tag{15}$$

where $\ln Y_t$ is the logarithm of a real exchange rate (domestic price of foreign currency) at time t ; $\ln \overline{P}_t$ and $\ln P_t$ are the logarithms of foreign and domestic price levels, respectively. The above algorithm leads to the results depicted in Table 1 on the following page.

In Table 1 the estimated values of d are reported in parentheses and the calculated values of the $W(\hat{d}, n)$ statistic are also reported. In the same table (*) and (**) represent significance levels at 1 per cent and 5 per cent, respectively. In most cases the estimate value of d is not affected by the choice of n .

According to the results, at the 1-per-cent and 5-per-cent significance levels the real exchange rates associated with Mauritius and Swaziland are not fractionally integrated. In addition, South Africa's real exchange rate is not an $I(d)$ process. Tanzania's real exchange rate was found to be invertible and stationary-fractionally integrated. Other currencies were found to be mean-reverting but nonstationary-fractionally integrated.

Table 1 $W(\hat{d}, n)$ based on the demeaned SADC real exchange rates

	T	$W(\hat{d},2)$	$W(\hat{d},3)$	$W(\hat{d},4)$	$W(\hat{d},5)$	$W(\hat{d},6)$	$W(\hat{d},7)$
Angola	130	1.79 (0.13)	1.84 (0.13)	2.45 (0.13)	2.74 (0.13)	3.23 (0.14)	4.18 (0.15)
Botswana	198	2.94 (0.03)	3.32 (0.03)	3.35 (0.03)	3.72 (0.03)	3.81 (0.03)	3.83 0.03
Madagascar	159	0.36 (-0.02)	4.92 (-0.02)	5.37 (-0.02)	5.40 (-0.02)	4.50 (-0.02)	6.25 (-0.01)
Malawi	198	0.01 (-0.01)	0.62 (-0.01)	3.74 (-0.01)	4.38 (-0.01)	5.23 (-0.01)	6.00 (-0.01)
Mauritius	198	0.36 (0.14)	15.77* (0.14)	24.52* (0.13)	25.83* (0.13)	30.30* (0.13)	30.32* (0.13)
Mozambique	155	0.84 (0.16)	5.63 (0.16)	6.37 (0.16)	6.77 (0.16)	12.53* (0.16)	12.64* (0.17)
South Africa	198	4.77** (0.09)	5.14 (0.1)	10.08** (0.1)	11.57** (0.1)	13.43** (0.1)	14.72** (0.09)
Swaziland	192	18.99* (-0.05)	19.36* (-0.05)	25.67* (-0.05)	25.89* (-0.06)	26.84* (-0.07)	27.38* (-0.08)
Tanzania	139	1.13 (-0.46)	1.91 (-0.46)	2.56 (-0.45)	4.87 (-0.45)	5.88 (-0.44)	6.12 (-0.45)
Zambia	197	1.18 (0.03)	1.19 (0.03)	1.20 (0.03)	1.34 (0.03)	2.06 (0.03)	6.44 (0.03)
$\chi^2_{n-1,1\%}$		6.63	9.21	11.34	13.28	15.09	16.81
$\chi^2_{n-1,5\%}$		3.84	5.99	7.81	9.49	11.1	12.60

5 Conclusions

This paper sought to determine the mean-reversion properties of SADC real exchange rates by means of a class test of fractional integration developed by Hinich and Chong (2007). At the 5-per-cent significance level, the null hypothesis could not be rejected that the real exchange rate associated with Angola, Botswana, Malawi, Tanzania, and Zambia were $I(d)$ processes. In the case of Mozambique the null hypothesis could be rejected when n was either 6 or 7. In addition, at the 5-per-cent significance level, the real exchange rates associated with Mauritius, Swaziland and South Africa were found not to be fractionally integrated.

References

Baillie, R.T. 1996. Long memory processes and fractional integration in econometrics. *Journal of Econometrics*, Vol. 73, 5-59.

Baillie, R.T. and Kapetanios, G. 2006. Nonlinear models with strongly dependent processes and applications to forward premia and real exchange rates, *Queen Mary University of London, Working Paper* 570.

Cheung, Y.W. 1993. Tests for fractional integration: A Monte Carlo investigation, *Journal of Time Series Analysis,* 14, 331-345.

Diebold, F. X and. Rudebusch, G.D. 1991. On the power of Dickey-Fuller tests against fractional alternatives, *Economics Letters*, Vol. 35: 2, 155-160.

Granger, CW.J and Joyeux, R. 1980. An introduction to long-range time series models and fractional differencing. *Journal of Time Series Analysis,* 15-29.

Hinich, M.J and Chong T.L. 2007. A class test for fractional integration, *Studies in Nonlinear Dynamics and Econometric*s, Vol. 11, Issue 2, Article 5.

Kapetanios, G. and Shin Y. 2003. Testing for nonstationary long memory against nonlinear ergodic models, *Queen Mary University of London Working Paper* number 500.

Mokoena T. M. 2007. Taking the puzzle out of the purchasing power parity puzzle: an application in respect of South Africa. *South African Journal of Economics,* Vol. 75:1 March.

Robinson, P. M. 2003. *Time series with long memory,* Oxford: Oxford University Press.

Rogoff, K. 1996. The purchasing power parity puzzle, *Journal of Economic Literature,* Vol. 34, 647-668.

Smallwood, A. D. 2005. Joint tests for non-linearity and long memory: The case of purchasing power parity. *Studies in Nonlinear Dynamics and Econometrics,* Vol. 9:2, Article 7.